

Recitation #21: Taylor series

Warm up:

Find the Taylor series for:

(a) $27x^2 - 3x + 17$ about $a = 1$.

(b) $\sin(2x)$ about $a = \frac{\pi}{8}$.

Group work:

Problem 1 Find a power series (and interval of convergence) for each of the following functions

(a) $f(x) = x^3 \sin(x^5)$

(c) $f(x) = \frac{1}{(3 - 5x^2)^4}$

(b) $f(x) = \frac{1}{(1 + x)^4}$

(d) $f(x) = \sin^{-1}(x^5)$

Problem 2 Find a function (closed expression) for the following series and the interval on which the function and the series are equal.

$$x + x^4 + \frac{1}{2}x^7 + \frac{1}{6}x^{10} + \frac{1}{24}x^{13} + \dots$$

Problem 3 Compute the sum of the following series (Hint: You should use Taylor series.)

(a) $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \dots$

(b) $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \dots$