Particle Physics Phenomenology exercise 1

1. Propose and implement an algorithm to select x randomly according to the distribution
$$f(x) = \frac{\sin^2 x}{x^2} \quad , x \geq 0 \ .$$

2. Consider the integration of a function $f(x) = \cos(\pi x/2)$.
a) Analytically, what is the integral $\int_0^1 f(x)\,\mathrm{d}x$?
b) Using hit-and-miss Monte Carlo, with $f(x) \leq 1$, study how rapidly the integration converges to the correct answer, analytically and numerically.
c) Instead using that $f(x) \leq g(x) = 1 - x^2$, so that
$$\int_0^1 f(x)\,\mathrm{d}x = \int_0^1 \frac{f(x)}{g(x)}\,g(x)\,\mathrm{d}x \ ,$$
how much is the convergence improved?

3. Find analytically the probability distribution for the product of three random numbers (each uniform between 0 and 1), and confirm this numerically by showing that the ratio of Monte Carlo results to the analytic answer is close to unity.

4. A "radioactive decay" process is given by
$$\frac{\mathrm{d}N}{\mathrm{d}t}(t) = -c(t)\,N(t) \quad \text{with } c(t) = 1 + \cos t.$$
a) Solve analytically to find $N(t)$ given $N(0)$.
b) Do a Monte Carlo simulation of the process, with the veto algorithm, using that $c(t) \leq 2$.
c) Check that the two agree.

5. Open `http://home.thep.lu.se/Pythia` in a browser and fetch `pythia8230.tgz` (in the Installation section) and "PYTHIA 8.2 Worksheet" (in the Documentation section) to a suitable place on your disk. Open the worksheet in a PDF viewer, read section 1 of it, and install+check according to section 2.
Comment: The program should be possible to install and use on any laptop with a valid C++ compiler, but the standard instructions are only for Linux and Mac OS X systems. See appendix D of the worksheet.
Disclaimer: the primitive histogramming methods that come with the program (see appendix B.1) should be enough for the exercises here. It is also possible to use ROOT for histogramming if you already have a ROOT installation, see instructions on the "ROOT usage" page of the html manual. It can be tricky to make it work, however, in particular if you want to go beyond simple histograms.

6. Work your way through sections 3 – 5 of the worksheet, including Appendix A.
Erratum: on p. 4 `mymain01.exe` → `mymain01` .