# Recitation 27: Cross products - Instructor Notes

## Warm up:

If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are vectors in 3-space $\mathbb{R}^3$, which of the following make sense?

(a) $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$ (d) $(\vec{a} \cdot \vec{b}) + \vec{c}$ (g) $\vec{a} \cdot (\vec{b} \times \vec{c})$

(b) $(\vec{a} \cdot \vec{b})\vec{c}$ (e) $(\vec{a} \times \vec{b}) + \vec{c}$ (h) $\vec{a} \times (\vec{b} \cdot \vec{c})$

(c) $(\vec{a} \times \vec{b}) \cdot \vec{c}$ (f) $\vec{a} \cdot (\vec{b} + \vec{c})$ (i) $(\vec{a} \times \vec{b})\vec{c}$

**Instructor Notes:** This problem can be split up among the groups if the instructor likes (with maybe 3 or so per group).

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## Group work:

**Problem 1** *Given three dimensional vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$, use dot product or cross product notation to describe the following vectors:*

(a) *The vector projection of $\vec{w}$ onto $\vec{u}$.*

(b) *A vector orthogonal to both $\vec{u}$ and $\vec{v}$.*

(c) *A vector with the length of $\vec{v}$ and the direction of $\vec{w}$.*

(d) *A vector orthogonal to $\vec{u} \times \vec{v}$ and $\vec{w}$.*

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**Instructor Notes:** This problem and the Warm-up are meant to force the students to make sense of scalar vs. vector quantities, as well as what quantities the dot and cross products produce.

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**Problem 2** *Find a vector of length 7 that is perpendicular to both* $\langle 5, -1, 8\rangle$ *and* $\langle -2, 10, 5\rangle$.

**Instructor Notes:** Using cross product to find perpendicular vectors.

**Problem 3** *Find the area of the triangle in* $\mathbb{R}^3$ *with vertices at* $P(2, -1, 0)$, $Q(1, 1, 4)$ *and* $R(2, -1, 6)$.

**Instructor Notes:** Students should know that we can find the areas of triangles and parallelograms in $\mathbb{R}^3$ by using the cross product.

**Problem 4** *A wrench that is* 30*cm long lies along the positive y-axis and grips a bolt at the origin. A force is applied in the direction* $\langle 0, 3, -4\rangle$ *at the end of the wrench. Find the magnitude of the force needed to supply* $100J$ *of torque to the bolt.*

**Instructor Notes:** One goal in this problem is for students to make sense of the right-hand rule. The students need to know which direction of rotation tightens or loosens a bolt.