

Recitation #26: Vectors in three dimensions and Dot Products

Warm up:

If $\vec{u} = \hat{i} - 2\hat{j}$ and $\vec{v} = 3\hat{i} + 4\hat{k}$, find $\vec{u} \cdot \vec{v}$.

Group work:

Problem 1 Solve the following problems:

- (a) Which of the points $(6, 2, 3)$, $(-5, -1, 4)$, and $(0, 3, 8)$ is closest to the xz -plane? Which point lies on the yz -plane?
- (b) Write an equation of the circle of radius 2 centered at $(-3, 4, 1)$ that lies in a plane parallel to the xy -plane.
- (c) Describe the sphere $x^2 + y^2 + z^2 + 6x - 14y - 2z = 5$ (ie, find its center and radius).
- (d) Find a vector whose magnitude is 311 and is in the same direction as the vector $\langle 3, -6, 7 \rangle$.

Problem 2 Find a vector (in the xy -plane) with length 4 that makes a $\frac{\pi}{3}$ radian angle with the vector $\langle 3, 4 \rangle$.

Problem 3 Answer the following questions about $\text{proj}_{\vec{v}} \vec{u}$.

- (a) Is $\text{proj}_{\vec{v}} \vec{u}$ a vector of the form $c\vec{v}$ or $c\vec{u}$ (where c is a real number)? ie, is $\text{proj}_{\vec{v}} \vec{u}$ parallel to \vec{u} or \vec{v} ?
 - (b) If $\vec{u} = 5\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{v} = 2\hat{i} - 4\hat{j} + 4\hat{k}$, find $\text{proj}_{\vec{v}} \vec{u}$.
 - (c) For \vec{u} and \vec{v} from part (b), write \vec{u} as the sum of two perpendicular vectors, one of which is parallel to \vec{v} .
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Problem 4 A 500kg lead hangs from three cables of equal length that are located at the points $(-2, 0, 0)$, $(1, \sqrt{3}, 0)$, and $(1, -\sqrt{3}, 0)$. The load is located at $(0, 0, -2\sqrt{3})$. Find the vectors describing the forces on the cables due to the load.

Problem 5 Find the work done by a constant force of $10\hat{i} + 18\hat{j} - 6\hat{k}$ that moves an object up a ramp from $(2, 3, 7)$ to $(4, 9, 15)$. Assume that distance is in feet and force in pounds. Also, find the angle between the force and the ramp.

Problem 6 Suppose that the deli at the Tiny Sparrow grocery store sells roast beef for \$9 per pound, turkey for \$4 per pound, salami for \$5 per pound, and ham for \$7 per pound. For lunches this week, Sam the sandwich maker buys 1.5 pounds of roast beef, 2 pounds of turkey, no salami, and half a pound of ham. How can you use a dot product to compute Sam's total bill from the deli?
