Particle Physics Phenomenology
7. Hadronization

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Introduction

Perturbative → nonperturbative ⟹ not calculable from first principles!

Model building = ideology + "cookbook"

Common approaches:

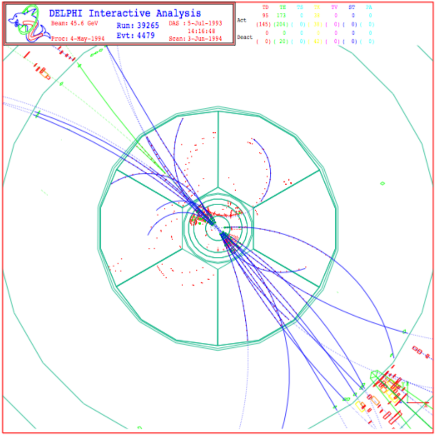
1) **String** Fragmentation
(most ideological)

2) **Cluster** Fragmentation
(simplest?)

3) **Independent** Fragmentation
(most cookbook)

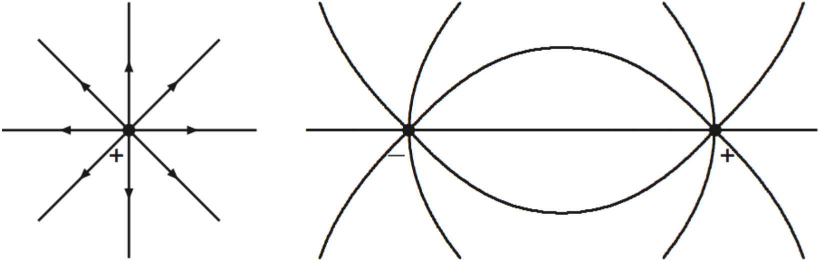
4) Local Parton–Hadron Duality
(limited applicability)

Best studied in
$e^+e^- \to \gamma^*/Z^0 \to q\overline{q}$



The QED potential

In QED, field lines go all the way to infinity



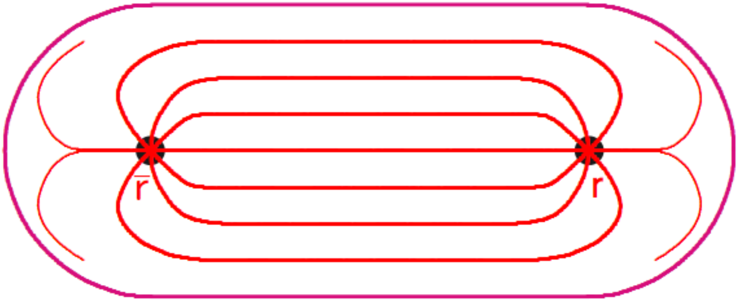
since photons cannot interact with each other.

Potential is simply additive:

$$V(\mathbf{x}) \propto \sum_i \frac{1}{|\mathbf{x} - \mathbf{x}_i|}$$

The QCD potential – 1

In QCD, for large charge separation, field lines seem to be compressed to tubelike region(s) ⇒ **string(s)**



by self-interactions among soft gluons in the "vacuum".

(Non-trivial ground state with quark and gluon "condensates". Analogy: vortex lines in type II superconductor)

The QCD potential – 2

Gives linear confinement with string tension:
$F(r) \approx \mathrm{const} = \kappa \approx 1\ \mathrm{GeV/fm} \quad \Longleftrightarrow \quad V(r) \approx \kappa r$

Separation of transverse and longitudinal degrees of freedom
$\Rightarrow$ simple description as 1+1-dimensional object – string –
with Lorentz invariant formalism

At short distances perturbation theory should be valid
$\Rightarrow$ Coulomb potential:

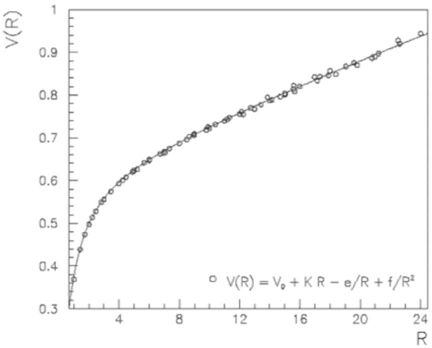
$$V(r) \approx -\frac{4}{3}\frac{\alpha_s}{r} + \kappa r \approx -\frac{0.13}{r} + r$$

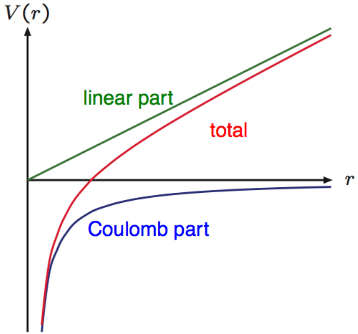
(for $\alpha_s \approx 0.5$, r in fm and V in GeV)

$V(0.4\,\mathrm{fm}) \approx 0$: Coulomb important for internal structure of hadrons, but not for particle production (?)

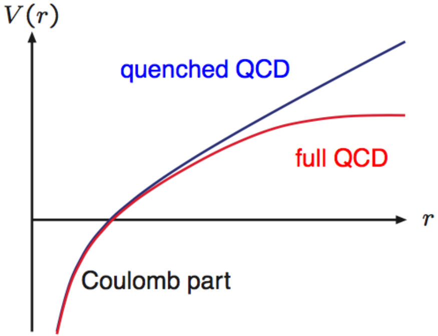
The QCD potential – 3

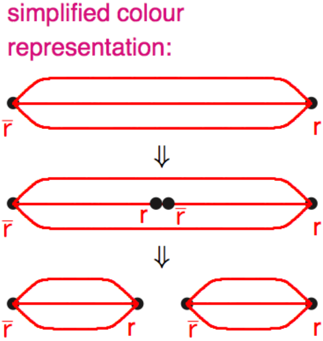
Linear confinement confirmed e.g. by quenched lattice QCD





Real world (??, or at least unquenched lattice QCD)
$\Longrightarrow$ nonperturbative string breakings $\mathrm{gg}\ldots \to \mathrm{q\overline{q}}$





String motion – 1

The Lund Model: core idea

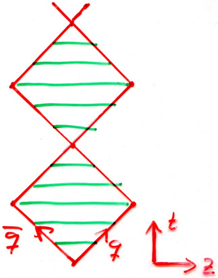
Use only linear potential $V(r) \approx \kappa r$ to trace string motion and let string fragment by repeated $\mathrm{q}\overline{\mathrm{q}}$ breaks.

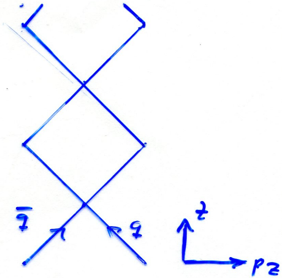
Linearity between space–time and energy–momentum gives

$$\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}E}{\mathrm{d}t}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}t}\right| = \kappa$$

$(c = 1)$ for a $\mathrm{q}\overline{\mathrm{q}}$ pair flying apart along the $\pm z$ axis.
But signs relevant: the q moving in the $+z$ direction has $\mathrm{d}z/\mathrm{d}t = +1$ but $\mathrm{d}p_z/\mathrm{d}t = -\kappa$.

The simple yo-yo:





t	$(E;p_z)_{\mathrm{q}}$	$(E;p_z)_{\overline{\mathrm{q}}}$	$(E;p_z)_{\mathrm{string}}$
$t=0$	$\frac{E_{\mathrm{cm}}}{2}(1;1)$	$\frac{E_{\mathrm{cm}}}{2}(1;-1)$	$(0;0)$
$0<t<\frac{E_{\mathrm{cm}}}{2\kappa}$	$\left(\frac{E_{\mathrm{cm}}}{2}-\kappa t\right)(1;1)$	$\left(\frac{E_{\mathrm{cm}}}{2}-\kappa t\right)(1;-1)$	$2\kappa t(1;0)$
$t=\frac{E_{\mathrm{cm}}}{2\kappa}$	$(0;0)$	$(0;0)$	$E_{\mathrm{cm}}(1;0)$
$\frac{E_{\mathrm{cm}}}{2\kappa}<t<\frac{E_{\mathrm{cm}}}{\kappa}$	$\left(\kappa t-\frac{E_{\mathrm{cm}}}{2}\right)(1;-1)$	$\left(\kappa t-\frac{E_{\mathrm{cm}}}{2}\right)(1;1)$	$2(E_{\mathrm{cm}}-\kappa t)(1;0)$
$t=\frac{E_{\mathrm{cm}}}{\kappa}$	$\frac{E_{\mathrm{cm}}}{2}(1;-1)$	$\frac{E_{\mathrm{cm}}}{2}(1;1)$	$(0;0)$

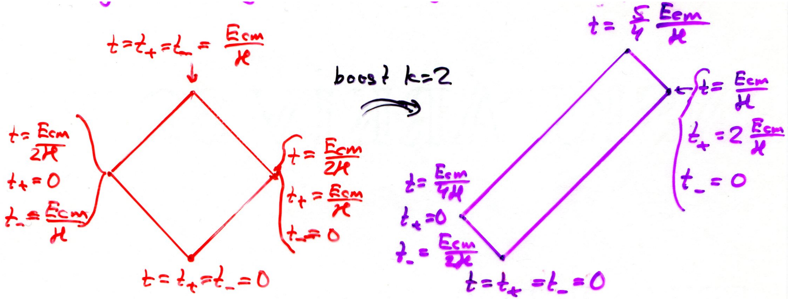
i.e. $\Delta t = E_{\mathrm{cm}}/\kappa$ bring partons back to original positions,
but $p_{z,\mathrm{q}} \leftrightarrow p_{z,\overline{\mathrm{q}}}$ so full period is $\Delta t = 2E_{\mathrm{cm}}/\kappa$.

String motion – 3

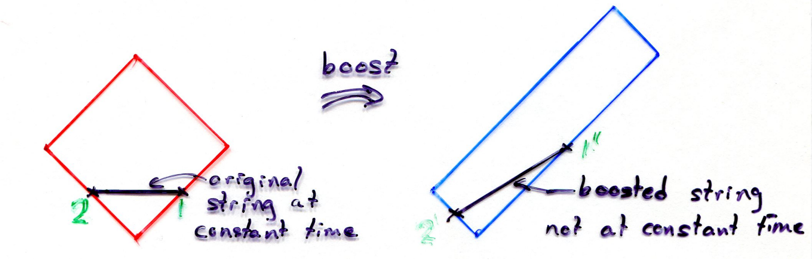
Boost $+\beta$ along z axis best expressed by lightcone variables

$$t_\pm = t \pm z \quad \Rightarrow \quad t'_\pm = k^{\pm 1}\, t_\pm$$
$$p_\pm = E \pm p_z \quad \Rightarrow \quad p'_\pm = k^{\pm 1}\, p_\pm$$

where $k = \sqrt{(1+\beta)/(1-\beta)}$.
Note that $p'_+ p'_- = p_+ p_- = E^2 - p_z^2 = m_\perp^2$ ($= m^2$ for $p_\perp = 0$).
Hence easy to obtain string motion in longitudinally boosted frame:



Paradox? If the momentum shift is traced in time, the string only carrries energy E, i.e. $p_z = 0$, independently of frame, but if a string piece is boosted it ought to acquire $p'_z = \gamma\beta E$.
Answer: string is extended object, watch out about simultaneity

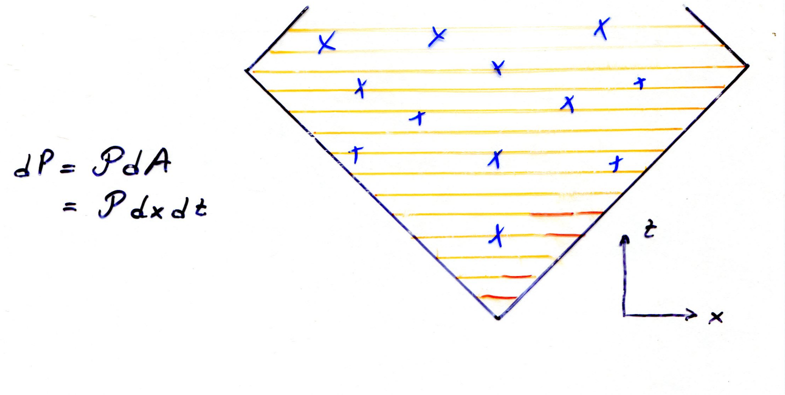


$$
\begin{aligned}
p_{+,\mathrm{string}} &= \kappa\,\Delta t_+ = \kappa(t_{+,1} - t_{+,2}) \\
p_{-,\mathrm{string}} &= \kappa\,\Delta t_- = \kappa(t_{-,2} - t_{-,1}) = -\kappa(t_{-,1} - t_{-,2}) \\
E_{\mathrm{string}} &= (p_+ + p_-)_{\mathrm{string}}/2 = \kappa(z_1 - z_2) \\
p_{z,\mathrm{string}} &= (p_+ - p_-)_{\mathrm{string}}/2 = \kappa(t_1 - t_2)
\end{aligned}
$$

1974: the first (semi-)realistic hadronization model

Assume fragmentation local, and string homogeneous.

Thus constant probability per unit string area of breaking.



The Artru-Mennessier model – 1

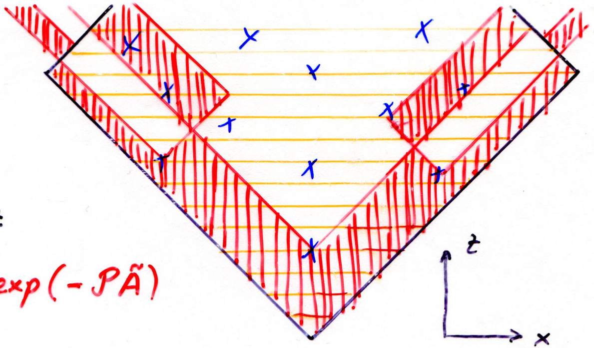
1974: the first (semi-)realistic hadronization model

Assume fragmentation local, and string homogeneous.

