

Worksheet #1 Answers

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Detailed Solutions follow this sheet!

- I.
- a) $8(2x-7)^3$
 - b) $\frac{1}{4}e^{\frac{1}{4}x}$
 - c) $28x^3 - \frac{3}{5}x^{-\frac{4}{5}} - \frac{4}{5}x^{-3}$
 - d) $\frac{2 - \sin x}{2x + \cos x}$
 - e) $2e^{-x} - 2xe^{-x}$
 - f) $\frac{3(4-x)^{\frac{1}{2}} \sec^2 3x + \frac{1}{2}(4-x)^{-\frac{1}{2}} \tan 3x}{4-x}$
 - g) $-4e^{4x} \csc(e^{4x}) \cot(e^{4x})$
 - h) $3[\ln(4x^3 - 2x)]^2 \cdot \frac{12x^2 - 2}{4x^3 - 2x}$
 - i) $2x^{-\frac{1}{2}} e^{4x^{\frac{1}{2}}}$
 - j) $4e^{x \sin x} (\sin x + x \cos x)$
 - k) $54x^8 + \frac{1}{2}x^{-5} - \frac{4}{3}(2x-1)^{-4/3}$
 - l) $-24x(3x^2-1)^{-3}$

- II.
- a) $\frac{3}{5}x^5 - \frac{3}{5}x^{5/3} - \frac{14}{6}x^{6/7} + C$
 - b) Cannot Integrate
 - c) 1
 - d) $-3e^{-\frac{1}{3}x} + C$
 - e) Cannot Integrate yet
 - f) $x^2 - \frac{3}{2}\ln|x| + C$
 - g) $\frac{1}{4}\sec 4x + 15 \tan \frac{1}{5}x + C$
 - h) $\frac{5}{6}$
 - i) $\frac{2}{3}e^{3/2} - \frac{2}{3}$

j) $-\frac{1}{3} \cot 3x + C.$

k) Cannot integrate!

l) $-\frac{2}{9} x^{-1} + C$

Worksheet # 1 Solutions

I. a) $y = (2x-7)^4$

$$y' = 4(2x-7)^3 (2x-7)'$$

$$y' = 4(2x-7)^3 \cdot 2$$

$$\boxed{y' = 8(2x-7)^3}$$

← It's ok just to write this! The previous steps are meant to make sure everyone's on the same page.

b) $y = e^{\frac{x}{4}}$

$$y = e^{\frac{1}{4}x} \leftarrow \text{It's helpful to separate numbers and variables. Once again, you don't have to show this if you're comfortable with the algebra}$$

$$y' = e^{\frac{1}{4}x} \left(\frac{1}{4}x\right)'$$

$$\boxed{y' = \frac{1}{4}e^{\frac{1}{4}x}}$$

c) $y = 7x^4 - 3\sqrt{x} + \frac{2}{5x^2} \leftarrow$

$$y = 7x^4 - 3x^{\frac{1}{2}} + \frac{2}{5} \frac{1}{x^2} \leftarrow$$

$$y = 7x^4 - 3x^{\frac{1}{2}} + \frac{2}{5}x^{-2}$$

$$\boxed{y' = 28x^3 - \frac{3}{5}x^{-\frac{1}{2}} - \frac{4}{5}x^{-3}}$$

— In order to apply $\frac{d}{dx}(x^n) = nx^{n-1}$, we have to rewrite all of these terms as fractional/negative powers of x !

