Matching and Merging

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Particle Physics Phenomenology

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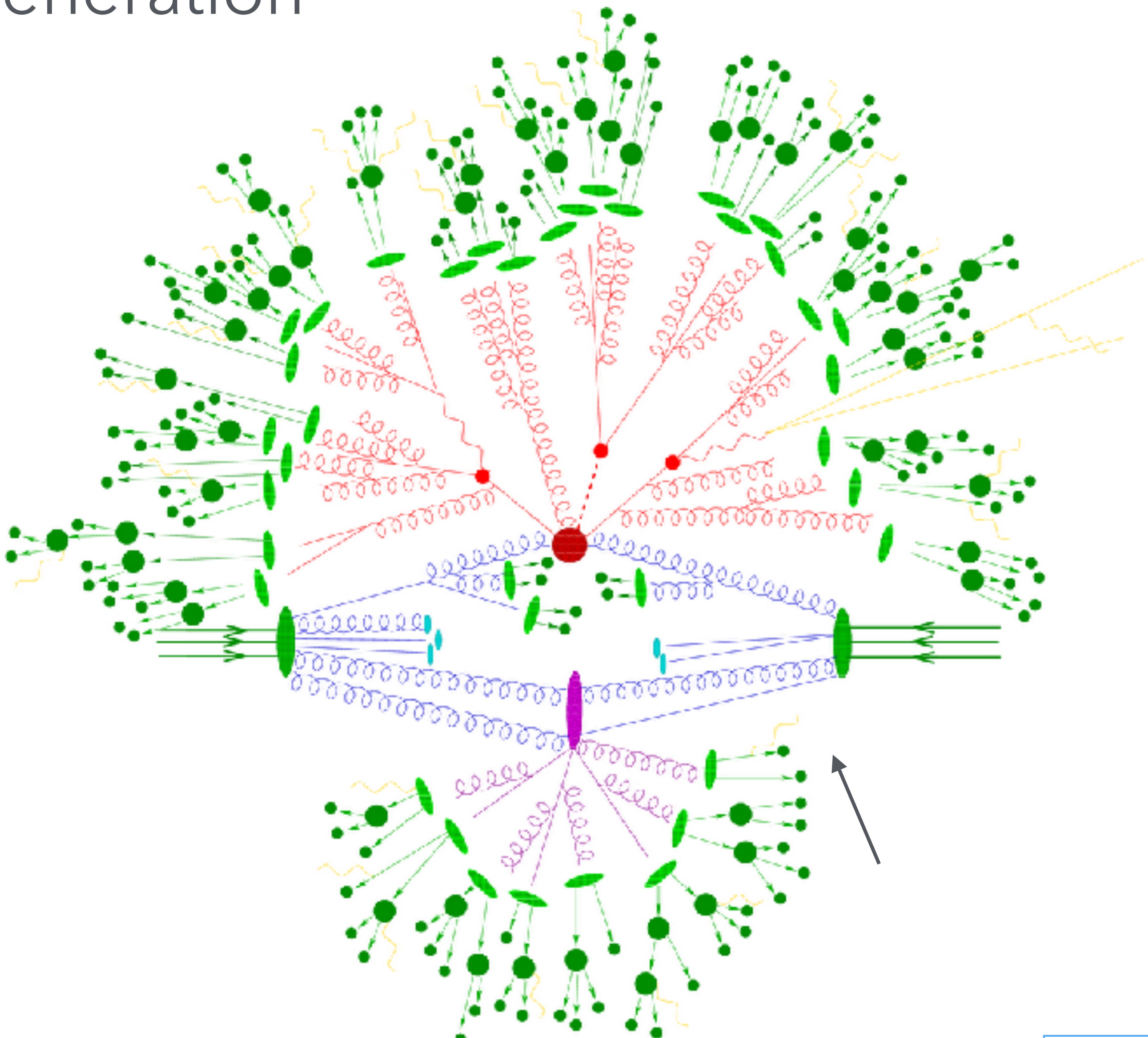
Johannes Bellm , 13.2.2018

Outline

1. Recap on Matrix elements and Parton Shower
2. Scale dependence
3. LO Merging
4. NLO Matching
5. NLO Merging

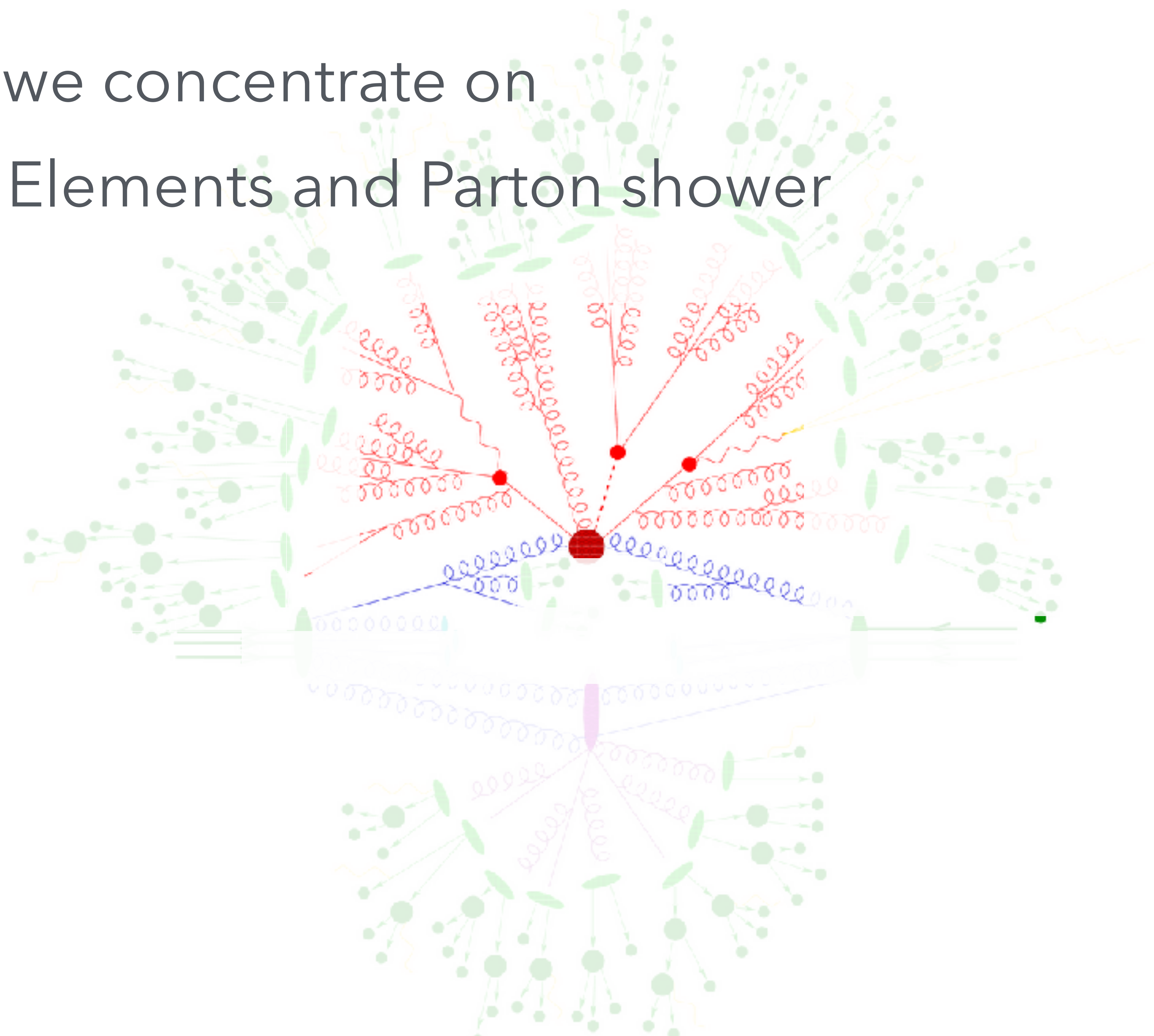
Event Generation



picture: arXiv:1411.4085

Today we concentrate on

Matrix Elements and Parton shower



Recap and Comments

We start with a short recap on matrix elements and parton shower and their connection. To remind you about the terminology and to review.

Warning ⚠️ and apologies:

– In todays lecture many of the formula are simplified versions.

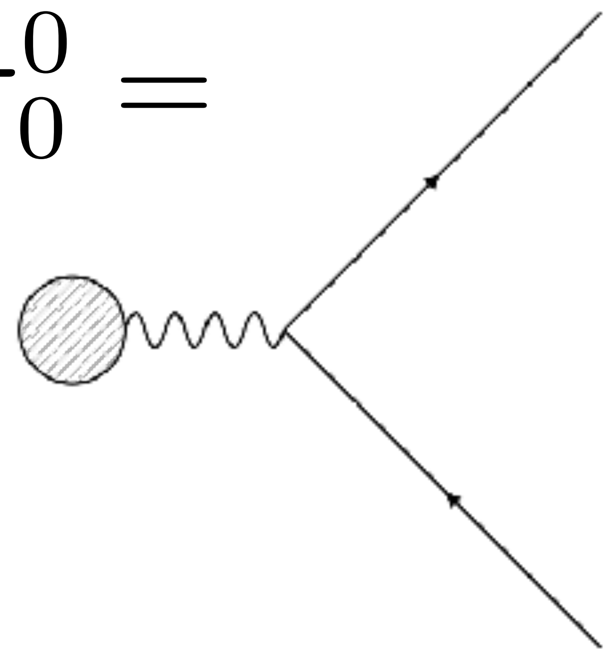
– The historical picture is altered in order to be more pedagogical.

– The references are given in terms of arXiv or inspireHEP record numbers not the Journals.. Links are provided.

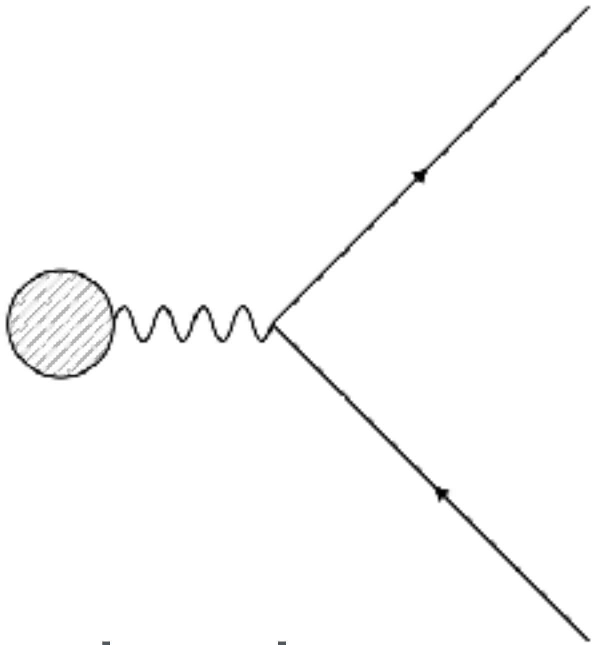
– Green means affiliation to Lund (either work was partially done here or person is currently here).

