Conceptual Problems Involving Partial Sums

The following questions provide practice with concepts involving partial sums. These are important and should be studied and understood in preparation for the second midterm.

I: True or False

Directions: CIRCLE ALL of the statements that *MUST* be TRUE. No explanation is necessary. Note that there may be several statements that are true for each question!

Problem 1: Suppose $\{a_n\}_{n\geq 1}$ is a sequence and $\displaystyle\sum_{n=1}^{\infty} a_n$ converges to $L > 0$. Let $s_n = \displaystyle\sum_{k=1}^{n} a_k$.

A. $\displaystyle\lim_{n\to\infty} a_n = L$

B. $\displaystyle\lim_{n\to\infty} a_n = 0$

C. $\displaystyle\lim_{n\to\infty} s_n = 0$

D. $\displaystyle\lim_{n\to\infty} s_n = L$

E. $\displaystyle\sum_{n=1}^{\infty} s_n$ MUST diverge.

F. $\displaystyle\sum_{n=1}^{\infty}(a_n + 1) = L + 1$

G. The divergence test tells us $\displaystyle\sum_{n=1}^{\infty} a_n$ converges to L.

Problem 2: Suppose that $\{a_n\}_{n\geq 1}$ is a *decreasing* sequence. Let $s_n = \displaystyle\sum_{k=1}^{n} a_k$ and suppose $\displaystyle\lim_{n\to\infty} s_n$ does not exist.

A. $\displaystyle\lim_{n\to\infty} a_n$ does not exist.

B. $\displaystyle\sum_{k=1}^{\infty} a_k$ could converge.

C. $\displaystyle\sum_{n=1}^{\infty} s_n$ MUST diverge.

D. $\{s_n\}$ MUST be monotonic.

E. $\{s_n\}$ MUST be bounded.

F. $\displaystyle\lim_{n\to\infty} s_n = -\infty$

G. The divergence test applied to $\displaystyle\sum_{k=1}^{\infty} a_k$ would guarantee that $\displaystyle\sum_{k=1}^{\infty} a_k$ diverges.

Problem 3: Suppose that $\{a_n\}_{n\geq 1}$ and $a_n > 0$ for all $n \geq 1$. Let $s_n = \displaystyle\sum_{k=1}^{n} a_k$ and suppose $\displaystyle\lim_{n\to\infty} s_n = L$.

A. $\displaystyle\sum_{k=1}^{\infty} a_k = L$

B. $\displaystyle\lim_{n\to\infty} a_n = 0$

C. $\{s_n\}$ MUST be monotonic.

D. $\{s_n\}$ MUST be bounded.

E. $\displaystyle\sum_{n=1}^{\infty}(a_n - L) = 0$

F. $\displaystyle\sum_{n=1}^{\infty} s_n$ MUST diverge.

G. The divergence test applied to $\sum_{k=1}^{\infty} a_k$ would guarantee that $\sum_{k=1}^{\infty} a_k$ converges.

II: Short Answer

Directions: Provide a brief response to the following questions.

Problem 4 (Exploring the Relationship Between $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} s_k$)

For a sequence $\{a_n\}_{n\geq 1}$ let $s_n = \sum_{k=1}^{n} a_k$ denote its sequence of partial sums.

a) Given that $\sum_{k=1}^{\infty} a_k$ converges, what can be said about $\sum_{k=1}^{\infty} s_k$?

b) Given that $\sum_{k=1}^{\infty} a_k$ diverges, what can be said about $\sum_{k=1}^{\infty} s_k$?

c) Given that $\sum_{k=1}^{\infty} s_k$ converges, what can be said about $\sum_{k=1}^{\infty} a_k$?

d) Given that $\sum_{k=1}^{\infty} s_k$ diverges, what can be said about $\sum_{k=1}^{\infty} a_k$?

Problem 5 For a sequence $\{a_n\}_{n\geq 1}$ let $s_n = \sum_{k=1}^{n} a_k$ denote its sequence of partial sums. Now, suppose that $\{a_n\}_{n\geq 1}$ is a sequence such that $s_n = \dfrac{2n-1}{3n+1}$.

a) Find $a_1 + a_2 + a_3 + a_4$.

b) Find $a_5 + a_6$.

c) Determine whether $\lim_{n\to\infty} a_n$ exists. If it does, find its value.

d) Determine whether $\lim_{n\to\infty} s_n$ exists. If it does, find its value.

e) Determine whether $\sum_{k=1}^{\infty} a_k$ converges or diverges. If it converges, find the value to which it converges, or state that there is not enough information to determine this.

f) Determine whether $\sum_{k=1}^{\infty} s_k$ converges or diverges. If it converges, find the value to which it converges, or state that there is not enough information to determine this.

Problem 6 For a sequence $\{a_n\}_{n\geq 1}$ let $s_n = \sum_{k=1}^{n} a_k$ denote its sequence of partial sums. Now, suppose that $\{a_n\}_{n\geq 1}$ is a sequence such that $s_n = \dfrac{4n^2+9}{1-2n}$.

a) Find $a_1 + a_2 + a_3$.

b) Find $a_8 + a_9 + a_{10}$.

c) Determine whether $\sum_{k=1}^{\infty} a_k$ converges or diverges. If it converges, find the value to which it converges, or state that there is not enough information to determine this.

d) Determine whether $\sum_{k=1}^{\infty} s_k$ converges or diverges. If it converges, find the value to which it converges, or state that there is not enough information to determine this.