

## Recitation #27: Vectors in Two and Three Dimensions and Dot Products

### Warm up:

**Problem 1** Sketch the vectors  $\mathbf{u} = \langle 1, -1 \rangle$  and  $\mathbf{v} = \langle 2, 0 \rangle$ . Now using your sketch of these vectors, sketch  $\mathbf{u} - 2\mathbf{v}$ .

---

**Problem 2** If  $\vec{u} = \hat{i} - 2\hat{j}$  and  $\vec{v} = 3\hat{i} + 4\hat{k}$ , find  $\vec{u} \cdot \vec{v}$ .

---

### Group work:

**Problem 3** Suppose that  $\mathbf{u} = \langle 5, -1 \rangle$  and  $\mathbf{v} = \langle 2, 3 \rangle$ . Find the following quantities:

- (a)  $-\mathbf{v}$
  - (b)  $3\mathbf{u} - 4\mathbf{v}$
  - (c)  $|\mathbf{u}|$
- 

**Problem 4** Suppose that  $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$  in a 2-dimensional vector space. Find the following:

- (a) A unit vector in the same direction of  $\mathbf{u}$ .
  - (b) All unit vectors parallel to  $\mathbf{u}$ . (How does differ from part (a)?)
  - (c) Two vector parallel to  $\mathbf{u}$  with length 10.
  - (d) Two non-zero vectors perpendicular to  $\mathbf{u}$ .
-

Recitation #27: Vectors in Two and Three Dimensions and Dot Products

**Problem 5** Solve the following problems:

- (a) Which of the points  $(6, 2, 3)$ ,  $(-5, -1, 4)$ , and  $(0, 3, 8)$  is closest to the  $xz$ -plane? Which point lies on the  $yz$ -plane?
- (b) Write an equation of the circle of radius 2 centered at  $(-3, 4, 1)$  that lies in a plane parallel to the  $xy$ -plane.
- (c) Describe the sphere  $x^2 + y^2 + z^2 + 6x - 14y - 2z = 5$  (ie, find its center and radius).
- (d) Find a vector whose magnitude is 311 and is in the same direction as the vector  $\langle 3, -6, 7 \rangle$ .

**Problem 6** Find a vector (in the  $xy$ -plane) with length 4 that makes a  $\frac{\pi}{3}$  radian angle with the vector  $\langle 3, 4 \rangle$ .

**Problem 7** Answer the following questions about  $\text{proj}_v u$ .

- (a) Is  $\text{proj}_v u$  a vector of the form  $c\vec{v}$  or  $c\vec{u}$  (where  $c$  is a real number)? ie, is  $\text{proj}_v u$  parallel to  $\vec{u}$  or  $\vec{v}$ ?
- (b) If  $\vec{u} = 5\hat{i} + 6\hat{j} - 3\hat{k}$  and  $\vec{v} = 2\hat{i} - 4\hat{j} + 4\hat{k}$ , find  $\text{proj}_v u$ .
- (c) For  $\vec{u}$  and  $\vec{v}$  from part (b), write  $\vec{u}$  as the sum of two perpendicular vectors, one of which is parallel to  $\vec{v}$ . Verify that the other vector is perpendicular to  $\vec{v}$ .

## Challenge Problem

**Problem 8** Suppose that the deli at the Tiny Sparrow grocery store sells roast beef for \$9 per pound, turkey for \$4 per pound, salami for \$5 per pound, and ham for \$7 per pound. For lunches this week, Sam the sandwich maker buys 1.5 pounds of roast beef, 2 pounds of turkey, no salami, and half a pound of ham. How can you use a dot product to compute Sam's total bill from the deli?