

1.2.10 Express the vector field $\mathbf{A} = 3\mathbf{u}_x + 4\mathbf{u}_y + 5\mathbf{u}_z$ in spherical coordinates. Check your answer using MATLAB.

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ &= \sqrt{3^2 + 4^2 + 5^2} \\ &\approx 7.07 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right) \\ &= \tan^{-1} \left(\frac{\sqrt{3^2 + 4^2}}{5} \right) \\ &\approx 0.785 \end{aligned}$$

$$\begin{aligned} \phi &= \tan^{-1} \left(\frac{y}{x} \right) \\ &= \tan^{-1} \left(\frac{4}{3} \right) \\ &\approx 0.927 \end{aligned}$$

The final vector in spherical coordinates is $\mathbf{A} \approx 7.07\mathbf{u}_r + 0.785\mathbf{u}_\theta + 0.927\mathbf{u}_\phi$.