Linear Regression

Data Set

h2o

```
ames <- AmesHousing::make_ames()
ames.h2o <- as.h2o(ames)
```

stratified (*Sale_Price*) training sample

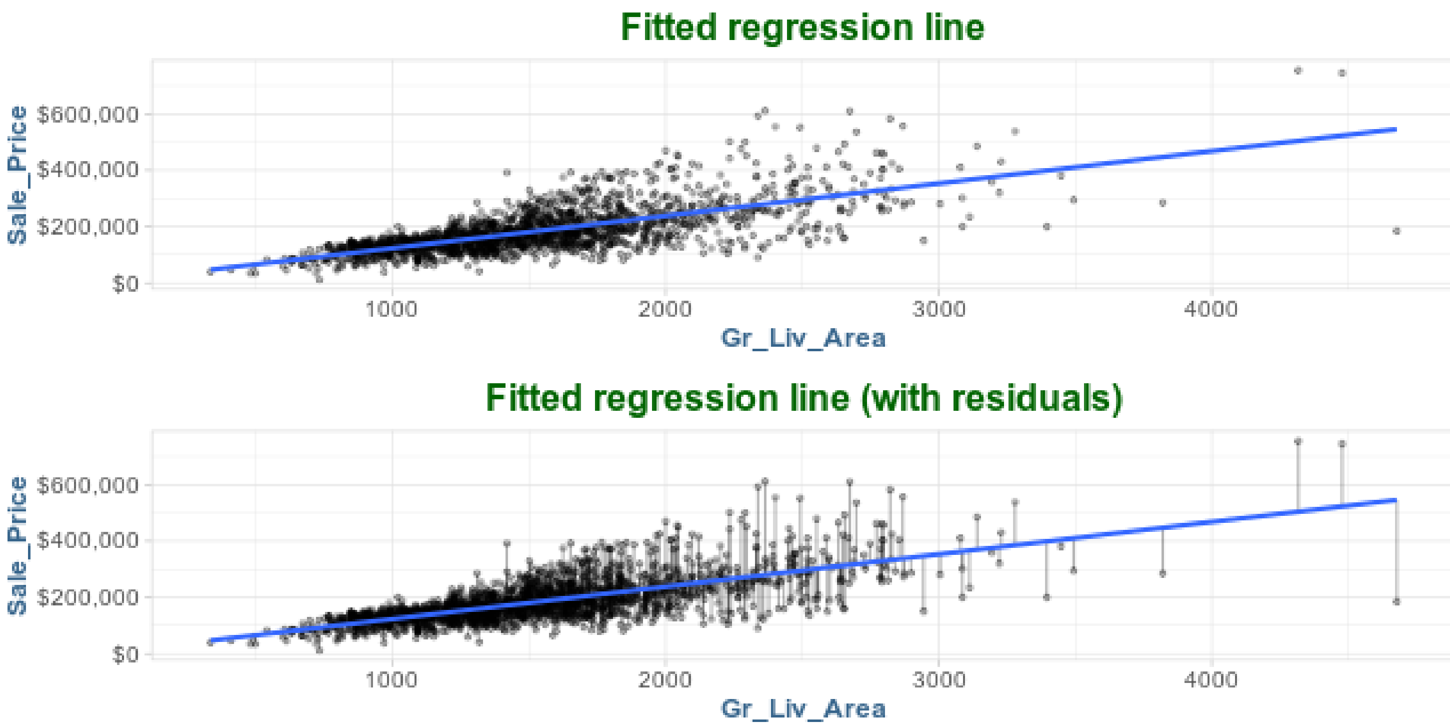
```
set.seed(123)

split <- initial_split(ames, prop = 0.7,
                       strata = "Sale_Price")

ames_train <- training(split)
ames_test <- testing(split)
```