Thus constant probability per unit string area of breaking.

$$dP = \mathcal{P}\,dA = \mathcal{P}\,dx\,dt$$

$$dP = \mathcal{P}\,dA\,\exp(-\mathcal{P}\tilde{A})$$



But string cannot break where it has already broken

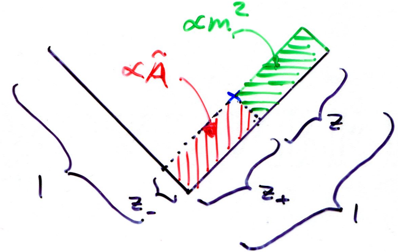
$\Rightarrow$ remove vertices in forward lightcone of another

$\Rightarrow$ dampening factor $\exp(-\mathcal{P}\tilde{A})$,

where $\tilde{A}$ is string area in the backwards lightcone

The Artru-Mennessier model – 2

Consider break that produces "rightmost" hadron, i.e. smallest z_-



$$
\begin{aligned}
z_\pm &= \frac{p_\pm}{E_{\mathrm{cm}}} = \frac{\kappa t_\pm}{E_{\mathrm{cm}}} \\
z &= 1 - z_+ \\
m^2 &= z\, z_-\, E_{\mathrm{cm}}^2 \\
\Rightarrow z_- &= \frac{m^2}{z E_{\mathrm{cm}}^2}
\end{aligned}
$$

Can rewrite breakup probability, introducing $b = \mathcal{P}/2\kappa^2$

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}z_+\,\mathrm{d}z_-} &= \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}z\,\mathrm{d}t} \left| \frac{\mathrm{d}(z,t)}{\mathrm{d}(z_+,z_-)} \right| = b\, E_{\mathrm{cm}}^2 \exp(-b\, E_{\mathrm{cm}}^2\, z_+\, z_-) \\
\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}z\,\mathrm{d}m^2} &= \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}z_+\,\mathrm{d}z_-} \left| \frac{\mathrm{d}(z_+,z_-)}{\mathrm{d}(z,m^2)} \right| = \frac{b}{z} \exp\left(-b\, m^2\, \frac{1-z}{z} \right)
\end{aligned}
$$

Can be repeated for not-yet-considered region to the right
$\Rightarrow$ iterative structure, with $z = p_{+,\mathrm{hadron}}/p_{+,\mathrm{remaining}}$

The Artru-Mennessier model – 3

Mass spectrum

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}m^2} = \int_0^1 \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}z\,\mathrm{d}m^2}\,\mathrm{d}z \propto \int_0^1 \frac{\mathrm{d}z}{z}\,\exp\left(-b\,m^2\,\frac{1}{z}\right) = E_1(b\,m^2)$$

which is logarithmically divergent for $m^2 \to 0$.
Worse: nothing like a discrete hadronic mass spactrum!
And no "nice" way to introduce it!
Could be useful for cluster models (but is not currently used there).

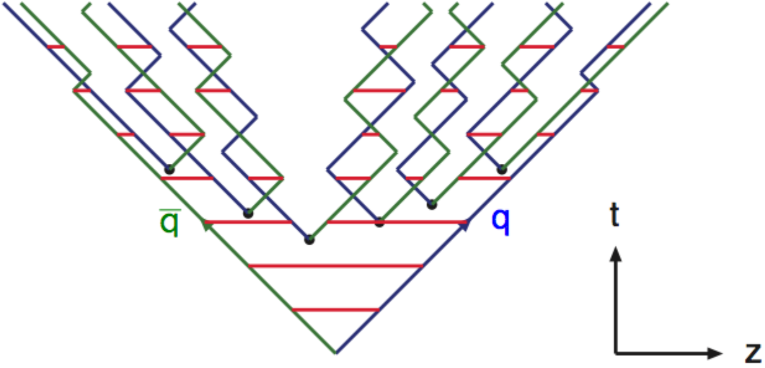
Fragmentation z spectrum for given mass:

$$\left.\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}z}\right|_{m\ \mathrm{fix}} \propto \left.\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}z\,\mathrm{d}m^2}\right|_{m^2\ \mathrm{fix}} \propto \frac{1}{z}\exp\left(-\frac{b\,m^2}{z}\right)$$

Note: larger $m^2 \Rightarrow$ stronger suppression of small $z \Rightarrow$ larger $\langle z \rangle$
$\Rightarrow$ harder energy spectrum.

The Lund Model

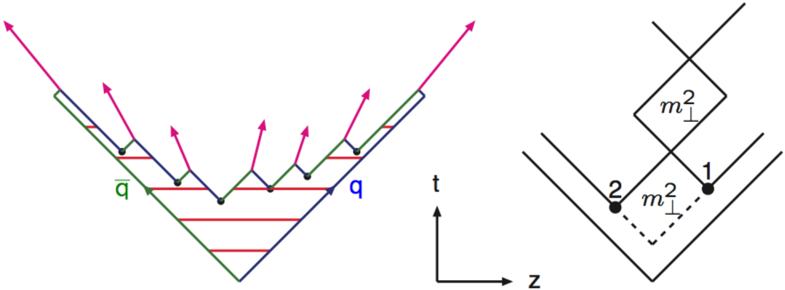
Motion of quarks and antiquarks in a $q\overline{q}$ system:



gives simple but powerful picture of hadron production
(with extensions to massive quarks, baryons, ...)
Seemingly same as Artru-Mennessier, but impose hadrons on mass shell, so not uncorrelated string breaks, *and yet similar end result*.

Where does the string break? – 1

Fragmentation starts in the middle and spreads outwards:



- Here $m_\perp^2$ fixed from hadron and $p_\perp$ selection (unlike AM).
- Lorentz covariant inside–out cascade.
- Breakup vertices causally disconnected
  $\Rightarrow$ iteration from ends inwards allowed!

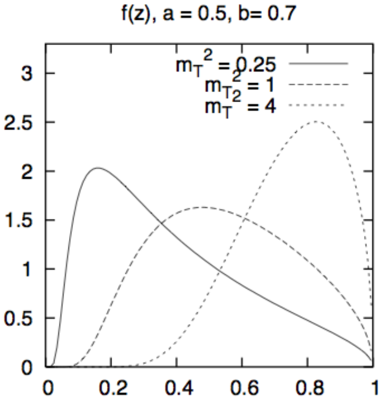
Where does the string break? – 2

Breakup vertices causally disconnected
⇒ can proceed in arbitrary order
⇒ *left–right symmetry*

$$\begin{aligned}\mathcal{P}(1,2) &= \mathcal{P}(1) \times \mathcal{P}(1 \to 2) \\ &= \mathcal{P}(2) \times \mathcal{P}(2 \to 1)\end{aligned}$$

⇒ Lund symmetric fragmentation function:

$f(z) \propto (1-z)^a \exp(-bm_\perp^2/z)/z$

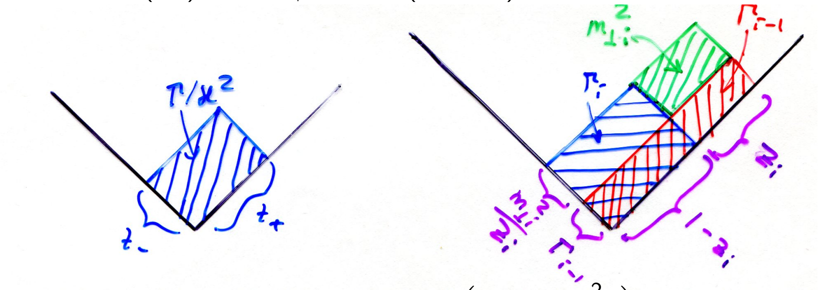


Lund–Bowler modified shape for heavy quarks:

$$f(x) \propto \frac{1}{z^{1+bm_q^2}} \exp\left(-\frac{bm_\perp^2}{z}\right) .$$

Where does the string break? – 3

Define $\Gamma = (\kappa\tau)^2 = \kappa^2 t_+ t_- = \kappa^2(t^2 - z^2)$



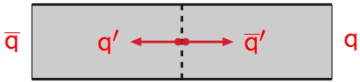
$$
\begin{aligned}
\text{Then} \qquad & \Gamma_i = (1 - z_i)\left(\Gamma_{i-1} + \frac{m_{\perp i}^2}{z_i}\right) \\
\text{with} \qquad & \Gamma_0 = 0
\end{aligned}
$$

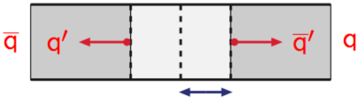
Equilibrium for $i \to \infty$ ($\sum m_{\perp i}^2$ large):

$$P(\Gamma) \propto \Gamma^a \exp(-b\Gamma)\,\mathrm{d}\Gamma$$

with a and b as for $f(z)$, independently of $m_{\perp i}^2$,
but fluctuations up and down from equilibrium value.

