

Recitation #8 - 3.2 Working with Derivatives

Warm up:

- (1) If $f'(2)$ exists, then $\lim_{x \rightarrow 2} f(x)$
- (a) must exist, but more information is needed if we want to find it.
 - (b) is equal to $f(2)$.
 - (c) is equal to $f'(2)$.
 - (d) need not exist.
- (2) The statement $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ means
- (a) $\frac{0}{0} = 1$.
 - (b) the tangent line to $y = \sin x$ at $(0, 0)$ has slope 1.
 - (c) you can cancel the x 's.
 - (d) for all x near 0, $\sin x = x$.
 - (e) for all x near 0, $\sin x \approx x$.

Group work:

Problem 1 Look at the following work for finding the derivative of $f(x) = \frac{x}{x-5}$ at the point $x = 3$ using the definition of the derivative. Adapt the following

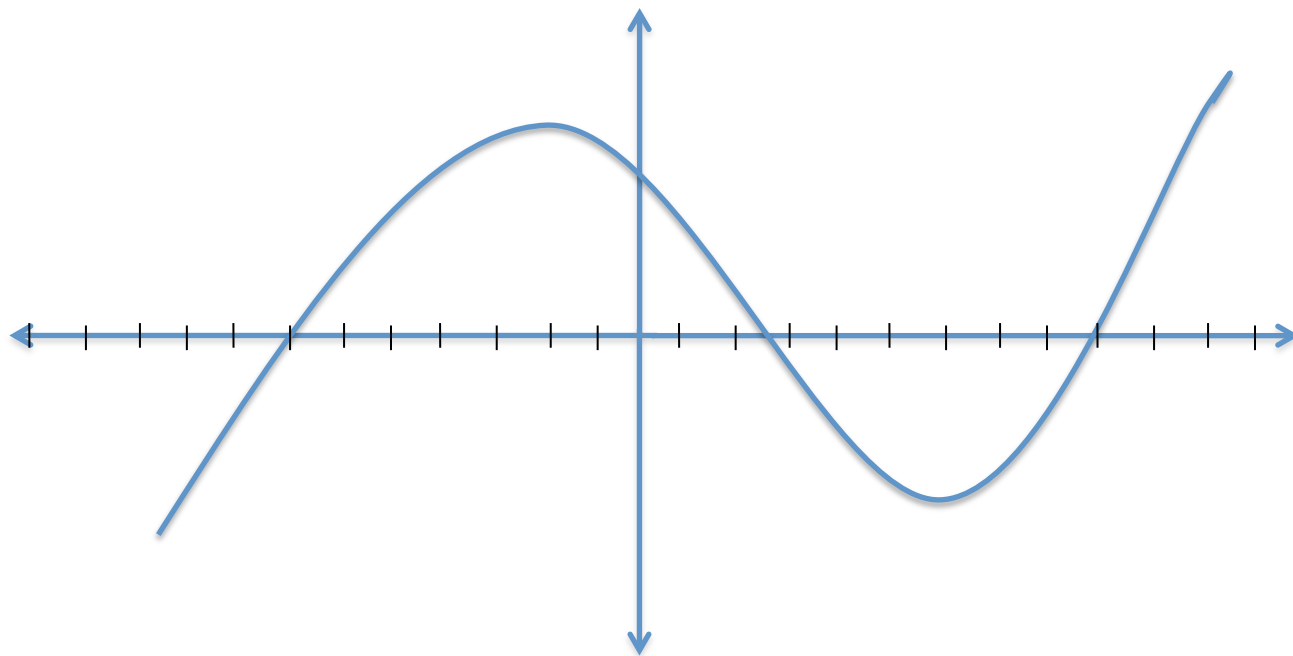
work to find $f'(x)$.

$$\begin{aligned}
 f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{\frac{3+h}{(3+h)-5} - \frac{3}{3-5}}{h} \\
 &= \lim_{h \rightarrow 0} \left[\frac{3+h}{(3+h)-5} - \frac{3}{3-5} \right] \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \left[\frac{(3+h)(3-5)}{[(3+h)-5](3-5)} - \frac{3[(3+h)-5]}{(3-5)[(3+h)-5]} \right] \cdot \frac{1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(3+h)(3-5) - 3[(3+h)-5]}{[(3+h)-5](3-5)h} \\
 &= \lim_{h \rightarrow 0} \frac{3 \cdot 3 + 3 \cdot (-5) + 3h - 5h - [3 \cdot 3 + 3h + 3 \cdot (-5)]}{[(3+h)-5](3-5)h} \\
 &= \lim_{h \rightarrow 0} \frac{9 - 15 + 3h - 5h - 9 + 3h + 15}{[(3+h)-5](3-5)h} \\
 &= \lim_{h \rightarrow 0} \frac{-5h}{[(3+h)-5](3-5)h} \\
 &= \lim_{h \rightarrow 0} \frac{-5}{[(3+h)-5](3-5)} = \frac{-5}{[(3+0)-5](3-5)} \\
 &= \frac{-5}{(3-5)^2} = \frac{-5}{4}
 \end{aligned}$$

Problem 2 Given the following graph of $f(x)$, find the values where $f(x)$ is zero, positive/negative, increasing/decreasing, and where $f(x)$ has its highest and lowest values. Additionally, find where the graph of $f(x)$ is the steepest. Then, without sketching the graph of $f'(x)$, determine for which values of x is $f'(x)$ zero, positive/negative, and where $f'(x)$ has the highest and lowest values.

