Particle Physics Phenomenology
8. QCD jets and jet algorithms

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At the beginning was ...

- Spear (SLAC): mid-70'ies, $\mathrm{e^+e^-} \to \mathrm{q\overline{q}}$ should have $1 + \cos^2\theta$ angular distribution if quarks have spin $1/2$.
  Solution: Sphericity.
- Fixed-target pp experiments study alignment of collision.
  Solution: Thrust.
- PETRA (DESY): early 80'ies, $\mathrm{e^+e^-} \to \mathrm{q\overline{q}g}$, establish g.
  First solution: extend Sphericity and Thrust families.
  Second solution: clustering algorithms, e.g. LUCLUS, JADE, Durham $k_\perp$. All rotationally symmetric.
- $\mathrm{Sp\overline{p}S}$ (CERN): need to separate beam jets from high$p_\perp$ ones.
  First solution: cone jets in (η, φ) space, e.g. UA1.
  (Second solution: clustering $\sim$ like Durham, but in (η, φ).)
- Tevatron (Fermilab): cone algorithms, increasingly messy.
- LHC: return of clustering with new safer and faster algorithms.
  Anti-$k_\perp$ "is" infrared safe return to UA1 cone algorithm.

Sphericity

View as eigenvector problem, e.g. rotation axes of irregular 3D body. Here spanned up by the $\mathbf{p}_i$ of "all" particles in event.

$$S^{ab} = \frac{\sum_i p_i^a p_i^b}{\sum_i p_i^2} \quad a, b = x, y, z$$

S^{ab} has three eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$ with $\lambda_1 + \lambda_2 + \lambda_3 = 1$.

Sphericity $S = \frac{3}{2}(\lambda_2 + \lambda_3)$, $0 \leq S \leq 1$.
$S = 0$: two back-to-back pencil jets, e.g. $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$.
$S = 1$: spherically symmetric distribution.

Aplanarity $A = \frac{3}{2}\lambda_3$, $0 \leq A \leq \frac{1}{2}$.
$A = 0$: all particles in one plane.
$A = 1/2$: like $S = 1$.

Problem: collinear unsafe!
E.g. different answer if $\pi^0 \to \gamma\gamma$ counted as one or two particles.

Linearized Sphericity

Collinear safe alternative, used in same way but with

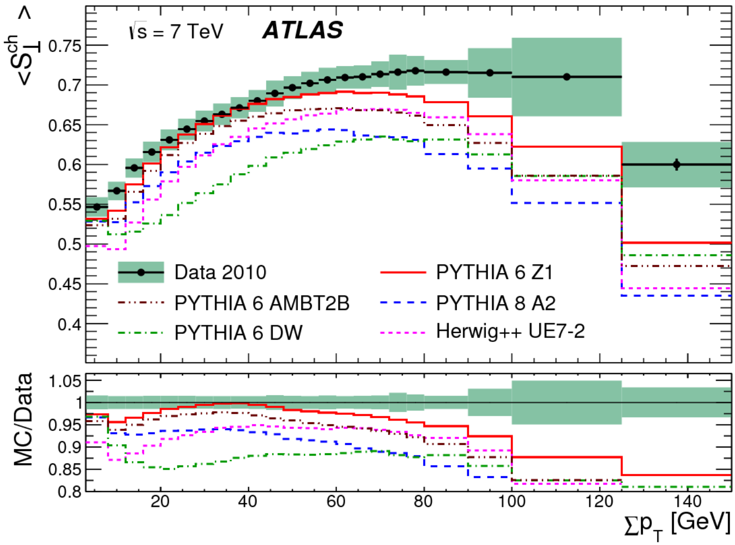
$$L^{ab} = \frac{\sum_i \frac{p_i^a p_i^b}{|\mathbf{p}_i|}}{\sum_i |\mathbf{p}_i|} \quad a, b = x, y, z$$

No proper name: some confusion!

Additional measures:
$C = 3(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)$
$D = 27\lambda_1\lambda_2\lambda_3$
used to characterize 3- and 4-jet topologies, respectively.

(Linearized) Sphericity family not normally used in pp,
since beam jets dominate structure.
Solution: set all $p_i^z = 0$ so only transverse structure studied.
Modified "2D" $S = 2\lambda_2$ and no A.

2D Sphericity at the LHC



Competition between more $\sum p_\perp$ by more particles or by jets?

Thrust

Thrust is computationally more demanding optimization

$$T = \max_{|\mathbf{n}|=1} \frac{\sum_i |\mathbf{p}_i \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$

with **n** for maximum is called Thrust axis.
$1/2 < T < 1$, with $T = 1$ for two back-to-back pencil jets and $T = 1/2$ for a spherically symmetric distribution.

$$\begin{aligned}
\mathrm{Major} &= \max_{|\mathbf{n}'|=1, \mathbf{n}'\mathbf{n}=0} \frac{\sum_i |\mathbf{p}_i \mathbf{n}'|}{\sum_i |\mathbf{p}_i|} \\
\mathrm{Minor} &= \frac{\sum_i |\mathbf{p}_i \mathbf{n}''|}{\sum_i |\mathbf{p}_i|} \quad \text{with } \mathbf{n}''\mathbf{n} = \mathbf{n}''\mathbf{n}' = 0 \\
\mathrm{Oblateness} &= \mathrm{Major} - \mathrm{Minor}
\end{aligned}$$

Major and Oblateness again useful for 3-jet structure,
Minor for 4-jet one.

The rest of these lectures

- The jet algorithms developed by Salam, Cacciari, Soyez and collaborators have set the standard for LHC
- The FastJet package provides standard implementations of many algorithms, see `http://fastjet.fr`
- An excellent summary of algorithms and physics can be found in G. Salam, *Towards Jetography*, Eur. Phys. J. C67 (2010) 637 [arXiv:0906.1833 [hep-ph]]
- Two excellent lectures can be downloaded from

  `http://conference.ippp.dur.ac.uk/conferenceOtherViews.py?confId=309`

  We will use (parts of) these in the following