

# Preinforme 3 Lab MAT023 S2 CC

## Valores de $\alpha, \beta, \gamma$

$\alpha := 4$   
 $\beta := 4$   
 $\gamma := 7$

## Ejercicio I.

Considere la siguiente ecuación diferencial de primer orden:

$$\alpha x^2 - \gamma xy + y^2 - y'(xy - \beta x^2 - \alpha y^2)$$

- Encuentre explícitamente la solución de la ecuación presentada.
- Presente una gráfica de la solución de la ecuación para valores arbitrarios de la constante de integración (fijados por usted).

## Solución:

Primero normalizaremos la edo y la signamos a una función de  $y$ ,  $x$  y las constantes del lab.

$$(\alpha x^2 - \gamma xy + y^2) - y'(xy - \beta x^2 - \alpha y^2) = 0$$

Por la forma de la Edo se detecta que es homogénea y que realizando el cambio de variable  $y = xz(x)$ , esta se transformará en una Edo de variables separables.

$$F[y_] := \frac{\alpha x^2 - \gamma xy + y^2}{(xy - \beta x^2 - \alpha y^2)} - D[y, x]$$

`Solve[F[x z[x]] == 0, {z'[x]}]`

$$\left\{ \left\{ z'[x] \rightarrow \frac{-4 + 3 z[x] - 4 z[x]^3}{x (4 - z[x] + 4 z[x]^2)} \right\} \right\}$$

Resulta algo feo pero de variable separable.

```
Edo := First[First[Solve[F[x z[x]] == 0, {z'[x]}]][[1]]] ==  
Last[First[Solve[F[x z[x]] == 0, {z'[x]}]][[1]]]
```

```
DSolve[Edo, z, x]
```

Resolvemos la edo.

```
First[DSolve[Edo, z[x], x]][[2]][[1]] e-(First[DSolve[Edo, z[x], x]][[1]]) - x
```

$$-x + e^{-\frac{1}{3} \text{RootSum}\left[4 - 3 \#1 + 4 \#1^3 \&, \frac{4 \text{Log}[-\#1 + z[x]] - \text{Log}[-\#1 + z[x]] \#1 + 4 \text{Log}[-\#1 + z[x]] \#1^2}{-1 + 4 \#1^2} \&\right]} C[1]$$

Nos devolvemos a la variable original.

$z[x] := y / x$

$$-x + e^{-\frac{1}{3} \text{RootSum}\left[4 - 3 \#1 + 4 \#1^3 \&, \frac{4 \text{Log}[-\#1 + z[x]] - \text{Log}[-\#1 + z[x]] \#1 + 4 \text{Log}[-\#1 + z[x]] \#1^2}{-1 + 4 \#1^2} \&\right]} / N C[1]$$

$c1 := 1$

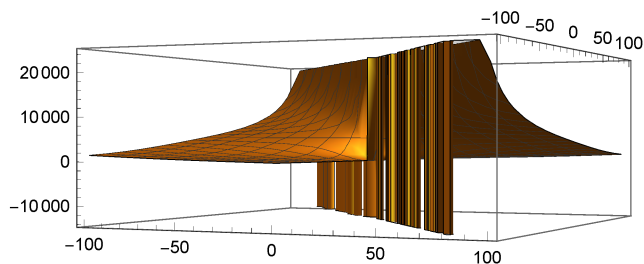
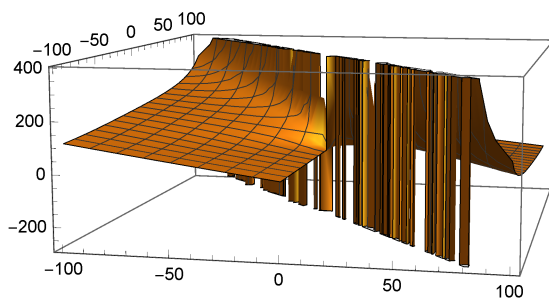
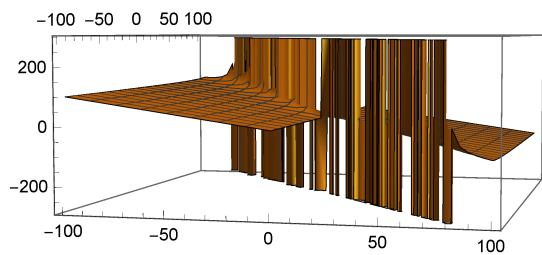
$c2 := 10$

