

1.4.9 Show that $\nabla \times \mathbf{A} = 0$ if $\mathbf{A} = \frac{1}{\rho} \mathbf{u}_\rho$ in cylindrical coordinates.

$$\begin{aligned}
 \nabla \times \mathbf{A} &= \left(\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \mathbf{u}_\rho \\
 &\quad + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) \mathbf{u}_\phi \\
 &\quad + \frac{1}{\rho} \left(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \right) \mathbf{u}_z \\
 &= \left(\frac{1}{\rho} \frac{\partial}{\partial \phi}(0) - \frac{\partial}{\partial z}(0) \right) \mathbf{u}_\rho \\
 &\quad + \left(\frac{\partial}{\partial z} \left(\frac{1}{\rho} \right) - \frac{\partial}{\partial \rho}(0) \right) \mathbf{u}_\phi \\
 &\quad + \frac{1}{\rho} \left(\frac{\partial}{\partial \rho}(\rho \cdot 0) - \frac{\partial}{\partial \phi}(0) \right) \mathbf{u}_z \\
 &= 0
 \end{aligned}$$