Recitation #13: Direction fields and Separable Differential Equations - Instructor Notes

Warm up:

Which of the following differential equations are separable?

(a) $y' = \dfrac{ty}{t^2+1}$,

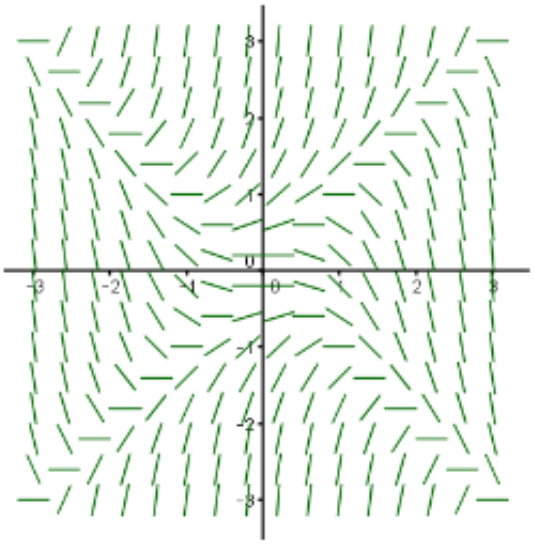
(b) $\dfrac{dy}{dx} = x^2\sin(3y) - x^2$,

(c) $y' = t^2 - y$.

Instructor Notes:

Group work:

Problem 1 (a) *The following is a direction field for the differential equation* $\dfrac{dy}{dx} = y^2 - x^2$.



Sketch the solution such that $y\left(\frac{1}{2}\right) = 1$.

(b) *Use Euler's Method to give a numerical estimate to the solution of the differential equation* $y' = y^2 - t^2$ *at* $y(2)$ *that goes through the point* $\left(\frac{1}{2}, 1\right)$. *Use* $\Delta t = 0.5$.

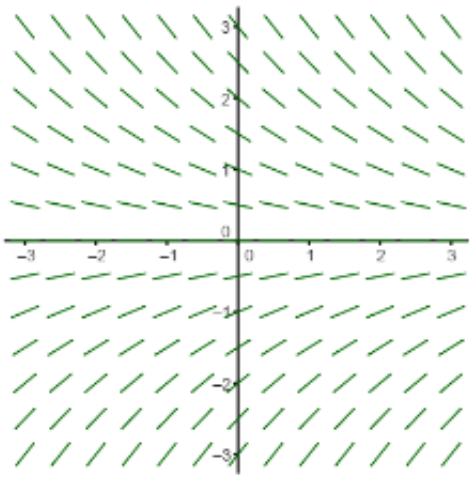
Instructor Notes: The major point here is that (a) and (b) are the same problem, presented with two different representations.

Problem 2 *Describe why the following direction field could be the direction field for the differential equation*

$$\frac{dy}{dt} = y\cos(t)$$

but ***not*** *for*

$$\frac{dy}{dt} = y\sin(t) \qquad \text{or} \qquad \frac{dy}{dt} = t\cos(y).$$



Instructor Notes: Students should examine when t varies across quadrants, combined with the sign of y at points (t, y) (with $y' = t \cos y$, y is going through "quadrants"). Checking where $y' = 0$ can reveal why the other two differential equations are not satisfied. This could all just be a whole class discussion with the instructor bringing up strategies.

Problem 3 *Match each of the following differential equations with a corresponding direction field (if it is present):*

i. $y' = \dfrac{t}{2+y}$

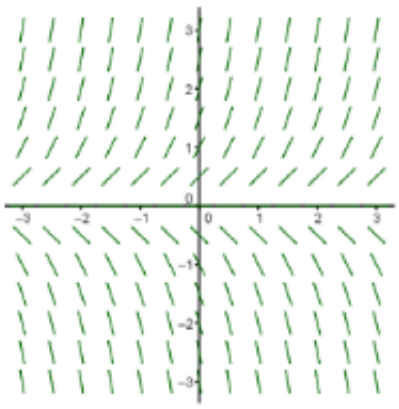
ii. $y' = \cos(t+y)$

iii. $y' = 1 + y^2$

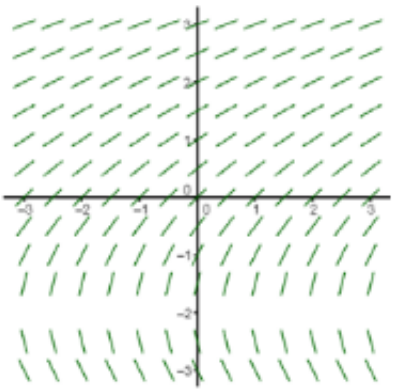
iv. $y' = ty$

b.

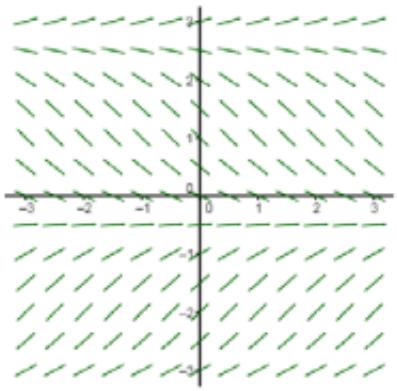
b.



c.



d.



Instructor Notes: Several strategies exist. Make sure to ask what special quality direction fields of autonomous differential equations have. Depending on time, this also could all be done as a whole class.

Problem 4 *Which of the following are separable differential equations? For those that are, solve them, assuming that* $y(4) = 5$.

(a) $y' = x^2 + y^2$

(b) $y' = x + xy^2$

(c) $y' = e^{2x-y}$

Instructor Notes: Part (b) is the only non-separable equation. Students may need help recognizing that they can divide by the entire right side of both sides. Also, some results may only define y implicitly.