

OSKAR Theory of Operation

Version history:

Revision	Date	Modification
1	2012-04-27	Creation.
2	2012-05-03	Clarified polarisation convention, updated definition of R and G matrices.
3	2012-05-14	Added polarisation examples.
4	2012-10-22	Added description of the addition of uncorrelated system noise.
5	2013-11-26	Fixed description of interferometer phase term.
6	2014-08-12	Updated description of addition of uncorrelated noise.

1 Introduction

This document describes the methods that OSKAR-2 uses to perform simulations, and all users of OSKAR-2 should read it carefully. If there are any problems, please notify us by sending an email to oskar@oerc.ox.ac.uk so that the simulation software can be updated.

2 Interferometer Simulation

OSKAR-2 uses the Radio Interferometer Measurement Equation of Hamaker, Bregman & Sault (1996) to produce simulated visibility data. This equation describes the effect due to all visible sources (indexed by s) on the complex, polarised visibility $\langle \mathbf{V}_{p,q} \rangle$ for a baseline between stations p and q . The time average, as denoted by the angle brackets, is implicitly derived from the source Stokes parameters, since they are also a measure of the average source polarisation.

The main terms in the Measurement Equation are Jones matrices: these are two-by-two complex quantities that effectively modify the original signal from each radio source, to give the signal that would be produced from each station due to that source. Each source has an additive effect on the measured visibilities, so the observed visibility on baseline p, q is given by the sum over all visible sources of the product of each set of Jones matrices with the source coherency matrix $\langle \mathbf{B} \rangle$. The Measurement Equation currently implemented in OSKAR-2 takes the form

$$\langle \mathbf{V}_{p,q} \rangle = \sum_s \mathbf{K}_{p,s} \mathbf{E}_{p,s} \mathbf{G}_{p,s} \mathbf{R}_{p,s} \langle \mathbf{B}_s \rangle \mathbf{R}_{q,s}^H \mathbf{G}_{q,s}^H \mathbf{E}_{q,s}^H \mathbf{K}_{q,s}^H,$$

where the superscript-H denotes Hermitian transpose.

The Jones matrices currently included in OSKAR-2 are:

- Parallactic angle rotation matrix (\mathbf{R})
- Element factor, or antenna field pattern matrix (\mathbf{G}). This is factored out where possible, but is strictly part of the station beam response.
- Array factor, or station beamforming matrix (\mathbf{E})
- Interferometer phase matrix (\mathbf{K})

For the following sections, in which the Jones terms are described, it will be helpful to introduce the coordinate systems used.

2.1 Coordinate Systems

Sources are specified in the equatorial system and have positions Right Ascension α and Declination δ . This spherical system has Cartesian (x', y', z') axes, where the x' axis points towards $\alpha = 0$, the z' axis points towards the North Celestial Pole (NCP) and the y' axis is perpendicular to both to make a right-handed system. The angle α increases from x' towards y' , and the angle δ increases from the $x'y'$ -plane towards z' . The equatorial system is shown in dashed black in Figure 1.

Antennas are specified in the local horizontal coordinate system. This spherical system has Cartesian (x, y, z) axes, where the x -axis points to the observer's geographic East, the y -axis points to geographic North, and the z -axis points to the observer's zenith. The local horizon is therefore in the xy -plane. The angle ϕ is the co-azimuth, and increases from x towards y , and the angle θ is the

polar angle, zenith distance, or co-elevation, which increases from z towards the xy -plane. The horizontal system is shown in red in Figure 1.

