1.2.13 For the vectors $\mathbf{A} = \mathbf{u_x} + 3\mathbf{u_y} + 5\mathbf{u_z}$, $\mathbf{B} = 2\mathbf{u_x} + 4\mathbf{u_y} + 6\mathbf{u_z}$, and $\mathbf{C} = 3\mathbf{u_x} + 4\mathbf{u_y} + 5\mathbf{u_z}$, show that $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \bullet \mathbf{C}) - \mathbf{C}(\mathbf{A} \bullet \mathbf{B})$. Check your answer using MATLAB.

We will compute each side independently and then show that they are equal.

$$\begin{aligned}
\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= \mathbf{A} \times ((B_y C_z - B_z C_y)\mathbf{u_x} \\
&\quad + (B_z C_x - B_x C_z)\mathbf{u_y} \\
&\quad + (B_x C_y - B_y C_x)\mathbf{u_z}) \\
&= \mathbf{A} \times ((4 \cdot 5 - 6 \cdot 4)\mathbf{u_x} + (6 \cdot 3 - 2 \cdot 5)\mathbf{u_y} + (2 \cdot 4 - 4 \cdot 3)\mathbf{u_z}) \\
&= \mathbf{A} \times (-4\mathbf{u_x} + 8\mathbf{u_y} - 4\mathbf{u_z}) \\
&= (A_y \cdot -4 - A_z \cdot 8)\mathbf{u_x} \\
&\quad + (A_z \cdot -4 - A_x \cdot -4)\mathbf{u_y} \\
&\quad + (A_x \cdot 8 - A_y \cdot -4)\mathbf{u_z} \\
&= (3 \cdot -4 - 5 \cdot 8)\mathbf{u_x} + (5 \cdot -4 - 1 \cdot -4)\mathbf{u_y} + (1 \cdot 8 - 3 \cdot -4)\mathbf{u_z} \\
&= -52\mathbf{u_x} - 16\mathbf{u_y} + 20\mathbf{u_z}
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}(\mathbf{A} \bullet \mathbf{C}) - \mathbf{C}(\mathbf{A} \bullet \mathbf{B}) &= \mathbf{B}((A_x \cdot C_x) + (A_y \cdot C_y) + (A_z \cdot C_z)) \\
&\quad - \mathbf{C}((A_x \cdot B_x) + (A_y \cdot B_y) + (A_z \cdot B_z)) \\
&= \mathbf{B}((1 \cdot 3) + (3 \cdot 4) + (5 \cdot 5)) \\
&\quad - \mathbf{C}((1 \cdot 2) + (3 \cdot 4) + (5 \cdot 6)) \\
&= 40\mathbf{B} - 44\mathbf{C} \\
&= (80 - 132)\mathbf{u_x} + (160 - 176)\mathbf{u_y} + (240 - 220)\mathbf{u_z} \\
&= -52\mathbf{u_x} - 16\mathbf{u_y} + 20\mathbf{u_z}
\end{aligned}$$

Both sides are equal to the same vector $\therefore\ \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \bullet \mathbf{C}) - \mathbf{C}(\mathbf{A} \bullet \mathbf{B})$.