Recitation #22: Working with Taylor series

You should memorize the Maclaurin series for $\cos(x), \sin(x), e^x$, and $\frac{1}{1-x}$. If you need the Maclaurin series for $\ln(1+x)$, $\arctan(x)$, or the binomial series these will be given to you.

$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k} \qquad -1 < x \leq 1$$

$$\arctan(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1} \qquad -1 \leq x \leq 1$$

Warm up:

True or False: To approximate $\frac{\pi}{3}$, one could substitute $x = \sqrt{3}$ into the Maclaurin series for $\tan^{-1} x$?

Group work:

Problem 1 *Use power series to evaluate the limit*

$$\lim_{x \to 0} \frac{\ln(1+x^2)}{1 - \cos x}$$

Problem 2 *Given that*

$$f(t) = \int_0^t x^2 \tan^{-1}(x^4)\, dx$$

approximate $f\left(\frac{1}{3}\right)$ *with the first four non-zero terms of a power series. Estimate how close this approximation is.*

Problem 3 *Use power series to determine a (series) solution to the initial value problem*

$$y'' - xy' + y = 0 \qquad y(0) = 1 \qquad y'(0) = 0$$

Problem 4 *Identify the function represented by the power series*

$$\sum_{k=0}^{\infty} \frac{k(k-1)x^k}{7^k}$$