Simple Linear Model

```
model1 <- lm(Sale_Price ~ Gr_Liv_Area, data = ames_train)
```

```
# Fitted regression line (full training)
p1 <- model1 %>%
   broom::augment() %>%
   ggplot(aes(Gr_Liv_Area, Sale_Price)) +
   geom_point(size = 1, alpha = 0.3) +
   geom_smooth(se = F, method = "lm") +
   scale_y_continuous(labels = scales::dollar) +
   ggtitle("Fitted regression line")

# Fitted regression line (restricted range)
p2 <- model1 %>%
   broom::augment() %>%
   ggplot(aes(Gr_Liv_Area, Sale_Price)) +
   geom_segment(aes(x = Gr_Liv_Area, y = Sale_Price,
                    xend = Gr_Liv_Area, yend = .fitted),
                alpha = .3) +
   geom_point(size = 1, alpha = 0.3) +
   geom_smooth(se = F, method = "lm") +
   scale_y_continuous(labels = scales::dollar) +
   ggtitle("Fitted regression line (with residuals)")
```

```r
grid.arrange(p1, p2, nrow = 2)
```




```r
summary(model1)
```

```

Call:
lm(formula = Sale_Price ~ Gr_Liv_Area, data = ames_train)

Residuals:
    Min      1Q  Median      3Q     Max 
-361143  -30668   -2449   22838  331357 

Coefficients:
            Estimate Std. Error t value            Pr(>|t|)    
(Intercept) 8732.938   3996.613   2.185               0.029 *  
Gr_Liv_Area  114.876      2.531  45.385 <0.0000000000000002 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 56700 on 2051 degrees of freedom
Multiple R-squared:  0.5011,    Adjusted R-squared:  0.5008 
F-statistic:  2060 on 1 and 2051 DF,  p-value: < 0.00000000000000022
```

```
[1] 56704.78
```

```
[1] 3215432370
```

Inference

The variability of an estimate is its *standard error (SE)*, the square root of its variance.

t-test for the coefficents are simply the estimated coefficent divided by the standard error (t value = Estimate / Std. Error)

t-test measure the number of standard deviations each coefficent is away from zero (basically abs(T) > 2 is significant at 95% conf)

The confidence interval for coefficents is:

$$\hat{\beta}_j \pm t_{1-\alpha/2,n-p} \hat{SE}(\hat{\beta}_j)$$

```r
confint(model1, level = .95)
```

```
                2.5 %     97.5 %
(Intercept) 895.0961 16570.7805
Gr_Liv_Area 109.9121   119.8399
```

Interpretation: We are 95% confident that each one unit increase in Gr_Liv_Area adds between 109.9 and 119.8 dollars to the sale price.

Linear Regression Assumptions:

- 1.) Independent observations
- 2.) The random errors have mean zero, and constant variance
- 3.) The random errors are normally distributed

Multiple Linear Regression

```r
(model2 <- lm(Sale_Price ~ Gr_Liv_Area + Year_Built, data = ames_train))
```

```

Call:
lm(formula = Sale_Price ~ Gr_Liv_Area + Year_Built, data = ames_train)

Coefficients:
(Intercept)  Gr_Liv_Area   Year_Built
-2123054.21        99.18      1093.48
```

```r
# Equivalent
(model2 <- update(model1, . ~ . + Year_Built))
```

```
Call:
lm(formula = Sale_Price ~ Gr_Liv_Area + Year_Built, data = ames_train)

Coefficients:
(Intercept)  Gr_Liv_Area   Year_Built
-2123054.21        99.18      1093.48
```

```r
round(coef(model2), 3)
```

```
 (Intercept)  Gr_Liv_Area   Year_Built
-2123054.207       99.176     1093.485
```

```r
summary(model3 <- lm(Sale_Price ~ Gr_Liv_Area + Year_Built + Gr_Liv_Area:Year_Built, data = ame
```

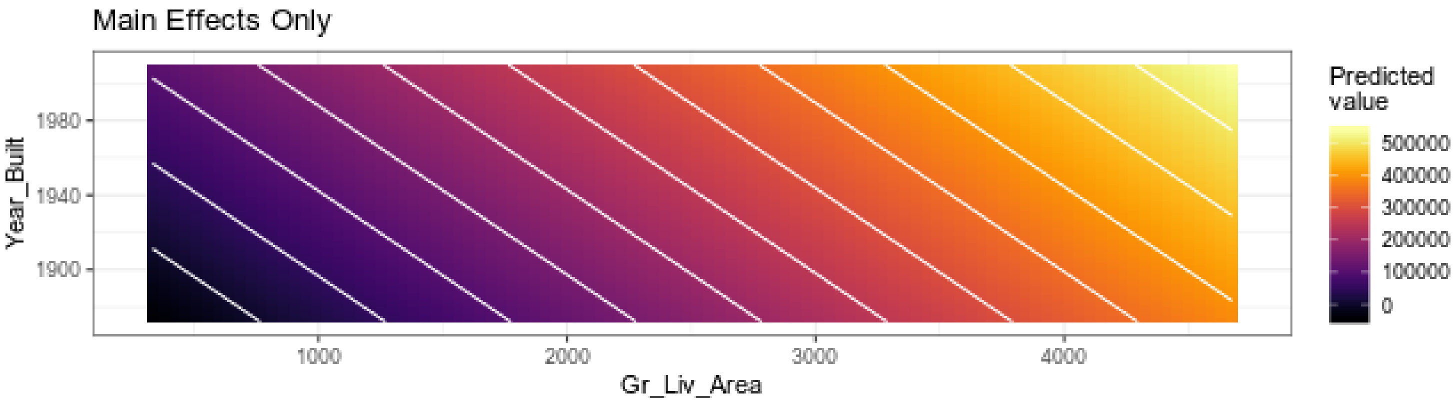
```
Call:
lm(formula = Sale_Price ~ Gr_Liv_Area + Year_Built + Gr_Liv_Area:Year_Built,
    data = ames_train)

Residuals:
    Min      1Q  Median      3Q     Max
-440543  -25191   -1896   17599  281542

Coefficients:
                           Estimate   Std. Error t value            Pr(>|t|)
(Intercept)            382194.30149 209192.64043   1.827              0.0678
Gr_Liv_Area             -1483.88103    125.65052 -11.810 <0.0000000000000002
Year_Built               -179.79795    106.40443  -1.690              0.0912
Gr_Liv_Area:Year_Built      0.80371      0.06378  12.601 <0.0000000000000002

(Intercept)            .
Gr_Liv_Area            ***
Year_Built             .
Gr_Liv_Area:Year_Built ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 44810 on 2049 degrees of freedom
Multiple R-squared:  0.6887,    Adjusted R-squared:  0.6883
F-statistic:  1511 on 3 and 2049 DF,  p-value: < 0.00000000000000022
```

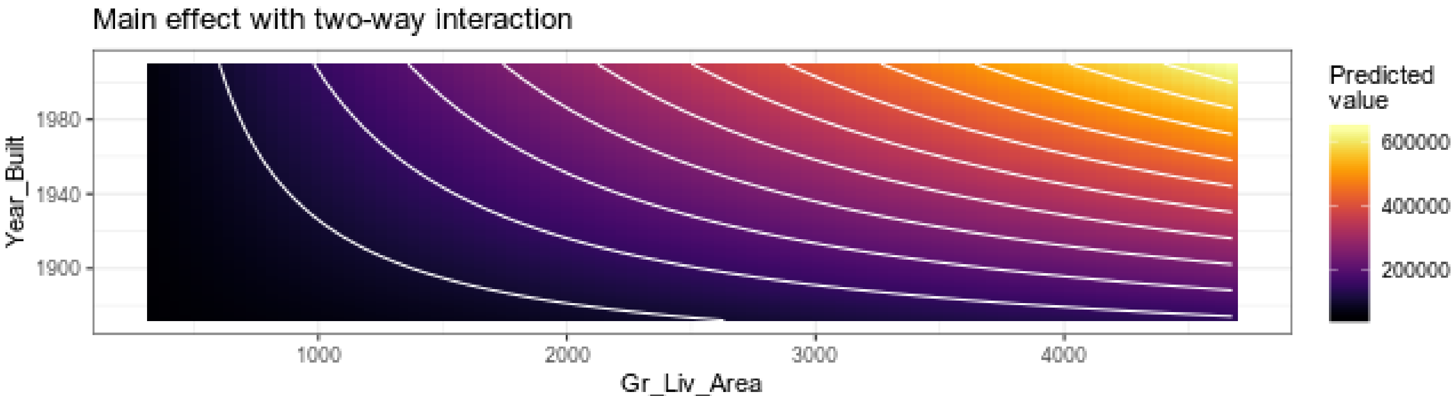
```r
round(coef(model3), 3)
```

```
           (Intercept)            Gr_Liv_Area             Year_Built
            382194.301              -1483.881               -179.798
Gr_Liv_Area:Year_Built
```

```
            0.804
```

```
fit1 <- lm(Sale_Price ~ Gr_Liv_Area + Year_Built, data = ames_train)
fit2 <- lm(Sale_Price ~ Gr_Liv_Area * Year_Built, data = ames_train)

# Regression plane
plot_grid <- expand.grid(
   Gr_Liv_Area = seq(from = min(ames_train$Gr_Liv_Area), to = max(ames_train$Gr_Liv_Area),
                     length = 100),
   Year_Built = seq(from = min(ames_train$Year_Built), to = max(ames_train$Year_Built),
                     length = 100)
)

plot_grid$y1 <- predict(fit1, newdata = plot_grid)
plot_grid$y2 <- predict(fit2, newdata = plot_grid)

# Level plots
p1 <- ggplot(plot_grid, aes(x = Gr_Liv_Area, y = Year_Built,
                      z = y1, fill = y1)) +
   geom_tile() +
   geom_contour(color = "white") +
   viridis::scale_fill_viridis(name = "Predicted\nvalue", option = "inferno") +
   theme_bw() +
   ggtitle("Main Effects Only")

p2 <- ggplot(plot_grid, aes(x = Gr_Liv_Area, y = Year_Built,
                      z = y2, fill = y2)) +
   geom_tile() +
   geom_contour(color = "white") +
   viridis::scale_fill_viridis(name = "Predicted\nvalue", option = "inferno") +
   theme_bw() +
   ggtitle("Main effect with two-way interaction")

gridExtra::grid.arrange(p1, p2, nrow = 2)
```





Full Model

```
model3 <- lm(Sale_Price ~ ., data = ames_train)
broom::tidy(model3)
```

```
# A tibble: 283 x 5
   term                                    estimate std.error statistic p.value
   <chr>                                      <dbl>     <dbl>     <dbl>   <dbl>
 1 (Intercept)                              -5.61e6 11261881.    -0.498   0.618
 2 MS_SubClassOne_Story_1945_and_Older       3.56e3     3843.     0.926   0.355
 3 MS_SubClassOne_Story_with_Finished_Atti…  1.28e4    12834.     0.997   0.319
 4 MS_SubClassOne_and_Half_Story_Unfinishe…  8.73e3    12871.     0.678   0.498
 5 MS_SubClassOne_and_Half_Story_Finished_…  4.11e3     6226.     0.660   0.509
 6 MS_SubClassTwo_Story_1946_and_Newer      -1.09e3     5790.    -0.189   0.850
 7 MS_SubClassTwo_Story_1945_and_Older       7.14e3     6349.     1.12    0.261
 8 MS_SubClassTwo_and_Half_Story_All_Ages   -1.39e4    11003.    -1.27    0.206
 9 MS_SubClassSplit_or_Multilevel           -1.15e4    10512.    -1.09    0.276
10 MS_SubClassSplit_Foyer                   -4.39e3     8057.    -0.545   0.586
# … with 273 more rows
```

Assessing Model Accuracy

Models 1/2/3:

```r
# Train model using 10-fold cross-validation

set.seed(123) # for reproducibility

(cv_model <- train(
   form = Sale_Price ~ Gr_Liv_Area,
   data = ames_train,
   method = "lm",
   trControl = trainControl(method = "cv", number = 10)
))
```

```
Linear Regression

2053 samples
   1 predictor

No pre-processing
Resampling: Cross-Validated (10 fold)
Summary of sample sizes: 1846, 1848, 1848, 1848, 1848, 1848, ...
Resampling results:

  RMSE      Rsquared   MAE
  56410.89  0.5069425  39169.09

Tuning parameter 'intercept' was held constant at a value of TRUE
```

```r
set.seed(123)

cv_model2 <- train(
   Sale_Price ~ Gr_Liv_Area + Year_Built,
   data = ames_train,
   method = "lm",
   trControl = trainControl(method = "cv", number = 10)
)

set.seed(123)

suppressWarnings({
   cv_model3 <- train(
      Sale_Price ~ .,
      data = ames_train,
      method = "lm",
      trControl = trainControl(method = "cv", number = 10)
   )
})
```