d) $y = \ln(2x + \cos x)$

$$y' = \frac{1}{2x + \cos x} (2x + \cos x)'$$

$$\boxed{y' = \frac{1}{2x + \cos x} (2 - \sin x)}$$

$$e) \quad y = 2xe^{-x}$$

$$y' = (2x)' e^{-x} + 2x (e^{-x})'$$

$$y' = 2e^{-x} + 2x e^{-x} (-x)'$$

$$\boxed{y' = 2e^{-x} - 2xe^{-x}}$$

$$f) \quad y = \frac{\tan 3x}{\sqrt{4-x}}$$

$$y = \frac{\tan 3x}{(4-x)^{1/2}}$$

$$y' = \frac{(\tan 3x)' (4-x)^{1/2} - \tan 3x [(4-x)^{1/2}]'}{[(4-x)^{1/2}]^2}$$

$$y' = \frac{\sec^2 3x \cdot (3x)' (4-x)^{1/2} - \tan 3x \left[\frac{1}{2} (4-x)^{-1/2} \cdot (4-x)' \right]}{(4-x)}$$

$$= \boxed{\frac{3 \sec^2 3x (4-x)^{1/2} + \frac{1}{2} \tan 3x (4-x)^{-1/2}}{4-x}}$$

$$g) \quad y = \csc(e^{4x})$$

$$y' = -\csc e^{4x} \cot e^{4x} (e^{4x})'$$

$$\boxed{y' = -\csc e^{4x} \cot e^{4x} (e^{4x} \cdot 4)}$$

$$h) y = [\ln(4x^3 - 2x)]^3$$

$$y' = 3[\ln(4x^3 - 2x)]^2 \cdot [\ln(4x^3 - 2x)]'$$

$$y' = 3[\ln(4x^3 - 2x)]^2 \cdot \frac{1}{4x^3 - 2x} (4x^3 - 2x)'$$

$$y' = 3[\ln(4x^3 - 2x)]^2 \cdot \frac{1}{4x^3 - 2x} (12x^2 - 2)$$

$$i) y = e^{4\sqrt{x}}$$

$$y = e^{4x^{1/2}}$$

$$y' = e^{4x^{1/2}} (4x^{1/2})'$$

$$y' = e^{4x^{1/2}} \cdot 2x^{-1/2}$$

$$j) y = 4e^{x \sin x}$$

$$y' = 4e^{x \sin x} (\underbrace{x \sin x}_{\text{Product Rule}})'$$

$$y' = 4e^{x \sin x} (\sin x + x \cos x)$$

$$k) y = 6x^9 - \frac{1}{8x^4} + \frac{2}{\sqrt[3]{2x-1}}$$

$$y = 6x^9 - \frac{1}{8} \frac{1}{x^4} + \frac{2}{(2x-1)^{1/3}}$$

$$y = 6x^9 - \frac{1}{8} x^{-4} + 2(2x-1)^{-1/3}$$

$$y' = 54x^8 + \frac{1}{2} x^{-5} - \frac{2}{3} (2x-1)^{-4/3} \cdot (2x-1)'$$

$$y' = 54x^8 + \frac{1}{2} x^{-5} - \frac{4}{3} (2x-1)^{-4/3}$$

$$e) \quad y = \frac{2}{(3x^2-1)^2}$$

$$y = 2(3x^2-1)^{-2}$$

$$y' = -4(3x^2-1)^{-3} (3x^2-1)'$$

$$y' = -4(3x^2-1)^{-3} (6x)$$

$$\boxed{y' = -24x(3x^2-1)^{-3}}$$

$$\text{II. a) } \int \left(3x^4 - 3\sqrt{x^2} + \frac{2}{\sqrt[4]{x}} \right) dx$$

$$= \int \left(3x^4 - x^{2/3} + 2 \cdot \frac{1}{x^{1/4}} \right) dx$$

$$= \int \left(3x^4 - x^{2/3} + 2x^{-1/4} \right) dx$$

$$= \boxed{\frac{3}{5}x^5 - \frac{3}{5}x^{5/3} - \frac{14}{6}x^{6/7} + C}$$

b) Cannot be integrated! (In fact, one can prove there's no elementary derivative of e^{x^2} !).

