Section 8.1: Basic Ideas of Differential Equations

Group work:

Problem 1 *Which of the following is a solution to the differential equation*

$$y'' + 9y = 0?$$

(a) $y = e^{3t} + e^{-3t}$

(b) $y = C(t^2 + t)$

(c) $y = \sin(3t) + 6$

(d) $y = 5\cos(3t) - 7\sin(3t)$

(e) $y = A\cos(3t) + B\sin(3t)$ *(where A and B are real numbers.)*

Solution: (a)

$$y = e^{3t} + e^{-3t} \qquad y' = 3e^{3t} - 3e^{-3t} \qquad y'' = 9e^{3t} + 9e^{-3t}$$

So,

$$\begin{aligned} y'' + 9y &= (9e^{3t} + 9e^{-3t}) + 9 \cdot (e^{3t} + e^{-3t}) \\ &= 18e^{3t} + 18e^{-3t} \neq 0. \end{aligned}$$

*Therefore, this is **not** a solution to $y'' + 9y = 0$.*

(b)

$$y = C(t^2 + t) \qquad y' = C(2t + 1) \qquad y'' = 2C$$

So,

$$\begin{aligned} y'' + 9y &= 2C + 9C(t^2 + t) \\ &= 9Ct^2 + 9Ct + 2C \neq 0 \text{ if } C \neq 0. \end{aligned}$$

*Therefore, this is **not** a solution to $y'' + 9y = 0$ unless $C = 0$, in which case we get the trivial solution.*

Learning outcomes:

(c)

$$y = \sin(3t) + 6 \qquad y' = 3\cos(3t) \qquad y'' = -9\sin(3t)$$

So,

$$\begin{aligned} y'' + 9y &= -9\sin(3t) + 9(\sin(3t) + 6) \\ &= 54 \neq 0. \end{aligned}$$

*Therefore, this is **not** a solution to* $y'' + 9y = 0$.

(d)

$$y = 5\cos(3t) - 7\sin(3t) \qquad y' = -15\sin(3t) - 21\cos(3t) \qquad y'' = -45\cos(3t) + 63\sin(3t)$$

So,

$$\begin{aligned} y'' + 9y &= -45\cos(3t) + 63\sin(3t) + 9(5\cos(3t) - 7\sin(3t)) \\ &= -45\cos(3t) + 63\sin(3t) + 45\cos(3t) - 63\sin(3t) = 0. \end{aligned}$$

*Therefore, this **is** a solution to* $y'' + 9y = 0$.

(e)

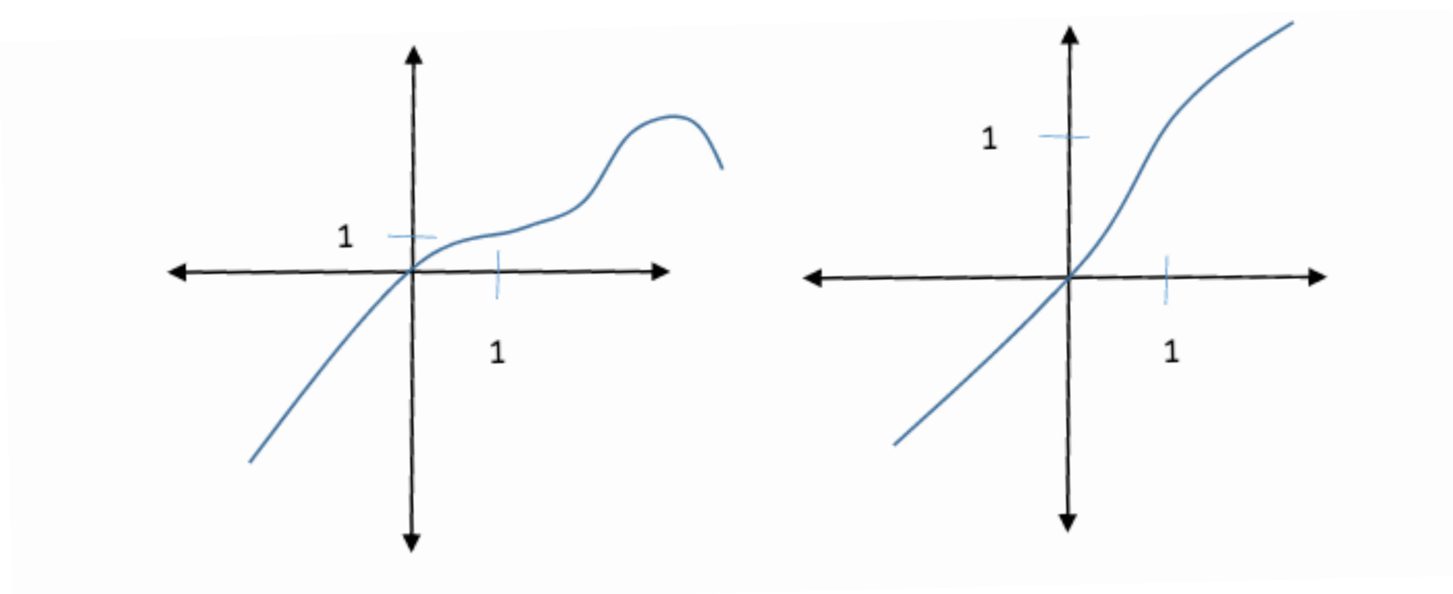
$$y = A\cos(3t) + B\sin(3t) \qquad y' = -3A\sin(3t) + 3B\cos(3t) \qquad y'' = -9A\cos(3t) - 9B\sin(3t)$$

