Recitation #16: The Divergence, Integral, Ratio and Root Tests

Group work:

Problem 1 *For each of the following, answer* **True** *or* **False***, and explain why.*

(a) *If* $\sum_{n=0}^{\infty} a_n$ *converges, then* $\sum_{n=0}^{\infty}(a_n + 0.001)$ *converges.*

(b) *Since* $\int_1^{\infty} x\sin(\pi x)\,dx$ *diverges then, by the Integral Test,* $\sum_{n=0}^{\infty} n\sin(\pi n)$ *diverges.*

(c) *Since* $\int_1^{\infty} \frac{1}{x^2}\,dx = 1$ *then, by the Integral Test,* $\sum_{k=1}^{\infty} \frac{1}{k^2} = 1$.

Problem 2 *Assume* $\sum_{k=0}^{\infty} a_k = L$ *and* $b_k = 8$ *for all* k.

(a) *What is* $\lim_{k\to\infty}(a_k + b_k)$?

(b) *What is* $\lim_{k\to\infty}\sum_{n=0}^{k}(a_n + b_n)$?

(c) *What is* $\lim_{k\to\infty}\sum_{n=0}^{k}(a_{n+1} - a_n)$?

Problem 3 *Determine if the following series converge or diverge.*

(a) $\sum_{n=1}^{\infty} \frac{(7n+1)^2 \cdot 2^n}{5^n}$

(b) $\sum_{n=1}^{\infty} a_n$, *where* $a_{n+1} = \frac{2n+5}{3n-1}\cdot a_n$ *and* $a_1 = 1$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^2+1}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(e) $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^3}{(3k)!}$

Problem 4 *How many terms are needed to estimate* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+1}$ *to within* 10^{-4}*? What is the estimate for the sum of the series?*