Recitation #22: Working with Taylor series

Warm up:

True or False: To approximate $\frac{\pi}{3}$, one could substitute $x=\sqrt{3}$ into the Maclaurin series for $\tan^{-1}x$?

Group work:

Problem 1 Use power series to evaluate the limit

$$\lim_{x \to 0} \frac{\ln(1+x^2)}{1 - \cos x}$$

Problem 2 Given that

$$f(t) = \int_0^t x^2 \tan^{-1}(x^4) \, dx$$

approximate $f\left(\frac{1}{3}\right)$ with the first four non-zero terms of a power series. Estimate how close this approximation is.

Problem 3 Identify the function represented by the power series

$$\sum_{k=0}^{\infty} \frac{k(k-1)x^k}{7^k}$$

Problem 4 Use power series to determine a (series) solution to the initial value problem

$$y'' - xy' + y = 0$$
 $y(0) = 1$ $y'(0) = 0$