ME C201 / MSE C286 PROJECT 6 - MODELING AND SIMULATION OF THE INFECTION ZONE FROM A COUGH (100 POINTS)

Due: Friday, May 7th, 2021, 11:59 pm.

Guidelines:

- Submit an electronic copy of your typed project on Gradescope before the due date.
- This project should follow the project instructions for this course. To receive full credit, your report must be neat and specific but does not need to be longer than a few pages.

Overview

The pandemic of 2020 has led to a huge interest of modeling and simulation of infectious diseases. One of the central questions is the potential infection zone produced by a cough. In this assignment, you will develop mathematical models to simulate the progressive time-evolution of the distribution of particles in space produced by a cough. From your simulations, we ask that you recommend safe distancing guidelines based on the range, distribution and settling time of the particles under the influence of gravity and drag from the surrounding air.

DROPLET DISPERSION FROM A COUGH

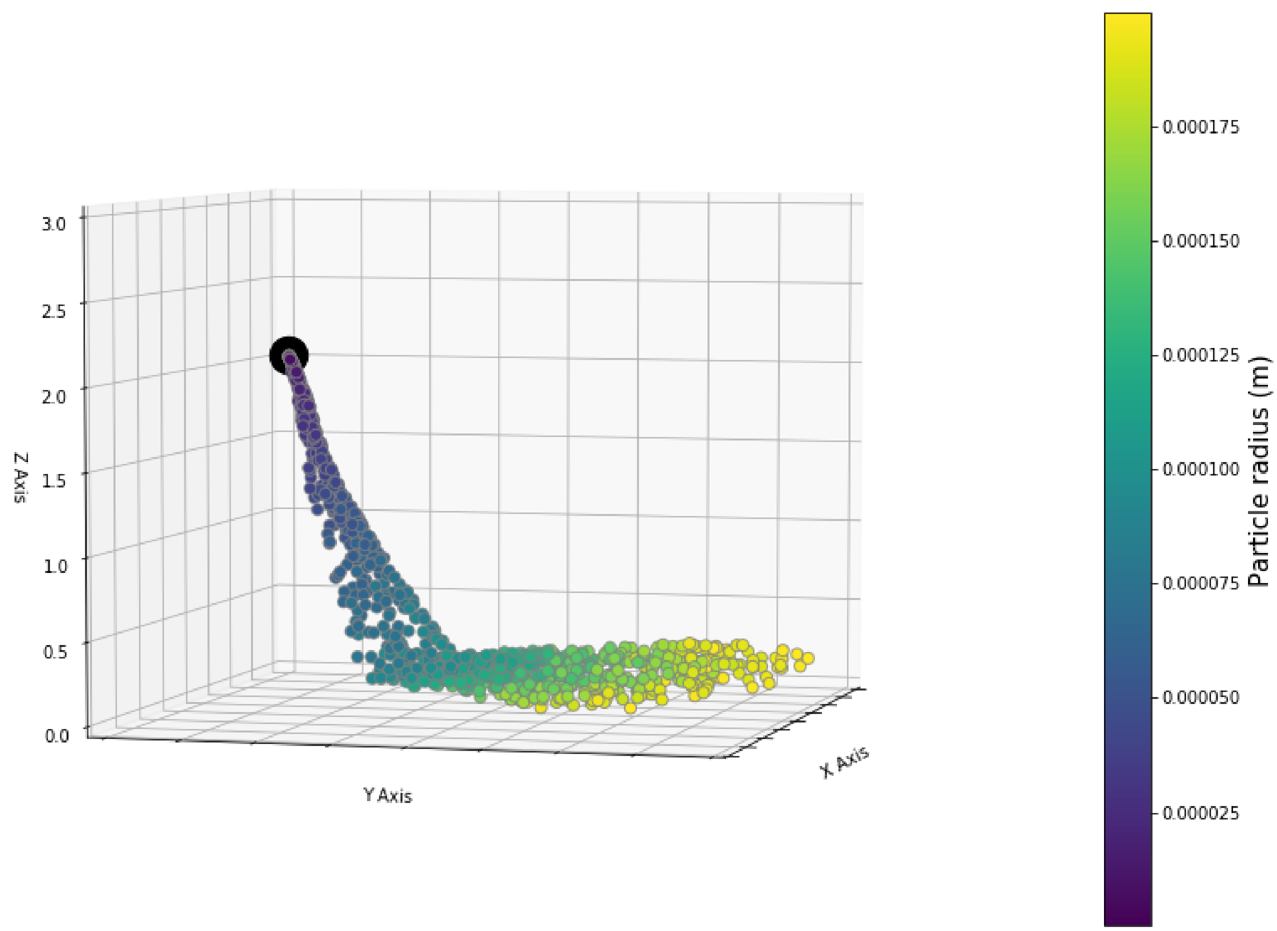

Figure 1: Droplet dispersion after 4 seconds.

Model Formulation

Particle Dynamics

In its most basic form, a cough can be considered as a high-velocity release of a random distribution of particles of various sizes, into an ambient atmosphere. For our particular simulation, assume the same initial velocity magnitude for all particles under consideration, with a random distribution of outward directions away from the source of the cough. This implies that a particle non-interaction approximation is appropriate. Thus, the inter-particle collisions are negligible. We also assume that the particles are spherical with a random distribution of radii $R_i, i = 1, 2, 3, ..., N = particles$. The masses are given by $m_i = \rho_i \frac{4}{3}\pi R_i^3$, where ρ_i is the density of the particles. We assume that the cough particles are quite small and that the amount of rotation, if any, contributes negligibly to the overall trajectory of the particles. The equation of motion for the ith particle in the system is

