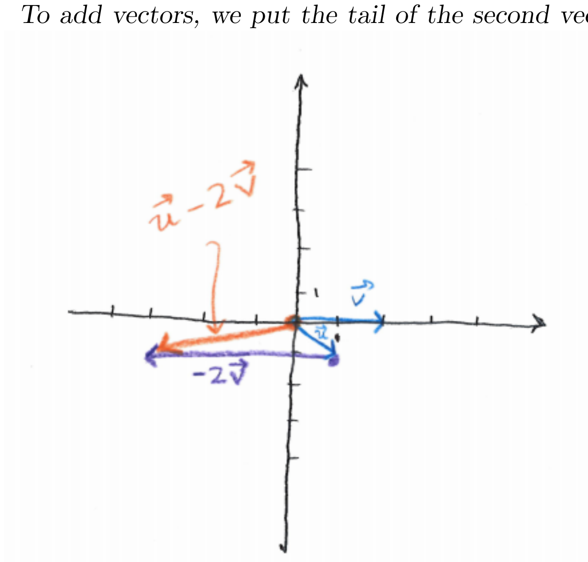
Section 12.1: Vectors in Two Dimensions

Warm up:

Problem 1 *Sketch the vectors* $\mathbf{u} = \langle 1, -1 \rangle$ *and* $\mathbf{v} = \langle 2, 0 \rangle$*. Now using your sketch of these vectors, sketch* $\mathbf{u} - 2\mathbf{v}$*.*

Solution: *To add vectors, we put the tail of the second vector on the head of the first.*



Group work:

Problem 2 *Suppose that* $\mathbf{u} = \langle 5, -1 \rangle$ *and* $\mathbf{v} = \langle 2, 3 \rangle$*. Find the following quantities:*

(a) $-\mathbf{v}$

Learning outcomes:

(b) $3\mathbf{u} - 4\mathbf{v}$

(c) $|\mathbf{u}|$

Solution: (a) $-\mathbf{v} = \langle -2, -3 \rangle$

(b) $3\mathbf{u} - 4\mathbf{v} = \langle 15, -3 \rangle - \langle 8, 12 \rangle = \langle 7, -15 \rangle$.

(c) $|\mathbf{u}| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$.

Problem 3 *Suppose that $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ in a 2-dimensional vector space. Find the following:*

(a) *A unit vector in the same direction of $\mathbf{u}$.*

(b) *All unit vectors parallel to $\mathbf{u}$. (How does differ from part (a)?)*

(c) *Two vectors parallel to $\mathbf{u}$ with length 10.*

(d) *Two non-zero vectors perpendicular to $\mathbf{u}$.*

Solution: (a) $|\mathbf{u}| = \sqrt{3^2 + (-4)^2} = 5$. *A unit vector in the same direction is* $\frac{\mathbf{u}}{|\mathbf{u}|} = \langle \frac{3}{5}, \frac{-4}{5} \rangle$.

(b) *Parallel unit vectors are* $\pm \frac{\mathbf{u}}{|\mathbf{u}|}$, *which are* $\langle \frac{3}{5}, \frac{-4}{5} \rangle$ *and* $\langle \frac{-3}{5}, \frac{4}{5} \rangle$. *Note that parallel vectors include vectors in the opposite direction.*

(c) *Since $\mathbf{u}$ has length 5, two parallel vectors of length 10 are $\pm 2\mathbf{u}$, which are* $\langle 6, -8 \rangle$ *and* $\langle -6, 8 \rangle$.

(d) *In 2 dimensions, we can find a perpendicular vector using what we know about finding a perpedicular line. In particular, we know that two lines are perpendicular if the slope of line 1 is equal to the negative reciprical of the slope of line 2. That is,* $m_1 = \frac{-1}{m_2}$. *To find the slope of our vector* $\mathbf{u}$, *we find the slope between the head of the vector at (3,-4) and the tail of the vector at (0,0). The slope of $\mathbf{u}$ is* $\frac{-4-0}{3-0} = \frac{-4}{3}$ *Therefore, the slope of a perpendicular vector will be* $\frac{3}{4}$. *One vector with this slope is* $\boxed{\mathbf{u_1} = 4\mathbf{i} + 3\mathbf{j}}$ *and another is* $\boxed{\mathbf{u_2} = -4\mathbf{i} - 3\mathbf{j}}$. *Note: In more that 2-dimensions, this technique won't work.*