Accuracy:

```
Call:
summary.resamples(object = resamples(list(model1 = cv_model, model2
 = cv_model2, model3 = cv_model3)))

Models: model1, model2, model3
Number of resamples: 10

MAE
           Min.  1st Qu.   Median     Mean  3rd Qu.     Max. NA's
model1 34457.58 36323.74 38943.81 39169.09 41660.81 45005.17    0
model2 28094.79 30594.47 31959.30 32246.86 34210.70 37441.82    0
model3 12458.27 15420.10 16484.77 16258.84 17262.39 19029.29    0

RMSE
           Min.  1st Qu.   Median     Mean  3rd Qu.     Max. NA's
model1 47211.34 52363.41 54948.96 56410.89 60672.31 67679.05    0
model2 37698.17 42607.11 45407.14 46292.38 49668.59 54692.06    0
model3 20844.33 22581.04 24947.45 26098.00 27695.65 39521.49    0

Rsquared
            Min.   1st Qu.    Median      Mean   3rd Qu.      Max. NA's
model1 0.3598237 0.4550791 0.5289068 0.5069425 0.5619841 0.5965793    0
model2 0.5714665 0.6392504 0.6800818 0.6703298 0.7067458 0.7348562    0
model3 0.7869022 0.9018567 0.9104351 0.8949642 0.9166564 0.9303504    0
```

