

1.3.6 Evaluate the closed-surface integral of the vector $\mathbf{A} = xyz\mathbf{u}_x + xyz\mathbf{u}_y + xyz\mathbf{u}_z$ over the cubical surface shown in Problem 1.3.5. *Note: Diagram not shown.*

The cube has side length 2, and one of its corners is at the origin.

$$\begin{aligned}
\oint \mathbf{A} \bullet d\mathbf{s} &= \int_{z=0}^2 \int_{y=0}^2 (xyz\mathbf{u}_x + xyz\mathbf{u}_y + xyz\mathbf{u}_z) \bullet (dydz\mathbf{u}_x) \\
&\quad + \int_{z=0}^2 \int_{y=0}^2 (xyz\mathbf{u}_x + xyz\mathbf{u}_y + xyz\mathbf{u}_z) \bullet (dydz(-\mathbf{u}_x)) \\
&\quad + \int_{z=0}^2 \int_{x=0}^2 (xyz\mathbf{u}_x + xyz\mathbf{u}_y + xyz\mathbf{u}_z) \bullet (dxdz\mathbf{u}_y) \\
&\quad + \int_{z=0}^2 \int_{x=0}^2 (xyz\mathbf{u}_x + xyz\mathbf{u}_y + xyz\mathbf{u}_z) \bullet (dxdz(-\mathbf{u}_y)) \\
&\quad + \int_{y=0}^2 \int_{x=0}^2 (xyz\mathbf{u}_x + xyz\mathbf{u}_y + xyz\mathbf{u}_z) \bullet (dxdy\mathbf{u}_z) \\
&\quad + \int_{y=0}^2 \int_{x=0}^2 (xyz\mathbf{u}_x + xyz\mathbf{u}_y + xyz\mathbf{u}_z) \bullet (dxdy(-\mathbf{u}_z)) \\
&= \int_{z=0}^2 \int_{y=0}^2 xyz dy dz - \int_{z=0}^2 \int_{y=0}^2 xyz dy dz \\
&\quad + \int_{z=0}^2 \int_{x=0}^2 xyz dx dz - \int_{z=0}^2 \int_{x=0}^2 xyz dx dz \\
&\quad + \int_{y=0}^2 \int_{x=0}^2 xyz dx dy - \int_{y=0}^2 \int_{x=0}^2 xyz dx dy \\
&= 0
\end{aligned}$$