1.2.5 Find the projection of a vector from the origin to a point defined at $(1, 2, 3)$ on the vector from the origin to a point defined at $(2, 1, 6)$. Find the angle between these two vectors. Check your answer using MATLAB.

Let $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 & 6 \end{pmatrix}$.

The projection of $\mathbf{A}$ onto $\mathbf{B}$ is equal to the scalar product of $\mathbf{A}$ and $\hat{\mathbf{B}}$, where $\hat{\mathbf{B}}$ is the unit vector pointing along $\mathbf{B}$.

$$\begin{aligned}
\hat{\mathbf{B}} &= \frac{\mathbf{B}}{||\mathbf{B}||} \\
&= \frac{2\mathbf{u_x} + \mathbf{u_y} + 6\mathbf{u_z}}{\sqrt{2^2 + 1^2 + 6^2}} \\
&= \frac{2}{\sqrt{41}}\mathbf{u_x} + \frac{1}{\sqrt{41}}\mathbf{u_y} + \frac{6}{\sqrt{41}}\mathbf{u_z}.
\end{aligned}$$

$$\begin{aligned}
\mathbf{A} \bullet \hat{\mathbf{B}} &= A_x\hat{B}_x + A_y\hat{B}_y + A_z\hat{B}_z \\
&= 1 \cdot \frac{2}{\sqrt{41}} + 2 \cdot \frac{1}{\sqrt{41}} + 3 \cdot \frac{6}{\sqrt{41}} \\
&= \frac{22}{\sqrt{41}} \\
&\approx 3.44
\end{aligned}$$

The angle between $\mathbf{A}$ and $\mathbf{B}$ may be computed using the definition of the scalar product.

$$\begin{aligned}
\mathbf{A} \bullet \mathbf{B} &= ABcos\theta \\
A_xB_x + A_yB_y + A_zB_z &= \sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}\cos\theta \\
\theta &= \arccos\frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}} \\
\theta &= \arccos\frac{1 \cdot 2 + 2 \cdot 1 + 3 \cdot 6}{\sqrt{1^2 + 2^2 + 3^2}\sqrt{2^2 + 1^2 + 6^2}} \\
\theta &= \arccos\frac{22}{\sqrt{574}} \\
\theta &\approx 23.3^\circ
\end{aligned}$$