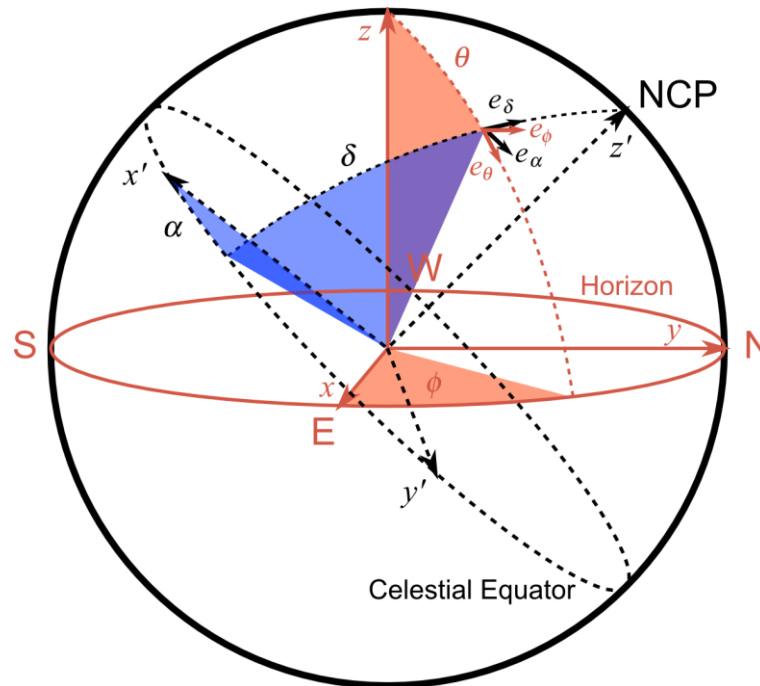


Figure 1: Horizontal and equatorial coordinate systems.

2.2 Source Brightness Matrix (B)

The source brightness (or coherency) matrix represents the intrinsic, unmodified radiation from an astronomical source. It is constructed using the source Stokes parameters (I, Q, U, V) , which completely describe the polarisation state of the radiation. Using the standard polarisation convention adopted by the International Astronomical Union for radio astronomy (IAU, 1974; see Figure 2), the polarisation axes are defined using the tangent plane to a point in the equatorial system. The polarisation angle is measured due east (counter-clockwise) from the direction to the North Celestial Pole, so that 100% Stokes +Q corresponds to North-to-South polarisation, 100% Stokes +U corresponds to North-East-to-South-West polarisation, and 100% Stokes +V corresponds to right-handed circular polarisation.

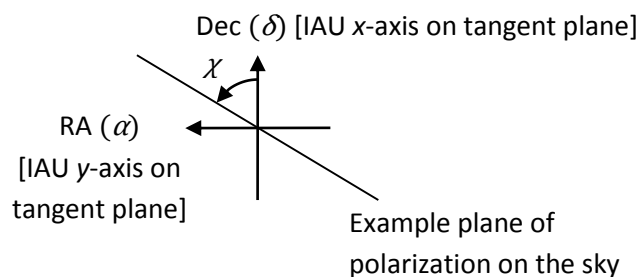


Figure 2: Polarisation axes in the equatorial system (IAU convention).

Using this convention, Hamaker & Bregman (1996) show that

$$\langle \mathbf{B} \rangle = \begin{bmatrix} I + Q & U + iV \\ U - iV & I - Q \end{bmatrix}.$$

2.3 Parallactic Angle Rotation (R)

The emission from each source must ultimately be expressed in the antenna frame, so the equatorial Stokes parameters must be transformed from components along the $(\hat{\mathbf{e}}_\delta, \hat{\mathbf{e}}_\alpha)$ directions to components along the $(\hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi)$ directions. This involves a rotation (intermixing) of only the Stokes Q and U parameters, using the parallactic angle at the position of the source: Stokes I and V remain unchanged.

The parallactic angle at a source position is defined as the angle between the direction of the North Celestial Pole and the local vertical on the sky (measured from north toward east), and depends on the observer's latitude φ and the source hour angle H and declination δ . The parallactic angle is

$$\psi_p = \arctan\left(\frac{\cos(\varphi) \sin(H)}{\sin(\varphi) \cos(\delta) - \cos(\varphi) \sin(\delta) \cos(H)}\right),$$

and the \mathbf{R} matrix corresponds to a normal 2D rotation by the parallactic angle.

$$\mathbf{R} = \begin{bmatrix} \cos(\psi_p) & -\sin(\psi_p) \\ \sin(\psi_p) & \cos(\psi_p) \end{bmatrix}.$$

2.4 Element Factor, or Antenna Field Pattern (G)

The polarisation response of each antenna is the main component of the system that will corrupt the true source polarisation. Assuming two dipoles labelled X and Y, which have their respective axes nominally along the x and y axes shown in Figure 1, then the matrix consists of four complex values that correspond to the average response of the X and Y dipoles in the θ and ϕ directions for all the antennas in the station. These four values are obtained by evaluating the fitted antenna patterns at the θ and ϕ positions of the source.

$$\mathbf{G} = \begin{bmatrix} g_\theta^X & g_\phi^X \\ g_\theta^Y & g_\phi^Y \end{bmatrix}$$

The θ -coordinate appears first here, since in the limit of $\theta = \phi = \psi_p = 0$ (near the zenith of the Earth's North Pole, where the equatorial and horizontal systems are effectively aligned), the $\hat{\mathbf{e}}_\theta$ -direction is parallel to a dipole on the ground oriented along the x -axis, and at this point the $\hat{\mathbf{e}}_\theta$ -direction is also anti-parallel to the $\hat{\mathbf{e}}_\delta$ -direction, which is the axis of zero polarisation according to the IAU (1974) definition.

