Recitation #18: Comparison Tests and Alternating Series - Instructor Notes

Warm up:

For each of the following, answer **True** or **False**, and explain why.

(a) If $a_n \geq 0$ and $\displaystyle\sum_{n=0}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=0}^{\infty} a_n^2$ converges.

(b) If $a_n, b_n \geq 0$ and both $\displaystyle\sum_{n=0}^{\infty} a_n$ and $\displaystyle\sum_{n=0}^{\infty} b_n$ converge, then $\displaystyle\sum_{n=0}^{\infty} a_n b_n$ converges.

Instructor Notes: Show that these series converge formally using the comparison test.

Group work:

Problem 1 (a) *Why can we not use the Comparison test with* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ *to show that* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2-5}$ *converges?*

(b) *Adjust* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ *to show that* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2-5}$ *converges via the Comparison Test.*

(c) *Give a convergent series we can use in the Limit Comparison Test to show that* $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2-5}$ *converges.*

Instructor Notes: This problem may be done as a quick whole class discussion. For (b), use something like $\dfrac{2}{k^2}$. Be sure to determine for which k the inequality will hold.

Problem 2 *Determine if the following series converge or diverge.*

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^3+1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+2n+1}{3n^4+1}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos^2 n}{n^3+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left[\left(1+\frac{1}{n}\right)^2 e^{-n}\right]$

Instructor Notes: These are all done using either the Comparison Test or the Limit Comparison Test. Parts (a) and (b) should be done with the limit comparison test (explain why). It is important to compare and contrast these two problems. Parts (c) and (d) should be done with the Comparison Test (again, explain why). The e^{-n} should be treated as a geometric series.

Problem 3 *Determine if the following series absolutely converge, conditionally converge, or diverge.*

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n+3}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)^n}{(2n)^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} n^2 e^{\frac{-n^3}{3}}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 5}{3^n+3^{-n}}$

(e) $\displaystyle\sum_{n=4}^{\infty} \frac{(-2)^n}{n}$

Instructor Notes: These problems use a mix of tests. One of the main goals is for students to get practice determining which test to use.

(a) Limit Comparison Test with the harmonic series

(b) Root Test

(c) Integral Test

(d) Limit Comparison Test

(e) Divergence Test. Be sure to talk about "pulling out the -1" to get an alternating series in standard form. Talk about how the Alternating Series Test and the Divergence Test will take care of conditional convergence for most (but not all) alternating series that they will see.

Problem 4 (a) *Find an upper bound for how close* $\displaystyle\sum_{k=0}^{4} \frac{(-1)^k k}{4^k}$ *is to the value of* $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k k}{4^k}$.

(b) *How many terms are needed to estimate* $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n!}$ *to within* 10^{-6}*?*

Instructor Notes: Split (a) and (b) among the groups.