Recitation #26: Vectors in three dimensions and Dot Products - Instructor Notes

Warm up:

If $\vec{u} = \hat{\imath} - 2\hat{\jmath}$ and $\vec{v} = 3\hat{\imath} + 4\hat{k}$, find $\vec{u} \cdot \vec{v}$.

Instructor Notes: Make sure that students realize that $\vec{u} = \langle 3, 4, 0 \rangle$ and not $\langle 3, 4 \rangle$.

Group work:

Problem 1 *Solve the following problems:*

(a) *Which of the points* $(6, 2, 3)$, $(-5, -1, 4)$, *and* $(0, 3, 8)$ *is closest to the* xz*-plane? Which point lies on the* yz*-plane?*

(b) *Write an equation of the circle of radius* 2 *centered at* $(-3, 4, 1)$ *that lies in a plane parallel to the* xy*-plane.*

(c) *Describe the sphere* $x^2 + y^2 + z^2 + 6x - 14y - 2z = 5$ *(ie, find its center and radius).*

(d) *Find a vector whose magnitude is* 311 *and is in the same direction as the vector* $\langle 3, -6, 7 \rangle$.

Instructor Notes:

Problem 2 *Find a vector (in the* xy*-plane) with length* 4 *that makes a* $\dfrac{\pi}{3}$ *radian angle with the vector* $\langle 3, 4 \rangle$.

Instructor Notes: The students need to assimilate a lot of information in this problem. They need to "name" the unknown vector (say $\langle a, b\rangle$). Then, they need to realize that both $\langle 3,4\rangle \cdot \langle 3,4\rangle = 3a+4b$ and that $\left|\langle 3,4\rangle\right| \cdot \left|\langle a,b\rangle\right| \cos\left(\frac{\pi}{3}\right) = 5 \cdot 4 \cdot \frac{1}{2}$, giving $3a+4b=10$. Lastly, they also need to realize that $a^2+b^2=16$. A picture illustrating that there could be two such vectors would be helpful.

Problem 3 *Answer the following questions about $proj_v u$.*

(a) *Is $proj_v u$ a vector of the form $c\vec{v}$ or $c\vec{u}$ (where c is a real number)? ie, is $proj_v u$ parallel to $\vec{u}$ or $\vec{v}$?*

(b) *If $\vec{u} = 5\hat{\imath} + 6\hat{\jmath} - 3\hat{k}$ and $\vec{v} = 2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$, find $proj_v u$.*

(c) *For $\vec{u}$ and $\vec{v}$ from part (b), write $\vec{u}$ as the sum of two perpendicular vectors, one of which is parallel to $\vec{v}$.*

Instructor Notes: Working with projections.

Problem 4 *A $500kg$ lead hangs from three cables of equal length that are located at the points $(-2,0,0)$, $(1,\sqrt{3},0)$, and $(1,-\sqrt{3},0)$. The load is located at $(0,0,-2\sqrt{3})$. Find the vectors describing the forces on the cables due to the load.*

Instructor Notes: Students need to take into account that the mass, and not the force, is given. The students should also take advantage of the fact that the vectors are all of the same magnitude.

Problem 5 *Find the work done by a constant force of $10\hat{\imath}+18\hat{\jmath}-6\hat{k}$ that moves an object up a ramp from $(2,3,7)$ to $(4,9,15)$. Assume that distance is in feet and force in pounds. Also, find the angle between the force and the ramp.*

Instructor Notes: Simple work question.

Problem 6 *Suppose that the deli at the Tiny Sparrow grocery store sells roast beef for* \$9 *per pound, turkey for* \$4 *per pound, salami for* \$5 *per pound, and ham for* \$7 *per pound. For lunches this week, Sam the sandwhich maker buys* 1.5 *pounds of roast beef,* 2 *pounds of turkey, no salami, and half a pound of ham. How can you use a dot product to compute Sam's total bill from the deli?*

Instructor Notes: