1.2.15 Show that we can use the vector definitions $\mathbf{A} \bullet \mathbf{B} = 0$ and $\mathbf{A} \times \mathbf{B} = \mathbf{0}$ to express that two vectors are perpendicular and parallel to each other respectively.

The definitions of the scalar and vector products are $\mathbf{A} \bullet \mathbf{B} = AB\cos\theta$ and $\mathbf{A} \times \mathbf{B} = AB\sin\theta\mathbf{u}_{\mathbf{A}\times\mathbf{B}}$, where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$. Assume that $\mathbf{A}$ and $\mathbf{B}$ are both nonzero. If either is the zero vector, then the vectors are neither perpendicular nor parallel.

When the scalar product is equal to zero, the value of θ must be $\pm\frac{\pi}{2}$ because $\cos\theta = 0$ for those values. Vectors are perpendicular when the angle between them is $\pm\frac{\pi}{2}$, so the vectors must be perpendicular when their scalar product is zero.

By a similar argument, when the vector product is equal to zero, the value of θ must be 0 or π because $\sin\theta = 0$ for those values. Vectors are parallel when the angle between them is 0 or π, so the vectors must be parallel when their vector product is equal to zero.