Section 7.8: Improper Integrals

Warm up:

True or False: It is possible for a region to be infinitely long but have a finite area.

Group work:

Problem 1 *Review of limits:*

(a) $\displaystyle\lim_{x\to-\infty}\left(3x^{-6}+e^{5x}+\frac{\sin x}{x^2+3}\right)$

(b) $\displaystyle\lim_{x\to\infty}\frac{x}{\sqrt{9x^2+4}}$

(c) $\displaystyle\lim_{x\to-\infty}\arctan x$

Problem 2 *Determine if the given integral converges or diverges. If it converges, find the value.*

$$\int_{-1}^{\infty}\frac{3}{2x+1}\,dx$$

Problem 3 (a) *Show that*

$$\frac{9}{2x^2+3x}=\frac{3}{x}-\frac{6}{2x+3}$$

(b) *Determine if the integral*

$$\int_{1}^{\infty}\frac{9}{2x^2+3x}\,dx$$

converges or diverges. If it converges, give the value that it converges to.

Learning outcomes:

Problem 4 (a) *Show that*

$$\frac{6x-8}{x^3+4x} = \frac{2x+6}{x^2+4} - \frac{2}{x}$$

(b) *Determine if the integral*

$$\int_3^\infty \frac{6x-8}{x^3+4x}\,dx$$

converges or diverges. If it converges, give the value that it converges to.

Problem 5 *Given that* $\dfrac{37}{(2x-1)(x^2+9)} = \dfrac{4}{2x-1} - \dfrac{2x+1}{x^2+9}$, *evaluate:*

$$\int_3^\infty \frac{37}{(2x-1)(x^2+9)}\,dx$$