$$m_i \ddot{\mathbf{r}}_i = \mathbf{\Psi}_i^{grav} + \mathbf{\Psi}_i^{drag} \tag{1}$$

where m_i is the mass of each droplet, $\ddot{\mathbf{r}}_i$ is the droplet's acceleration, and the two $\mathbf{\Psi}$ are gravitational and drag forces respectively.

Let the y axis be the verticle axis so we note that the gravitation force be defined as

$$\mathbf{\Psi}_i^{grav} = m_i(g_x, g_y, g_z) = (0, 0, -m_i g) \tag{2}$$

where g is the graviational constant. The drag force depends on the geomtry of the droplet and the properties of the surrounding medium:

$$\mathbf{\Psi}_i^{drag} = \frac{1}{2}\rho_a C_D ||\mathbf{v}^f - \mathbf{v}_i||(\mathbf{v}^f - \mathbf{v}_i)A_i \tag{3}$$

where C_D is the drag coefficient, A_i is the reference area, which for a sphere is $A_i = \pi R_i^2$, ρ_a is the density of the ambient fluid environment and $\mathbf{v}^f$ is the velocity of the surrounding medium, which in this case is air. As in Project 1, to determine the drag coefficient, we first determine the Reynolds number of the droplet Re:

$$Re = \frac{2R\rho_a \|\boldsymbol{v}^f - \boldsymbol{v}_i\|}{\mu_f} \tag{4}$$

Where μ_f is the viscosity of the surrounding medium. Then, the drag coefficient is a piecewise function of Re:

$$C_{Di} = \begin{cases} \frac{24}{Re}, & 0 < Re \leq 1 \\ \frac{24}{Re^{0.646}}, & 1 < Re \leq 400 \\ 0.5, & 400 < Re \leq 3 \times 10^5 \\ 0.000366 Re^{0.4275}, & 3 \times 10^5 < Re \leq 2 \times 10^6 \\ 0.18, & Re > 2 \times 10^6 \end{cases} \tag{5}$$

Numerical Integration

Thus we have the governing equation for the motion of the droplets, which we will numerically integrate using the Forward Euler method. We employ equation 1 and solve for $\ddot{\boldsymbol{r}}_i$. We use a Forward Euler scheme to integrate and find the position and velocity of some i-th droplet:

$$\boldsymbol{r}_i(t + \Delta t) = \boldsymbol{r}_i(t) + \Delta t \boldsymbol{v}_i(t) \tag{6}$$
$$\boldsymbol{v}_i(t + \Delta t) = \boldsymbol{v}_i(t) + \Delta t \ddot{\boldsymbol{r}}_i(t) \tag{7}$$

