Recitation # 2 Regions Between Curves

Group work:

Problem 1 Consider the region bounded by the curves $y = 2x^2 - 7x + 8$ and $y = x^2 - 4x + 18$.

- (a) Draw a sketch of the graphs.
- (b) Find the area between these curves.
- (c) Find the area of the region bounded by the curves $x = 2y^2 7y + 8$ and $x = y^2 4y + 18$.
- (d) Find the area of the region bounded by the curves
 - (i) $y = 2x^2 7x$ and $y = x^2 4x + 10$.
 - (ii) $y = 2x^2 7x 30$ and $y = x^2 4x 20$.
- (e) Find the area of the region bounded by the curves $y = 2x^2 7x + 8$, $y = x^2 4x + 18$, x = 1, and x = 6.

Instructor Notes: Have the students do (a) and (b), and then have a group present. Afterwards, discuss the variations (c) and (d) as a whole class.

Problem 2 Set up a single integral that computes the area of the region bounded by the curves (and be sure to draw a sketch of the graphs).

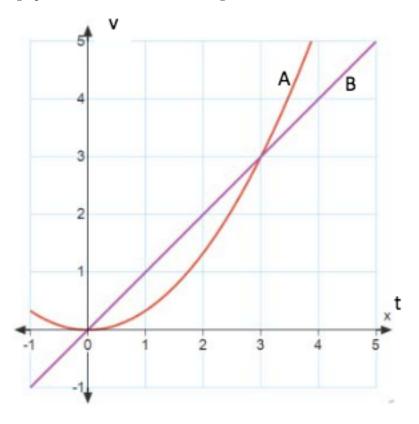
- (a) $x = y^2$ and y = 6 x
- (b) $y = x^2 + 6$ and y = 3x + 10

Instructor Notes: Split (a) and (b) among the groups. Note that (a) should be set up in terms of y while (b) should be set up in terms of x. Have groups present their solutions. Discuss with the students factors to consider when deciding whether to integrate in terms of x or y.

Problem 3 Two runners (A and B) run in a race in which the winner runs the farthest in 4 minutes. The runners' respective velocities are

$$v_A(t) = \frac{1}{3}t^2 \qquad v_B(t) = t$$

The graphs of the runners' velocities is given below.



- (a) Who is running the fastest 2 minutes into the race?
- (b) Who is winning the race 2 minutes into the race (and by how much)?
- (c) What special event occurs 3 minutes into the race?
- (d) Who wins the race (and by how much)?

Instructor Notes: Do this problem as a class discussion. The main point is to have students identify how each of the questions relates to the graph of the velocity functions.