Recitation #1 Chapter 1 - Precalculus Review (Solutions)

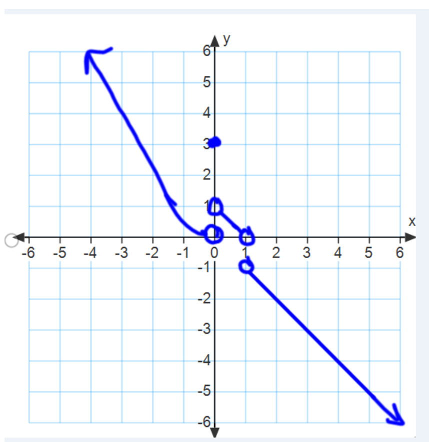
Warm up:

If f is always increasing, is f^{-1} always increasing?

Solution: Yes, if f is always increasing, then f^{-1} is always increasing. To prove this, suppose $x < y$. Then it follows from the definition of f^{-1} that $f(f^{-1}(x)) = x < y = f(f^{-1}(y))$. But since f is increasing, this implies that $f^{-1}(x) < f^{-1}(y)$.

Group work:

Problem 1 *Given the graph of the function f below, answer the following questions.*



(a) *What is the domain of f?*

Solution: $(-\infty, 1) \cup (1, \infty)$

(b) *What is the range of f?*

Solution: *The range is* $(-\infty, -1) \cup (0, \infty)$

(c) *What is $f(0)$? $f(1)$? $f(2)$?*

Solution: *$f(0) = 3$, $f(1)$ does not exist, $f(2) = -2$*

(d) *Does f have an inverse? Why or why not?*

Solution: *No, the function does not have an inverse. It is not one-to-one (ie, it does not pass the horizontal line test).*

Problem 2 *Find the inverse $y = f^{-1}(x)$ of the function. State the domain and range of the inverse.*

(a) $f(x) = x^2 - 4x - 5$ *(when $x \geq 2$).*

Solution: $f(x) = y = x^2 - 4x - 5$

$y - 5 = x^2 - 4x$

complete the square

$y + 5 + 4 = x^2 - 4x + 4 = (x-2)^2$

$\sqrt{y+9} = x - 2$

$2 + \sqrt{y+9} = x$

$f^{-1}(x) = 2 + \sqrt{x+9}$

The domain of $f^{-1}(x)$ is $[-9, \infty)$. The range is $[2, \infty)$.

(b) $f(x) = \sqrt[4]{x+2}$.

Solution: $y = \sqrt[4]{x+2}$

$y^4 = x + 2$

$y^4 - 2 = x$

$f^{-1}(x) = x^4 - 2$

The domain of $f^{-1}(x)$ is $[0, \infty)$. The range is $[-2, \infty)$.

(c) $f(x) = \dfrac{1}{(x+2)^2}$ *(when $x > -2$).*

Solution: $y = \dfrac{1}{(x+2)^2}$

$$(x+2)^2 = \frac{1}{y}$$

$$x + 2 = \sqrt{\frac{1}{y}} = \frac{1}{\sqrt{y}}$$

$x = \dfrac{1}{\sqrt{y}} - 2$

$f^{-1}(x) = \dfrac{1}{\sqrt{x}} - 2$

The domain of $f^{-1}(x)$ is $(0, \infty)$. The range is $(-2, \infty)$.

Problem 3 *Find all values of x which satisfy the equation.*

(a) $\log_x 25 = 2$

Solution: $x^2 = 25 \implies x = \pm 5$. *But the base of a logarithm is always a positive number different from 1, and therefore $x = 5$ is the only solution.*

(b) $7^x = 15$

Solution: $x = \log_7 15$.

Problem 4 *Find all values which satisfy the given equation.*

(a) $\cos(x) = 1$

Solution: *This is asking for the angles such that cosine of that angle equals 1. Thus, $x = 2\pi n$, where n is any integer.*

(b) $\sin(3\theta) = \dfrac{\sqrt{3}}{2}$ *for* $0 \le \theta \le 2\pi$

Solution: *Let $3\theta = x$, so that we have that*

$\dfrac{\sqrt{3}}{2} = \sin(x)$.

So $x = \dfrac{\pi}{3} + 2\pi n$ or $x = \dfrac{2\pi}{3} + 2\pi n$ for n any integer. Since $x = 3\theta$, we can solve for theta to obtain $\theta = \dfrac{\pi}{9} + \dfrac{2}{3}\pi n$ or $\theta = \dfrac{2\pi}{9} + \dfrac{2}{3}\pi n$, where n is again any integer.

We are only looking for solutions of θ in $[0, 2\pi]$, and so our solutions are

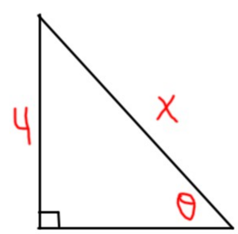
$\theta = \dfrac{\pi}{9}, \dfrac{2\pi}{9}, \dfrac{7\pi}{9}, \dfrac{8\pi}{9}, \dfrac{13\pi}{9}, \dfrac{14\pi}{9}$

Problem 5 (a) *Simplify the expression:* $\cos^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

Solution: $\sin\left(\frac{\pi}{2}\right) = 1$, *and so we are looking for* $\cos^{-1}(1)$. *The range of* $\cos^{-1}$ *is* $[0, \pi]$, *and so* $\cos^{-1}(1) = 0$.

(b) *Simplify the expression:* $\tan\left(\cos^{-1}\left(\frac{4}{x}\right)\right)$

Solution: *Let* $\theta = \cos^{-1}\left(\frac{4}{x}\right)$. *Then* $\cos\theta = \frac{4}{x}$. *Consider the corresponding right triangle*



Call the adjacent side y. *We use the Pythagorean Theorem to obtain*
$x^2 = 16 + y^2 \implies y = \sqrt{x^2 - 16}$.
Then
$$\tan\left(\cos^{-1}\left(\frac{4}{x}\right)\right) = \tan\theta = \frac{4}{y} = \frac{4}{\sqrt{x^2 - 16}}.$$