3.10 Derivatives of Inverse Trig Functions (Solutions)

Warm up:

Explain what each of the following means:

(a) $\sin^{-1}(x)$

(b) $(\sin(x))^{-1}$

(c) $\sin\left(x^{-1}\right)$

(d) $f^{-1}(x)$

(e) $f(x^{-1})$

(f) $(f(x))^{-1}$

Group work:

Problem 1 *Find the derivatives of the following functions:*

(a) $f(x) = \sec^{-1}(\sqrt{x})$.

(b) $g(x) = \ln(\sin^{-1}(x))$.

(c) $h(x) = \dfrac{1}{\tan^{-1}(x^2+4)}$.

Problem 2 *Find the slope of the tangent line to the curve $y = f^{-1}(x)$ at $(4, 7)$ if the slope of the tangent line to the curve $y = f(x)$ at $(7, 4)$ is $\dfrac{2}{3}$.*

Problem 3 *Suppose that $f(x)$ is a differentiable function which is one-to-one. Given the table of values below, find the value of $(f^{-1})'(7)$.*

x	1	7	11
$f(x)$	7	11	1
$f'(x)$	61	-17	71

Problem 4 *Find the derivative of f^{-1} at the following points without solving for f^{-1}.*

(a) $f(x) = x^2 + 1$ *(for $x \geq 0$) at the point* $(5, 2)$.

(b) $f(x) = x^2 - 2x - 3$ *(for $x \leq 1$) at the point* $(12, -3)$.