$c3 := 1000$

$\text{Plot3D}\left[-x + e^{-0.3333333333333333 \left( (0.40060067582662473 - 1.1577299553257219 i) \text{Log}\left[(-0.6230083252929541 - 0.6437531515674265 i) + c1, \{x, -100, 100\}, \{y, -100, 100\}\right]}\right)}\right]$

$\text{Plot3D}\left[-x + e^{-0.3333333333333333 \left( (0.40060067582662473 - 1.1577299553257219 i) \text{Log}\left[(-0.6230083252929541 - 0.6437531515674265 i) + c2, \{x, -100, 100\}, \{y, -100, 100\}\right]}\right)}\right]$

$\text{Plot3D}\left[-x + e^{-0.3333333333333333 \left( (0.40060067582662473 - 1.1577299553257219 i) \text{Log}\left[(-0.6230083252929541 - 0.6437531515674265 i) + c3, \{x, -100, 100\}, \{y, -100, 100\}\right]}\right)}\right]$



El isótopo radioactivo del Torio 234 se desintegra a una rapidéz proporcional a la cantidad existente en ese instante de tiempo. Si máx  $\{100\alpha, 100\beta, 100\gamma\}$  miligramos de este material se reducen a mín  $\{80\alpha, 80\beta, 80\gamma\}$  miligramos en un semana, ¿Cuánto Torio se tendrá a al cabo de 3 semanas? ¿Cuánto tiempo tiene que transcurrir para que la cantidad de Torio se reduzca a la mitad?

### Solución :

Analizando el problema detectamos las siguientes ecuaciones

$$\frac{dT}{dt} = -k T(t) \quad (\text{edo de variable separable})$$

$$T(0) = \text{máx} \{100\alpha, 100\beta, 100\gamma\}$$

$$T(t=1 \text{ semana}) = \text{mín} \{80\alpha, 80\beta, 80\gamma\}$$

$$T[0] := \text{Max}[100 \alpha, 100 \beta, 100 \gamma]$$

$$T[1] := \text{Min}[80 \alpha, 80 \beta, 80 \gamma]$$

$$H[t] = T[t], \text{ de otro modo no funcionaba}$$

$$H[t_] := T[0] e^{-t k}$$

$$k := \text{Last}[\text{Refine}[\text{Log}[T[0] / T[0] e^{-d}] == \text{Log}[T[1] / T[0]], \text{Assumptions} \rightarrow d > 0] // \text{FullSimplify}]$$

a. ¿Cuánto Torio se tendrá a al cabo de 3 semanas?

$$H[3] // N$$

$$66.8735$$

R: 66.8735 miligramos

b. ¿Cuánto tiempo tiene que transcurrir para que la cantidad de Torio se reduzca a la mitad?

$$\text{Solve}\left[\frac{T[0]}{2} == H[t], t\right]$$

$$\left\{\left\{t \rightarrow \frac{-\text{Log}\left[\frac{7}{2}\right] + \text{Log}[7]}{-2 \text{Log}[4] + \text{Log}[5] + \text{Log}[7]}\right\}\right\} // N$$

$$\{\{t \rightarrow 0.885518\}\}$$

R: Debe transcurrir 0.88 semanas, que son 6 días y 4 horas app.

### Ejercicio 3

Calcule la solución general de la siguiente ecuación diferencial:

$$y^{(6)} + \alpha y^{(4)} + 3y^{(3)} - \beta y^{(2)} + 2y^{(1)} - 5y = x^{\frac{7}{2}+1} \ln[x]$$

para  $x \in \mathbb{R}^+$ .

**Solución:**

Detectamos al mirar la edo que es una de orden superior no homogénea, por lo que para resolverla debemos

buscar la solución homogénea (mediante polinomio asociado) y luego aplicar variación de parámetros, puesto que no existe anulador para la parte no homogénea

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D[Y[x], {x, 6}] + α D[Y[x], {x, 4}] + 3 D[Y[x], {x, 3}] -
β D[Y[x], {x, 2}] + 2 D[Y[x], {x, 1}] - 5 Y[x] == x^(7/2) Log[x]

-5 Y[x] + 2 Y'[x] - 4 Y''[x] + 3 Y^(3)[x] + 4 Y^(4)[x] + Y^(6)[x] == x^(7/2) Log[x]

L[1_] := -5 + 2 1 - 4 1^2 + 3 1^3 + 4 1^4 + 1^6

Solve[-5 + 2 1 - 4 1^2 + 3 1^3 + 4 1^4 + 1^6 == 0, 1] // N

{{1 -> -1.42068}, {1 -> 0.956845}, {1 -> -0.0168415 - 0.872517 i},
{1 -> -0.0168415 + 0.872517 i}, {1 -> 0.248761 - 2.18354 i}, {1 -> 0.248761 + 2.18354 i}}

(* Con una horrible aproximación el polinomio que se exige abajo*)

(1 + 1.42) (1 - 0.95) (1 + 0.016 + 0.87 i)
(1 + 0.016 - 0.87 i) (1 - 0.24 + 2.18 i) (1 - 0.24 - 2.18 i)

Que nos da una solución homogénea como le sigue:

In[1]:= Yh := d1 e^(-1.42 x) + d2 e^(0.95 x) + d3 Sin[0.87 x] e^(-0.016 x) +
d4 Cos[0.87 x] e^(-0.016 x) + d5 Sin[2.18 x] e^(0.24 x) + d6 Cos[2.18 x] e^(0.24 x)

```

Haciendo variación de parámetros para obtener la particular. Esto implica calcular el wronskiano y luego las constantes del sistema

$$\text{In[2]:= } \mathbf{W} := \text{Det} \left[ \begin{pmatrix} e^{-1.42 x} & e^{0.95 x} & \sin[0.87 x] e^{-0.016 x} & \cos[0.87 x] e^{-0.016 x} \\ D[e^{-1.42 x}, x] & D[e^{0.95 x}, x] & D[\sin[0.87 x] e^{-0.016 x}, x] & D[\cos[0.87 x] e^{-0.016 x}, x] \\ D[e^{-1.42 x}, \{x, 2\}] & D[e^{0.95 x}, \{x, 2\}] & D[\sin[0.87 x] e^{-0.016 x}, \{x, 2\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 2\}] \\ D[e^{-1.42 x}, \{x, 3\}] & D[e^{0.95 x}, \{x, 3\}] & D[\sin[0.87 x] e^{-0.016 x}, \{x, 3\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 3\}] \\ D[e^{-1.42 x}, \{x, 4\}] & D[e^{0.95 x}, \{x, 4\}] & D[\sin[0.87 x] e^{-0.016 x}, \{x, 4\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 4\}] \\ D[e^{-1.42 x}, \{x, 5\}] & D[e^{0.95 x}, \{x, 5\}] & D[\sin[0.87 x] e^{-0.016 x}, \{x, 5\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 5\}] \end{pmatrix} \right]$$

$$\text{In[3]:= } \mathbf{g1} := \int \frac{1}{\mathbf{W}} \text{Det} \left[ \begin{pmatrix} 0 & e^{0.95 x} & \sin[0.87 x] e^{-0.016 x} & \cos[0.87 x] e^{-0.016 x} \\ 0 & D[e^{0.95 x}, x] & D[\sin[0.87 x] e^{-0.016 x}, x] & D[\cos[0.87 x] e^{-0.016 x}, x] \\ 0 & D[e^{0.95 x}, \{x, 2\}] & D[\sin[0.87 x] e^{-0.016 x}, \{x, 2\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 2\}] \\ 0 & D[e^{0.95 x}, \{x, 3\}] & D[\sin[0.87 x] e^{-0.016 x}, \{x, 3\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 3\}] \\ 0 & D[e^{0.95 x}, \{x, 4\}] & D[\sin[0.87 x] e^{-0.016 x}, \{x, 4\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 4\}] \\ x^{7/2} \log[x] & D[e^{0.95 x}, \{x, 5\}] & D[\sin[0.87 x] e^{-0.016 x}, \{x, 5\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 5\}] \end{pmatrix} \right] dx$$

$$\text{In[4]:= } g2 := \int \frac{1}{w} \text{Det} \left[ \begin{array}{cccc} e^{-1.42 x} & 0 & \sin[0.87 x] e^{-0.016 x} & \cos[0.87 x] e^{-0.016 x} \\ D[e^{-1.42 x}, x] & 0 & D[\sin[0.87 x] e^{-0.016 x}, x] & D[\cos[0.87 x] e^{-0.016 x}, x] \\ D[e^{-1.42 x}, \{x, 2\}] & 0 & D[\sin[0.87 x] e^{-0.016 x}, \{x, 2\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 2\}] \\ D[e^{-1.42 x}, \{x, 3\}] & 0 & D[\sin[0.87 x] e^{-0.016 x}, \{x, 3\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 3\}] \\ D[e^{-1.42 x}, \{x, 4\}] & 0 & D[\sin[0.87 x] e^{-0.016 x}, \{x, 4\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 4\}] \\ D[e^{-1.42 x}, \{x, 5\}] & x^{7/2} \log[x] & D[\sin[0.87 x] e^{-0.016 x}, \{x, 5\}] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 5\}] \end{array} \right] dx$$

$$\text{In[5]:= } g3 := \int \frac{1}{w} \text{Det} \left[ \begin{array}{cccc} e^{-1.42 x} & e^{0.95 x} & 0 & \cos[0.87 x] e^{-0.016 x} \\ D[e^{-1.42 x}, x] & D[e^{0.95 x}, x] & 0 & D[\cos[0.87 x] e^{-0.016 x}, x] \\ D[e^{-1.42 x}, \{x, 2\}] & D[e^{0.95 x}, \{x, 2\}] & 0 & D[\cos[0.87 x] e^{-0.016 x}, \{x, 2\}] \\ D[e^{-1.42 x}, \{x, 3\}] & D[e^{0.95 x}, \{x, 3\}] & 0 & D[\cos[0.87 x] e^{-0.016 x}, \{x, 3\}] \\ D[e^{-1.42 x}, \{x, 4\}] & D[e^{0.95 x}, \{x, 4\}] & 0 & D[\cos[0.87 x] e^{-0.016 x}, \{x, 4\}] \\ D[e^{-1.42 x}, \{x, 5\}] & D[e^{0.95 x}, \{x, 5\}] & x^{7/2} \log[x] & D[\cos[0.87 x] e^{-0.016 x}, \{x, 5\}] \end{array} \right] dx$$

$$\text{In[6]:= } g4 := \int \frac{1}{w} \text{Det} \left[ \begin{array}{cccc} e^{-1.42 x} & e^{0.95 x} & \text{Sin}[0.87 x] e^{-0.016 x} & 0 \\ D[e^{-1.42 x}, x] & D[e^{0.95 x}, x] & D[\text{Sin}[0.87 x] e^{-0.016 x}, x] & 0 \\ D[e^{-1.42 x}, \{x, 2\}] & D[e^{0.95 x}, \{x, 2\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 2\}] & 0 \\ D[e^{-1.42 x}, \{x, 3\}] & D[e^{0.95 x}, \{x, 3\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 3\}] & 0 \\ D[e^{-1.42 x}, \{x, 4\}] & D[e^{0.95 x}, \{x, 4\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 4\}] & 0 \\ D[e^{-1.42 x}, \{x, 5\}] & D[e^{0.95 x}, \{x, 5\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 5\}] & x^{7/2} \text{Log}[x] \end{array} \right] dx$$

$$\text{In[7]:= } g5 := \int \frac{1}{w} \text{Det} \left[ \begin{array}{cccc} e^{-1.42 x} & e^{0.95 x} & \text{Sin}[0.87 x] e^{-0.016 x} & \text{Cos}[0.87 x] \\ D[e^{-1.42 x}, x] & D[e^{0.95 x}, x] & D[\text{Sin}[0.87 x] e^{-0.016 x}, x] & D[\text{Cos}[0.87 x] e^{-0.016 x}, x] \\ D[e^{-1.42 x}, \{x, 2\}] & D[e^{0.95 x}, \{x, 2\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 2\}] & D[\text{Cos}[0.87 x] e^{-0.016 x}, \{x, 2\}] \\ D[e^{-1.42 x}, \{x, 3\}] & D[e^{0.95 x}, \{x, 3\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 3\}] & D[\text{Cos}[0.87 x] e^{-0.016 x}, \{x, 3\}] \\ D[e^{-1.42 x}, \{x, 4\}] & D[e^{0.95 x}, \{x, 4\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 4\}] & D[\text{Cos}[0.87 x] e^{-0.016 x}, \{x, 4\}] \\ D[e^{-1.42 x}, \{x, 5\}] & D[e^{0.95 x}, \{x, 5\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 5\}] & D[\text{Cos}[0.87 x] e^{-0.016 x}, \{x, 5\}] \end{array} \right] dx$$

$$\text{In[8]:= } \mathbf{g6} := \int \frac{1}{\mathbf{w}} \text{Det} \left[ \begin{array}{cccc} e^{-1.42 x} & e^{0.95 x} & \text{Sin}[0.87 x] e^{-0.016 x} & \text{Cos}[0.87 x] \\ D[e^{-1.42 x}, x] & D[e^{0.95 x}, x] & D[\text{Sin}[0.87 x] e^{-0.016 x}, x] & D[\text{Cos}[0.87 x] e^{-0.016 x}, x] \\ D[e^{-1.42 x}, \{x, 2\}] & D[e^{0.95 x}, \{x, 2\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 2\}] & D[\text{Cos}[0.87 x] e^{-0.016 x}, \{x, 2\}] \\ D[e^{-1.42 x}, \{x, 3\}] & D[e^{0.95 x}, \{x, 3\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 3\}] & D[\text{Cos}[0.87 x] e^{-0.016 x}, \{x, 3\}] \\ D[e^{-1.42 x}, \{x, 4\}] & D[e^{0.95 x}, \{x, 4\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 4\}] & D[\text{Cos}[0.87 x] e^{-0.016 x}, \{x, 4\}] \\ D[e^{-1.42 x}, \{x, 5\}] & D[e^{0.95 x}, \{x, 5\}] & D[\text{Sin}[0.87 x] e^{-0.016 x}, \{x, 5\}] & D[\text{Cos}[0.87 x] e^{-0.016 x}, \{x, 5\}] \end{array} \right] dx$$

**g1 // N**

$$-0.0205999 \left( -0.0493827 x^{9/2} \text{HypergeometricPFQ}[\{4.5, 4.5\}, \{5.5, 5.5\}, 1.42 x] + \right. \\ \left. \text{Log}[x] \left( 2.40075 \text{Erfi}[1.19164 \sqrt{x}] + \sqrt{x} \right) \right. \\ \left. \left( -3.22809 + 3.05593 x - 1.73577 x^2 + 0.704225 x^3 \right) (\text{Cosh}[1.42 x] + \text{Sinh}[1.42 x]) \right)$$

**g2 // N**

$$0.0474957 (\text{Cosh}[0.95 x] - 1. \text{Sinh}[0.95 x]) \\ \left( -0.0493827 x^{9/2} \text{HypergeometricPFQ}[\{4.5, 4.5\}, \{5.5, 5.5\}, -0.95 x] \right. \\ \left. (\text{Cosh}[0.95 x] + \text{Sinh}[0.95 x]) + \right. \\ \left. \text{Log}[x] \left( \sqrt{x} (-16.1141 - 10.2056 x - 3.87812 x^2 - 1.05263 x^3) + \right. \right. \\ \left. \left. 14.6517 \text{Erf}[0.974679 \sqrt{x}] (\text{Cosh}[0.95 x] + \text{Sinh}[0.95 x]) \right) \right)$$

**g3 // N**

$$-0.0474957 (\text{Cosh}[0.95 x] - 1. \text{Sinh}[0.95 x]) \\ \left( -0.0493827 x^{9/2} \text{HypergeometricPFQ}[\{4.5, 4.5\}, \{5.5, 5.5\}, -0.95 x] \right. \\ \left. (\text{Cosh}[0.95 x] + \text{Sinh}[0.95 x]) + \right. \\ \left. \text{Log}[x] \left( \sqrt{x} (-16.1141 - 10.2056 x - 3.87812 x^2 - 1.05263 x^3) + \right. \right. \\ \left. \left. 14.6517 \text{Erf}[0.974679 \sqrt{x}] (\text{Cosh}[0.95 x] + \text{Sinh}[0.95 x]) \right) \right)$$



**g4 // N**

$$\frac{1}{\sqrt{x}} 2.71828^{(0.-0.87 i) x} \left( (0.000917871 - 0.00310233 i) 2.71828^{(0.+0.87 i) x} x^5 \text{HypergeometricPFQ}[\{4.5, 4.5\}, \{5.5, 5.5\}, (0.016 - 0.87 i) x] + (0.000917871 + 0.00310233 i) 2.71828^{(0.+0.87 i) x} x^5 \text{HypergeometricPFQ}[\{4.5, 4.5\}, \{5.5, 5.5\}, (0.016 + 0.87 i) x] + (2.71828^{0.016 x} x ((0.318688 - 1.46566 i) + (0.846682 + 0.200473 i) x - (0.0751832 - 0.293362 i) x^2 - (0.0725778 + 0.0200295 i) x^3 + 2.71828^{(0.+1.74 i) x} ((0.318688 + 1.46566 i) + (0.846682 - 0.200473 i) x - (0.0751832 + 0.293362 i) x^2 - (0.0725778 - 0.0200295 i) x^3)) - (1.49846 + 0.297074 i) 2.71828^{(0.+0.87 i) x} \sqrt{(0.016 - 0.87 i) x} \text{Erfi}\left[\sqrt{(0.016 - 0.87 i) x}\right] - (1.49846 - 0.297074 i) 2.71828^{(0.+0.87 i) x} \sqrt{(0.016 + 0.87 i) x} \text{Erfi}\left[\sqrt{(0.016 + 0.87 i) x}\right] \right) \text{Log}[x]$$

**g5 // N**

$$\frac{1}{\sqrt{x}} 2.71828^{(-0.24-2.18 i) x} \left( (-0.000361042 - 0.000253778 i) 2.71828^{(0.24+2.18 i) x} x^5 \text{HypergeometricPFQ}[\{4.5, 4.5\}, \{5.5, 5.5\}, (-0.24 - 2.18 i) x] - (0.000361042 - 0.000253778 i) 2.71828^{(0.24+2.18 i) x} x^5 \text{HypergeometricPFQ}[\{4.5, 4.5\}, \{5.5, 5.5\}, (-0.24 + 2.18 i) x] + (x ((-0.00251692 - 0.00440074 i) + (0.00599304 - 0.00436204 i) x + (0.00437903 + 0.00480718 i) x^2 - (0.00269391 - 0.00305714 i) x^3 + 2.71828^{(0.+4.36 i) x} ((-0.00251692 + 0.00440074 i) + (0.00599304 + 0.00436204 i) x + (0.00437903 - 0.00480718 i) x^2 - (0.00269391 + 0.00305714 i) x^3)) - (0.00187889 - 0.000816341 i) 2.71828^{(0.24+2.18 i) x} \sqrt{(-0.24 - 2.18 i) x} \text{Erfi}\left[\sqrt{(-0.24 - 2.18 i) x}\right] - (0.00187889 + 0.000816341 i) 2.71828^{(0.24+2.18 i) x} \sqrt{(-0.24 + 2.18 i) x} \text{Erfi}\left[\sqrt{(-0.24 + 2.18 i) x}\right] \right) \text{Log}[x]$$

In[9]:= **Yp :=**

$$\mathbf{g1} e^{-1.42 x} + \mathbf{g2} e^{0.95 x} + \mathbf{g3} \sin[0.87 x] e^{-0.016 x} + \mathbf{g4} \cos[0.87 x] e^{-0.016 x} + \mathbf{g5} \sin[2.18 x] e^{0.24 x}$$

In[10]:= **J[x\_] = Yh + Yp**