1.4.14 In rectangular coordinates, verify that $\nabla \bullet (a\mathbf{A}) = \mathbf{A} \bullet \nabla a + a\nabla \bullet \mathbf{A}$ where $\mathbf{A} = xyz[\mathbf{u_x} + \mathbf{u_y} + \mathbf{u_z}]$ and $a = 3xy + 4zx$ by carrying out the detailed differentiations.

We will show that this equation is true by evaluating both sides separately and showing that they have the same value.

$$\begin{aligned}
\nabla \bullet (a\mathbf{A}) &= \nabla \bullet (3xy + 4zx)(xyz)(\mathbf{u_x} + \mathbf{u_y} + \mathbf{u_z}) \\
&= \nabla \bullet (3x^2y^2z + 4x^2yz^2)(\mathbf{u_x} + \mathbf{u_y} + \mathbf{u_z}) \\
&= \frac{\partial}{\partial x}(3x^2y^2z + 4x^2yz^2) + \frac{\partial}{\partial y}(3x^2y^2z + 4x^2yz^2) + \frac{\partial}{\partial x}(3x^2y^2z + 4x^2yz^2) \\
&= 6xy^2z + 8xyz^2 + 6x^2yz + 4x^2z^2 + 3x^2y^2 + 8x^2yz \\
&= 6xy^2z + 8xyz^2 + 14x^2yz + 4x^2z^2 + 3x^2y^2
\end{aligned}$$

$$\begin{aligned}
\mathbf{A} \bullet \nabla a + a\nabla \bullet \mathbf{A} &= \mathbf{A} \bullet \left(\frac{\partial a}{\partial x}\mathbf{u_x} + \frac{\partial a}{\partial y}\mathbf{u_y} + \frac{\partial a}{\partial z}\mathbf{u_z}\right) + a\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) \\
&= \frac{\partial a}{\partial x}A_x + \frac{\partial a}{\partial y}A_y + \frac{\partial a}{\partial z}A_z + a\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) \\
&= \frac{\partial}{\partial x}(3xy + 4zx)(xyz) + \frac{\partial}{\partial y}(3xy + 4zx)(xyz) + \frac{\partial}{\partial z}(3xy + 4zx)(xyz) \\
&\quad + (3xy + 4zx)\left(\frac{\partial}{\partial x}(xyz) + \frac{\partial}{\partial y}(xyz) + \frac{\partial}{\partial z}(xyz)\right) \\
&= (3y + 4z)(xyz) + (3x)(xyz) + (4x)(xyz) + (3xy + 4zx)(yz + xz + xy) \\
&= 3xy^2z + 4xyz^2 + 3x^2yz + 4x^2yz + 3xy^2z \\
&\quad + 4xyz^2 + 3x^2yz + 4x^2z^2 + 3x^2y^2 + 4x^2yz \\
&= 6xy^2z + 8xyz^2 + 14x^2yz + 4x^2z^2 + 3x^2y^2
\end{aligned}$$

The two sides of the equation have the same value. Therefore, the equation is true.