Recitation #4 - 2.3: Limit Laws

Warm up:

Below is a table listing all of the Limit Laws, followed by an argument of what the limit of $\dfrac{5x^3 - 4\sqrt{x}}{\sqrt{x^5 - 87}}$ as x approaches 3 must be. State which limit law is used to justify each step.

THEOREM 2.3 Limit Laws

Assume $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. The following properties hold, where c is a real number and $m > 0$ and $n > 0$ are integers.

1. **Sum** $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. **Difference** $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. **Constant multiple** $\lim_{x\to a} [c\, f(x)] = c \lim_{x\to a} f(x)$
4. **Product** $\lim_{x\to a} [f(x)\, g(x)] = \left[\lim_{x\to a} f(x)\right]\left[\lim_{x\to a} g(x)\right]$
5. **Quotient** $\lim_{x\to a} \left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, provided $\lim_{x\to a} g(x) \neq 0$
6. **Power** $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$
7. **Fractional power** $\lim_{x\to a} [f(x)]^{n/m} = \left[\lim_{x\to a} f(x)\right]^{n/m}$, provided $f(x) \geq 0$, for x near a, if m is even and n/m is reduced to lowest terms

$$\lim_{x\to 3}\left(\frac{5x^3-4\sqrt{x}}{\sqrt{x^5-87}}\right)$$

$$= \frac{\lim_{x\to 3}(5x^3-4\sqrt{x})}{\lim_{x\to 3}\sqrt{x^5-87}}$$

$$= \frac{5\lim_{x\to 3}(x^3) - 4\lim_{x\to 3}\sqrt{x}}{\sqrt{\lim_{x\to 3}(x^5-87)}}$$

$$= \frac{5(\lim_{x\to 3} x)^3 - 4\sqrt{3}}{\sqrt{\lim_{x\to 3}(x^5) - \lim_{x\to 3}(87)}}$$

$$= \frac{5(3)^3 - 4\sqrt{3}}{\sqrt{3^5-87}}$$

$$= \frac{135-4\sqrt{3}}{\sqrt{156}}$$

Know: $\lim_{x\to a}(x) = a$

and $\lim_{x\to a}(c) = c$, where c is a constant

Group work:

Problem 1 *Evaluate the following limits algebraically using the limit laws.*

(a) $\lim\limits_{x\to 6} \dfrac{4x^2 - 144}{x - 6}$

(b) $\lim\limits_{x\to 6} \dfrac{x - 6}{\sqrt{2x - 8} - 2}$

(c) $\lim\limits_{x\to 2} \dfrac{(3x - 2)^2 - 16}{x - 2}$

(d) $\lim\limits_{x\to 1} \dfrac{\sqrt{5x - 2} - \sqrt{3}}{x - 1}$

Problem 2 *Suppose* $f(x) = \begin{cases} x^2 - ax & \text{if } x < 3 \\ a2^x + 7 + a & \text{if } x > 3 \end{cases}$

Find a *so that* $\lim\limits_{x\to 3} f(x)$ *exists.*

Problem 3 *Sketch the graph of a function with the given properties. You need not find a formula for the function:*

$f(3) = -2, f(-2) = 3, f(5) = 6, \lim\limits_{x\to 5^-} f(x) = -1, \lim\limits_{x\to 5^+} f(x) = 4, \lim\limits_{x\to 3} f(x) = 7$

$\lim\limits_{x\to -2^-} f(x) = 3, \lim\limits_{x\to -2^+} f(x) = 0, \lim\limits_{x\to 1^+} f(x) = 5$
