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September 12, 2018

1 Model

The laser cavity is treated as the composition of five independent process—gain, nonlinearity, loss, dispersion, and modulation. Expressions for each component will be derived from the generalized nonlinear Schrödinger equation [1, 4, 6, 8],

$$\frac{\partial A}{\partial z} = -i\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2} + \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3} + i\gamma|A|^2A + \frac{1}{2}g(A)A - \alpha A, \tag{1}$$

where A is the complex amplitude of the pulse, β_2 and β_3 are the second and third order dispersion coefficients respectively, γ is the coefficient of nonlinearity, $g(A)$ is the gain, and α is the loss of the fiber.

1.1 Gain

Within the Er-doped gain fiber the other four processes are assumed to be negligible, and so (1) reduces to

$$\frac{\partial A}{\partial z} = \frac{1}{2}g(A)A \tag{2}$$

with

$$g(A) = \frac{g_0}{1 + E/E_{sat}},$$

where g_0 is a small signal gain, E_{sat} is the energy at which the gain begins to saturate, and

$$E = \int_{-\infty}^{\infty} |A|^2 \, dT,$$

is the energy of the pulse [2, 8]. We shall first transform (2) into an equation in terms of the energy. Taking

$$\frac{\partial A}{\partial z} = \frac{1}{2} \cdot \frac{g_0 A}{1 + E/E_{sat}},$$

multiplying by $\bar{A}$, the complex conjugate of A, and then adding the complex conjugate gives

$$\frac{d|A|^2}{dz} = \frac{g_0|A|^2}{1 + E/E_{sat}},$$

which after integrating becomes

$$\frac{dE}{dz} = \frac{g_0 E}{1 + E/E_{sat}}.$$

For $E \ll E_{sat}$ the energy grows exponentially, whereas for $E \gg E_{sat}$ the gain has saturated and so the energy grows linearly. A closed form of the energy can be found by separating and integrating yielding

$$E(z) = E_{sat} W_0 \left(\frac{E_0}{E_{sat}} e^{E_0/E_{sat}} e^{g_0 z} \right), \tag{3}$$

where W_0 is the Lambert W function. However, only the exiting energy is of interest, thus (3) can be written as

$$E' = E_{sat} W_0 \left(\frac{E}{E_{sat}} e^{E/E_{sat}} e^{g_0 L_g} \right),$$

where E is the energy of the incoming pulse, and E' is the energy after traveling through the length of the gain fiber. Since $E \sim |A|^2$ the gain in terms of the amplitude is given by

$$G(A) = \left[\frac{E_{sat}}{E} W_0 \left(\frac{E}{E_{sat}} e^{E/E_{sat}} e^{g_0 L_g} \right) \right]^{1/2} A. \tag{4}$$

1.2 Nonlinearity of Fiber

The effect of the nonlinearity of the fiber can also be found from (1):

$$\frac{\partial A}{\partial z} - i\gamma |A|^2 A = 0,$$

which similarly to the gain, can be transformed by multiplying by the complex conjugate of A, and adding its complex conjugate gives

$$\frac{\partial |A|^2}{\partial z} = 0. \tag{5}$$

This suggests the envelope of the pulse does not change as it travels through the fiber—a solution of the form $A = A_0 e^{i\varphi}$ can be assumed. Substituting this expression into (5) gives $\varphi = \gamma |A|^2 z$ therefore

$$F(A) = A e^{i\gamma |A|^2 L_f},$$

where L_f is the length of the fiber.

1.3 Loss

Two sources of loss exist within the laser circuit: the loss due to the output coupler and the optical loss due to the circuit. It will be assumed all loss occurs at a particular point in the circuit because of the extremely high reflectivity of the fiber the optical loss is negligible compared to the output coupler loss ***cite. The loss is then given as

$$L(A) = C e^{-\alpha L} A,$$

where C is the loss due to the output coupler.

