

**1.4.1** A hill can be modeled with the equation  $H = 10 - x^2 - 3y^2$  where  $H$  is the elevation of the hill. Find the path that a frictionless ball would take to experience the greatest change of elevation in the least change of horizontal position. Assume that the motion of the ball is unconstrained.

The path that the ball would take is equivalent to the gradient of  $H$ .

$$\begin{aligned}\nabla H &= \frac{\partial H}{\partial x} \mathbf{u}_x + \frac{\partial H}{\partial y} \mathbf{u}_y \\ &= \frac{\partial}{\partial x}(10 - x^2 - 3y^2) \mathbf{u}_x + \frac{\partial}{\partial y}(10 - x^2 - 3y^2) \mathbf{u}_y \\ &= -2x \mathbf{u}_x - 6y \mathbf{u}_y\end{aligned}$$