So,

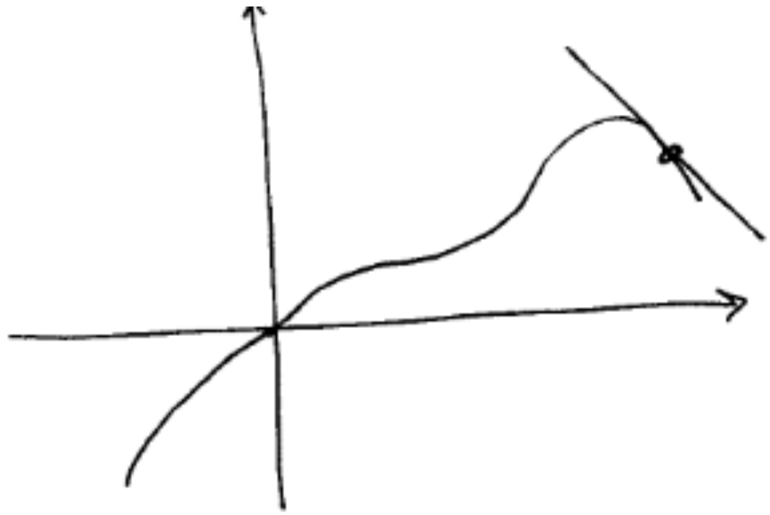
$$\begin{aligned} y'' + 9y &= -9A\cos(3t) - 9B\sin(3t) + 9(A\cos(3t) + B\sin(3t)) \\ &= 0. \end{aligned}$$

*Therefore, this **is** a solution to* $y'' + 9y = 0$.

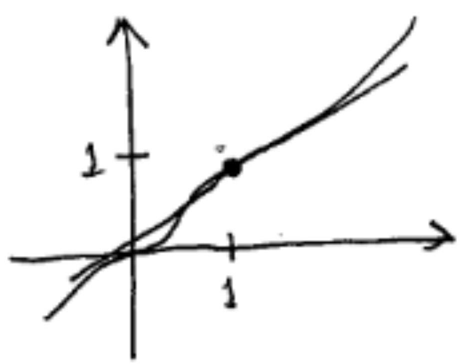
Problem 2 *Explain why the functions with the given graphs cannot be solutions of the differential equation* $y' = e^x(y-1)^2$.



Solution: *Since $y' = e^x(y-1)^2$, the derivative of y is always nonnegative. Thus, the first graph cannot satisfy this differential equation since it has a tangent line with a negative slope.*



The second graph cannot satisfy the differential equation since the slope of the tangent line at $x = 1$ is positive



but $\left[\dfrac{dy}{dx}\right]_{x=0} = e^0(1-1) = 0.$