See the examples in Section 2.8 for further discussion on this topic.

2.5 Array Factor (E)

The shape of the station beam is primarily governed by the projected spacing between individual antennas (the array factor), which is polarisation-independent. The E-Jones matrix in this case is a scalar matrix, which corresponds to the required value of the complex beam pattern: this is given by the weighted (beam-formed) sum of all antennas in the station, evaluated at the source position.

$$\mathbf{E} = e \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Using all antennas a , the complex scalar value e at the source position is given by the sum of the complex beamforming weights w multiplied by the complex antenna signals S due to the test source:

$$e = \sum_a w_a S_a$$

If the antennas in the station cannot be described by an average element pattern, then the \mathbf{G} matrix cannot be treated separately, and it is absorbed in the computation of \mathbf{E} .

2.6 Interferometer Phase (\mathbf{K})

The interferometer phase matrix depends only on the projected spacing between stations. This is polarisation-independent, so \mathbf{K} is a scalar matrix.

$$\mathbf{K} = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The phase k is (e.g. Thomson, Moran & Swenson, 2001)

$$k = \exp\{-2\pi i[ul + vm + w(n - 1)]\},$$

where u, v, w are the *station* coordinates in the plane perpendicular to the phase centre, and l, m, n are the direction cosines of the source relative to the phase centre. Using the normal conventions in radio astronomy, the u and l directions increase towards the East, the v and m directions increase towards the North, and the w and n directions increase towards the phase centre.

2.7 Visibilities to Stokes Parameters

Having obtained the simulated visibility correlation matrix

$$\langle \mathbf{V}_{p,q} \rangle = \begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} = \begin{bmatrix} I + Q & U + iV \\ U - iV & I - Q \end{bmatrix},$$

the Stokes parameters can then be recovered for the purposes of making images by rearranging the diagonal and off-diagonal elements:

$$I = \frac{1}{2}(XX + YY)$$

$$Q = \frac{1}{2}(XX - YY)$$

$$U = \frac{1}{2}(XY + YX)$$

$$V = -\frac{1}{2}i(XY - YX)$$

Note, however, that this conversion does not involve polarisation calibration in any way: additional corrections for the parallactic angle and antenna response would need to be made in order to recover the true source polarisation.

2.8 Interferometer Polarisation Example

To test that the polarisation simulation behaves as expected, we describe some simple examples where the geometry of the system is known in advance. In all cases, we choose the telescope longitude to be 0 degrees, and the simulation date to be close to the autumnal equinox (21 September 2000), so that the local sidereal time is approximately equal to the solar time. For these examples, we observe at midnight UTC, so that objects with zero right ascension will be on the local meridian.

At a selection of points in the sky and on the Earth, we simulate the observation of four different sources, emitting in pure (100% polarised) +Q, -Q, +U and -U directions in the equatorial (IAU) frame. We then make images of these sources in Stokes Q and U to show how the simulator will corrupt the source polarisation, even when using ideal dipoles.

2.8.1 Telescope near North Pole, observing towards (RA, Dec) = (90, 87)

Let the telescope be sited at latitude 89.9 degrees, observing in the direction of RA = 90 degrees (or 6 hours), Dec = 87 degrees. (These numbers were chosen to be close to the pole and the zenith, but not precisely at these singular points, so that all directions remain well-defined.) Almost everywhere in the sky at this latitude, the parallactic angle is very close to zero: for a source at this declination, \mathbf{R} is almost an identity matrix, so it can be ignored safely. The source at RA = 90 degrees lies in the Easterly direction at this point in time, or along the x -axis in the horizontal frame (see Figure 3, and caption).

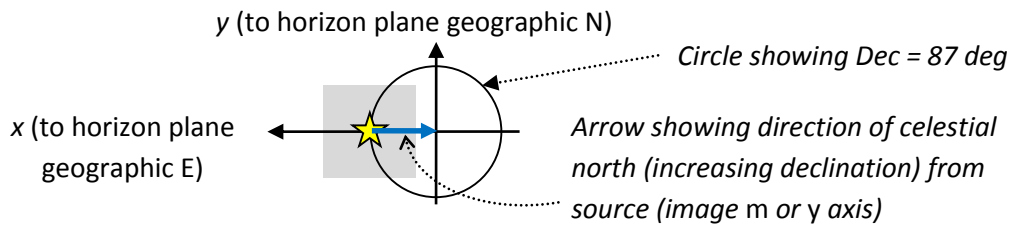


Figure 3: Geometry of source at (RA, Dec) = (90, 87) degrees, looking towards the zenith and pole from the observer's location. The shaded grey square shows an area of the sky that will be imaged using the normal conventions, and the blue arrow shows the direction that would actually appear as 'up' in the image (and hence defines the IAU axis of polarisation for the source, corresponding to Stokes +Q).

