Selective topics in BSM Physics: a theoretical perspective

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Particle Physics Phenomenology

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Outline

SM: a quick overview

Building blocks

(1) Define the gauge symmetry: $\mathcal{G}_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$

(2) Choose the representations of the matter content under the symmetry:

Matter	**Flavour**	$\mathcal{G}_{\text{SM}}$
$q_{L\alpha} \equiv \begin{pmatrix} u_{L\alpha} \\ d_{L\alpha} \end{pmatrix}$	$\begin{pmatrix} \mathsf{u}_L \\ \mathsf{d}_L \end{pmatrix}, \begin{pmatrix} \mathsf{c}_L \\ \mathsf{s}_L \end{pmatrix}, \begin{pmatrix} \mathsf{t}_L \\ \mathsf{b}_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, 1/6)$
$u_{R\alpha}$	$\mathsf{u}_R\,, \mathsf{c}_R\,, \mathsf{t}_R$	$(\mathbf{3}, \mathbf{1}, 2/3)$
$d_{R\alpha}$	$\mathsf{d}_R\,, \mathsf{s}_R\,, \mathsf{b}_R$	$(\mathbf{3}, \mathbf{1}, -1/3)$
$\ell_{L\alpha} \equiv \begin{pmatrix} \nu_{L\alpha} \\ e_{L\alpha} \end{pmatrix}$	$\begin{pmatrix} \nu_{L\mathsf{e}} \\ \mathsf{e}_L \end{pmatrix}, \begin{pmatrix} \nu_{L\mu} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{L\tau} \\ \tau_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1/2)$
$e_{R\alpha}$	$\mathsf{e}_R\,, \mu_R\,, \tau_R$	$(\mathbf{1}, \mathbf{1}, -1)$

(3) Choose the way symmetry is broken: $\mathcal{G}_{\text{SM}} \to SU(3)_c \times U(1)_Q$

Bosons	**Force**
G^a_μ	Strong
$W^\pm_\mu\,, Z^0_\mu$	Weak
A_μ	EM
$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	Yukawa-type $(\mathbf{1}, \mathbf{2}, 1/2)$

$Q = T_3 + Y$

The Lagrangian

The full Lagrangian: $\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Kin}}^{\text{gauge}} + \mathcal{L}_{\text{Kin}}^{\text{fermion}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{FP}}$

Sector	**Lagrangian**
$\mathcal{L}_{\text{kin}}^{\text{gauge}}$	$-\frac{1}{4}G^{a\mu\nu}G^{a}_{\mu\nu} - \frac{1}{4}W^{a\mu\nu}W^{a}_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$
$\mathcal{L}_{\text{kin}}^{\text{fermion}}$	$\overline{q^0_{L\alpha}}i\not{D}q^0_{L\alpha} + \overline{u^0_{R\alpha}}i\not{D}u^0_{R\alpha} + \overline{d^0_{R\alpha}}i\not{D}d^0_{R\alpha} + \overline{\ell^0_{L\alpha}}i\not{D}\ell^0_{L\alpha} + \overline{e^0_{R\alpha}}i\not{D}e^0_{R\alpha}$
$\mathcal{L}_{\text{Higgs}}$	$\left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right) - V\left(\phi\right)$
$\mathcal{L}_{\text{Yukawa}}$	$-Y^d_{\alpha\beta}\,\overline{q^0_{L\alpha}}\phi d^0_{R\beta} - Y^u_{\alpha\beta}\,\overline{q^0_{L\alpha}}\tilde{\phi}u^0_{R\beta} - Y^\ell_{\alpha\beta}\,\overline{\ell^0_{L\alpha}}\phi e^0_{R\beta} + \text{h.c.}$

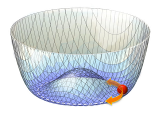
- $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^a_\mu G^b_\nu\,,(a,b,c=1,...,8)$;
- $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^a_\mu W^b_\nu\,(a,b,c=1,...,3)$;
- $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$Y^{u,d,\ell}$ Yuwaka coupling matrices and $\tilde{\phi} = i\tau_2\phi^*$

Aspects of SSB

$$V(\phi) = \mu_\phi^2\, \phi^\dagger\phi + \frac{\lambda_\phi}{2}(\phi^\dagger\phi)^2 = \frac{\lambda_\phi}{2}\left(\phi^\dagger\phi + \frac{\mu_\phi^2}{\lambda_\phi}\right)^2 + \text{const.}$$

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$



Higgs kinetic term induces masses to some of the gauge bosons

$$(D_\mu\phi)^\dagger (D^\mu\phi) \sim m_W^2 W_\mu^+ W^{\mu-} + \frac{1}{2} m_Z^2 Z_\mu^0 Z^{\mu 0} + \cdots$$

with ($T^\pm = (T^1 \pm iT^2)/\sqrt{2}$, $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$)

$$\begin{aligned} D_\mu =& \partial_\mu - ig_s G_\mu^a \frac{\lambda_a}{2} - igW_\mu^a \frac{\tau_a}{2} - ig' B_\mu Y \\ =& \partial_\mu - ig_s G_\mu^a \frac{\lambda_a}{2} - ig\left(W_\mu^+ T_+ + W_\mu^- T_-\right) - ieA_\mu Q - \frac{ig}{c_W} Z_\mu^0 \left(T_3 - s_W^2 Q\right) \end{aligned}$$

Gauge boson mass: $m_W^2 = \frac{g^2 v^2}{4}\,, \quad m_Z^2 = \frac{g^2 v^2}{4c_W^2}\,, m_A = 0$ and $m_G = 0\,.$

The flavour sector

Charged Current:
$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} \left(\overline{u^0_{L\alpha}} \gamma^\mu d^0_{L\alpha} W^+_\mu + \overline{e^0_{L\alpha}} \gamma^\mu \nu^0_{L\alpha} W^-_\mu \right) + \text{h.c.}$$

Neutral Current:
$$\mathcal{L}_{\text{NC}} = e Q_f \overline{f^0} \gamma^\mu f^0 A_\mu + \frac{g}{c_W} \overline{f^0} \gamma^\mu \left(g^f_V - g^f_A \gamma_5 \right) f^0 Z_\mu$$

with
$$g^f_V = \frac{1}{2} T^f_3 - s^2_W Q_f \,, \quad g^f_A = \frac{1}{2} T^f_3 \,,$$

WBTs:
$$\left\{ \begin{array}{l} q^0_L = W^q_L q'_L \,,\, u^0_R = W^u_R u'_R \,,\, d^0_R = W^d_R d'_R \,, \\ \ell^0_L = W^\ell_L \ell'_L \,,\, e^0_R = W^e_R e'_R \,, \end{array} \right. \longrightarrow \left\{ \begin{array}{l} Y'_u = W^{q\dagger}_L Y_u W^u_R \,, \\ Y'_d = W^{q\dagger}_L Y_d W^d_R \,, \\ Y'_e = W^{\ell\dagger}_L Y_e W^e_R \,. \end{array} \right.$$

flavour basis I:
$$\left\{ \begin{array}{l} Y_u = U^u_L \lambda_u V^{u\dagger}_R \\ Y_d = U^d_L \lambda_d V^{d\dagger}_R \\ Y_e = U^e_L \lambda_e V^{e\dagger}_R \end{array} \right.$$

WBTs
$$\begin{array}{c} W^q_L = U^d_L \,,\, W^u_R = V^u_R \,,\, W^d_R = V^d_R \\ W^\ell_L = U^e_L \,,\, W^e_R = V^e_R \end{array}$$

flavour basis II:
$$\left\{ \begin{array}{l} Y'_u = V^\dagger_{CKM} \lambda_u \\ Y'_d = \lambda_d \\ Y'_e = \lambda_e \end{array} \right.$$

The flavour sector

Mass: $-\mathcal{L}_{\text{mass}} = M^e_{\alpha\beta}\,\overline{e^0_{L\alpha}}e^0_{R\beta} + M^u_{\alpha\beta}\,\overline{u^0_{L\alpha}}u^0_{R\beta} + M^d_{\alpha\beta}\,\overline{d^0_{L\alpha}}d^0_{R\beta} + \text{h.c.}$

hff: $-\mathcal{L}_{\text{hff}} = \frac{h}{\sqrt{2}}Y^\ell_{\alpha\beta}\,\overline{e^0_{L\alpha}}e^0_{R\beta} + \frac{h}{\sqrt{2}}Y^u_{\alpha\beta}\,\overline{u^0_{L\alpha}}u^0_{R\beta} + \frac{h}{\sqrt{2}}Y^d_{\alpha\beta}\,\overline{d^0_{L\alpha}}d^0_{R\beta} + \text{h.c.}$

$$M^f = \frac{v}{\sqrt{2}}Y^f\,, \quad \text{with} \quad f = \{u, d, e\}\,.$$

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}}\left(\overline{u_{L\alpha}}\,(V_{\text{CKM}})_{\alpha\beta}\,\gamma^\mu d_{L\beta}W^+_\mu + \overline{e_{L\alpha}}\gamma^\mu \nu_{L\alpha}W^-_\mu\right) + \text{h.c.}\,,$$

with

$$V_{\text{CKM}} \equiv U_L^{u\dagger}U_L^d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \text{and} \quad V_{\text{CKM}} \longrightarrow K_u^\dagger V_{\text{CKM}} K_d$$

