Assignment 1

Circle detection in binary dot images

1.1 Introduction

The goal of this exercises was to find a points which could form a circle in a black-white image, consisting of random-noise. While the definition of the problem might not sound serious, research for possible solutions carried out.[2] proposes Harmony Search, an optimization method inspired by music.In [1] authors propose randomized algorithm, incorporating also edge detection.

1.2 Approach

To describe approach which was used to solve the problem, firstly some notation should be stated. A point in a 2 dimensional space is defined as follows:

$$p_i = (u_i, k_i)$$

where i is an index of i-th point Equation of a circle is defined as follows:

$$(x - x_i)^2 + (y - y_i)^2 = r^2$$

where i is an index of i-th point

The algorithm is based on collecting all of the points (that is to say, checking which pixel of an image is black, when we operate on white background or white, when the colors are reversed). In the next step random sample of three points is performed. Having three points selected, it is possible to obtain circle equation. Basing on [4] and high school mathematics materials available in the internet, such as [3], following equations are obtained:

$$x_i = \frac{(x_1^2 + y_1^2)(y_2 - y_3) + (x_2^2 + y_2^2)(y_3 - y_1) + (x_3^2 + y_3^2)(y_1 - y_2)}{2(x_1(y_2 - y_3) - y_1(x_2 - x_3) + x_2 y_3 - x_3 y_2)}$$

$$y_i = \frac{(x_1^2 + y_1^2)(x_3 - x_2) + (x_2^2 + y_2^2)(x_1 - x_3) + (x_3^2 + y_3^2)(x_2 - x_1)}{2(x_1(y_2 - y_3) - y_1(x_2 - x_3) + x_2 y_3 - x_3 y_2)}$$

$$r = \sqrt{(x - x_i)^2 + (y - y_i)^2)}$$

In every iteration of the algorithm basing on sampled points a circle equation is estimated. In further step it is verified how many points lay near formed circle. Some treshold must be put to measure closeness to the obtained circle. For this purpose following equation was used:

$$|(x - x_i)^2 + (y - y_i)^2 - r^2| \leq 0.5$$

Formula above describes how far the point could lay from the obtained circle, regardless of direction (inside or outside the circle) to be considered.

Algorithm 1.1: Algorithm pseudocode

Result: Best parameters for circle

Set following variables:

$$max\_number\_of\_points\_near\_circle = 0$$

$$best\_radius = 0$$

$$best\_x = 0$$

$$best\_y = 0$$

for $i \leftarrow 0$ **to** $number\_of\_iterations$ **do**
    Randomly sample three points

$$number\_of\_points\_close\_to\_circle = 0$$

    if *if not collinear* **then**
        Calculate circle equation
        foreach *point in points* **do**
            if *meet criterion* **then**

$$number\_of\_points\_close\_to\_circle + +$$

        end
        if $number\_of\_points\_close\_to\_circle > max\_number\_of\_points\_near\_circle$
         then
            max_number_of_points_near_circle=number_of_points_close_to_circle
            set best_x,best_y,best_radius
end

1.3   Results

1.4   Research question A

Given specified amount of points being total number of points, m and points which belongs to circle n, probability of drawing a 3 circle points in a random draw could be described as follows:

$$p = \frac{\binom{n}{3}}{\binom{m}{3}}$$

1.5 Research question B

The above algorithm could be somehow extended in such way that not only the best one with largest number of points in neighbourhood could be stored but a couple of them. If these circles are different then their parameters, that is center and radius should be different. The first circle with biggest number of points close to it and also with significantly different parameters can be potentially other circle.

References

[1] Yazan Alomari, Siti Norul Huda Sheikh Abdullah, and Khairuddin Omar. "Randomized Circle Detection Performance Based on Image Difficulty Levels and Edge Filters". In: *FIRA RoboWorld Congress*. Springer. 2013, pp. 361–374 (cit. on p. 1).

[2] Jaco Fourie. "Robust circle detection using Harmony Search". *Journal of Optimization* 2017 (2017) (cit. on p. 1).

[3] Robert Eisele. *Create a circle out of three points.* URL: https://www.xarg.org/2018/02/create-a-circle-out-of-three-points/ (cit. on p. 1).

[4] Wikipedia contributors. *Ellipse— Wikipedia, The Free Encyclopedia.* [Online; accessed 22-July-2004]. URL: https://en.wikipedia.org/wiki/Ellipse#Circles (cit. on p. 1).