When making an image of this source (with the source at the centre), the usual conventions apply: The direction to the North Celestial Pole will be 'up' and the direction of increasing right ascension will be 'left,' so using Figure 3 as a guideline, the marked x -axis would be pointing 'down,' and the blue arrow would be pointing 'up.' (The marked x -axis is in the horizon plane, and is unrelated to the image l -coordinate or baseline u -coordinate.)

Ignoring all except the G-Jones term, the measurement equation for this source is given by:

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} = \begin{bmatrix} g_{\theta}^X & g_{\phi}^X \\ g_{\theta}^Y & g_{\phi}^Y \end{bmatrix} \begin{bmatrix} I + Q & U + iV \\ U - iV & I - Q \end{bmatrix} \begin{bmatrix} g_{\theta}^X & g_{\theta}^Y \\ g_{\phi}^X & g_{\phi}^Y \end{bmatrix}$$

At this source position, the \mathbf{G} matrix is approximately:

$$\mathbf{G} \cong \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The **G** matrix is almost an identity matrix here, because the dipole oriented along the x -axis will respond maximally to radiation polarised in the θ -direction (which is essentially parallel to x for this source), and the dipole oriented along the y -axis will respond maximally to radiation polarised in the ϕ -direction (parallel to y for this source).

If the source emits in pure Stokes +Q ($I = 1, Q = 1$, IAU frame of the sky), then the measured values will be

$$Q_m = \frac{1}{2}(XX - YY) = \frac{1}{2}(I + Q - (I - Q)) \cong \frac{1}{2}(1 + 1 - (1 - 1)) \cong 1$$

$$U_m = \frac{1}{2}(XY + YX) \cong \frac{1}{2}(0 + 0) \cong 0$$

If the source emits in pure Stokes -Q ($I = 1, Q = -1$, IAU frame of the sky), then the measured values will be

$$Q_m = \frac{1}{2}(XX - YY) = \frac{1}{2}(I + Q - (I - Q)) \cong \frac{1}{2}(1 - 1 - (1 - (-1))) \cong -1$$

$$U_m = \frac{1}{2}(XY + YX) \cong \frac{1}{2}(0 + 0) \cong 0$$

If the source emits in pure Stokes +U ($I = 1, U = 1$, IAU frame of the sky), then the measured values will be

$$Q_m = \frac{1}{2}(XX - YY) = \frac{1}{2}(I + Q - (I - Q)) \cong \frac{1}{2}(1 - 1) \cong 0$$

$$U_m = \frac{1}{2}(XY + YX) = \frac{1}{2}(U + U) \cong \frac{1}{2}(1 + 1) \cong 1$$

If the source emits in pure Stokes -U ($I = 1, U = -1$, IAU frame of the sky), then the measured values will be

$$Q_m = \frac{1}{2}(XX - YY) = \frac{1}{2}(I + Q - (I - Q)) \cong \frac{1}{2}(1 - 1) \cong 0$$

$$U_m = \frac{1}{2}(XY + YX) = \frac{1}{2}(U + U) \cong \frac{1}{2}(-1 - 1) \cong -1$$

This is one of the few examples where the **G**-matrix is an identity matrix, and the instrumental response closely corresponds to the intrinsic polarisation properties of the source.

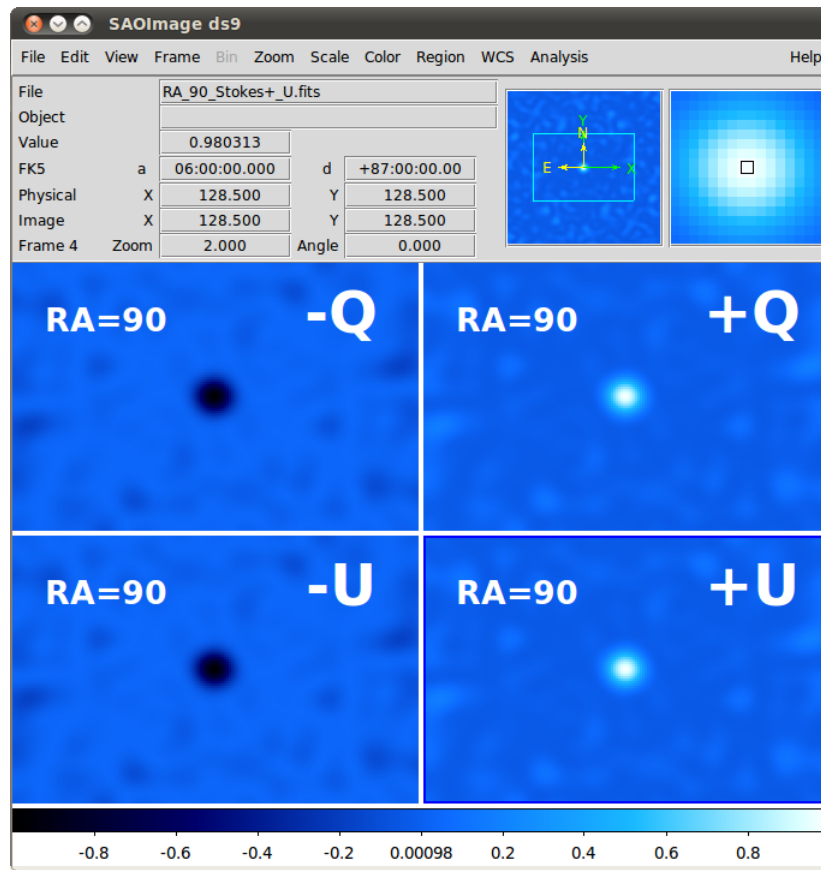


Figure 4: Images of a source at (RA, Dec) = (90, 87) for an observer near the North Pole. The four non-zero polarisations in the images correspond closely to the intrinsic source polarisation, labelled in the top-right of each panel.

2.8.2 Telescope near North Pole, observing towards (RA, Dec) = (0, 87)

Now let the telescope be placed at the same latitude of 89.9 degrees, but observing in the direction of RA = 0 degrees, Dec = 87 degrees. The source at RA = 0 degrees now lies in the Southerly direction, or along the $-y$ -axis in the horizontal frame (see Figure 5). The orientation of each dipole has not changed, but the axis of polarisation angle (as defined by the IAU in the tangent plane to the sky) has now been effectively rotated by 90 degrees with respect to the antennas, so the response of the instrument will no longer correspond to the intrinsic source polarisation.

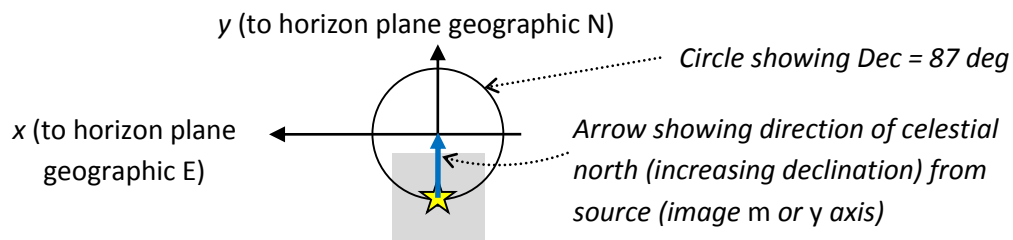


Figure 5: Geometry of source at (RA, Dec) = (0, 87) degrees, looking towards the zenith and pole from the observer's location. The shaded grey square shows an area of the sky that will be imaged using the normal conventions, and the blue arrow shows the direction that would actually appear as 'up' in the image (and hence defines the IAU axis of polarisation for the source, corresponding to Stokes +Q).

Ignoring all except the G-Jones term, the measurement equation for this source is given by:

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} = \begin{bmatrix} g_{\theta}^X & g_{\phi}^X \\ g_{\theta}^Y & g_{\phi}^Y \end{bmatrix} \begin{bmatrix} I + Q & U + iV \\ U - iV & I - Q \end{bmatrix} \begin{bmatrix} g_{\theta}^X & g_{\phi}^Y \\ g_{\theta}^Y & g_{\phi}^X \end{bmatrix}$$

At this source position, the \mathbf{G} matrix is approximately:

$$\mathbf{G} \cong \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The \mathbf{G} matrix takes this form at this point, because the dipole oriented along the x -axis will respond maximally to radiation polarised in the ϕ -direction (which is essentially parallel to x for this source), and the dipole oriented along the y -axis will respond maximally to radiation polarised in the θ -direction (anti-parallel to y for this source, hence the negative sign).

