Section 9.5: The Divergence, Integral, Ratio and Root Tests - Solutions

Warm up:

For each of the following, answer **True** or **False**, and explain why.

(a) If $a_n \geq 0$ and $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty} a_n^2$ converges.

(b) If $a_n, b_n \geq 0$ and both $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ converge, then $\sum_{n=0}^{\infty} a_n b_n$ converges.

Problem 1 (a) *Why can we not use the Comparison test with* $\sum_{k=1}^{\infty} \frac{1}{k^2}$ *to show that* $\sum_{k=1}^{\infty} \frac{1}{k^2-5}$ *converges?*

(b) *Adjust* $\sum_{k=1}^{\infty} \frac{1}{k^2}$ *to show that* $\sum_{k=1}^{\infty} \frac{1}{k^2-5}$ *converges via the Comparison Test.*

(c) *Give a convergent series we can use in the Limit Comparison Test to show that* $\sum_{k=1}^{\infty} \frac{1}{k^2-5}$ *converges.*

Group work:

Problem 2 *Determine if the following series converge or diverge.*

(a) $\sum_{n=1}^{\infty} \frac{(7n+1)^2 \cdot 2^n}{5^n}$

Learning outcomes:

(b) $\displaystyle\sum_{n=1}^{\infty} a_n$, where $a_{n+1} = \dfrac{2n+5}{3n-1} \cdot a_n$ and $a_1 = 1$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^2+1}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(e) $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^3}{(3k)!}$

Problem 3 *Determine if the following series converge or diverge.*

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^3+1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^4+1}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos^2 n}{n^3+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left[\left(1+\frac{1}{n}\right)^2 e^{-n}\right]$