Model Concerns / Assumptions

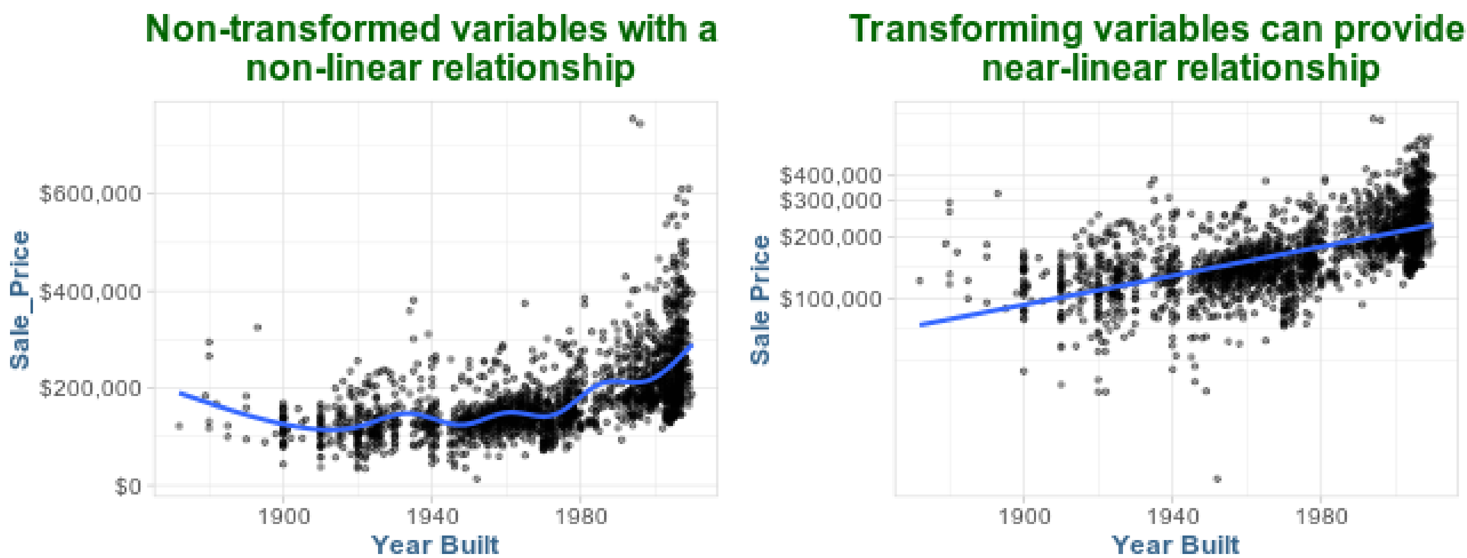
1.) Linear Relationships

Possible solution to non-linear relationship is by variable transformations:

```r
p1 <- ggplot(ames_train, aes(Year_Built, Sale_Price)) +
   geom_point(size = 1, alpha = .4) +
   geom_smooth(se = F) +
   scale_y_continuous(labels = scales::dollar) +
   xlab("Year Built") +
   ggtitle(paste("Non-transformed variables with a \n", "non-linear relationship"))

p2 <- ggplot(ames_train, aes(Year_Built, Sale_Price)) +
   geom_point(size = 1, alpha = .4) +
   geom_smooth(se = F, method = "lm") +
   scale_y_log10("Sale Price", labels = scales::dollar, breaks = seq(0, 400000, by = 100000)) +
```

```
  xlab("Year Built") +
  ggtitle(paste("Transforming variables can provide a \n", "near-linear relationship"))

gridExtra::grid.arrange(p1, p2, nrow = 1)
```

```
`geom_smooth()` using method = 'gam' and formula 'y ~ s(x, bs = "cs")'
```



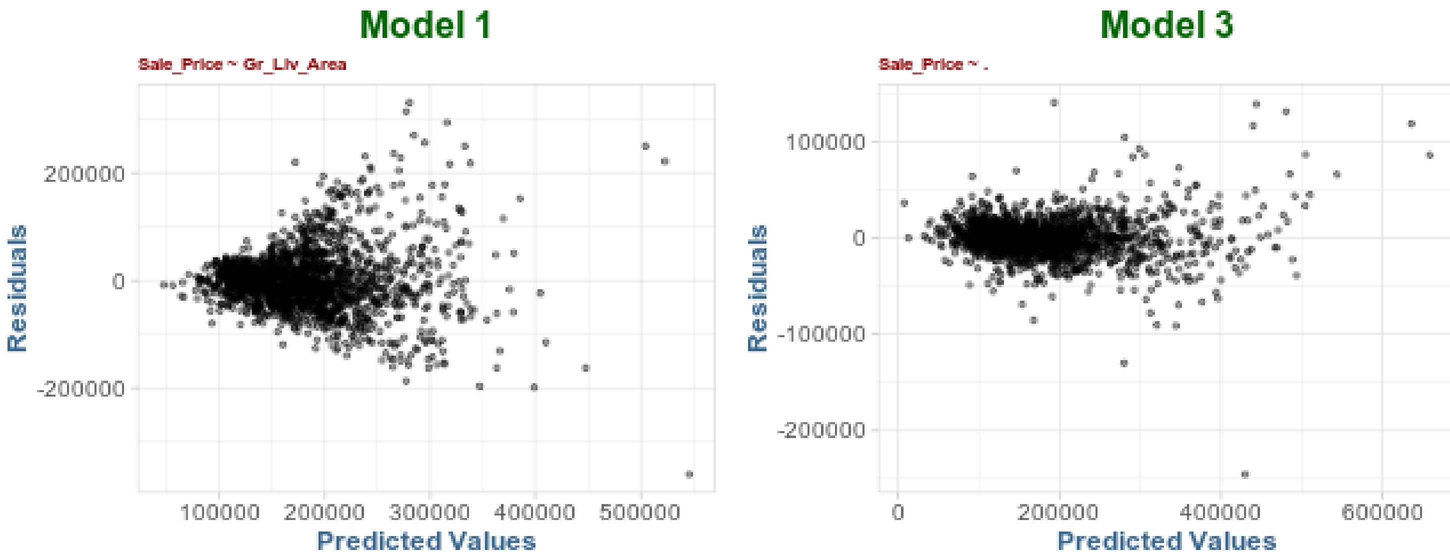
2.) Constant variance among residuals

Linear models assume constant variance amoung error terms (*homoscedasticity*).