If the source emits in pure Stokes +Q ($I = 1, Q = 1$, IAU frame of the sky), then the measured values will be

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} \cong \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1+1 & 0 \\ 0 & 1-1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong \begin{bmatrix} 0 & 0 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

$$Q_m = \frac{1}{2}(XX - YY) \cong \frac{1}{2}(0 - 2) \cong -1$$

$$U_m = \frac{1}{2}(XY + YX) \cong 0$$

If the source emits in pure Stokes -Q ($I = 1, Q = -1$, IAU frame of the sky), then the measured values will be

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} \cong \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1-1 & 0 \\ 0 & 1+1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$Q_m = \frac{1}{2}(XX - YY) \cong \frac{1}{2}(2 - 0) \cong 1$$

$$U_m = \frac{1}{2}(XY + YX) \cong 0$$

If the source emits in pure Stokes +U ($I = 1, U = 1$, IAU frame of the sky), then the measured values will be

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} \cong \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$Q_m = \frac{1}{2}(XX - YY) \cong \frac{1}{2}(1 - 1) \cong 0$$

$$U_m = \frac{1}{2}(XY + YX) \cong \frac{1}{2}(-1 - 1) \cong -1$$

If the source emits in pure Stokes -U ($I = 1, U = -1$, IAU frame of the sky), then the measured values will be

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} \cong \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cong \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$Q_m = \frac{1}{2}(XX - YY) \cong \frac{1}{2}(1 - 1) \cong 0$$

$$U_m = \frac{1}{2}(XY + YX) \cong \frac{1}{2}(1 + 1) \cong 1$$

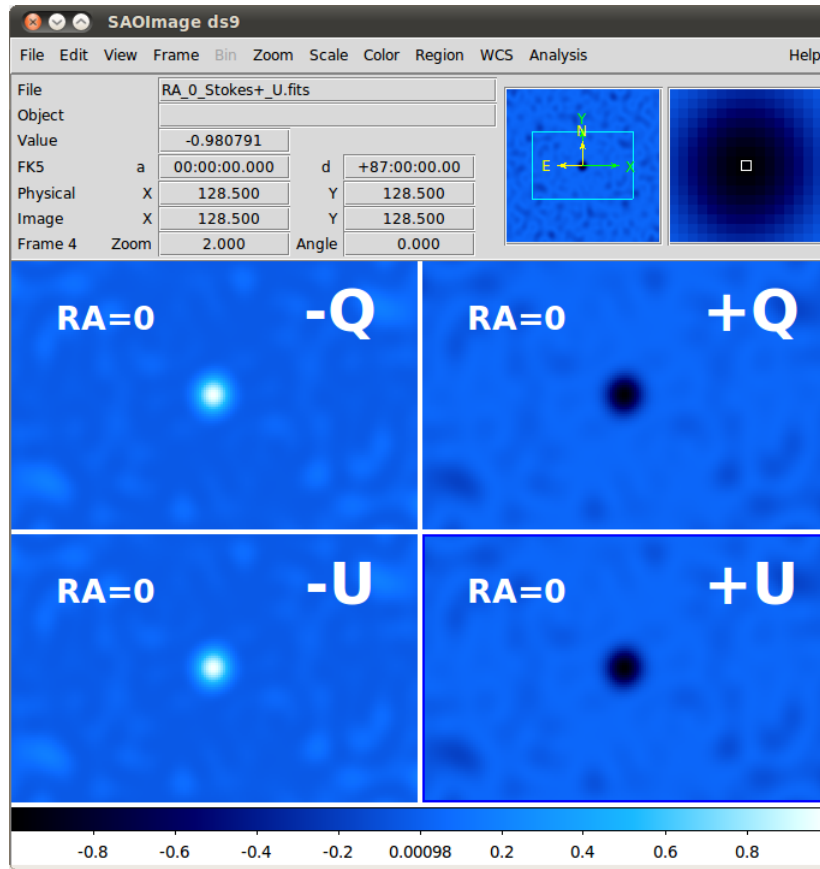


Figure 6: Images of a source at (RA, Dec) = (0, 87) for an observer near the North Pole. The four non-zero polarisations in the images show a sign change with respect to the intrinsic source polarisation, labelled in the top-right of each panel.

2.8.3 Telescope on Equator, observing towards (RA, Dec) = (1, 0)

In this final example, the telescope is at latitude 0 degrees (on the Earth's equator), observing in the direction of RA = 1 degree, Dec = 0 degrees (again, almost directly overhead). The source at RA = 1 degree again lies in the Easterly direction, or along the x -axis in the horizontal frame (see Figure 7). The main difference between this example and that in Section 2.8.1 is that the parallactic angle is no longer nearly zero, but is now -90 degrees.