1.4 Dispersion

The dominant dispersion is due to the chirped fiber Bragg grating (CFBG)—the dispersion due to the fiber is negligible in comparison [**agrawalother**]. The dispersive pieces of (1) give

$$\frac{\partial A}{\partial z} = -i\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2} + \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3}, \tag{6}$$

but, the third order effects of dispersion will be neglected ***cite. Although, under some circumstances they may need to be considered since the CFBG is highly dispersive [5]. Taking the Fourier transform of (6) gives

$$\frac{\partial \mathcal{F}\left\{A\right\}}{\partial z} = \frac{1}{2}i\beta_2\omega^2\mathcal{F}\left\{A\right\}.$$

The Fourier transform of A can be found by integrating and

$$D(A) = \mathcal{F}^{-1}\left\{e^{i\frac{1}{2}\beta_2\omega^2 L_D}\mathcal{F}\left\{A\right\}\right\}$$

is the effect of the CFBG on the pulse.

1.5 Modulation

Pick a function that is bump-like, infinitely differentiable (I don't know how important that is), and has a maximum of 1. We shall pick

$$M(A) = e^{-T^2/2T_M^2}A,$$

where T_M is a characteristic width of the modulation, since its Fourier transform is itself. ***I don't know how to write this.

1.6 Combining the Pieces

$$\mathcal{L}(A) = M(D(L(F(G(A)))))$$

A solution such that $|\mathcal{L}(A)|^2 = |A|^2$ is sought.

2 Non-Dimensionalization

$$T = T_M\widetilde{T}, \quad E = E_{sat}\widetilde{E}, \quad A = \left(\frac{E_{sat}}{T_M}\right)^{1/2}\widetilde{A}, \quad \omega = \frac{\widetilde{\omega}}{T_M}$$

$$G(A) = \left(\frac{W_0\left(aEe^E\right)}{E}\right)^{1/2}A.$$

Parameter	Value
$\beta_2^g L_D$	10–2000ps^2
g_0	1–10m^{-1}
β_2^f	20–50ps^2/km
γ	0.001–0.01W^{-1}m^{-1}
E_{sat}	10^{4}pJ
L_G	3m

$$D(A) = \mathcal{F}^{-1}\left\{e^{is^2\omega^2}\mathcal{F}\left\{A\right\}\right\}$$

$$F(A) = Ae^{ib|A|^2},$$

$$L(A) = hA,$$

$$M(A) = e^{-T^2/2}A$$

$$a = e^{g_0 L_g} \quad s^2 = \frac{\beta_2 L_D}{2T_M^2}, \quad b = \gamma L_F \frac{E_{sat}}{T_M}, \quad h = Ce^{-\alpha L}$$