How does the string break?





String breaking modelled by tunneling:

$$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right) = \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_{q}^2}{\kappa}\right)$$

1. common Gaussian $p_\perp$ spectrum
2. suppression of heavy quarks
   $\mathrm{u\overline{u}} : \mathrm{d\overline{d}} : \mathrm{s\overline{s}} : \mathrm{c\overline{c}} \approx 1 : 1 : 0.3 : 10^{-11}$
3. diquark $\sim$ antiquark $\Rightarrow$ simple model for baryon production

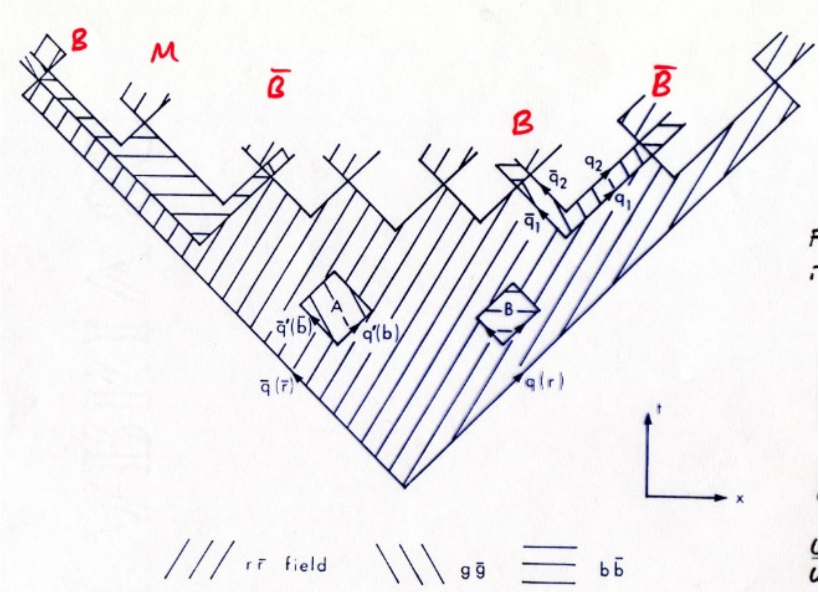
Flavour composition

Combination of q from one break and $\overline{\mathrm{q}}$ (qq) gives meson (baryon).
Many uncertainties in selection of hadron species, e.g.:

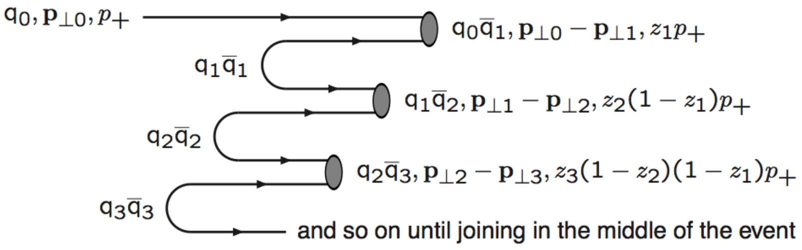
- Spin counting suggests vector:pseudoscalar $= 3{:}1$, but $m_\rho \gg m_\pi$, so empirically $\sim$1:1.
- Also for same spin $m_{\eta'} \gg m_\eta \gg m_{\pi^0}$ gives mass suppression. String model unpredictive in understanding of hadron mass effects $\Rightarrow$ many "materials constants".
- There is one V and one PS for each $\mathrm{q}\overline{\mathrm{q}}$ flavour set, but baryons are more complicated, e.g. $\mathrm{uuu} \Rightarrow \Delta^{++}$ whereas $\mathrm{uds} \Rightarrow \Lambda^0$, Σ^0 or Σ^{*0}. SU(6) (flavour$\times$spin) Clebsch-Gordans needed; affects surrounding flavours.
- Simple diquark model too simpleminded; produces baryon–antibaryon pairs nearby in momentum space.

Many parameters, 10–20 depending on how you count.

The popcorn model for baryon production



The iterative ansatz

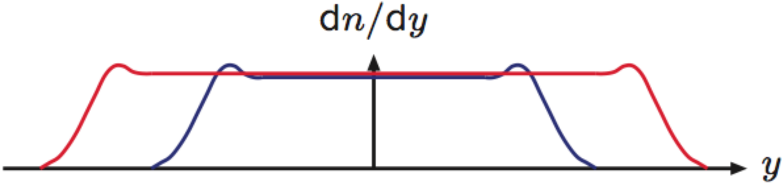


In each step:

1. pick $\mathrm{q}\overline{\mathrm{q}}$ flavour and combine to hadron flavour
2. pick spin etc. to uniquely specify hadron (ρ/π, e.g.)
3. pick hadron mass, possibly according to Breit-Wigner
4. pick $\mathrm{q}\overline{\mathrm{q}}$ $p_\perp$ and combine to hadron $p_\perp$
5. pick z from $f(z)$ for given $m_\perp^2$
6. construct full kinematics of hadron

The rapidity plateau

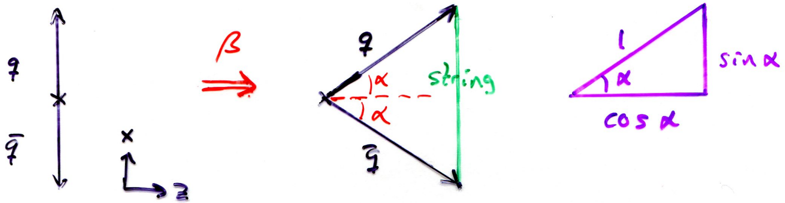
Scaling in lightcone $p_{\pm} = E \pm p_z$ (for $\mathrm{q\overline{q}}$ system along z axis) implies flat central rapidity plateau + some endpoint effects:



$\langle n_{\mathrm{ch}} \rangle \approx c_0 + c_1 \ln E_{\mathrm{cm}}$

$\sim$ Poissonian multiplicity distribution (but depends on a and b)

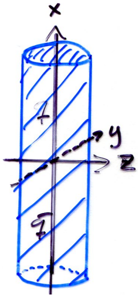
Transverse string boosts – 1



with $\cos\alpha = \beta$, $\sin\alpha = 1/\gamma$.
In boosted frame

$$\begin{aligned}
\frac{\Delta\ell_{\mathrm{string}}}{\Delta t} &= 2\,\sin\alpha \\
\frac{\Delta E_{\mathrm{string}}}{\Delta\ell_{\mathrm{string}}} &= \gamma\,\frac{(\Delta E_{\mathrm{string}})_{\mathrm{rest}}}{\Delta\ell_{\mathrm{string}}} = \gamma\,\kappa = \frac{\kappa}{\sin\alpha} \\
\frac{\Delta E_{\mathrm{string}}}{\Delta t} &= \frac{\Delta E_{\mathrm{string}}}{\Delta\ell_{\mathrm{string}}}\,\frac{\Delta\ell_{\mathrm{string}}}{\Delta t} = \frac{\kappa}{\sin\alpha}\,2\,\sin\alpha = 2\,\kappa
\end{aligned}$$

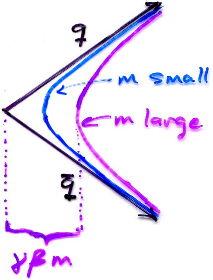
i.e. energy change per time independent of boost (α, β)



Hadron production occurs in roughly cylindrical volume surrounding string axis.
Much of production at moderate $p_{\mathrm{longitudinal}} = p_x$, thus m and $p_{\mathrm{transverse}} = \sqrt{\mathrm{p}_y^2 + p_z^2}$ relevent.

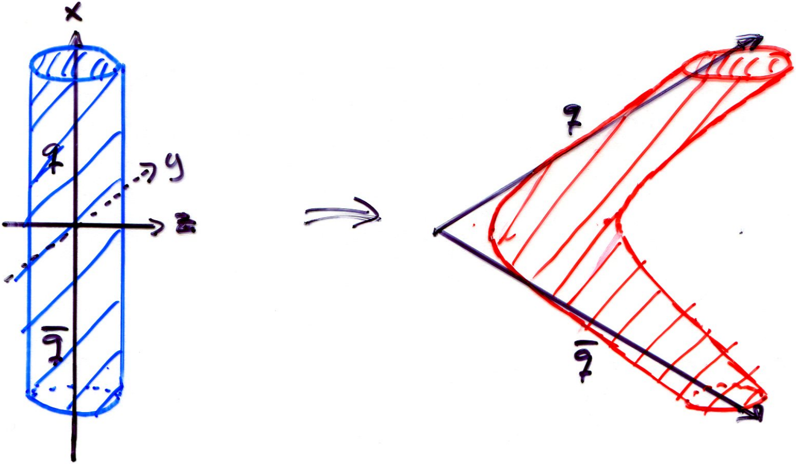
First case: $m \neq 0$, $p_{\mathrm{transverse}} = 0$

$$\begin{aligned} E &= \sqrt{m^2 + p_x^2} \\ E' &= \gamma \sqrt{m^2 + p_x^2} \\ p'_x &= p_x \\ p'_z &= \gamma\beta \sqrt{m^2 + p_x^2} \end{aligned}$$



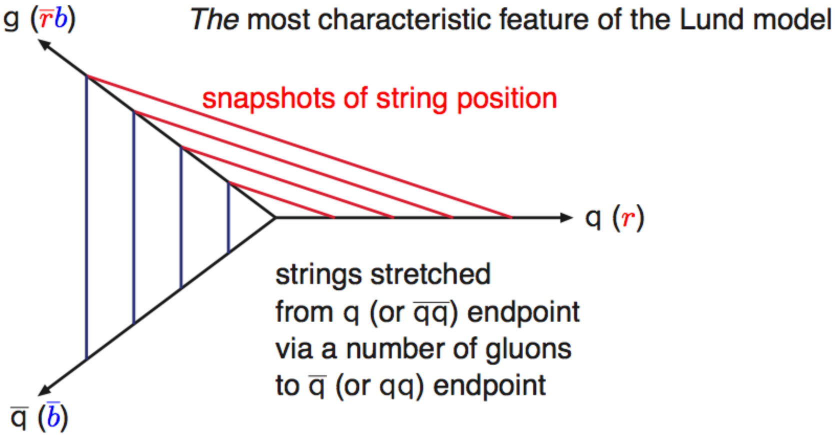
with ${p'}_z^2 - \gamma^2\beta^2 {p'}_x^2 = \gamma^2\beta^2 m^2$,
so production along hyperbola,
with asymptotes given by massless case

Transverse string boosts – 3



Particle production boosted into region in between the two colour-connected q and $\overline{\mathrm{q}}$!

The Lund gluon picture – 1



Gluon = kink on string, carrying energy and momentum

The Lund gluon picture – 2

Gluon = kink on string

Force ratio gluon/ quark = 2,
cf. QCD $N_C/C_F = 9/4$, $\rightarrow 2$ for $N_C \rightarrow \infty$
No new parameters introduced for gluon jets!

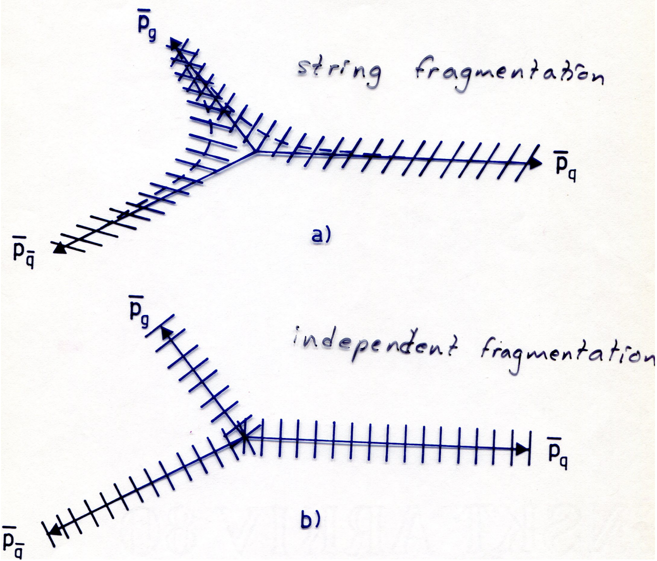
so

- Few parameters to describe energy-momentum structure!
- Many parameters to describe flavour composition!

String piece ≈ dipole

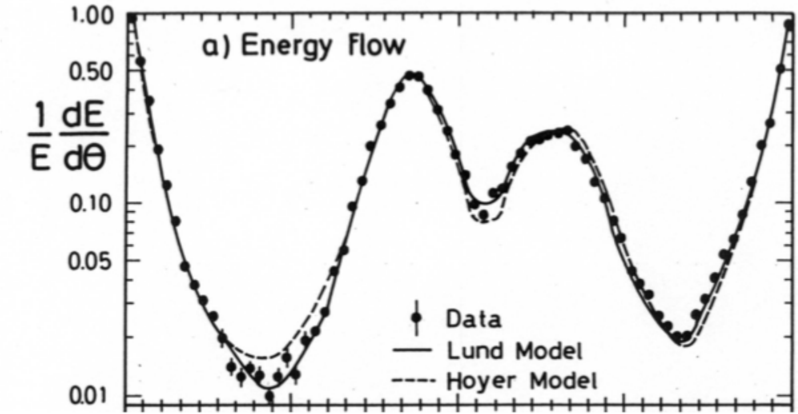
One-to-one correspondence between how strings and how colour dipoles are stretched between colour charges in $N_C \rightarrow \infty$ limit.
Dipole: emission in perturbative regime.
String: "emission" in nonperturbative regime.
String picture 5 years ahead...

The string challenged Independent Fragmentation



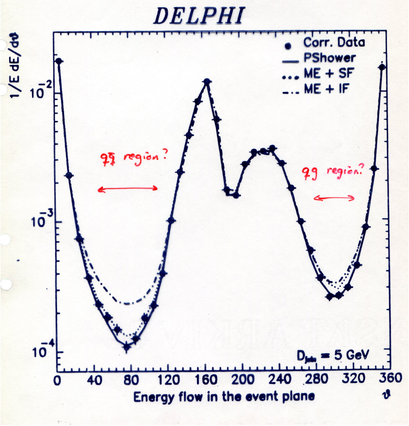
The string effect – 2

String effect (JADE, 1980) $\approx$ coherence in nonperturbative context



Further numerous and detailed tests at LEP disfavour independent fragmentation, so nowadays of historical interest only.