Start simple

$$d\sigma_0^0 =$$



Start simple

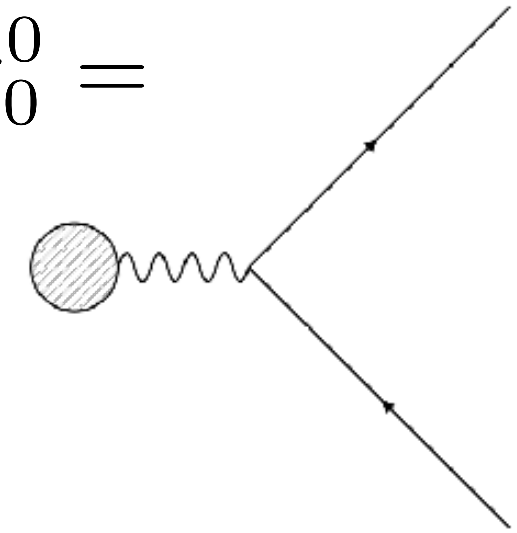


Feynman gave a set of ‚simple' rules
how to get cross sections from Lagrangians
(applying them is complicated)

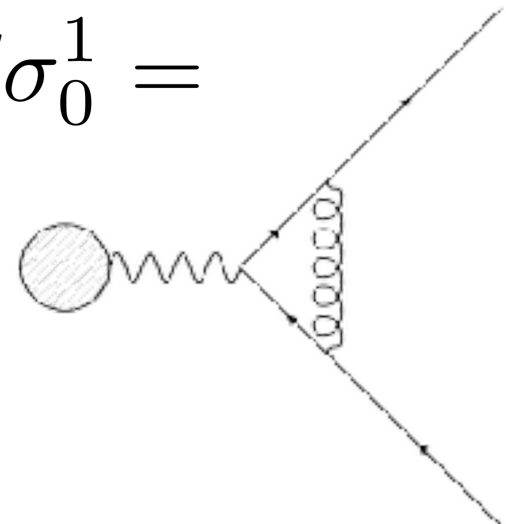
- extract rules from Lagrangian
- draw diagrams and apply rules
- average incoming possibilities
- sum all final possibilities (once)

Next-to-leading order

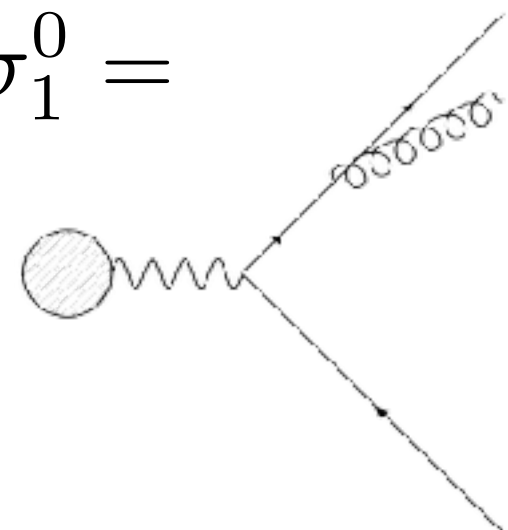
$$d\sigma_0^0 =$$



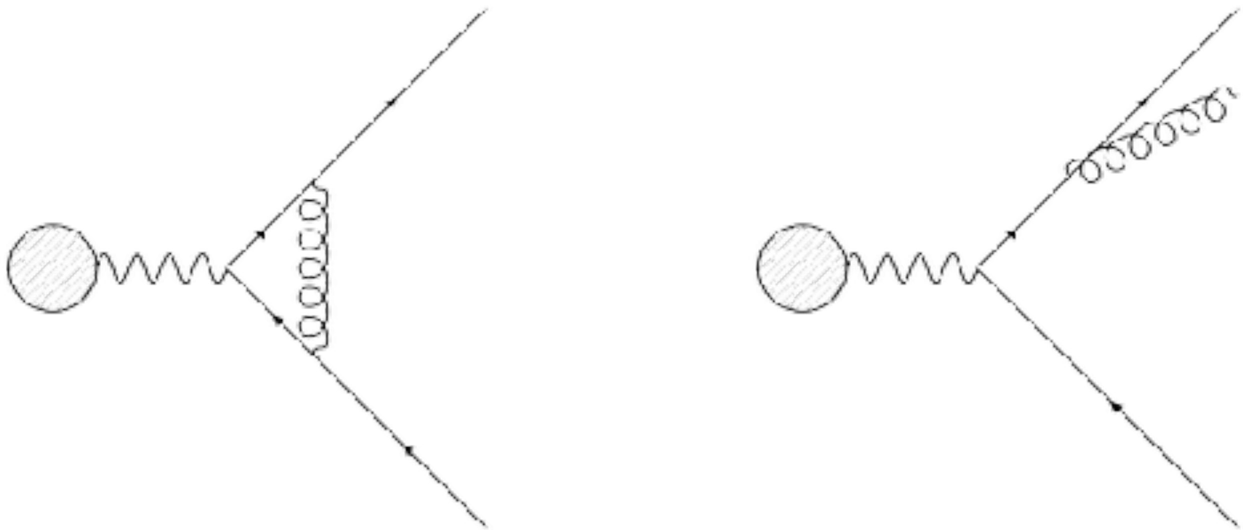
$$d\sigma_0^1 =$$

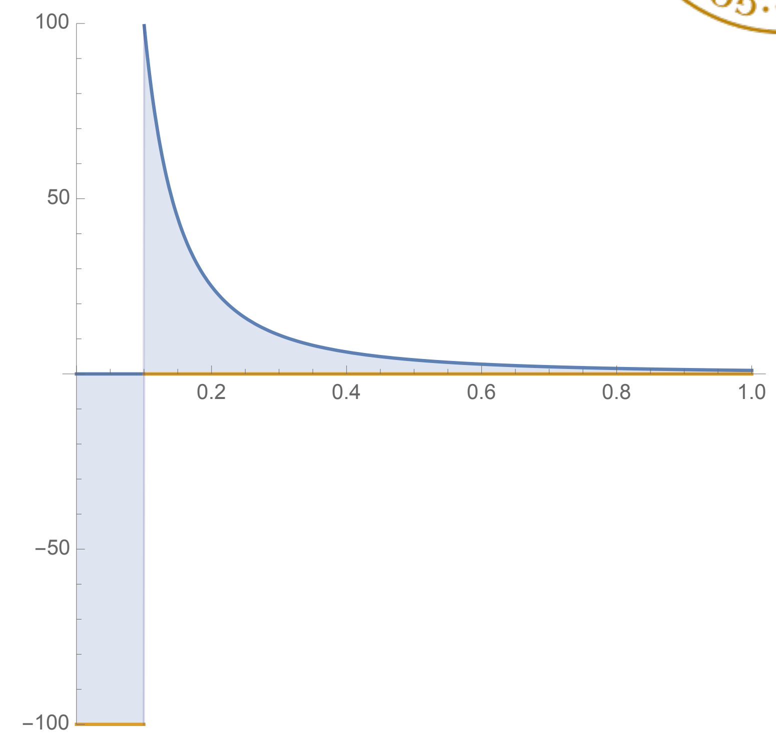


$$d\sigma_1^0 =$$



Next-to-leading order



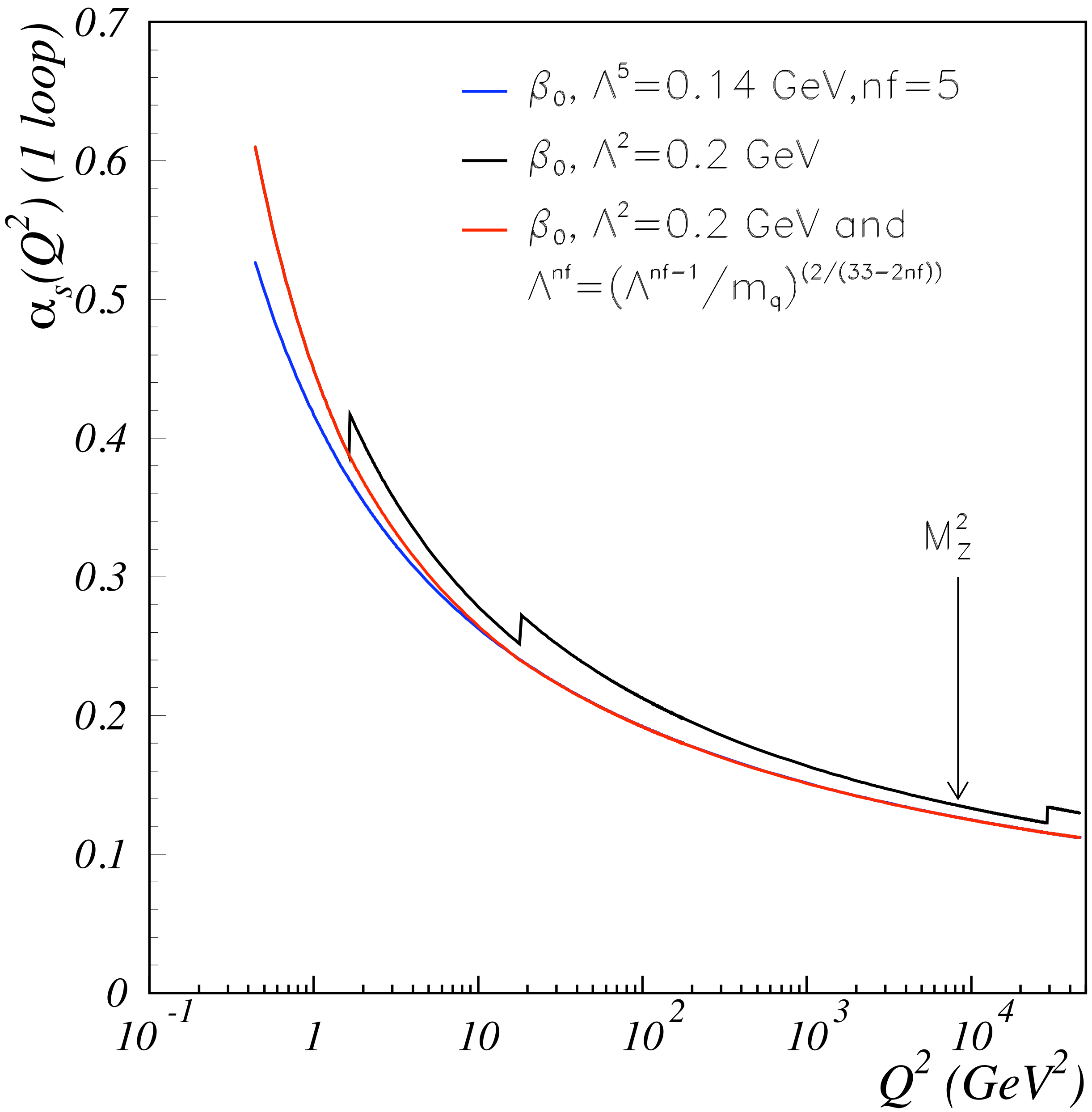


But: Divergences arise!

UV –> Solved by renormalisation –> scale dependent parameters.

IR/C –> Solved by summing all contributions to same order and same observable –> incl. vs. excl. observables !?

Scale Dependent Parameters (UV)



Renormalization Group Equation (RGE):

$$\mu_R^2 \frac{\partial \alpha_S}{\partial \mu_R^2} = \beta(\alpha_S)$$

$$\beta(\alpha_S) = -\alpha_S^2(b_0 + b_1\alpha_S + b_2\alpha_S^2 + \dots)$$

$$b_0 = \frac{11C_A - 2n_f}{12\pi}$$

$$\alpha_S(q^2) = \frac{\alpha_S(\mu_R^2)}{1 + \alpha_S(\mu_R^2) b_0 \log \frac{q^2}{\mu_R^2}}$$

– "Running Coupling"
– If nf < 17 –> asymptotic freedom.

Nobel Price 2004: record/81238 and record/81351, Video

Review on QCD running arXiv:1604.08082

Scale Dependent Cross Sections (UV)

NLO cross section dependence on renormalisation scale:

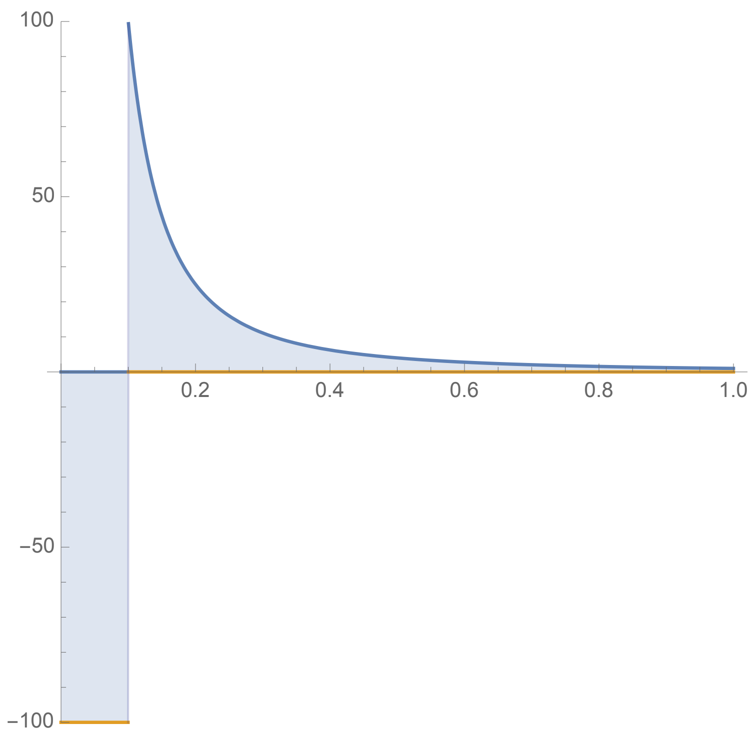
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi} = \overset{d\sigma_0^0 =}{\alpha_{\mathrm{S}}^{N}(\mu_{\mathrm{R}})\, B} + \overset{d\sigma_0^1 =}{\alpha_{\mathrm{S}}^{N+1}(\mu_{\mathrm{R}}) \left[V + N\, b_0\, \log \frac{\mu_{\mathrm{R}}^2}{Q^2}\, B \right]} + \overset{d\sigma_1^0 =}{\alpha_{\mathrm{S}}^{N+1}(\mu_{\mathrm{R}})\, R}$$

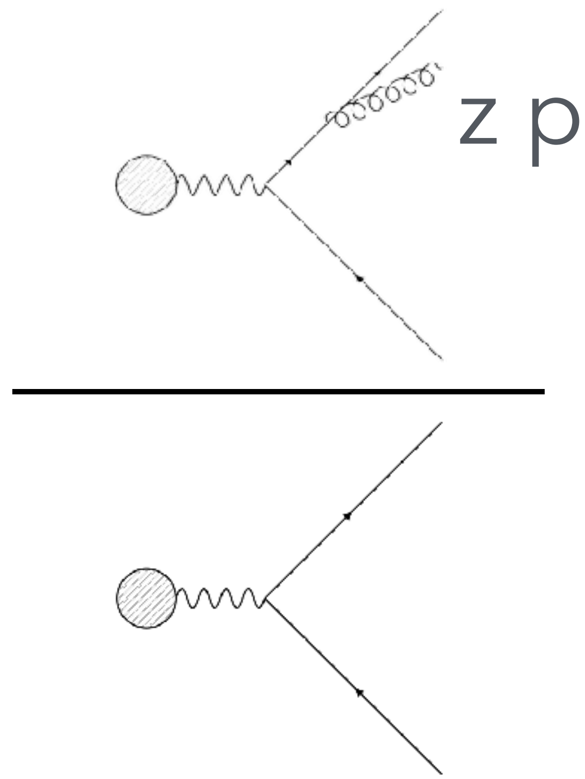
– renormalisation scale dependence moved to $\mathcal{O}(\alpha_S^{N+2})$
– Q is process dependent
– Large correction if scale chosen poorly at LO
– No universal recipe

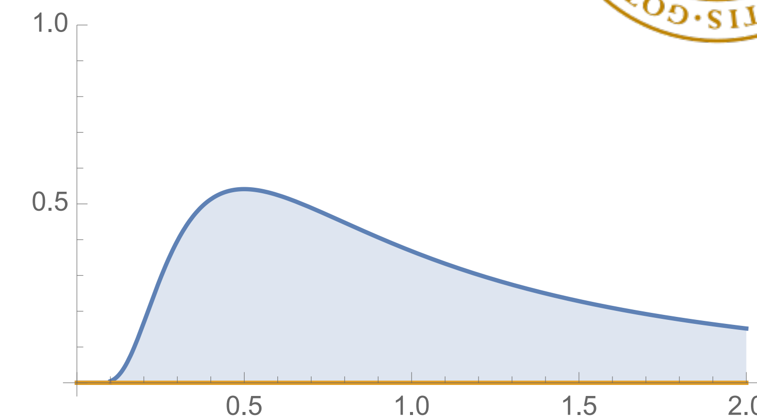
Warning: Other sources of large corrections
e.g. new channels.

MINLO arXiv:1206.3572

Construct (no-)emission probability (IR/C)







$$\mathrm{d}\mathcal{P}_{\tilde{ij}\to ij} = \frac{\alpha_S}{2\pi}\frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\,\mathrm{d}z\,P_{\tilde{ij}\to ij}\left(z,\tilde{q}\right)$$

$$P_{q\to qg}(z) = \frac{\alpha_S}{2\pi} C_F \frac{1+z^2}{1-z}$$

$$\mathcal{P}\left(\text{resolved}\right) + \mathcal{P}\left(\text{unresolved}\right) = 1$$

$$\Delta_{\tilde{ij}\to ij}\left(\tilde{q},\tilde{q}_h\right) = \exp\left\{-\int_{\tilde{q}}^{\tilde{q}_h}\frac{\mathrm{d}\tilde{q}'^2}{\tilde{q}'^2}\int \mathrm{d}z\,\frac{\alpha_S\left(z,\tilde{q}'\right)}{2\pi}\,P_{\tilde{ij}\to ij}\left(z,\tilde{q}'\right)\Theta\left(\mathbf{p}_\perp^2>0\right)\right\}$$

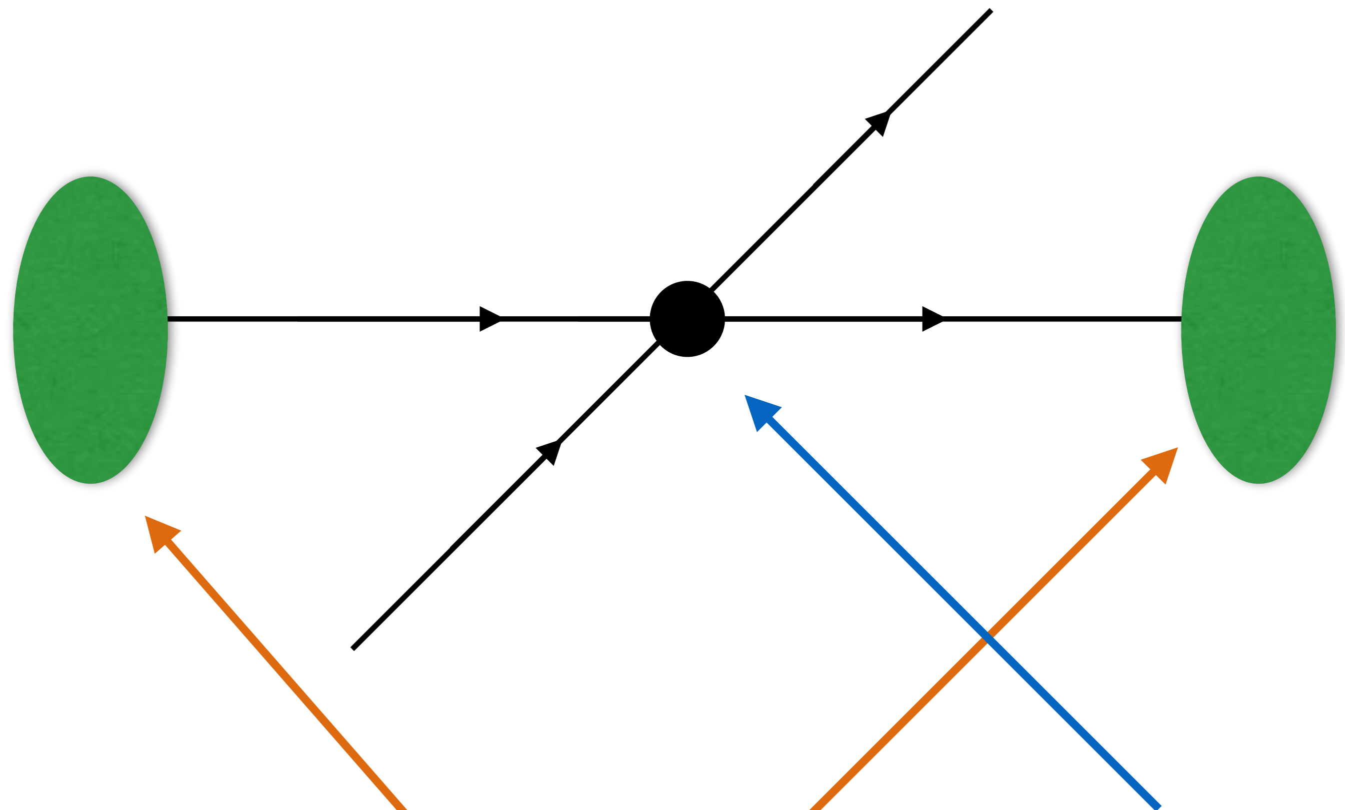
$$\Delta\left(\tilde{q},\tilde{q}_h\right) = \mathcal{R}$$

(Over-) simplified parton shower:

1. Factorize ME and phase space to get splitting probability. (IR and C Approximation)
2. Use radiative decay formula to exponentiate and get no emission probability.

Another Collinear Problem

Another factorization (incoming hadrons):



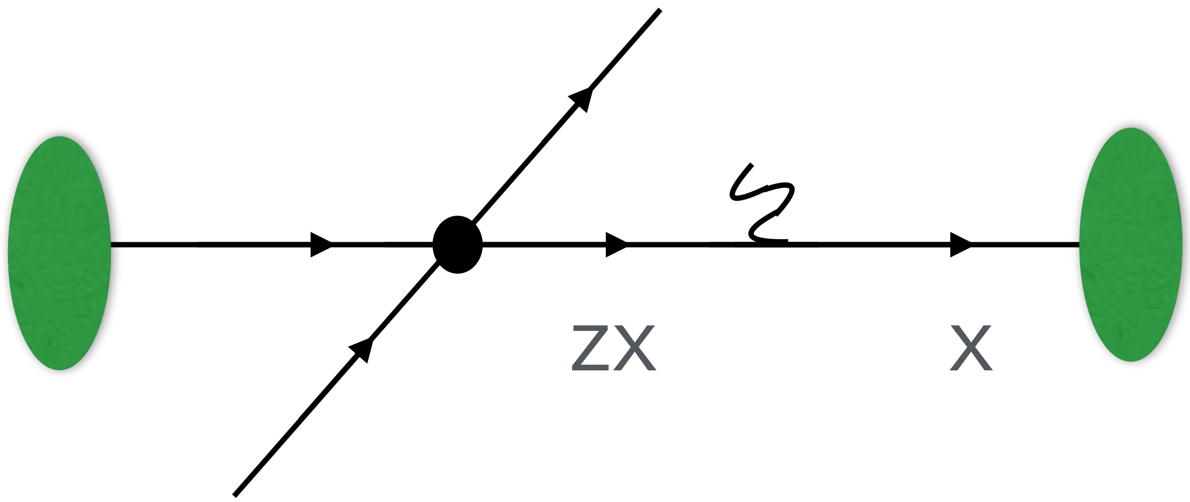
$$\mathrm{d}\sigma_{h_1 h_2} = \sum_{i,j} \int_0^1 \mathrm{d}x_i \int_0^1 \mathrm{d}x_j \sum_f \int \mathrm{d}\Phi_f \, f_{i/h_1}(x_i, \mu_F^2) \, f_{j/h_2}(x_j, \mu_F^2) \, \frac{\mathrm{d}\hat{\sigma}_{ij\to f}}{\mathrm{d}x_i \, \mathrm{d}x_j \, \mathrm{d}\Phi_f}$$

