Recitation # 6 - Physical Applications - Instructor Notes

Group work:

Problem 1 *A metal bar 8 meters long has density (in kg/m) x meters from the left end of the bar given by*

$$\rho(x) = \begin{cases} x^2 & \text{if } 0 \le x \le 3 \\ \dfrac{4}{x} & \text{if } 3 < x \le 8. \end{cases}$$

Find the mass of the portion of the bar from 2 meters to 5 meters from its left end.

Instructor Notes: Give students about 5 minutes to work on this problem in groups before discussing.

Problem 2 *Assume that a spring, whose equilibrium length is 1 meter, obeys Hooke's law. It requires $25J$ of work to compress this spring to $0.7m$ in length.*

(a) *Find the work required to compress the spring to $0.6m$ in length.*

(b) *Find the work required to stretch the spring from $1.4m$ to $1.8m$.*

Instructor Notes: You may need to remind students beforehand that Hooke's law says that the force exerted by the spring is $F(x) = k\,x$, where k is some constant dependent on the spring and x is the displacement from the equilibrium position.

Problem 3 *A bucket with mass $100kg$ when filled with oil is lifted at a constant rate by a pulley from the bottom to the top of a 60-meter hole.*

(a) *Assuming the chain used to lift the bucket has negligible mass, compute the work needed to lift the bucket from the bottom of the hole to the top.*

(b) *Assuming the chain used to lift the bucket has an evenly-distributed mass of* $12kg$*, set up an integral that will compute the amount of work needed to lift the bucket from the bottom to the top of the hole.*

(c) *Assuming the chain used to lift the bucket has an evenly-distributed mass of* $12kg$ *and that oil is leaking out of a hole in the bucket at a constant rate such that the bucket and the remaining oil has a mass of* $27kg$ *when it reaches the top, set up an integral that will compute the amount of work to lift the bucket from the bottom of the hole to the top (assuming that the bucket is lifted at a constant rate).*

Instructor Notes: Have students work on part (a). Then discuss part (a) as a class before having the students work on part (b). Note that the problem just asks for students to set up the integral. Discuss part (b) as a class before having students continue to do part (c).

Problem 4 *A truncated conical tank that is* 8 *feet tall,* 5 *feet across the top, and* 3 *feet across the bottom is filled with water (water weighs* 62.5 *pounds per cubic foot).*

(a) *Set up an integral that will compute the work needed to pump the water out of the top of the tank.*

(b) *Set up an integral that will compute the work needed to pump the water out of a pipe that sticks* 2 *feet above the top of the tank.*

Instructor Notes: I imainge that it will be unlikely that you complete this exercise.
