Section 12.5: Lines and Curves in Space

Warm up:

Group work:

Problem 1 *Find a vector-valued function for the line segment connecting the points $P = (-3, 7, 6)$ and $Q = (5, -4, 7)$ in such a way that the value at $t = 0$ is P and the value at $t = 1$ is Q. Also, find the point two-thirds of the way from P to Q.*

Solution: *The line segment $\vec{r}(t)$ from P to Q is*

$$\begin{aligned}\vec{r}(t) &= (1-t)P + tQ \\ &= (1-t)\langle -3, 7, 6\rangle + t\langle 5, -4, 7\rangle \\ &= \boxed{\langle -3+8t, 7-11t, 6+t\rangle \quad \text{for } 0 \le t \le 1}.\end{aligned}$$

The point two-thirds of the way from P to Q is

$$\begin{aligned}\vec{r}\left(\frac{2}{3}\right) &= \left\langle -3 + 8\left(\frac{2}{3}\right), 7 - 11\left(\frac{2}{3}\right), 6 + \frac{2}{3}\right\rangle \\ &= \boxed{\left\langle \frac{7}{3}, -\frac{1}{3}, \frac{20}{3}\right\rangle}\end{aligned}$$

Problem 2 *Find a vector-valued function for the line through the point $(1, -2, 3)$ that is perpendicular to the lines*

$$\vec{r}_1(t) = \langle 7, 8, -2\rangle + t\langle 3, 5, 7\rangle \quad \text{and} \quad \vec{r}_2(s) = \langle 4, -3, -7\rangle + s\langle 4, 9, -1\rangle$$

Solution: *Let $\vec{v}_1 = \langle 3, 5, 7\rangle$ and $\vec{v}_2 = \langle 4, 9, -1\rangle$. Then $\vec{v}_1$ is parallel to the line $\vec{r}_1$, and similarly for $\vec{v}_2$ and $\vec{r}_2$. So a vector perpendicular to both of the*

Learning outcomes:

lines $\vec{r}_1$ and $\vec{r}_2$ is

$$\begin{aligned}\vec{n} = \vec{v}_1 \times \vec{v}_2 &= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 5 & 7 \\ 4 & 9 & -1 \end{vmatrix} \\ &= (-5 - 63)\hat{\imath} - (-3 - 28)\hat{\jmath} + (27 - 20)\hat{k} \\ &= \langle -68, 31, 7 \rangle.\end{aligned}$$

So the equation of the line through $(1, -2, 3)$ and perpendicular to both $\vec{r}_1$ and $\vec{r}_2$ is

$$\begin{aligned}\vec{r}_3(t) &= \langle 1, -2, 3 \rangle + t\langle -68, 31, 7 \rangle \\ &= \boxed{\langle 1 - 68t, -2 + 31t, 3 + 7t \rangle \quad \textit{for } -\infty < t < \infty}\end{aligned}$$

Problem 3 *Show that the curve $\vec{r} = \langle t\cos t, t\sin t, t \rangle$ lies completely on the cone $z^2 = x^2 + y^2$.*

Solution: *We just need to check that the components of $\vec{r}$ satisfies the given equation. So we compute*

$$\begin{aligned}x^2 + y^2 &= (t\cos t)^2 + (t\sin t)^2 \\ &= t^2\cos^2 t + t^2\sin^2 t \\ &= t^2(\cos^2 t + \sin^2 t) \\ &= t^2 \\ &= z^2.\end{aligned}$$