Airmass Calculator

This calculator was designed to calculate the airmass X that an observer at non-zero elevations y_{obs} (height above sea-level) would measure for objects at various altitudes α (angular heights above the horizon) compared to an observer at sea-level. By definition, the airmass for a target at the zenith is unity at sea-level, and we define relative airmass such that $X < 1$ for targets at the zenith at elevations $y_{obs} > 0$.

Some definitions: We define the zenith angle $z = \pi/2 - \alpha$, the radius of the earth $R_e = 6371$ km, the height above sea-level of the observer y_{obs}, the height above sea-level where the atmosphere becomes negligible $y_{atm} = 100$ km (the "Karman line"), the atmospheric density ρ which is a function of elevation, the column density σ and the path length of light through the atmosphere to the observer s.

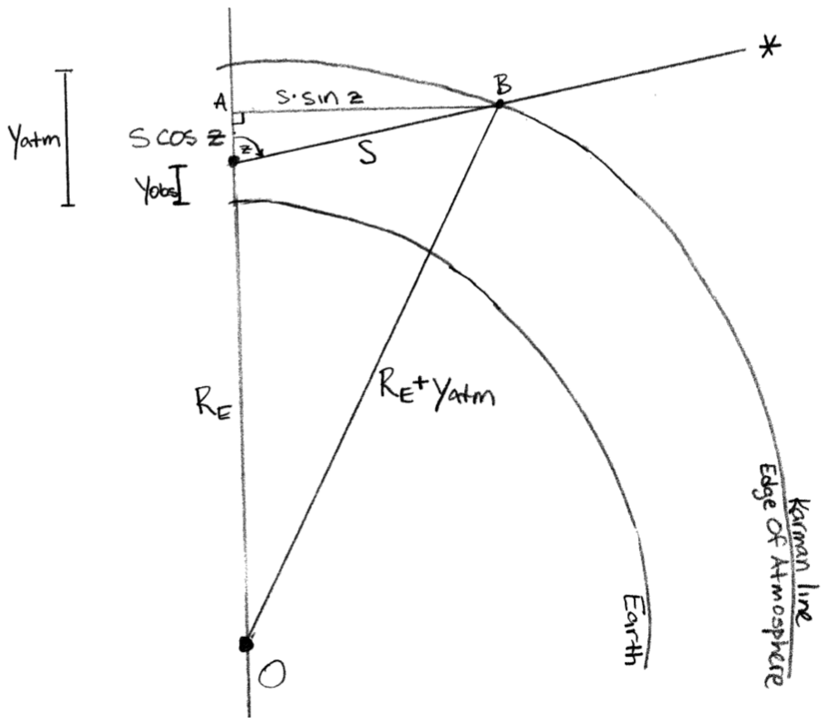

Figure 1: Geometric defintions

From triangle OAB, we see that

$$(R_e + y_{obs} + s\cos z)^2 + (s\sin z)^2 = (R_e + y_{atm})^2 \tag{1}$$

Manipulating this relation we can find the quadratic in s

$$s^2 + 2\cos z(R_e + y_{obs})s + y_{obs}^2 - y_{atm}^2 + 2R_e(y_{obs} - y_{atm}) = 0. \tag{2}$$

The roots are

$$s = \pm\sqrt{(R_e + y_{obs})^2\cos^2 z - y_{obs}^2 + y_{atm}^2 - 2R_e(y_{obs} - y_{atm})} - (R_e + y_{obs})\cos z, \tag{3}$$

and we take only the positive solution since we are solving for a positive path length s.

The column density of atmosphere along the path s for an atmosphere of variable density ρ is

$$\sigma = \int \rho \, ds. \tag{4}$$

Notice that in Eqn. 3 we have found $s = s(y_{atm})$, we can say more generally that the path length to a given height in the atmosphere y is

$$s(y) = \sqrt{(R_e + y_{obs})^2 \cos^2 z - y_{obs}^2 + y^2 - 2R_e(y_{obs} - y)} - (R_e + y_{obs}) \cos z, \tag{5}$$

and we can change variables in Eqn. 4 to exchange s for height in the atmosphere y, using $ds = \frac{ds}{dy} dy$,

$$\frac{ds}{dy} = \frac{R_e + y}{\sqrt{(R_e + y_{obs})^2 \cos^2 z - y_{obs}^2 + y^2 - 2R_e(y_{obs} - y)}}, \tag{6}$$

$$\sigma(y_{obs}, z) = \int_{y_{obs}}^{y_{atm}} \frac{\rho(y) \;\; (R_e + y) \;\; dy}{\sqrt{(R_e + y_{obs})^2 \cos^2 z - y_{obs}^2 + y^2 - 2R_e(y_{obs} - y)}} \tag{7}$$

The density of the atmosphere as a function of height $\rho(y)$ is interpolated from the US Standard Atmosphere (1976 version).

The relative airmass X given by the calculator for a target at zenith angle z_0, for an observer at elevation y_0 is

$$X = \frac{\sigma(y_{obs} = y_0, z = z_0)}{\sigma(y_{obs} = 0, z = 0)}. \tag{8}$$

Below we compare this definition of X (labelled "Morris") to the simple $X \approx \sec z$ approximation often used, as in the package "SkyCalc":

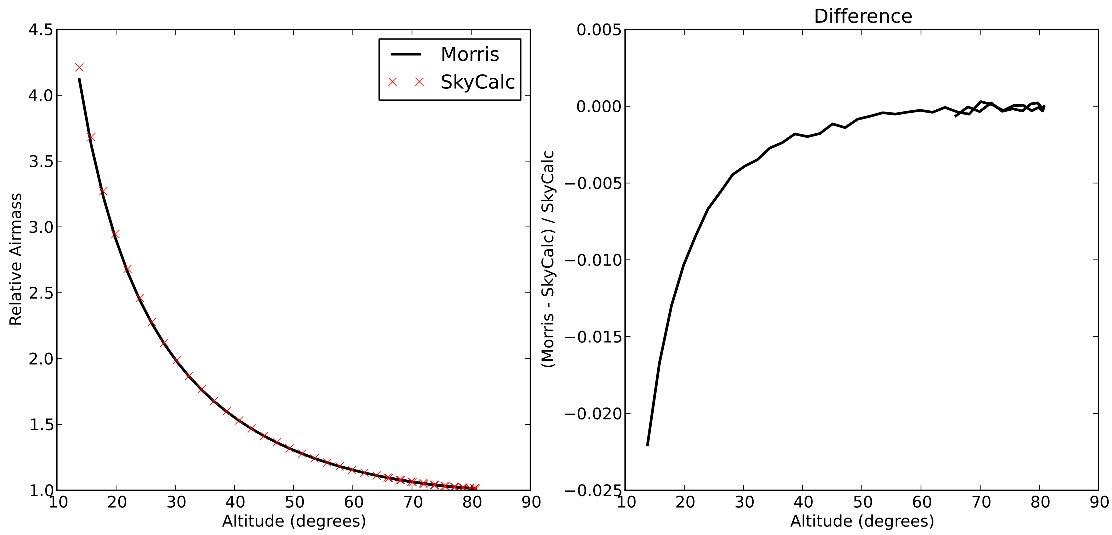

Figure 2: Validation by comparison to the common approximation $X \approx \sec z$.