Sections 12.2: Vectors in Three Dimensions

Group work:

Problem 1 *Solve the following problems:*

(a) *Which of the points* $(6, 2, 3)$, $(-5, -1, 4)$, *and* $(0, 3, 8)$ *is closest to the* xz*-plane? Which point lies on the* yz*-plane?*

(b) *Write an equation of the circle of radius* 2 *centered at* $(-3, 4, 1)$ *that lies in a plane parallel to the* xy*-plane.*

(c) *Describe the sphere* $x^2 + y^2 + z^2 + 6x - 14y - 2z = 5$ *(ie, find its center and radius).*

(d) *Find a vector whose magnitude is* 311 *and is in the same direction as the vector* $\langle 3, -6, 7\rangle$.

Solution: (a) *The* xz*-plane has equation* $y = 0$. *The distance from a point* (a, b, c) *to* $y = 0$ *is just* $|b|$. *So*

$(6, 2, 3)$ *has distance* 2

$(-5, -1, 4)$ *has distance* 1

$(0, 3, 8)$ *has distance* 3

Therefore, the point $(-5, -1, 4)$ *is closest to the* xz*-plane.*

The yz*-plane is* $x = 0$, *and so the point* $(0, 3, 8)$ *is on the* yz*-plane.*

(b) *A plane parallel to the* xy*-plane has equation* $z = \#$. *We are looking for such a plane containing the point* $(-3, 4, 1)$, *and so the plane is* $z = 1$. *Therefore, the equation is*

$$\boxed{(x+3)^2 + (y-4)^2 = 4\ ,\ z = 1.}$$

(c) *Let* $\vec{v} = \langle 3, -6, 7\rangle$. *Then*

$$\begin{aligned} |\vec{v}| &= \sqrt{3^2 + (-6)^2 + 7^2} \\ &= \sqrt{9 + 36 + 49} \\ &= \sqrt{94}. \end{aligned}$$

Learning outcomes:

So a unit vector in the same direction as $\vec{v}$ *is*

$$\frac{1}{\sqrt{94}}\langle 3, -6, 7\rangle$$

and therefore a vector with magnitude 311 *in the same direction as* v *is*

$$\boxed{\frac{311}{\sqrt{94}}\langle 3, -6, 7\rangle}$$