Protons are complicated, composite, non-perturbative objects

Factories cross section calculation into PDF and ME

Another Collinear Problem

NLO:

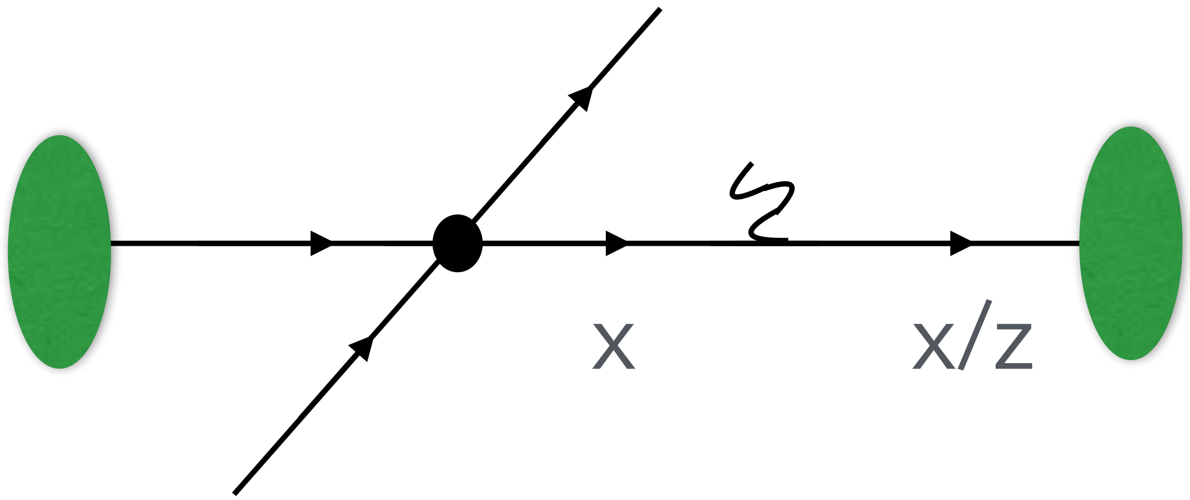


$$V \sim dx f(x) \frac{dq^2}{q^2} dz P(z) B(x)$$

$$R \sim dx f(x) \frac{dq^2}{q^2} dz P(z) B(zx)$$

Mismatch between virtual and real correction…

Another Collinear Problem



$$V \sim dx f(x) \frac{dq^2}{q^2} dz P(z) B(x)$$

$$R \sim dx f(x/z) \frac{dq^2}{q^2} \frac{dz}{z} P(z) B(x)$$

Redifine PDFs, here with cutoff (not usual):

$$f_B(x) = f(x, \mu_F) - \frac{\alpha_S}{2\pi} \int_0^{\mu_F^2} \frac{dq^2}{q^2} \int_0^1 dz \left[f(x/z) P(z)/z - f(x) P(z) \right]$$

Collinear divergency needs renormalisation of PDF to absorb the remaining part (universal).

I did not specify **q**, which is scheme dependent.
Usually the integration of the virtual part is even performed in d dimensions leaving behind ~ $\frac{1}{\epsilon} (\frac{\mu_F^2}{\mu^2})^\epsilon = \frac{1}{\epsilon} e^{\epsilon \log \frac{\mu_F^2}{\mu^2}}$

e.g. Eq. 6.6 in Catani, Seymour arXiv:9605323

DGLAP

Scale dependent PDFs:

$$\frac{\partial f(x,\mu_F^2)}{\partial \log \mu_F^2} = \frac{\alpha_S}{2\pi}\int \frac{dz}{z}\left[f(x/z)P_+(z)\right] + \mathcal{O}(\alpha_S^2)$$

D: inspirehep:126153

GL: inspirehep:73449

AP: inspirehep:119585

For parton showers this in the end means can be rewritten to:

$$\Delta_{\widetilde{ij}\to ij}(x,\tilde{q},\tilde{q}_h) = \exp\left\{-\int_{\tilde{q}}^{\tilde{q}_h}\frac{\mathrm{d}\tilde{q}'^2}{\tilde{q}'^2}\int_x^{z_+}\mathrm{d}z\,\frac{\alpha_S(z,\tilde{q}')}{2\pi}\,P_{\widetilde{ij}\to ij}(z,\tilde{q}')\boxed{\frac{\frac{x}{z}f_{\widetilde{ij}}\left(\frac{x}{z},\tilde{q}'\right)}{x f_i(x,\tilde{q}')}}\Theta\left(\mathbf{p}_\perp^2>0\right)\right\}$$

Initial State Showers inspirehep:212950

From DGLAP to initial state showers

$$\Delta(t) = \exp\left[-\int_{t_0}^{t}\int dz \frac{\alpha_S}{2\pi} P(z)\right] \qquad t = q^2$$

$$t\frac{\partial}{\partial t} f(x,t) = \int \frac{dz}{z}\frac{\alpha_S}{2\pi} P(z) f(x/z,t) + \frac{f(x,t)}{\Delta(t)} t\frac{\partial}{\partial t}\Delta(t)$$

$$t\frac{\partial}{\partial t}\left(\frac{f(x,t)}{\Delta}\right) = \frac{1}{\Delta}\int \frac{dz}{z}\frac{\alpha_S}{2\pi} P(z) f(x/z,t)$$

$$f(x,t) = \frac{\Delta(t)}{\Delta(t_0)} f(x,t_0) + \int_{t_0}^{t} \frac{dt'}{t'}\frac{\Delta(t)}{\Delta(t')}\int \frac{dz}{z}\frac{\alpha_S}{2\pi} P(z) f(x/z,t')$$

$$1 = \boxed{\frac{\Delta(t)}{\Delta(t_0)}\frac{f(x,t_0)}{f(x,t)}} + \int_{t_0}^{t} \frac{dt'}{t'}\int \frac{dz}{z}\frac{\alpha_S}{2\pi} P(z) \frac{f(x/z,t')}{f(x,t')}\boxed{\frac{\Delta(t)}{\Delta(t')}\frac{f(x,t')}{f(x,t)}}$$

From Ellis , Stirling, Webber, “QCD and Colliders“

Summary 1

1. Divergences at NLO
2. Divergences transformed in
   – scale dependent parameters
   – or a IR/C save observable definition.
3. "Wrong" scale choice at LO is partially cured at NLO
4. Parton Showers use IR/C approx. to mimic multiple higher orders.

Can we improve the description?

Yes we can! Correct IR/C approximation with actual matrix elements!
And possibly higher orders (loops).

Problem: The parton shower already approximates both.
Solution: Avoid double counting of similar contributions.

Merging: Calculate the higher multiplicities with merging scale
but multiply with no emission probability of shower.

Matching: Expand shower to first order in coupling.
Subtract from NLO configuration. Start shower.

Matching and Merging, whats the difference??

Loops and Legs

If we could calculate everything, we would. But…

Loops

$$
\begin{array}{ccccccccc}
\mathcal{O}(\alpha_S^{N}) & & \mathcal{O}(\alpha_S^{N+1}) & & \mathcal{O}(\alpha_S^{N+2}) & & \mathcal{O}(\alpha_S^{N+3}) & & \\
d\sigma_0^0 & + & d\sigma_0^1 & + & d\sigma_0^2 & + & d\sigma_0^3 & + & \ldots \\
 & & d\sigma_1^0 & + & d\sigma_1^1 & + & d\sigma_1^2 & + & \ldots \\
 & & & & d\sigma_2^0 & + & d\sigma_2^1 & + & \ldots \\
 & & & & & & d\sigma_3^0 & + & \ldots \\
 & & & & & & & & \ldots
\end{array}
$$

Legs

Loops and Legs

Todays standards:

Loops

$$\begin{array}{cccc}
\mathcal{O}(\alpha_S^N) & \mathcal{O}(\alpha_S^{N+1}) & \mathcal{O}(\alpha_S^{N+2}) & \mathcal{O}(\alpha_S^{N+3}) \\
d\sigma_0^0 & + \, d\sigma_0^1 & + \, d\sigma_0^2 & + \, d\sigma_0^3 & + \ldots \\
 & d\sigma_1^0 & + \, d\sigma_1^1 & + \, d\sigma_1^2 & + \ldots \\
 & & d\sigma_2^0 & + \, d\sigma_2^1 & + \ldots \\
 & & & d\sigma_3^0 & + \ldots \\
 & & & & \ldots
\end{array}$$

Legs

Automated (~199?)

Automated SM (~200?)

Most 2->2

pp ->jj (~'16)

Some inclusive processes gg->h

Loops and Legs

Parton shower approximation:

"Loops"

$$
\begin{array}{llllll}
\mathcal{O}(\alpha_S^N) & \mathcal{O}(\alpha_S^{N+1}) & \mathcal{O}(\alpha_S^{N+2}) & \mathcal{O}(\alpha_S^{N+3}) & & \\
d\sigma_0^0 & \Delta_\mu^Q & & & & + \\
d\sigma_0^0 & \Delta_{q_1}^Q P_1 & \Delta_\mu^{q_1} & & & + \\
d\sigma_0^0 & \Delta_{q_1}^Q P_1 & \Delta_{q_2}^{q_1} & P_2 \Delta_\mu^{q_2} & & + \\
d\sigma_0^0 & \Delta_{q_1}^Q P_1 & \Delta_{q_2}^{q_1} & P_2 \Delta_{q_3}^{q_2} & P_3 \Delta_\mu^{q_3} & + \\
 & & & & & \ldots
\end{array}
$$

"Legs"

$$P_1 \approx \frac{d\sigma_1^0}{d\sigma_0^0} \qquad \Delta_\mu^Q + \int_\mu^Q dq_1 \Delta_{q_1}^Q P_1(q_1) = 1$$

Loops and Legs

Transform to LO merging: $$P_1 \approx \frac{d\sigma_1^0}{d\sigma_0^0}$$

"Loops"

$\mathcal{O}(\alpha_S^N)$ $\mathcal{O}(\alpha_S^{N+1})$ $\mathcal{O}(\alpha_S^{N+2})$ $\mathcal{O}(\alpha_S^{N+3})$

$$d\sigma_0^0 \;\; \Delta_\mu^Q \; +$$

$$d\sigma_0^0 \;\; \Delta_{q_1}^Q \frac{d\sigma_1^0}{d\sigma_0^0} \;\; \Delta_\mu^{q_1} \; +$$

$$d\sigma_0^0 \;\; \Delta_{q_1}^Q \frac{d\sigma_1^0}{d\sigma_0^0} \;\; \Delta_{q_2}^{q_1} \;\; \frac{d\sigma_2^0}{d\sigma_1^0} \Delta_\mu^{q_2} \; +$$

$$d\sigma_0^0 \;\; \Delta_{q_1}^Q \frac{d\sigma_1^0}{d\sigma_0^0} \;\; \Delta_{q_2}^{q_1} \;\; \frac{d\sigma_2^0}{d\sigma_1^0} \Delta_{q_3}^{q_2} \;\; \frac{d\sigma_3^0}{d\sigma_2^0} \Delta_\mu^{q_3} \; +$$

$$\cdots$$

"Legs"

LO Merging

"Loops"

$$
\begin{array}{ccccccccc}
\mathcal{O}(\alpha_S^N) & & \mathcal{O}(\alpha_S^{N+1}) & & \mathcal{O}(\alpha_S^{N+2}) & & \mathcal{O}(\alpha_S^{N+3}) & & \\
d\sigma_0^0 & \Delta_\mu^Q & & & & & & & + \\
 & \Delta_{q_1}^Q & d\sigma_1^0 & \Delta_\mu^{q_1} & & & & & + \\
 & \Delta_{q_1}^Q & & \Delta_{q_2}^{q_1} & d\sigma_2^0 & \Delta_\mu^{q_2} & & & + \\
 & \Delta_{q_1}^Q & & \Delta_{q_2}^{q_1} & & \Delta_{q_3}^{q_2} & d\sigma_3^0 & \Delta_\mu^{q_3} & + \\
 & & & & & & & & \dots
\end{array}
$$