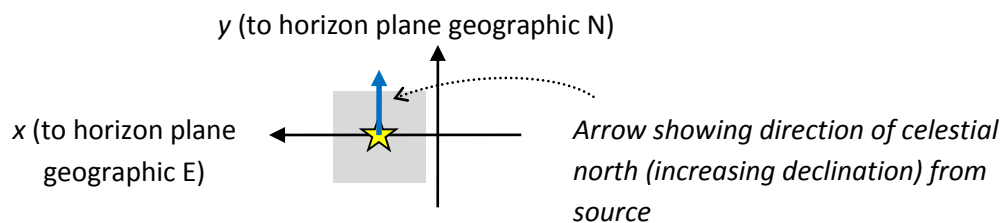


Figure 7: Geometry of source at (RA, Dec) = (1, 0) degrees, looking towards the zenith from the observer's location. The shaded grey square shows an area of the sky that will be imaged using the normal conventions, and the blue arrow shows the direction that would actually appear as 'up' in the image (and hence defines the IAU axis of polarisation for the source, corresponding to Stokes +Q).

Ignoring all except the R- and G-Jones terms, the measurement equation for this source is given by:

$$\begin{bmatrix} XX & XY \\ YX & YY \end{bmatrix} = \begin{bmatrix} g_\theta^X & g_\phi^X \\ g_\theta^Y & g_\phi^Y \end{bmatrix} \begin{bmatrix} \cos(\psi_p) & -\sin(\psi_p) \\ \sin(\psi_p) & \cos(\psi_p) \end{bmatrix} \begin{bmatrix} I + Q & U + iV \\ U - iV & I - Q \end{bmatrix} \begin{bmatrix} \cos(\psi_p) & \sin(\psi_p) \\ -\sin(\psi_p) & \cos(\psi_p) \end{bmatrix} \begin{bmatrix} g_\theta^X & g_\theta^Y \\ g_\phi^X & g_\phi^Y \end{bmatrix}$$

While the **G** matrix is almost equal to the identity matrix for the same reasons as described in Section 2.8.1, the **R** matrix is now given by:

$$\mathbf{R} \cong \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Since **R** in this case is equal to **G** in Section 2.8.2, it follows that the source polarisation will be modified in the same way.

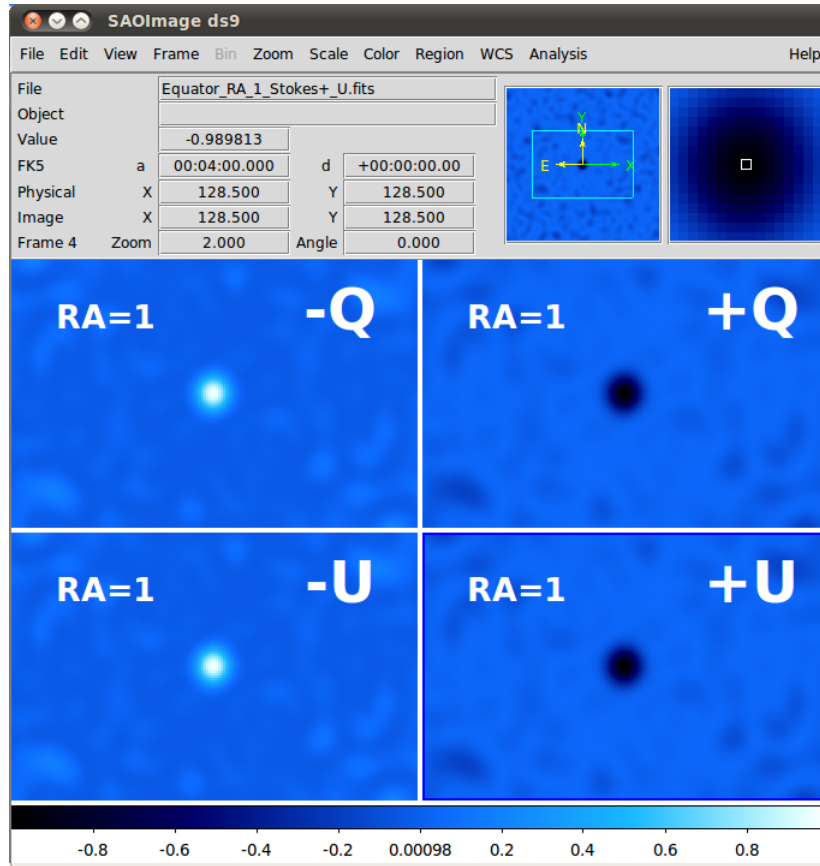


Figure 8: Images of a source at (RA, Dec) = (1, 0) for an observer on the Equator. The four non-zero polarisations in the images show a sign change with respect to the intrinsic source polarisation, labelled in the top-right of each panel.

