Recitation #21: Taylor series - Instructor Notes

Warm up:

Find the Taylor series for:

(a) $27x^2 - 3x + 17$ about $a = 1$.

(b) $\sin(2x)$ about $a = \dfrac{\pi}{8}$.

Instructor Notes: Here, they need to compute the Taylor series by computing derivatives and recognizing patterns.

Part (a) is an opportunity to show the students that a polynomial is already a Maclaurin series. Use derivatives to figure out the Taylor series about $a = 1$, and then show them that the answer simplifies back to the original problem.

Part (b) is a continuation from the previous recitation where they already found the approximating polynomial. The students may have a problem finding the pattern in the derivatives (especially with the alternating part).

Group work:

Problem 1 *Find a power series (and interval of convergence) for each of the following functions*

(a) $f(x) = x^3 \sin(x^5)$

(b) $f(x) = \dfrac{1}{(1+x)^4}$

(c) $f(x) = \dfrac{1}{(3-5x^2)^4}$

(d) $f(x) = \sin^{-1}(x^5)$

Instructor Notes: Students should use the known Maclaurin series in various ways. You might want to give the hint in part (d) that $\dfrac{d}{dx}\sin^{-1}(x) = \dfrac{1}{\sqrt{1-x^2}}$.

Problem 2 *Find a function (closed expression) for the following series and the interval on which the function and the series are equal.*

$$x + x^4 + \frac{1}{2}x^7 + \frac{1}{6}x^{10} + \frac{1}{24}x^{13} + \dots$$

Instructor Notes: The students need to rewrite $f(x)$ in summation notation (factoring out an x) and seeing the series for xe^{x^3}.

Problem 3 *Compute the sum of the following series (Hint: You should use Taylor series.)*

(a) $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \dots$

(b) $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \dots$

Instructor Notes: The goal here is for students to realize that Taylor series gives them a tool for finding the exact sum of a series.