Recitation #15: Infinite Series, Divergence and Integral Tests

Problem 1 *Suppose* $\{a_n\}_{n\geq 1}$ *is a sequence and* $\displaystyle\sum_{n=1}^{\infty} a_n$ *converges to* $L > 0$. *Let* $s_n = \displaystyle\sum_{k=1}^{n} a_k$. *Circle all of the statements that* MUST *be true.*

A. $\lim_{n\to\infty} a_n = L$ B. $\lim_{n\to\infty} a_n = 0$ C. $\lim_{n\to\infty} s_n = 0$

D. $\lim_{n\to\infty} s_n = L$ E. $\displaystyle\sum_{n=1}^{\infty} s_n$ *MUST diverge.* F. $\displaystyle\sum_{n=1}^{\infty}(a_n + 1) = L + 1$

G. *The divergence test tells us* $\displaystyle\sum_{n=1}^{\infty} a_n$ *converges to* L.

Problem 2 *For each of the following, answer* **True** *or* **False**, *and explain why.*

(a) *If* $\displaystyle\sum_{n=0}^{\infty} a_n$ *converges, then* $\displaystyle\sum_{n=0}^{\infty}(a_n + 0.001)$ *converges.*

(b) *Since* $\displaystyle\int_1^{\infty} x\sin(\pi x)\,dx$ *diverges then, by the Integral Test,* $\displaystyle\sum_{n=0}^{\infty} n\sin(\pi n)$ *diverges.*

(c) *Since* $\displaystyle\int_1^{\infty} \frac{1}{x^2}\,dx = 1$ *then, by the Integral Test,* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} = 1$.

Problem 3 *Assume* $\displaystyle\sum_{k=0}^{\infty} a_k = L$ *and* $b_k = 8$ *for all* k.

(a) *What is* $\lim_{k\to\infty}(a_k + b_k)$?

(b) *What is* $\displaystyle\lim_{k\to\infty}\sum_{n=0}^{k}(a_n + b_n)$?

(c) *What is* $\displaystyle\lim_{k\to\infty}\sum_{n=0}^{k}(a_{n+1} - a_n)$?

Problem 4 *Determine if the following series converge or diverge. If they converge, find the sum.*

(a) $\displaystyle\sum_{k=0}^{99} 2^k + \sum_{k=100}^{\infty} \frac{1}{2^k}$

(b) $\displaystyle\sum_{k=4}^{\infty} \frac{5 \cdot 4^{k+3}}{7^{k-2}}$

(c) $\displaystyle\sum_{k=0}^{\infty} e^{5-2k}$

(d) $\displaystyle\sum_{i=1}^{\infty} \left(\frac{2}{i^2+2i}\right)$ *Hint:* $\dfrac{2}{i^2+2i} = \dfrac{1}{i} - \dfrac{1}{i+2}$ *by partial fractions*

Problem 5 *Determine if the following series converge or diverge.*

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^2+1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

Problem 6 *For a sequence* $\{a_n\}_{n\geq 1}$ *let* $s_n = \displaystyle\sum_{k=1}^{n} a_k$ *denote its sequence of partial sums. Now, suppose that* $\{a_n\}_{n\geq 1}$ *is a sequence such that* $s_n = \dfrac{4n^2+9}{1-2n}$.

(a) *Find* $a_1 + a_2 + a_3$.

(b) *Find* $a_8 + a_9 + a_{10}$.

(c) *Determine whether* $\displaystyle\sum_{k=1}^{\infty} a_k$ *converges or diverges. If it converges, find the value to which it converges, or state that there is not enough information to determine this.*

(d) *Determine whether* $\displaystyle\sum_{k=1}^{\infty} s_k$ *converges or diverges. If it converges, find the value to which it converges, or state that there is not enough information to determine this.*
