Recitation #22: Working with Taylor series - Instructor Notes

Warm up:

True or False: To approximate $\frac{\pi}{3}$, one could substitute $x = \sqrt{3}$ into the Maclaurin series for $\tan^{-1} x$?

Instructor Notes: Remind students that a power series representation for a function is not the exact same as the function.

Group work:

Problem 1 *Use power series to evaluate the limit*

$$\lim_{x \to 0} \frac{\ln(1 + x^2)}{1 - \cos x}$$

Instructor Notes: Using power series to evaluate a limit. Tell students that they may have to do such a limit and be specifically told not to use L'hôpital's rule.

Problem 2 *Given that*

$$f(t) = \int_0^t x^2 \tan^{-1}(x^4)\, dx$$

approximate $f\left(\frac{1}{3}\right)$ *with the first four non-zero terms of a power series. Estimate how close this approximation is.*

Instructor Notes: Error for power series.

Problem 3 *Identify the function represented by the power series*

$$\sum_{k=0}^{\infty} \frac{k(k-1)x^k}{7^k}$$

Instructor Notes: Most of these types of problems come from $\dfrac{1}{1-x} = \sum_k x^k$.

Problem 4 *Use power series to determine a (series) solution to the initial value problem*

$$y'' - xy' + y = 0 \qquad y(0) = 1 \qquad y'(0) = 0$$

Instructor Notes: