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Finance Research Essay

The Impact of Security Design on Information Aggregation: An Experimental Study

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Abstract

This paper uses an experimental markets framework to provide novel insights into how a security design that incorporates the verification mechanism of a decision problem can significantly improve human decision-making, and consequently market efficiency. Specifically, I show that Computational Complexity Theory explains both the behaviours of humans and markets when the underlying problem is hard. Moreover, I provide experimental evidence that markets do not need to reflect all information, but only the most crucial pieces of information about the underlying problem, to achieve efficiency, as first conjectured by Hayek (1945). This is in contrast to popular market efficiency hypotheses, notably the Efficient Market Hypothesis and the Rational Expectations Hypothesis, that assume efficiency requires all information to be revealed in market prices.

Declaration

This essay is the sole work of the author whose name appears on the title page. It contains no material which the author has previously submitted for assessment at the University of Melbourne or elsewhere. To the best of the author's knowledge, the essay contains no material previously written or published by another person except where reference is made in the text of the essay.

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Lastly, I am grateful to my family for giving me the strength to keep going.

Contents

"The most significant fact about [the price] system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action."

- Friedrich Hayek, *The Use of Knowledge in Society*

Introduction

Information aggregation refers to the markets' ability to collect, aggregate, and reveal individual pieces of information in the market price. This is an important and compelling concept in finance that has been much debated since the 1970s. Indeed, little has been known about the underlying mechanism of information aggregation and the extent of informational efficiency in markets.

Current studies on experimental finance attempt to address this issue by examining information aggregation under the framework of Computational Complexity Theory (CCT; see e.g., Bossaerts and Murawski, 2017 and Bossaerts, Bowman, Huang, Murawski, Tang, and Yadav, 2018). These studies provide evidence that a relationship exists between market efficiency, human decision-making and computational complexity.

Importantly, Bossaerts et al. (2018) find that market efficiency decreases as the underlying problem, the Knapsack Problem (KP), increases in difficulty. The KP is an optimisation problem where the objective is to maximise the value of a Knapsack from a set of items given a weight constraint. This is analogous to many ubiquitous financial decision-making problems, including portfolio selection and assets selection for securitisation (Kellerer, Pferschy, and Pisinger, 2004, chap. 15).

While such an optimisation problem is central to human decision-making, it is in fact extremely difficult for humans and for algorithms to solve. Thus, I propose an alternative approach to study information aggregation under the CCT framework, where I transform the KP from an optimisation problem into its decision variant (hereinafter referred to as KP-DEC).

The KP-DEC is a decision problem where a proposed solution can be verified against a specified profit constraint, where the Knapsack is subject to a weight constraint. This mechanism is known as the *verification mechanism*, and is intrinsic to decision problems. In other words, the problem becomes a yes-no question, which is fundamentally different to the KP where the task is to find the optimal solution that maximises the Knapsack value.

How a verification mechanism impacts individual decision-making and information aggregation in the market is an important question yet to be answered in exisiting literature. The findings will provide novel insights into how a specific security design that incorporates a verification mechanism can improve informational efficiency in markets. Further, the results can have implications for computer science about how a market mechanism can be used to solve decision problems that are computationally complex.

In order to examine this overarching question, I adapt the experimental design studied in Bossaerts et al. (2018), where participants are asked to solve KP-DEC instances while simultaneously trading securities that corresponded to each instance in an online marketplace. The payoff of each security is directly linked to the solvability of the respective KP-DEC instance. A security pays \$1 if the corresponding KP-DEC instance satisfies a given profit constraint and \$0 otherwise. In other words, we can interpret the market price of a given security as the probability of a KP-DEC instance being solvable.

The results of my study are fourfold. First, market prices aggregate information about the solvability of the KP-DEC instances, and converge to their fair values despite asymmetric information in the market.

Second, individuals learn from market prices and improve their ability to solve the KP-DEC instances over time. Individuals exert more effort, whether it be physical or mental, to solve a KP-DEC instance when they observe a positive pricing signal in the market.

Third, computational complexity affects both individual performance and information aggregation in the market. That is, individual decision-making and informational efficiency deteriorates as computational complexity increases. Thus, the CCT framework applies to humans and markets.

Fourth, human decision-making improves when an optimisation problem is transformed into a decision problem. Specifically, the proportion of individuals who found the optimal solution to the KP increases from 26.9% in the KP instance studied in Bossaerts et al. (2018) to 45.98% in the decision variant, where the profit constraint of a KP-DEC instance is equal to the solution to the KP. Further, multi-asset markets facilitate the verification mechanism of the KP-DEC and provide a feedback channel for individuals to improve their individual performance. This suggests that a security design that incorporates a verification mechanism can help individuals to solve an optimisation problem by iteratively verifying whether a specified profit constraint is satisfiable or not.

In essence, my findings show that market efficiency can be achieved without perfect information revelation. This is contrary to prevailing market efficiency theories, including the Rational Expectations Equilibrium and the Efficient Market Hypothesis (Grossman and Stiglitz, 1980; Fama, 1991). Instead, market efficiency only requires the most crucial information to be reflected in the market price as proposed by Hayek (1945) who highlights the nature of dispersed information in an economy.

The following paper is organised as follows. Section 1 provides a literature review on experimental finance, information aggregation, and computational complexity. Section 2 states the research questions and hypotheses. Section 3 and 4 outline the experimental design and the methodology used for data analysis. Section 5 and 6 analyse and discuss the main findings. Lastly, Section 7 concludes the paper and provides suggestions for further research opportunities.

1 Literature Review and Prerequisites

One of the most prominent fields in finance revolves around the notion of information aggregation and market efficiency. According to the Efficient Market Hypothesis (EMH), an efficient market is one where all available information is revealed in the market price (Fama, 1991, p. 1575). Similarly, the Rational Expectations Equilibrium (REE) model assumes that individual traders have access to all information in the market (Grossman and Stiglitz, 1980).

Yet, Hayek (1945) argues that this notion of market efficiency- by assuming "perfect knowledge" in the market- is misleading. Instead, he proposes an alternative theory of decentralised knowledge to account for the dispersed nature of information in an economy, and suggests that an efficient market only requires the most crucial information to be revealed in the market price (Hayek, 1945, p. 527).

On the other hand, emerging studies provide experimental evidence that market prices converge to an equilibrium that is congruent with the Noisy Rational Expectation Equilibrium (NREE) model (e.g., Plott and Sunder, 1982, 1988; Bossaerts, Frydman, and Ledyard, 2013; Page and Siemroth, 2018; Bossaerts et al., 2018). They show that the presence of information asymmetry introduces noise in the market, which results in imperfect information revelation of the market price. Other theoretical studies in favour of the NREE framework also model noise in the form of information asymmetry and dispersed heterogeneous information. [1][2]

Clearly, the extent of information aggregation is of great concern in the exisiting literature. Even without artificial imposition of noise, such that all individuals have the same public information, noise can simply arise from deviations in human

[1]See e.g., Grossman and Stiglitz, 1980 and Ostrovsky, 2012 for noise in the form of information asymmetry.

[2]See e.g., Diamond and Verrecchia, 1981 for noise in the form of dispersed heterogenous information.

decision-making. This is expected given that human decision-making is known to be complex and that individuals are limited in their cognitive resources (Fellner, Güth, and Maciejovsky, 2009).

For example, if all individuals are given the same problem to solve, one would not expect that all individuals would approach the problem in a similar manner, and specifically, not all of them would find a solution to that problem. Consequently, individuals have different pieces information about the solution to the problem at hand (Hayek, 1945). This creates noise in the market, whether it be in the form of information asymmetry or dispersed heterogeneous information.

Thus, noise exists in the market simply as a result of deviations in individual's ability to solve problems. By increasing the complexity of the underlying problem, one would expect that there will be more noise in the market.

In fact, existing literature shows that human decision-making is related to computational complexity, where human performance decreases as a problem increases in computational complexity (Bossaerts and Murawski, 2017; Yadav, Murawski, Sardina, and Bossaerts, 2018).

We can therefore use the CCT framework to quantify and measure noise in terms of complexity in human decision-making by varying the difficulty of a problem. The CCT is a framework that measures the difficulty of a problem based on its computational complexity (Bossaerts and Murawski, 2017). Here, computational complexity refers to the amount of time required for a computer to solve the problem (Bossaerts and Murawski, 2017).

In modern decision-making theories, human decision-making is often modelled as an optimisation problem. That is, individuals are assumed to be rational and behave "as if" they are optimisers- they will always choose the best available option to a problem (Shiller, 1999). Particularly in finance and economics, it is assumed that markets are formed by "rational investors" who always choose the optimal strategy that maximises their utility function.

Accordingly, Bossaerts et al. (2018) use such an optimisation problem- the KP, to study the relationship between human decision-making, information aggregation and computational complexity. Formally, the KP is a 0/1 combinatorial optimisation problem that involves maximising the value of a Knapsack given a set of items with different values and weights subject to a capacity constraint of the Knapsack (Kellerer et al., 2004, chap. 1). A solution to the KP needs to satisfy:

- maximise $\sum_{i \in S} \widehat{\upsilon}_i$ subject to $\sum_{i \in S} \widehat{\omega}_i \leq \widehat{c}$

where i, S, ω_i, υ_i, and $\widehat{c}$ denote a set of items, all possible Knapsack sets that satisfy the capacity constraint, item weight, item value, and capacity constraint, respectively (Kellerer et al., 2004, p. 3). The solution to a KP instance will be referred to as the *optimal solution* in this paper.

Under the CCT framework, the KP is known to be *NP-hard*, where finding the optimal solution is computationally intractable[3]. Additionally, there is no known algorithm that can determine whether the solution is optimal or not in polynomial time. Thus, the KP is a computationally complex problem, that is also proven to be extremely difficult for humans to solve Bossaerts et al. (2018).

The KP is specifically studied in Bossaerts et al. (2018) given its relevance to finance. Specifically, the KP is analogous to many ubiquitous financial decision-making problems, including portfolio selection. Portfolio selection involves finding an optimal combination of assets with given costs, risks and returns, that maximises

[3] This means that the computational time required to solve a problem increases non-polynomially (NP), e.g., 2^n, instead of n^2 (Kellerer et al., 2004, chap. 1).

the return of an investment portfolio subject to a risk appetite and budget constraint (Markowitz, 1952; Kellerer et al., 2004, chap. 15).

In their experiment, Bossaerts et al. (2018) link the payoff of a security to an item's allocation in the optimal solution to a given KP instance. A security pays off $1 if the corresponding item is in the optimal Knapsack and zero otherwise. Using this experimental design, Bossaerts et al. (2018) find that market prices only partially reflect private information in the presence of computational complexity. They argue that market prices become a noisy aggregator of individuals' beliefs as first suggested by Diamond and Verrecchia (1981), and the realised equilibrium is consistent with the NREE. In other words, the realised market prices are found to be noisy.

Interestingly, Bossaerts et al. (2018) find that participants in their experiment were only interested in improving their solution to get *closer* to the optimal solution of the KP. In other words, participants were *satisficing*. Satisficing is a heuristic under the framework of bounded rationality, as first suggested by Simon (1955).[4] This shows that humans are limited in their cognitive resources, and that they do not behave "as if" they are optimisers when solving optimisation problems. The standard assumption- that individuals are rational- is hypothetical and does not properly encapsulate the process of human decision-making (Hayek, 1945). In fact, Hayek (1945) argues that such an assumption "disregard[s] everything that is important and significant in the real world."(p. 530).[5]

Therefore, I propose an alternative study to Bossaerts et al. (2018) where an

[4]Bounded rationality is an alternative theory for human decision-making where individuals need not optimise (Simon, 1955). Rather, they are assumed to rely on heuristics to find a suboptimal solution that satisfies a given set of constraints (e.g., minimum return requirement given a budget constraint for an investment portfolio) that are acceptable under the given circumstances (Fellner et al., 2009).

[5] Hayek (1945) refers to the fact that human knowledge is imperfect and that information is dispersed in a market.

individual is faced with a decision problem that is consistent with the notion of *satisficing*. In particular, I will transform the KP into its decision variant known as the KP-DEC.

Formally, the KP-DEC is a decision problem that corresponds to a binary question, where a proposed solution is either yes or no (Bossaerts and Murawski, 2017). It involves solving a Knapsack that satisfies a given profit constraint subject to a capacity constraint, where the proposed solution can be easily verified. A solution to the KP-DEC needs to satisfy:

- $\sum_{i \in S} \widehat{\omega}_i \leq \widehat{c}$, that is, the weight of the Knapsack is less than or equal to the capacity constraint; and
- $\sum_{i \in S} \widehat{\upsilon}_i \geq \widehat{p}$, that is, the value of the Knapsack is greater than or equal to the profit constraint,

where i, S, $\omega, \upsilon, \widehat{c}$, and $\widehat{p}$ denote a set of items, the total Knapsack sets that satisfy the constraints, item weight, item value, capacity constraint, and profit constraint, respectively (Yadav et al., 2018, p. 2). In the KP-DEC, one is asked to verify whether a proposed solution satisfies the specified profit and capacity constraint. This is clearly different from the KP, which only has a capacity constraint and the goal is to maximise the value of the constrained Knapsack.

Specifically, the key underlying difference between the optimisation problem and the decision problem is the presence of a verification mechanism in the decision problem. A solution to a decision problem can be easily verified against a specified profit constraint in polynomial time (Kellerer et al., 2004, chap. 1). However, finding a solution itself (i.e., finding a combination of items that satisfies the constraints of the Knapsack) can still be computationally intractable. This property is known as *NP-complete*, where a KP-DEC instance is at least as difficult as its optimisation case, but not any more difficult (Arora and Barak, 2009).

The profit constraint of a solvable KP-DEC instance therefore establishes the lower bound for the optimal solution to the corresponding KP instance (Arora and Barak, 2009). In other words, the optimal solution of the KP instance is at least as the value of the profit constraint of a solvable KP-DEC instance.

Therefore, by iteratively solving the KP-DEC instances with different profit constraints, one will eventually find the optimal solution to the KP instance with the same Knapsack item set and weight constraint (Kellerer et al., 2004, chap. A). This implies that if the KP-DEC is solvable, then the KP is also solvable (Kellerer et al., 2004, chap. A). This is an important concept in the context of the CCT and information aggregation: if we can provide evidence that the KP-DEC is easily solvable by the markets, then we can also show that the KP is easily solvable by such mechanism. Put simply, markets can iteratively solve the KP-DEC to find the optimal solution to the KP.

To measure the complexity of the KP-DEC instance, I will use two metrics. The first metric measures the computational time required to solve a given KP-DEC instance. This is known as the propagation value, which can be computed by using an algorithm, i.e., Minizinc.

The other metric is the phase transition metric proposed by Yadav et al. (2018) based on their findings on the phase transition phenomenon for the KP-DEC. The metric is defined as the log ratio of its normalised capacity (c) and normalised profit (p), where normalised capacity is the capacity constraint of the Knapsack over the sum of item weights, and normalised profit is the profit constraint of the KP-DEC instance over the sum of item values.

The phase transition phenomenon is the sudden transition where the "probability that a random instance has a solution changes from zero to one" (Yadav et al., 2018,

p. 3). It is located where the metric is approximately 0 (see Fig. 1.) That is, where the profit and capacity constraints are not overly relaxed or restrictive, so that it is difficult to determine whether the KP-DEC instance is solvable or not (Yadav et al., 2018).

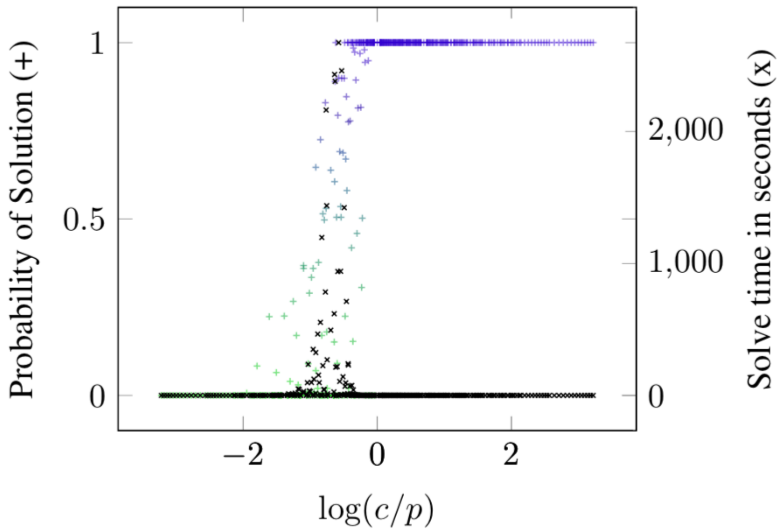


Figure 1: Phase Transition in the KP-DEC measured by log(c/p).

Adapted from "Phase transition in the knapsack problem", by N. Yadav, C. Murawski, S. Sardina, & P, Bossaerts, 2018, *working paper*, p. 8. Adapted with permission.

One can think of the phase transition phenomenon with respect to the freezing of water (Moore and Mertens, 2011). At the critical temperature of 0 degrees Celsius, water changes from a liquid to a solid, where this phase transition is energy intensive (Moore and Mertens, 2011). Similarly, a random instance that is located in the phase transition region (see the dark green region in Fig. 2) is known to be computationally-costly on average. In other words, this region is where the instance complexity is the highest on average. The phase transition phenomenon is widely known in physics, as well as in many decision problems in computer science (Moore and Mertens, 2011).

Yadav et al. (2018) show that the computational resources required to solve a random KP-DEC instance increase on average as the instance location gets closer to the phase transition region (see Fig. 3b). Importantly, this is also true for humans, where individual performance decreases and effort (in terms of time spent at each possible set of Knapsack items) increases as instance complexity increases (Yadav et al. (2018); see Fig. 3a). Whether the phase transition phenomenon affects informational efficiency within markets remains an open question. Thus, my paper will examine this question by incorporating the verification mechanism of the KP-DEC into the security design.

In my experiment, the securities will have a strike price that will be linked to the profit constraint of a given KP-DEC instance. Whether a security pays off will then depend on the 'the solvability' of the corresponding KP-DEC instance. Via trading, private information about the KP-DEC instance (e.g. is the KP-DEC instance solvable or not?) will be reflected in market prices. Thus, we can interpret market prices of a given security as the probability of the respective KP-DEC instance satisfying its profit constraint.

The proposed security design is analogous to a digital option. A digital option is a derivative security that provides two possible payoff outcomes: a fixed payoff, e.g. \$1 if the price of the underlying asset exceeds the strike price at maturity, or zero otherwise. A digital option is purchased if one believes that the underlying asset will exceed the strike price in the future. In other words, the market price of a digital option reveals the market's expectation of the underlying asset exceeding the strike price in the future.

Clearly, the security design of my experiment will be inherently different to that of Bossaerts et al. (2018), as the presence of a verification mechanism transforms the market efficiency problem entirely. Whether such a security design can actually improve information aggregation remains an open question.

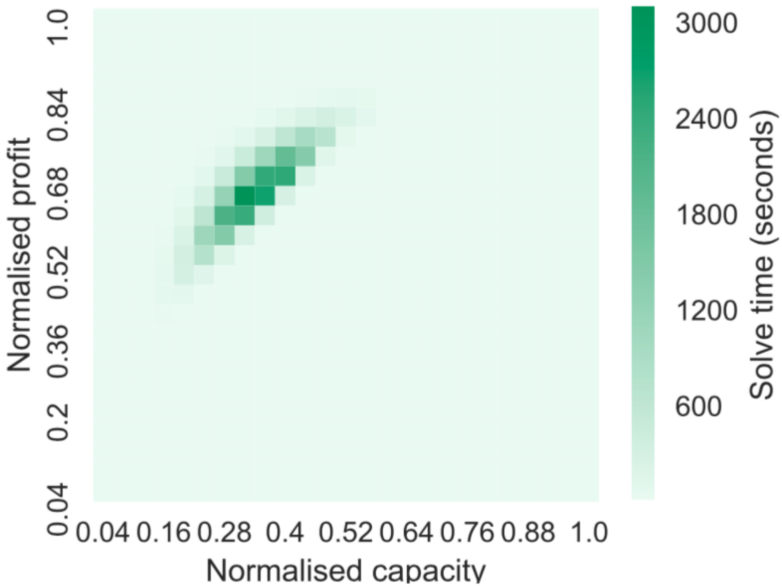


Figure 2: The amount of computational time required for the algorithm to solve a KP-DEC instance relative to its normalised capacity and normalised profit.

Adapted from "Phase transition in the knapsack problem", by N. Yadav, C. Murawski, S. Sardina, & P, Bossaerts, 2018, *working paper*, p. 8. Adapted with permission.

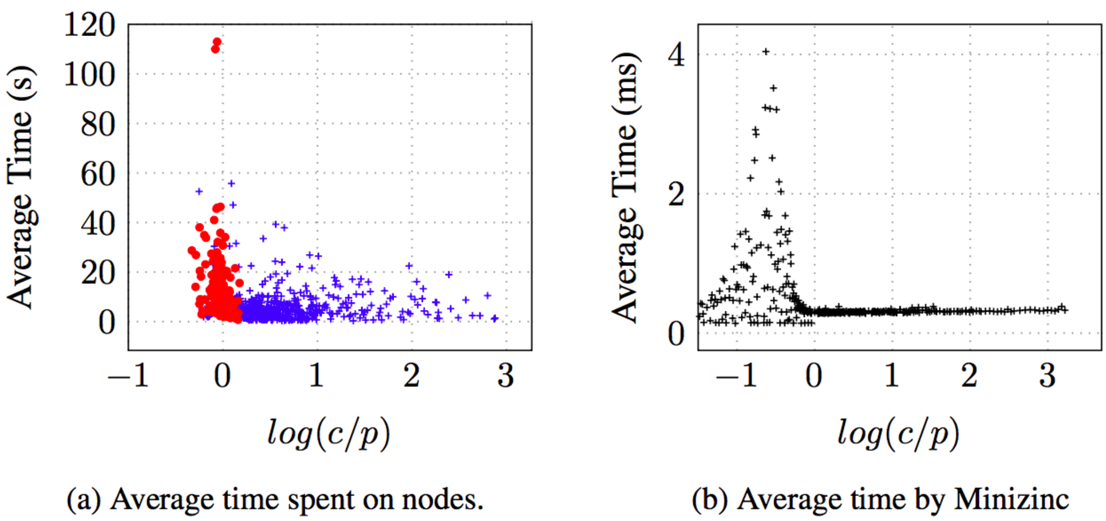


Figure 3: Solvability of the KP-DEC measured by log(c/p). **(a)** Average time spent on a KP-DEC instance search path node by humans. **(b)** Average time spent by Minizinc.

Adapted from "Phase transition in the knapsack problem", by N. Yadav, C. Murawski, S. Sardina, & P, Bossaerts, 2018, *working paper*, p. 8. Adapted with permission

2 Research Questions and Hypotheses

I propose the following four research questions and hypotheses to examine the extent of information aggregation in the market under varying levels of instance complexity of the KP-DEC.

2.1 Information Aggregation under Computational Complexity

Q1: Does the market aggregate individual traders' information about the solvability of KP-DEC instances into market prices?

H1: The market price of a given security aggregates individual traders' information about the solvability of the corresponding KP-DEC instance, and eventually converges to its fair value.

2.2 Learning from Market Prices

Q2: Do subjects learn from market prices and thereby improve their individual performance?

H2: Subjects learn from market prices and improve their individual performance by updating solutions that appear suboptimal according to market prices.

2.3 Varying Instance Complexity

Q3a: How does the complexity of a given KP-DEC instance affect individual performance?

H3a: Individual performance decreases as instance complexity measured by its relative location with respect to the phase transition increases.

Q3b: How does the complexity of a given KP-DEC instance affect information aggregation in the market?

H3b: The informativeness of market prices, i.e., the speed of price convergence, decreases as the instance complexity measured by its relative location with respect to the phase transition increases.

2.4 Security Design: The Impact of a Verification Mechanism

Q4: Does a security design that incorporates a KP-DEC's verification mechanism improve individual performance and/or information aggregation in the market?

H4: The presence of a verification mechanism in the KP-DEC improves both individual performance as well as information aggregation relative to the corresponding optimisation problem.

2.4.1 Remarks

To make a comparison between the KP-DEC and the KP, I will use one particular instance of the KP studied in Bossaerts et al. (2018). Specifically, I will transform their instance with a Sahni-k[6] of three into a KP-DEC instance whose profit constraint equals the optimal solution to the KP instance. Sahni-k of three was selected as Bossaerts et al. (2018) discover that individuals struggle to solve the KP from this level of instance complexity onwards.

[6] Sahni-k is a metric used to measure instance complexity of the KP, see Bossaerts et al. (2018) for further details.

3 Experimental Design

3.1 Participants

Participants were recruited from the University of Melbourne for five experimental sessions. To be eligible, participants were not allowed to have any previous experience with earlier "Knapsack experiments". The final sample include a total of 87 participants, with the number of participants in each experimental session ranging from 13 to 19 (see Table 1).[7] The average age of participants was 23 (age range: 18 to 46, standard deviation = 4, and the gender ratio was 29 males: 58 females).[8]

Table 1: Summary Statistics per Experimental Session

This table shows the summary statistics per experimental session. This includes the average final earnings per participant, the standard deviation of final earnings, and total number of participants.

Session	**Average Final Earnings**	**sd ($)**	**No. of Participants**
1	$46.53	0.77	19
2	$44.37	2.09	19
3	$46.59	0.71	17
4	$50.54	0.88	13
5	$48.42	2.41	19
Overall	$47.08	2.53	Total= 87

[7] The relative small number of participants may raise concerns regarding the scalability and relevance of the experimental results to the real world. However, **?** find that fifteen participants are sufficient for experiments to appropriately model real markets.

[8] The study was approved by the University of Melbourne Human Research Ethics Committee (Ethics ID: 1852128.2) and was conducted in accordance with the World Medical Association Declaration of Helsinki. All participants provided written informed consent.

3.2 Task

Each experimental session consisted of six trading rounds and six corresponding KP trials. In each KP trial, participants were asked to carry out a Knapsack task while simultaneously trading four securities, which corresponded to the respective KP-DEC instances, in an online marketplace.

A Knapsack task included a set of ten items with given values and weights, and four KP-DEC instances with different profit constraints that were subject to the same item set and capacity constraint. Participants were asked to concurrently perform a trading task, which involved trading securities based on the solvability of the corresponding KP-DEC instances in the online marketplace.

The KP-DEC instances were made available electronically on a computer interface, where participants could try out different solutions.[9] The software recorded every move of an item into and out of the Knapsack. Importantly, the software did not indicate if a proposed solution satisfied a given profit constraint or if it was optimal.

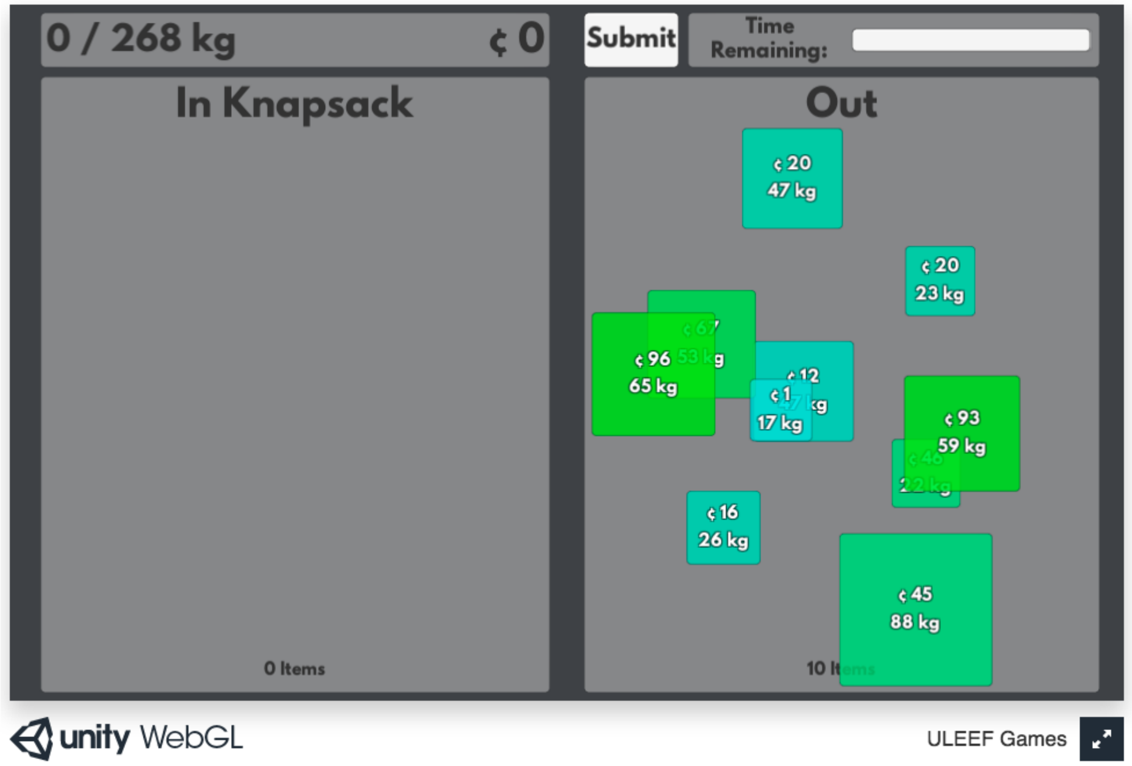
The four securities could be traded on an online market platform called Ad-Hoc-Markets, where the platform recorded every order and trade in the marketplace.[10] Ad-Hoc-Markets uses the continuous double-sided open-book system that is similar to exisiting stock exchange markets around the world.

Participants were given a maximum of 10 minutes in each round to use the Knapsack solver and to trade in the marketplace simultaneously.[11] The user interfaces of the Knapsack solver and the online market platform are shown in Fig. 4.

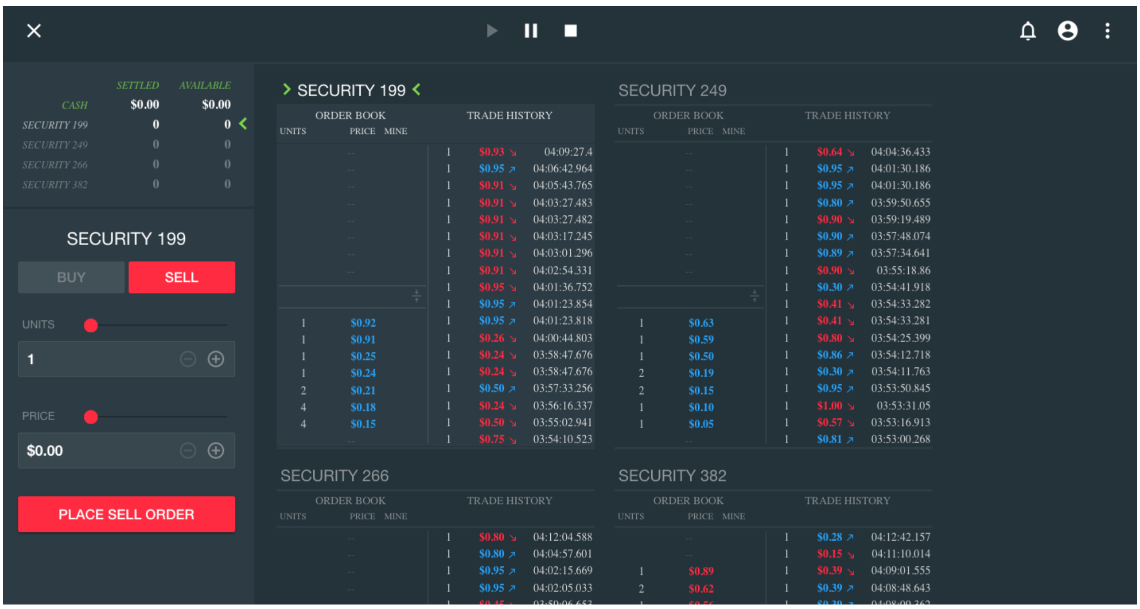
[9] The application is part of a game suite called ULEEF GAMES, which can be accessed at http://bmmlab.org/games.

[10] See http://www.adhocmarkets.com.

[11] A trading round was ended early if majority of the participants agreed to end the round given that there was no trade in the market for at least 30 seconds.



(a) Knapsack Solver



(b) Ad-Hoc-Markets

Figure 4: User Interfaces

Each experimental session started with the reading of the instructions, followed by a practice round, which jointly lasted about one hour.[12] This was to ensure that participants had ample time to familiarise themselves with the Knapsack solver, the market platform, and how to exploit, through trading, the knowledge acquired from attempting to solve a KP-DEC instance. After the practice round, participants were given a 10-minute break before starting the six official rounds of the experiment. In total, each experimental session lasted approximately two-and-a-half hours.

3.3 Instance Selection

Within a KP trial, there were four KP-DEC instances with different profit constraints that shared the same item set and capacity constraint.[13] As a result, the four KP-DEC instances also corresponded to the same underlying KP instance, where the optimal solution to the KP instance was the same for the four KP-DEC instances. In other words, all four of the KP-DEC instances have the same maximum Knapsack value in a given KP trial. Thus, I defined the maximum value of the Knapsack in a given KP trial as the *optimal solution* to the corresponding KP instance.

Each KP-DEC instance was mapped to one of the four securities in a KP trial, where the profit constraint of a given KP-DEC instance was attached to the corresponding security as its strike price. The securities were ordered in an ascending manner of its strike price in each trading round. The security design was similar to that of a digital option: the security paid \$1 if the profit constraint of the corresponding KP-DEC was attainable and zero otherwise.

For one KP trial (KP Trial 3), I transformed the KP instance studied in Bossaerts

[12] See Appendix A for a handout of the Instructions.

[13] See Appendix B for the normalised capacity and normalised profit of each KP-DEC instance used in the experiment

et al. (2018), which has a Sahni-k equal to three, into a KP-DEC instance where the profit constraint corresponded to the optimal solution to the KP instance. This formed the basis for the remaining three KP-DEC instances in the trial, of which they shared the same item set and capacity constraint. Consequently, I was able to make a comparison between the KP and the KP-DEC.

For the remaining five KP trials, I selected the c and p based on their locations in the phase transition (see Fig. 2).[14] I specifically chose the normalised values of two trials to be inside the phase transition, and those of the remaining three trials to be outside the phase transition. This allowed me to examine the effect of computational complexity of the KP-DEC instances on individual and market performance.

Then, for each set of c and p, I chose the Knapsack parameters (i.e., the Knapsack item set and capacity constraint) that had the median propagation value from a sample of Knapsacks simulated in Yadav et al. (2018).[15] The Knapsack parameters formed the basis of instance selection for the five KP trials.

I then selected the profit constraint of one KP-DEC instance to be equal, or at least centred around, the maximum value of the Knapsack given its capacity constraint. This profit constraint was the *reference point* for choosing the remaining three profit constraints.

I then determined the desired payoff structure for each corresponding trading round (i.e., whether a KP-DEC instance should satisfy its profit constraint or not in each KP trial). This was important to ensure that the payoff structure of each trading round was random. The remaining three profit constraints were chosen based on the payoff structure and propagation value.

[14] The parameters of each KP trial can be found in Appendix B denoted as Security 0.

[15] Note that there are many combinations of item set and capacity constraint that correspond to a given normalised capacity and normalised profit.

Specifically, the profit constraints were chosen according to the Knapsack values closest to *reference point* that corresponded to the payoff structure, where the respective propagation values were sufficiently lower than 20, and if possible, lower than five. This was to ensure that the effect of instance complexity of the KP-DEC could be easily distinguished between the easy and hard instances.

Fig. 5 shows the computational complexity of the selected KP-DEC instances in terms of c and p (See Fig. 2 for comparison). Evidently, it characterises the phase transition phenomenon of the KP-DEC, where the propagation value is the highest when log(c\p) is near 0 (see Fig. 6, which is similar to Fig. 3b).

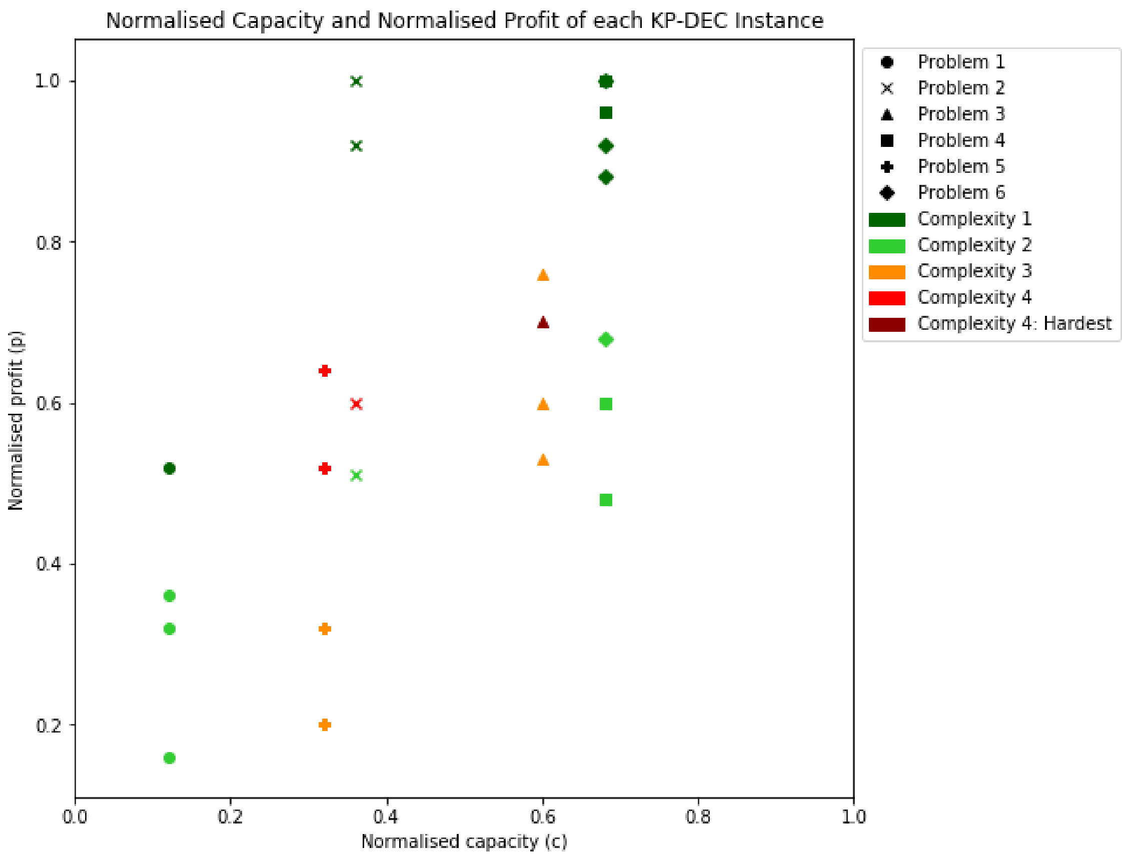


Figure 5: Propagation value of each KP-DEC instance relative to c and p. It characterises the phase transition phenomenon of the KP-DEC. See Figure 2 for comparison.

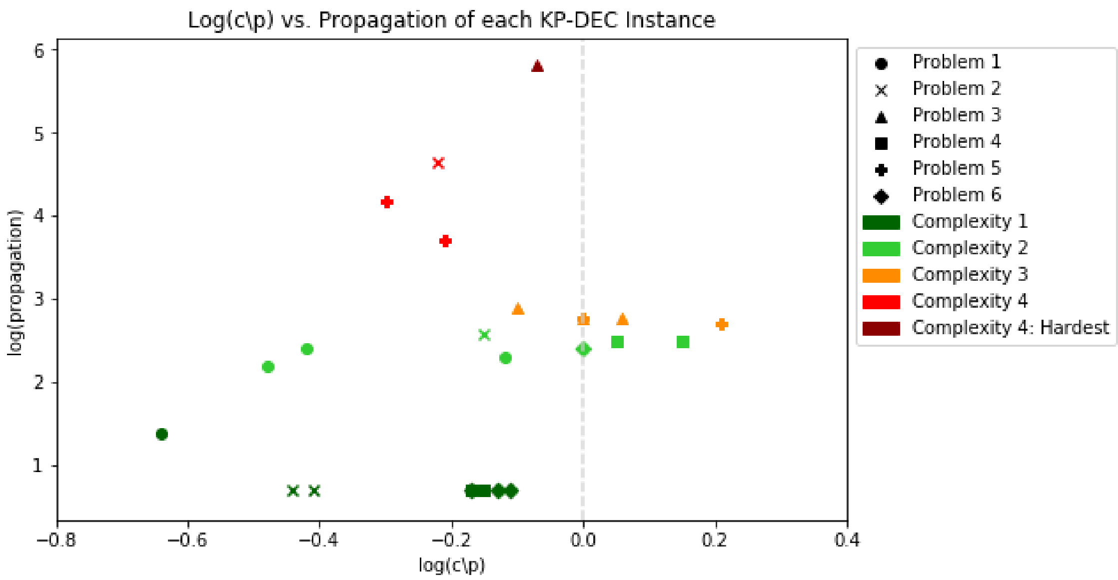


Figure 6: Propagation value relative to log(c\p) of each KP-DEC instance. See Fig. 3b for comparison

3.4 Initial Allocations

The initial allocations of securities were designed to induce trade in the marketplace. Participants were divided into two groups, Group I and Group II. In the first round, Group I (odd numbered participants) was endowed with four shares each of the first and third security; Group II (even numbered participants) was endowed with four shares each of the second and forth security. Then, the initial allocation of securities were alternated between groups for the remaining rounds, such that in the second round, Group I had Group II's initial allocation, and Group II had Group I's, and so forth.

Overall, the average initial allocations of securities were similar for all participants with respect to the security payoffs. That is, participants were endowed with equal numbers of shares of securities that paid \$1 and securities that paid zero across the six trading rounds.

3.5 Incentives

In every round, all participants were endowed with \$8 of cash, and four shares each of two securities based on their group allocation (refer to 3.4 Initial Allocations). The price range of a share of each security was between \$0 and \$1. It is important to note that short sales were not permitted in trading.

Participants were informed that their earnings could increase by buying shares of a security that corresponded to a solvable KP-DEC instance and selling shares of a security that corresponded to an insolvable instance.[16]

To align participant incentives with performance, final earnings were determined based on two "payment rounds" that were randomly selected at the end of the experimental session.[17] Final earnings consisted of: (i) the amount of final cash holdings of each payment round; (ii) liquidating dividends from each share held at the end of each payment round; (iii) a fixed reward of \$2 for submitting at least one proposed solution through the Knapsack solver for each KP task; and (iv) a show-up fee of \$10. The average final earnings across the five experimental sessions was \$47.08 (final earnings range: \$42 to \$52, standard deviation = \$2.53, see Table 1).

Notice that the fixed reward for submitting a solution was independent of the number of submissions per KP task, and also independent of the correctness of the submissions. This was designed in order to make it impossible for participants to hedge themselves via trading in the marketplace and simultaneously submitting solutions through the Knapsack solver.

[16] See Appendix A for Instructions.

[17] See https://www.random.org/ for the random number generator used in the experiment

4 Data Analysis

The following section outlines the methodology used for data analysis in this paper. It is important to note that the observations collected from the experiment were aggregated into 15 second time intervals for data analysis purposes. This is because there was limited activity in the trading and Knapsack tasks, so using the original dataset or time intervals less than 15 seconds created analytical issues, including noisy data and insufficient observations. For individual-specific analysis, I used limit market orders (i.e., bid and ask orders) rather than traded prices as a proxy for pricing signals in markets, as the multi-asset markets were relatively illiquid (See Appendix F).[18]

Table 2: Summary Statistics of Aggregated Knapsack and Market Activities in each KP Trial

This table shows the summary statistics per KP Trial aggregated across all experimental sessions. This includes the number of moves made in the Knapsack solver, the number of market bid orders submitted, the number of market ask orders submitted as the number of market traded orders executed in each KP Trial. For each of the activity, I computed the total activity level across all experimental sessions and average activity per participant (N = 87).

KP Trial	**No. of moves in the Knapsack task**		**No. of market bid orders**		**No. of market ask orders**		**No. of market traded orders**	
	Total	**Avg.**	**Total**	**Avg.**	**Total**	**Avg.**	**Total**	**Avg.**
1	22384	258	898	11	613	8	503	6
2	29802	343	1033	12	590	7	514	6
3	29104	335	892	11	867	10	519	6
4	25664	295	1041	12	577	7	513	6
5	24120	278	866	10	762	9	497	6
6	29116	335	970	12	770	9	464	6

[18] Illiquidity is not a major concern here, as markets were competitive and functioned efficiently.

4.1 On Information Aggregation under Computational Complexity

For each security in each KP trial of an experimental session, I checked whether the market traded price converged to its fair value or not. I then aggregated the dataset and computed the average market traded price per 15 second time interval to check for overall price convergence.

Further, I examined how subjects used the information about the solvability of a given KP-DEC instance to trade in the market. I split the dataset into two subsamples based on the payoffs of the securities (\$1 or \$0). The general regression is as follows:

$$\begin{aligned}(bid/ask)\ orders_{i,p,j,t} = \alpha_i + \rho_t + \beta_0 \Delta var_{p,j,t} + \\ \beta_1 \Delta var_{p,j,t-1} + \ldots + \\ \beta_k \Delta var_{p,j,t-k} + \epsilon_{i,p,j,t}\end{aligned} \tag{1}$$

where $(bid/ask)\ orders$, i, p, j and t denote the number of bid or ask orders submitted by an individual, an individual, a trading round, a security, and a given 15 second time interval; α_i and ρ_t denote the individual and time fixed effects, respectively; Δvar denotes the change in the individual performance measure (see below); and k is the number of lagged terms of the explanatory variable. I used the first difference in the individual performance measure, as the running maximum of an individual tends to increase over time (see Appendix C).

The individual performance measure is defined as the difference between the running maximum of an individual's Knapsack value and the profit constraint of a given KP-DEC instance, normalised by that profit constraint.[19] Formally:

$$var_{i,p,j,t} = \left[\frac{Running\ Maximum_i}{Profit\ Constraint_{p,j}} - 1\right]_t \tag{2}$$

where $var,\ i,\ p,\ j$ and t denote the individual performance measure, an individual, a trading round, a security, and a given 15 second time interval. A positive (negative) measure of individual performance indicates that the individual's Knapsack value is greater (less) than the profit constraint of a given KP-DEC instance.

For securities that paid \$1, the subsample included the number of bid orders submitted by an individual conditional on positive individual performance measure. This regression therefore examines how a participant reacts in the market when his Knapsack value exceeds a specific profit constraint in a KP trial.

Intuitively, a participant will submit bid orders if his individual performance measure is positive. This is because the participant is certain that the fair value of the corresponding security is \$1. Thus, a *rational* participant will profit by buying shares of the security when its price is below \$1.

For securities that paid \$0, the subsample included the number of ask orders submitted by an individual conditional on negative individual performance measure. I then took the absolute value of the negative individual performance measure so that the interpretation of the coefficient signs would be consistent with the Equation 1. This regression therefore examines how a participant reacts in the market when his Knapsack value is below a specific profit constraint in a KP trial.

The rationale is that a participant will submit ask orders when his individual performance measure is negative. However, the participant can never be sure that

[19] Running maximum or cumulative maximum refers to the best available value of an individual's Knapsack over time.

the fair value of the corresponding security is indeed zero, as he can never be sure that a solution does not exist that exceeds the profit constraint. Thus, the participant will be more hesitate to sell shares of the security above the price of zero.

Equation 1 is modelled using the Poisson model to account for the discrete nature of the dependent variables (i.e., the number of limit orders submitted by an individual is count data).[20]

4.2 On Learning from Market Prices

To examine the flow of information from markets to individuals, I used two regression analyses. First, I studied how individual effort in the Knapsack task is affected by a change in market price using a subsample of the dataset conditional on the individual Knapsack value being lower than the optimal solution to the corresponding KP instance. I then regressed the change of market bid order of a security on the number of moves made by an individual in the Knapsack solver. Here, the dependent variable is used as a proxy for measuring individual (physical) effort. Formally,

$$\begin{aligned} moves_{i,p,j,t} = \alpha_i + \rho_t + \beta_0 \Delta market\ bid_{p,j,t} + \beta_1 \Delta market\ bid_{p,j,t-1} \\ + \ldots + \beta_k \Delta market\ bid_{p,j,t-k} + \epsilon_{i,p,j,t} \end{aligned} \tag{3}$$

where $moves$, i, p, j and t denote the number of moves in the Knapsack solver, an individual, a trading round, a security, and a given 15 second time interval; α_i and ρ_t denote the individual and time fixed effects, respectively; $\Delta market\ bid$ denotes the change in market bid order; and k is the number of lagged terms of the explanatory

[20] Alternative models, such as the negative binomial model for count data, were also tested in the analysis, but they failed to converge due to the complexity of the model using many fixed effects parameters.

variable. I took the first difference of market bid order, as it has the tendency to increase over time.

Equation 3 is modelled using the Poisson model to account for the discrete nature of the dependent variable (i.e., the number of moves in the Knapsack solver is count data).[21]

Second, I examined how participants learn from market prices and thereby improve their individual performance in the Knapsack task using a dynamic fixed effects model. Formally:

$$\begin{aligned} \Delta var_{i,p,j,t} = \alpha_i + \rho_t + \beta_0 \Delta market\ bid_{p,j,t} \ + \beta_1 \Delta market\ bid_{p,j,t-1} \\ + \ldots + \beta_k \Delta market\ bid_{p,j,t-k} + \epsilon_{i,p,j,t} \end{aligned} \tag{4}$$

where Δvar, i, p, j and t denote the change in the individual performance measure using Equation 2, an individual, a trading round, a security, and a given 15 second time interval; α_i and ρ_t denote the individual and time fixed effects, respectively; $\Delta market\ bid$ denotes the change in market bid order; and k is the number of lagged terms of the explanatory variable. I took the first differences of the variables as they exhibit trending, non-stationary behaviour over time (see Appendix C and Appendix D).

In addition, I calculated the proportion of subjects who submitted the optimal solution to the corresponding KP instance in each KP trial. This can be interpreted as the instance solvability of the KP.

[21] Alternative models, such as the negative binomial model for count data, were also tested in the analysis, but they failed to converge due to the complexity of the model using many fixed effects parameters.

4.3 On Varying Instance Complexity

The informativeness of market prices was quantified by a price convergence measure using integration. This measure computes the area under or above the curve of the average traded price of a security across all experimental sessions per 15 second time interval, relative to its payoff.

For a security that paid \$1, price convergence was measured as the area above the curve of the average traded price. For a security that paid \$0, it was measured as the area below the curve of the average traded price.

The price convergence measure was then normalised by the total trading time of each security in the market. The price convergence measure is the greatest (smallest) when the speed of price convergence is the slowest (fastest).

I also ran a fixed effects regression model to examine the relationship between instance complexity of the KP-DEC and price convergence for each security in each session. Formally,

$$\log(npc_{s,p,j}) = \alpha_s + \rho_p + D_k + \beta_k \log(propagation_{p,j}) \times D_k + \epsilon_{s,p,j} \tag{5}$$

where npc, s, p and j denote the normalised price convergence measure, a session, a trading round, and a security; α_s and ρ_p denote the fixed effects for session and KP trial, respectively; D denotes the dummy variables for the difficulty ranking of each KP-DEC instance within a KP trial where one indicates the easiest instance and four indicates the hardest instance; k denotes the difficulty ranking of securities within a KP trial; and *propagation* is the propagation value of each KP-DEC instance. The lowest difficulty formed the baseline of the regression.

I included dummy variables for the difficulty ranking of instances within a KP trial to separate the easy KP-DEC instances from the difficult ones. Here, I am most interested in the KP-DEC instances with a difficulty ranking of four, as these instances were used as the *reference points* for instance selection. They were considered to be the hardest instances within each KP trial based on their propagation values.

Further, these instances have a greater variation in propagation value relative to other difficulty rankings, as the remaining KP-DEC instances were purposefully selected to be easy and have similar in their propagation values. Thus, the coefficients of the dummy variables for ranking one, two and three should be insignificant.

4.4 On Security Design: The Impact of a Verification Mechanism

To compare the KP and its decision variant, the KP-DEC, I examined the instance solvability of each problem in terms of the proportion of subjects who submitted the option solution to the KP instance with a Sahni-k of three studied in Bossaerts et al. (2018). The optimal solution to the KP instance with a Sahni-k of three is equivalent to the profit constraint of Security 3 in KP Trial 3. This measure therefore allows me to study the impact of a verification mechanism on individual, as well as market performance.

5 Results

5.1 On Information Aggregation under Computational Complexity

All securities except one converged to their fair values (either \$1 or \$0) in every experimental session.[22] The exception was Security 2 in KP Trial 6, where participants seemed to have a prior expectation of the securities' payoffs.[23]

I also examined how participants use their private information about the solvability of a KP-DEC instance to trade in the market using Equation 1. The results are twofold. First, for securities that paid \$1, I found that there is a statistically positive relationship between the change in individual performance and the number of bid orders submitted by a participant ($\beta_0 = 1.928$, $p < 0.001$). While the lagged terms of the explanatory variables are all insignificant.

Second, for securities that paid \$0, there is a positive relationship between the change in individual performance and number of ask orders submitted by a participant for the first and fourth lagged term ($\beta_1 = 1.080$, $p < 0.001$; $\beta_4 = 0.834$, $p < 0.01$). However, the effects are much smaller in magnitude when compared to the market bid order case ($\beta_0 = 1.928$, $p < 0.001$). Additionally, the second lagged term of the explanatory variable has a significantly negative effect on the number of ask orders submitted by a participant ($\beta_2 = -0.651$, $p < 0.05$). While the remaining lagged terms are insignificant.

[22] See Appendix E for the aggregated trading activity of each KP trial.

[23] Subjects appeared to assume that this particular security would pay \$1 given the payoff patterns of the previous trading rounds. Yet, the payoff patterns were random and independent of one another. Thus, the subjects were confused about the fair value of Security 2, which resulted in the price of Security 2 being extremely noisy.

Table 3: Regression 1.1

This table reports the regression results for equation 1 using a Poisson model. For each participant i of security j in each KP trial p at time t, I estimate equation 1 using a subsample conditional on securities that paid \$1 and positive individual performance measure ($\Delta var_{i,p,j,t} > 0$), divided into 15 second time intervals. *Bid Orders* is the number of bid orders submitted by participant i. Δvar is the change in the individual performance measure defined using equation 2 with 5 lagged terms. The last six rows report the fixed effects used in the regression, the number of observations in the dataset, and the model selection criteria (log-likelihood, AIC, and BIC). Robust standard errors are reported in parentheses below the coefficient estimates.

	$Bid\ Orders_{i,p,j,t}$
$\Delta var_{i,p,j,t}$	1.928***
	(0.473)
$\Delta var_{i,p,j,t-1}$	−0.048
	(0.552)
$\Delta var_{i,p,j,t-2}$	0.066
	(0.510)
$\Delta var_{i,p,j,t-3}$	−0.171
	(0.488)
$\Delta var_{i,p,j,t-4}$	−0.160
	(0.462)
$\Delta var_{i,p,j,t-5}$	0.143
	(0.320)
Constant	−4.203***
	(0.201)
Individual Fixed Effects	Yes
Time Fixed Effects	Yes
Observations	55,994
Log Likelihood	-8,442
Akaike Inf. Crit.	16,903
Bayesian Inf. Crit.	16,983

Note: + p<0.1; * p<0.05; ** p<0.01; *** p<0.001

Table 4: Regression 1.2

This table reports the regression results for equation 1 using a Poisson model. For each participant i of security j in each KP trial p at time t, I estimate equation 1 using a subsample conditional on securities that paid \$0 and the absolute value of the negative individual performance measure ($\Delta var_{i,p,j,t} = |\Delta var_{i,p,j,t} < 0|$), divided into 15 second time intervals. *Ask Orders* is the number of ask orders submitted by participant i. Δvar is the change in the absolute value of the individual performance measure defined using equation 2 with 5 lagged terms. The last six rows report the fixed effects used in the regression, the number of observations in the dataset, and the model selection criteria (log-likelihood, AIC, and BIC). Robust standard errors are reported in parentheses below the coefficient estimates.

	$Ask\ Orders_{i,p,j,t}$
$\Delta var_{i,p,j,t}$	0.316
	(0.235)
$\Delta var_{i,p,j,t-1}$	1.080^{***}
	(0.254)
$\Delta var_{i,p,j,t-2}$	-0.651^{*}
	(0.290)
$\Delta var_{i,p,j,t-3}$	0.131
	(0.302)
$\Delta var_{i,p,j,t-4}$	0.834^{**}
	(0.256)
$\Delta var_{i,p,j,t-5}$	-0.019
	(0.197)
Constant	-5.247^{***}
	(0.307)
Individual Fixed Effects	Yes
Time Fixed Effects	Yes
Observations	55,994
Log Likelihood	-6,864
Akaike Inf. Crit.	13,746
Bayesian Inf. Crit.	13,826
Note:	+ p<0.1; * p<0.05; ** p<0.01; *** p<0.001

5.2 On Learning from Market Prices

On the aggregate, the running maximum of the average participant improved over time, and the average participant found the optimal solution to the corresponding KP instance in four of the six KP trials (KP Trial 2, 4, 5, and 6). This is shown graphically in Appendix C.

Using regression analysis, I find that there is a significantly positive relationship between the number of moves in the Knapsack task and the contemporaneous term of the change in market bid of a security in a given market ($\beta_0 = 0.305$, $p < 0.001$). While there is a statistically negative relationship between the number of moves in the Knapsack task and the four lagged terms of the change in market bid ($\beta_1 = -0.414$, $p < 0.001$; $\beta_2 = -0.657$, $p < 0.001$; $\beta_3 = -0.571$, $p < 0.001$; $\beta_4 = -0.341$, $p < 0.001$). The fifth lagged term is insignificant.

This suggests that a change in market bid induces an individual to exert more effort in the Knapsack task, however the effect is short-lived, as individual effort decreases after the first 15 seconds. The results of Equation 3 can be shown in Table 5.

Further, there is a significantly positive relationship between the change in market bid of a given security and the change in individual performance in the Knapsack, where individual performance is defined in Equation 2 (see Table 6). I found that a 1% increase in the market bid in the first 15 seconds improves an individual's Knapsack value by 4.3% ($\beta_0 = 0.043$, $p < 0.01$). Further, the first lagged term of the explanatory variable has a greater positive effect on individual performance of 4.8% ($\beta_0 = 0.048$, $p < 0.01$). While the remaining lagged terms are insignificant at the 5% significance level.

Table 5: Regression 2

This table reports the regression results for equation 3 using a Poisson model. For each participant i of security j in each KP trial p at time t, I estimate equation 3 using a subsample contional on the running maximum of participant i's Knapsack value being lower than the profit constraint of security j, divided into 15 second time intervals. $Moves$ is the number of moves by participant i in the Knapsack solver. $\Delta Market\ bid$ is the change in market bids with 5 lagged terms. The last six rows report the fixed effects used in the regression, the number of observations in the dataset, and the model selection criteria (log-likelihood, AIC, and BIC). Robust standard errors are reported in parentheses below the coefficient estimates.

	$Moves_{i,p,j,t}$
$\Delta Market\ bid_{p,j,t}$	0.305^{***}
	(0.061)
$\Delta Market\ bid_{p,j,t-1}$	-0.414^{***}
	(0.064)
$\Delta Market\ bid_{p,j,t-2}$	-0.657^{***}
	(0.057)
$\Delta Market\ bid_{p,j,t-3}$	-0.571^{***}
	(0.051)
$\Delta Market\ bid_{p,j,t-4}$	-0.341^{***}
	(0.050)
$\Delta Market\ bid_{p,j,t-5}$	-0.024
	(0.038)
Constant	-1.349^{***}
	(0.399)
Individual Fixed Effects	Yes
Time Fixed Effects	Yes
Observations	29,058
Log Likelihood	-71,456
Akaike Inf. Crit.	142,930
Bayesian Inf. Crit.	143,004
Note:	+ p<0.1; * p<0.05; ** p<0.01; *** p<0.001

Table 6: Regression 3

This table reports the regression results for equation 4 using a fixed effects model. For each participant i of security j in each KP trial p at time t, I estimate equation 4 using the entire sample divided into 15 second time intervals. Δvar is the change in the individual performance measure. $\Delta Market\ bid$ is the change in market bids with 5 lagged terms. The last six rows report the fixed effects used in the regression, the number of observations in the dataset, and the model selection criteria (log-likelihood, AIC, and BIC). Robust standard errors are reported in parentheses below the coefficient estimates.

	$\Delta var_{i,p,j,t}$
$\Delta Market\ bid_{p,j,t}$	0.043^{**}
	(0.015)
$\Delta Market\ bid_{p,j,t-1}$	0.048^{**}
	(0.017)
$\Delta Market\ bid_{p,j,t-2}$	0.020
	(0.016)
$\Delta Market\ bid_{p,j,t-3}$	0.020
	(0.014)
$\Delta Market\ bid_{p,j,t-4}$	0.023^{+}
	(0.013)
$\Delta Market\ bid_{p,j,t-5}$	0.018^{+}
	(0.010)
Constant	-0.001
	(0.002)
Individual Fixed Effects	Yes
Time Fixed Effects	Yes
Observations	29,058
Log Likelihood	17,353
Akaike Inf. Crit.	-34,687
Bayesian Inf. Crit.	-34,604
Note:	+ p<0.1; * p<0.05; ** p<0.01; *** p<0.001

5.3 On Varying Instance Complexity

KP Trial	Proportion of subjects who submitted the optimal solution
1	89.66%
2	72.41%
3	45.98%
4	87.36%
5	85.06%
6	91.95%

Table 7: Proportion of subjects who submitted the optimal solution to the corresponding KP instance in each KP Trial

I found that the proportion of subjects who submitted the optimal solution to the corresponding KP decreased as the instance complexity of its decision variant measured by propagation value increased. The proportion ranged from 45.98% for the KP-DEC instance with highest propagation value (KP Trial 3) to 91.95% for the KP-Trial which has the lowest average propagation value of an instance (KP Trial 6), see Table 7.[24] Given that subjects were not incentivised to submit their best solution in the Knapsack task, I speculate that the proportion of subjects who found the optimal solution to the corresponding KP instance is even higher in reality.

Using regression analysis, I found that there is only a statistically positive relationship between price convergence and instance complexity when the difficulty ranking of instances within a KP trial is the highest ($\beta_4 = 0.086$, $p < 0.01$; see Table 8 and Fig. 7). While difficulty rankings two and three have no effect on price convergence in the market.

This is expected given my instance selection procedure, where only the KP-DEC instances that were used as the *reference points* are considered to be difficult and

[24] Refer to Appendix E for the proportion of subjects who submitted a solution at different Knapsack values.

Table 8: Regression 4

This table reports the regression results for equation 5 using a fixed effects model. For each security j in each KP trial p of each session s, I estimate equation 5 using the respetive normalised price convergence measure and propagation value. npc is the normalised price convergence measure. Propagation is the propagation value. D2, D3, and D4 are dummies for the difficulty ranking of the KP-DEC instances within a KP trial. The lowest difficulty forms the baseline and is thus not explicitly reported. The last six rows report the fixed effects used in the regression, the number of observations in the dataset, and the model selection criteria (log-likelihood, AIC, and BIC). Robust standard errors are reported in parentheses below the coefficient estimates. Five observations were missing from the dataset as there was no trade in the market.

	$\log(npc_{s,p,j})$
$\log(propagation_{p,j})$	-0.064^{*}
	(0.025)
D2	-0.077
	(0.059)
D3	0.073
	(0.074)
D4	-0.123
	(0.075)
$\log(propagation_{p,j}) \times D2$	0.054
	(0.033)
$\log(propagation_{p,j}) \times D3$	-0.019
	(0.034)
$\log(propagation_{p,j}) \times D4$	0.086^{**}
	(0.029)
Constant	0.717^{***}
	(0.046)
Session Fixed Effects	Yes
Problem Fixed Effects	Yes
Observations	115
Log Likelihood	61
Akaike Inf. Crit.	-100
Bayesian Inf. Crit.	-70
Note:	+ p<0.1; * p<0.05; ** p<0.01; *** p<0.001

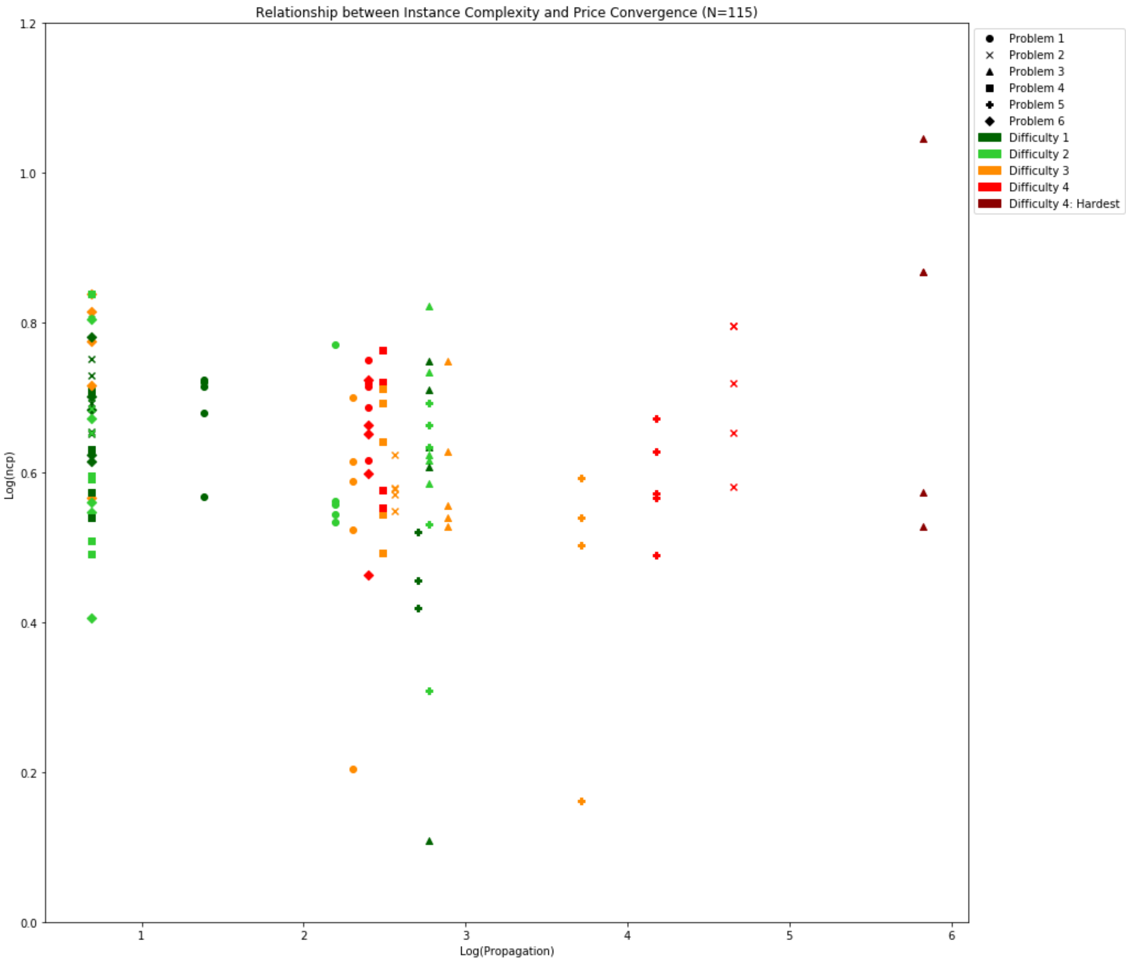


Figure 7: Relationship between instance complexity and normalised price convergence (N=115)

are of interest in this regression. While the remaining instances were purposefully selected to be relatively easy. Interestingly, I found that instance complexity has a significantly negative effect on the price convergence measure for the lowest instance difficulty ranking ($\beta_1 = -0.064$, $p < 0.05$). This could suggest that there is a learning effect where subjects become more skilled in solving the KP-DEC instances when the propagation values of the KP-DEC instances are relatively low and similar.

Additionally, my results show that market performance, in terms of the price convergence measure, decreased as the location of the KP-DEC instance approached the phase transition region (see Fig. 8). That is, price convergence was the slowest when log(c\p) was near the left of zero.

Overall, instance complexity has a negative effect on both individual and market performance, where individual performance, measured in terms of instance solvability, decreases and price converges at a slower rate (i.e., the price convergence measure increases) when instance complexity of the KP-DEC increases.

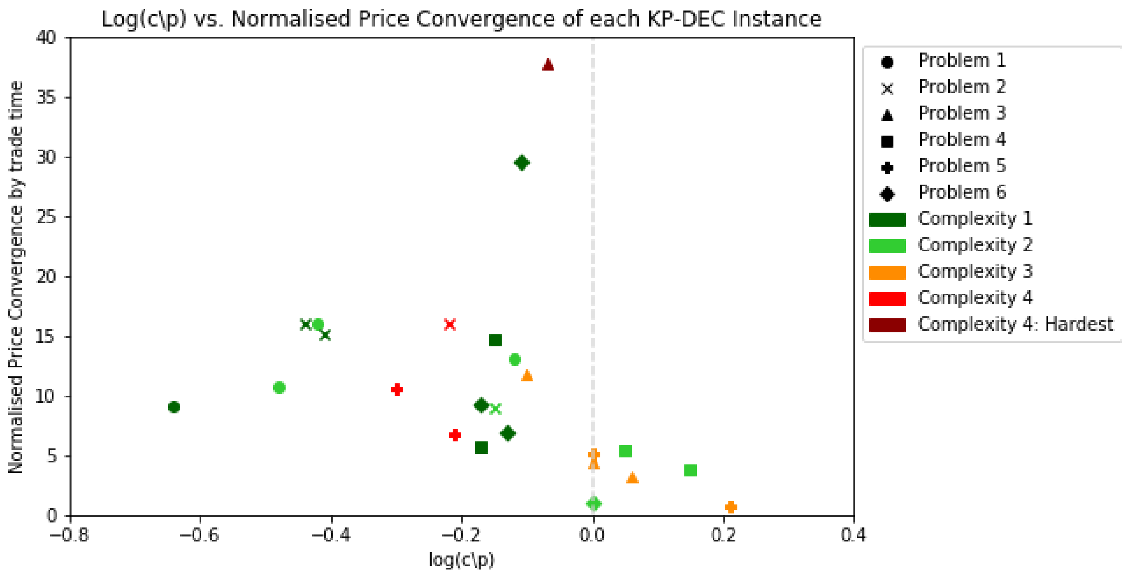


Figure 8: Normalised price convergence by trade time relative to log(c\p) of each KP-DEC instance. See Fig. 6 for comparison with the performance of Minizinc relative to log(c\p).

5.4 On Security Design: The Impact of a Verification Mechanism

I examined the proportion of subjects who submitted the optimal solution to the corresponding KP instance in my experiment relative to that studied in Bossaerts et al. (2018). In my experiment, 45.98% of the subjects found the optimal solution, while only 26.9% of the subjects did in Bossaerts et al. (2018). Further, Security 3 in KP Trial 3, which corresponded to the KP-DEC instance where the profit constraint equals to the optimal solution to the KP, converged to its fair value. This differs to Bossaerts et al. (2018), where they find that the equivalent KP instance, which has a Sahni-k of three, converged to the NREE price.

6 Discussion and Implications

6.1 On Information Aggregation under Computational Complexity

Q1: Does the market aggregate individual traders' information about the solvability of KP-DEC instances into market prices?

Given that the underlying problem was computationally complex (i.e., KP-DEC is *NP-complete*), the subjects' ability to solve the KP-DEC instances varied. This resulted in the emergence of information asymmetry in markets, where some subjects had superior information about the solvability of a given KP-DEC instance over the others. Consequently, the presence of information asymmetry introduced noise in markets, which caused prices to fluctuate in the beginning of a trading round.

However, noise was subsequently eliminated via continuous trading, where market prices (of most securities) converged to their fair values.[25] Thus, the presence of noise did not affect markets' ability to aggregate and reflect information about the solvability of the KP-DEC instances.

My results are therefore consistent with the hypothesis that market prices converged to their fair values. This is contrary to Bossaerts et al. (2018) who conclude that market prices are never fully revealing when there is asymmetric information under computational complexity. In other words, Bossaerts et al. (2018) find that market prices converge to the NREE prices, which is not the case in my study.

Further, my results are inconsistent with the EMH and the REE model. I find that markets were informationally efficient even when market prices did not reveal

[25] Security 2 in KP Trial 6 was an exception.

all information about a given KP-DEC instance. For instance, market prices did not reveal the items that should be included in the Knapsack to satisfy the profit constraint of a given KP-DEC instance. In fact, market prices only aggregated and reflected what was necessarily important for the subjects to solve the KP-DEC instance, i.e., whether the profit constraint of a KP-DEC instance was solvable or not.

This finding provides evidence that market efficiency does not depend on the amount of information, but the quality of information being aggregated in market prices, which is consistent with Hayek (1945). Importantly, market prices only need to reveal crucial information about the underlying KP-DEC instance for participants to correctly solve the problem. In this case, the crucial information is the instance solvability of the KP-DEC.

Additionally, the regression results in Table 3 and Table 4 show that subjects react quickly to an increase in individual performance by submitting more bid orders within the first 15 seconds of improvement. While there seems to be a delayed reaction for submitting ask orders when an individual performance is negative. This is expected, as participants knew with certainty that a security paid $1 when their proposed Knapsack solution exceeded the profit constraint of the corresponding KP-DEC instance.

On the other hand, participants could never be certain about the solvability of a given KP-DEC instance when their proposed Knapsack solution was below the profit constraint. Therefore, participants were more hesitant to submit ask orders than bid orders as a result of uncertainty about the instance solvability of the KP-DEC. This could explain the regression results where the number of ask orders submitted by an individual has the tendency to increase then decrease, and then increase again over time given a positive change in the individual performance measure computed using Equation 2 (see Table 4).

Overall, my findings show that participants fully understood the experimental tasks, and that the multi-asset markets were competitive under the CCT framework. Participants submitted bid and ask orders according to their private information, and there were rarely any arbitrage opportunities in the multi-asset markets (76 arbitrage occurrences, which was 0.88% of the total market activity, N=8684).[26]

[26] Arbitrage opportunities were either immediately exploited, immediately cancelled, or left unexploited.

6.2 On Learning from Market Prices

Q2: Do subjects learn from market prices and thereby improve their individual performance?

To answer this question, I examined the regression results of Equation 3 and Equation 4 concurrently in order to provide insights into how subjects learn from market prices and improve their individuals performance. On the aggregate, a positive change in market bid order encourages an individual to expend more effort in the Knapsack task, which thereby improve his individual performance in the first 15 seconds. However, in the subsequent 15 seconds, the individual exerts less effort in the Knapsack task but his Knapsack value continues to improve. After the first 30 seconds, individual effort continues to decrease with no improvements in individual performance in the Knapsack task (See Table 5 and Table 6).

One explanation for these results could be that subjects first approached a KP-DEC instance using the sampling method as proposed by Savage (1972). However, given that sampling is known to be ineffective for solving computationally complex problems (Bossaerts et al., 2018), I conjecture that subjects also recognised this limitation and subsequently reduced their *physical* effort in solving the Knapsack task over time.

Instead of randomly trying out different combinations of items in the Knapsack solver, subjects could be cognitively engaged in solving the KP-DEC instance (e.g., mentally adding Knapsack item values to the Knapsack). This could explain why there was an increase in individual performance even when subjects made less moves in the Knapsack solver. Clearly, this type of mental effort was not captured by the measure of individual effort using the number of moves made by a subject in the Knapsack solver, given that it only accounts for physical, observable effort. However, the exact reasoning for these results remains inconclusive.

Overall, my results show that markets provide a form of feedback mechanism for subjects to improve their Knapsack solution over time. This consequently motivates them to exert more effort, whether physically or mentally, to find the optimal solution that maximises the Knapsack value in a KP trial.

6.3 On Varying Instance Complexity

Q3a: How does the complexity of a given KP-DEC instance affect individual performance?

Consistent with existing literature (Bossaerts and Murawski, 2017; Bossaerts et al., 2018; Yadav et al., 2018), individual performance decreases as a given KP-DEC instance increases in computational complexity. This finding suggests that it is difficult for subjects to find a solution to a given KP-DEC instance as instance complexity increases in propagation value. This in turn affects the subjects' ability to verify their proposed solution, which resulted in a decline in the proportion of subjects who found the optimal solution to the corresponding KP instance. I therefore conclude that the CCT framework can be used to model human decision-making, which is consistent with current studies on the relationship between computational complexity and human decision-making.

In addition, my results show that an increase in instance complexity creates a greater knowledge gap between the informed subjects who solved a given KP-DEC instance and the uninformed subjects. This in turn introduces more noise in the market as a result of greater information asymmetry, and consequently decreases the informativeness of market prices.

Q3b: How does the complexity of a given KP-DEC instance affect information aggregation in the market?

I find that there is a positive relationship between propagation value and the price convergence measure for instances with a difficulty ranking of four. This finding suggests that price converges at a slower rate as propagation value increases conditional on KP-DEC instances that are the most difficult in each KP trial.

Further, I find that markets behave similarly to algorithms under computational complexity, where price convergence is the slowest when the location of the KP-DEC instance is closest to the phase transition region, i.e., when $\log(c\backslash p)$ is to the left of zero (see Fig. 6 and Fig. 8). Importantly, markets are affected by the instance complexity of the KP-DEC as characterised by the phase transition. This finding aligns with Yadav et al. (2018) who find that both algorithms and humans perform worse in a KP-DEC instance when $\log(c\backslash p)$ gets closer to zero (see Fig. 3 for comparison). Thus, the CCT framework can be used to explain the behaviours of humans, as well as markets.

Interestingly, for KP-DEC instances with the lowest propagation value within a KP trial, the relationship between instance complexity and the price convergence measure is negative. This could imply that there is a learning effect where participants improved their ability in perform the two experimental tasks over time. This finding suggest that a small variation in propagation values has little impact on the subjects' ability to solve a given KP-DEC instance [27], and this in turn has little impact on markets' ability to aggregate information in prices. Further, when the KP-DEC instances are easy (i.e., propagation values are sufficiently lower than 20), the informativeness of market prices seem to be affected by factors other than the variation in instance complexity of the KP-DEC.

[27]This could be the fact that humans use heuristics in decision-making, which makes them insensitive to small changes in computational complexity (Simon, 1955).

6.4 On Security Design: The Impact of a Verification Mechanism

Q4: Does a security design that incorporates a KP-DEC's verification mechanism improve individual performance and/or information aggregation in the market?

My results show that individual performance improved significantly when subjects were asked to solve a decision problem instead of an optimisation problem (instance solvability of the KP increased from 26.9% in Bossaerts et al. (2018) to 45.98% in decision variant.). This finding suggests that the presence of a verification mechanism, which is inherent to decision problems, has a positive impact on individual decision-making when the underlying problem is complex.

Given the nature of decision problems, the optimal solution to a given KP instance can be attained by repeatedly solving the KP-DEC instances with different profit constraints but share the same item set and weight constraint. Therefore, the security design used in the experiment (i.e., a digital option) had an inherent feature where the securities' payoffs are interdependent in a market.

For instance, if a participant discovered that a profit constraint of a given KP-DEC instance was satisfiable, then it follows that the KP-DEC instances with lower profit constraints were also satisfiable. The presence of a verification mechanism therefore removed any uncertainty about the payoffs of the securities with equal or lower strike prices. Consequently, the participant would bid for those securities based on his private information about the *interdependency* of securities, which would then be reflected in market prices.

For uninformed subjects, who have yet to solve a given KP-DEC instance, he could infer from those market prices and solve the KP-DEC instances accordingly. For instance, if a security was trading at $1, the uninformed subject would know that any KP-DEC instances with a profit constraint lower than the strike price of that security would also pay \$1. Thus, the uninformed subject would focus his effort on solving KP-DEC instances with higher profit constraints instead, where the instance solvability was still uncertain (i.e., the security price is between \$0 and \$1).

The market was able to eliminate the uncertainty surrounding the solvability of a given KP-DEC instance by revealing the informed subjects' minimum achievable Knapsack value in market prices. However, it is important to understand that this did not work reversely. That is, participants could not be certain that a security that was trading at \$0 indicated that the profit constraint of the corresponding KP-DEC instance was unsolvable.

Therefore, the market played an important role in facilitating the verification mechanism by guiding individuals towards the most relevant KP-DEC instance at hand where the solvability is difficult to determine (i.e., when $\log(c \backslash p)$ is near zero). This is shown in my results where price convergence was the slowest when a KP-DEC instance was located around the phase transition region. This shows that the participants exerted more effort collectively to try and determine the solvability of the KP-DEC instance, while the remaining KP-DEC instances were less active (see Appendix F).

Participants were incentivised to solve the difficult KP-DEC instance as it was profitable to trade on superior private information. This consequently improved the informational efficiency of the market as the solvability of the KP-DEC instance imposed either a lower or upper bound for the optimal solution to the corresponding KP instance. That is, the profit constraint of a solvable KP-DEC instance sets the minimum value for the optimal solution to the KP instance; while an insolvable KP-DEC instance sets the upper limit.

My results are therefore inconsistent with Bossaerts et al. (2018) who find that market prices converged to the NREE prices. This suggests that the verification mechanism of the KP-DEC improves individual performance in the Knapsack task, which translates into better information aggregation in the market. The multi-asset market seems to act as a feedback channel for uninformed subjects to verify their proposed solution and for informed subjects to check if higher profit constraints are achievable.

Further, a security design that incorporates the verification mechanism of a decision problem has an inherent feature where the securities are interdependent. As a result, market prices reveal information about the solvability of other KP-DEC instances to individuals, which consequently improves informational efficiency of a multi-asset market.

7 Conclusion

This paper used an experimental markets framework to study the impact of security design on information aggregation under computational complexity. Specifically, I examined the KP-DEC, which is the decision variation of the KP, where there exists a verification mechanism for individuals to check whether a proposed solution satisfies a specified profit constraint or not. Such a verification mechanism was incorporated in the security design in the form of a digital option. Specifically, each security had a strike price equals the profit constraint of the corresponding KP-DEC instance, and the payoff of the security was directly linked to the solvability of the KP-DEC instance. The findings of my paper are fourfold.

First, my results show that markets aggregate and reflect individual traders' information about the solvability of a given KP-DEC instance in market prices. Importantly, most securities ultimately converged to their fair values even when there is information asymmetry in the market.

Second, subjects learn from market prices and improve their individual performance over time. In fact, subjects react very quickly to pricing signals by exerting more effort in the knapsack task in order to improve their individual performance. Further, participants then utilise the newly acquired information about the solvability of a KP-DEC to trade accordingly in the market.

Third, my findings show that the CCT framework can be applied to humans, where human decision-making deteriorates as the difficulty of the underlying problem increases in computational complexity. This in turn reduces markets' ability to aggregate and reflect information prices in terms of their convergence. That is, the speed of price convergence decreases as the instance complexity of the KP-DEC increases.

Lastly, most subjects, whether knowingly or unknowingly, maximise the value of the Knapsack as if it was an optimisation problem, despite the fact that the KP instance was transformed into multiple KP-DEC instances with different profit constraints. This finding provides novel insights into how transforming an optimisation problem into a decision problem that incorporates a verification mechanism can significantly improve one's decision-making. Further, a security design that incorporates a verification mechanism is facilitated by a multi-asset market, which potentially provides additional information about the underlying problem. This in turn improves information aggregation in the market.

In essence, my findings show that markets do not need to reflect all information, but only the most crucial information about the underlying problem, in order to be informationally efficient. This is contrary to popular market efficiency hypotheses, notably the EMH and REE, but consistent with the conjecture made by Hayek (1945).

7.1 Limitations and Further Research

Given the experimental nature of the study, there are some limitations that can be improved in future studies. First, a more comprehensive comparison between the KP and the corresponding KP-DEC is called for, as my study only transformed one KP instance into its decision variant. Thus, it provided very limited insights into their differences in terms of information aggregation within markets.

Second, there needs to be a better measure for individual effort that accounts for the mental costs endured by an individual during the Knapsack task. This would be useful given that the KP-DEC is a cognitively-intensive problem for humans. However, finding a reliable proxy for mental effort can be difficult.

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Appendices

A Experimental Instructions

THE UNIVERSITY OF MELBOURNE

BRAIN MIND AND MARKETS LABORATORY

INSTRUCTIONS

Summary

Over 6 rounds, you will be able to trade securities in an online marketplace. Each share of a traded security pays off $1, if there exists a solution of the "knapsack problem" that satisfies the threshold condition of that security. The knapsack problem is a problem where one is asked to find the right items from a collection of possible items that fit a given knapsack while reaching a certain value.

During the experiment, you will have access to two websites. One will be an online trading platform, and the other will be a website where you can try out solutions to the knapsack problem at hand, to determine the values of a particular knapsack, and thus whether you should buy or sell securities. You earn money by buying shares of securities whose threshold value *can* be achieved given the knapsack at hand and by selling shares of securities whose threshold value *cannot* be achieved with the given knapsack. Additionally, you will earn a fixed $2 reward for each suggested solution to the knapsack problem you submit (independently of whether this solution is correct or not).

At the end of the experiment, your earnings from trading will be determined by two randomly selected trading rounds. Your performance in each round will be equal to the sum of remaining cash and the payoffs from your final share holdings in that round, plus the reward for submitting a suggested solution to the knapsack problem.

Setting: Knapsack Problems

In our knapsack problems, one is given a list of 10 items and is asked to optimally load them in a knapsack. Each item has a *weight* and a *value*. The knapsack has a *weight limitation* that may prevent one from loading all items in the knapsack. Given this limitation, one is asked to determine whether the set of items *can* achieve a pre-specified threshold value. The total value of a proposed knapsack equals the sum of the values of its individual items.

You can think of the problem as asking the following question: *Given the weight constraint of the knapsack, does there exist a combination of items that* ***at least*** *achieves a total knapsack value of X?*

In each of the 6 rounds (plus a practice round) you can see the corresponding knapsack problem online, at http://bmmlab.org/games. Log into the website with the ID and password you are given, and navigate to "Play Knapsack Game," where you pick the problem corresponding to the market from the drop-down list (see Table 1 below for the list of problem identifiers). An example of the interface is shown in Figure 1 below. You will be asked to refresh this webpage and login again between each round, in order to navigate to the next problem.

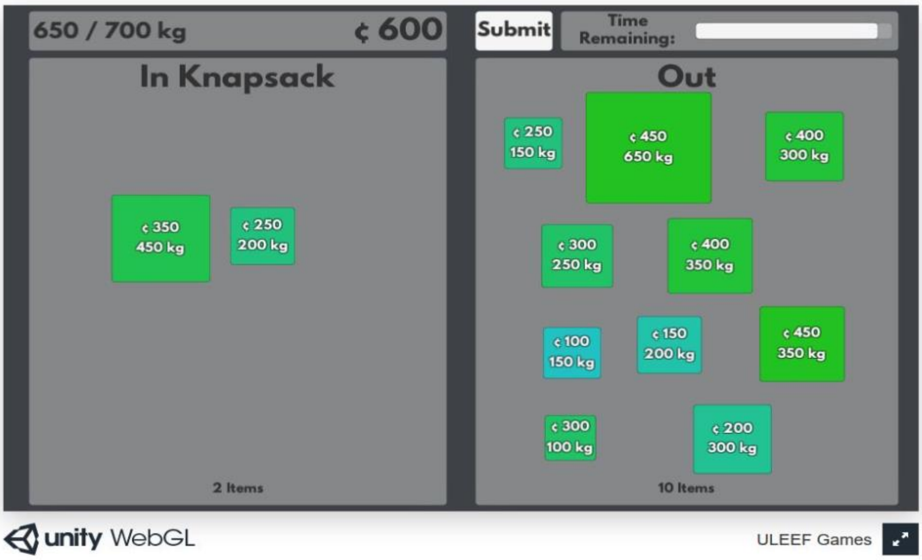

Figure 1: Knapsack problem interface

Round	Knapsack Problem Name	Market Name
Practice	KP-Trial	KPM-Trial
1	KP-Ant	KPM-Ant
2	KP-Beaver	KPM-Beaver
3	KP-Camel	KPM-Camel
4	KP-Dolphin	KPM-Dolphin
5	KP-Elephant	KPM-Elephant
6	KP-Frog	KPM-Frog

Table 1: Knapsack problem and market names

You can try for yourself which items you think should be in the knapsack, by moving the items you want in the knapsack in the “IN” panel, while keeping the others in the “OUT” panel (which is to the right in the above picture, but at other times will be to the left). Items are identified by their WEIGHT (their size increases with weight) and VALUE (their colour changes from blue to green as value increases). Before the round ends, you should SUBMIT your suggested solution by clicking on the “Submit” button. You have to submit your solution at the latest by the end of trading. (There is a white bar next to the submit button which indicates “time remaining”; please ignore it.) You earn $2 when you submit your suggested solution, independent of whether it is correct or not.

Important: You are not allowed to access any webpages other than the two you are instructed to use (the knapsack problem and the market pages)! Failure to do so will lead to exclusion from the experiment.

Earnings from Trading in the Online Market

You can earn money by *trading in a market* where shares can be bought or sold. There will be 4 types of traded securities in each market, each with a different threshold value attached to it (see Table 2 below for an example). You will be provided with a separate document called **Securities**, which includes all the tradeable securities and their corresponding threshold values for each trading round.

Security	Threshold Value
Security 30	30
Security 50	50
Security 70	70
Security 90	90

Table 2: Example of tradeable securities and their corresponding threshold values

After markets close, **each share** of a security will pay off **$1** *if* there exists a knapsack whose total value is equal or higher than the threshold value of that security. For example, let us assume that a knapsack has a maximum total value of 52. In this case, each share of 'Security 30' and of 'Security 50' (see Table 2 above) would pay $1, whereas 'Security 70' and 'Security 90' would expire worthless. In other words, only the shares of securities whose attached threshold value can be satisfied by the given knapsack will pay off at the end of trading.

If your solution to the knapsack problem at hand matches or exceeds the corresponding threshold value of a security, you can earn money by *buying* shares of that security. On the other hand, if your solution to the knapsack problem has a lower value than the threshold value of another security, you could earn money by *selling* shares of that security. For instance, if your optimal knapsack reaches a maximum value of 89, you would prefer to buy shares of 'Security 30', 'Security 50', and 'Security 70'. If the maximum total value of that knapsack is indeed 89, you should be selling shares of 'Security 90'.

Your total earnings from each trading round will consist of (i) the amount of final cash holdings, plus (ii) the sum of payoffs from each share that you hold at the end of trading. At the end of the experiment, **two** rounds will be randomly selected as "payment rounds".

Trading in the Online Market

Trading takes place through an electronic trading platform called *Adhoc-Markets*. In *Adhoc-Markets* you submit *limit orders*, which are orders to buy or sell at a price you determine, or, if possible, at any better price. Transactions take place from the moment a buy order with a higher price crosses a sell order with a lower price or vice versa. Orders remain valid until you cancel them or the marketplace closes. You will be given ample opportunity to train yourself in submitting and canceling orders.

You can access *Adhoc-Markets* as follows: use your logon information sheet and log onto https://adhocmarkets.com/ using the ID labelled "Trading Market" and the same password you have used to access the knapsack problem. You should then navigate to the market name for the corresponding round (see Table 1 above for a list of market names). Each market will be open for a pre-determined time period (approximately 10 minutes). Instructors will notify you at halftime as well as one minute before markets close.

	Website	Suggested Web Browser
Knapsack Problem	http://bmmlab.org/games	Firefox
Online Market	https://adhocmarkets.com/	Chrome

Table 3: Websites to use and suggested Web Browsers

B Parameters of Each KP-DEC Instance (N=24)

KP Trial	Security	Normlised capacity (c)	Normalised profit (p)	log(c\p)	Propagation value
1	0	0.12	0.52	-0.64	
	1	0.12	0.16	-0.12	10
	2	0.12	0.32	-0.42	11
	3	0.12	0.36	-0.48	9
	4	0.12	0.52	-0.64	4
2	0	0.36	0.6	-0.22	
	1	0.36	0.51	-0.15	13
	2	0.36	0.6	-0.22	105
	3	0.36	0.92	-0.41	2
	4	0.36	1	-0.44	2
3	0	0.6	0.68	-0.05	
	1	0.6	0.53	0.06	16
	2	0.6	0.6	0	16
	3	0.6	0.7	-0.07	339
	4	0.6	0.76	-0.1	18
4	0	0.68	0.6	0.05	
	1	0.68	0.48	0.15	12
	2	0.68	0.6	0.05	12
	3	0.68	0.96	-0.15	2
	4	0.68	1	-0.17	2
5	0	0.32	0.52	-0.21	
	1	0.32	0.2	0.21	15
	2	0.32	0.32	0	16
	3	0.32	0.52	-0.21	41
	4	0.32	0.64	-0.3	65
6	0	0.68	0.68	0	
	1	0.68	0.68	0	11
	2	0.68	0.88	-0.11	2
	3	0.68	0.92	-0.13	2
	4	0.68	1	-0.17	2

Table 9: Parameters of each KP-DEC instance. Security 0 refers to the parameters used for selecting the KP trials.

C Average Individual Running Maximum of the Knapsack Value in each KP Trial

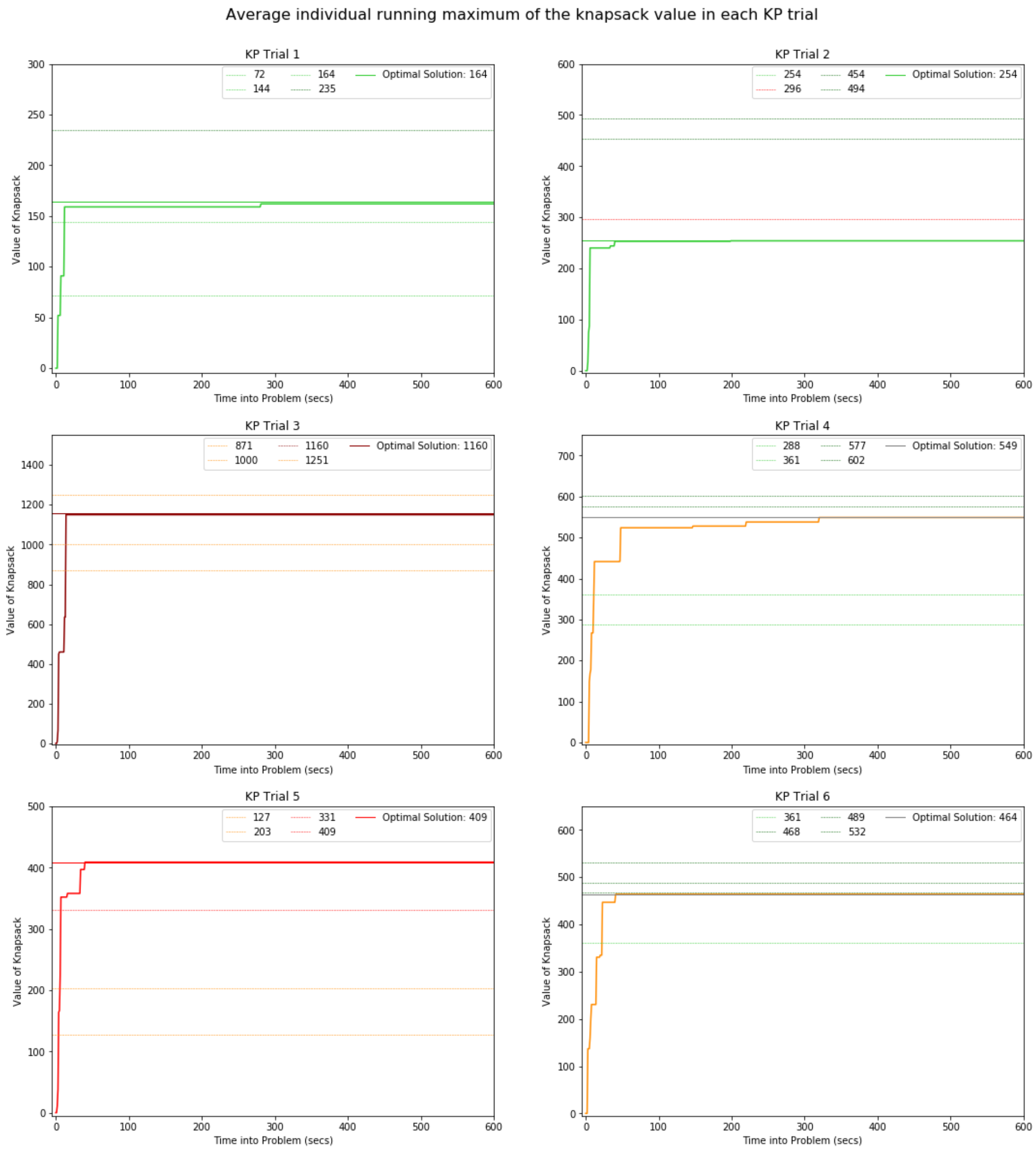

Figure 9: Average individual running maximum of the knapsack value in each KP trial

D Average Individual Moves in the Knapsack Task of each KP Trial

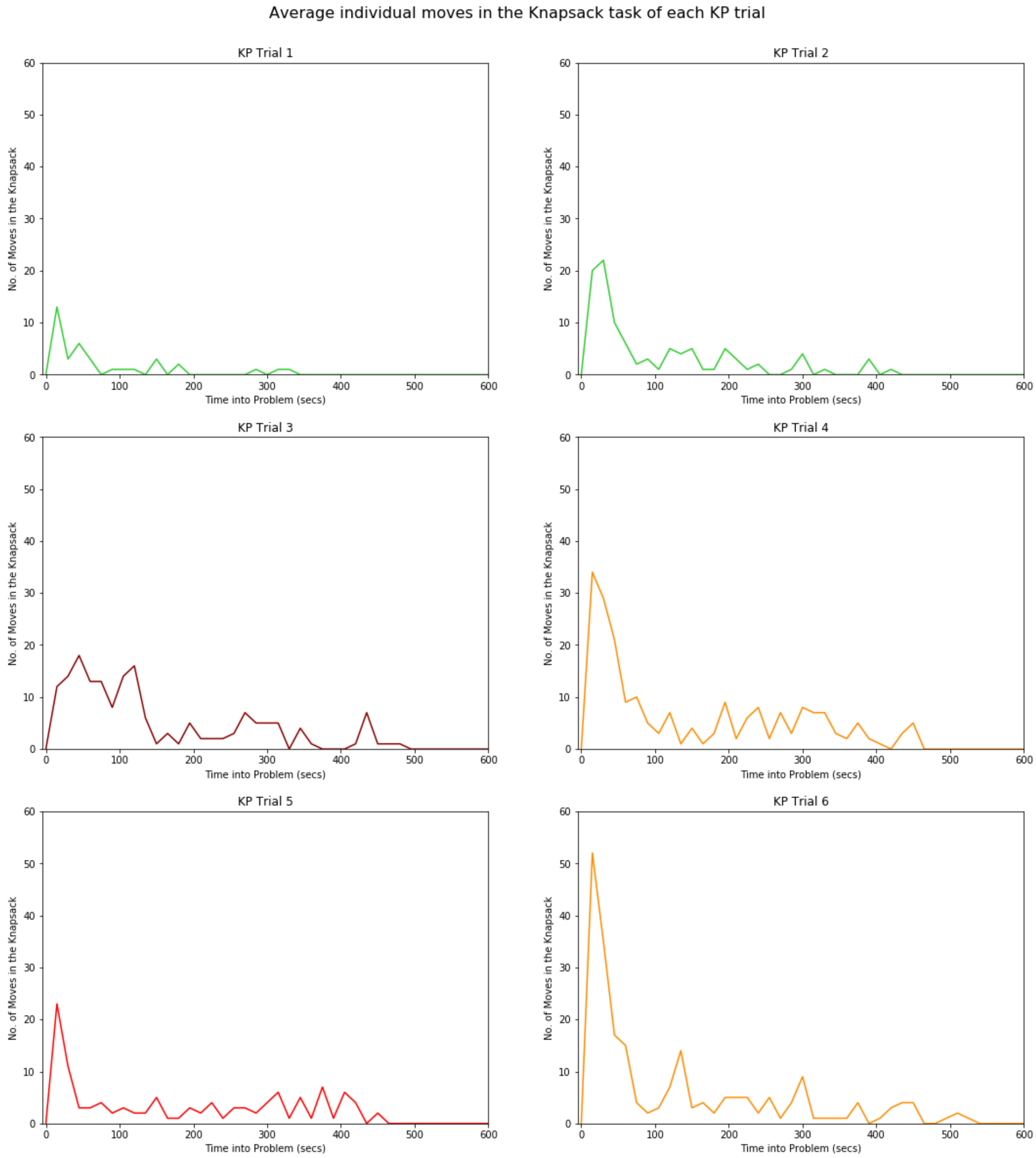

Figure 10: Average individual moves in the Knapsack task of each KP trial

E Instance Solvability of the KP-DEC

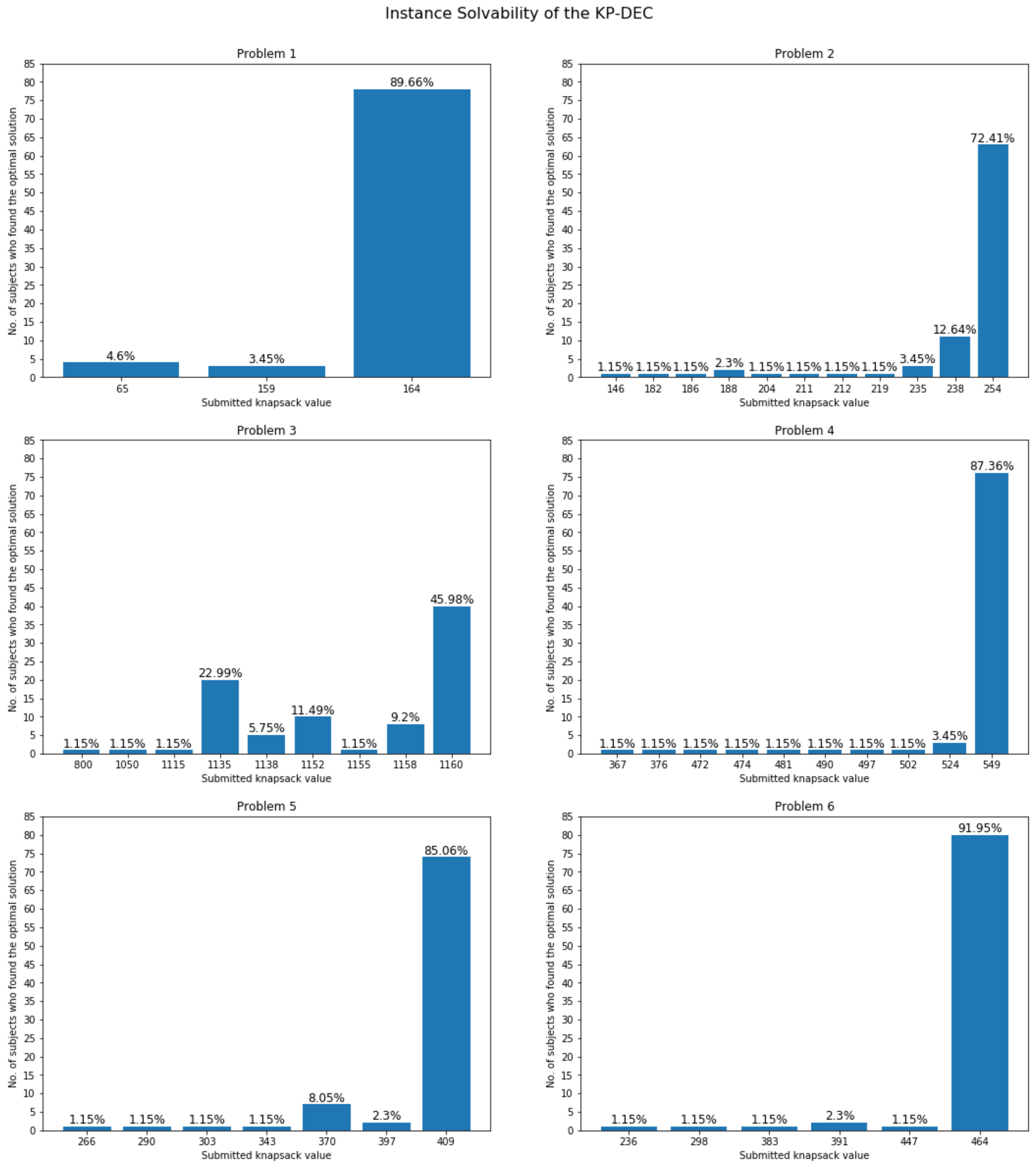

Figure 11: Submitted Solutions by subjects

F Aggregated Trading Activity of Each KP Trial

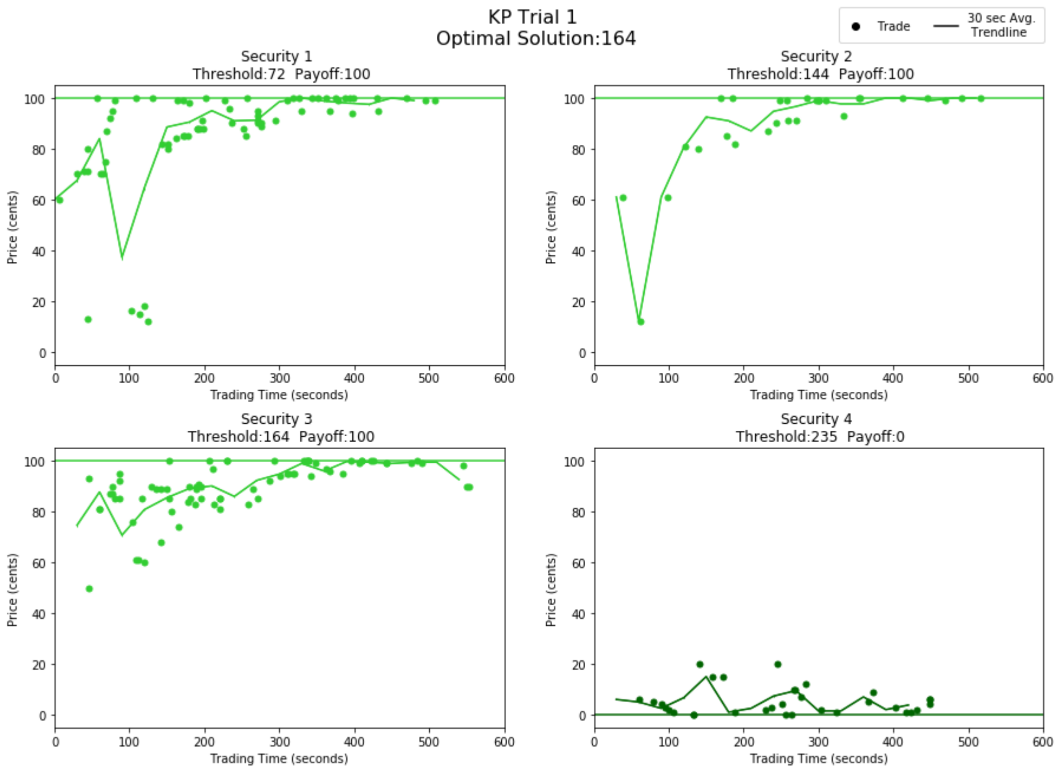

Figure 12: KP Trial 1: Aggregated Trading Activity.

