

1.3.4 Calculate the work required to move a mass m against a force field $\mathbf{F} = \rho\phi\mathbf{u}_\phi$ if the radius of the circle is a and $0 \leq \phi \leq 2\pi$. *Note: Diagram not shown.*

By inspection of the diagram, the labeled points are $a = (0, 0)$, $b = (2, 0)$, and $c = (2, \pi)$ in cylindrical coordinates. To find the work required to move along the path, we need to compute the path integral of the force field.

$$\begin{aligned}
 W &= \int \mathbf{F} \cdot d\mathbf{l} \\
 &= \int \rho\phi\mathbf{u}_\phi \cdot \rho d\phi\mathbf{u}_\phi \\
 &= \int_{\phi=0}^{2\pi} \rho^2 \phi d\phi \\
 &= a^2 \left. \frac{\phi^2}{2} \right|_{\phi=0}^{2\pi} \\
 &= a^2 \frac{4\pi^2}{2} \\
 &= 2a^2\pi^2
 \end{aligned}$$