

Section - 2.5: Limits at Infinity

Warm up:

Match functions 1-6 with graphs A-F in the figure without using a graphing utility.

1. $f(x) = \frac{x}{x^2 + 1}$

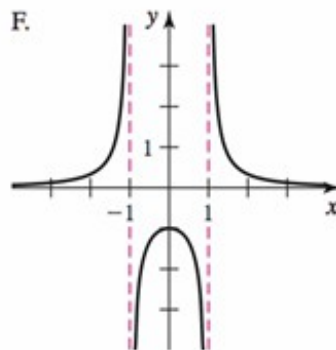
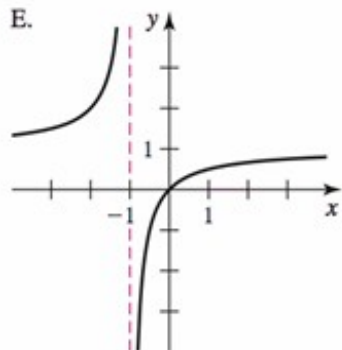
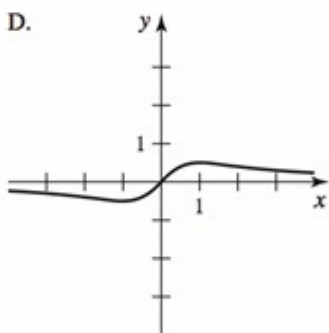
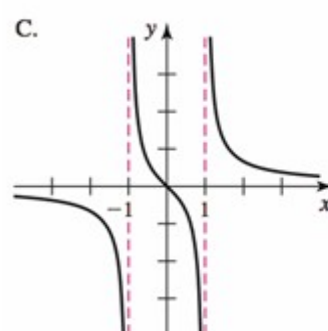
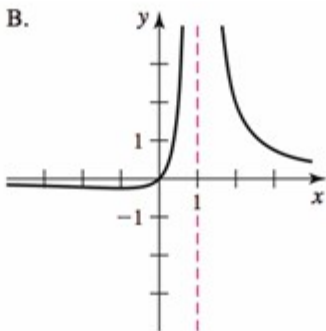
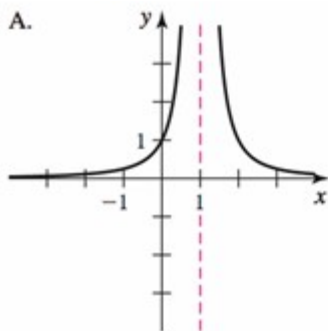
2. $f(x) = \frac{x}{x^2 - 1}$

3. $f(x) = \frac{1}{x^2 - 1}$

4. $f(x) = \frac{x}{(x - 1)^2}$

5. $f(x) = \frac{1}{(x - 1)^2}$

6. $f(x) = \frac{x}{x + 1}$



Group work:

Problem 1 Find any vertical or horizontal asymptotes for the given function. Be sure to tell where the function crosses its horizontal or vertical asymptote. Also, when finding vertical asymptotes, be sure to say how the function approaches the asymptote on each side:

(a) $f(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5}$

(b) $f(x) = \sqrt{x^2 + 8x + 1} - x$

Problem 2 Find any vertical, horizontal, or slant asymptotes for the function $f(x) = \frac{x^2 + 7x + 11}{x - 3}$. Be sure to tell where the function crosses its horizontal or vertical asymptotes. Also, when finding vertical asymptotes, be sure to say how the function approaches the asymptote on each side.

Problem 3 Sketch the graph of a function with all of the following properties:

$$\begin{aligned} \lim_{x \rightarrow -2^-} f(x) = \infty, f(-2) = 7, f(1) = 2, \lim_{x \rightarrow \infty} f(x) = 3, \lim_{x \rightarrow -\infty} f(x) = -\infty, \\ \lim_{x \rightarrow 5} f(x) = \infty, \lim_{x \rightarrow 9} f(x) = 3, f(9) = 1, \lim_{x \rightarrow -3^-} f(x) = \infty, \lim_{x \rightarrow -3^+} f(x) = -\infty, \\ f(4) \text{ is undefined, } f(x) = 3 \text{ for } x > 9 \end{aligned}$$