Recitation # 11: Improper Integrals and Differential Equations

Warm up:

True or False: It is possible for a region to be infinitely long but have a finite area.

Group work:

Problem 1 Review of limits:

(a)
$$\lim_{x \to -\infty} \left(3x^{-6} + e^{5x} + \frac{\sin x}{x^2 + 3} \right)$$

(b)
$$\lim_{x \to \infty} \frac{x}{\sqrt{9x^2 + 4}}$$

(c)
$$\lim_{x \to -\infty} \arctan x$$

Problem 2 Determine if the given integral converges or diverges. If it converges, find the value.

$$\int_{-1}^{\infty} \frac{3}{2x+1} \, dx$$

Problem 3 Which of the following is a solution to the differential equation y'' + 9y = 0?

(a)
$$y = e^{3t} + e^{-3t}$$

(b)
$$y = C(t^2 + t)$$

(c)
$$y = \sin(3t) + 6$$

(d)
$$y = 5\cos(3t) - 7\sin(3t)$$

(e)
$$y = A\cos(3t) + B\sin(3t)$$
 (where A and B are real numbers.)

Problem 4 Explain why the functions with the given graphs cannot be solutions of the differential equation $y' = e^x(y-1)^2$.

