Recitation #1 Chapter 1 - Precalculus Review (Solutions)

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Warm up:

If f is always increasing, is f^{-1} always increasing?

Solution: Yes, if f is always increasing, then f^{-1} is always increasing. To prove this, suppose $x < y$. Then it follows from the definition of f^{-1} that $f(f^{-1}(x)) = x < y = f(f^{-1}(y))$. But since f is increasing, this implies that $f^{-1}(x) < f^{-1}(y)$.

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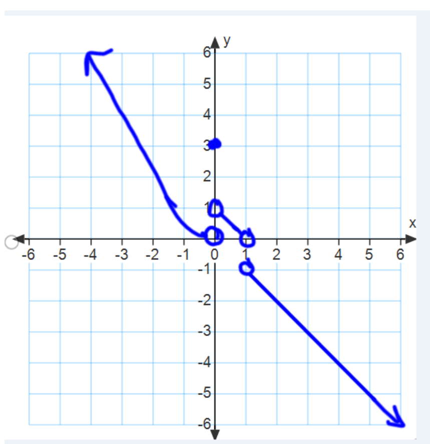
Group work:

Problem 1 *Given the graph of the function f below, answer the following questions.*



(a) *What is the domain of f?*

Solution: $(-\infty, 1) \cup (1, \infty)$

(b) *What is the range of f?*

Solution: *The range is* $(-\infty, -1) \cup (0, \infty)$

(c) *What is* $f(0)$*?* $f(1)$*?* $f(2)$*?*

Solution: $f(0) = 3$, $f(1)$ *does not exist,* $f(2) = -2$

(d) *Does* f *have an inverse? Why or why not?*

Solution: *No, the function does not have an inverse. It is not one-to-one (ie, it does not pass the horizontal line test).*

Problem 2 *Find the inverse* $y = f^{-1}(x)$ *of the function. State the domain and range of the inverse.*

(a) $f(x) = x^2 - 4x - 5$ *(when* $x \geq 2$*).*

Solution: *First notice that*

$$f(x) = y = x^2 - 4x - 5$$

$$y - 5 = x^2 - 4x$$

Then completing the square and solving for x*:*

$$y + 5 + 4 = x^2 - 4x + 4 = (x - 2)^2$$

$$\sqrt{y + 9} = x - 2$$

$$2 + \sqrt{y + 9} = x$$

$$f^{-1}(x) = 2 + \sqrt{x + 9}$$

Thus the domain of $f^{-1}(x)$ *is* $[-9, \infty)$*. The range is* $[2, \infty)$*.*

(b) $f(x) = \sqrt[4]{x + 2}$.

Solution:

$$y = \sqrt[4]{x + 2}$$

$$y^4 = x + 2$$

$$y^4 - 2 = x$$

$$f^{-1}(x) = x^4 - 2$$

Thus, the domain of $f^{-1}(x)$ *is* $[0, \infty)$*. The range is* $[-2, \infty)$*.*

(c) $f(x) = \dfrac{1}{(x+2)^2}$ *(when $x > -2$).*

Solution:

$$\begin{aligned} y &= \frac{1}{(x+2)^2} \\ (x+2)^2 &= \frac{1}{y} \\ x+2 &= \sqrt{\frac{1}{y}} = \frac{1}{\sqrt{y}} \\ x &= \frac{1}{\sqrt{y}} - 2 \\ f^{-1}(x) &= \frac{1}{\sqrt{x}} - 2 \end{aligned}$$

Thus, the domain of $f^{-1}(x)$ is $(0, \infty)$. The range is $(-2, \infty)$.

Problem 3 *Find all values of x which satisfy the equation.*

(a) $\log_x 25 = 2$

Solution: $x^2 = 25 \implies x = \pm 5$. *But the base of a logarithm is always a positive number different from 1, and therefore $x = 5$ is the only solution.*

(b) $7^x = 15$

Solution: $x = \log_7 15$.

Problem 4 *Find all values which satisfy the given equation.*

(a) $\cos(x) = 1$

Solution: *This is asking for the angles such that cosine of that angle equals 1. Thus, $x = 2\pi n$, where n is any integer.*

(b) $\sin(3\theta) = \dfrac{\sqrt{3}}{2}$ *for $0 \le \theta \le 2\pi$*

Solution: *Let $3\theta = x$, so that we have*

$$\frac{\sqrt{3}}{2} = \sin(x)$$

. Then $x = \frac{\pi}{3} + 2\pi n$ or $x = \frac{2\pi}{3} + 2\pi n$ for n any integer. Since $x = 3\theta$, we can solve for theta to obtain $\theta = \frac{\pi}{9} + \frac{2}{3}\pi n$ or $\theta = \frac{2\pi}{9} + \frac{2}{3}\pi n$, where n is again any integer. We are only looking for solutions of θ in $[0, 2\pi]$, and so our solutions are

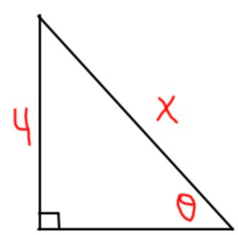
$$\theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}.$$

Problem 5 (a) *Simplify the expression:* $\cos^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

Solution: $\sin\left(\frac{\pi}{2}\right) = 1$*, and so we are looking for $\cos^{-1}(1)$. The range of $\cos^{-1}$ is $[0, \pi]$, and so $\cos^{-1}(1) = 0$.*

(b) *Simplify the expression:* $\tan\left(\sin^{-1}\left(\frac{4}{x}\right)\right)$

Solution: *Let $\theta = \sin^{-1}\left(\frac{4}{x}\right)$. Then $\sin\theta = \frac{4}{x}$. Consider the corresponding right triangle*



Call the adjacent side y. We use the Pythagorean Theorem to obtain

$$x^2 = 16 + y^2 \implies y = \sqrt{x^2 - 16}$$

. Then

$$\tan\left(\sin^{-1}\left(\frac{4}{x}\right)\right) = \tan\theta = \frac{4}{y} = \frac{4}{\sqrt{x^2 - 16}}$$

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