Brady Metherall

September 10, 2018

1 Model

Each component of the laser is treated separately and all processes will be composed to represent a single circuit of the laser.

1.1 Gain

As the pulse travels through the gain media the change in energy is

$$\frac{dE}{dz} = \frac{g_0 E}{1 + E/E_{sat}},$$

where g_0 is a small signal gain, and E_{sat} is the energy at which the gain starts to saturate [1]. For $E \ll E_{sat}$ the energy grows exponentially, whereas for $E \gg E_{sat}$ the gain has saturated and so the energy grows linearly. The energy can be solved analytically by separating and integrating yielding

$$E(z) = E_{sat} W_0 \left(\frac{E_0}{E_{sat}} e^{E_0/E_{sat}} e^{g_0 z} \right), \tag{1}$$

where W_0 is the Lambert W function. However, only the exiting energy is of interest, thus (1) can be written as

$$E' = E_{sat} W_0 \left(\frac{E}{E_{sat}} e^{E/E_{sat}} e^{g_0 L_g} \right),$$

where E is the energy of the incoming pulse, and E' is the energy after traveling through the length of the gain media. Since the energy can be expressed as

$$E = \int_{-\infty}^{\infty} |A(T)|^2 dT,$$

it can be shown that

$$\frac{E'}{E} = \left(\frac{A'}{A} \right)^2,$$

and so the gain in terms of the amplitude is given by

$$G(A) = \left[\frac{E_{sat}}{E} W_0 \left(\frac{E}{E_{sat}} e^{E/E_{sat}} e^{g_0 L_g} \right) \right]^{1/2} A. \tag{2}$$

1.2 Dispersion

Expressions for the amplitude as the pulse travels through the grating as well as the fibre can be derived from

$$i\frac{\partial A}{\partial z} - \frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2} + \gamma|A|^2 A = 0, \tag{3}$$

the non-linear Schrödinger equation [2]. In the case of the dispersive element γ is assumed to be negligible. Thus, (3) reduces to

$$i\frac{\partial A}{\partial z} - \frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2} = 0,$$

and after taking the Fourier transform

$$\frac{\partial \mathcal{F}\left\{A\right\}}{\partial z} = \frac{1}{2}i\beta_2\omega^2\mathcal{F}\left\{A\right\}.$$

The Fourier transform of A can be found and evaluated at $z = L_d$ yielding

$$D(A) = \mathcal{F}^{-1}\left\{e^{\frac{1}{2}i\beta_2\omega^2 L_D}\mathcal{F}\left\{A\right\}\right\}$$

as the effect of the dispersive element on the pulse.

1.3 Fibre

The effect of the fibre can also be found from (3), by assuming the dispersion of the fibre is negligible when compared to the dispersive element the non-linear Schrödinger equation simplifies to

$$\frac{\partial A}{\partial z} - i\gamma|A|^2 A = 0,$$

or after multiplying by the complex conjugate of A,

$$\frac{\partial A}{\partial z}\bar{A} - i\gamma|A|^4 = 0. \tag{4}$$

Adding (4) to its complex conjugate gives

$$\frac{\partial |A|^2}{\partial z} = 0, \tag{5}$$

this suggests the envelope of the pulse does not change as it travels through the fibre, a solution of the form $A = A_0 e^{i\varphi}$ can be assumed. Substituting this expression into (5) gives $\varphi = \gamma|A|^2 z$ therefore

$$F(A) = Ae^{i\gamma|A|^2 L_f},$$

where L_f is the length of the fibre.

1.4 Loss

Two sources of loss exist within the laser circuit: the loss due to the output coupler and the optical loss due to the circuit. It will be assumed all loss occurs at a particular point in the circuit, however, the model can easily be modified to account for the optical loss between each component. The loss is then given as

$$L(A) = Ce^{-\alpha L}A,$$

where C is the loss due to the output coupler, and α is a characteristic loss per length of the fibre.

1.5 Modulation

Pick

$$M(A) = e^{-T^2/2T_M^2}A$$

since its Fourier transform is itself

2 Non-Dimensionalization

$$T = T_M\widetilde{T}, \quad E = E_{sat}\widetilde{E}, \quad A = \left(\frac{E_{sat}}{T_M}\right)^{1/2}\widetilde{A}, \quad \omega = \frac{\widetilde{\omega}}{T_M}$$