3 Beam Pattern Polarisation Order

When computing a station beam pattern, the result returned by OSKAR-2 is actually the combination of **EG**, the array factor and the element factor. The four polarisation planes in the image cube are in the same order as that used to construct the **G** matrix, so the order is $X_\theta, X_\phi, Y_\theta, Y_\phi$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} e \cdot g_\theta^X & e \cdot g_\phi^X \\ e \cdot g_\theta^Y & e \cdot g_\phi^Y \end{bmatrix}.$$

4 Addition of Uncorrelated System Noise

When performing an interferometer simulation, OSKAR has the option of adding uncorrelated Gaussian noise. This is implemented by the addition of a complex value drawn from a random Gaussian distribution to the visibility samples for each baseline, time integration, frequency channel and polarisation. The Gaussian distributions are specified by a standard deviation (defined as a function of frequency) for each station in the interferometer, representing the un-polarised sensitivity of that station.

The standard deviation of the Gaussian distributions can be specified in three different ways:

1. As the un-polarised RMS flux density (σ), in Jy.
2. As the system equivalent flux density, or system sensitivity (S_{sys}), in Jy.
3. As the system temperature (T_{sys}), in K, the station effective area ($A_{\text{eff}}^{\text{station}}$), in m^2 , and the system efficiency η .

If specified by methods 2 or 3, the standard deviation is then evaluated based on standard formulae as described by Thomson, Moran & Swenson and Wrobel & Walker.

The noise power per unit bandwidth, received in one polarisation of an antenna from an un-polarised source of system equivalent flux density S_{sys} , is given by

$$k_B T_{\text{sys}} = \frac{S_{\text{sys}} A_{\text{eff}} \eta}{2}$$

Here, T_{sys} is the system temperature, A_{eff} is the effective area of the antenna, η is the system efficiency, and k_B is the Boltzmann constant.

The RMS noise on a given baseline can then be expressed in terms of the system equivalent flux densities S_p and S_q of antennas (or stations) p and q that make up the baseline by

$$\sigma_{p,q} = \sqrt{\frac{S_p S_q}{2 \Delta \nu \tau_{\text{acc}}}}$$

Here, $\Delta \nu$ is the bandwidth and τ_{acc} is the correlator accumulation time. Note the term $2 \Delta \nu \tau_{\text{acc}}$ represents the number of independent samples of the signal for a band-limited signal sampled at the Nyquist rate.

This equation can be re-expressed in terms of the individual system temperatures T_p and T_q , effective areas A_p and A_q and system efficiencies η_p and η_q of antennas (or stations) which make up the baseline as

$$\sigma_{p,q} = k_B \sqrt{\frac{2 T_p T_q}{A_p A_q \eta_p \eta_q \Delta \nu \tau_{\text{acc}}}}$$

Equally, given values of the RMS on individual baselines σ_p and σ_q , the baseline RMS is given by

$$\sigma_{p,q} = \sqrt{\sigma_p \sigma_q}.$$

4.1 Noise in the Synthesised Map

For an array with n_b antenna pairs which observes for a length of time τ_0 , the total number of independent data points in the u, v plane for a single polarisation is

$$n_d = n_b \frac{\tau_0}{\tau_{\text{acc}}}$$

and therefore the noise in the image or map will decrease by a factor $\sqrt{n_d}$.

If we consider the special case where the system temperature, effective area, and system efficiency are the same for an array of n_a antennas observing for total time τ_0 , the following equation describes the total noise in the image plane of a single polarisation image.

$$\sigma_{\text{im}} = \frac{2k_B T_{\text{sys}}}{A_{\text{eff}} \eta \sqrt{n_a(n_a - 1)} \Delta\nu \tau_0}$$

This can be expressed in terms of the RMS noise on a given baseline as

$$\sigma_{\text{im}} = \frac{\sigma_{p,q}}{\sqrt{\frac{n_a(n_a - 1)}{2} \frac{\tau_0}{\tau_{\text{acc}}}}} = \frac{\sigma_{p,q}}{\sqrt{n_d}}$$

Note that for measurements comprised of dual-polarisation data such as Stokes-I,Q,U,V the image RMS will be reduced by a further factor of $\sqrt{2}$.

5 References

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