Recitation # 11: Partial fractions and Improper Integrals - Instructor Notes

Warm up:

True or False: It is possible for a region to be infinitely long but have a finite area.

Instructor Notes:

Group work:

Problem 1 *Without determining the coefficients, write the partial fraction decomposition of the following rational function:*

$$\frac{5x^{13} - 6x^{12} + 7x^3 - 5x - 18}{(2x-3)(5x+9)^3(x^2+9x+19)(x^2+9x+21)^2}$$

Instructor Notes: Make sure that the students do not attempt to perform the long division. We are only looking for the **form** of the decomposition, which will be a cubic polynomial followed by a sum of rational functions. Note that $x^2 + 9x + 19$ can be factored over the reals while $x^2 + 9x + 21$ cannot. This is a good time to talk about the descriminant.

Problem 2 *Evaluate:*

$$\int \frac{7x^3 + 18x + 9}{x^4 + 9x^2}\, dx$$

Hint: If $f(x) = 7x^3 + 18x + 9$*, then* $f(2) = 101$*,* $f(1) = 34$*, and* $f(-1) = -16$*.*

Instructor Notes: First, note that students often have difficulty understanding that $x^2 = (x-0)^2$ is a perfect square of a linear factor $(x-0)$. The hint should help the students quickly solve for the unknowns. Using $x = 0$ (not mentioned, but easily evaluated), one of the unknowns is immediately known. Using 1 and -1 and adding the resulting equations finds a second unknown. Using 2 will give them two equations and two unknowns to find the other two. The decomposition is

$$\frac{2}{x} - \frac{1}{x^2} + \frac{5x-1}{x^2+9}.$$

Problem 3 *Review of limits:*

(a) $\lim_{x\to-\infty} \left(3x^{-6} + e^{5x} + \dfrac{\sin x}{x^2+3}\right)$

(b) $\lim_{x\to\infty} \dfrac{x}{\sqrt{9x^2+4}}$

(c) $\lim_{x\to-\infty} \arctan x$

Instructor Notes: Review of limits.

Problem 4 *In each of the following, determine if the given integral converges or diverges. If it converges, find the value.*

(a) $\displaystyle\int_{-1}^{\infty} \frac{3}{2x+1}\,dx$

(b) $\displaystyle\int_{-\infty}^{\infty} xe^{-x}\,dx$

(c) $\displaystyle\int_{6}^{\infty} \frac{2-4x}{2x^2-13x+20}\,dx$

Instructor Notes: Make sure that students write in the limit notation throughout their work, with the limit taken after the definite integral has been evaluated.

(a) has a vertical asymptote as well as a limit going to infinity.

(b) is by parts, and L'Hospital's Rule will be useful for the limit.

(c) will need partial fractions. The coefficients of the decomposition are rigged to be opposites of each other, so that one can use properties of logarithms to aid in taking the limit.

Problem 5 *Find the volume of the solid whose base is the region where* $x \geq 1$, $y \geq 0$, *and below the curve* $y = \dfrac{1}{x^4}$, *and whose cross sections perpendicular to the* x*-axis are squares.*

Instructor Notes:
