3.3 Rules of Differentiation (Solutions)

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Warm up:

Solution:

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Group work:

Problem 1 *Use the "short-cut derivative rules" to compute the derivatives of the following functions:*

(a) $f(x) = \sqrt{x}$

Solution:

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}.$$

So

$$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}.$$

(b) $f(x) = \dfrac{5}{x^2}$

Solution:

$$f(x) = \frac{5}{x^2} = 5x^{-2}.$$

So

$$f'(x) = 5(-2)x^{-2-1} = -10x^{-3} = \frac{-10}{x^3}.$$

(c) $f(x) = x^5 + 4x^3 + \pi$

Solution:

$$f'(x) = 5x^{5-1} + 4(3)x^{3-1} + 0 = 5x^4 + 12x^2.$$

Note that $\dfrac{d}{dx}(\pi) = 0$ *because* π *is a constant.*

Problem 2 *Find the slope of the function $f(x) = 2x^3 - 5x^2 + 7x - 9$ at $x = 3$ two ways. First, by finding it directly by the limit definition and, secondly, by using the "derivative short-cut" to find $f'(x)$ and then evaluating it at $x = 3$. Then, find the equation of the tangent line to the graph of $f(x)$ at $x = 3$.*

Solution: $f'(3) = \lim_{x\to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x\to 3} \frac{(2x^3 - 5x^2 + 7x - 9) - (54 - 45 + 21 - 9)}{x - 3} = \lim_{x\to 3} \frac{(2x^3 - 5x^2 + 7x - 9) - 21}{x - 3} = \lim_{x\to 3} \frac{2x^3 - 5x^2 + 7x - 30}{x - 3} = \lim_{x\to 3} \frac{(x - 3)(2x^2 + x + 10)}{x - 3} = \lim_{x\to 3} (2x^2 + x + 10) = 18 + 3 + 10 = 31.$

Using the "short-cut rules", $f'(x) = 6x^2 - 10x + 7$ *and so* $f'(3) = 6(9) - 10(3) + 7 = 54 - 30 + 7 = 31.$

The tangent line to the graph of $f(x)$ *at* $x = 3$ *has slope* $m = 31$ *and goes through the point* $(3, 21)$. *Thus, the equation of the tangent line is* $y - 21 = 31(x - 3)$, *or* $y = 31x - 72$.