Recitation 23: Parametric equations & Polar coordinates - Instructor Notes

Warm up:

Describe the motion given by $x = 8$, $y = 7\sin(t)$ for all t.

Instructor Notes: One variable is held constant while the other oscillates up and down.

Group work:

Problem 1 *Try to figure out the shape of the following curve and then eliminate the parameter and check your intuition.*

$$x = \ln t - 1 \qquad y = (\ln t)^2$$

Instructor Notes: It is not hard to see that $y = (x+1)^2$. So this is a parabola.

Problem 2 *Find parametric equations for the path of a particle moving around the circle*

$$(x-3)^2 + (y+7)^2 = 4$$

(a) *one time around clockwise starting at* $(5, -7)$.

(b) *three times around counterclockwise starting at* $(5, -7)$.

(c) *halfway around clockwise starting at* $(-1, -7)$.

Instructor Notes: Practice with the formula for parameterizing a circle. It is useful to point out that the same curve can have many different parameterizations.

Problem 3 *Find the intersection point(s) of the lines*

$$x = -6 + 9t, \qquad y = 3 - 2t \tag{1}$$

and

$$x = 3 + t, \qquad y = -4 - 2t. \tag{2}$$

Do they intersect at the same time?

Instructor Notes: The key point here is that intersections can occur at different "times".

Problem 4 *Consider the curve defined by the parameterization* $x = t^2$, $y = t^3 - 3t$. *Show that this curve has two tangent lines at* $(3, 0)$, *and find the equations of the tangent lines there.*

Instructor Notes: The main idea here is that a parametric curve can have several tangent lines at the same point (in (x, y) coordinates).

Problem 5 *Plot the following (polar) points in the* xy*-plane and then rewrite them as rectangular coordinates.*

(a) $\left(3, \frac{5\pi}{4}\right)$ (b) $\left(3, -\frac{5\pi}{4}\right)$ (c) $\left(-3, \frac{5\pi}{4}\right)$ (d) $\left(-3, -\frac{5\pi}{4}\right)$

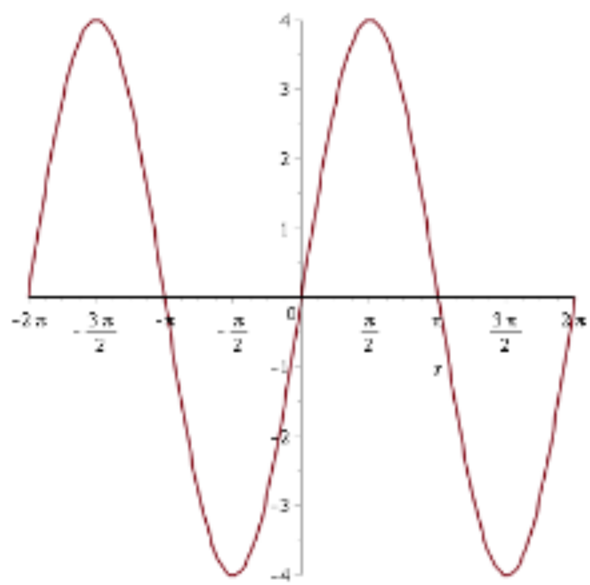
Instructor Notes: Some students will be familiar with polar coordinates while others will have never worked with them before. So it is very important to establish how these coordinates are measured.

Students sometimes have a difficult time dealing with negative values for r as well as with working with these while graphing. Both this and the next problem force the students to deal with these issues.

Problem 6 *Rewrite the rectangular point $(3, 5)$ in polar coordinates in three different ways.*

Instructor Notes: See problem 5.

Problem 7 *The graph of the curve $r = 4 \sin\theta$ is a circle. Use the graph below to sketch this circle. Can you verify this algebraically? What is the period of the polar curve? Is $0 \le \theta \le 2\pi$ necessary to complete the graph?*



Instructor Notes: Students sometimes have difficulty with the "Cartesian to Polar" graphing method given in the textbook. This will be important in the next section when they need to visualize curves in order to integrate in polar coordinates.

Problem 8 *Graph $r = 2 + 4\cos\theta$ using the "Cartesian-to-Polar" method.*

Instructor Notes: See problem 7.