3 Results

$$\left(\frac{W(aEe^E)}{E}\right)^{1/2} S(s)h = 1$$

$$E = \frac{S(s)^2h^2}{1 - S(s)^2h^2} \ln\left(aS(s)^2h^2\right)$$

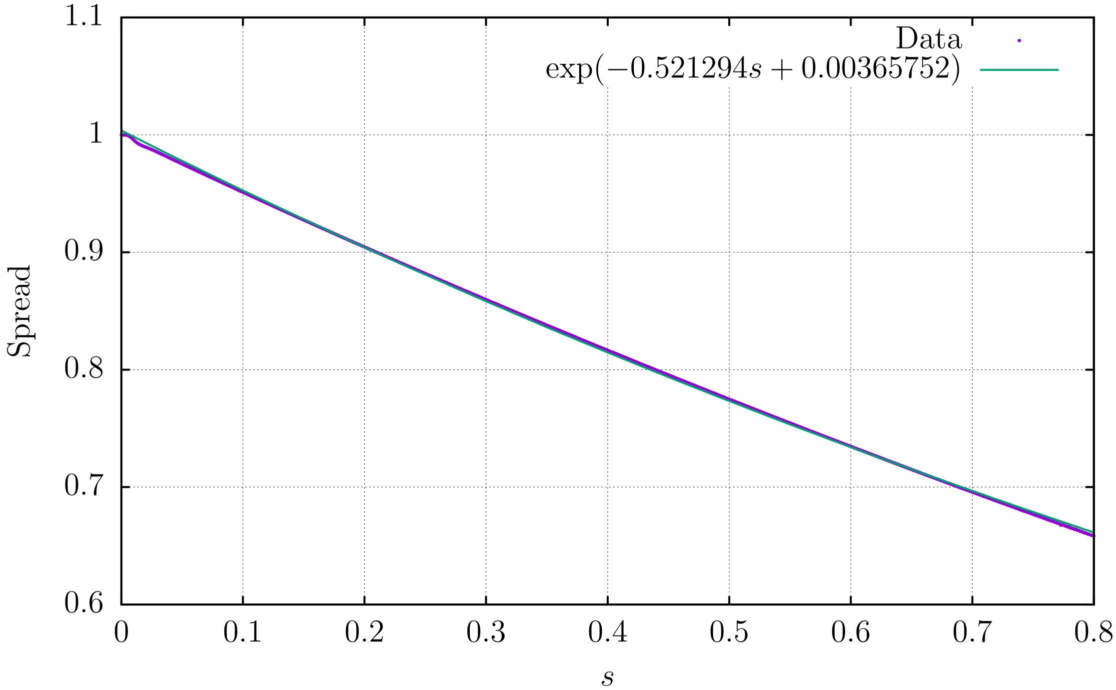

Figure 1:

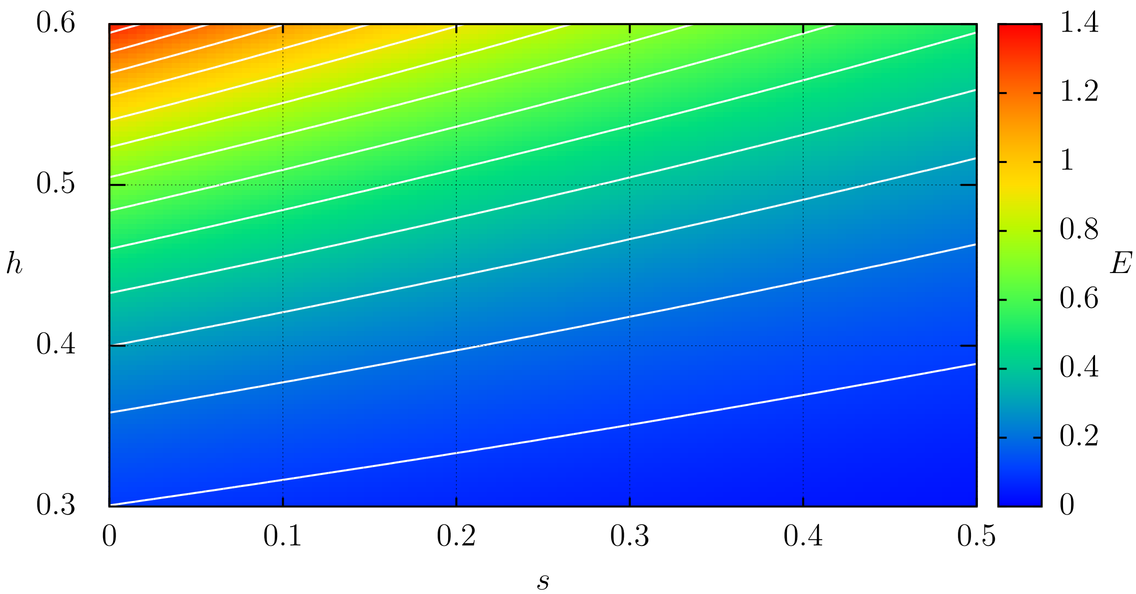

Figure 2: a=30

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