Symbols

Sets

Name	Domains	Description
m, mi, mj	*	Machine, ie A,B,C
u	*	Unit, ie A1
machine	m, u	Machine-unit, mapping of unit to machine (ex, A1 to A)
p, pi, pj	*	Product
o, oi, oj	*	Job Number
c	*	Product Category
product_category	p, c	Product Category assignment
order	mi, mj	process order pairs, ex A,B

Parameters

Name	Domains	Description
t	p, m	Processing times for each product on each machine
penalty		Penalty for consecutive same product types
max_t		Big M scalar for upper bound set as the max total time

Variables

Name	Domains	Description
y	u, o, p	1 if product P is scheduled on machine-unit U for job number O
z	u, c	1 if the same product category is scheduled on the same machine-unit U for consecutive jobs
x_start	u, o, p	Start time of product P on machine-unit U for job number O
obj		Objective function - max processing time + penalties
x_makespan		Max processing time for the objective function

Equations

Name	Domains	Description
set_next_start	u, o, p	Set the start time on or after the end time of previous job (O) per unit U
link_constraint	u, o, p	Link variables x_start and y together
set_sequence	u, o	
assign_unique_product	u, o	
find_consecutive_products	u, o, c	
set_process_order	p, mi, mj	Processing order, ie machine A must be processed before B and B before C
force_all_scheduled	p, m	

Name	Domains	Description
set_makespan	p, m	Calculate the max processing time (makespan)
set_obj		Calculate the objective function

Equation Definitions

set_next_start$_{u,oj,pj}$

$$\text{x\_start}_{u,oj,pj} + \text{max\_t} \cdot (1 - \text{y}_{u,oj,pj}) \geq \sum_{oi,pi|(\text{ord(oi)}=(\text{ord(oj)}-1))} (\text{x\_start}_{u,oi,pi} + \sum_{m,machine_{m,u}} (\text{t}_{pi,m} \cdot \text{y}_{u,oi,pi})) \qquad \forall u, oj, pj \mid (\text{ord(oj)} > 1)$$

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link_constraint$_{u,o,p}$

$$\text{x\_start}_{u,o,p} \leq \text{max\_t} \cdot \text{y}_{u,o,p} \qquad \forall u, o, p$$

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set_sequence$_{u,oi}$

$$\sum_{p} (|o| \cdot \text{y}_{u,oi,p}) \geq \sum_{p,oj|(\text{ord(oi)}=(\text{ord(oj)}-1))} \text{y}_{u,oj,p} \qquad \forall u, oi \mid (\text{ord(oi)} < |o|)$$

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assign_unique_product$_{u,o}$

$$\sum_{p} \text{y}_{u,o,p} \leq 1 \qquad \forall u, o$$

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find_consecutive_products$_{u,oj,c}$

$$\sum_{p,oi,product\_category_{p,c}|(\text{ord(oi)}=(\text{ord(oj)}-1))} \text{y}_{u,oi,p} + \sum_{p,product\_category_{p,c}} \text{y}_{u,oj,p} \leq \text{z}_{u,c} + 1 \qquad \forall u, oj, c$$

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set_process_order$_{p,order_{mi,mj}}$

$$\sum_{u,o,machine_{mi,u}} (\text{x\_start}_{u,o,p} + \text{t}_{p,mi} \cdot \text{y}_{u,o,p}) \leq \sum_{u,o,machine_{mj,u}} \text{x\_start}_{u,o,p} \qquad \forall p, order_{mi,mj}$$

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force_all_scheduled$_{p,m}$

$$\sum_{u,o,machine_{m,u}} \text{y}_{u,o,p} = 1 \qquad \forall p, m$$

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set_makespan$_{p,m}$

$$\sum_{o,u,machine_{m,u}} (\text{x\_start}_{u,o,p} + \text{t}_{p,m} \cdot \text{y}_{u,o,p}) \leq \text{x\_makespan} \qquad \forall p, m \mid (\text{ord(m)} = |m|)$$

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set_obj

$$\text{obj} = \text{x\_makespan} + \sum_{u,c} (\text{penalty} \cdot \text{z}_{u,c})$$

$\text{x\_start}_{u,o,p} \geq 0 \ \forall u, o, p$
$\text{y}_{u,o,p} \in \{0, 1\} \ \forall u, o, p$
$\text{z}_{u,c} \in \{0, 1\} \ \forall u, c$
$\text{x\_makespan} \geq 0 \ \forall$