Legs
and
"Legs"

MLM arXiv:0108069

CKKW arXiv:0109231

CKKW-L arXiv:0112284

LO Merging

Step back: LO Merging with one additional leg:

“Loops”

$$
\begin{array}{lllllll}
\mathcal{O}(\alpha_S^N) & & \mathcal{O}(\alpha_S^{N+1}) & & \mathcal{O}(\alpha_S^{N+2}) & \mathcal{O}(\alpha_S^{N+3}) & \\
d\sigma_0^0 & \Delta_\mu^Q & & & & & + \\
 & \Delta_{q_1}^Q d\sigma_1^0 & \Delta_\mu^{q_1} & & & & + \\
 & \Delta_{q_1}^Q d\sigma_1^0 & \Delta_{q_2}^{q_1} & P_2\Delta_\mu^{q_2} & & & + \\
 & \Delta_{q_1}^Q d\sigma_1^0 & \Delta_{q_2}^{q_1} & P_2\Delta_{q_3}^{q_2} & P_3\Delta_\mu^{q_3} & & + \\
 & & & & & & \ldots
\end{array}
$$

Legs
and
“Legs”

LO Merging

LO Merging with one additional leg:

"Loops"

$$\begin{array}{ccccc}
\mathcal{O}(\alpha_S^N) & \mathcal{O}(\alpha_S^{N+1}) & \mathcal{O}(\alpha_S^{N+2}) & \mathcal{O}(\alpha_S^{N+3}) & \\
d\sigma_0^0 \;\; \Delta_\mu^Q & & & & + \\
\boxed{\Delta_{q_1}^Q \, d\sigma_1^0} & \Delta_\mu^{q_1} & & & + \\
\Delta_{q_1}^Q \, d\sigma_1^0 & \Delta_{q_2}^{q_1} & P_2 \Delta_\mu^{q_2} & & + \\
\Delta_{q_1}^Q \, d\sigma_1^0 & \Delta_{q_2}^{q_1} & P_2 \Delta_{q_3}^{q_2} & P_3 \Delta_\mu^{q_3} & + \\
 & & & & \cdots
\end{array}$$

Legs
and
"Legs"

LO Merging

Lets discuss $\boxed{\Delta^{Q}_{q_1} d\sigma^0_1}$ a bit closer.

The shower produces the emission and no-emission simultaneously.
$d\sigma^0_1$ corrects and produces the emission probability.

So we need an expression for no-emission probability.

Idea: Get it from analytic resumation.

In e+e- collisions with only final state emission the jet structure and no-emission probability was calculated for clustering partons with measure:

$$y_{kl} = 2(1 - \cos\theta_{kl}) \min(E_k^2, E_l^2)/s$$

Will come back: 1991: kT Algorithm inspirehep:317695

LO Merging

Lets discuss $\boxed{\Delta^{Q}_{q_1} d\sigma^0_1}$ a bit closer.

The result with $y_{cut} = q^2/Q^2$

q –> q g: $$-\log \Delta^{Q}_{q_1} \approx \int_{q_1^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi} C_F \left(\log \frac{Q^2}{q^2} - \frac{3}{2} \right)$$

g –> g g: $$-\log \Delta^{Q}_{q_1} \approx \int_{q_1^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi} C_A \left(\log \frac{Q^2}{q^2} - \frac{11}{6} \right)$$

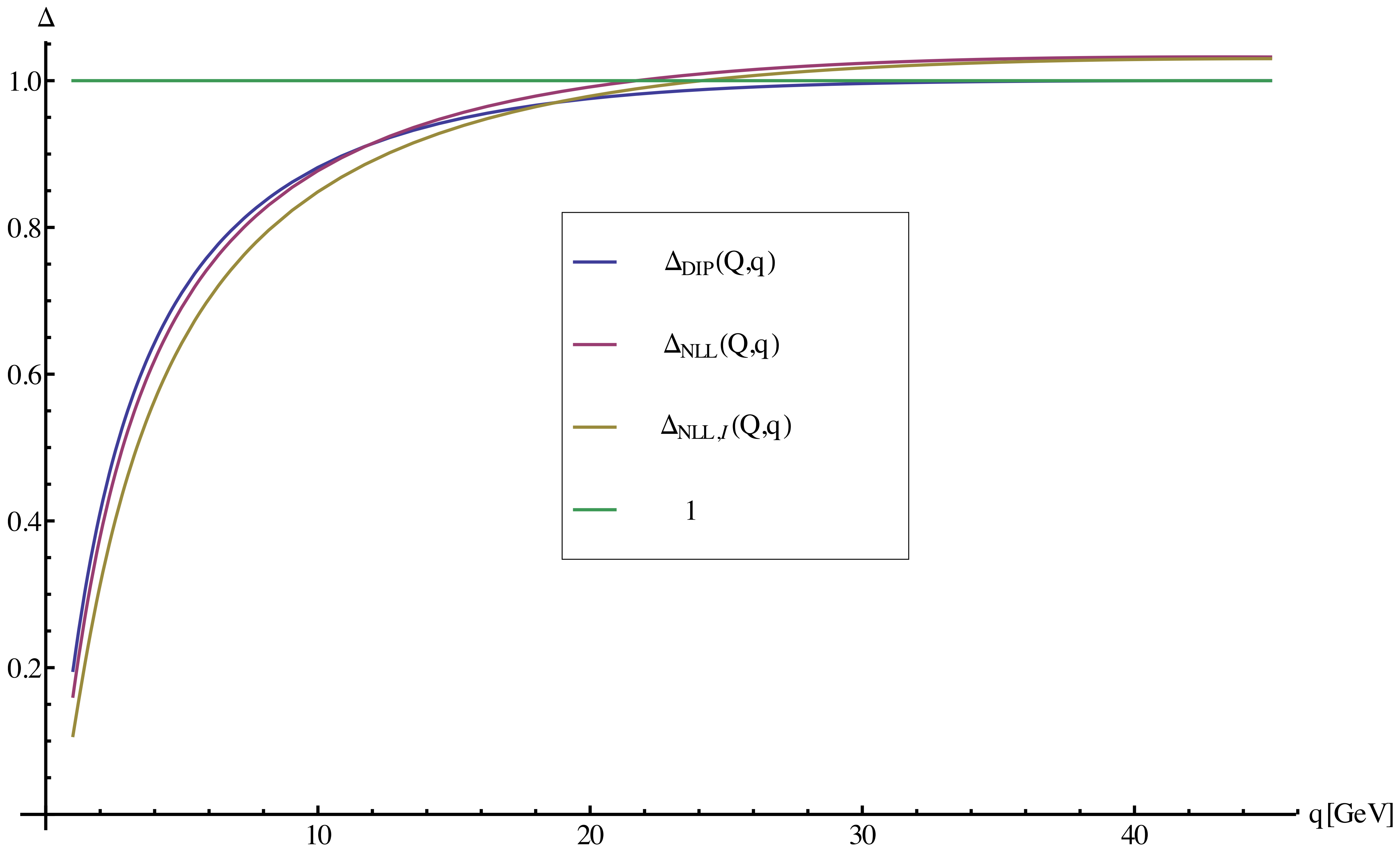
g –> q q: $$-\log \Delta^{Q}_{q_1} \approx \int_{q_1^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi} \frac{2}{3} n_f$$

This can now be used to reweighs the events to make them **exclusive**.
For stability and since in the collinear approximation the shower is pretty good, one introduces a **merging scale**.

CKKW arXiv:0109231

LO Merging

$$-\log \Delta_{q_1}^{Q} \approx \int_{q_1^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi} C_F \left(\log \frac{Q^2}{q^2} - \frac{3}{2} \right)$$



CKKW arXiv:0109231

LO Merging

Lets discuss $\boxed{\Delta_{q_1}^{Q} d\sigma_1^0}$ a bit closer.

Now we have the no-emission probabilities…

But $$d\sigma_n^0(\mu_R) = \frac{\alpha_S^{X+n}(\mu_R)}{\alpha_S^{X+n}(\mu'_R)} d\sigma_1^0(\mu'_R)$$

Shower scales:

$$d\sigma_2^0 \to \frac{\alpha_S(q_2)}{\alpha_S(Q)} \frac{\alpha_S(q_1)}{\alpha_S(Q)} d\sigma_2^0(Q)$$

Similar for PDFs, but need to care about momentum fractions.

CKKW arXiv:0109231

LO Merging

$$
\begin{array}{llllll}
d\sigma_0^0 & \Delta_\mu^Q & & & & \\
 & \Delta_{q_1}^Q\, d\sigma_1^0 & \Delta_\mu^{q_1} & & & + \\
 & \Delta_{q_1}^Q & \Delta_{q_2}^{q_1} & d\sigma_2^0 \Delta_\mu^{q_2} & & + \\
 & \Delta_{q_1}^Q & \Delta_{q_2}^{q_1} & \Delta_{q_3}^{q_2} & d\sigma_3^0 \Delta_\mu^{q_3} & + \\
 & & & & & \dots
\end{array}
$$

LO Merging:

The algorithm:

1. Calculate cross section for X+n partons.
2. Cluster according to Jet algorithm with a measure you can calculate the no-emission probability.
3. Multiply with no-emission for the received history.
4. Multiply with scale dependent ratios.
5. If n<N make the state exclusive towards merging scale.
6. Allow further emissions only below merging scale (veto above).

LO Merging (Exercise)

In exercise we calculate $\boxed{\Delta_{q_1}^{Q} d\sigma_1^0}$ for hadron colliders and compare spectrum to parton shower.

Here we need that:

$$\Pi(t1, t2; x_2) = \frac{f_b(x_2, t_1)}{f_b(x_2, t_2)} \frac{\Delta_b(t_2, t_0)}{\Delta_b(t_1, t_0)}$$

With this and the splitting function:

$$\frac{\alpha_s[p_\perp(z, t_1)]}{2\pi} \frac{x_2/z f_a(x_2/z, t_1)}{x_2 f_b(x_2, t_1)} P_{a\to bc}(z)$$

We write down $\boxed{\Delta_{q_1}^{Q} d\sigma_1^0}$ for hadron colliders.

