Particle Physics Phenomenology exercise 4

1. a) Show that the eikonal expression

$$\mathrm{d}\sigma_{n+1} = \mathrm{d}\sigma_n \, \frac{\mathrm{d}^3 k}{(2\pi)^3 2\omega} \left| \sum_{i=1}^{n} g_s \, \mathbf{T}_i \, \frac{p_i}{p_i k} \right|^2$$

leads to

$$\frac{\mathrm{d}\sigma_3}{\sigma_2} \propto \frac{\mathrm{d}E_3}{E_3} \, \mathrm{d}\Omega_3 \, \frac{a_{12}}{a_{13} a_{23}} \; .$$

b) Show that the $\mathrm{q\bar{q}g}$ matrix element, as a function of x_1 and x_2, can be recast into the same form in the soft-gluon limit.
Hint for part b: it is enough that you show this correspondence in the CM frame. Then, in the soft-gluon limit, you are allowed to assume that partons 1 and 2 are back-to-back, which simplifies the variable transformation.

2. The three most important QCD $2 \to 2$ processes $\mathrm{qq'} \to \mathrm{qq'}$, $\mathrm{qg} \to \mathrm{qg}$ and $\mathrm{gg} \to \mathrm{gg}$ are all dominated by gluon exchange. The cross sections

$$\begin{aligned}
\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}(\mathrm{qq'} \to \mathrm{qq'}) &= \frac{\pi \alpha_\mathrm{s}^2}{\hat{s}^2} \frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \\
\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}(\mathrm{qg} \to \mathrm{qg}) &= \frac{\pi \alpha_\mathrm{s}^2}{\hat{s}^2} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} - \frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} \right) \\
\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}}(\mathrm{gg} \to \mathrm{gg}) &= \frac{\pi \alpha_\mathrm{s}^2}{\hat{s}^2} \frac{9}{4} \left(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} \right)
\end{aligned}$$

are thus approximately proportional. Find a combination of parton distributions that it is therefore possible to access in $\mathrm{pp}/\mathrm{p\bar{p}}$ collisions.

3. a) Check that

$$P_{\mathrm{q} \to \mathrm{qg}} = \frac{4}{3} \left[\frac{1 + z^2}{(1 - z)_+} + \frac{3}{2} \delta(1 - z) \right]$$

preserves the number of quarks.
b) Why don't you need to use the + prescription or the δ term in an event generator?

4. At a scale Q_0^2 a quark carries a fraction x_0 of the proton momentum. Study how $\langle x \rangle$ is changed for $Q^2 > Q_0^2$. Use fixed α_s for simplicity.
Hint: use the moments formalism, picking the right moment n. Remember to use the + prescription for the DGLAP kernel.

5. Use the Durham or Apfel web interface to plot the $xu(x)$, $xd(x)$, $xg(x)$ and $x\overline{u}(x)$ distributions of some suitable PDF set at a $Q^2 = 10^4$ GeV^2 and down to $x = 10^{-4}$.
Hint: prescale $xg(x)$ suitably to simplify viewing, or use a vertical log scale.