Particle Physics Phenomenology exercise 11

1. Show that in the $SU(2)_L \times U(1)_Y$ gauge theory, if there are n scalar multiplets ϕ_i, with dimension d_i, hypercharge Y_i, and vacuum expectation value of the neutral components v_i, then the parameter ρ is, at tree level

$$\rho = \frac{\sum_i \left[\frac{d_i^2 - 1}{4} - Y_i^2\right] v_i^2}{2\sum_i Y_i^2 v_i^2}$$

Hint: Remember that for the angular momentum group, i.e. $SU(2)$, one has $J^2 = (J_3)^2 + \frac{1}{2}(J^+J^- + J^-J^+) = j(j+1)\mathbb{I}$, with $J^\pm = J^1 \pm iJ^2$ and j the total spin of the representation.

2. (a) Find the light-neutrino spectrum in the type-I seesaw with just 2 ν_R fields? How many Majorana CP violating phases are present?

(b) Show that in type-I seesaw, in the basis were the charged leptons mass matrix is diagonal, one can always parameterize the neutrino Dirac mass matrix as

$$m_D = iU_{PMNS}\, d_\nu^{1/2}\, R^T\, d_R^{1/2}\, V_R^\dagger$$

with $d_{\nu,R}$ the light and heavy neutrino masses, respectively. V_R and U_{PMNS} are unitary matrices and R is a complex orthogonal matrix. This is known as Casas-Ibarra parameterization.

3. Take a general $1 \to 2$ decay of a massive particle to two massless ones, violating lepton number. Show that no leptonic CP asymmetry can be generated at tree-level. What happens when one-loop corrections are introduced?

Hint: You can show this for the type-I leptogenesis scenario.

4. Take the most minimal composite Higgs, i.e. $\frac{SU(3)}{SU(2)\times U(1)}$. The Coset space has 4 generators that can be identified with the Higgs degrees of freedom. The $SU(3)$ algebra is $[T^a, T^b] = if_{abc}T^c$ with $T^a = \lambda_a/2$ and

$$\lambda_1 = \begin{pmatrix} & 1 & \\ 1 & & \\ & & \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} & -i & \\ i & & \\ & & \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} & & 1 \\ & & \\ 1 & & \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} & & -i \\ & & \\ i & & \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} & & \\ & & 1 \\ & 1 & \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} & & \\ & & -i \\ & i & \end{pmatrix}$$

The first line represents the unbroken generators. Construct the Goldstone matrix in the unitary gauge, i.e. $\pi_6 = h$ all others zero. Show that the custodial symmetry is violated at tree-level. For a $\Delta\rho \sim 10^{-4}$ what would that imply to the compositeness scale?

Hint: Look at the "pions" kinetic term $f^2/4\mathrm{Tr}[d_\mu d^\mu]$, with $d_\mu = d_\mu^{\hat{a}} T^{\hat{a}} = -2i\mathrm{Tr}[U^\dagger D_\mu U T^{\hat{a}}]T^{\hat{a}}$, with $T^{\hat{a}}$ the broken generators. Use $Q = T_3 + Y$ with $Y = \frac{1}{\sqrt{3}}T_8$.