# Section 11.2: Polar coordinates

## Group work:

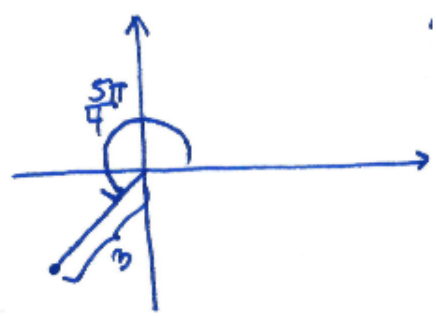
**Problem 1** *Plot the following (polar) points in the $xy$-plane and then rewrite them as rectangular coordinates.*

(a) $\left(3, \frac{5\pi}{4}\right)$ (b) $\left(3, -\frac{5\pi}{4}\right)$ (c) $\left(-3, \frac{5\pi}{4}\right)$ (d) $\left(-3, -\frac{5\pi}{4}\right)$

***Solution:*** (a)

$$x = 3\cos\left(\frac{5\pi}{4}\right) = -\frac{3\sqrt{2}}{2}$$

$$y = 3\sin\left(\frac{5\pi}{4}\right) = -\frac{3\sqrt{2}}{2}$$

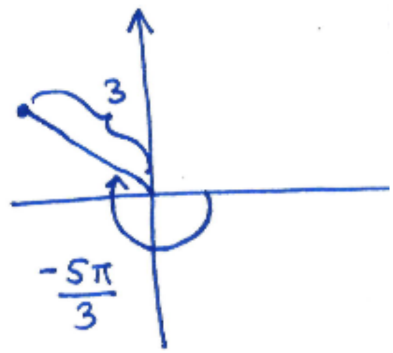



(b)

$$x = 3\cos\left(-\frac{5\pi}{4}\right) = -\frac{3\sqrt{2}}{2}$$

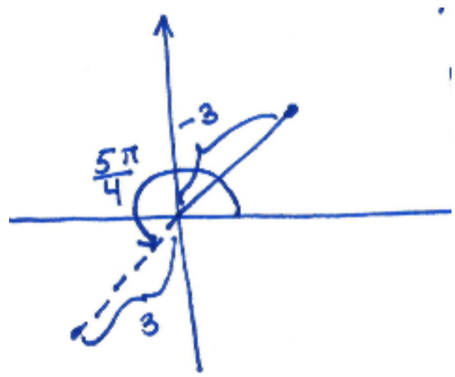
$$y = 3\sin\left(-\frac{5\pi}{4}\right) = \frac{3\sqrt{2}}{2}$$

Learning outcomes:



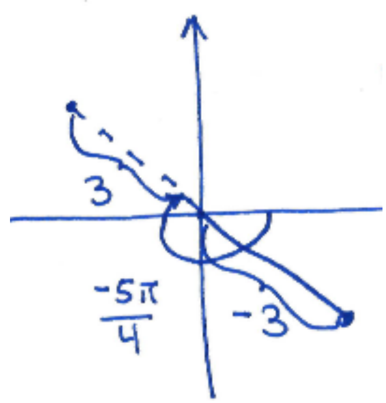

(c)

$$x = -3\cos\left(\frac{5\pi}{4}\right) = \frac{3\sqrt{2}}{2}$$
$$y = -3\sin\left(\frac{5\pi}{4}\right) = \frac{3\sqrt{2}}{2}$$




(d)

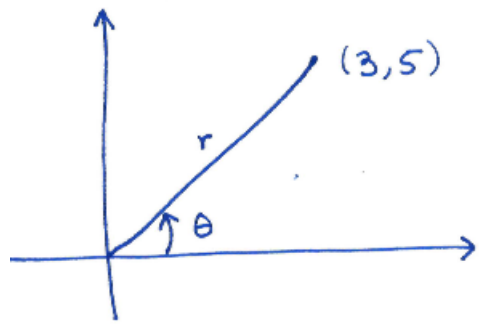
$$x = -3\cos\left(-\frac{5\pi}{4}\right) = \frac{3\sqrt{2}}{2}$$
$$y = -3\sin\left(-\frac{5\pi}{4}\right) = -\frac{3\sqrt{2}}{2}$$



