

Section 10.3: Taylor series

Warm up:

Find the Taylor series for:

(a) $27x^2 - 3x + 17$ centered at $a = 0$.

(b) $27x^2 - 3x + 17$ centered at $a = 1$.

Group work:

Problem 1 Find a Maclaurin series (and interval of convergence) for

$$f(x) = x^3 \sin(x^5)$$

Problem 2 Find the first four non-zero terms of the Maclaurin Series for

$$xe^{x^2} + \cos(x^3)$$

Problem 3 Find a function (closed expression) for the following series and the interval on which the function and the series are equal.

$$x + x^4 + \frac{1}{2}x^7 + \frac{1}{6}x^{10} + \frac{1}{24}x^{13} + \dots$$

Problem 4 Compute the sum of the following series (Hint: You should use Taylor series.)

(a) $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \dots$

(b) $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \dots$

Problem 5 Find the Taylor Series for $\sin(2x)$ about $a = \frac{\pi}{8}$.

Hint: Recall from a previous recitation that

$$p_3(x) = \frac{\sqrt{2}}{2} + \sqrt{2} \left(x - \frac{\pi}{8}\right) - \sqrt{2} \left(x - \frac{\pi}{8}\right)^2 - \frac{2\sqrt{2}}{3} \left(x - \frac{\pi}{8}\right)^3$$
