Bayesian Nonparametrics and Random Trees: Multi Object Tracking Problem

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October 15, 2018

Problem Statement and Objective

Problem in hand :

- Transition kernel $p_{\theta_k}(x_k|x_{k-1})$ due to state equation $x_k = f(x_{k-1}, n_k)$ with parameters θ_k.
- Observation (measurement) equations (Likelihood density) $p(z_k|x_k)$ due to measurement equation $z_k = g(x_k, \nu_k)$.

Objective:

1. Use a dependent Diffusion process on a tree as prior on θ_k.
2. Find trajectory of each object by tracing the paths on the tree.
3. Predict and update the number of objects at each time step.

Diffusion Process

- Draw $\theta_0 \sim P_{\theta_0}$.
- Generate N data points as follows:
  - Points start at $t = 0$ and follow a diffusion process to time $t = 1$.
  - Branching probability: At a branch point,

$$\mathbb{P}(\textit{selecting } k\textit{th path}) = \frac{n_k - \beta}{m + \eta}$$
$$\mathbb{P}(\textit{diverging}) = \frac{\eta - \beta K}{m + \eta}$$

    where:
    n_k: number of samples which previously took branch k.
    K: current number of branches from this branch point.
    $m = \Sigma_{k=1}^{K} n_k$: number of samples which previously took the current path.
    β, η are concentration and discount hyperparameters.
  - Probability of diverging between $[t, t + dt]$:
    $\frac{\Gamma(m-\beta)}{\Gamma(m+1-\eta)} \int_{[t,t+dt]} dH(s)$, $H(t)$ is cumulative Hazard function.

Survival Analysis

Fix P_{θ_0} as the base distribution on the parameters, α is the hyperparameter. Draw $N_0 \sim poisson(\mu(\Theta))$.

- **At time** $k-1$: Given:
  1. N_{k-1}
  2. $V_{k-1} = \{\theta^*_{1,k-1}, \ldots, \theta^*_{N_{k-1},k-1}\}$
  3. $V_{B,k-1}$ is the set of all branch nodes that are connected to a leaf at time $k-1$.
  4. $S_{a,k-1}$ is the set of siblings with the common parent branch node a.

- **Transitioning from time $k-1$ to k:**
  1. Define $V_{k|k-1} \subset V_{k-1}$ to be the set of survived object parameters.
  2. $V_{B,k|k-1} \subset V_{B,k-1}$ is the set of survived branch nodes.
  3. $S_{a,k|k-1} \subset S_{a,k-1}$ is the set of survived siblings with the common parent branch node a.
  4. $N_{B,k|k-1}$ is the total number of survived points after transition.

- **At time k:**
  1. For $\theta_{i,k|k-1} \in V_{k|k-1}$ (W.L.O.G $\theta_{i,k|k-1} \in S_{a,k|k-1}$.)
  2. Draw $\tilde{N}_{i,k|k-1} \sim poisson(\frac{p_a \times \alpha}{2|S_{a,k|k-1}|})$ and generate $\tilde{N}_{i,k|k-1}$ points given $\theta_{i,k|k-1}$ based on a diffusion process.
  3. Draw $\tilde{N}_{\delta,k|k-1} \sim poisson(p_\delta \times \alpha)$ and draw $\tilde{N}_{\delta,k|k-1}$ new points from P_{θ_0}.

Output at time k:

Set $\tilde{N}_k = \Sigma_i \tilde{N}_{i,k|k-1}$.

Set $\tilde{V}_k = \{\theta_1, \dots \theta_{\tilde{N}_k}\}$.

Note1: Assign probability vector $\mathbf{p}_{branch\ node} = [p_a]_{a \in V_{B,k|k-1} \cup \delta}$ to the survived branch nodes as follows (γ, ζ are hyperparameters):

$$p_a = \begin{cases} \frac{|S_{a,k-1}|+|S_{a,k|k-1}|-\gamma}{N_{B,k|k-1}-1+\Sigma_{a \in V_{B,k|k-1}}|V_{a,k-1}|+\zeta} & a \in V_{B,k|k-1} \\ \frac{\zeta-|V_{B,k|k-1}|\gamma}{N_{B,k|k-1}-1+\Sigma_{a \in V_{B,k|k-1}}|V_{a,k-1}|+\zeta} & a = \delta \end{cases}$$

Note2: if all the leaves connected to a branch node disappear, the branch node is removed from the set of branch nodes.

Inference

Receive measurement vectors $\mathbf{z}_{\ell,k}$ for $\ell = 1, \ldots, L_k$ at time k.

- Update:
  - $\mathbf{x}_{l,k}|\theta_{l,k} \sim G(\theta_{l,k})$. ($\theta_{l,k}$ is drawn from the described process.)
  - $\mathbf{z}_{\ell,k}|\mathbf{x}_{l,k}, \theta_{l,k} \sim F(\mathbf{x}_{l,k}, \theta_{l,k})$
- $N_k \leftarrow \tilde{N}_k$
  Note that all the new generated parameters may not be used.
- $V_k = \{\theta^*_{1,k}, \ldots, \theta^*_{N_k,k}\}$