Recitation 28: Cross Products and Lines and Curves in Space

Warm up:

If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are vectors in 3-space $\mathbb{R}^3$, which of the following make sense?

(a) $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$

(b) $(\vec{a} \cdot \vec{b})\vec{c}$

(c) $(\vec{a} \times \vec{b}) \cdot \vec{c}$

(d) $(\vec{a} \cdot \vec{b}) + \vec{c}$

(e) $(\vec{a} \times \vec{b}) + \vec{c}$

(f) $\vec{a} \cdot (\vec{b} + \vec{c})$

(g) $\vec{a} \cdot (\vec{b} \times \vec{c})$

(h) $\vec{a} \times (\vec{b} \cdot \vec{c})$

(i) $(\vec{a} \times \vec{b})\vec{c}$

Group work:

Problem 1 *Given three dimensional vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$, use dot product or cross product notation to describe the following vectors:*

(a) *The vector projection of $\vec{w}$ onto $\vec{u}$.*

(b) *A vector orthogonal to both $\vec{u}$ and $\vec{v}$.*

(c) *A vector with the length of $\vec{v}$ and the direction of $\vec{w}$.*

(d) *A vector orthogonal to $\vec{u} \times \vec{v}$ and $\vec{w}$.*

Problem 2 *Let $\vec{u} = \langle 5, -1, 8 \rangle$ and $\vec{v} = \langle -2, 10, 5 \rangle$.*

(a) *Find a vector that is perpendicular to both $\vec{u}$ and $\vec{v}$.*

(b) *Verify that your answer is perpendicular to both $\vec{u}$ and $\vec{v}$*

(c) *Find a vector of length 7 perpendicular to both $\vec{u}$ and $\vec{c}$.*

Problem 3 *Find the area of the triangle in $\mathbb{R}^3$ with vertices at $P(2, -1, 0)$, $Q(1, 1, 4)$ and $R(2, -1, 6)$.*

Problem 4 *Find a vector-valued function for the line segment connecting the points $P = (-3, 7, 6)$ and $Q = (5, -4, 7)$ in such a way that the value at $t = 0$ is P and the value at $t = 1$ is Q. Also, find the point two-thirds of the way from P to Q.*

Problem 5 *Find a vector-valued function for the line through the point $(1, -2, 3)$ that is perpendicular to the lines*

$$\vec{r}_1(t) = \langle 7, 8, -2 \rangle + t\langle 3, 5, 7 \rangle \quad \text{and} \quad \vec{r}_2(s) = \langle 4, -3, -7 \rangle + s\langle 4, 9, -1 \rangle$$

Problem 6 *Show that the curve $\vec{r} = \langle t\cos t, t\sin t, t \rangle$ lies completely on the cone $z^2 = x^2 + y^2$.*

Challenge Problems

Problem 7 *Find the distance from the point $P(-1, 4, 3)$ to the line $\langle 8 + t, 3 - 3t, -26t \rangle$. Hint: The distance from the point to the line is the distance from the point P and the closest point on the line.*

Problem 8 *Match each of the following curves to the corresponding vector-valued function.*

(a) $\langle 3, t^2, 5\rangle$

(b) $\langle 3, t^2, t\rangle$

(c) $\langle 3, \sin t, \cos t\rangle$

(d) $\langle 3t, 5\sin t, 5\cos t\rangle$

(e) $\langle \sin t, \cos t, 2\cos t\rangle$

(f) $\langle 2\cos t, \sin t, \cos(3t)\rangle$

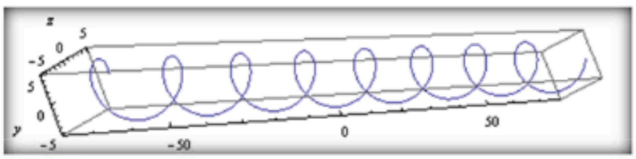

Figure 1

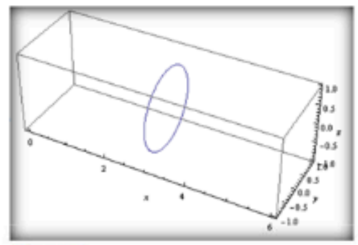

Figure 2

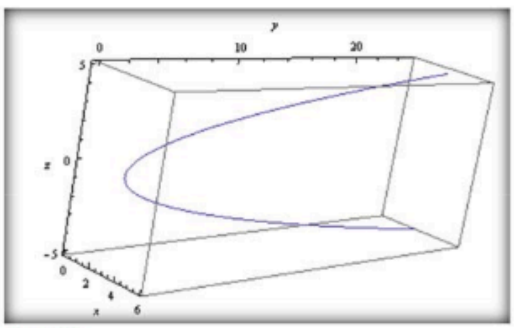

Figure 3

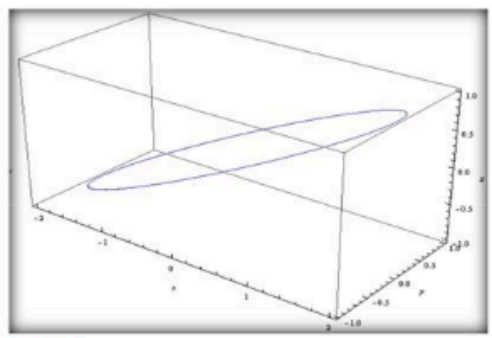

Figure 4

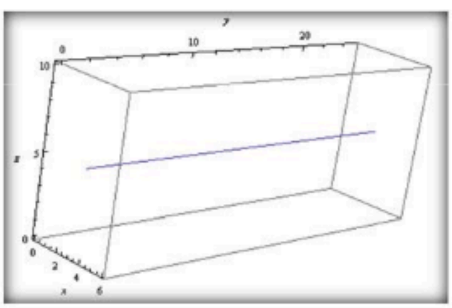

Figure 5

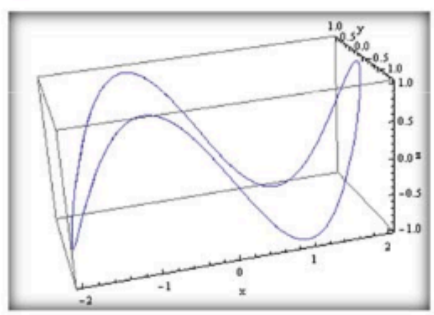

Figure 6