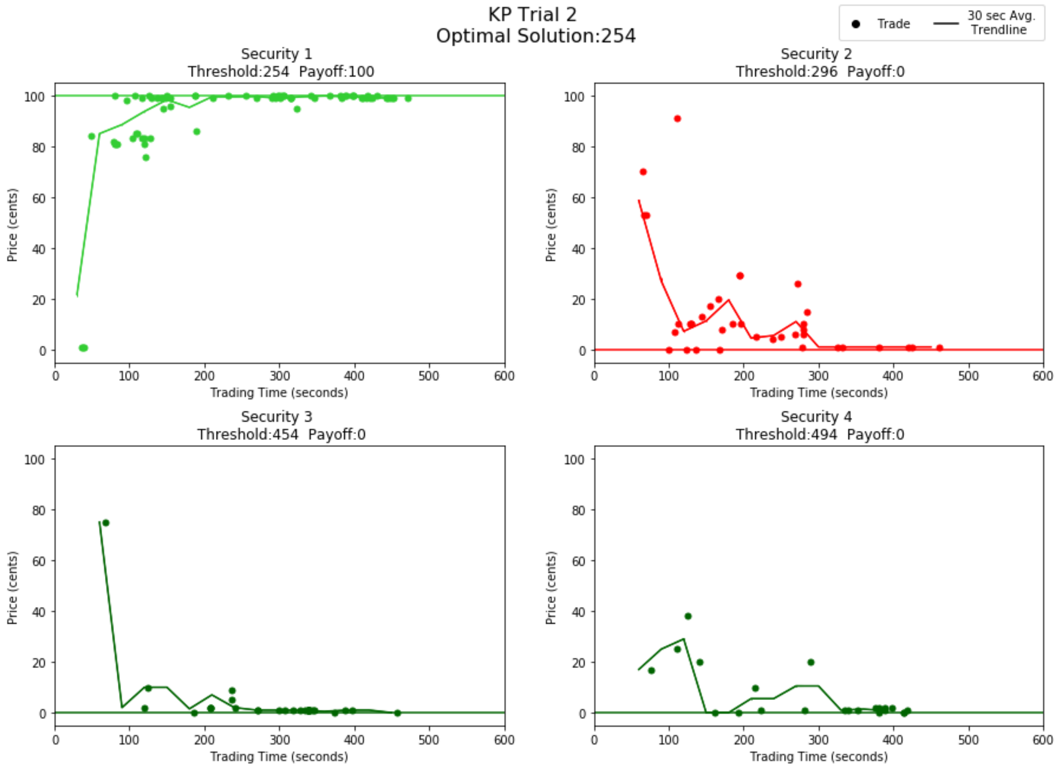

Figure 13: KP Trial 2: Aggregated Trading Activity.

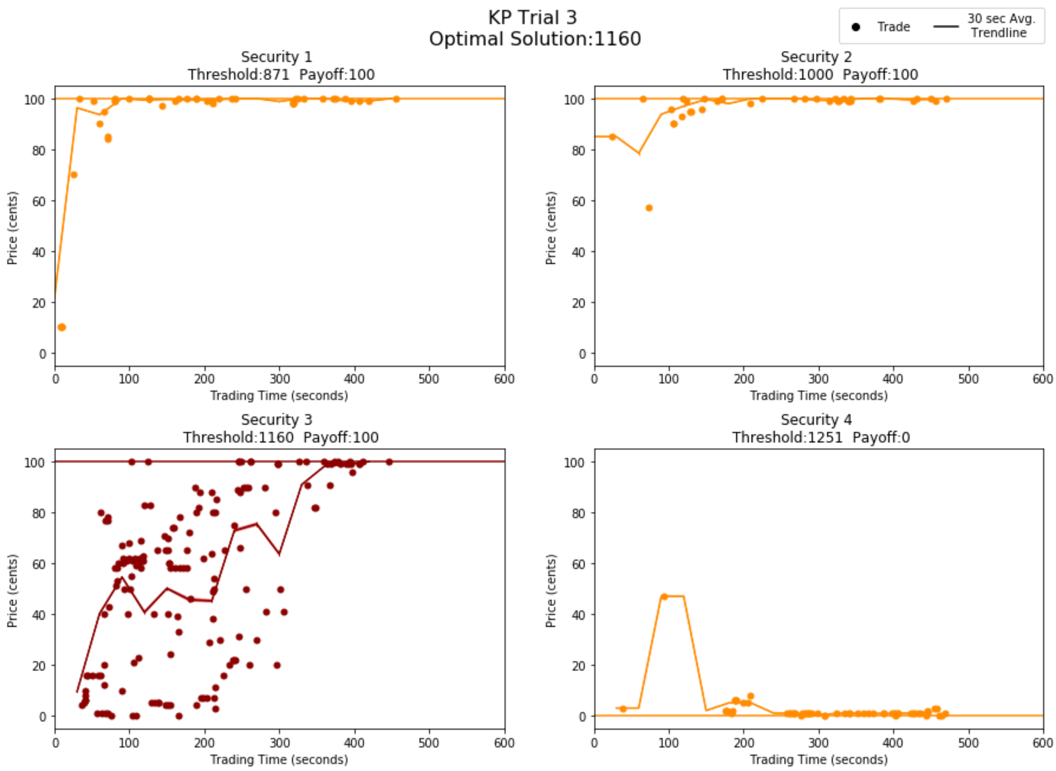

Figure 14: KP Trial 3: Aggregated Trading Activity.

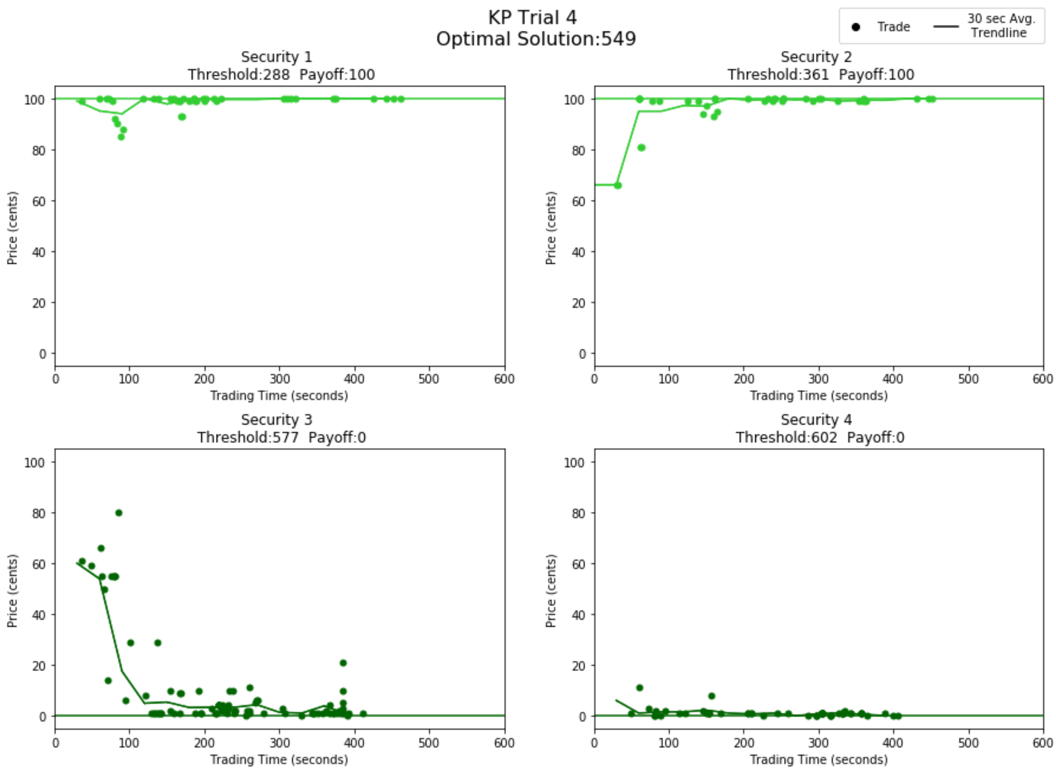

Figure 15: KP Trial 4: Aggregated Trading Activity.

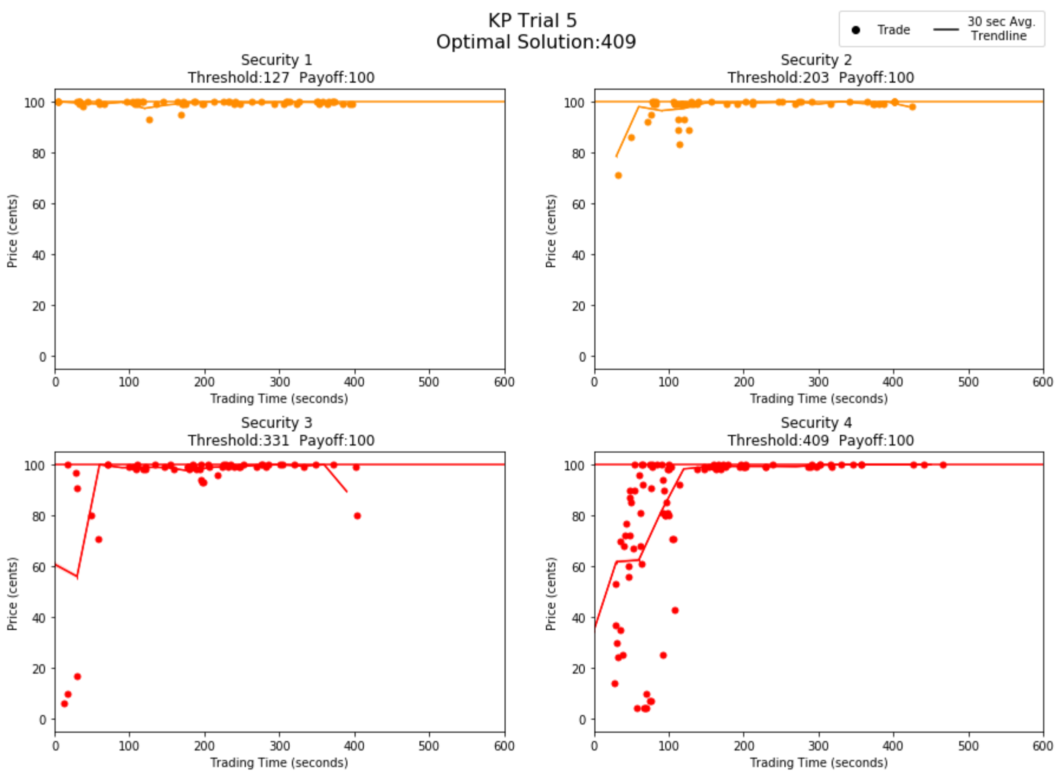

Figure 16: KP Trial 5: Aggregated Trading Activity.

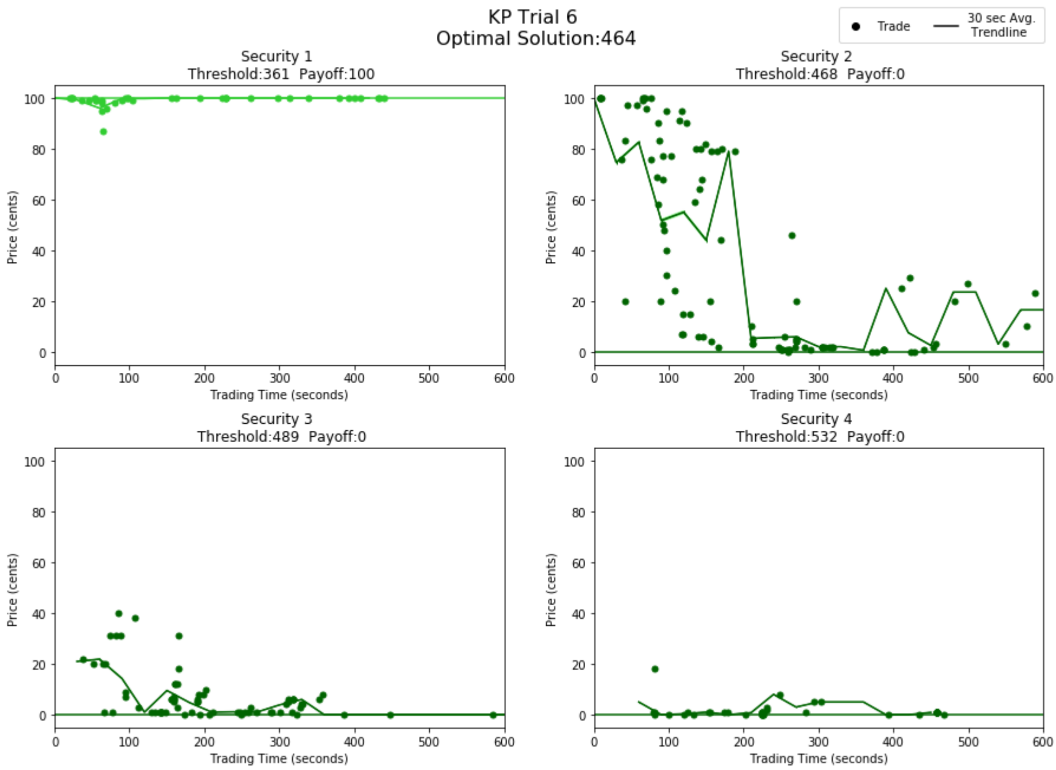

Figure 17: KP Trial 6: Aggregated Trading Activity.