Particle Physics Phenomenology exercise 2

1. Show that a four-vector can be written in the form

$$p = (m_\perp \cosh y; p_\perp \cos\varphi, p_\perp \sin\varphi, m_\perp \sinh y) \ .$$

   Note simplification for $m = 0$.

2. Work out the form of the four-body phase space if written in terms of two separate intermediate states X and Y, i.e. $a + b \to X + Y,\ X \to c + d,\ Y \to e + f$.

3. Show that, in a $2 \to 2$ process with massive incoming and outgoing particles,

$$s + t + u = \sum_{i=1}^{4} m_i^2 \ .$$

4. Calculate the kinematically allowed t range for massive outgoing (but not incoming) particles in a $2 \to 2$ process. What is the product of these two limits?

5. Assume a resonance of mass M at rest. It decays isotropically to two massless particles. Calculate the $p_\perp$ spectrum of these particles. (The answer is the famous Jacobian peak, used e.g. to discover $\mathrm{W}^\pm$.) Draw the shape schematically.

6. The cross section for the process $\mathrm{q\overline{q}} \to \gamma\mathrm{g}$ is

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} = \frac{8\pi}{9}\, e_\mathrm{q}^2 \alpha_\mathrm{em}\, \alpha_\mathrm{s}\, \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2 \hat{t} \hat{u}}$$

   Explain qualitatively how such an expression could come about.

7. Study the mass dependence of the pseudorapidity dip as follows. Let PYTHIA generate 8 TeV LHC pp inelastic nondiffractive events, `SoftQCD:nonDiffractive = on` and plot the y and η distributions (methods `pythia.event[i].y()` and `.eta()`) for $\pi^\pm, \mathrm{K}^\pm, \mathrm{p}/\overline{\mathrm{p}}$ (`.idAbs()` equal to 211, 321 and 2212, respectively. To avoid plotting also the incoming protons, check that the particles are final-state ones (`.isFinal()`).