STRATIFICATIONS OF TOPOLOGICAL SPACES, AND ∞-CATEGORIES

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ABSTRACT. I am interested in Stratifications in general. There does not seem to be an exceeding amount of literature on the subject, so I am interested in compiling notes on the subject as I learn. The hope is to compile notes on stratifications of topological spaces, conical stratifications, and stratifications of stable infinity categories.

CONTENTS

0. STRATIFICATIONS

0.1. Posets.

Definition 0.1.1 (Posets)**.** A poset is

+) A set P

+) A binary relation $\leq$

such that the binary relation satisfies the following properties. Let

-) (Reflexive)-$p \leq p$

-) (Antisymmetry)-If $p \leq p'$ and $p' \leq p$, then $p = p'$

-) (Transitive)-$p \leq p'$ and $p' \leq p''$ then $p \leq p''$

Example 0.1.2. The most basic example of a poset is a finite set, with a linear ordering. A linear ordering is a binary operation such that for all p and p' in P, either $p \leq p'$ or $p' \leq p$. Since every set is isomorphic to a set $\{0, \ldots, n\}$, we defined the following poset $[n]$, which gives the usual ordering to the set $\{0, \ldots, n\}$

$$[n] := \{0 \leq \cdots \leq n\}$$

Definition 0.1.3. A map between posets P and P'

+) A map $f : \mathsf{P} \to \mathsf{P}'$

such that

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-) The map f induces a map on the binary relation $\leq_P$ to $\leq_{\mathsf{P}'}$

0.2. **Posets as Categories.**

Definition 0.2.1 (Poset)**.** A poset is

+) A category P

such that

-) Either $\mathsf{Hom}(p, p') = \emptyset$ or $\mathsf{Hom}(p, p') \simeq *$

-) If $\mathsf{Hom}(p, p') = *$ and $\mathsf{Hom}(p', p) \simeq *$ then $p = p'$

Remark 0.2.2. The transitive condition of a poset P is the condition that there is a composition rule for the category, namely if $\mathsf{Hom}(p, p') \simeq \mathsf{Hom}(p', p'') \simeq *$, then it must be the case that $\mathsf{Hom}(p, p'') \simeq *$.