**Problem 2** *Rewrite the rectangular point* $(3, 5)$ *in polar coordinates in three different ways.*

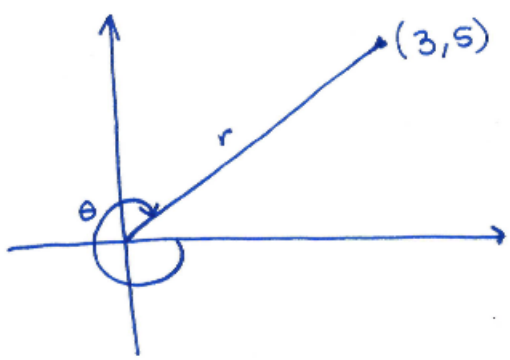
***Solution:*** *i.*

$$r = \sqrt{3^2 + 5^2} = \sqrt{34} \qquad \theta = \arctan\left(\frac{5}{3}\right)$$



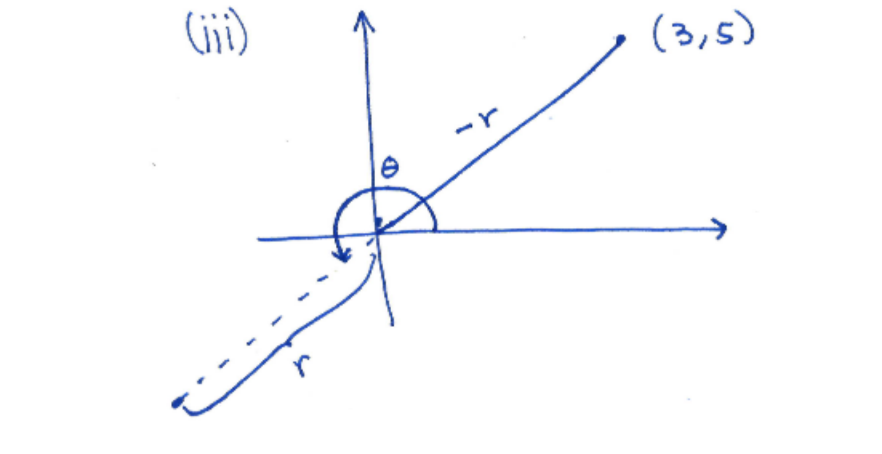

*ii.*

$$r = \sqrt{34} \qquad \theta = \arctan\left(\frac{5}{3}\right) - 2\pi$$

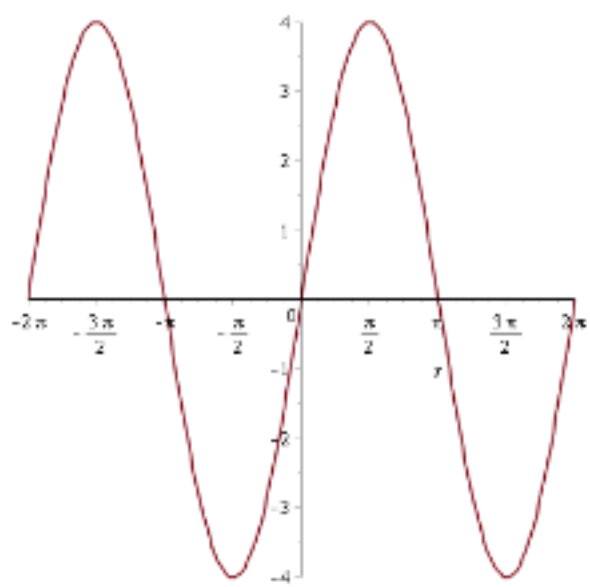



*iii.*

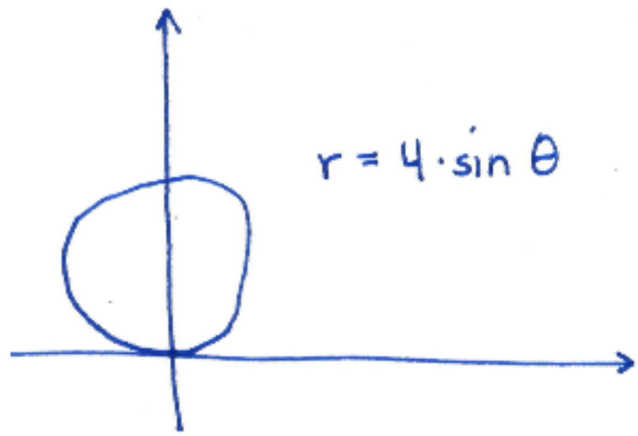
$$r = -\sqrt{34} \qquad \theta = \arctan\left(\frac{5}{3}\right) + \pi$$




**Problem 3** *The graph of the curve $r = 4\sin\theta$ is a circle. Use the graph below to sketch this circle. Can you verify this algebraically? What is the period of the polar curve? Is $0 \le \theta \le 2\pi$ necessary to complete the graph?*



