Recitation #16: The Divergence, Integral, Ratio and Root Tests - Instructor Notes

Group work:

Problem 1 *For each of the following, answer* **True** *or* **False**, *and explain why.*

(a) *If* $\sum_{n=0}^{\infty} a_n$ *converges, then* $\sum_{n=0}^{\infty}(a_n + 0.001)$ *converges.*

(b) *Since* $\int_1^{\infty} x\sin(\pi x)\,dx$ *diverges then, by the Integral Test,* $\sum_{n=0}^{\infty} n\sin(\pi n)$ *diverges.*

(c) *Since* $\int_1^{\infty} \frac{1}{x^2}\,dx = 1$ *then, by the Integral Test,* $\sum_{k=1}^{\infty} \frac{1}{k^2} = 1$.

Instructor Notes: For part (b) we need $f(x)$ to be a decreasing function for the Integral Test to (necessarily) hold. All groups should do all of the parts.

Problem 2 *Assume* $\sum_{k=0}^{\infty} a_k = L$ *and* $b_k = 8$ *for all* k.

(a) *What is* $\lim_{k\to\infty}(a_k + b_k)$*?*

(b) *What is* $\lim_{k\to\infty}\sum_{n=0}^{k}(a_n + b_n)$*?*

(c) *What is* $\lim_{k\to\infty}\sum_{n=0}^{k}(a_{n+1} - a_n)$*?*

Instructor Notes: This question was adapted from midterm #2 in Spring 2013. Students had difficulty distinguishing between a question dealing with sequences vs. a question dealing with series.

Problem 3 *Determine if the following series converge or diverge.*

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(7n+1)^2 \cdot 2^n}{5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} a_n$, *where* $a_{n+1} = \dfrac{2n+5}{3n-1} \cdot a_n$ *and* $a_1 = 1$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^2+1}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(e) $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^3}{(3k)!}$

Instructor Notes: Let the students experiment with what tests to use. Perhaps give two problems per group.

Problem 4 *How many terms are needed to estimate* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+1}$ *to within* 10^{-4}*? What is the estimate for the sum of the series?*

Instructor Notes: This should be a straightforward calculation, but calculators may be needed.