Notice the cone shape in Model 1:

```
df1 <- broom::augment(cv_model$finalModel, data = ames_train)

p1 <- ggplot(df1, aes(.fitted, .resid)) +
  geom_point(size = 1, alpha = .4) +
  xlab("Predicted Values") +
  ylab("Residuals") +
  ggtitle("Model 1", subtitle = "Sale_Price ~ Gr_Liv_Area")

df2 <- broom::augment(cv_model3$finalModel, data = ames_train)

p2 <- ggplot(df2, aes(.fitted, .resid)) +
  geom_point(size = 1, alpha = .4) +
  xlab("Predicted Values") +
  ylab("Residuals") +
  ggtitle("Model 3", subtitle = "Sale_Price ~ .")
```

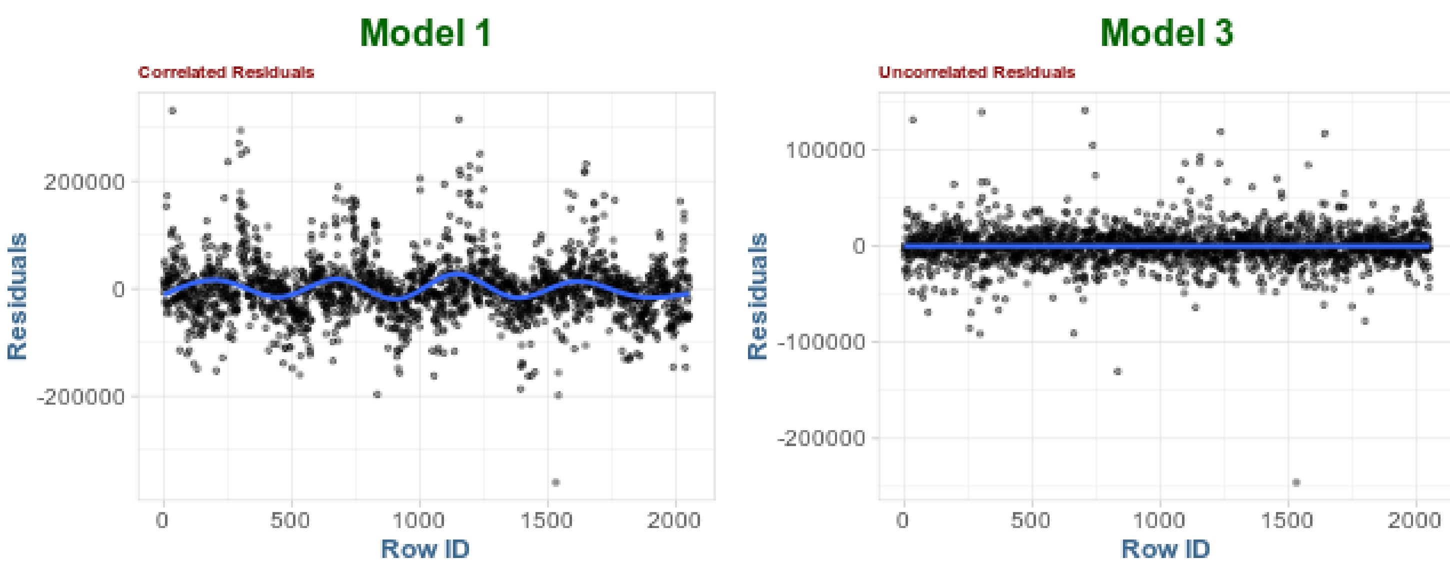
```r
gridExtra::grid.arrange(p1, p2, nrow =1)
```



3.) No autocorrelation

Residuals should be uncorrelated and i.i.d.

```
`geom_smooth()` using method = 'gam' and formula 'y ~ s(x, bs = "cs")'
`geom_smooth()` using method = 'gam' and formula 'y ~ s(x, bs = "cs")'
```



Model 1 has a distict pattern to the residuals (due to being ordered by neighborhood, which is unaccounted for in the model)