* If you tried to integrate this, take the derivative of your answer. Is it equal to e^{x^2} ?

$$c) \int_0^{\pi/6} 4 \sin 2x \, dx$$

$$= -4 \cdot \frac{1}{2} \cos 2x \Big|_0^{\pi/6}$$

$$= -2 \cos \frac{\pi}{3} - (-2 \cos 0)$$

$$= -2 \cdot \frac{1}{2} + 2$$

$$= \boxed{1}$$

$$d) \int e^{-\frac{x}{3}} dx$$

$$= \int e^{-\frac{1}{3}x} dx$$

$$= \boxed{-3 e^{-\frac{1}{3}x} + C}$$

e) Cannot integrate yet! (We will learn how to do this later in the semester though!)

(For those curious: $\int \ln x \, dx = x \ln x - x + C$; you can check this by differentiating $x \ln x - x + C$!)

$$f) \int \frac{4x^3 - 3x}{2x^2} dx$$

DO NOT WRITE $\int \frac{4x^3 - 3x}{2x^2} dx = \frac{\int (4x^3 - 3x) dx}{\int 2x^2 dx}$

We need to write this as a sum of powers of x !

$$\begin{aligned} \int \frac{4x^3 - 3x}{2x^2} dx &= \int \left(\frac{4x^3}{2x^2} - \frac{3x}{2x^2} \right) dx \\ &= \int \left(\frac{4}{2} \cdot \frac{x^3}{x^2} - \frac{3}{2} \cdot \frac{x}{x^2} \right) dx \\ &= \int \left(2x - \frac{3}{2} \cdot \frac{1}{x} \right) dx \\ &= \boxed{x^2 - \frac{3}{2} \ln|x| + C} \end{aligned}$$

$$g) \int (\sec 4x \tan 4x + 3 \sec^2 \frac{1}{5}x) dx$$

$$= \int \sec 4x \tan 4x \, dx + \int 3 \sec^2 \frac{1}{5}x \, dx$$

$$= \boxed{\frac{1}{4} \sec 4x + 3 \cdot \left(5 \tan \frac{1}{5} x\right) + C}$$

$$h) \int_1^4 (\sqrt{x} - 1)^2 dx$$

↓ FOIL

$$= \int_1^4 (x - 2\sqrt{x} + 1) dx$$

$$= \int_1^4 (x - 2x^{1/2} + 1) dx$$

$$= \left[\frac{1}{2} x^2 - \frac{4}{3} x^{3/2} + x \right]_1^4$$

$$= \left[\frac{1}{2} (4)^2 - \frac{4}{3} (4)^{3/2} + 4 \right] - \left[\frac{1}{2} (1)^2 - \frac{4}{3} (1)^{3/2} + 1 \right]$$

$$= \boxed{\frac{5}{6}}$$

$$i) \int_0^1 \sqrt{e^{3x}} dx$$

$$= \int_0^1 e^{\frac{3}{2}x} dx$$

$$= \left[\frac{2}{3} e^{\frac{3}{2}x} \right]_0^1$$

$$= \frac{2}{3} e^{\frac{3}{2}} - \frac{2}{3} e^0$$

$$= \boxed{\frac{2}{3} e^{3/2} - \frac{2}{3}}$$

$$j) \int \cot^2 3x \sec^2 3x dx$$

$$= \int \frac{\cos^2 3x}{\sin^2 3x} \sec^2 3x dx$$

$$= \int \frac{1}{\sin^2 3x} dx$$

$$(j) = \int \csc^2 3x \, dx$$

$$= \boxed{-\frac{1}{3} \cot 3x + C}$$

k) Cannot integrate! (This can be shown to possess no elementary antiderivative!).

* If you tried to antidifferentiate this, differentiate your answer. Is it equal to $\cos \sqrt{x}$?

$$l) \int \frac{2}{(3x)^2} \, dx$$

$$= \int \frac{2}{9x^2} \, dx$$

$$= \int \frac{2}{9} \frac{1}{x^2} \, dx$$

$$= \int \frac{2}{9} x^{-2} \, dx$$

$$= \boxed{-\frac{2}{9} x^{-1} + C}$$