Simulation Parameters

Particle Generation

Let us assume the total mass of all of the droplets in a cough is $M^{total} = 0.00005kg$, with a mean particle radius of $\bar{R} = 0.0001m$ with variations according to

$$R_i = \bar{R} \times (1 + A \times \xi_i) \tag{8}$$

where $A = 0.9975$ and a random variable $-1 \leq \xi_i \leq 1$. The algorithm used for particle generation should be:

- $M = 0$
- Start loop: $i = 1, P_n$
- $R_i = \bar{R} \times (1 + A \times \xi_i)$
- $M = M + m_i = M + \rho_i \frac{4}{3}\pi R_i^3$
- If $M \leq M^{total}$ then stop (determines $P_n =$ particles)
- End loop

Initial Trajectories

The initial trajectories we determined from the following algorithm

- Specify the relative direction 'cone' parameters: $\mathbf{N}^c = (N_x^c, N_y^c, N_z^c)$,
- For each particle, $i = 1, 2, 3, ..., P_n$, construct a (perturbed) trajectory vector:

$$\mathbf{N}_i = (N_x^c + A_x^c \times \eta_{ix}, N_y^c + A_y^c \times \eta_{iy}, N_z^c + A_z^c \times \eta_{iz}), \tag{9}$$

  where the random cone parameters are bounded: $-1 \leq \eta_{ix} \leq 1$, $0 \leq \eta_{iy} \leq 1$, $-1 \leq \eta_{iz} \leq 1$.
- For each particle, normalize the trajectory vector:

$$\mathbf{n}_i = \frac{1}{||\mathbf{N}_i||_2}(N_{ix}, N_{iy}, N_{iz}). \tag{10}$$

- For each particle, the velocity vector is constructed by a projection onto the normal vector:

$$\mathbf{v}_i = V^c \mathbf{n}_i \tag{11}$$

Implementation Tips

- As with assignment 1, consider only computing the forces on the droplets that are still airborne, while dropletes with position $r_y <= 0$ are immobilized and reset to 0.
- Rather than plotting all of the droplets, randomly choose 1000 droplets to display in your plots.

Deliverables

1. Plot the trajectory of the droplets in 3D at time $t = [0.25s, 0.5s, 2.0s, 4.0s]$ using **colorbar()** to indicate range of droplets' radii.
2. Report the maximum distance traveled from the source and give a recommendation for practicing safe social distancing.
3. Comment on how the particle size affects the physical range and settling time of the particle.

To submit your project:

1. Upload your report PDF to Gradescope under the assignment name: *Spring 2021, Assignment 7 Report*
2. Upload the complete Matlab or Python scripts (or your preferred language) to Gradescope under the assignment name: *Spring 2021, Assignment 7 Code*

VARIABLE GLOSSARY

Symbol	Type	Units	Value	Description
r_0	Vector	m	[0, 0, 2]	Standing height
t_{tot}	Scalar	s	4	Total simulation time
dt	Scalar	s	1e-4	Time step
$V^c_{t=0}$	Scalar	m/s	35	Cough velocity
V^f	Vector	m/s	[0, 0, 0]	Velocity of surrounding medium
ρ_i	Scalar	kg/m^3	1000	Density of droplets
ρ_a	Scalar	kg/m^3	1.225	Density of air
M^{total}	Scalar	kg	0.00005	Total mass $\sum_{i=1}^{P_n} m_i$
$\bar{R}$	Scalar	m	0.0001	Mean particle radius
A	Scalar	none	0.9975	Deviatoric constant
$\mathbf{N}^c$	vector	none	[0, 1, 0]	Trajectory cone
$\mathbf{A}^c$	vector	none	[1, 0.5, 1]	Deviatoric constant for trajectory
μ_f	scalar	$Pa \cdot s$	1.8e-5	Viscosity coefficient for air
g	vector	m/s^2	[0, 0, -9.81]	Gravitational constant