

Section 12.4: Cross Products

Warm up:

If \vec{a} , \vec{b} , and \vec{c} are vectors in 3-space \mathbb{R}^3 , which of the following make sense?

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|--|--|--|
| (a) $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$ | (d) $(\vec{a} \cdot \vec{b}) + \vec{c}$ | (g) $\vec{a} \cdot (\vec{b} \times \vec{c})$ |
| (b) $(\vec{a} \cdot \vec{b})\vec{c}$ | (e) $(\vec{a} \times \vec{b}) + \vec{c}$ | (h) $\vec{a} \times (\vec{b} \cdot \vec{c})$ |
| (c) $(\vec{a} \times \vec{b}) \cdot \vec{c}$ | (f) $\vec{a} \cdot (\vec{b} + \vec{c})$ | (i) $(\vec{a} \times \vec{b})\vec{c}$ |

Group work:

Problem 1 Given three dimensional vectors \vec{u} , \vec{v} , and \vec{w} , use dot product or cross product notation to describe the following vectors:

- The vector projection of \vec{w} onto \vec{u} .
- A vector orthogonal to both \vec{u} and \vec{v} .
- A vector with the length of \vec{v} and the direction of \vec{w} .
- A vector orthogonal to $\vec{u} \times \vec{v}$ and \vec{w} .

Problem 2 Let $\vec{u} = \langle 5, -1, 8 \rangle$ and $\vec{v} = \langle -2, 10, 5 \rangle$.

- Find a vector that is perpendicular to both \vec{u} and \vec{v} .
- Verify that your answer is perpendicular to both \vec{u} and \vec{v} .
- Find a vector of length 7 perpendicular to both \vec{u} and \vec{v} .

Problem 3 Find the area of the triangle in \mathbb{R}^3 with vertices at $P(2, -1, 0)$, $Q(1, 1, 4)$ and $R(2, -1, 6)$.