

Recitation # 6 - Physical Applications

Group work:

Problem 1 A metal bar 8 meters long has density (in kg/m) x meters from the left end of the bar given by

$$\rho(x) = \begin{cases} x^2 & \text{if } 0 \leq x \leq 3 \\ \frac{4}{x} & \text{if } 3 < x \leq 8. \end{cases}$$

Find the mass of the portion of the bar from 2 meters to 5 meters from its left end.

Problem 2 Assume that a spring, whose equilibrium length is 1 meter, obeys Hooke's law. It requires 25J of work to compress this spring to 0.7m in length.

- (a) Find the work required to compress the spring to 0.6m in length.
 - (b) Find the work required to stretch the spring from 1.4m to 1.8m.
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Problem 3 A truncated conical tank that is 8 feet tall, 5 feet across the top, and 3 feet across the bottom is filled with water (water weighs 62.5 pounds per cubic foot).

- (a) Set up an integral that will compute the work needed to pump the water out of the top of the tank.
 - (b) Set up an integral that will compute the work needed to pump the water out of a pipe that sticks 2 feet above the top of the tank.
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Problem 4 A bucket with mass 100kg when filled with oil is lifted at a constant rate by a pulley from the bottom to the top of a 60-meter hole.

- (a) Assuming the chain used to lift the bucket has negligible mass, compute the work needed to lift the bucket from the bottom of the hole to the top.
 - (b) Assuming the chain used to lift the bucket has an evenly-distributed mass of 12kg, set up an integral that will compute the amount of work needed to lift the bucket from the bottom to the top of the hole.
 - (c) Assuming the chain used to lift the bucket has an evenly-distributed mass of 12kg and that oil is leaking out of a hole in the bucket at a constant rate such that the bucket and the remaining oil has a mass of 27kg when it reaches the top, set up an integral that will compute the amount of work to lift the bucket from the bottom of the hole to the top (assuming that the bucket is lifted at a constant rate).
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