Recitation #13: An overview of sequences and series

Warm up:

For each of the following sequences, list the first four terms (start each with n=1).

(a)
$$a_{n+1} = \frac{1}{2} \left(a_n + \frac{2}{a_n} \right), \ a_1 = 1.$$

(b)
$$a_n = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{(2n)! \cdot 2n!}$$
, Recall that $n! = 1 \cdot 2 \cdot 3 \cdot 4 \dots (n-1) \cdot n$.

Group work:

Problem 1 Give an explicit formula for each of the following sequences:

(a)
$$\frac{2}{3}, \frac{-2}{7}, \frac{2}{11}, \frac{-2}{15}, \dots$$

(b)
$$-2, 6, -24, 120, -720, \dots$$

(c)
$$2, 8, 26, 80, 242, \dots$$

Problem 2 For the sequence $a_k = (2 - k)^k$

- (a) calculate and list a_0 , a_1 , a_2 , a_3 , and a_4 .
- (b) Starting with k = 0, calculate and list $S_0 = \sum_{k=0}^{0} a_k$, $S_1 = \sum_{k=0}^{1} a_k$, $S_2 = \sum_{k=0}^{2} a_k$, $S_3 = \sum_{k=0}^{3} a_k$, and $S_4 = \sum_{k=0}^{4} a_k$. Write S_n in summation form and write S_{∞} in summation form.

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Problem 3 For each of the following, write the series in the form $\sum_{k=0}^{\infty} a_k$.

- (a) $0.2 + 0.06 + 0.018 + 0.0054 + \dots$
- (b) $\frac{1}{2} + \frac{1}{3^2} \frac{1}{3} \frac{1}{4^2} + \frac{1}{4} + \frac{1}{5^2} \frac{1}{5} \frac{1}{6^2} \dots$

Problem 4 Reindex the series

$$\sum_{k=0}^{\infty} \frac{5}{(k+2)(k+1)}$$

in the form $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=-4}^{\infty} c_k$.

Problem 5 If
$$\sum_{k=0}^{\infty} a_k = 6$$
 and $a_n = \frac{3}{2^n}$, what is $\sum_{k=4}^{\infty} a_k$?