

Recitation #1 Chapter 1 - Precalculus Review (Solutions)

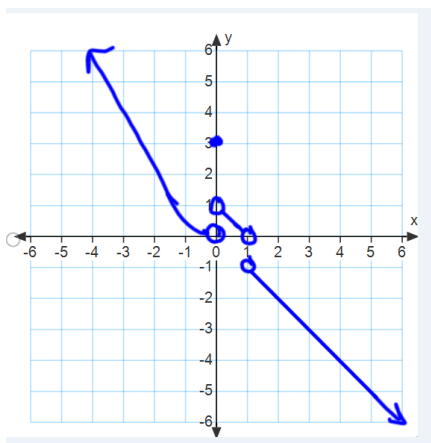
Warm up:

If f is always increasing, is f^{-1} always increasing?

Solution: Yes, if f is always increasing, then f^{-1} is always increasing. To prove this, suppose $x < y$. Then it follows from the definition of f^{-1} that $f(f^{-1}(x)) = x < y = f(f^{-1}(y))$. But since f is increasing, this implies that $f^{-1}(x) < f^{-1}(y)$.

Group work:

Problem 1 Given the graph of the function f below, answer the following questions.



(a) What is the domain of f ?

Solution: $(-\infty, 1) \cup (1, \infty)$

(b) What is the range of f ?

Solution: The range is $(-\infty, -1) \cup (0, \infty)$

(c) What is $f(0)$? $f(1)$? $f(2)$?

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Solution: $f(0) = 3$, $f(1)$ does not exist, $f(2) = -2$

(d) Does f have an inverse? Why or why not?

Solution: No, the function does not have an inverse. It is not one-to-one (ie, it does not pass the horizontal line test).

Problem 2 Find the inverse $y = f^{-1}(x)$ of the function. State the domain and range of the inverse.

(a) $f(x) = x^2 - 4x - 5$ (when $x \geq 2$).

Solution: $f(x) = y = x^2 - 4x - 5$

$$y - 5 = x^2 - 4x$$

complete the square

$$y + 5 + 4 = x^2 - 4x + 4 = (x - 2)^2$$

$$\sqrt{y + 9} = x - 2$$

$$2 + \sqrt{y + 9} = x$$

$$f^{-1}(x) = 2 + \sqrt{x + 9}$$

The domain of $f^{-1}(x)$ is $[-9, \infty)$. The range is $[2, \infty)$.

(b) $f(x) = \sqrt[4]{x + 2}$.

Solution: $y = \sqrt[4]{x + 2}$

$$y^4 = x + 2$$

$$y^4 - 2 = x$$

$$f^{-1}(x) = x^4 - 2$$

The domain of $f^{-1}(x)$ is $[0, \infty)$. The range is $[-2, \infty)$.

(c) $f(x) = \frac{1}{(x + 2)^2}$ (when $x > -2$).

Solution: $y = \frac{1}{(x + 2)^2}$

$$(x + 2)^2 = \frac{1}{y}$$

$$x + 2 = \sqrt{\frac{1}{y}} = \frac{1}{\sqrt{y}}$$

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$$x = \frac{1}{\sqrt{y}} - 2$$

$$f^{-1}(x) = \frac{1}{\sqrt{x}} - 2$$

The domain of $f^{-1}(x)$ is $(0, \infty)$. The range is $(-2, \infty)$.

Problem 3 Find all values of x which satisfy the equation.

(a) $\log_x 25 = 2$

Solution: $x^2 = 25 \implies x = \pm 5$. But the base of a logarithm is always a positive number different from 1, and therefore $x = 5$ is the only solution.

(b) $7^x = 15$

Solution: $x = \log_7 15$.

Problem 4 Find all values which satisfy the given equation.

(a) $\cos(x) = 1$

Solution: This is asking for the angles such that cosine of that angle equals 1. Thus, $x = 2\pi n$, where n is any integer.

(b) $\sin(3\theta) = \frac{\sqrt{3}}{2}$ for $0 \leq \theta \leq 2\pi$

Solution: Let $3\theta = x$, so that we have that

$$\frac{\sqrt{3}}{2} = \sin(x).$$

So $x = \frac{\pi}{3} + 2\pi n$ or $x = \frac{2\pi}{3} + 2\pi n$ for n any integer. Since $x = 3\theta$, we can solve for theta to obtain $\theta = \frac{\pi}{9} + \frac{2}{3}\pi n$ or $\theta = \frac{2\pi}{9} + \frac{2}{3}\pi n$, where n is again any integer.

We are only looking for solutions of θ in $[0, 2\pi]$, and so our solutions are

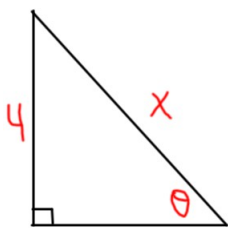
$$\theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}$$

Problem 5 (a) Simplify the expression: $\cos^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

Solution: $\sin\left(\frac{\pi}{2}\right) = 1$, and so we are looking for $\cos^{-1}(1)$. The range of \cos^{-1} is $[0, \pi]$, and so $\cos^{-1}(1) = 0$.

(b) Simplify the expression: $\tan\left(\sin^{-1}\left(\frac{4}{x}\right)\right)$

Solution: Let $\theta = \sin^{-1}\left(\frac{4}{x}\right)$. Then $\sin \theta = \frac{4}{x}$. Consider the corresponding right triangle



Call the adjacent side y . We use the Pythagorean Theorem to obtain $x^2 = 16 + y^2 \implies y = \sqrt{x^2 - 16}$.

Then

$$\tan\left(\sin^{-1}\left(\frac{4}{x}\right)\right) = \tan \theta = \frac{4}{y} = \frac{4}{\sqrt{x^2 - 16}}.$$