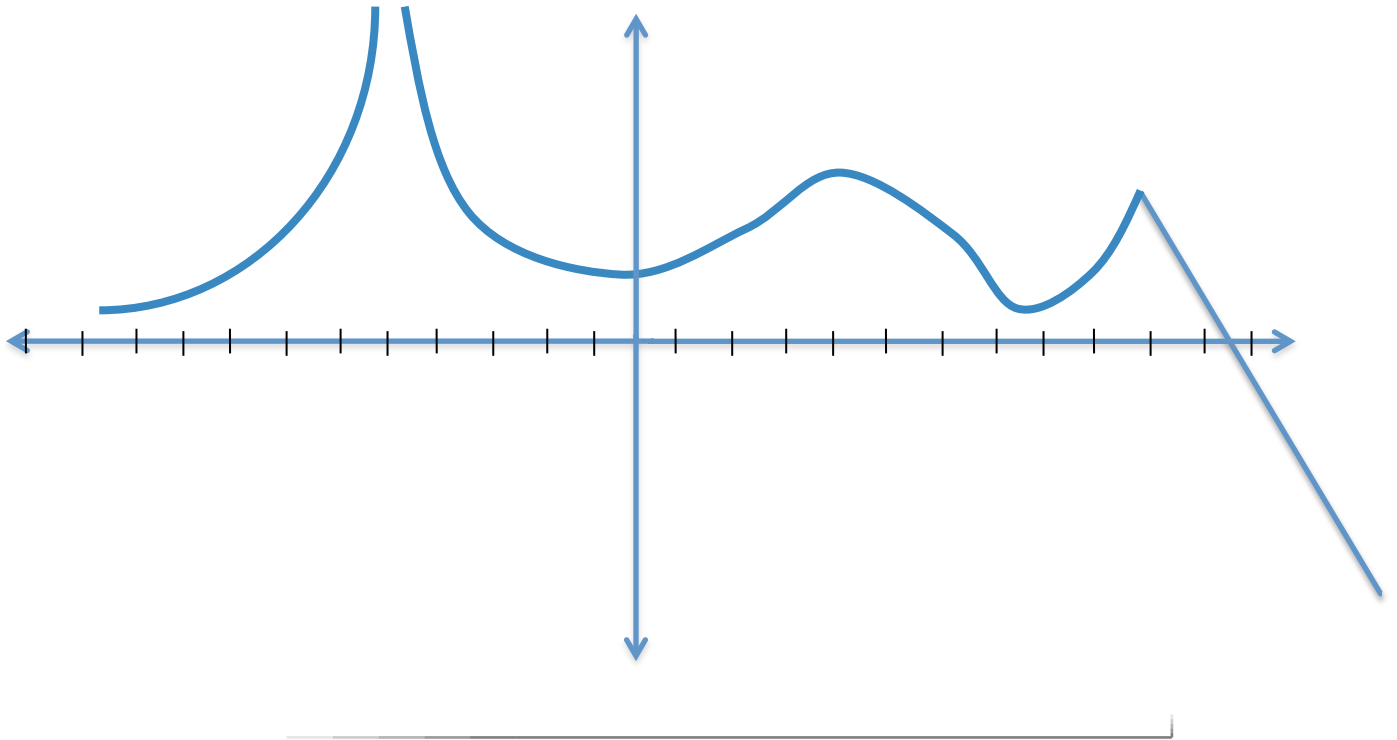
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Then sketch $f'(x)$.



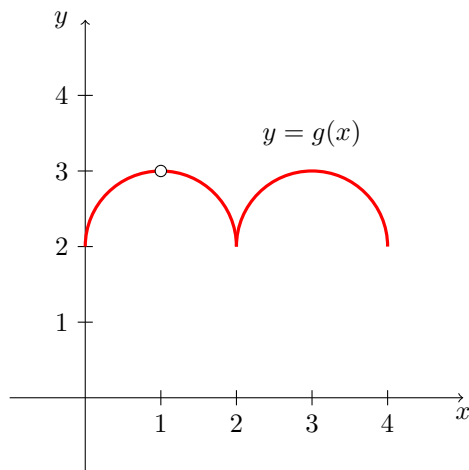
Problem 3 Given the graph of $y = f(x)$, sketch the graph of the derivative $f'(x)$.

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Problem 4 Use the graph of g in the figure to do the following.

- (a) Find the values of x in $(0, 4)$ at which g is not continuous.
- (b) Find the values of x in $(0, 4)$ at which g is not differentiable.



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