1.4.11 In rectangular coordinates, verify that $\nabla \bullet \nabla \times \mathbf{A} = 0$ where $\mathbf{A} = x^2y^2z^2\left[\mathbf{u_x} + \mathbf{u_y} + \mathbf{u_z}\right]$ by carrying out the detailed differentiations.

$$\begin{aligned}
\nabla \bullet \nabla \times \mathbf{A} &= \nabla \bullet \left[\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\mathbf{u_x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\mathbf{u_y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\mathbf{u_z}\right] \\
&= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \\
&= \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}(x^2y^2z^2) - \frac{\partial}{\partial z}(x^2y^2z^2)\right) \\
&\quad + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial z}(x^2y^2z^2) - \frac{\partial}{\partial x}(x^2y^2z^2)\right) \\
&\quad + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial x}(x^2y^2z^2) - \frac{\partial}{\partial y}(x^2y^2z^2)\right) \\
&= \frac{\partial}{\partial x}\left(2x^2yz^2 - 2x^2y^2z\right) + \frac{\partial}{\partial y}\left(2x^2y^2z - 2xy^2z^2\right) + \frac{\partial}{\partial z}\left(2xy^2z^2 - 2x^2yz^2\right) \\
&= 4xyz^2 - 4xy^2z + 4x^2yz - 4xyz^2 + 4xy^2z - 4x^2yz \\
&= 0
\end{aligned}$$