

## Recitation #14: An overview of sequences and series

### Group work:

**Problem 1** For each of the following sequences, list the first four terms (start each with  $n = 1$ ).

(a)  $a_{n+1} = \frac{1}{2} \left( a_n + \frac{2}{a_n} \right), a_1 = 1.$

(b)  $a_n = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{(2n)! \cdot 2n!},$  Recall that  $n! = 1 \cdot 2 \cdot 3 \cdot 4 \dots (n-1) \cdot n.$

**Problem 2** Give an explicit formula for each of the following sequences:

(a)  $\frac{2}{3}, \frac{-2}{7}, \frac{2}{11}, \frac{-2}{15}, \dots$

(b)  $-2, 6, -24, 120, -720, \dots$

(c)  $2, 8, 26, 80, 242, \dots$

**Problem 3** For the sequence  $a_k = (2 - k)^k$

(a) calculate and list  $a_0, a_1, a_2, a_3,$  and  $a_4.$

(b) Starting with  $k = 0$ , calculate and list  $S_0 = \sum_{k=0}^0 a_k, S_1 = \sum_{k=0}^1 a_k, S_2 = \sum_{k=0}^2 a_k, S_3 = \sum_{k=0}^3 a_k,$  and  $S_4 = \sum_{k=0}^4 a_k.$  Write  $S_n$  in summation form and write  $S_\infty$  in summation form.

**Problem 4** For each of the following, write the series in the form  $\sum_{k=0}^{\infty} a_k.$

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(a)  $0.2 + 0.06 + 0.018 + 0.0054 + \dots$

(b)  $\frac{1}{2} + \frac{1}{3^2} - \frac{1}{3} - \frac{1}{4^2} + \frac{1}{4} + \frac{1}{5^2} - \frac{1}{5} - \frac{1}{6^2} \dots$

**Problem 5** Reindex the series

$$\sum_{k=0}^{\infty} \frac{5}{(k+2)(k+1)}$$

in the form  $\sum_{k=1}^{\infty} a_k$  and  $\sum_{k=-4}^{\infty} c_k$ .

**Problem 6** If  $\sum_{k=0}^{\infty} a_k = 6$  and  $a_n = \frac{3}{2^n}$ , what is  $\sum_{k=4}^{\infty} a_k$ ?