

## Section 10.2: Properties of Power Series

### Warm up:

**Problem 1** Suppose that  $\sum_{k=0}^{\infty} c_k(x+5)^k$  converges when  $x = -9$  and diverges when  $x = -1$ . What can be said about the convergence and divergence of the following series?

(a)  $\sum_{k=0}^{\infty} c_k$

(b)  $\sum_{k=0}^{\infty} c_k(-5)^k$

(c)  $\sum_{k=0}^{\infty} c_k(5)^k$

### Group work:

**Problem 2** If the series  $\sum_{k=0}^{\infty} a_k(x-2)^k$  has an interval of convergence of  $[-4, 8)$ , determine the interval of convergence of the following series:

(a)  $\sum_{k=300}^{\infty} a_k(x-2)^k$

(b)  $\sum_{k=0}^{\infty} a_k x^k$

(c)  $\sum_{k=0}^{\infty} \left( a_k(x-2)^k + \left(\frac{1}{7}\right)^k x^k \right)$

**Problem 3** For each of the following, find the domain of  $f(x)$  (i.e. find the interval of convergence).

(a)  $f(x) = \sum_{k=1}^{\infty} \frac{(3x-2)^k}{k \cdot 3^k}$

(c)  $f(x) = \sum_{k=2}^{\infty} \frac{x^{3k+2}}{(\ln k)^k}$

(b)  $f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k^2+1}} x^k$

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**Problem 4** In each of the following, give a power series (with an interval of convergence) for the given function. Assume that we know  $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$  on  $(-1, 1)$ .

(a)  $f(x) = \frac{3}{5x-2}$

(b)  $f(x) = \frac{3x^4}{5x^3-2}$

**Problem 5** Consider  $f(x) = \sum_{k=0}^{\infty} \frac{2^k x^k}{(k+1)^3}$ .

(a) Write out  $p_3(x)$ , the cubic polynomial which is the first three terms of this power series.

(b) Find  $p'_3(x)$  and  $f'(x)$  and compare your answers.

(c) Find  $\int p_3(x) dx$  and  $\int f(x) dx$  and compare your answers.

**Problem 6** Give a power series (with interval of convergence) for the given functions.

(a)  $f(x) = \frac{1}{1+x^2}$

(b)  $f(x) = \tan^{-1}(x)$

(c)  $f(x) = \tan^{-1}(3x^2)$