Particle Physics Phenomenology exercise 7

1. Study the space–time picture for a yo-yo which originally has

$$\begin{aligned} p_{\mathrm{q}} &= k\,\frac{E_{\mathrm{cm}}}{2}\,(1;0,0,1) \\ p_{\overline{\mathrm{q}}} &= \frac{1}{k}\,\frac{E_{\mathrm{cm}}}{2}\,(1;0,0,-1) \end{aligned}$$

   for $k \neq 1$, by applying $\mathrm{d}p_z/\mathrm{d}t = \pm\kappa$.
   Show that it agrees with what is obtained by boosting an original system with $k = 1$.

2. Consider the string piece spanned between two points (t_1, z_1) and (t_2, z_2) along the string, where $z_1 > z_2$. Show that the relations

$$\begin{aligned} E &= \kappa(z_1 - z_2) \\ p_z &= \kappa(t_1 - t_2) \end{aligned}$$

   hold for a string piece "at rest", i.e. with $t_1 = t_2$. Thereafter boost along the z axis to show that it holds generally.

3. The charm mass varies between different approaches (e.g. "current algebra" vs. "constituent"), in the range 1.2 – 1.6 GeV. Estimate the range of tunneling suppression factors that would correspond to.

4. Assume that all string branchings produce particles with $m_\perp = 1$ GeV and that the fragmentation function is $f(z) = \delta(z - 1/2)$. Estimate the typical $\Gamma = (\kappa\tau)^2$ value, and from there the typical proper time at which strings break.
   Hint: use the recursive formula

$$\Gamma_i = (1 - z_i)\left(\Gamma_{i-1} + \frac{m_{\perp i}^2}{z_i}\right)$$

   with equilibrium corresponding to $\Gamma_i = \Gamma_{i-1}$.

5. Consider a back-to-back two-parton system, e.g. from $\mathrm{e}^+\mathrm{e}^-$ annihilation. Show that the absolute momentum of a jet is approximately $|\mathbf{p}_{\mathrm{jet}}| \approx E_{\mathrm{jet}} - \text{constant}$, where $E_{\mathrm{jet}} \approx E_{\mathrm{cm}}/2$. How is the constant related to the typical Γ value introduced above? What does it imply about the jet mass?
   Hint: consider how one string break in the middle of the event splits the system into two parts.

6. Compare the average multiplicity of a "Mercedes" three-jet event ($E_{\mathrm{q}} = E_{\overline{\mathrm{q}}} = E_{\mathrm{g}} = E_{\mathrm{cm}}/3$) with a two-jet event of the same energy ($E_{\mathrm{q}} = E_{\overline{\mathrm{q}}} = E_{\mathrm{cm}}/2$). Assume that the multiplicity per string piece is proportional to its effective rapidity range in its rest frame, defined for an average transverse hadron mass $\langle m_\perp \rangle$.