

Recitation #18: Comparison Tests and Alternating Series

Warm up:

For each of the following, answer **True** or **False**, and explain why.

- (a) If $a_n \geq 0$ and $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty} a_n^2$ converges.
- (b) If $a_n, b_n \geq 0$ and both $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ converge, then $\sum_{n=0}^{\infty} a_n b_n$ converges.

Group work:

Problem 1 (a) Why can we not use the Comparison test with $\sum_{k=1}^{\infty} \frac{1}{k^2}$ to show

that $\sum_{k=1}^{\infty} \frac{1}{k^2 - 5}$ converges?

(b) Adjust $\sum_{k=1}^{\infty} \frac{1}{k^2}$ to show that $\sum_{k=1}^{\infty} \frac{1}{k^2 - 5}$ converges via the Comparison Test.

(c) Give a convergent series we can use in the Limit Comparison Test to show that $\sum_{k=1}^{\infty} \frac{1}{k^2 - 5}$ converges.

Problem 2 Determine if the following series converge or diverge.

(a) $\sum_{n=0}^{\infty} \frac{n^2 + 2n + 1}{3n^3 + 1}$

(c) $\sum_{n=0}^{\infty} \frac{\cos^2 n}{n^3 + 1}$

(b) $\sum_{n=0}^{\infty} \frac{n^2 + 2n + 1}{3n^4 + 1}$

(d) $\sum_{n=1}^{\infty} \left[\left(1 + \frac{1}{n} \right)^2 e^{-n} \right]$

Problem 3 Determine if the following series absolutely converge, conditionally converge, or diverge.

$$\begin{array}{lll}
 \text{(a)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n+3} & \text{(c)} \sum_{n=1}^{\infty} (-1)^{n+1} n^2 e^{\frac{-n^3}{3}} & \text{(e)} \sum_{n=4}^{\infty} \frac{(-2)^n}{n} \\
 \text{(b)} \sum_{n=1}^{\infty} \frac{(-1)^n (n+1)^n}{(2n)^n} & \text{(d)} \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 5}{3^n + 3^{-n}} &
 \end{array}$$

Problem 4 (a) Find an upper bound for how close $\sum_{k=0}^4 \frac{(-1)^k k}{4^k}$ is to the value

$$\text{of } \sum_{k=0}^{\infty} \frac{(-1)^k k}{4^k}.$$

(b) How many terms are needed to estimate $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n!}$ to within 10^{-6} ?