***Solution:*** *Graphing this equation in the picture below, we see that this is a circle with radius 2 and center $(0, 2)$.*




*To verify this algebraically,*

$$\begin{aligned} 4 = 4\sin\theta \quad &\Longrightarrow \quad r^2 = 4r\sin\theta \\ &\Longrightarrow \quad x^2 + y^2 = 4y \\ &\Longrightarrow \quad x^2 + y^2 - 4y = 0 \\ &\Longrightarrow \quad x^2 + y^2 - 4y + 4 = 4 \\ &\Longrightarrow \quad x^2 + (y-2)^2 = 2^2. \end{aligned}$$

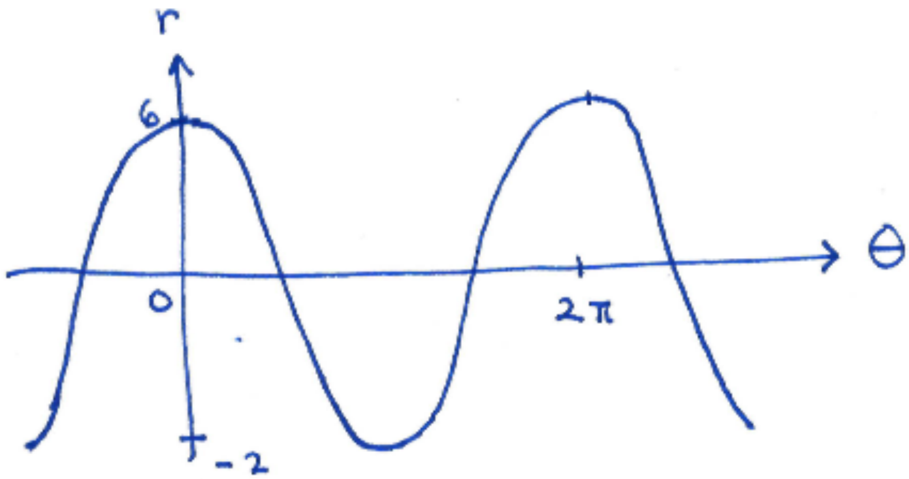
*To find the period of the polar curve, we convert both $x$ and $y$ into parametric equations with parameter $\theta$.*

$$\begin{aligned} x &= r\cos\theta = 4\sin\theta\cos\theta = 2\sin(2\theta) \\ y &= r\sin\theta = 4\sin^2\theta = 4\cdot\frac{1}{2}(1-\cos(2\theta)) = 2(1-\cos(2\theta)). \end{aligned}$$

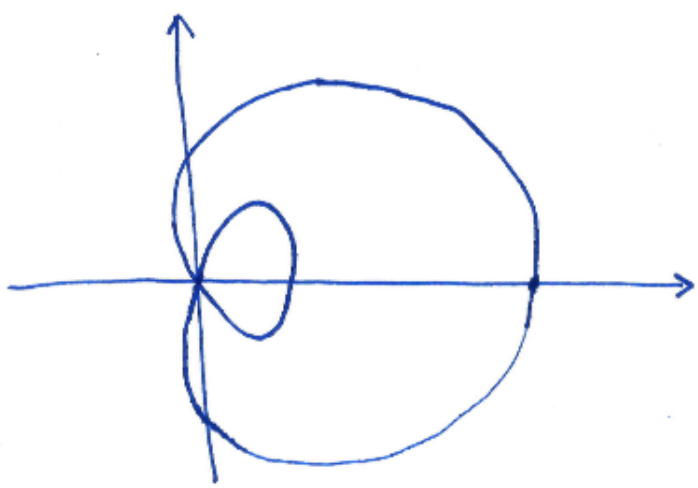
*The period of both $\sin(2\theta)$ and $\cos(2\theta)$ is $\pi$. So the entire graph of the equation $r = 4\sin\theta$ is traversed over the region $0 \le \theta < \pi$.*

**Problem 4** *Graph $r = 2 + 4\cos\theta$ using the "Cartesian-to-Polar" method.*

***Solution:*** *First, we graph $r = 2 + 4\cos\theta$ as if $r$ and $\theta$ were Cartesian coordinates.*




*We then use this to draw the following graph in the $xy$-plane*



*If it helps, here are the parametric equations for this graph*

$$x = r\cos\theta = (2 + 4\cos\theta)\cos\theta = 2\cos\theta + 4\cos^2\theta = 2(\cos\theta + 1 + \cos(2\theta))$$
$$y = 4\sin\theta = (2 + 4\cos\theta)\sin\theta = 2\sin\theta + 2\sin(2\theta).$$