Recitation #8 - 3.2 Working with Derivatives

Warm up:

(1) If $f'(2)$ exists, then $\lim_{x\to 2} f(x)$

- (a) must exist, but more information is needed if we want to find it.
- (b) is equal to $f(2)$.
- (c) is equal to $f'(2)$.
- (d) need not exist.

(2) The statement $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$ means

- (a) $\dfrac{0}{0} = 1$.
- (b) the tangent line to $y = \sin x$ at $(0, 0)$ has slope 1.
- (c) you can cancel the x's.
- (d) for all x near 0, $\sin x = x$.
- (e) for all x near 0, $\sin x \approx x$.

Group work:

Problem 1 *Look at the following work for finding the derivative of* $f(x) = \dfrac{x}{x-5}$ *at the point* $x = 3$ *using the definition of the derivative. Adapt the following* *work to find $f'(x)$.*

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{\frac{3+h}{(3+h)-5} - \frac{3}{3-5}}{h}$$

$$= \lim_{h \to 0} \left[\frac{3+h}{(3+h)-5} - \frac{3}{3-5} \right] \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \left[\frac{(3+h)(3-5)}{[(3+h)-5](3-5)} - \frac{3[(3+h)-5]}{(3-5)[(3+h)-5]} \right] \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{(3+h)(3-5) - 3[(3+h)-5]}{[(3+h)-5](3-5)h}$$

$$= \lim_{h \to 0} \frac{3 \cdot 3 + 3 \cdot (-5) + 3h - 5h - [3 \cdot 3 + 3h + 3 \cdot (-5)]}{[(3+h)-5](3-5)h}$$

$$= \lim_{h \to 0} \frac{\cancel{9} - \cancel{15} + \cancel{3h} - 5h - \cancel{9} - \cancel{3h} + \cancel{15}}{[(3+h)-5](3-5)h}$$

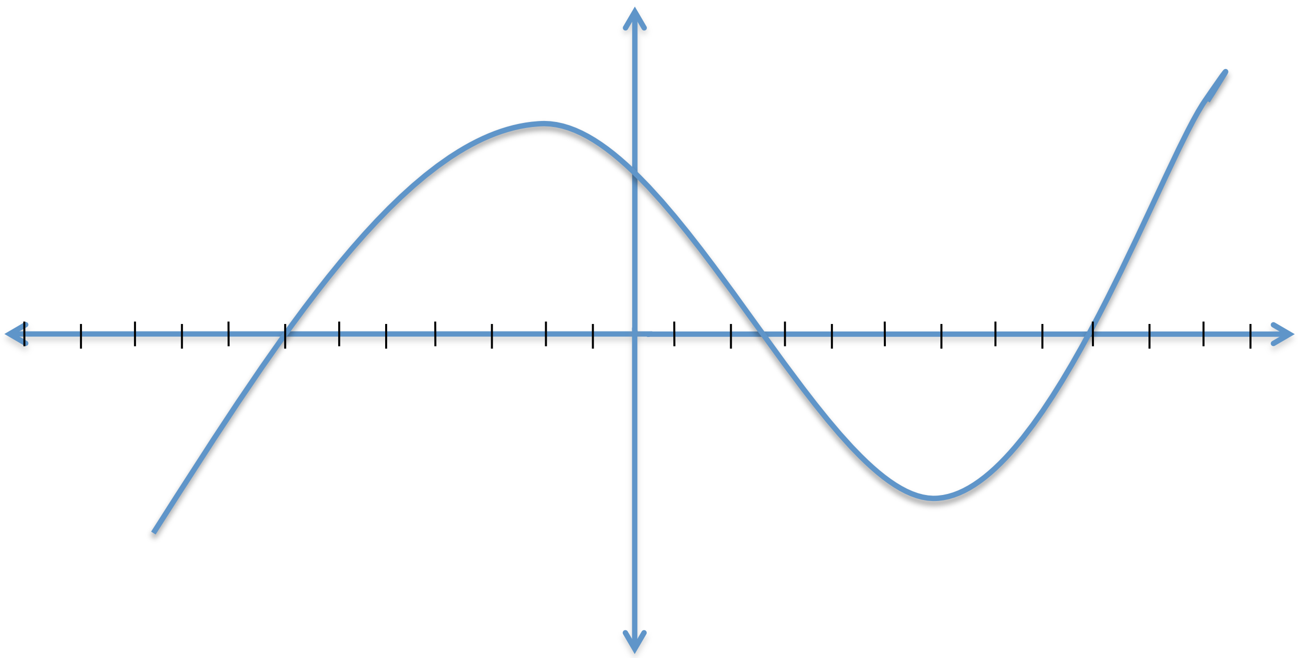
$$= \lim_{h \to 0} \frac{-5\cancel{h}}{[(3+h)-5](3-5)\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-5}{[(3+h)-5](3-5)} = \frac{-5}{[(3+0)-5](3-5)}$$