CKKW for initial states arXiv:0205283

LO Merging

But:

1. If shower evolution variable is not the cutting variable.
2. Does shower produce the same no emission probability?

Solution:

"Loops"

$$\begin{array}{ccccc}
\mathcal{O}(\alpha_S^N) & \mathcal{O}(\alpha_S^{N+1}) & \mathcal{O}(\alpha_S^{N+2}) & \mathcal{O}(\alpha_S^{N+3}) & \\
d\sigma_0^0\ \Delta_\mu^Q & & & & + \\
\Delta_{q_1}^Q d\sigma_1^0 & \Delta_\mu^{q_1} & & & + \\
\Delta_{q_1}^Q & \Delta_{q_2}^{q_1} & d\sigma_2^0 \Delta_\mu^{q_2} & & + \\
\Delta_{q_1}^Q & \Delta_{q_2}^{q_1} & \Delta_{q_3}^{q_2} & d\sigma_3^0 \Delta_\mu^{q_3} & + \\
 & & & & \cdots
\end{array}$$

Legs and "Legs"

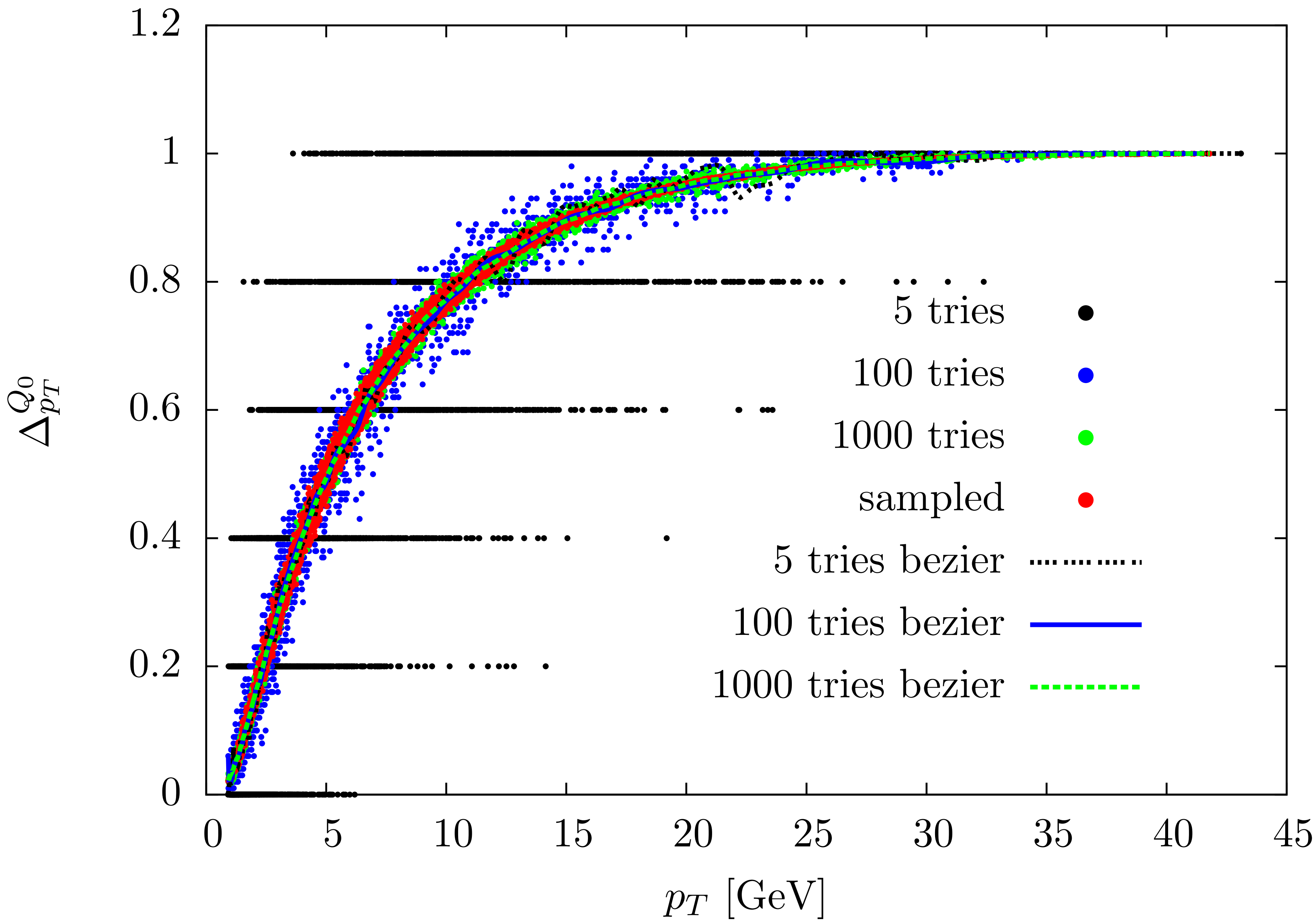
Ask the shower for the no-emission probabilities with trial emissions.

Reconstruct shower history, then start shower from intermediate states, remove event if first emission to hard.

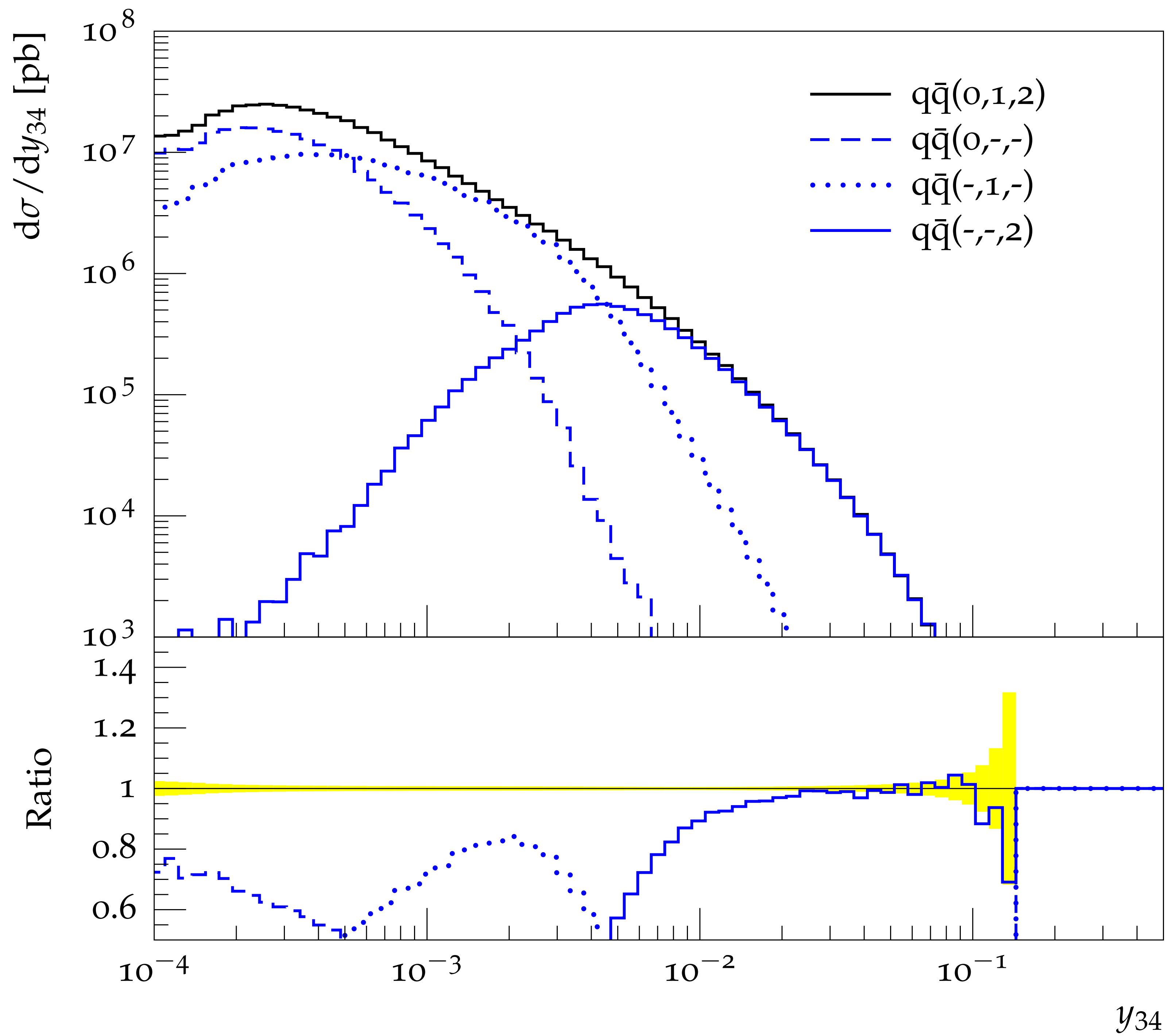
CKKW-L arXiv:0112284

LO Merging

Ask the shower for the no-emission probabilities with trial emissions.



Multiple LO merged: Example e+e– –> j j [j j]



Loops and Legs

This was all for merging for now. Lets see whats matching.

Loops

$\mathcal{O}(\alpha_S^N)$ $\mathcal{O}(\alpha_S^{N+1})$ $\mathcal{O}(\alpha_S^{N+2})$ $\mathcal{O}(\alpha_S^{N+3})$

$$
\begin{array}{rrrr}
d\sigma_0^0 + & d\sigma_0^1 + & d\sigma_0^2 + & d\sigma_0^3 + \dots \\
 & d\sigma_1^0 + & d\sigma_1^1 + & d\sigma_1^2 + \dots \\
 & & d\sigma_2^0 + & d\sigma_2^1 + \dots \\
 & & & d\sigma_3^0 + \dots \\
 & & & \dots
\end{array}
$$

Legs

Exact not easily extendable to X+n legs.
But the cross section and the scale dependence is improved.

Try to include one NLO calculation.
Beware of **double counting**!!

"Loops"

$\mathcal{O}(\alpha_S^N)$ $\mathcal{O}(\alpha_S^{N+1})$ $\mathcal{O}(\alpha_S^{N+2})$ $\mathcal{O}(\alpha_S^{N+3})$

$$
\begin{array}{lllll}
d\sigma_0^0 \; \Delta_\mu^Q & & & & + \\
d\sigma_0^0 \; \Delta_{q_1}^Q P_1 & \Delta_\mu^{q_1} & & & + \\
d\sigma_0^0 \; \Delta_{q_1}^Q P_1 & \Delta_{q_2}^{q_1} & P_2 \Delta_\mu^{q_2} & & + \\
d\sigma_0^0 \; \Delta_{q_1}^Q P_1 & \Delta_{q_2}^{q_1} & P_2 \Delta_{q_3}^{q_2} & P_3 \Delta_\mu^{q_3} & + \\
 & & & & \dots
\end{array}
$$

"Legs"

Loops and Legs

Double counting: The shower already approximates the loops and legs. Adding a loop and a leg from NLO virtual and reals requires to remove shower parts first.

Matching: Expand the approximation to same order as the correct result. Subtract approximation and add correct result.

Shower "Virtual"

$$d\sigma_0^0 \Delta_\mu^Q = d\sigma_0^0 \left(1 - \int_\mu^Q dq_1 \Delta_{q_1}^Q P(q_1)\right) \to -d\sigma_0^0 \int_\mu^Q dq_1 P(q_1)$$

Shower "Real"

$$d\sigma_0^0 \Delta_{q_1}^Q P_1 \Delta_\mu^{q_1} \to d\sigma_0^0 P_1$$

MC@NLO arXiv:0204244

$\mathcal{O}(\alpha_S^N)$	$\mathcal{O}(\alpha_S^{N+1})$	$\mathcal{O}(\alpha_S^{N+2})$	$\mathcal{O}(\alpha_S^{N+3})$	
$d\sigma_0^0 \quad \Delta_\mu^Q$				$+$
$d\sigma_0^0 \quad \Delta_{q_1}^Q P_1$	$\Delta_\mu^{q_1}$			$+$
$d\sigma_0^0 \quad \Delta_{q_1}^Q P_1$	$\Delta_{q_2}^{q_1}$	$P_2 \Delta_\mu^{q_2}$		$+$
$d\sigma_0^0 \quad \Delta_{q_1}^Q P_1$	$\Delta_{q_2}^{q_1}$	$P_2 \Delta_{q_3}^{q_2}$	$P_3 \Delta_\mu^{q_3}$	$+$
				$\cdots$

$$\Delta_\mu^Q + \int_\mu^Q dq_1 \Delta_{q_1}^Q P_1(q_1) = 1$$

Matching: **MC@NLO**

Virtual **Shower "Virtual"**

$$d\sigma_0^1 + d\sigma_0^0 \int^Q dq_1 P_1(q_1) \qquad \mathcal{O}(\phi_X)$$

$$d\sigma_1^0 - d\sigma_0^0 \qquad P_1(q_1)\theta(Q-q_1)\mathcal{O}(\phi_{X+1})$$

Real **Shower "Real"**

– Observables are here shower starting conditions.
– Without additional shower and Born cross section the formula make sense.
– Both parts can get negative, e.g. : Shower "Real" > Real
– Do we need to calculate Shower "Virtuals" and virtual correction together?