3 Results

$$\left(\frac{W(aEe^E)}{E}\right)^{1/2} S(s)h = 1$$

$$E = \frac{S(s)^2h^2}{1 - S(s)^2h^2}\ln\left(aS(s)^2h^2\right)$$

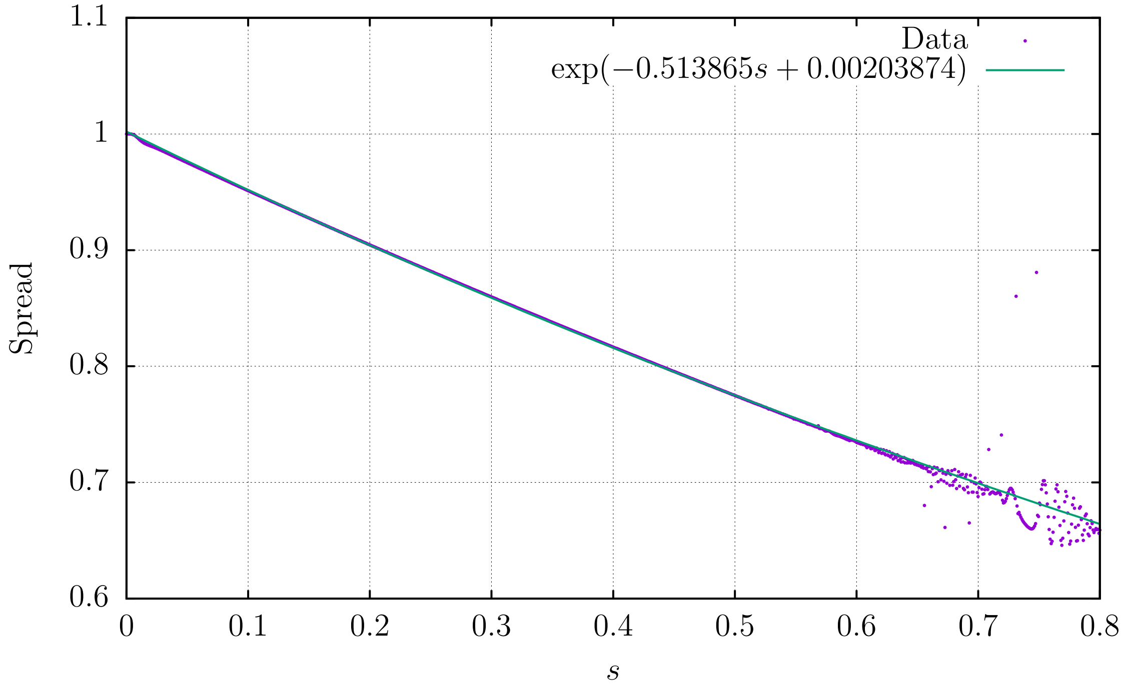

Figure 1:

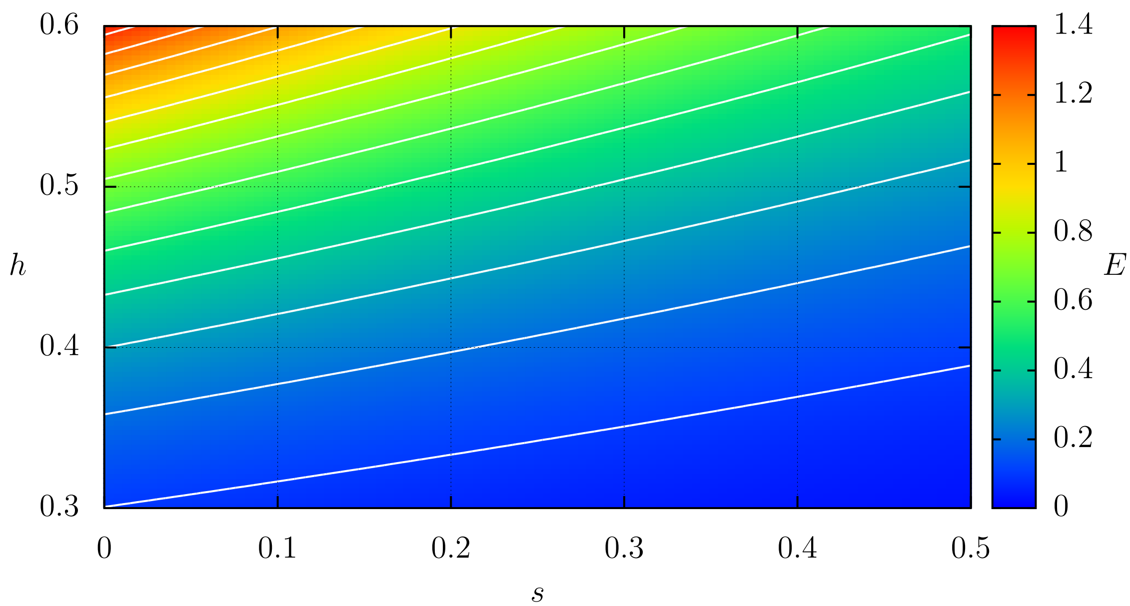

Figure 2: a=30

References

[1] Laser Fundamentals

[2] Nonlinear Effects in Optical Fibers