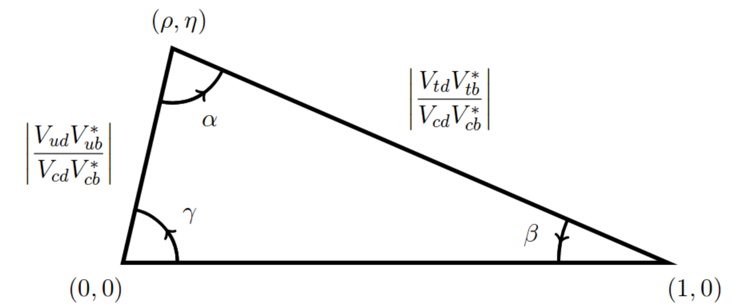
CKM param. $= 9 - (6-1) = 4$ (3 angles, 1 phase)

The flavour sector

The result of a global fit gives

$$|V_{CKM}| = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

The Jarlskog invariant is $J \equiv \mathrm{Im}(V_{us}V_{cb}V_{ub}^*V_{cs}^*) = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$.



$$\text{Area} = \frac{|J|}{2}$$

Custodial Symmetry

Returning to the Higgs potential. $V(\phi) = f(\phi^\dagger \phi)$

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix} \to \phi^\dagger \phi \equiv \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 \equiv \vec{\Phi}^T . \vec{\Phi} \leftarrow \vec{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}$$

$\vec{\Phi}$ is a real 4 dim. vector field $\Rightarrow O(4) = SO(4) \rtimes Z_2$ is the largest symmetry in the potential.

$$\boxed{SO(4) \simeq \frac{SU(2)_L \times SU(2)_R}{Z_2}} \to \begin{cases} \boxed{SO(4)} \\ \quad v^a \to S^{ab} v^b \,, \quad |v|^2 = \text{const.} \\ \\ \boxed{SU(2)_L \times SU(2)_R} \\ \quad \Sigma \to L \Sigma R^\dagger, \quad \det(\Sigma) = -|v|^2 = \text{const.} \\ \quad \Sigma \equiv \sigma^a v^a \text{ and } \sigma^a = (i\vec{\sigma}, \mathbb{I}) \end{cases}$$

$$\Sigma = \sigma^a \phi^a = \begin{pmatrix} \phi_4 + i\phi_3 & \phi_2 + i\phi_1 \\ -\phi_2 + i\phi_1 & \phi_4 - i\phi_3 \end{pmatrix} = \begin{pmatrix} \varphi^{0*} & \varphi^+ \\ -\varphi^- & \varphi^0 \end{pmatrix} = \left(\begin{array}{c|c} \tilde{\phi} & \phi \end{array} \right)$$

$$\boxed{\mathcal{L}(\phi) = \frac{1}{2} \langle (D^\mu \Sigma)^\dagger D_\mu \Sigma \rangle - \frac{\lambda_\phi}{8} \left(\langle \Sigma^\dagger \Sigma \rangle - v^2 \right)^2}$$

Custodial Symmetry

Symmetry breaking pattern:

$$\boxed{SU(2)_L \times SU(2)_R \longrightarrow SU(2)_{L+R}} \qquad \langle \Sigma \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Performing a polar decomposition

$$\boxed{\Sigma(x) \equiv \frac{1}{\sqrt{2}} \left[v + H(x) \right] U(\varphi(x))} \quad \text{with} \quad U(\varphi(x)) = \exp\left(i\vec{\sigma}.\vec{\varphi}/v \right)$$

Taking the limit of heavy Higgs field

$$\mathcal{L}(\phi) = \frac{v^2}{4} \langle (D^\mu U)^\dagger \, D_\mu U \rangle \quad \text{with} \quad D_\mu U \equiv \partial_\mu U - i g W^a_\mu \frac{\tau_a}{2} U - i g' U \frac{\sigma_3}{2} B_\mu$$

In the unitary gauge $U = 1$

$$\mathcal{L}(\phi) \quad \longrightarrow \quad m_W^2 W^+_\mu W^{\mu -} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$ Massive gauge bosons: no Higgs!

Custodial Symmetry

Sources of Custodial breaking:

- B_μ not a full adjoint of $SU(2)_R$;
- $m_t \neq m_b$, i.e. $\begin{pmatrix} t_L & b_L \end{pmatrix} U(\varphi(x)) \begin{pmatrix} m_t t_R \\ m_b b_R \end{pmatrix}$

"Custotial measure": $\boxed{\rho \equiv \frac{m_W^2}{c_W^2 m_Z^2}}$

In the SM:

- Tree-level: $\rho = 1$;
- Loop-level: $\rho \simeq 1 - \frac{G_F}{8\sqrt{2}\pi^2}\left[\frac{(m_t^2 - m_b^2)^2}{m_b^2} - \frac{11}{3} m_Z^2 s_W^2 \ln \frac{m_H^2}{m_Z^2}\right] + \cdots$

Loss of Unitarity

Take a massive spin-1 gauge boson: $A^\mu(x) = \int \frac{d^4k}{(2\pi)^4} \epsilon^\mu \text{exp}(ik_\nu x^\nu)$

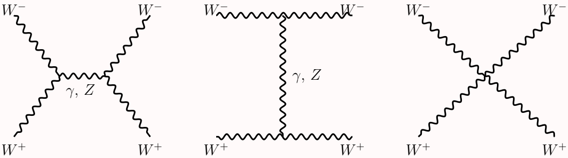
Polarization vectors: $\epsilon_\mu \epsilon^\mu = -1$ and $k_\mu \epsilon^\mu = 0$

$$\epsilon^\mu_\pm = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)\,, \quad \epsilon^\mu_L = \left(\frac{k}{M}, 0, 0, \frac{E}{M}\right) \simeq \frac{k^\mu}{M} + \mathcal{O}(M/E)$$

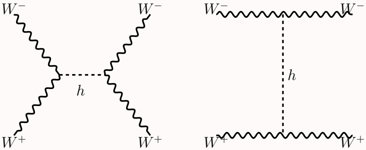
At high energies, the longitudinal polarization aligns with the momentum of the gauge boson. This causes a growth in the scattering amplitude that is incompatible with unitarity.

$$W_L^+ W_L^- \to W_L^+ W_L^-$$

- L-polarization $\sim E$
- Triple-gauge vertex $\sim E$
- Gauge propagator $\sim (1 + E^2)/E^2$
- 4-point amp. $\sim \mathcal{O}(E^4)$
- Z, γ amp. $\sim \mathcal{O}(E^4)$ ($p^\mu p^\nu / M^2$ cancel out in each diagram)



$$A_{\text{gauge}} = 0\,,\; B_{\text{gauge}} \neq 0\,,\; C_{\text{gauge}} \neq 0$$



$$A_{\text{higgs}} = 0\,,$$
$$B_{\text{higgs}} = -B_{\text{gauge}}\,,$$
$$C_{\text{higgs}} \neq 0$$

$$\mathcal{M} \simeq \mathsf{A}\left(\frac{s}{4m_W^2}\right)^2 + \mathsf{B}\left(\frac{s}{4m_W^2}\right) + \mathsf{C} + \mathcal{O}\left(m_W^2/s\right)$$

Unitarity restoration

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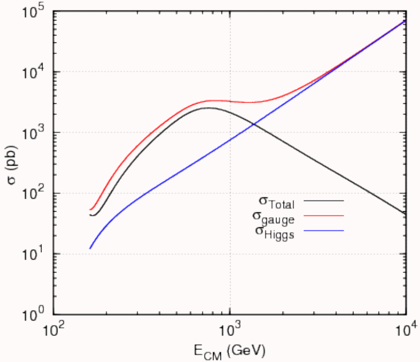


Partial wave decomposition:

$$\mathcal{M} = 16\pi \sum_{j=0} (2j+1) a_j \mathcal{P}_j(\cos\theta)$$

$$a_j = \frac{1}{32\pi} \int_{-1}^{1} \mathcal{M}\, \mathcal{P}_j(\cos\theta) d(\cos\theta)$$

Optical theorem:

$$\sigma = \frac{1}{s} \mathrm{Im}(\mathcal{M}(\theta = 0)) \Rightarrow \mathrm{Re}\, a_j \leq \frac{1}{2}$$

Equivalence theorem: $W_L W_L \to W_L W_L \quad \longrightarrow \quad \varphi^+\varphi^- \to \varphi^+\varphi^-$

$$\mathcal{M} \simeq \frac{m_H^2}{v^2} \left[2 + \frac{m_H^2}{s - m_H^2} + \frac{m_H^2}{t - m_H^2} \right] \quad \Rightarrow \quad \boxed{m_H^2 < 2\pi v^2 \simeq (870\,\mathrm{GeV})^2}$$

Landau pole and stability bound

Quartic coupling running:

$$\frac{d\lambda}{d\ln Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}(2g_2^4 + (g_2 + g_1)^2)\right]$$

Strong λ coupling

$$\lambda(Q^2) \simeq \frac{\lambda(v^2)}{1 - \frac{3}{4\pi^2}\lambda(v^2)\ln\frac{Q^2}{v^2}} \;\Rightarrow\; Q_{LP} \simeq v\,\mathrm{exp}\left[\frac{4\pi^2 v^2}{3m_H^2}\right]$$ Perturbativity breaks

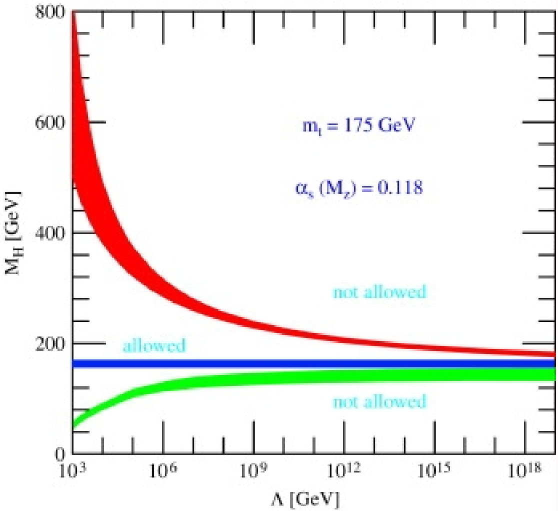
Turning the argument around: $Q_{LP} \sim 10^{19}$ GeV requires $m_H \leq 180$ GeV

Small λ coupling

$$\lambda(Q^2) \simeq \lambda(v^2) + \frac{1}{16\pi^2}\left[-\frac{12m_t^4}{v^4} + \frac{3}{16}(2g_2^4 + (g_2 + g_1)^2)\right]\ln\frac{Q^2}{v^2}$$

$$\lambda(Q^2_{\text{stable}}) = 0 \;\Rightarrow\; m_H = \begin{cases} 70\,\text{GeV} & \text{for } Q_{\text{stable}} = 10^3\,\text{GeV} \\ 130\,\text{GeV} & \text{for } Q_{\text{stable}} = 10^{16}\,\text{GeV}\end{cases}$$ Stable vacuum

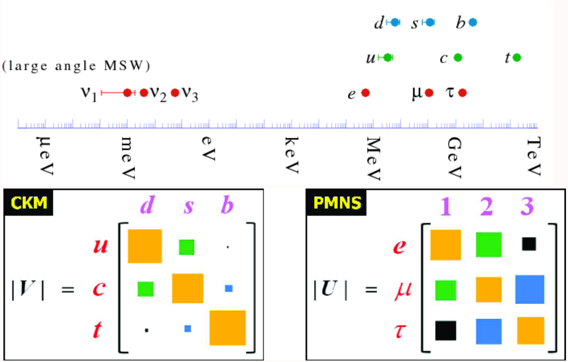
Landau pole and stability bound

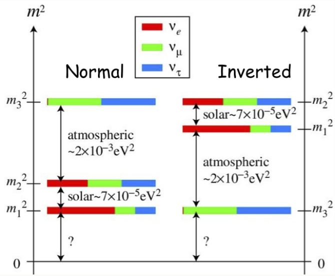



Neutrino masses
Fermions Hierarchies
Baryon asymmetry
The SM works!
. . .
Light Higgs
Dark matter
CP violation

Origin of neutrino masses and matter in the Universe

"Trivial" New Physics



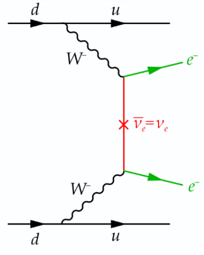


No way to generate neutrino masses and mixing in the SM!

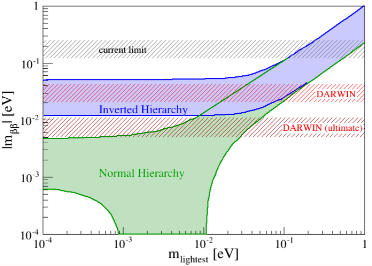
Dirac vs. Majorana

Two independent 4-spinor ψ_{LR}. Construct Lorentz-invariant bilinear

- Dirac: $-m\left(\overline{\psi_R}\psi_L + \text{h.c}\right) = -m\bar{\psi}\psi$ with $\psi = \psi_L + \psi_R$
- Majorana: $\frac{1}{2}m\psi_L^T C^{-1}\psi_L + \text{h.c.} = -\frac{1}{2}m\bar{\psi}\psi$ with $\psi = \psi_L + (\psi_L)^c$
  $(\psi_L)^c \equiv C\overline{\psi_L}^T$ Majorana condition: $\psi = \psi^c$



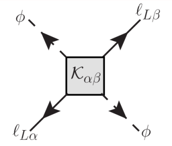
$$n + n \to p^+ p^+ e^- e^- \left(\bar{\nu}_e \nu_e\right)$$



Giving mass to neutrinos

Make a decision: Dirac or Majorana?

- Look at higher dim. operators;
- Only one at $d = 5$. Weinberg operator.

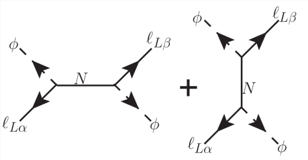


$$\mathcal{L}_{d=5} = \frac{\mathcal{K}_{\alpha\beta}}{\Lambda}\overline{\ell_{L\alpha}}\tilde{\phi}\tilde{\phi}^T\ell^c_{L\beta}$$
$$\longrightarrow \overline{\nu}_{L\alpha}\nu^c_{L\beta}\frac{(\phi^0)^2}{\Lambda}$$

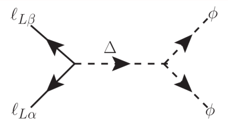
Majorana neutrino mass

Combinations: $\mathbf{2}\otimes\mathbf{2}\otimes\mathbf{2}\otimes\mathbf{2} = (\mathbf{3}\oplus\mathbf{1})\otimes(\mathbf{3}\oplus\mathbf{1})$

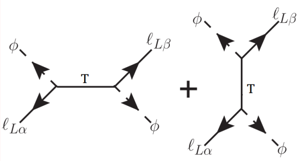
$\mathcal{O}_I = (\overline{\ell_{L\alpha}}\tilde{\phi})_{\mathbf{1}}(\tilde{\phi}\ell^c_{L\beta})_{\mathbf{1}}$ $\quad$ $\mathcal{O}_{II} = (\overline{\ell_{L\alpha}}\ell^c_{L\beta})_{\mathbf{3}}(\tilde{\phi}\tilde{\phi})_{\mathbf{3}}$ $\quad$ $\mathcal{O}_{III} = (\overline{\ell_{L\alpha}}\tilde{\phi})_{\mathbf{3}}(\tilde{\phi}\ell^c_{L\beta})_{\mathbf{3}}$



type-I (77')



type-II (80')



type-III (89')

The operator $(\overline{\ell_{L\alpha}}\ell^c_{L\beta})_{\mathbf{1}}(\tilde{\phi}\tilde{\phi})_{\mathbf{1}} = 0$, since $(\tilde{\phi}\tilde{\phi})_{\mathbf{1}} = 0$ (anti-symm singlet)

Type-I Seesaw

Simply add $\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$ to SM

$$\begin{aligned}\mathcal{L}_I^\nu =& -(\mathbf{m_D})_{\alpha i}\overline{\nu_{L\alpha}^0}\nu_{Ri}^0 - \frac{1}{2}(\mathbf{M}_R)_{ij}\overline{(\nu_{Ri}^0)^c}\nu_{Rj}^0 + \text{h.c.}\\ =& -\frac{1}{2}\overline{n_L^0}\mathcal{M}(n_L^0)^c + \text{h.c.}\end{aligned}$$

with

$$n_L^0 \equiv \begin{pmatrix}\nu_L^0\\ (\nu_R^0)^c\end{pmatrix}, \quad \boxed{\mathcal{M} = \begin{pmatrix}\mathbb{O} & \mathbf{m}_D\\ \mathbf{m}_D^T & \mathbf{M}_R\end{pmatrix}}$$

$n_L^0 = \mathbf{U}n_L = \begin{pmatrix}\nu_{lL} & \nu_{hL}\end{pmatrix}^T$ then $\mathbf{U}^\dagger\mathcal{M}\mathbf{U}^* = \mathbf{d}_{n_L}$

Number of ν_R not specified. But you need at least 2!

Type-I Seesaw

$$\mathcal{L}_\nu^I = \frac{g}{\sqrt{2}} \left[\overline{e_L^0} \gamma^\mu W_\mu^- \left(\mathbb{I} - \frac{1}{2} \boldsymbol{\Delta}_l \right) \nu_{lL}^0 + \overline{e_L^0} \gamma^\mu W_\mu^- \boldsymbol{\Delta}_h^{CC} \nu_{hL}^0 + \text{h.c.} \right]$$
$$- \frac{1}{2} \overline{\nu_{lL}^0} \mathbf{m}_{I\nu} (\nu_{lL}^0)^c - \frac{1}{2} \overline{\nu_{hL}^0} \mathbf{M}_{I\nu} (\nu_{hL}^0)^c + \text{h.c.} + \cdots$$

Seesaw formula: $\mathbf{m}_{I\nu} \simeq -\mathbf{m}_D \mathbf{M}_R^{-1} \mathbf{m}_D^T\,, \quad \mathbf{M}_{\mathbf{I}\nu} \simeq \mathbf{M}_R$

Unitarity violation: $\boldsymbol{\Delta}_l \simeq \mathbf{m}_D \mathbf{M}_R^{-1} (\mathbf{M}_R^*)^{-1} \mathbf{m}_D^\dagger\,, \quad \boldsymbol{\Delta}_h^{CC} \simeq \mathbf{m}_D \mathbf{M}_R^{-1}$

Typical scales for $\mathbf{m}_{I\nu} \sim 0.1$ eV

- $\mathbf{m}_D \sim 0.1$ eV, $M_R \to 0$. Neutrinos become Dirac particles;
- $\mathbf{m}_D \sim 1$ GeV, $M_R \to 10^{9,10}$ GeV. Generation of a cosmic matter-antimatter asymmetry;
- $\mathbf{m}_D \sim 10^{-4}$ GeV, $M_R \sim 1$ TeV. Right-handed neutrinos then become kinematically accessible in colliders;
- $\mathbf{m}_D \sim 10$ eV, $M_R \sim 1$ keV. Right-handed neutrinos might be a viable candidate for dark matter.

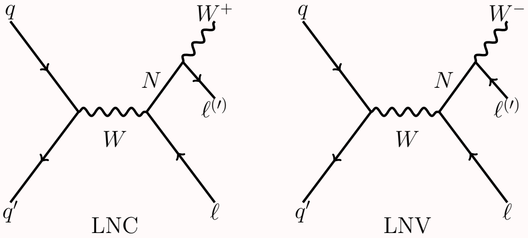
Type-I Seesaw

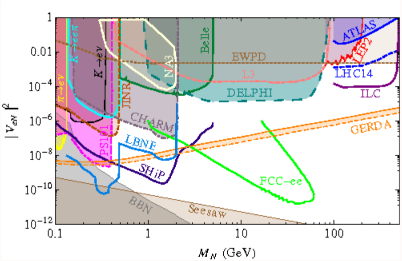
$$V_{\text{PMNS}} \equiv U_L^{e\dagger} U_L^{\nu} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} e^{i\alpha/2} & & \\ & e^{i\beta/2} & \\ & & 1 \end{pmatrix}$$

$$V_{\text{PMNS}} \longrightarrow K_e^{\dagger} V_{\text{PMNS}}$$

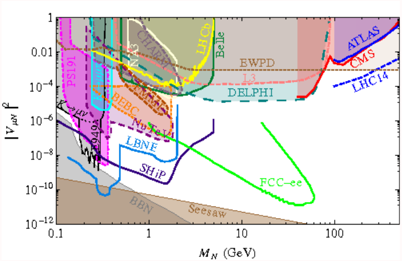
PMNS param. $= 9 - (3) = 6$ (3 angles, 3 phases)

At colliders

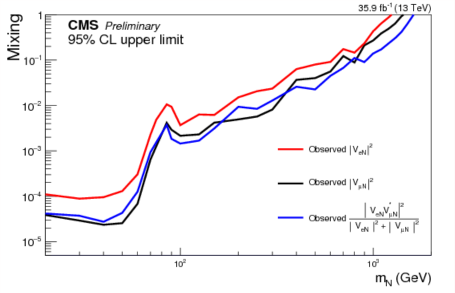




$|V_{eN}|^2$
M_N (GeV)
ATLAS
LEP2
LHC14
ILC
EWPD
L3
DELPHI
Belle
NA3
CHARM
JINR
PS191
LBNE
SHiP
FCC-ee
GERDA
BBN
Seesaw



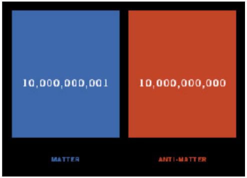
$|V_{\mu N}|^2$
M_N (GeV)
ATLAS
CMS
LHC14
EWPD
L3
DELPHI
Belle
LHCb
CHARM
NA3
PS191
E949
BEBC
NuTeV
LBNE
SHiP
FCC-ee
BBN
Seesaw



35.9 fb^{-1} (13 TeV)
CMS Preliminary
95% CL upper limit
Mixing
m_N (GeV)
Observed $|V_{eN}|^2$
Observed $|V_{\mu N}|^2$
Observed $\frac{|V_{eN}V^*_{\mu N}|^2}{|V_{eN}|^2+|V_{\mu N}|^2}$

Baryon asymmetry of the Universe

Baryon Asymmetry: $\eta_B \equiv \dfrac{n_B - n_{\bar{B}}}{n_\gamma} \simeq (6.2 \pm 0.15) \times 10^{-10}$ [WMAP]



Dynamical generation $\Rightarrow$ Sakharov conditions:

(1) Baryon number violation;

(2) C and CP violation;

(3) Departure from thermal equilibrium.

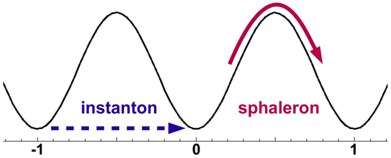
Baryon number violation

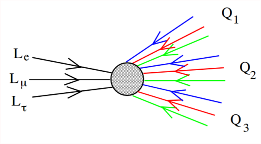
$$\hat{B} = \frac{1}{3}\sum_i \int d^3x : \psi_i^\dagger(\vec{x},t)\psi_i(\vec{x},t) : , \quad [B,\mathcal{H}] = 0 \quad \rightarrow \quad B(t) = 0$$

[Rubakov,Shaposhnikov,1996] The three Sakharov conditions do not guarantee the generation of a baryon asymmetry in the decay of heavy particles.

['t Hooft,1976] In the SM $\partial_\mu j_B^\mu \neq 0\,,\ \partial_\mu j_L^\mu \neq 0$ (quantum). Anomalies

- $T < T_{ew}$: $\Gamma \sim e^{-2m_W/(\alpha_W T)}$; $\quad T > T_{ew}$: $\Gamma \sim \kappa\alpha_W^5 T^4$





but

$$\partial_\mu(j_B^\mu - j_L^\mu) \propto \sum_{\text{gen.}} \left[\sum_{\text{quarks}} (Y(Q_L) - Y(Q_R)) - \sum_{\text{leptons}} (Y(L_L) - Y(L_R)) \right] = 0$$

$B - L$ violation

C and CP violation

$$\boxed{X \to Y + B} \quad \text{with} \quad n_B = 1$$

The net rate of baryon productions goes like

$$\boxed{\Delta B \propto \Gamma(\bar{X} \to \bar{Y} + \bar{B}) - \Gamma(X \to Y + B)}$$

- C invariance

$$\Gamma(\bar{X} \to \bar{Y} + \bar{B}) = \Gamma(X \to Y + B) \quad \Rightarrow \quad \Delta B = 0$$

- C violation but CP invariance
  Take the example: $X \to q_L q_L$ and $X \to q_R q_R$

$$\mathcal{CP}: \quad q_L \to \overline{q_R}\,, \quad \mathcal{C}: \quad q_L \to \overline{q_L}$$

  - C violation: $\Gamma(X \to q_L q_L) \neq \Gamma(\bar{X} \to \overline{q_L q_L})$
  - CP invariance: $\Gamma(X \to q_L q_L) = \Gamma(\bar{X} \to \overline{q_R}\,\overline{q_R})$ $(L \leftrightarrow R)$

$$\boxed{\Gamma(X \to q_L q_L) + \Gamma(X \to q_R q_R) = \Gamma(\bar{X} \to \overline{q_R}\,\overline{q_R}) + \Gamma(\bar{X} \to \overline{q_L}\,\overline{q_L})}$$

No net asymmetry in quarks, as long as the initial state has equal numbers of X and $\bar{X}$

Departure from thermal equilibrium

$$\mathcal{C}\,\hat{B}\,\mathcal{C}^{-1} = -\hat{B}\,, \quad \mathcal{CP}\,\hat{B}\,\mathcal{CP}^{-1} = -\hat{B}\,, \quad \mathcal{CPT}\,\hat{B}\,\mathcal{CPT}^{-1} = -\hat{B}$$

$$\begin{aligned}
\langle \hat{B}(t) \rangle_T =& \mathrm{Tr}\left[e^{-\mathcal{H}/T}\hat{B}(t)\right] = \mathrm{Tr}\left[e^{-\mathcal{H}/T}\mathcal{CPT}^{-1}\mathcal{CPT}\hat{B}(t)\right] \\
=& \mathrm{Tr}\left[e^{-\mathcal{H}/T}\mathcal{CPT}\hat{B}(t)\mathcal{CPT}^{-1}\right] = -\mathrm{Tr}\left[e^{-\mathcal{H}/T}\hat{B}(t)\right] \\
=& -\langle \hat{B}(t) \rangle_T \quad \Rightarrow \quad \langle \hat{B}(t) \rangle_T = 0
\end{aligned}$$

- In thermal equilibrium $\Gamma(X \to Y + B) = \Gamma(Y + B \to X)$;
- No net baryon asymmetry can be produced;

  Take X a heavy particle $(M_X > T)$ at the time of decay $(\tau = 1/\Gamma)$.

  $E_{Y+B} \sim T$, then there is no phase space for the inverse decay.
  $\Gamma(Y + B \to X) \sim e^{-M_X/T}$ is Boltzmann-suppressed.

Seesaw type-I Baryogenesis through leptogenesis

Baryon asymmetry: $\boxed{\eta_B \simeq \epsilon_1 \times \eta \times \kappa}$

- ϵ_1: leptonic-CP asymmetry;
- η: washout factor;
- κ: chemical equilibrium.

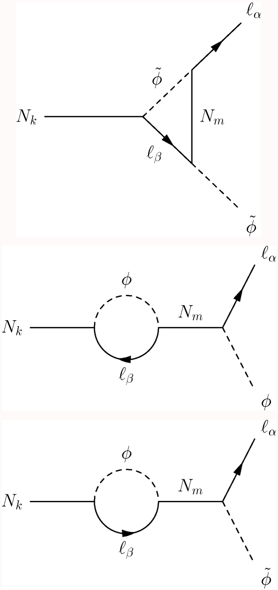
Amplitude: $\mathcal{M} = c_0\mathcal{A}_0 + c_1\mathcal{A}_1$

$$\begin{aligned}\epsilon_1 =& \Delta L \frac{\Gamma(N_1 \to \ell\phi^*) - \Gamma(N_1 \to \bar{\ell}\phi)}{\Gamma_{tot}} \\ \propto& \,\mathrm{Im}(c_0^* c_1)\mathrm{Im}(\mathcal{A}_0^*\mathcal{A}_1)\end{aligned}$$

Davidson-Ibarra bound [hep-ph/0202239]

$$|\epsilon_1| \leq 10^{-6}\left(\frac{M_1}{10^{10}\,\text{GeV}}\right)\left(\frac{m_{max} - m_{min}}{m_{atm}}\right)$$

implies $M_1 \geq 10^{8,9}$ GeV



The Higgs mass hierarchy problem
and
Naturalness

Natural scale hierarchy

Dimensional transmutation: Λ_{QCD} arises naturally

$$\frac{dg_s}{d\ln\mu} = \beta(g) < 0 \quad \rightarrow \quad \boxed{\Lambda_{QCD} \sim \Lambda_{UV} e^{-g_c^2/g_{UV}^2}}$$

The philosophy underlying theories of dynamical EWSB is that the weak scale has a similar dynamical and natural origin.
We look for a dynamical origin of a very small physical parameter in a theory in which no initial small input parameters occur

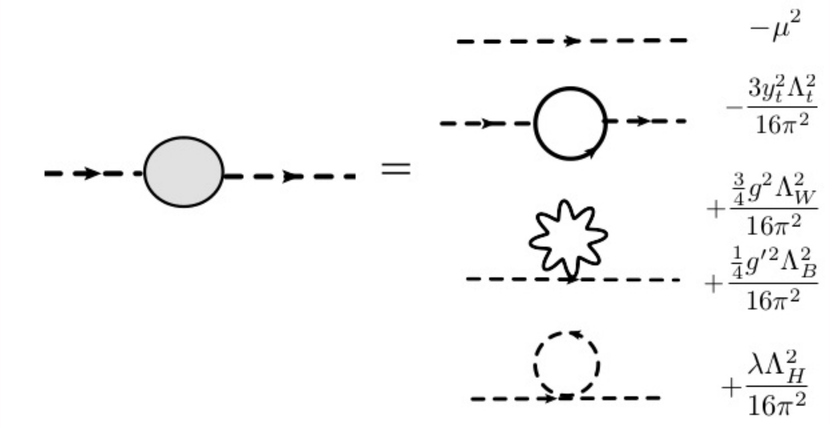
$$\boxed{\Lambda_{QCD}/M_{\text{Planck}} \sim 10^{-20}}$$

Technically natural if radiative corrections to it are multiplicative.
Examples:

- Spin 1 (vector) fields: mass protected by gauge symmetry;
- Spin 1/2 (fermion) fields: mass protected by chiral symmetry;

Symmetry that protects the small value of a parameter are called a custodial symmetry. Do we have a **custodial symmetry** for Spin 0 (scalar) fields?

Why is the Higgs so light?



Theories with light scalars

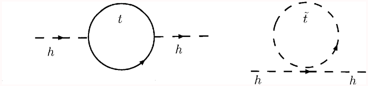
The scalar boson mass is typically subject to large additive renormalizations. The important exceptions to this are:

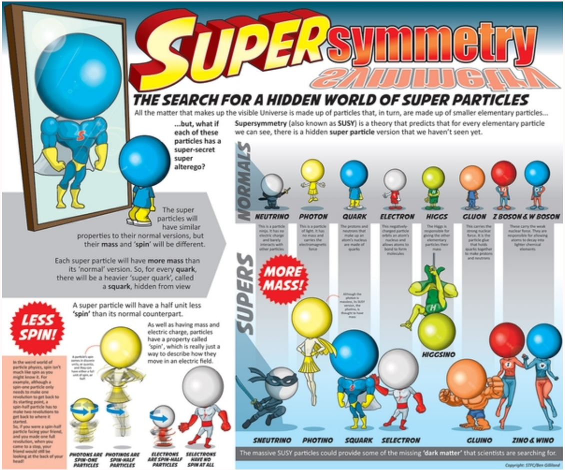
(i) Nambu-Goldstone bosons which can have technically natural low masses due to their spontaneously broken chiral symmetry;

(ii) Composite scalars which only form at a strong scale such as Λ_{QCD} and could receive only additive renormalizations of order Λ_{QCD};

(iii) A technically natural mechanism for having fundamental low mass scalars is also provided by SUSY because the scalars are then associated with fermionic superpartners. The chiral symmetries of these superpartner fermions then protect the mass scale of the scalars so long as SUSY is intact.

SUSY partitles cancel Λ^2 corrections





Scaling up QCD: Technicolor

Consider two quarks flavour

$$\mathcal{L}_f = \overline{q_L}^i i\not{D} q_L^i + \overline{q_R}^i i\not{D} q_R^i \quad \text{with} \quad q^i = u,\, d$$

First, ignore EW gauge interactions. Above Λ_{QCD} quarks are free. Below they condensate:

$$\langle \overline{q_L}^i q_R^j \rangle = \Lambda_{QCD}^3 \delta_{ij} \neq 0$$

The system change phase

$$SU(2)_L \times SU(2)_R \times U(1) \longrightarrow SU(2) \times U(1)$$

As a result: 3 Goldstone bosons $U = e^{i\pi^i T^i / f_\pi}$. π^i are the Goldstone bosons, T^i are the broken generators, $f_\pi = 93 MeV$ is the pion decay constant. The lowest order Lagrangian is

$$\mathcal{L}_\pi = \frac{f_\pi^2}{2} \mathrm{Tr}\left[(\partial_\mu U)^\dagger \partial^\mu U \right]$$

Scaling up QCD: Technicolor

Turn on the gauge interaction, i.e. gauge $SU(2)_L$ and T^3 of the $SU(2)_R$

$$\partial_\mu U \to D_\mu U = \left(\partial_\mu - \boxed{igW_\mu^a T^a + ig' BT^3}\right) U$$

The gauge part of the pion Lagrangian now reads

$$\begin{aligned}\mathcal{L}_\pi \supset \quad & \frac{f_\pi^2}{2}\text{Tr}\left|\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) + gW_\mu^3 T^3 - g' B_\mu T^3\right|^2 \\ = \quad & \frac{f_\pi^2}{4}\left[g^2 W_\mu^+ W^{-\mu} + \frac{1}{2}\left(gW_\mu^3 - g' B_\mu\right)\left(gW^{\mu 3} - g' B^\mu\right)\right]\end{aligned}$$

EWSSB via QCD: $\boxed{m_W^2 = \frac{g^2 f_\pi^2}{4}}$ and $\boxed{m_Z^2 = \frac{(g^2 + g'^2) f_\pi^2}{4}}$

The problem: $\boxed{f_\pi = 93\,\text{MeV} \ll v = 246\,\text{GeV}}$

Scaling up QCD: Technicolor

Main idea: scale up this QCD toy model

- Introduce a new gauge group (Technicolor) which gets strongly coupled at $\Lambda_{TC} \sim$ TeV;
- Introduce fermions which are charged under this group and form a condensate.
- The fermion Lagrangian must have a global symmetry group $\mathcal{G}$;
- The techniquark condensate must break $\mathcal{G}$ to $\mathcal{H}$,

$$\mathcal{G} \supset SU(2) \times U(1) \quad \longrightarrow \quad \mathcal{H} \supset U(1)_{em}$$

- The technipions are the Goldstone bosons of this breaking which have to give mass to W and Z

Higgs potential in Composite Higgs Models

Assumptions:

- Higgs is a composite, with compositeness scale f;
- vev hierarchy $v/f < 1$. Allow expansion in powers of h/f;
- The potential is (fully or partially) radiatively generated;

$$\boxed{V(h) = \frac{g_{SM}^2 \Lambda^2}{16\pi^2}\left(-a|h|^2 + b\frac{|h|^4}{2f^2}\right)}$$

$$\left[\begin{array}{l}\text{typical SM coupling: } g_{SM}^2 \sim N_c y_t^2 \\ \text{cutting scale: } \Lambda \sim m_*\end{array}\right.$$

$$\left[\begin{array}{l}(246\,\text{GeV})^2 = v^2 = \dfrac{a}{b} f^2 \\ (125\,\text{GeV})^2 = m_h^2 = 4bv^2 \dfrac{g_{SM}^2}{16\pi^2}\dfrac{\Lambda^2}{f^2}\end{array}\right.$$

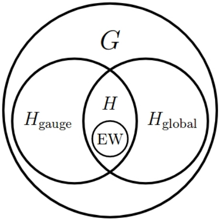
We can classify composite Higgs models in terms of scalar potential

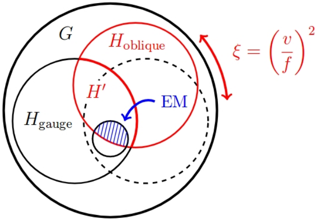
Composite Higgs Models potential classification

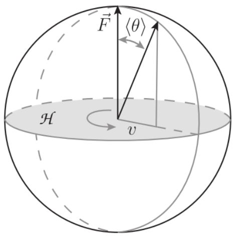
- Tree-level mass and quartic:
  $a = \mathcal{O}(1)$, $b = \mathcal{O}(1)$, $g_* \sim 4\pi$ [Technicolor]
  - Typical too large Higgs mass;
  - $v \sim f$;
  - Large tuning
- Loop-level mass, tree-level quartic:
  $a = \mathcal{O}(1)$, $b = \mathcal{O}(16\pi^2/g_*^2)$, $g_* \ll 4\pi^2$ [Little Higgs]
  - Natural scale hierarchy: $v^2/f^2 \simeq g_*^2/(16\pi^2) \ll 1$;
  - Price to pay: $m_h \sim 2vg_{SM} \sim 500\,$GeV ($g_{SM} \sim 1$);
  - Mild tuning
- Loop-level mass and quartic:
  $a = \mathcal{O}(1)$, $b = \mathcal{O}(1)$, $g_* \ll 4\pi$ [Holographic/pNGB composite Higgs]
  - Tuning parameter $\xi = v^2/f^2$;
  - Higgs automatically light once tuning is fixed.
  - Inspired by AdS/CFT correspondence. Some strongly interacting theories can be described by weakly coupled AdS duals.
  - Recently, several works in the direction of 4D strongly coupled theories. Lattice results.

pNGB Higgs scalar potential

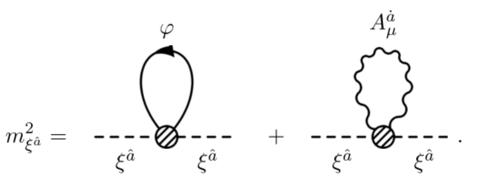
Loop-level mass and quartic term $\Rightarrow$ $V(h)_{\text{tree}} = 0$







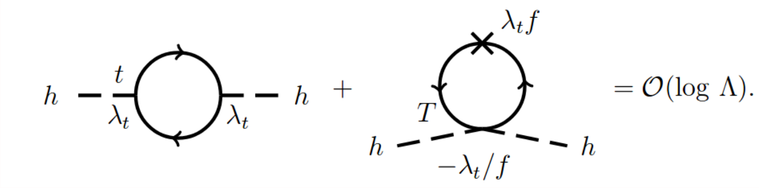
pNGB mass



- Gauge contributions: expected to be positive. Pushes the minimum to be at $\langle\theta\rangle = 0$, i.e. electroweak symmetry is not broken;
- Top contribution: expected to be negative. Pushes the minimum to at $\langle\theta\rangle = \pi/2$, i.e. electroweak symmetry broken by a condensate at the electroweak scale. Technicolor limit;

Collective symmetry breaking

- Higgs lightness explained by appealing to the Goldstone shift symmetry;
- One must break this shift symmetry in order generate the Higgs potential; this generically reintroduces a dependence on the cutoff, $\Lambda = 4\pi f$;
- One can separate the scales v and f by introducing new particles which cancel the quadratic divergences at one-loop order. Unlike supersymmetry, these partner particles carry the same spin as the Standard Model particles whose virtual contributions are to be canceled;
- Top-partner will play an important role in composite models



A powerful tool: The CCWZ formalism

[Callan, Coleman, Wess, Zumino]

- The CCWZ formalism allows to understand some properties of the confined theory without solving exactly the underlying dynamics;
- Consider a generic, weakly or strongly coupled theory with a Lagrangian invariant under linearly realized transformations of a group G and a vacuum state $\vec{f}$ which is only invariant under a certain subgroup $H \subset G$;
- We can split the generators of G into:
  - T^a: Generators of the subgroup H; (unbroken)
  - T^i: Generators that span G/H; (broken)
- The broken generators have a set of fields associated with them. (Goldstone bosons). Parametrised into

$$\boxed{U(\pi) = \exp\left[i\frac{\sqrt{2}}{f}\pi_i T^i\right]}$$

Goldstone matrix

A powerful tool: The CCWZ formalism

- U acts as a link between the broken group G and unbroken H. The goldstones transform non-linearly and non-homogeneously

$$U \to U' = gUh^{-1} \quad \Rightarrow \quad \pi'^i = \pi^i + a^i + \cdots$$

- The invariance under G transformations implies an invariance under the shift of the Goldstone fields by a constant vector a^i. This symmetry forbids the Goldstone fields to have any potential and consequently allows for any VEV

$$\vec{f}' = e^{i\langle \pi'^i T^i \rangle} \vec{f}$$

A powerful tool: The CCWZ formalism

- The theory under consideration is by construction invariant under the linearly realized group H, therefore besides the goldstones it can contain any H-multiplets, which we collectively denote ψ

$$U_{Ii} \to g_{IJ} U_{Jj} h^{-1}_{ji} \,, \quad \psi_i \to D(h(g,U))_{ij} \psi_j$$

- U first index can only be contracted with another U:
  - First combinations $U^\dagger U$. Trivial
  - Next combination $iU^\dagger \partial_\mu U$. (Maurer-Cartan form)

- We then get

$$iU^\dagger \partial_\mu U \equiv -d^i_\mu T^i - e^a_\mu T^a$$

$$\begin{cases} e_\mu = e^a_\mu T^a \to h(e_\mu - i\partial_\mu)h^{-1} & \text{gauge field} \\ d_\mu = d^i_\mu T^i \to h d_\mu h^{-1} \end{cases}$$

- With d_μ one can build the pions kinetic term

$$\mathcal{L}_{kin} = \frac{f^2}{2} \mathrm{Tr} \left[d_\mu d^\mu \right]$$

Being more concrete

Consider a strongly coupled theory with a global symmetry $SO(5)$ (we shall need and extra $U(1)_X$) which is spontaneously broken to $SO(4)$ at scale f

- The Goldstone bosons live in $SO(5)/SO(4)$ →**4 d.o.f.**
- Pion matrix in the unitary gauge ($\pi = (0, 0, 0, \langle h\rangle + h)$)

$$U(\pi) = \exp\left[\frac{i\sqrt{2}}{f}\pi_i T^i\right] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos\dfrac{\langle h\rangle + h}{f} & \sin\dfrac{\langle h\rangle + h}{f} \\ 0 & 0 & 0 & -\sin\dfrac{\langle h\rangle + h}{f} & \cos\dfrac{\langle h\rangle + h}{f} \end{pmatrix}$$

Being more concrete

Kinetic term for the Higgs

$$\begin{aligned}\mathcal{L}_\Pi = & \frac{f^2}{4} d^i_\mu d^{i\mu} \\ = & \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{4} f^2 \sin^2 \frac{\langle h \rangle + h}{f} \left(W_\mu W^\mu + \frac{1}{2c_W^2} Z_\mu Z^\mu \right)\end{aligned}$$

Canonical Kinetic Higgs term. The gauge boson masses are

$$\boxed{m_W^2 = c_W^2 m_Z^2 = \frac{g^2}{4} f^2 \sin^2 \frac{\langle h \rangle}{f}}$$

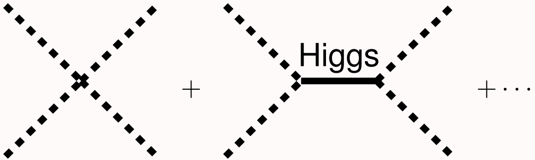
VEV: $\boxed{v = 246\,\text{GeV} = f \sin \frac{\langle h \rangle}{f} \equiv f \sin \epsilon}$

Unitarity again

The Higgs couplings to gauge are modified

$$g_{hVV}/g_{hVV}^{SM} = \sqrt{1-\epsilon^2}\,, \quad g_{hhVV}/g_{hhVV}^{SM} = 1 - 2\epsilon^2\,, \cdots$$

Therefore, the divergent high energy behavior of the $W_L W_L$ scattering amplitude is reintroduced. We need New Physics at the scale $\simeq 4\pi f$ (compositeness scale)

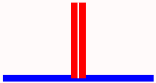


$$\mathcal{A} \simeq \frac{s}{v^2}\left(1-\sqrt{1-\epsilon^2}\right) - (1-\epsilon^2)\frac{m_h^2}{v^2}\frac{s}{s-m_h^2} + \cdots$$

Fermion masses: two approaches

The bilinear approach. (as in Technicolor)
[Dimopoulos, Susskind], [Eichten, Lane]

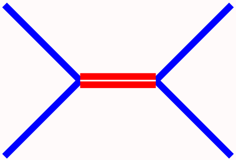
at the UV: $\dfrac{\lambda_t}{\Lambda_{UV}^{d-1}} \overline{q}_L \mathcal{O} t_R +$ h.c.



$[\mathcal{O}] = d$ and carries the Higgs quantum numbers. Running down to Λ (where the dynamics of SSB kicks in)

$$m_t \simeq \lambda_t v \left(\frac{\Lambda}{\Lambda_{UV}} \right)^{d-1}$$

Alert: dangerous 4-fermion operators [Dimopoulus, Ellis]



Fermion masses: two approaches

The linear approach (Partial Compositeness) [Kaplan]

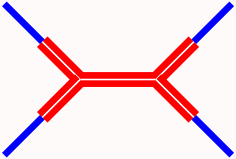
at the UV: $\frac{\lambda_{q_L}}{\Lambda_{UV}^{dL-5/2}}\overline{\mathcal{O}_R}q_L+\frac{\lambda_{t_R}}{\Lambda_{UV}^{dL-5/2}}\overline{\mathcal{O}_L}t_R+$h.c.

$[\mathcal{O}_{L,R}]=d_{L,R}$, fermionic operators carrying quarks quantum numbers.

$$m_t \simeq \lambda_{q_L}\lambda_{t_R} v \left(\frac{\Lambda}{\Lambda_{UV}}\right)^{d_L+d_R-5}$$

$|\text{SM}\rangle = \cos\varphi\,|\text{elementary}\rangle + \sin\varphi\,|\text{composite}\rangle$

Better: alleviates the 4-fermion operators



Sort of GIM protection

$(\bar{q}q)^2\frac{\sin^4\varphi}{M_*^2}$

Fermion masses: two approaches

Both cases:

- SM couplings break the full global symmetry G of the strongly coupled theory;
- The breaking can be modeled by spurion fields, formally transforming under G;
- This amounts to embedding the SM fermions into an incomplete multiplet of a representation of G

The MCHM: $SO(5)/SO(4)$

Look among the smallest irreps of $SO(5)$ (SM embedding)

$$g_{hff}/g_{hff}^{SM} = \begin{cases} \sqrt{1-\epsilon^2} & \text{spinorial } \mathbf{4} \text{ of } SO(5) \\ \dfrac{1-2\epsilon^2}{\sqrt{1-\epsilon^2}} & \text{vectorial } \mathbf{5} \text{ of } SO(5) \end{cases}$$

[Agashe et al.]

Elementary fermions
5 of $SO(5)$

$$q_L^5 = \frac{1}{\sqrt{2}}\begin{pmatrix} id_L \\ d_L \\ iu_L \\ -u_L \\ 0 \end{pmatrix}$$

$$u_R^5 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ u_R \end{pmatrix}$$

Composite fermions
5 of $SO(5)$

$$\begin{aligned} \psi &= \begin{pmatrix} Q \\ \tilde{U} \end{pmatrix} \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix} iD - iX_{5/3} \\ D + X_{5/3} \\ iU + iX_{2/3} \\ -U + X_{2/3} \\ \sqrt{2}\tilde{U} \end{pmatrix} \end{aligned}$$

We need extra $U(1)_X$ to reproduce quark charges. New vector-like composite fermions with $X = 2/3$ charge

	U	$X_{2/3}$	D	$X_{5/3}$	$\tilde{U}$
$SO(4)$	$\mathbf{4}$	$\mathbf{4}$	$\mathbf{4}$	$\mathbf{4}$	$\mathbf{1}$
$U(1)_Q$	$2/3$	$2/3$	$-1/3$	$5/3$	$2/3$

$$\mathcal{L} = \mathcal{L}_{comp} + \mathcal{L}_{mix} + \mathcal{L}_{elem}$$

$$-\mathcal{L} = \left(M_1 e^{i\phi} \overline{\tilde{T}}_L \tilde{T}_R + M_4 \overline{\tilde{Q}}_L \tilde{Q}_R \right) + y_{L4} f \, \overline{q^5}^I_L U_{Ii} Q^i_R + y_{L1} f \, \overline{q^5}^I_L U_{I5} \tilde{T}_R$$
$$+ y_{R4} f \, \overline{t^5}^I_R U_{Ii} Q^i_L + y_{R1} f \, \overline{t^5}^I_R U_{I5} \tilde{T}_L + \text{h.c.}$$

$$m_t \simeq f \sin \frac{2\langle h \rangle}{f} \frac{f \left| y_{L1} y_{R1} M_4 - e^{i\phi} y_{L4} y_{R4} M_1 \right|}{2\sqrt{2} M_T M_{\tilde{T}}} \,, \quad D_M \simeq \text{diag}(M_4, M_T, M_{\tilde{T}})$$

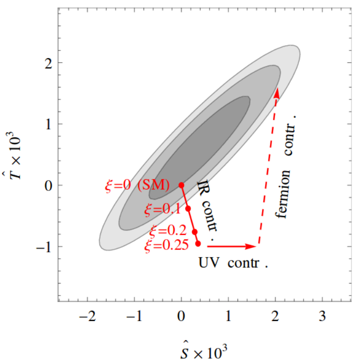
$$M_T = \sqrt{M_4^2 + f^2 y_{L4}^2}, \; M_{\tilde{T}} = \sqrt{M_1^2 + f^2 y_{R1}^2}$$

EW constraints

Non-renormalizable theory, log div. from Goldstone bosons is no longer canceled by the physical Higgs

$$\Delta\hat{S} = \frac{g^2}{192\pi^2}\xi \ln\frac{m_\rho^2}{m_H^2} \simeq 1.4\times 10^{-3}\xi$$

$$\Delta\hat{T} = -\frac{3g'^2}{64\pi^2}\xi \ln\frac{m_\rho^2}{m_H^2} \simeq -3.8\times 10^{-3}\xi$$

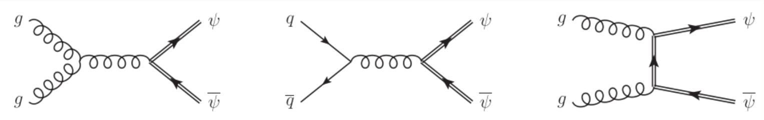


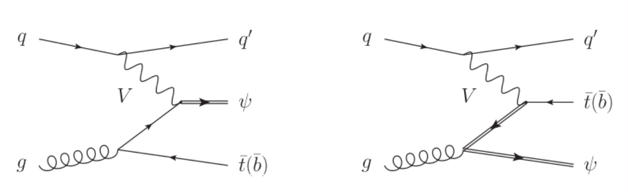
Vector resonances: $\Delta\hat{S} \simeq m_W^2\left(\frac{1}{m_\rho^2}+\frac{1}{m_a^2}\right)$

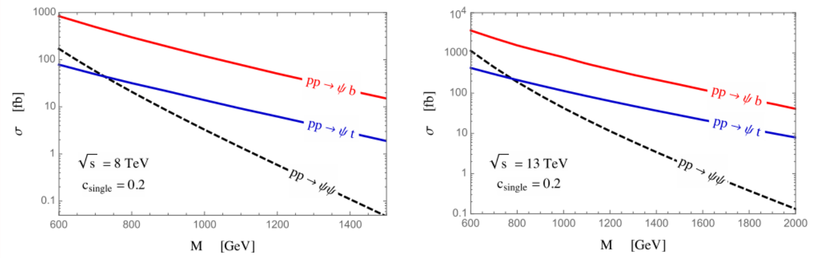
Fermion resonances: $\Delta\hat{S} = \frac{g^2 N_c}{24\pi^2}(1-c_L^2-c_R^2)\xi\ln\frac{m_\rho^2}{m_4^2}$

$$\Delta\hat{T} = \frac{N_c}{16\pi^2}\frac{y_L^4 f^2}{m^2}\xi \sim \frac{N_c}{16\pi^2}y_t^2\xi \sim 2\times 10^{-2}\xi$$

Top partners



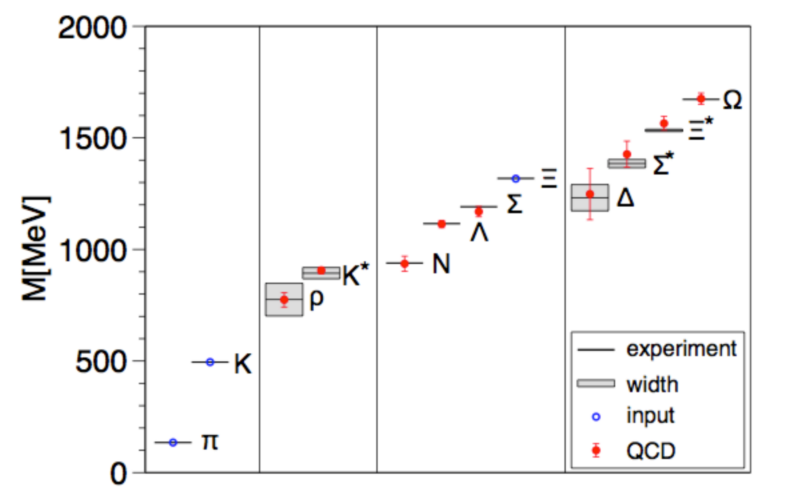




[1506.01961]

Light composite fermionic partners?

Can we have light fermions (baryons) in composite models? Not in QCD!



[1412.6393]

't Hooft anomaly matching condition

Spontaneously broken chiral symmetries $\Rightarrow$ massless (Goldstone) bosons

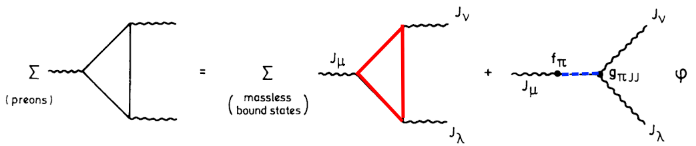
Unbroken chiral symmetries $\Rightarrow$ massless fermions? ['t Hooft 80]

- Chiral group G;
- Strongly coupled dynamics (maybe confinement) at Λ scale;
- $E \gg \Lambda$: elementary fermions;
- $E \sim \Lambda$: bound states. Most are massive ($\sim \Lambda$), some massless;
- $E \ll \Lambda$: Massive state decouple, only massless remain;

't Hooft anomaly matching condition

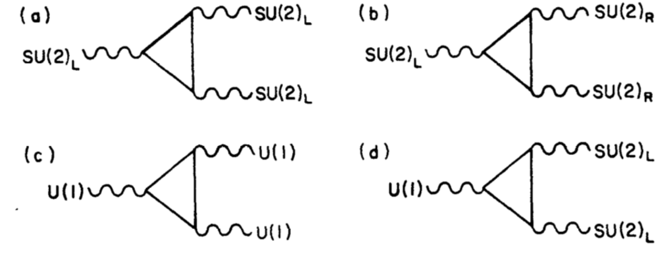
Now:

- "Weakly gauge" chiral symmetry;
- Chiral currents may be anomalous, i.e. $\partial_\mu j^\mu_{\text{chiral}} \neq 0$;
- If so, invent some new fermions χ that only couple to chiral gauge bosons and cancel anomalies;
- At $E \ll \Lambda$ only chiral gauge bosons, fermions χ and massless bound states are present;
- Nothing has happened from high energies down to low energies to spoil gauge symmetry;
- The anomaly contribution from massless bound states must be the same as the free theory fermions, otherwise the symmetry must be spontaneously broken.



[Frishman,Schwimmer,Banks,Yankielowicz 81] [Coleman,Gross 82]

QCD with 2 light flavours



- $(a) \equiv 0$; $(b) \equiv 0$; $(c) = 0$; $(d) \propto \text{Tr}[Q_B T_a T_b] = \sum_r n_r Q_B(r) C(r)$

	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_B$	$U(1)[SU(2)L]^2$
q_L	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{1}$	$1/3$	$3 \times (1/3) \times (1/2)$
q_R	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2}$	$1/3$	$3 \times (1/3) \times (0)$
$B_L \equiv q_L q_R q_R$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	1	$1 \times (1) \times (1/2)$
$B_R \equiv q_L q_L q_R$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	1	$1 \times (1) \times (0)$

In 2 flavour QCD we could have massless Baryonic states. Not true for $N_c \geq 3$!

Underline theories

Building an underlying theory that contains both a composite Higgs and composite top partners is not an easy task, as many conditions need to be satisfied: [Ferretti, Karateev]

- Simple hypercolor group (G_{HC})
- Asymptotically free theories
- Absence of gauge anomalies and Witten's global anomalies
- Symmetry breaking pattern: $G_F \to H_F \supset C_{cus} \supset G_{SM}$
- The most attractive channel (MAC) should not break neither G_{HC} nor G_{cus}
- $G/H \ni (\mathbf{1},\mathbf{2},\mathbf{2})_0$ of G_{cus}. (the Higgs boson)
- Fermionic hypercolor singlets $\in (\mathbf{3},\mathbf{2})_{1/6}$ and $(\mathbf{3},\mathbf{1})_{2/3}$ of G_{SM} (at least 3rd family)
- B and L symmetry

Underline theories

We shall consider models with two chiral fermion species, each with n_i flavours:

Global symmetry: $\boxed{U(n_\psi) \times U(n_\chi)}$

- Colourless ψ, which produce the Higgs as a pNGB, after condensation occurs;
- Colourfull χ, since we want to obtain the top partners.

EW coset

- Complex: $\dfrac{SU(4) \times SU(4)'}{SU(4)_D}$
- Pseudoreal: $\dfrac{SU(4)}{Sp(4)}$
- Real: $\dfrac{SU(5)}{SO(5)}$

Colour coset

- Complex: $\dfrac{SU(3) \times SU(3)'}{SU(3)_D}$
- Pseudoreal: $\dfrac{SU(6)}{Sp(6)}$
- Real: $\dfrac{SU(6)}{SO(6)}$

Underline theories

Coset	HC	ψ	χ	$-q_\chi/q_\psi$	Y_χ	Model
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{SO(6)}$	$SO(7)$	$5\times\mathbf{F}$	$6\times\mathbf{Spin}$	$5/6$	$1/3$	M1
	$SO(9)$	$5\times\mathbf{F}$	$6\times\mathbf{Spin}$	$5/12$	$1/3$	M2
	$SO(7)$	$5\times\mathbf{Spin}$	$6\times\mathrm{F}$	$5/6$	$2/3$	M3
	$SO(9)$	$5\times\mathbf{Spin}$	$6\times\mathrm{F}$	$5/3$	$2/3$	M4
$\frac{SU(5)}{SO(5)} \times \frac{SU(6)}{Sp(6)}$	$Sp(4)$	$5\times\mathbf{A}_2$	$6\times\mathbf{F}$	$5/3$	$1/3$	M5
$\frac{SU(5)}{SO(5)} \times \frac{SU(3)^2}{SU(3)}$	$SU(4)$	$5\times\mathbf{A}_2$	$3\times(\mathbf{F},\overline{\mathbf{F}})$	$5/3$	$1/3$	M6
	$SO(10)$	$5\times\mathbf{F}$	$3\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$5/12$	$1/3$	M7
$\frac{SU(4)}{Sp(4)} \times \frac{SU(6)}{SO(6)}$	$Sp(4)$	$4\times\mathbf{F}$	$6\times\mathbf{A}_2$	$1/3$	$2/3$	M8
	$SO(11)$	$4\times\mathbf{Spin}$	$6\times\mathbf{F}$	$8/3$	$2/3$	M9
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(6)}{SO(6)}$	$SO(10)$	$4\times(\mathbf{Spin},\overline{\mathbf{Spin}})$	$6\times\mathbf{F}$	$8/3$	$2/3$	M10
	$SU(4)$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$6\times\mathbf{A}_2$	$2/3$	$2/3$	M11
$\frac{SU(4)^2}{SU(4)} \times \frac{SU(3)^2}{SU(3)}$	$SU(5)$	$4\times(\mathbf{F},\overline{\mathbf{F}})$	$3\times(\mathbf{A}_2,\overline{\mathbf{A}_2})$	$4/9$	$2/3$	M12

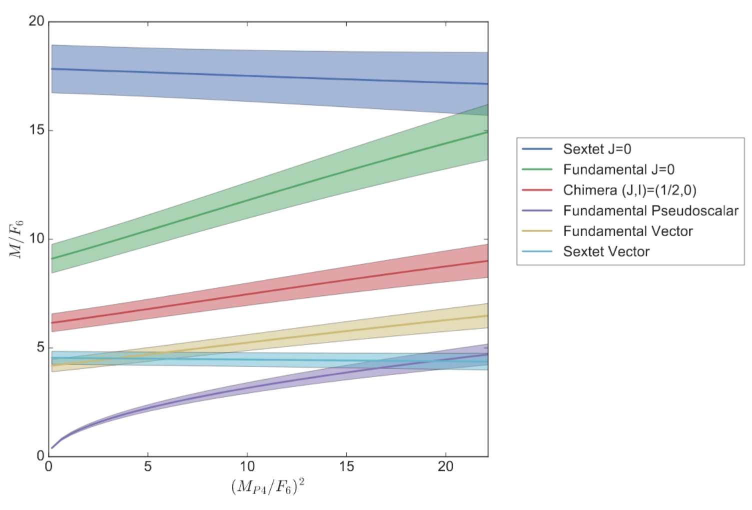
Always 2 $U(1)$s that are spontaneously broken: $U(1)_\psi$, $U(1)_\chi$.
One combination of the two has an anomaly with the G_{HC}

$$U(1)_{\psi,\chi}G^2_{HC} \neq 0 \quad \Rightarrow \quad \boxed{\left[U(1)_\psi + U(1)_\chi\right]} \, G^2_{HC} \neq 0$$

For the anomaly free $U(1)$, associated to the light pNGB

Lattice predictions

First lattice results for model M11 [1801.05809]

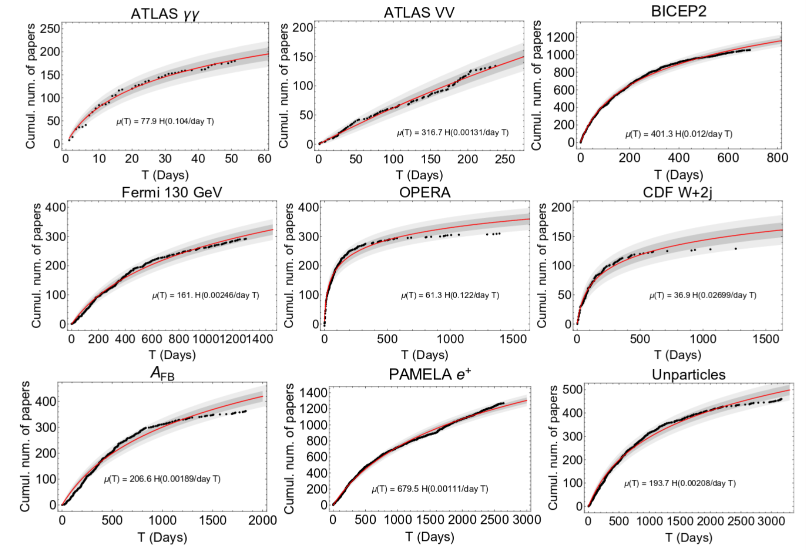


"Chimera"=top-partner not as light as desired!

Final words

Final words

A Theory of Ambulance Chasing [1603.01204]



Final words

"...time evolution of the dynamical system in question is driven only by a few out of a large number of degrees of freedom (in this case the interest in the topic and the number of available ideas)"

"Forecasting the total number of preprints on an ambulance chasing topic could be useful to particle physics journals. The forecast would allow journals to anticipate the load of submissions for publication and hence improve the overall effectiveness of the rejection process."

Positive final note:
SM works beautifully...
but there is still lots of room for NP.
It's the right time to get our hands dirty!