Recitation #13: Direction fields and Separable Differential Equations

Warm up:

Which of the following differential equations are separable?

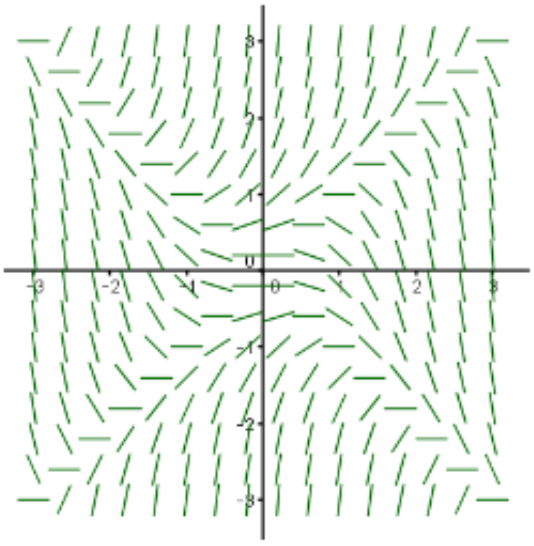
(a) $y' = \dfrac{ty}{t^2+1}$,

(b) $\dfrac{dy}{dx} = x^2\sin(3y) - x^2$,

(c) $y' = t^2 - y$.

Group work:

Problem 1 (a) *The following is a direction field for the differential equation* $\dfrac{dy}{dx} = y^2 - x^2$.



Sketch the solution such that $y\left(\dfrac{1}{2}\right) = 1$.

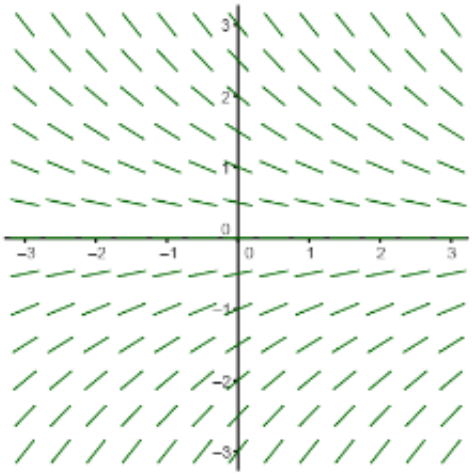
(b) *Use Euler's Method to give a numerical estimate to the solution of the differential equation* $y' = y^2 - t^2$ *at* $y(2)$ *that goes through the point* $\left(\frac{1}{2}, 1\right)$. *Use* $\Delta t = 0.5$.

Problem 2 *Describe why the following direction field could be the direction field for the differential equation*

$$\frac{dy}{dt} = y\cos(t)$$

but ***not*** *for*

$$\frac{dy}{dt} = y\sin(t) \qquad \text{or} \qquad \frac{dy}{dt} = t\cos(y).$$



Problem 3 *Match each of the following differential equations with a corresponding direction field (if it is present):*

i. $y' = \dfrac{t}{2+y}$

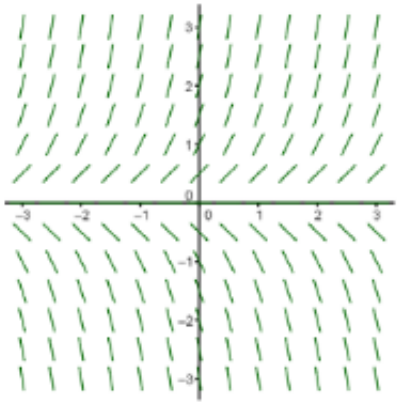
ii. $y' = \cos(t+y)$

iii. $y' = 1 + y^2$

iv. $y' = ty$

b.

b.



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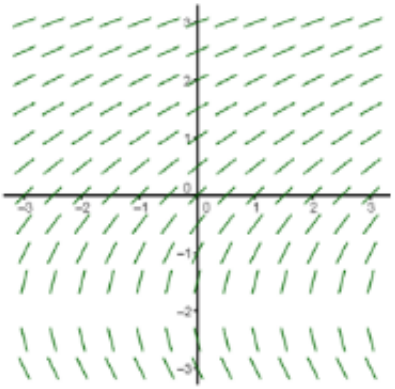

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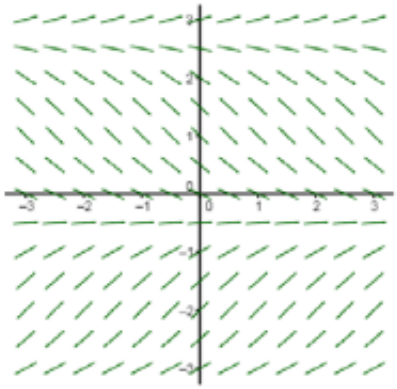

Problem 4 *Which of the following are separable differential equations? For those that are, solve them, assuming that* $y(4) = 5$.

(a) $y' = x^2 + y^2$

(b) $y' = x + xy^2$

(c) $y' = e^{2x-y}$