MC@NLO arXiv:0204244

Intermezzo: Subtraction

Virtual **Int. Subtraction**

$$d\sigma_0^1 \mathcal{O}(\phi_X) + d\sigma_0^0 \oplus \mathcal{I}_1 \mathcal{O}(\phi_X)$$

$$d\sigma_1^0 \mathcal{O}(\phi_{X+1}) - d\sigma_0^0 \oplus D_1 \mathcal{O}(\tilde{\phi}_X(\phi_{X+1}))$$

Real **Subtraction**

- Auxiliary/helper cross sections subtracted in four dimensions and added integrated in d-dimensions. Sum is zero. $\int d\tilde{\phi}_X = \int d\phi_X$
- Construction independent or process. Integrate once, reuse.
- $\oplus$ stands here for more than multiplication $< M|T_i T_j|M >$.
  $< M|T_i T_j|M >$ in large color limit encoded in showers.
- Event interpretation hardly possible as $\mathcal{O}(\phi_{X+1}) \neq \mathcal{O}(\tilde{\phi}_X(\phi_{X+1}))$

e.g. Catani, Seymour arXiv:9605323

Matching: **MC@NLO**

Virtual **Int. Subt.** **Shower "Virtual"** **Subtraction**

$$(d\sigma_0^1 + d\sigma_0^0 \oplus \mathcal{I}_1) + d\sigma_0^0 \int dq_1 \{P_1(q_1)\theta(Q - q_1) - D_1\}\mathcal{O}(\phi_X)$$

$$d\sigma_1^0 - d\sigma_0^0 \qquad\qquad P_1(q_1)\theta(Q - q_1)\mathcal{O}(\phi_{X+1})$$

Real **Shower "Real"**

– Technical "Trick" of adding zero cross section gives nice picture.
– Different parts are all at defined phase space points.
– No counter events with different kinematic.
– Event interpretation possible.
– Can still be negative.
– Can be unstable if P differs strongly from D.
– More and more showers appeared to remove P – D.
(e.g. CSS or Vincia, but not the only motivation..)

MC@NLO arXiv:0204244

Matching: **POWHEG** Positive Weight Hardest Emission Generator

Modify Shower to get: **Real = Shower "Real"** for hardest emission.

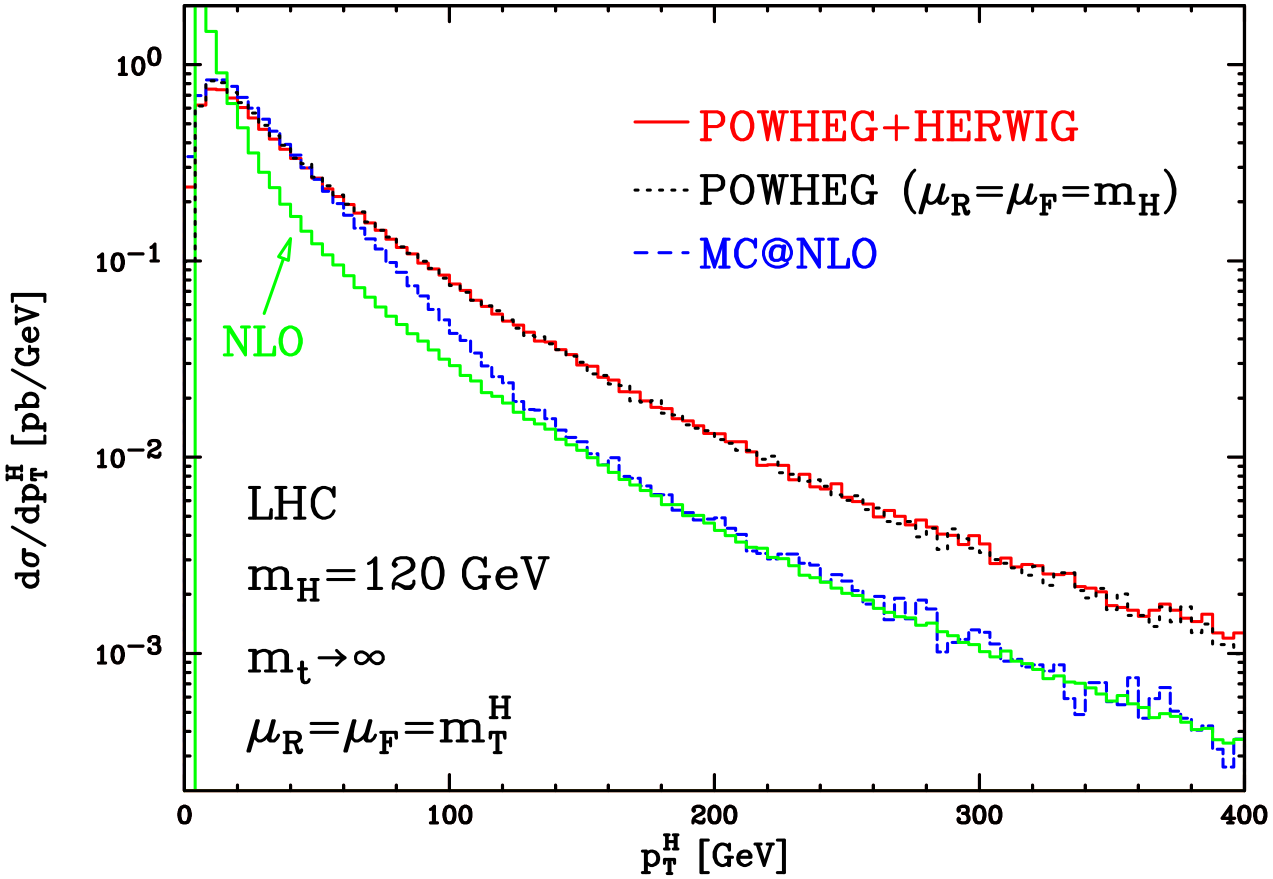
Virtual **Int. Subt.** **Shower "Virtual"** **Subtraction**

$$(d\sigma_0^1 + d\sigma_0^0 \oplus \mathcal{I}_1) + \int dq_1 \{d\sigma_1^0 - d\sigma_0^0 \oplus D_1\} \mathcal{O}(\phi_X)$$

– Again all in one kinematic.
– Sum mostly positive –> improves efficiency in event generation.
– Now hardest emission can emit in full phase space.
– Efficient implementation usually process by process.
– Possible problems with cuts on ϕ_X, as ϕ_{X+1} needs to be filled.

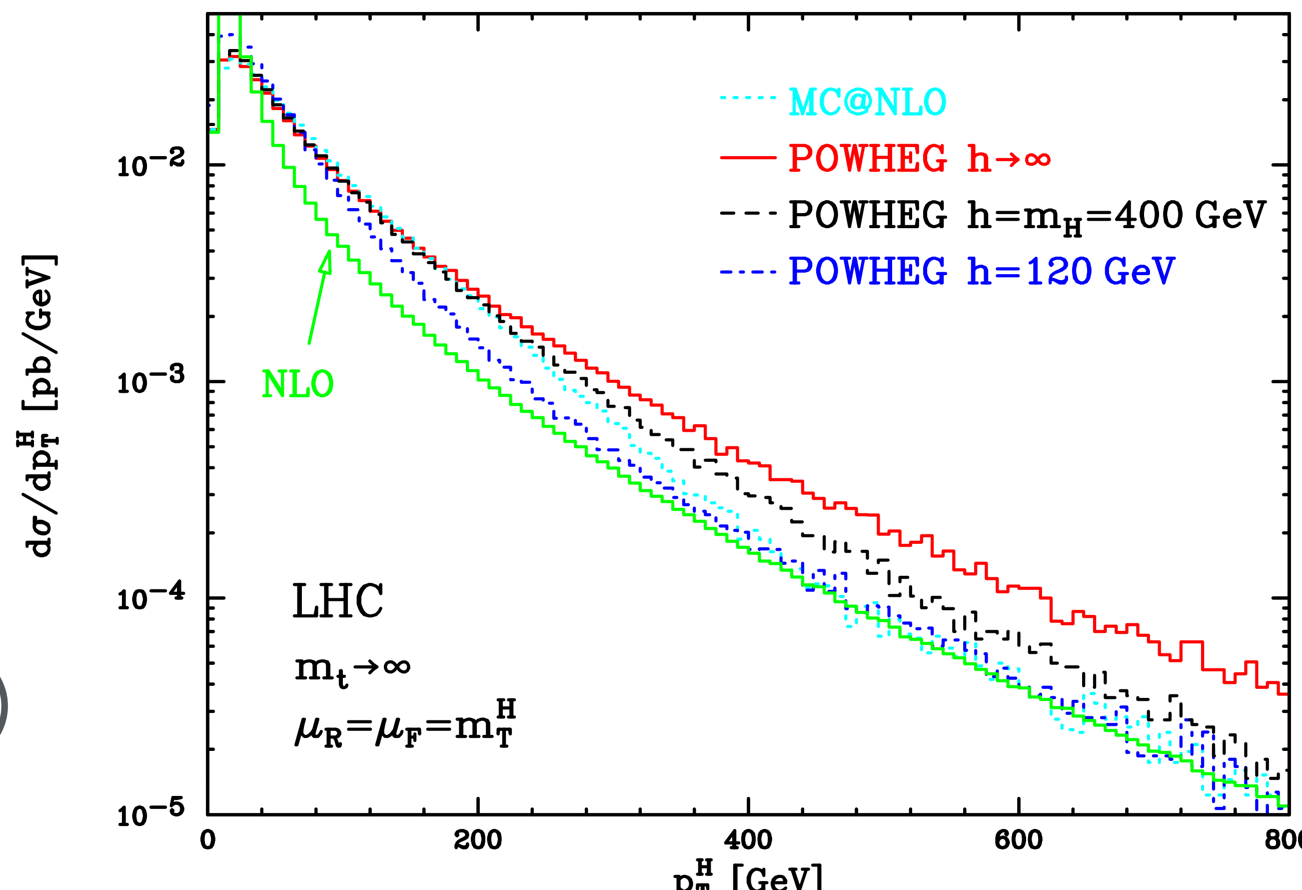
Nason arXiv:0409146
Powheg arXiv:0709.2092

Matching: **MC@NLO vs. POWHEG**



– MC@NLO starts shower from born-like structure.
– POWHEG fills full phase space with B+V+R.
– hdamp introduced to reduce tails to only R.

Personal remarks:
– Green line „NLO" ?
– In paper they argue that hdamp not needed here.
– Experiments now even tune the value of hdamp. (???)



Picture from: arXiv:0812.0578

Summary 2

LO Merging

* Produce exclusive states.

+ Extendable to multi legs
+ Positive weights
+ Multiple LO jets well described.

– LO
– merging scale

NLO Matching

* Include NLO corrections to cross section.
* Identify expansion of shower.
* remove double counting by subtraction.

+ NLO cross section
+ Only one additional jet at LO

MC@NLO

* original method
* Simplification if shower ~D

+ dont modify peak

– negative weights

POWHEG

* Modify shower to emit with R

+ positive weights.

