Recitation # 11: Partial fractions and Improper Integrals

Warm up:

True or False: It is possible for a region to be infinitely long but have a finite area.

Group work:

Problem 1 *Without determining the coefficients, write the partial fraction decomposition of the following rational function:*

$$\frac{5x^{13} - 6x^{12} + 7x^3 - 5x - 18}{(2x-3)(5x+9)^3(x^2+9x+19)(x^2+9x+21)^2}$$

Problem 2 *Evaluate:*

$$\int \frac{7x^3 + 18x + 9}{x^4 + 9x^2}\,dx$$

Hint: If $f(x) = 7x^3 + 18x + 9$*, then* $f(2) = 101$*,* $f(1) = 34$*, and* $f(-1) = -16$*.*

Problem 3 *Review of limits:*

(a) $\displaystyle\lim_{x\to-\infty}\left(3x^{-6} + e^{5x} + \frac{\sin x}{x^2+3}\right)$

(b) $\displaystyle\lim_{x\to\infty}\frac{x}{\sqrt{9x^2+4}}$

(c) $\displaystyle\lim_{x\to-\infty}\arctan x$

Problem 4 *In each of the following, determine if the given integral converges or diverges. If it converges, find the value.*

(a) $\displaystyle\int_{-1}^{\infty} \frac{3}{2x+1}\,dx$

(b) $\displaystyle\int_{-\infty}^{\infty} xe^{-x}\,dx$

(c) $\displaystyle\int_{6}^{\infty} \frac{2-4x}{2x^2-13x+20}\,dx$

Problem 5 *Find the volume of the solid whose base is the region where* $x \geq 1$, $y \geq 0$, *and below the curve* $y = \dfrac{1}{x^4}$, *and whose cross sections perpendicular to the* x*-axis are squares.*