1.2.12 For the vectors $\mathbf{A} = \mathbf{u_x} + \mathbf{u_y} + \mathbf{u_z}$, $\mathbf{B} = 2\mathbf{u_x} + 2\mathbf{u_y} + 2\mathbf{u_z}$, and $\mathbf{C} = 3\mathbf{u_x} + 3\mathbf{u_y} + 3\mathbf{u_z}$, show that $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \bullet \mathbf{C}) - \mathbf{C}(\mathbf{A} \bullet \mathbf{B})$. Check your answer using MATLAB.

We will compute each side independently and then show that they are equal.

$$
\begin{aligned}
\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= \mathbf{A} \times ((B_y C_z - B_z C_y)\mathbf{u_x} \\
&\quad + (B_z C_x - B_x C_z)\mathbf{u_y} \\
&\quad + (B_x C_y - B_y C_x)\mathbf{u_z}) \\
&= \mathbf{A} \times ((2 \cdot 3 - 2 \cdot 3)\mathbf{u_x} + (2 \cdot 3 - 2 \cdot 3)\mathbf{u_y} + (2 \cdot 3 - 2 \cdot 3)\mathbf{u_z}) \\
&= \mathbf{A} \times (0\mathbf{u_x} + 0\mathbf{u_y} + 0\mathbf{u_z}) \\
&= (A_y \cdot 0 - A_z \cdot 0)\mathbf{u_x} + (A_z \cdot 0 - A_x \cdot 0)\mathbf{u_y} + (A_x \cdot 0 - A_y \cdot 0)\mathbf{u_z} \\
&= 0\mathbf{u_x} + 0\mathbf{u_y} + 0\mathbf{u_z}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{B}(\mathbf{A} \bullet \mathbf{C}) - \mathbf{C}(\mathbf{A} \bullet \mathbf{B}) &= \mathbf{B}((A_x \cdot C_x) + (A_y \cdot C_y) + (A_z \cdot C_z)) \\
&\quad - \mathbf{C}((A_x \cdot B_x) + (A_y \cdot B_y) + (A_z \cdot B_z)) \\
&= \mathbf{B}((1 \cdot 3) + (1 \cdot 3) + (1 \cdot 3)) \\
&\quad - \mathbf{C}((1 \cdot 2) + (1 \cdot 2) + (1 \cdot 2)) \\
&= 9\mathbf{B} - 6\mathbf{C} \\
&= (18 - 18)\mathbf{u_x} + (18 - 18)\mathbf{u_y} + (18 - 18)\mathbf{u_z} \\
&= 0\mathbf{u_x} + 0\mathbf{u_y} + 0\mathbf{u_z}
\end{aligned}
$$

Both sides are equal to the zero vector $\therefore \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \bullet \mathbf{C}) - \mathbf{C}(\mathbf{A} \bullet \mathbf{B})$.