– power shower or hdamp needed

NLO Merging (many ambiguities/choices)

Here, unitariesed NLO merging

"Loops"

$\mathcal{O}(\alpha_S^N)$ $\mathcal{O}(\alpha_S^{N+1})$ $\mathcal{O}(\alpha_S^{N+2})$ $\mathcal{O}(\alpha_S^{N+3})$

$$
\begin{array}{llllr}
d\sigma_0^0\,\Delta_\mu^Q & & & & + \\
\Delta_{q_1}^Q\, d\sigma_1^0 & \Delta_\mu^{q_1} & & & + \\
\Delta_{q_1}^Q & \Delta_{q_2}^{q_1} & d\sigma_2^0 \Delta_\mu^{q_2} & & + \\
\Delta_{q_1}^Q & \Delta_{q_2}^{q_1} & \Delta_{q_3}^{q_2} & d\sigma_3^0 \Delta_\mu^{q_3} & + \\
 & & & & \dots
\end{array}
$$

Legs and "Legs"

Replace with

$$\Delta_\mu^Q + \int_\mu^Q dq_1 \Delta_{q_1}^Q P_1(q_1) = 1$$

NL3 arXiv:0811.2912

MEPS@NLO arXiv:1207.5031

FxFx arXiv:1209.6215

UNLOPS arXiv:1211.7278

Plätzer arXiv:1211.5467

H71 arXiv:1705.06700

NLO Merging

"Loops"

$$
\begin{array}{llll}
d\sigma_0^0 \; (1-\int_\mu^Q \Delta_{q_1}^Q P_1) & & & + \\
\Delta_{q_1}^Q d\sigma_1^0 \; (1-\int_\mu^{q_1} \Delta_{q_2}^{q_1} P_2) & & & + \\
\Delta_{q_1}^Q & \Delta_{q_2}^{q_1} \; d\sigma_2^0 (1-\int_\mu^{q_2} \Delta_{q_3}^{q_2} P_3) & & + \\
\Delta_{q_1}^Q & \Delta_{q_2}^{q_1} & \Delta_{q_3}^{q_2} \; d\sigma_3^0 (1-\int_\mu^{q_3} \Delta_{q_4}^{q_3} P_4) & + \\
 & & & \ldots
\end{array}
$$

Legs
and
"Legs"

NLO Merging

"Loops"

"Legs"

$$
\begin{array}{llll}
d\sigma_0^0 \left(1 - \int_\mu^Q \Delta_{q_1}^Q P_1\right) & & & + \\
\Delta_{q_1}^Q \, d\sigma_1^0 \left(1 - \int_\mu^{q_1} \Delta_{q_2}^{q_1} P_2\right) & & & + \\
\Delta_{q_1}^Q \quad \Delta_{q_2}^{q_1} \, d\sigma_2^0 \left(1 - \int_\mu^{q_2} \Delta_{q_3}^{q_2} P_3\right) & & & + \\
\Delta_{q_1}^Q \quad \Delta_{q_2}^{q_1} \quad \Delta_{q_3}^{q_2} \, d\sigma_3^0 \left(1 - \int_\mu^{q_3} \Delta_{q_4}^{q_3} P_4\right) & & & + \\
 & & & \ldots
\end{array}
$$

Mismatch between emission and no-emission!

NLO Merging

Unitariesed LO merging:

“Loops”

“Legs”

$$
\begin{aligned}
&\left(d\sigma_0^0 - \int_\mu^Q \Delta_{q_1}^Q d\sigma_1^0\right) &+\\
&\Delta_{q_1}^Q \left(d\sigma_1^0 - \int_\mu^{q_1} \Delta_{q_2}^{q_1} d\sigma_2^0\right) &+\\
&\Delta_{q_1}^Q \quad \Delta_{q_2}^{q_1} \left(d\sigma_2^0 - \int_\mu^{q_2} \Delta_{q_3}^{q_2} d\sigma_3^0\right) &+\\
&\Delta_{q_1}^Q \quad \Delta_{q_2}^{q_1} \quad \Delta_{q_3}^{q_2} \left(d\sigma_3^0 - \int_\mu^{q_3} \Delta_{q_4}^{q_3} d\sigma_4^0\right) &+\\
& &\ldots
\end{aligned}
$$

Replace no-emission with (1 – emission)

NLO Merging

Expand unitariesed LO merging (simplified!):

"Loops"

$$
\begin{aligned}
&(d\sigma_0^0 - \int_\mu^Q d\sigma_1^0) && + \\
&\Delta_{q_1}^Q (d\sigma_1^0 - \int_\mu^{q_1} d\sigma_2^0) && + \\
&\Delta_{q_1}^Q \Delta_{q_2}^{q_1} (d\sigma_2^0 - \int_\mu^{q_2} d\sigma_3^0) && + \\
&\Delta_{q_1}^Q \Delta_{q_2}^{q_1} \Delta_{q_3}^{q_2} (d\sigma_3^0 - \int_\mu^{q_3} d\sigma_4^0) && + \\
& && \ldots
\end{aligned}
$$

Legs and "Legs"

NLO Merging

Add $d\sigma_n^1$ and $d\sigma_{n+1}^0$ but subtract expansion (simplified!):

Loops and "Loops"

$$
\begin{array}{l}
(d\sigma_0^1 + \int_\mu^Q d\sigma_1^0) + \\
\Delta_{q_1}^Q (d\sigma_1^1 + \int_\mu^{q_1} d\sigma_2^0) + \\
\Delta_{q_1}^Q \Delta_{q_2}^{q_1} (d\sigma_2^1 + \int_\mu^{q_2} d\sigma_3^0) + \\
\Delta_{q_1}^Q \Delta_{q_2}^{q_1} \Delta_{q_3}^{q_2} (d\sigma_3^1 + \int_\mu^{q_3} d\sigma_4^0) + \\
\qquad \ldots
\end{array}
$$

Legs and "Legs"

Thats part of the addition. Now unitarize once more.

NLO Merging

NLO cross section

$$\boxed{\left(d\sigma_0^1 + \int_\mu^Q dq_1 \quad d\sigma_1^0\right)} - \int_\mu^Q dq_1 \Delta_{q_1}^Q \left(d\sigma_1^1 + \int_\mu^{q_1} dq_2 \quad d\sigma_2^0\right) +$$

$$\Delta_{q_1}^Q \left(d\sigma_1^1 + \int_\mu^{q_1} dq_2 \quad d\sigma_2^0 - \int_\mu^{q_1} dq_2 \Delta_{q_2}^{q_1} \left(d\sigma_2^1 + \int_\mu^{q_2} dq_3 \quad d\sigma_3^0\right)\right) +$$

$$\Delta_{q_1}^Q \quad \Delta_{q_2}^{q_1} \left(d\sigma_2^1 + \int_\mu^{q_2} dq_3 \quad d\sigma_3^0 - \int_\mu^{q_2} dq_3 \Delta_{q_3}^{q_1} \left(d\sigma_3^1 + \int_\mu^{q_3} dq_4 \quad d\sigma_4^0\right)\right) +$$

$$\Delta_{q_1}^Q \quad \Delta_{q_2}^{q_1} \quad \Delta_{q_3}^{q_2} \left(d\sigma_3^1 + \int_\mu^{q_3} dq_4 \quad d\sigma_4^0 - \int_\mu^{q_3} dq_4 \left(d\sigma_4^1 + \int_\mu^{q_4} dq_5 \quad d\sigma_5^0\right)\right) +$$

$$\ldots$$

– Red and blue (…) parts cancel in inclusive cross section.

– NLO cross section restored.

– Subtraction not discussed here (makes it really messy).

NLO Merging

Thats not all!

What scales to choose?

Did we expand the shower history? Not yet!

$$
\begin{aligned}
&(d\sigma_0^0 - \int_\mu^Q d\sigma_1^0) &+\\
&\boxed{\Delta_{q_1}^Q (d\sigma_1^0} - \int_\mu^{q_1} d\sigma_2^0) &+\\
&\Delta_{q_1}^Q \quad \Delta_{q_2}^{q_1} \; (d\sigma_2^0 - \int_\mu^{q_2} d\sigma_3^0) &+\\
&\Delta_{q_1}^Q \quad \Delta_{q_2}^{q_1} \quad \Delta_{q_3}^{q_2} \; (d\sigma_3^0 - \int_\mu^{q_3} d\sigma_4^0) &+\\
&&\ldots
\end{aligned}
$$

NLO Merging

Thats not all!

What scales to choose?

Did we expand the shower history? Not yet!

We discussed:

$$d\sigma_2^0 \to \frac{\alpha_S(q_2)}{\alpha_S(Q)} \frac{\alpha_S(q_1)}{\alpha_S(Q)} d\sigma_2^0(Q)$$

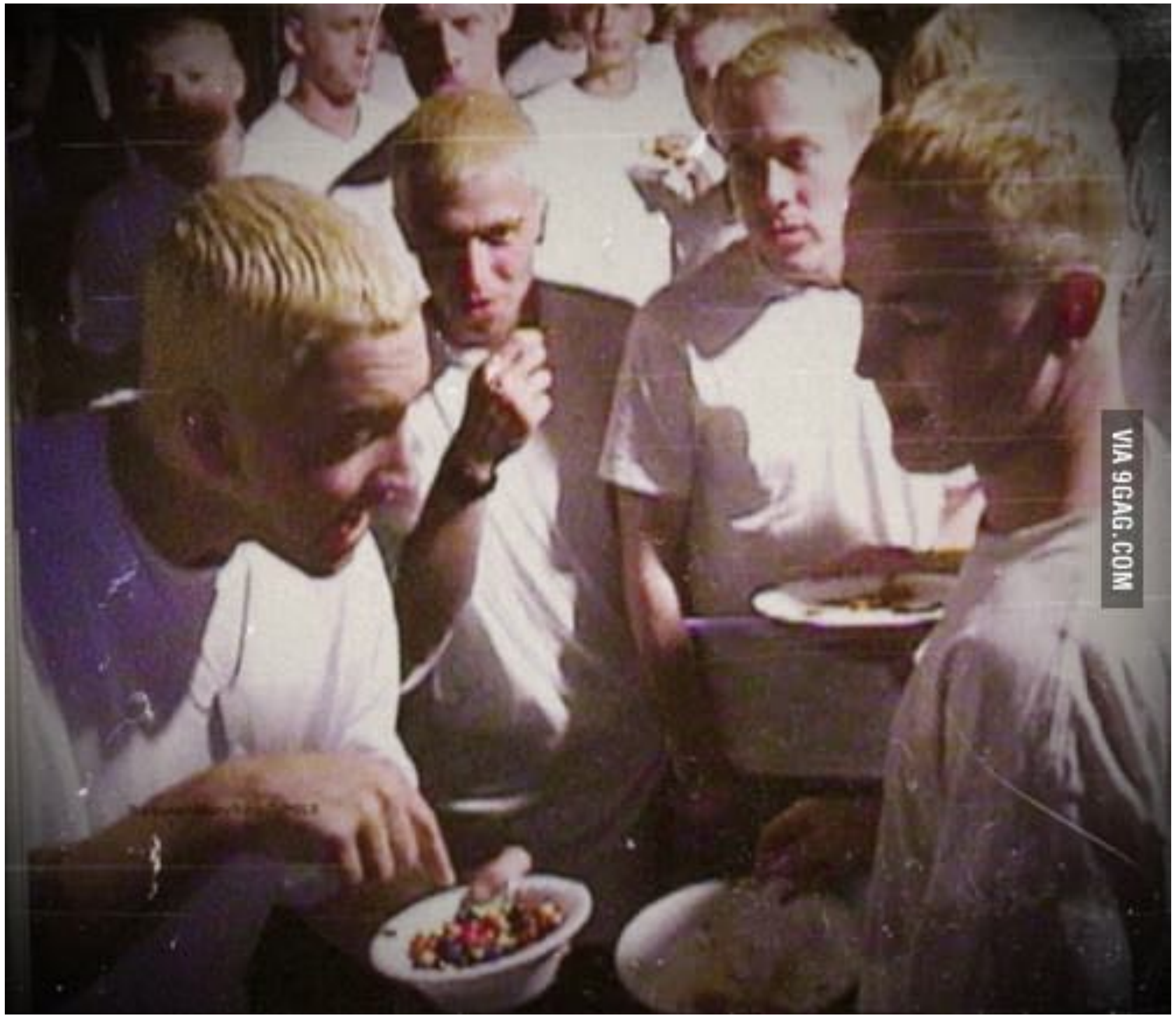
Expanded:

$$(1 + \alpha_S(Q) b_0 \log \frac{q_1^2}{Q^2} + \alpha_S(Q) b_0 \log \frac{q_2^2}{Q^2} + \ldots) d\sigma_2^0(Q)$$

This dictates what scales to use.

Similar for PDFs and Sudakov expansion.

Thank you!



Eminem eating M&Ms with other Eminems