The model for the amplitude in the 'average' model is given by

$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \frac{\epsilon}{2}T^2 A + \frac{g}{2}A \tag{1}$$

with $\beta_2 \in \mathbb{R}$ defining the dispersion, $\epsilon \in \mathbb{R}$, $\epsilon > 0$, determining the modulation and $g \in \mathbb{R}$, $g > 0$ giving the gain. The ansatz for the amplitude is a function of the form

$$A(T,z) = \left(\frac{P_0}{1 - iC}\right)^{1/2} \exp\left(-\frac{\delta\Omega^2 T^2}{2(1 - iC)}\right) \mathrm{e}^{i\psi z} \tag{2}$$

so that the modulus $|A|$ and the phase $A/|A|$ are given by

$$|A| = \left(\frac{P_0}{\sqrt{1 + C^2}}\right)^{1/2} \exp\left(-\frac{\delta\Omega^2 T^2}{2(1 + C^2)}\right), \qquad \frac{A}{|A|} = \left(\frac{1 + iC}{\sqrt{1 + C^2}}\right)^{1/2} \exp\left(-i\frac{\delta\Omega^2 T^2 C}{2(1 + C^2)}\right) \mathrm{e}^{i\psi z}.$$

The quantity $C \in \mathbb{R}$ is known as the chirp and contributes a constant phase of θ where $2\theta = \arctan C$, P_0 is the maximum value of $|A|^2$ at $C = 0$ (zero chirp), $\psi \in \mathbb{R}$ is the accumulated phase, and $\delta\Omega^2$ is the spectral half-width of $|A|^2$ since[1]

$$\hat{A}(\omega, z) = \frac{1}{2\pi}\int_{\infty}^{\infty} A(t,z)\mathrm{e}^{i\omega t}\,\mathrm{d}t = \left(\frac{P_0}{2\pi\delta\Omega^2}\right)^{1/2} \exp\left(-\frac{(1 - iC)\omega^2}{2\delta\Omega^2}\right) \mathrm{e}^{i\psi z},$$

and $|\hat{A}|^2(\delta\Omega, z) = \mathrm{e}^{-1}|\hat{A}|^2(0, z)$. The corresponding half-width of the pulse duration comes from the expression for $|A|$ and gives

$$\delta T = \frac{\sqrt{1 + C^2}}{\delta\Omega}. \tag{3}$$

In the case of the expression (1), applying (2) gives the condition

$$\begin{aligned} i\psi &= -i\frac{\beta_2}{2}\delta\Omega^2(1 - iC)^{-1}\left(-1 + \delta\Omega^2(1 - iC)^{-1}T^2\right) - \frac{\epsilon}{2}T^2 + \frac{g}{2} \\ &= \frac{\beta_2\delta\Omega^2}{2(1 + C^2)^2}(-C(1 + C^2) + 2\delta\Omega^2 T^2 C) - \frac{\epsilon}{2}T^2 + \frac{g}{2} \\ &\quad + i\frac{\beta_2\delta\Omega^2}{2(1 + C^2)^2}(1 + C^2 + \delta\Omega^2 T^2(C^2 - 1)). \end{aligned}$$

This gives four conditions by utilizing that $\psi \in \mathbb{R}$. In detail,

$$\begin{aligned} \mathcal{O}(T^2)_{\mathrm{Im}} &: 0 = C^2 - 1, & \mathcal{O}(1)_{\mathrm{Im}} &: \psi = \frac{\beta_2\delta\Omega^2}{2(1 + C^2)}, \\ \mathcal{O}(T^2)_{\mathrm{Re}} &: \epsilon = \frac{2\beta_2\delta\Omega^4 C}{(1 + C^2)^2}, & \mathcal{O}(1)_{\mathrm{Re}} &: g = \frac{\beta_2\delta\Omega^2 C}{(1 + C^2)}. \end{aligned}$$

Starting with $\mathcal{O}(1)_{\mathrm{Re}}$ we note that $g > 0$ implies that $\operatorname{sgn}(\beta_2 C) = \operatorname{sgn}(\beta_2)\operatorname{sgn}(C) = 1$. From $\mathcal{O}(T^2)_{\mathrm{Im}}$, $C = \pm 1$ and therefore $C = \operatorname{sgn}(\beta_2)$, $\beta_2 C = |\beta_2|$ and $\epsilon > 0$. We also see that the representation (2) as a classical solution of (1) imposes a subclass of solutions whereby $g = (\epsilon|\beta_2|/2)^{1/2}$. One is left with a two parameter family of solutions to (2) with

$$\delta\Omega^2 = \left(\frac{2\epsilon}{|\beta_2|}\right)^{1/2}, \qquad \psi = \operatorname{sgn}(\beta_2)\left(\frac{\epsilon|\beta_2|}{8}\right)^{1/2}, \qquad \delta T^2 = \left(\frac{2|\beta_2|}{\epsilon}\right)^{1/2}.$$

[1] If $f(t) = \mathrm{e}^{-\alpha t^2}$ then $\hat{f}(\omega) = \frac{1}{2\pi}\int_{\mathbb{R}} f(t)\mathrm{e}^{i\omega t}\,\mathrm{d}t = \frac{1}{2\pi}(\frac{\pi}{\alpha})^{1/2}\mathrm{e}^{-\omega^2/4\alpha}$.

Since (1) is linear in A, any value of the peak power, P_0, is admissible. In practice however, at high power levels the gain drops as the fibre saturates. One can model this with

$$g(P_0) = \frac{g_0}{1 + \frac{\text{power in fibre}}{\text{saturation power}}} - \alpha \tag{4}$$

where g_0 is the low-power gain and α represents the net losses in the laser cavity. The power in the fibre depends on the frequency f and modulus of the pulse so that

$$f \int_{-\infty}^{\infty} |A(s)|^2 \, \mathrm{d}s = \frac{\sqrt{\pi} f}{\delta\Omega} P_0 = \Delta P_0, \qquad \Delta = \frac{\sqrt{\pi} f}{\delta\Omega} = \frac{\sqrt{\pi} f \delta T}{\sqrt{1 + C^2}}$$

where Δ is the duty cycle of the pulse. Denoting the saturation power as P_{sat}, (4) can be inverted to give

$$P_0 = \frac{P_{\text{sat}}}{\sqrt{\pi} f} \left(\frac{|\beta_2|}{2\epsilon} \right)^{1/4} \left(g_0 \left(\left(\frac{\epsilon |\beta_2|}{2} \right)^{1/2} + \alpha \right)^{-1} - 1 \right).$$