Particle Physics Phenomenology exercise 3

1. a) The splitting kernel
$$P_{\mathrm{g}\to\mathrm{q}\bar{\mathrm{q}}} \propto z^2 + (1-z)^2$$
is related to the QED angular relation
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}(\mathrm{e}^+\mathrm{e}^- \to \gamma^* \to \mathrm{q}\bar{\mathrm{q}}) \propto 1 + \cos^2\theta \ .$$
Show how one may go from one to the other.
Hint: Use fraction of the lightcone variable $p^+ = E + p_z$ as z definition.
b) How could correspondingly the splitting kernel
$$P_{\mathrm{q}\to\mathrm{qg}} \propto \frac{1+z^2}{1-z}$$
be interpreted in terms of angles?

2. a) Show that the eikonal expression
$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \, \frac{\mathrm{d}^3k}{(2\pi)^3 2\omega} \left| \sum_{i=1}^{n} g_s \, \mathbf{T}_i \, \frac{p_i}{p_i k} \right|^2$$
leads to
$$\frac{\mathrm{d}\sigma_3}{\sigma_2} \propto \frac{\mathrm{d}E_3}{E_3} \, \mathrm{d}\Omega_3 \, \frac{a_{12}}{a_{13}a_{23}} \ .$$
b) Show that the $\mathrm{q}\bar{\mathrm{q}}\mathrm{g}$ matrix element, as a function of x_1 and x_2, can be recast into the same form in the soft-gluon limit.
Hint for part b: it is enough that you show this correspondence in the CM frame. Then, in the soft-gluon limit, you are allowed to assume that partons 1 and 2 are back-to-back, which simplifies the variable transformation.

3. Do a toy Monte Carlo simulation, i.e. write your own simple code from scratch, using the veto algorithm, to estimate the number of branchings of a quark at LEP1 energies, $E_{\mathrm{cm}} = 91$ GeV $(= Q_{\max})$. Assume a fix $\alpha_{\mathrm{s}} = 0.15$, a lower evolution cutoff $Q_{\min} = Q_0 = 1$ GeV, and that only emissions with $z < 0.99$ are to be studied.

4. a) Use e^+e^- annihilation as an environment for the clean study of final-state QCD radiation. Specifically study how the average number of final-state partons increases with $E_{\rm cm}$, say 25, 50 100 and 200 GeV.
   Hint: some useful settings are

```
Beams:idA = 11
Beams:idB = -11
Beams:eCM = ...
PDF:lepton = off
WeakSingleBoson:ffbar2gmZ = on
HadronLevel:all = off
23:onMode = off
23:onIfAny = 1 2 3 4 5
```

   (or only `23:onIfAny = 1 2` to come closer to a comparison with quark jets at the LHC).
   b) Compare it with the number of gluons emitted directly from the q or $\overline{\rm q}$.
   Hint: these need not be final, only have a mother parton that is a q or $\overline{\rm q}$.