

**1.4.10** Show that  $\nabla \times \mathbf{A} = 0$  if  $\mathbf{A} = r^2 \mathbf{u}_r$  in spherical coordinates.

$$\begin{aligned}
\nabla \times \mathbf{A} &= \frac{1}{r \sin \theta} \left( \frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right) \mathbf{u}_r \\
&\quad + \frac{1}{r} \left( \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right) \mathbf{u}_\theta \\
&\quad + \frac{1}{r} \left( \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right) \mathbf{u}_\phi \\
&= \frac{1}{r \sin \theta} \left( \frac{\partial}{\partial \theta} (0 \cdot \sin \theta) - \frac{\partial}{\partial \phi} (0) \right) \mathbf{u}_r \\
&\quad + \frac{1}{r} \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} (r^2) - \frac{\partial}{\partial r} (r \cdot 0) \right) \mathbf{u}_\theta \\
&\quad + \frac{1}{r} \left( \frac{\partial}{\partial r} (r \cdot 0) - \frac{\partial}{\partial \theta} (r^2) \right) \mathbf{u}_\phi \\
&= 0
\end{aligned}$$