Recitation #1 - Review of Integration

The following worksheet is designed to help review and/or sharpen your ability to differentiate and integrate functions encountered in a typical Calculus 1 course.

Group work:

Problem 1 *Consider the function $f(x) = e^{2x}$. We know that $\frac{d}{dx}\left(e^{2x}\right) = 2e^{2x}$ by the Chain Rule, and this lets us easily conclude that $\int e^{2x}\,dx = \frac{1}{2}e^{2x}$. This could of course be verified by u-substitution (if you know/remember this technique), but can also be understood the following way:*

The symbol $\int e^{2x}\,dx$ represents a function whose derivative is e^{2x}. Since taking a derivative of e^{2x} results in multiplying e^{2x} by 2, when we antidifferentiate e^{2x}, we must multiply by $\frac{1}{2}$.

You must be careful with this type of thought! Indeed, it works only when the argument of the function (in this case, the expression in the exponent) is LINEAR. "Linear in x" means the argument is of the form $ax + b$ in x!

(a) *Calculate $\frac{d}{dx}\left(e^{x^2}\right)$.*

Solution: $\frac{d}{dx}\left(e^{x^2}\right) = e^{x^2}\left(2^x\right)' = e^{x^2}\left(2x\right)$

(b) *Suppose a student tries to apply the above logic to compute $\int e^{x^2}\,dx$. The student concludes that since $\frac{d}{dx}e^{x^2}\,dx = 2xe^{x^2}$, then:*

$$\int e^{x^2}\,dx = \frac{1}{2x}e^{x^2} \tag{1}$$

Since you know that $\int e^{x^2}\,dx$ is a function whose derivative is e^{x^2}, prove this student wrong by differentiating his/her answer (i.e. the RHS of Eqn 1).

Learning outcomes:

Solution: *First, the student forgot "+C" !*
Also, if the student is correct, then the derivative of his/her answer should be e^{x^2}*, but:*

$$\frac{d}{dx}\left(\frac{1}{2x}e^{x^2}\right) = \frac{\left(e^{x^2}\right)' \cdot 2x - e^{x^2}\,(2x)'}{(2x)^2} = \frac{4x^2e^{x^2} - 2e^{x^2}}{4x^2}$$

This is not e^{x^2}*!*

(c) *What insight does this reveal as to why this students' answer is wrong? Why can we think of antidifferentiating* e^{2x} *differently than antidifferentiating* e^{x^2}*?*

Solution: *Note that to differentiate* $\frac{1}{2x}e^{x^2}$*, we need the quotient rule!*
When we differentiate $\frac{1}{2}e^{x^2}$*, we don't need the quotient rule because* $\frac{1}{2}$ *is just a constant.*
Whenever the arguement is linear in x, the derivative of the argument will be a constant, so we won't need the quotient rule. Indeed, if $F'(x) = f(x)$*, then* $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$ *since*

$$\frac{d}{dx}\left(\frac{1}{a}F(ax+b)\right) = \frac{1}{a}F'(ax+b)\cdot(ax+b)' = \frac{1}{a}f(ax+b)\cdot a = f(ax+b)$$

Problem 2 *Categorize the following integrals into ones you know how to integrate after taking Calculus 1 and learning section 7.1 and ones you do not know how to integrate. If you have time, try to evaluate the integrals.*

a) $\int \left(3x^4 - \sqrt[3]{x^2} + \frac{2}{\sqrt[7]{x}}\right) dx$	*b)* $\int e^{x^2}\,dx$	*c)* $\int_0^{\pi/6} 4\sin(2x)\,dx$
d) $\int e^{-\frac{x}{3}}\,dx$	*e)* $\int \ln x\,dx$	*f)* $\int \frac{4x^3-3x}{2x^2}\,dx$
g) $\int \left(\sec(4x)\tan(4x) + 3\sec^2\frac{x}{5}\right) dx$	*h)* $\int_1^4 (\sqrt{x}-1)^2\,dx$	*i)* $\int_0^1 \sqrt{e^{3x}}\,dx$
j) $\int \cot^2(3x)\sec^2(3x)\,dx$	*k)* $\int \cos\sqrt{x}\,dx$	*l)* $\int \frac{2}{(3x)^2}$

Solution: *See attached solutions section II*

On your own:

Problem 3 *Differentiate the following functions.*

a) $y = (2x-7)^4$	*b)* $y = e^{\frac{x}{4}}$	*c)* $y = 7x^4 - 3\sqrt[5]{x} + \dfrac{2}{5x^2}$
d) $y = \ln(2x + \cos x)$	*e)* $y = 2xe^{-x}$	*f)* $y = \dfrac{\tan(3x)}{\sqrt{4-x}}$
g) $y = \csc\left(e^{4x}\right)$	*h)* $y = [\ln(4x^3 - 2x)]^3$	*i)* $y = e^{4\sqrt{x}}$
j) $y = 4e^{x\sin x}$	*k)* $y = 6x^9 - \dfrac{1}{8x^4} + \dfrac{2}{\sqrt[3]{2x-1}}$	*l)* $y = \dfrac{2}{(3x^2-1)^2}$

Solution: *See attached solutions section I*