The string effect – 3



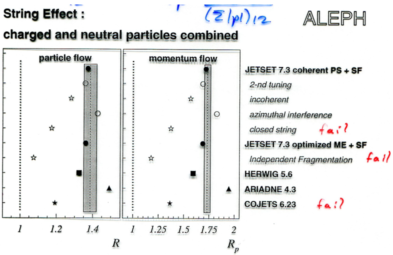
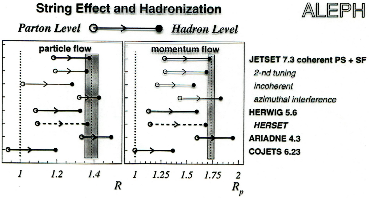
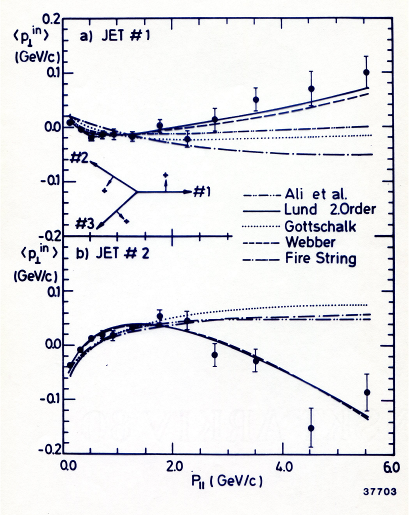
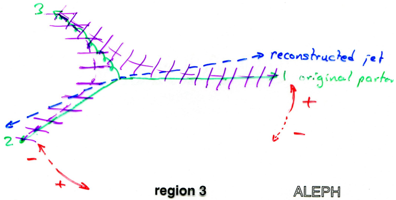


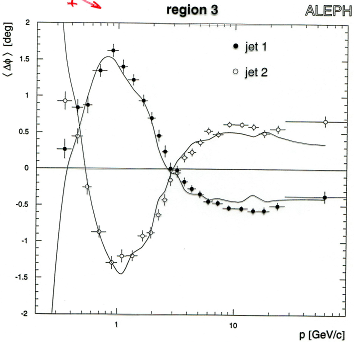
Figure 5: Values of inter-jet particle flow and momentum flow ratios of corrected data (the shaded bands indicate the total error) and various QCD models.



The string effect – 4

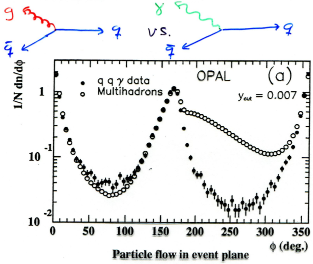




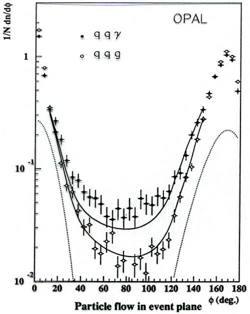


"Jets are crooked"

Strings vs. perturbative dipoles – 1

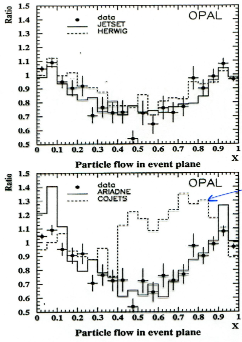


OPAL, CERN-PPE/95-83 EPS 332



coherence + LPHD:
Azimov
Dokshitzer
Khoze
Troyan

$\frac{n(q\bar{q}g)}{n(q\bar{q}\gamma)}$



breakdown of model

x = scaled angle between jet 1 and 2

Figure 6: Ratio of charged particle flows in three-jet and two-jet radiative events with respect to the reduced angle X for various Monte Carlo models: JETSET coherent parton shower with string fragmentation, HERWIG, ARIADNE and COJETS.

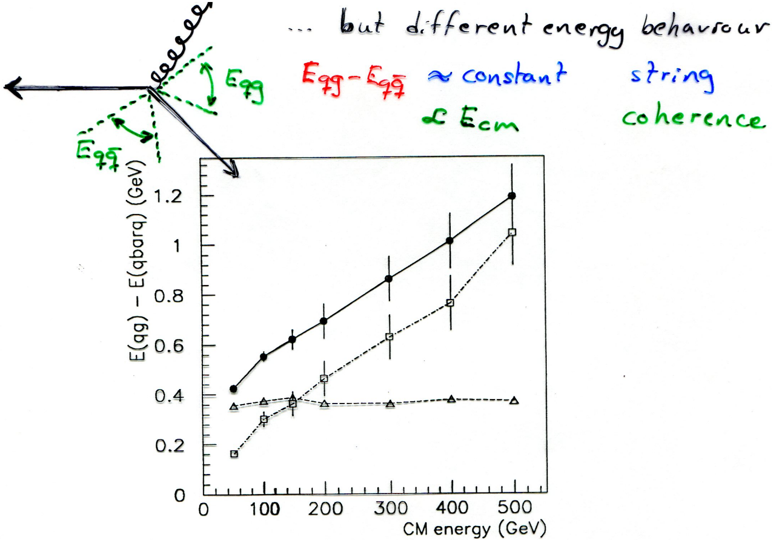
OPAL, CERN-PPE/95-83 EPS 332

Reference value	
Data	0.71±0.03
Monte Carlo, with detector simulation	
JETSET	0.76±0.03
ERT (Data $q\bar{q}\gamma$)	0.71±0.03
HERWIG	0.82±0.02
ARIADNE	0.70±0.03
COJETS	1.08±0.03
JETSET, without detector simulation	
coherent, SF	0.73±0.03
incoherent, SF	0.91±0.03
coherent, IF	1.01±0.03
incoherent, IF	1.11±0.03

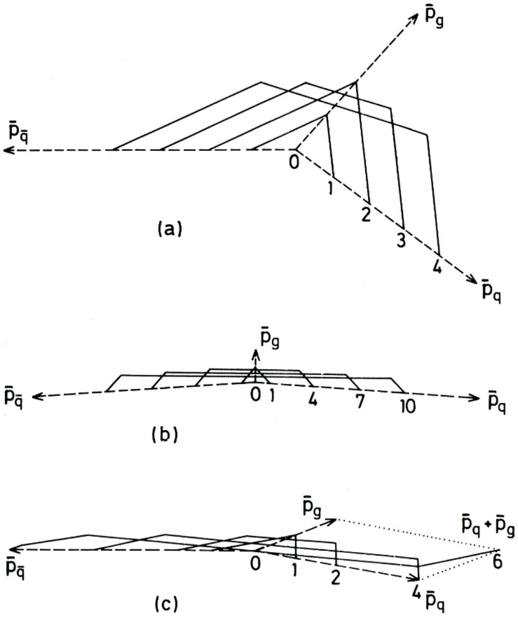
Table 2: Ratio R of particle flows, compared to models

both pert & nonpert effects enter

Strings vs. perturbative dipoles– 2



Infrared and collinear safety of string fragmentation



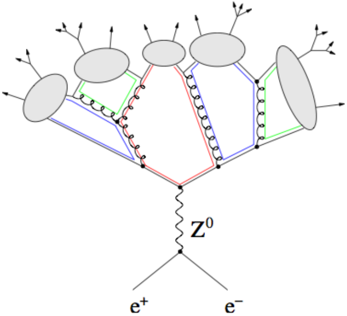
Emission of a soft or collinear gluon only negligibly perturbs string motion/evolution.

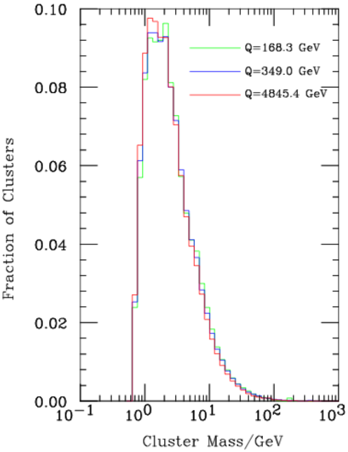
Therefore string fragmentation is soft and collinear safe.

Technically, tracing the string motion for many nearby gluons can become messy, prompting simplifications.

The HERWIG Cluster Model

"Preconfinement":
colour flow is local
in coherent shower evolution





1. Introduce forced $\mathrm{g} \to \mathrm{q}\overline{\mathrm{q}}$ branchings
2. Form colour singlet clusters
3. Clusters decay isotropically to 2 hadrons according to phase space weight $\sim (2s_1 + 1)(2s_2 + 1)(2p^*/m)$

Cluster Model problems

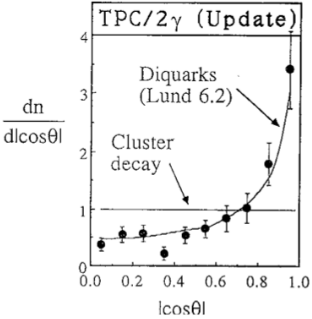
1 Tail to very large-mass clusters (e.g. if no emission in shower); if large-mass cluster $\rightarrow$ 2 hadrons then incorrect hadron momentum spectrum, crazy four-jet events
$\Longrightarrow$ split big cluster into 2 smaller along "string" direction; daughter-mass spectrum $\Rightarrow$ iterate if required;
$\sim$ 15% of primary clusters are split,
but give $\sim$ 50% of final hadrons

2 Isotropic baryon decay inside cluster
$\Longrightarrow$ splittings $\mathrm{g} \rightarrow \mathrm{qq} + \overline{\mathrm{qq}}$

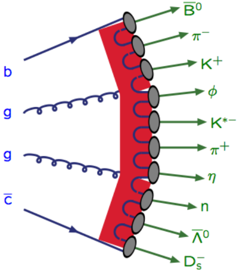
3 Too soft charm/bottom spectra
$\Longrightarrow$ anisotropic leading-cluster decay

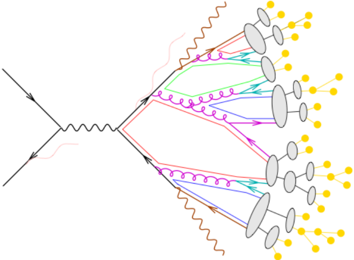
4 Charge correlations still problematic
$\Longrightarrow$ all clusters anisotropic (?)

5 Sensitivity to particle content
$\Longrightarrow$ only include complete multiplets



String vs. Cluster





program	PYTHIA	HERWIG
model	string	cluster
energy–momentum picture	powerful	simple
	predictive	unpredictive
parameters	few	many
flavour composition	messy	simple
	unpredictive	in-between
parameters	many	few

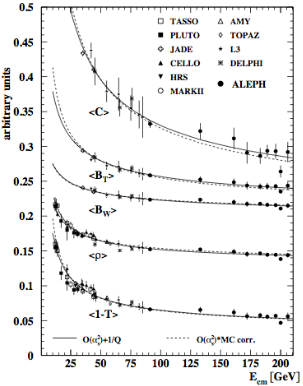
Local Parton–Hadron Duality (LPHD)

Analytic approach:
Run shower down to to $Q \approx \Lambda_{\mathrm{QCD}}$
(or m_{hadron}, if larger)

"Hard Line": each parton $\equiv$ one hadron

"Soft Line": local hadron density
$\propto$ parton density

describes momentum spectra $\mathrm{d}n/\mathrm{d}x_p$
and semi-inclusive particle flow,
but fails for identified particles



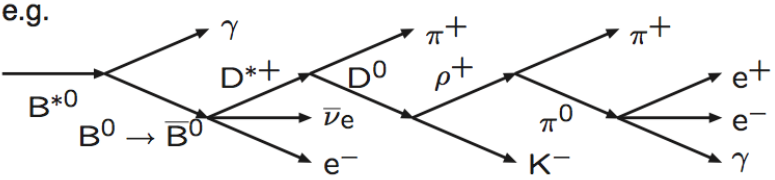
+ "renormalons" (power corrections)
$\langle 1 - T \rangle = a\,\alpha_{\mathrm{s}}(E_{\mathrm{cm}}) + b\,\alpha_{\mathrm{s}}^2(E_{\mathrm{cm}}) + c/E_{\mathrm{cm}}$

Not Monte Carlo, **not** for arbitrary quantities

at LHC only relevant for high-$p_{\perp}$ jets, not for MB/UE

Decays

Unspectacular/ungrateful but necessary:
this is where most of the final-state particles are produced!
Involves hundreds of particle kinds and thousands of decay modes.



- $\mathrm{B}^{*0} \to \mathrm{B}^0\gamma$: electromagnetic decay
- $\mathrm{B}^0 \to \overline{\mathrm{B}}^0$ mixing (weak)
- $\overline{\mathrm{B}}^0 \to \mathrm{D}^{*+}\overline{\nu}_{\mathrm{e}}\mathrm{e}^-$: weak decay, displaced vertex, $|\mathcal{M}|^2 \propto (p_{\overline{\mathrm{B}}}p_{\overline{\nu}})(p_{\mathrm{e}}p_{\mathrm{D}^*})$
- $\mathrm{D}^{*+} \to \mathrm{D}^0\pi^+$: strong decay
- $\mathrm{D}^0 \to \rho^+\mathrm{K}^-$: weak decay, displaced vertex, ρ mass smeared
- $\rho^+ \to \pi^+\pi^0$: ρ polarized, $|\mathcal{M}|^2 \propto \cos^2\theta$ in ρ rest frame
- $\pi^0 \to \mathrm{e}^+\mathrm{e}^-\gamma$: Dalitz decay, $m(\mathrm{e}^+\mathrm{e}^-)$ peaked

Gluon vs. quark jets – 1

Perturbatively

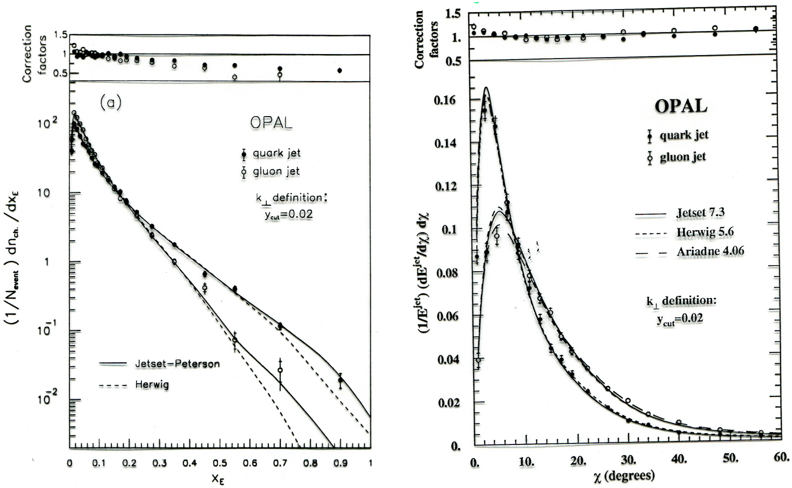
$$
\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}} &\propto C_F \frac{1+z^2}{1-z} \approx C_F \frac{2}{1-z} \\
P_{\mathrm{g}\to\mathrm{gg}} &\propto N_C \frac{(1-z(1-z))^2}{z(1-z)} \approx N_C \frac{2}{1-z} \\
\frac{P_{\mathrm{g}\to\mathrm{gg}}}{P_{\mathrm{q}\to\mathrm{qg}}} &\approx \frac{N_c}{C_F} = \frac{9}{4}
\end{aligned}
$$

Nonperturbatively a gluon is shared between two string pieces.