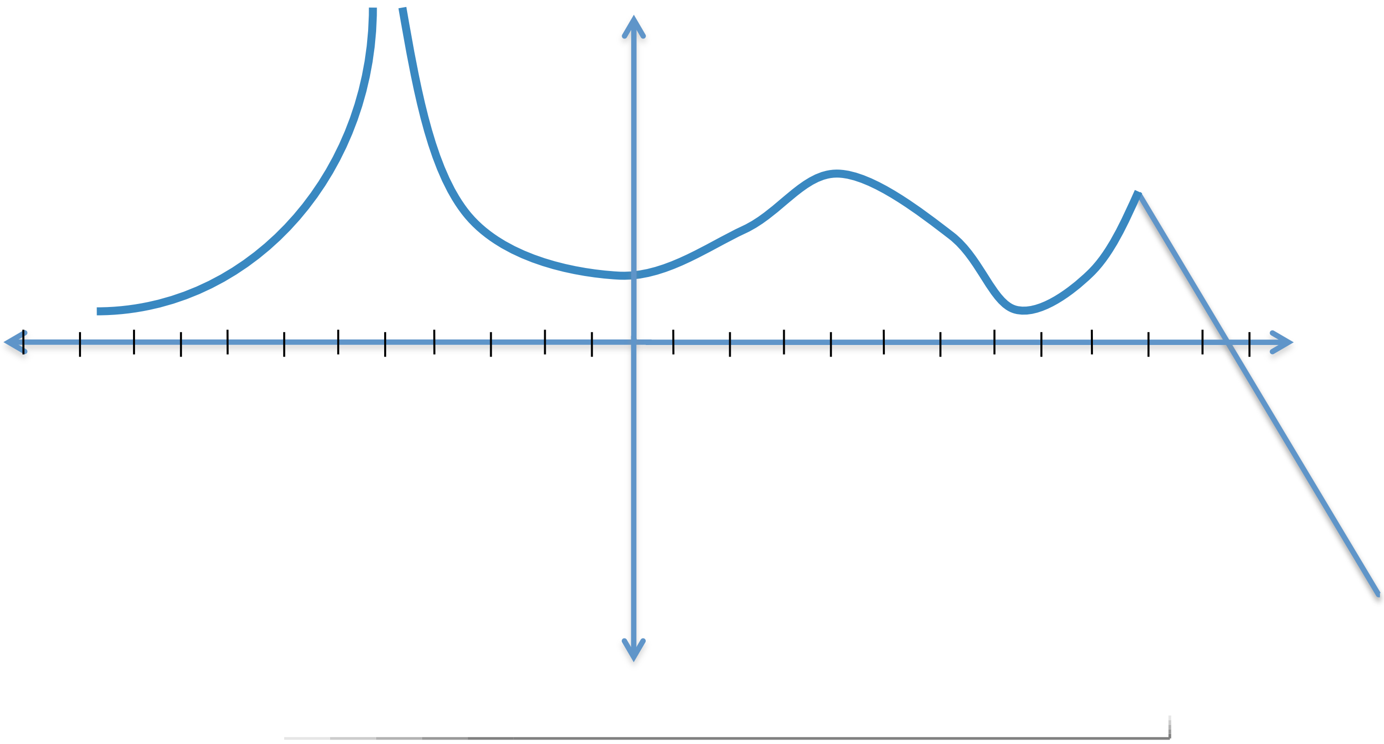
$$= \frac{-5}{(3-5)^2} = \frac{-5}{4}$$

Problem 2 *Given the following graph of $f(x)$, find the values where $f(x)$ is zero, positive/negative, increasing/decreasing, and where $f(x)$ has its highest and lowest values. Additionally, find where the graph of $f(x)$ is the steepest. Then, without sketching the graph of $f'(x)$, determine for which values of x is $f'(x)$ zero, positive/negative, and where $f'(x)$ has the highest and lowest values.*

Then sketch $f'(x)$.



Problem 3 *Given the graph of $y = f(x)$, sketch the graph of the derivative $f'(x)$.*



Problem 4 *Use the graph of g in the figure to do the following.*

(a) *Find the values of x in $(0, 4)$ at which g is not continuous.*

(b) *Find the values of x in $(0, 4)$ at which g is not differentiable.*