1.4.13 In rectangular coordinates, verify that $\nabla \times (a\mathbf{A}) = (\nabla a) \times \mathbf{A} + a\nabla \times \mathbf{A}$ where $\mathbf{A} = xyz[\mathbf{u_x} + \mathbf{u_y} + \mathbf{u_z}]$ and $a = 3xy + 4zx$ by carrying out the detailed differentiations.

We will show that this equation is true by evaluating both sides separately and showing that they have the same value.

$$
\begin{aligned}
\nabla \times (a\mathbf{A}) &= \nabla \times \left[(3x^2y^2z + 4x^2yz^2)\mathbf{u_x} + (3x^2y^2z + 4x^2yz^2)\mathbf{u_y} + (3x^2y^2z + 4x^2yz^2)\mathbf{u_z}\right] \\
&= \left(\frac{\partial}{\partial y}(3x^2y^2z + 4x^2yz^2) - \frac{\partial}{\partial z}(3x^2y^2z + 4x^2yz^2)\right)\mathbf{u_x} \\
&\quad + \left(\frac{\partial}{\partial z}(3x^2y^2z + 4x^2yz^2) - \frac{\partial}{\partial x}(3x^2y^2z + 4x^2yz^2)\right)\mathbf{u_y} \\
&\quad + \left(\frac{\partial}{\partial x}(3x^2y^2z + 4x^2yz^2) - \frac{\partial}{\partial y}(3x^2y^2z + 4x^2yz^2)\right)\mathbf{u_z} \\
&= (6x^2yz + 4x^2z^2 - 3x^2y^2 - 8x^2yz)\mathbf{u_x} \\
&\quad + (3x^2y^2 + 8x^2yz - 6xy^2z - 8xyz^2)\mathbf{u_y} \\
&\quad + (6xy^2z + 8xyz^2 - 6x^2yz - 4x^2z^2)\mathbf{u_z} \\
&= (-2x^2yz + 4x^2z^2 - 3x^2y^2)\mathbf{u_x} \\
&\quad + (3x^2y^2 + 8x^2yz - 6xy^2z - 8xyz^2)\mathbf{u_y} \\
&\quad + (6xy^2z + 8xyz^2 - 6x^2yz - 4x^2z^2)\mathbf{u_z}
\end{aligned}
$$

$$
\begin{aligned}
(\nabla a) \times \mathbf{A} + a\nabla \times \mathbf{A} &= \left(\frac{\partial a}{\partial x}\mathbf{u_x} + \frac{\partial a}{\partial y}\mathbf{u_y} + \frac{\partial a}{\partial z}\mathbf{u_z}\right) \times \mathbf{A} \\
&\quad + a\left[\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\mathbf{u_x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\mathbf{u_y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\mathbf{u_z}\right] \\
&= \left(\frac{\partial a}{\partial y}A_z - \frac{\partial a}{\partial z}A_y\right)\mathbf{u_x} + \left(\frac{\partial a}{\partial z}A_x - \frac{\partial a}{\partial x}A_z\right)\mathbf{u_y} + \left(\frac{\partial a}{\partial x}A_y - \frac{\partial a}{\partial y}A_x\right)\mathbf{u_z} \\
&\quad + a\left[\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\mathbf{u_x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\mathbf{u_y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\mathbf{u_z}\right] \\
&= (3x \cdot xyz - 4x \cdot xyz)\mathbf{u_x} \\
&\quad + (4x \cdot xyz - (3y + 4z)xyz)\,\mathbf{u_y} \\
&\quad + ((3y + 4z)xyz - 3x \cdot xyz)\,\mathbf{u_z} \\
&\quad + (3xy + 4zx)\,((xz - xy)\mathbf{u_x} + (xy - yz)\mathbf{u_y} + (yz - xz)\mathbf{u_z}) \\
&= (-x^2yz)\mathbf{u_x} + (4x^2yz - 3xy^2z - 4xyz^2)\mathbf{u_y} + (3xy^2z + 4xyz^2 - 3x^2yz)\mathbf{u_z} \\
&\quad + (3x^2yz - 3x^2y^2 + 4x^2z^2 - 4x^2yz)\mathbf{u_x} \\
&\quad + (3x^2y^2 - 3xy^2z + 4x^2yz - 4xyz^2)\mathbf{u_y} \\
&\quad + (3xy^2z - 3x^2yz + 4xyz^2 - 4x^2z^2)\mathbf{u_z} \\
&= (-2x^2yz - 3x^2y^2 + 4x^2z^2)\mathbf{u_x} \\
&\quad + (8x^2yz - 6xy^2z - 8xyz^2 + 3x^2y^2)\mathbf{u_y} \\
&\quad + (6xy^2z + 8xyz^2 - 6x^2yz - 4x^2z^2)\mathbf{u_z}
\end{aligned}
$$

The two sides of the equation have the same value. Therefore, the equation is true.