Recitation #18: Comparison Tests and Alternating Series

Warm up:

For each of the following, answer **True** or **False**, and explain why.

(a) If $a_n \geq 0$ and $\displaystyle\sum_{n=0}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=0}^{\infty} a_n^2$ converges.

(b) If $a_n, b_n \geq 0$ and both $\displaystyle\sum_{n=0}^{\infty} a_n$ and $\displaystyle\sum_{n=0}^{\infty} b_n$ converge, then $\displaystyle\sum_{n=0}^{\infty} a_n b_n$ converges.

Group work:

Problem 1 (a) *Why can we not use the Comparison test with* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ *to show that* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2-5}$ *converges?*

(b) *Adjust* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ *to show that* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2-5}$ *converges via the Comparison Test.*

(c) *Give a convergent series we can use in the Limit Comparison Test to show that* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2-5}$ *converges.*

Problem 2 *Determine if the following series converge or diverge.*

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^3+1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^4+1}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos^2 n}{n^3+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left[\left(1+\frac{1}{n}\right)^2 e^{-n}\right]$

Problem 3 *Determine if the following series absolutely converge, conditionally converge, or diverge.*

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n+3}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)^n}{(2n)^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} n^2 e^{\frac{-n^3}{3}}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 5}{3^n + 3^{-n}}$

(e) $\displaystyle\sum_{n=4}^{\infty} \frac{(-2)^n}{n}$

Problem 4 (a) *Find an upper bound for how close* $\displaystyle\sum_{k=0}^{4} \frac{(-1)^k k}{4^k}$ *is to the value of* $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k k}{4^k}$.

(b) *How many terms are needed to estimate* $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n!}$ *to within* 10^{-6}*?*