1.3.8 Find the surface area of a cylindrical surface by setting up and evaluating the integral $\oint \mathbf{A} \bullet \mathbf{ds}$ where $\mathbf{A} = \mathbf{u}_\rho + 2\mathbf{u_z}$. *Note: Diagram not shown.*

The radius of the cylinder is a. In the following steps, the first surface integral covers the wall of the cylinder, and the second covers the top and bottom. The top and bottom are identical, so the integral is doubled.

$$
\begin{aligned}
\oint \mathbf{A} \bullet \mathbf{ds} &= \int_{\phi=0}^{2\pi} \int_{z=0}^{\mathscr{L}} (\mathbf{u}_\rho + 2\mathbf{u_z}) \bullet (\rho \mathrm{d}\phi \mathrm{d}z \mathbf{u}_\rho + \rho \mathrm{d}\rho \mathrm{d}\phi \mathbf{u_z}) \\
&\quad + 2 \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} (\mathbf{u}_\rho + 2\mathbf{u_z}) \bullet (\rho \mathrm{d}\phi \mathrm{d}z \mathbf{u}_\rho + \rho \mathrm{d}\rho \mathrm{d}\phi \mathbf{u_z}) \\
&= \int_{\phi=0}^{2\pi} \int_{z=0}^{\mathscr{L}} \rho \mathrm{d}z \mathrm{d}\phi + \int_{\phi=0}^{2\pi} \int_{z=0}^{\mathscr{L}} 2\rho \mathrm{d}\rho \mathrm{d}\phi \\
&\quad + 2 \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \rho \mathrm{d}\phi \mathrm{d}z + 2 \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} 2\rho \mathrm{d}\rho \mathrm{d}\phi \\
&= \int_{\phi=0}^{2\pi} \rho z \Big|_{z=0}^{\mathscr{L}} \mathrm{d}\phi + 2 \int_{\phi=0}^{2\pi} \frac{2\rho^2}{2} \Big|_{\rho=0}^{a} \mathrm{d}\phi \\
&= \int_{\phi=0}^{2\pi} \rho \mathscr{L} \mathrm{d}\phi + 2 \int_{\phi=0}^{2\pi} \frac{2a^2}{2} \mathrm{d}\phi \\
&= \rho \mathscr{L} \phi \Big|_{\phi=0}^{2\pi} + 2\phi a^2 \Big|_{\phi=0}^{2\pi} \\
&= 2\pi \rho \mathscr{L} + 4\pi a^2 \\
&= 2\pi (a\mathscr{L} + 2a^2)
\end{aligned}
$$