Recitation #14: Sequences

Warm up:

Find the limit of the following sequences as n tends to ∞.

(a) $a_n = \dfrac{n^{1000}}{2^n}$

(b) $b_n = \cos(n\pi)$

(c) $c_n = \cos(n!\pi)$

Group work:

Problem 1 *Find the limit of the given sequence. Also, determine if it is a geometric sequence.*

(a) $a_n = \dfrac{n^2}{2^n}$

(b) $a_n = \dfrac{1}{3^n}$

(c) $a_n = \left(\dfrac{1}{n}\right)^4$

(d) $a_n = \dfrac{e^n + (-3)^n}{5^n}$

(e) $a_n = 3^{\frac{1}{n}}$

Problem 2 *Show that*

$$\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n}\right)$$

exists by proving that $a_n = \sqrt{n+1} - \sqrt{n}$ is a bounded monotonic sequence. A hint is to show that $f(x) = \sqrt{x+1} - \sqrt{x}$ is a decreasing function by showing that $f'(x) < 0$.

Problem 3 *For each of the following sequences, find the limit as the number of terms approaches infinity.*

(a) $a_n = \left(\dfrac{n+1}{2n}\right)\left(\dfrac{n-2}{n}\right)^{\frac{n}{2}}$

(b) $a_n = \sqrt[n]{3^{2n+1}}$

(c) $a_n = \left(\sqrt{n^2+7} - n\right)$

(d) $a_n = \dfrac{(2n+3)!}{5n^3(2n)!}$

(e) $a_n = (2^n + 3^n)^{\frac{1}{n}}$
Hint: $a_n \geq (0 + 3^n)^{\frac{1}{n}} = 3$ *and*
$a_n \leq (2 \cdot 3^n)^{\frac{1}{n}} = 2^{\frac{1}{n}} \cdot 3$

(f) $a_n = \dfrac{n^{365} + 5^n}{8^n + n^3}$
