1.3.3 Calculate the work required to move a mass m against a force field $\mathbf{F} = \rho\mathbf{u}_\rho + \rho\phi\mathbf{u}_\phi$ along the path *abc*. *Note: Diagram not shown.*

By inspection of the diagram, the labeled points are $a = (0, 0)$, $b = (2, 0)$, and $c = (2, \pi)$ in cylindrical coordinates. To find the work required to move along the path along the path, we need to compute the path integral of the force field.

$$\begin{aligned}
W_{abc} &= \int_a^b \mathbf{F} \bullet \mathbf{dl} + \int_b^c \mathbf{F} \bullet \mathbf{dl} \\
&= \int_a^b (\rho\mathbf{u}_\rho + \rho\phi\mathbf{u}_\phi) \bullet (\mathrm{d}\rho\mathbf{u}_\rho + \rho\mathrm{d}\phi\mathbf{u}_\phi) \\
&\quad + \int_b^c (\rho\mathbf{u}_\rho + \rho\phi\mathbf{u}_\phi) \bullet (\mathrm{d}\rho\mathbf{u}_\rho + \rho\mathrm{d}\phi\mathbf{u}_\phi) \\
&= \int_a^b \rho\mathrm{d}\rho + \int_a^b \rho^2\phi\mathrm{d}\phi + \int_b^c \rho\mathrm{d}\rho + \int_b^c \rho^2\phi\mathrm{d}\phi \\
&= \int_{\rho=0}^2 \rho\mathrm{d}\rho + \int_{\phi=0}^0 \rho^2\phi\mathrm{d}\phi + \int_{\rho=2}^2 \rho\mathrm{d}\rho + \int_{\phi=0}^\pi \rho^2\phi\mathrm{d}\phi \\
&= \frac{\rho^2}{2}\bigg|_{\rho=0}^2 + 2^2\frac{\phi^2}{2}\bigg|_{\phi=0}^\pi \\
&= 2 + \frac{4\pi^2}{2} \\
&= 2(1 + \pi^2)
\end{aligned}$$