

1.4.4 Calculate the divergence of the vector $\mathbf{A} = x^3y \sin(\pi z)\mathbf{u}_x + xy \sin(\pi z)\mathbf{u}_y + x^2y^2z^2\mathbf{u}_z$ at the point $(1,1,1)$.

Let $p = (1 \ 1 \ 1)$.

$$\begin{aligned}
 (\nabla \bullet \mathbf{A})|_p &= \left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) \Big|_p \\
 &= \left(\frac{\partial}{\partial x}(x^3y \sin(\pi z)) + \frac{\partial}{\partial y}(xy \sin(\pi z)) + \frac{\partial}{\partial z}(x^2y^2z^2) \right) \Big|_p \\
 &= (3x^2y \sin(\pi z) + x \sin(\pi z) + 2zx^2y^2)|_p \\
 &= 3 \cdot 1^2 \cdot 1 \cdot \sin(\pi \cdot 1) + 1 \cdot \sin(\pi \cdot 1) + 2 \cdot 1 \cdot 1^2 \cdot 1^2 \\
 &= 3 + 1 + 2 \\
 &= 6
 \end{aligned}$$