Worksheet #1 Answers

Detailed Solutions follow this sheet!

I. a) $8(2x-7)^3$

b) $\frac{1}{4}e^{\frac{1}{4}x}$

c) $28x^3 - \frac{3}{5}x^{-\frac{4}{5}} - \frac{4}{5}x^{-3}$

d) $\dfrac{2-\sin x}{2x+\cos x}$

e) $2e^{-x} - 2xe^{-x}$

f) $\dfrac{3(4-x)^{1/2}\sec^2 3x + \frac{1}{2}(4-x)^{-1/2}\tan 3x}{4-x}$

g) $-4e^{4x}\csc(e^{4x})\cot(e^{4x})$

h) $3[\ln(4x^3-2x)]^2 \cdot \dfrac{12x^2-2}{4x^3-2x}$

i) $2x^{-1/2}e^{4x^{1/2}}$

j) $4e^{x\sin x}(\sin x + x\cos x)$

k) $54x^8 + \frac{1}{2}x^{-5} - \frac{4}{3}(2x-1)^{-4/3}$

l) $-24x(3x^2-1)^{-3}$

II. a) $\frac{3}{5}x^5 - \frac{3}{5}x^{5/3} - \frac{14}{6}x^{6/7} + C$

b) Cannot Integrate

c) 1

d) $-3e^{-\frac{1}{3}x} + C$

e) Cannot Integrate yet

f) $x^2 - \frac{3}{2}\ln|x| + C$

g) $\frac{1}{4}\sec 4x + 15\tan\frac{1}{5}x + C$

h) $\frac{5}{6}$

i) $\frac{2}{3}e^{3/2} - \frac{2}{3}$

j) $-\frac{1}{3}\cot 3x + C$.

k) Cannot integrate!

l) $-\frac{2}{9}x^{-1} + C$

III. ⟨See solutions sheet⟩

Worksheet # 1 Solutions

I. a) $y = (2x-7)^4$

$y' = 4(2x-7)^3(2x-7)'$

$y' = 4(2x-7)^3 \cdot 2$

$\boxed{y' = 8(2x-7)^3}$ ← It's ok just to write this! The previous steps are meant to make sure everyone's ~~on~~ the same page.

b) $y = e^{\frac{x}{4}}$

$y = e^{\frac{1}{4}x}$ ← It's helpful to separate numbers and variables. Once again, you don't have to show this if you're comfortable with the algebra

$y' = e^{\frac{1}{4}x}\left(\frac{1}{4}x\right)'$

$\boxed{y' = \frac{1}{4}e^{\frac{1}{4}x}}$

c) $y = 7x^4 - 3\sqrt[5]{x} + \frac{2}{5x^2}$ ←

$y = 7x^4 - 3x^{\frac{1}{5}} + \frac{2}{5}\frac{1}{x^2}$ ←

In order to apply $\frac{d}{dx}(x^n) = nx^{n-1}$, we have to rewrite all of these terms as fractional / negative powers of x!

$y = 7x^4 - 3x^{\frac{1}{5}} + \frac{2}{5}x^{-2}$

$\boxed{y' = 28x^3 - \frac{3}{5}x^{-\frac{4}{5}} - \frac{4}{5}x^{-3}}$

d) $y = \ln(2x + \cos x)$

$y' = \frac{1}{2x+\cos x}(2x+\cos x)'$

$\boxed{y' = \frac{1}{2x+\cos x}(2-\sin x)}$

e) $y = 2xe^{-x}$

$y' = (2x)' e^{-x} + 2x (e^{-x})'$

$y' = 2e^{-x} + 2x\, e^{-x} (-x)'$

$$\boxed{y' = 2e^{-x} - 2xe^{-x}}$$

f) $y = \dfrac{\tan 3x}{\sqrt{4-x}}$

$y = \dfrac{\tan 3x}{(4-x)^{1/2}}$

$y' = \dfrac{(\tan 3x)' (4-x)^{1/2} - \tan 3x \left[(4-x)^{1/2}\right]'}{\left[(4-x)^{1/2}\right]^2}$

$y' = \dfrac{\sec^2 3x \cdot (3x)' (4-x)^{1/2} - \tan 3x \left[\frac{1}{2} (4-x)^{-1/2} \cdot (4-x)' \right]}{(4-x)}$

$$= \boxed{\dfrac{3\sec^2 3x\,(4-x)^{1/2} + \frac{1}{2}\tan 3x\,(4-x)^{-1/2}}{4-x}}$$

g) $y = \csc(e^{4x})$

$y' = -\csc e^{4x} \cot e^{4x} (e^{4x})'$

$$\boxed{y' = -\csc e^{4x} \cot e^{4x} (e^{4x} \cdot 4)}$$

h) $y = [\ln(4x^3 - 2x)]^3$

$y' = 3[\ln(4x^3-2x)]^2 \cdot [\ln(4x^3-2x)]'$

$y' = 3[\ln(4x^3-2x)]^2 \ \frac{1}{4x^3-2x} \ (4x^3-2x)'$

$\boxed{y' = 3[\ln(4x^3-2x)]^2 \cdot \frac{1}{4x^3-2x} \ (12x^2-2)}$

i) $y = e^{4\sqrt{x}}$

$y = e^{4x^{1/2}}$

$y' = e^{4x^{1/2}} \ (4x^{1/2})'$

$\boxed{y' = e^{4x^{1/2}} \cdot 2x^{-1/2}}$

j) $y = 4e^{x \sin x}$

$y' = 4e^{x\sin x} \ \underbrace{(x \sin x)'}_{\text{Product Rule}}$

$\boxed{y' = 4e^{x\sin x} (\sin x + x\cos x)}$

k) $y = 6x^9 - \frac{1}{8x^4} + \frac{2}{\sqrt[3]{2x-1}}$

$y = 6x^9 - \frac{1}{8}\frac{1}{x^4} + \frac{2}{(2x-1)^{1/3}}$

$y = 6x^9 - \frac{1}{8}x^{-4} + 2(2x-1)^{-1/3}$

$y' = 54x^8 + \frac{1}{2}x^{-5} - \frac{2}{3}(2x-1)^{-4/3} \cdot (2x-1)'$

$\boxed{y' = 54x^8 + \frac{1}{2}x^{-5} - \frac{4}{3}(2x-1)^{-4/3}}$

e) $y = \frac{2}{(3x^2-1)^2}$

$y = 2(3x^2-1)^{-2}$

$y' = -4(3x^2-1)^{-3}(3x^2-1)'$

$y' = -4(3x^2-1)^{-3}(6x)$

$\boxed{y' = -24x(3x^2-1)^{-3}}$

II. a) $\int \left(3x^4 - \sqrt[3]{x^2} + \frac{2}{\sqrt[7]{x}}\right) dx$

$= \int \left(3x^4 - x^{2/3} + 2 \cdot \frac{1}{x^{1/7}}\right) dx$

$= \int \left(3x^4 - x^{2/3} + 2x^{-1/7}\right) dx$

$= \boxed{\frac{3}{5}x^5 - \frac{3}{5}x^{5/3} - \frac{14}{6}x^{6/7} + C}$

b) $\boxed{\text{Cannot be integrated!}}$ (In fact, one can prove there's no elementary derivative of e^{x^2}!).

* If you tried to integrate this, take the derivative of your answer. Is it equal to e^{x^2}?

c) $\int_0^{\pi/6} 4 \sin 2x \, dx$

$= -4 \cdot \frac{1}{2} \cos 2x \Big|_0^{\pi/6}$

$= -2 \cos \frac{\pi}{3} - (-2\cos 0)$

$= -2 \cdot \frac{1}{2} + 2$

$= \boxed{1}$

d) $\int e^{-\frac{x}{3}}\, dx$

$= \int e^{-\frac{1}{3}x}\, dx$

$= \boxed{-3e^{-\frac{1}{3}x} + C}$

e) $\boxed{\text{Cannot integrate yet!}}$ (We will learn how to do this later in the semester though!)

(For those curious: $\int \ln x\, dx = x\ln x - x + C$; you can check this by differentiating $x\ln x - x + C$!).

f) $\int \frac{4x^3 - 3x}{2x^2}\, dx$

DO NOT WRITE $\int \frac{4x^3-3x}{2x^2}\, dx = \frac{\int (4x^3 - 3x)\, dx}{\int 2x^2\, dx}$!

We need to write this as a sum of powers of x!

$$\int \frac{4x^3-3x}{2x^2}\, dx = \int \left(\frac{4x^3}{2x^2} - \frac{3x}{2x^2}\right) dx$$

$$= \int \left(\frac{4}{2}\cdot\frac{x^3}{x^2} - \frac{3}{2}\,\frac{x}{x^2}\right) dx$$

$$= \int \left(2x - \frac{3}{2}\,\frac{1}{x}\right) dx$$

$$= \boxed{x^2 - \frac{3}{2}\ln|x| + C}$$

g) $\int \left(\sec 4x \tan 4x + 3\sec^2 \frac{1}{5}x\right) dx$

$= \int \sec 4x \tan 4x\, dx + \int 3\sec^2 \frac{1}{5}x\, dx$

$$= \boxed{\frac{1}{4}\sec 4x + 3\cdot\left(5\tan\tfrac{1}{5}x\right) + C}$$

h) $\int_1^4 (\sqrt{x}-1)^2\,dx$

$\downarrow$ FOIL

$$= \int_1^4 (x - 2\sqrt{x} + 1)\,dx$$

$$= \int_1^4 (x - 2x^{1/2} + 1)\,dx$$

$$= \left[\frac{1}{2}x^2 - \frac{4}{3}x^{3/2} + x \right|_1^4$$

$$= \left[\frac{1}{2}(4)^2 - \frac{4}{3}(4)^{3/2} + 4\right] - \left[\frac{1}{2}(1)^2 - \frac{4}{3}(1)^{3/2} + 1\right]$$

$$= \boxed{\frac{5}{6}}$$

i) $\int_0^1 \sqrt{e^{3x}}\,dx$

$$= \int_0^1 e^{\frac{3}{2}x}\,dx$$

$$= \left[\frac{2}{3}\cdot e^{\frac{3}{2}x}\right|_0^1$$

$$= \frac{2}{3}e^{\frac{3}{2}} - \frac{2}{3}e^0$$

$$= \boxed{\frac{2}{3}e^{3/2} - \frac{2}{3}}$$

j) $\int \cot^2 3x \sec^2 3x\,dx$

$$= \int \frac{\cos^2 3x}{\sin^2 3x}\sec^2 3x\,dx$$

$$= \int \frac{1}{\sin^2 3x}\,dx$$

(j) $= \int \csc^2 3x \, dx$

$= \boxed{-\frac{1}{3}\cot 3x + C}$

k) $\boxed{\text{Cannot integrate!}}$ (This can be shown to possess no elementary antiderivative!).

* If you tried to antidifferentiate this, differentiate your answer. Is it equal to $\cos\sqrt{x}$?

l) $\int \frac{2}{(3x)^2} \, dx$

$= \int \frac{2}{9x^2} \, dx$

$= \int \frac{2}{9} \frac{1}{x^2} \, dx$

$= \int \frac{2}{9} x^{-2} \, dx$

$= \boxed{-\frac{2}{9} x^{-1} + C}$

III. 1 a) $\frac{d}{dx}\left(e^{x^2}\right) = e^{x^2}(x^2)'$

$= \boxed{2xe^{x^2}}$

b) First off, the student forgot $+C$!

Also, if the student is correct, then the derivative of his/her answer should be e^{x^2}, but:

$$\frac{d}{dx}\left(\frac{1}{2x}e^{x^2}\right) = \frac{d}{dx}\left(\frac{e^{x^2}}{2x}\right) = \frac{(e^{x^2})' \cdot 2x - e^{x^2}(2x)'}{(2x)^2}$$

$$= \boxed{\frac{4x^2e^{x^2} - 2e^{x^2}}{4x^2}}$$

This is NOT e^{x^2}!!!

c) Note that to differentiate $\frac{1}{2x} e^{x^2}$, we need the quotient rule! When we differentiate $\frac{1}{2} e^{2x}$, we don't need quotient rule!

When the argument is linear in x, the deriv. of the argument will be a constant, so we won't need quotient rule!

Indeed, if $F'(x) = f(x)$, then

$$\int f(ax+b)\, dx = \frac{1}{a} F(ax+b) + C$$

since $\frac{d}{dx}\left[\frac{1}{a} F(ax+b)\right] = \frac{1}{a} \cdot F'(ax+b) \cdot (ax+b)'$

$$= \frac{1}{\cancel{a}} f(ax+b) \cdot \cancel{a}$$

2 a) $$\frac{d}{dx}\left(\frac{x^7 - x^3}{x^4} + C\right) = \frac{(x^7-x^3)' x^4 - (x^7-x^3)(x^4)'}{(x^4)^2}$$

$$= \frac{(7x^6 - 3x^2)x^4 - (x^7 - x^3)4x^3}{x^8}$$

$$= \frac{7x^{10} - 3x^6 - 4x^{10} + 4x^6}{x^8}$$

$$= \frac{3x^{10} + x^6}{x^8}$$

This is certainly NOT the original integrand!

b) When we differentiate our result, we'd need to use quotient rule!

c) $$\int \frac{7x^6 - 3x^2}{4x^3}\, dx = \int \left(\frac{7}{4}\frac{x^6}{x^3} - \frac{3}{4}\frac{x^2}{x^3}\right) dx = \int \frac{7}{4}x^3 - \frac{3}{4}\frac{1}{x}\, dx$$

$$= \boxed{\frac{7}{16}x^4 - \frac{3}{4}\ln|x| + C}$$