1.4.12 In rectangular coordinates, verify that $\nabla \times \nabla a = \mathbf{0}$ where $a = 3x^2y + 4z^2x$ by carrying out the detailed differentiations.

$$\begin{aligned}
\nabla \times \nabla a &= \nabla \times \left(\frac{\partial a}{\partial x}\mathbf{u_x} + \frac{\partial a}{\partial y}\mathbf{u_y} + \frac{\partial a}{\partial z}\mathbf{u_z} \right) \\
&= \left(\frac{\partial}{\partial y}\frac{\partial a}{\partial z} - \frac{\partial}{\partial z}\frac{\partial a}{\partial y} \right)\mathbf{u_x} + \left(\frac{\partial}{\partial z}\frac{\partial a}{\partial x} - \frac{\partial}{\partial x}\frac{\partial a}{\partial z} \right)\mathbf{u_y} + \left(\frac{\partial}{\partial x}\frac{\partial a}{\partial y} - \frac{\partial}{\partial y}\frac{\partial a}{\partial x} \right)\mathbf{u_z} \\
&= \left(\frac{\partial}{\partial y}\frac{\partial}{\partial z}(3x^2y + 4z^2x) - \frac{\partial}{\partial z}\frac{\partial}{\partial y}(3x^2y + 4z^2x) \right)\mathbf{u_x} \\
&\quad + \left(\frac{\partial}{\partial z}\frac{\partial}{\partial x}(3x^2y + 4z^2x) - \frac{\partial}{\partial x}\frac{\partial}{\partial z}(3x^2y + 4z^2x) \right)\mathbf{u_y} \\
&\quad + \left(\frac{\partial}{\partial x}\frac{\partial}{\partial y}(3x^2y + 4z^2x) - \frac{\partial}{\partial y}\frac{\partial}{\partial x}(3x^2y + 4z^2x) \right)\mathbf{u_z} \\
&= 0\mathbf{u_x} + (8z - 8z)\mathbf{u_y} + (6x - 6x)\mathbf{u_z} \\
&= \mathbf{0}
\end{aligned}$$