Recitation # 11: Improper Integrals and Differential Equations

Warm up:

True or False: It is possible for a region to be infinitely long but have a finite area.

Group work:

Problem 1 *Review of limits:*

(a) $\lim_{x\to-\infty}\left(3x^{-6}+e^{5x}+\dfrac{\sin x}{x^2+3}\right)$

(b) $\lim_{x\to\infty}\dfrac{x}{\sqrt{9x^2+4}}$

(c) $\lim_{x\to-\infty}\arctan x$

Problem 2 *Determine if the given integral converges or diverges. If it converges, find the value.*

$$\int_{-1}^{\infty}\frac{3}{2x+1}\,dx$$

Problem 3 (a) *Show that*

$$\frac{9}{2x^2+3x}=\frac{3}{x}-\frac{6}{2x+3}$$

(b) *Determine if the integral*

$$\int_{1}^{\infty}\frac{9}{2x^2+3x}\,dx$$

converges or diverges. If it converges, give the value that it converges to.

Problem 4 (a) *Show that*

$$\frac{6x-8}{x^3+4x} = \frac{2x+6}{x^2+4} - \frac{2}{x}$$

(b) *Determine if the integral*

$$\int_3^\infty \frac{6x-8}{x^3+4x}\,dx$$

converges or diverges. If it converges, give the value that it converges to.

Problem 5 *Which of the following is a solution to the differential equation* $y'' + 9y = 0$*?*

(a) $y = e^{3t} + e^{-3t}$

(b) $y = C(t^2 + t)$

(c) $y = \sin(3t) + 6$

(d) $y = 5\cos(3t) - 7\sin(3t)$

(e) $y = A\cos(3t) + B\sin(3t)$ *(where A and B are real numbers.)*

Problem 6 *Explain why the functions with the given graphs cannot be solutions of the differential equation* $y' = e^x(y-1)^2$*.*