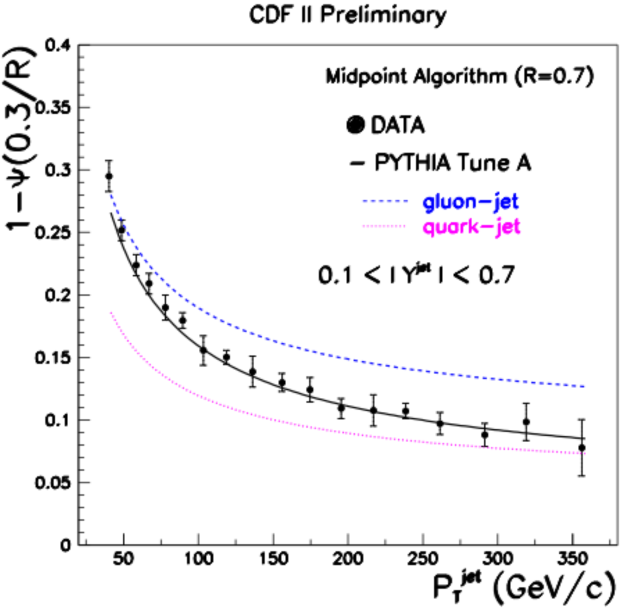
Consequences:

- $\langle n_{\mathrm{g\ jet}} \rangle / \langle n_{\mathrm{q\ jet}} \rangle \to 2$ when energy large.
  But asymptotia far away; expect ~ 1.5 at typical energies.
- Fragmentation function softer.
- Angular spread larger.
- $p_\perp$ distribution about the same.

Gluon vs. quark jets – 2

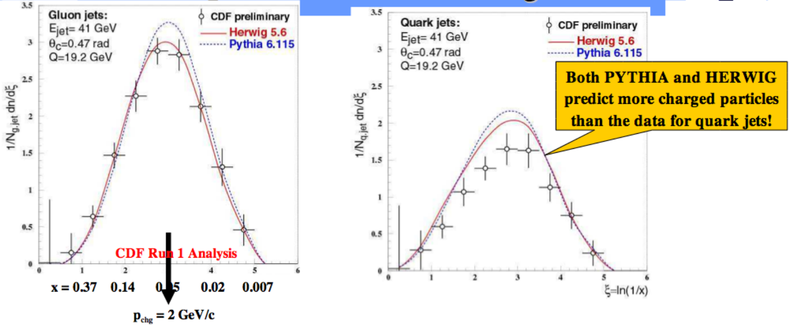


Gluon vs. quark jets – 3



Gluon vs. quark jets – 4 (Rick Field)

Distribution of Particles in Quark and Gluon Jets



Momentum distribution of charged particles in gluon jets. HERWIG 5.6 predictions are in a good agreement with CDF data. PYTHIA 6.115 produces slightly more particles in the region around the peak of distribution.

Momentum distribution of charged particles in quark jets. Both HERWIG and PYTHIA produce more particles in the central region of distribution.

Heavy flavours: the dead cone

Recall eikonal expression for soft-gluon radiation

$$\frac{\mathrm{d}\sigma_{\mathrm{q\bar{q}g}}}{\sigma_{\mathrm{q\bar{q}}}} \propto (-1)\left(\frac{p_1}{p_1p_3}-\frac{p_2}{p_2p_3}\right)^2\frac{\mathrm{d}^3p_3}{E_3}$$

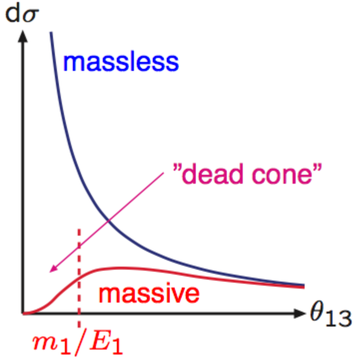
$$\propto \left(\frac{2p_1p_2}{(p_1p_3)(p_2p_3)}-\frac{m_1^2}{(p_1p_3)^2}-\frac{m_2^2}{(p_2p_3)^2}\right)E_3\,\mathrm{d}E_3\,d\cos\theta_{13}$$

For θ_{13} small

$$\frac{\mathrm{d}\sigma_{\mathrm{q\bar{q}g}}}{\sigma_{\mathrm{q\bar{q}}}} \propto \frac{\mathrm{d}\omega}{\omega}\frac{\mathrm{d}\theta_{13}^2}{\theta_{13}^2}\left(\frac{\theta_{13}^2}{\theta_{13}^2+m_1^2/E_1^2}\right)^2$$

$$= \frac{\mathrm{d}\omega}{\omega}\frac{\theta_{13}^2\,\mathrm{d}\theta_{13}^2}{(\theta_{13}^2+m_1^2/E_1^2)^2}$$

so "dead cone" for $\theta_{13} < m_1/E_1$



Heavy flavours: fragmentation ansatz

Less radiation matched by less energy loss in hadronization.

- Lund (generic for all hadrons; universal a and b)

$$f(z) \propto \frac{1}{z} (1-z)^a \exp\left(-\frac{b m_\perp^2}{z}\right)$$

- Bowler (based on Artru-Mennessier)

$$f(z) \propto \frac{1}{z^{1+b m_{\mathrm{Q}^2}}} \exp\left(-\frac{b m_\perp^2}{z}\right)$$

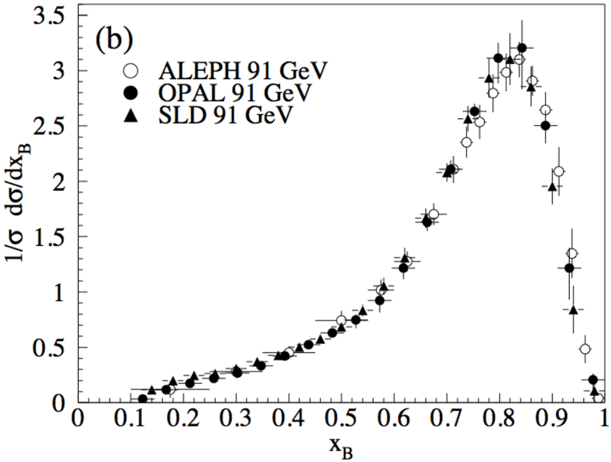
- SLAC (Peterson et al.)

$$f(z) \propto \frac{1}{z\left(1-\frac{1}{z}-\frac{\epsilon_{\mathrm{Q}}}{1-z}\right)^2} \quad \text{with } \epsilon_{\mathrm{Q}} \approx \frac{m_{\mathrm{q}}^2}{m_{\mathrm{Q}}^2}$$

Rule of thumb, to be used with caution:

$$\langle x_E \rangle = \left\langle \frac{E_{\text{heavy hadron}}}{E_{\text{jet}}} \right\rangle \approx 1 - \frac{1\ \mathrm{GeV}}{m_{\mathrm{Q}}}$$

Heavy flavours: fragmentation data



But note that a heavy hadron decays to many secondaries,
filling up "dead cone" and
giving "normally-soft" light-hadron spectra.