Recitation #21: Taylor series

Warm up:

Find the Taylor series for:

(a) $27x^2 - 3x + 17$ centered at $a = 0$.

(b) $27x^2 - 3x + 17$ centered at $a = 1$.

Group work:

Problem 1 *Find a Maclaurin series (and interval of convergence) for*

$$f(x) = x^3 \sin(x^5)$$

Problem 2 *Find the first four non-zero terms of the Maclaurin Series for*

$$xe^{x^2} + \cos(x^3)$$

Problem 3 *Find a function (closed expression) for the following series and the interval on which the function and the series are equal.*

$$x + x^4 + \frac{1}{2}x^7 + \frac{1}{6}x^{10} + \frac{1}{24}x^{13} + \dots$$

Problem 4 *Compute the sum of the following series (Hint: You should use Taylor series.)*

(a) $1 - \ln 2 + \dfrac{(\ln 2)^2}{2!} - \dfrac{(\ln 2)^3}{3!} + \dots$

(b) $3 + \dfrac{9}{2!} + \dfrac{27}{3!} + \dfrac{81}{4!} + \dots$

Problem 5 *Find the Taylor Series for* $\sin(2x)$ *about* $a = \dfrac{\pi}{8}$.
Hint: Recall from a previous recitation that
$p_3(x) = \dfrac{\sqrt{2}}{2} + \sqrt{2}\left(x - \dfrac{\pi}{8}\right) - \sqrt{2}\left(x - \dfrac{\pi}{8}\right)^2 - \dfrac{2\sqrt{2}}{3}\left(x - \dfrac{\pi}{8}\right)^3$