Recitation #20: Properties of power series

Warm up:

Suppose that $\sum_{k=0}^{\infty} c_k(x+5)^k$ converges when $x=-9$ and diverges when $x=-1$. What can be said about the convergence and divergence of the following series?

(a) $\sum_{k=0}^{\infty} c_k$ (b) $\sum_{k=0}^{\infty} c_k(-5)^k$ (c) $\sum_{k=0}^{\infty} c_k(5)^k$

Group work:

Problem 1 *If the series* $\sum_{k=0}^{\infty} a_k(x-2)^k$ *has an interval of convergence of* $[-4,8)$, *determine the interval of convergence of the following series:*

(a) $\sum_{k=300}^{\infty} a_k(x-2)^k$ (b) $\sum_{k=0}^{\infty} a_k x^k$ (c) $\sum_{k=0}^{\infty} \left(a_k(x-2)^k + \left(\frac{1}{7}\right)^k x^k \right)$

Problem 2 *For each of the following, find the domain of* $f(x)$ *(i.e. find the interval of convergence).*

(a) $f(x) = \sum_{k=1}^{\infty} \frac{(3x-2)^k}{k \cdot 3^k}$

(b) $f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k^2+1}} x^k$

(c) $f(x) = \sum_{k=2}^{\infty} \frac{x^{3k+2}}{(\ln k)^k}$

Problem 3 *In each of the following, give a power series (with an interval of convergence) for the given function. Assume that we know* $\sum_{k=0}^{\infty} x^k = \dfrac{1}{1-x}$ *on* $(-1,1)$.

(a) $f(x) = \dfrac{3}{5x-2}$

(b) $f(x) = \dfrac{3x^4}{5x^3-2}$

Problem 4 *Consider* $f(x) = \sum_{k=0}^{\infty} \dfrac{2^k x^k}{(k+1)^3}$.

(a) *Write out* $p_3(x)$*, the cubic polynomial which is the first three terms of this power series.*

(b) *Find* $p_3'(x)$ *and* $f'(x)$ *and compare your answers.*

(c) *Find* $\int p_3(x)\,dx$ *and* $\int f(x)\,dx$ *and compare your answers.*

Problem 5 *Give a power series (with interval of convergence) for the given functions.*

(a) $f(x) = \dfrac{1}{1+x^2}$

(b) $f(x) = \tan^{-1}(x)$

(c) $f(x) = \tan^{-1}(3x^2)$
