

Recitation #10 - 3.5 Derivatives of Trig Functions (Solutions)

Warm up:

Solution:

Group work:

Problem 1 Find the following limits:

(a) $\lim_{x \rightarrow 0} \frac{\sin(8x)}{x}$

Solution:

(b) $\lim_{x \rightarrow 0} \frac{\cos^2(x) - 1}{4x}$

Solution:

(c) $\lim_{x \rightarrow 0} \frac{x}{\tan(5x)}$

Solution:

Problem 2 Find the derivative of the following functions:

(a) $f(x) = \frac{x+5}{7x^6 + \cot(x)}$

Solution:
$$f'(x) = \frac{(7x^6 + \cot(x))(1) - (x+5)(42x^5 - \csc^2(x))}{(7x^6 + \cot(x))^2} = f'(x) = \frac{7x^6 + \cot(x) - (x+5)(42x^5 - \csc^2(x))}{(7x^6 + \cot(x))^2}.$$

(b) $f(x) = \sin(x) \cos(x)$

Solution:
$$f'(x) = (\cos(x))(\cos(x)) + (\sin(x))(-\sin(x)) = \cos^2(x) - \sin^2(x).$$

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(c) $f(x) = \frac{e^x \tan(x)}{\sec(x) + 2}$

Solution:
$$f'(x) = \frac{(\sec(x) + 2)(e^x \tan(x) + e^x \sec^2(x)) - e^x \tan(x)(\sec(x) \tan(x))}{(\sec(x) + 2)^2} = \frac{e^x[(\sec(x) + 2)(\tan(x) + \sec^2(x)) - \sec(x) \tan^2(x)]}{(\sec(x) + 2)^2}.$$

(d) $f(x) = \sin(x) \cos(x) e^{3x}$

Solution:
$$\begin{aligned} f'(x) &= \frac{d}{dx}(\sin(x) \cos(x)) e^{3x} + (\sin(x) \cos(x)) \frac{d}{dx}(e^{3x}) \\ &= (\cos^2(x) - \sin^2(x)) e^{3x} + 3e^{3x} \sin(x) \cos(x) \\ &= e^{3x}(\cos^2(x) + 3 \sin(x) \cos(x) - \sin^2(x)). \end{aligned}$$

Problem 3 Solution:

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