Recitation #18: Comparison Tests and Alternating Series

Warm up:

For each of the following, answer **True** or **False**, and explain why.

(a) If $a_n \geq 0$ and $\displaystyle\sum_{n=0}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=0}^{\infty} a_n^2$ converges.

(b) If $a_n, b_n \geq 0$ and both $\displaystyle\sum_{n=0}^{\infty} a_n$ and $\displaystyle\sum_{n=0}^{\infty} b_n$ converge, then $\displaystyle\sum_{n=0}^{\infty} a_n b_n$ converges.

Group work:

Problem 1 (a) *Why can we not use the Comparison test with* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ *to show that* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2 - 5}$ *converges?*

(b) *Adjust* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ *to show that* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2 - 5}$ *converges via the Comparison Test.*

(c) *Give a convergent series we can use in the Limit Comparison Test to show that* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2 - 5}$ *converges.*

Problem 2 *Determine if the following series converge or diverge.*

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2 + 2n + 1}{3n^3 + 1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2 + 2n + 1}{3n^4 + 1}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos^2 n}{n^3 + 1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left[\left(1 + \frac{1}{n}\right)^2 e^{-n} \right]$