

Recitation #14: An overview of sequences and series - Instructor Notes

Warm up:

For each of the following sequences, list the first four terms (start each with $n = 1$).

(a) $a_{n+1} = \frac{1}{2} \left(a_n + \frac{2}{a_n} \right), a_1 = 1.$

(b) $a_n = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{(2n)! \cdot 2n!},$ Recall that $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-1) \cdot n.$

Instructor Notes: For this handout, the problems are rather short. I suggest having all groups do each part.

Group work:

Problem 1 Give an explicit formula for each of the following sequences:

(a) $\frac{2}{3}, \frac{-2}{7}, \frac{2}{11}, \frac{-2}{15}, \dots$

(b) $-2, 6, -24, 120, -720, \dots$

(c) $2, 8, 26, 80, 242, \dots$

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Problem 2 For the sequence $a_k = (2 - k)^k$

(a) calculate and list $a_0, a_1, a_2, a_3,$ and $a_4.$

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- (b) Starting with $k = 0$, calculate and list $S_0 = \sum_{k=0}^0 a_k$, $S_1 = \sum_{k=0}^1 a_k$, $S_2 = \sum_{k=0}^2 a_k$, $S_3 = \sum_{k=0}^3 a_k$, and $S_4 = \sum_{k=0}^4 a_k$. Write S_n in summation form and write S_∞ in summation form.

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Problem 3 For each of the following, write the series in the form $\sum_{k=0}^{\infty} a_k$.

- (a) $0.2 + 0.06 + 0.018 + 0.0054 + \dots$
(b) $\frac{1}{2} + \frac{1}{3^2} - \frac{1}{3} - \frac{1}{4^2} + \frac{1}{4} + \frac{1}{5^2} - \frac{1}{5} - \frac{1}{6^2} \dots$

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Problem 4 Reindex the series

$$\sum_{k=0}^{\infty} \frac{5}{(k+2)(k+1)}$$

in the form $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=-4}^{\infty} c_k$.

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Problem 5 If $\sum_{k=0}^{\infty} a_k = 6$ and $a_n = \frac{3}{2^n}$, what is $\sum_{k=4}^{\infty} a_k$?

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