Hypothesis Testing, Exercise 7.1

If a one tailed test had been done instead, it likely would have used a null hypothesis like below:

Null Hypothesis (H0): The mean impurity for Filter Agent 1 is greater than or equal to the mean impurity for Filter Agent 2 ($\mu 1 \geq \mu 2$).

Alternative Hypothesis (H1): The mean impurity for Filter Agent 1 is less than the mean impurity for Filter Agent 2 ($\mu 1 < \mu 2$). This would mean Filter Agent 1 is more effective.

The conclusion would then depend on the one-tailed p-value. If the mean impurity for Filter Agent 1 was higher than for Filter Agent 2, the result would be declared not significant immediately as the data would be consistent with the null hypothesis, making a p-value calculation unnecessary. However, if the mean impurity for Filter Agent 1 was lower than for Filter Agent 2, I would check the p-value. Should the one-tailed p-value be less than the chosen significance level (e.g., 0.05), I would reject the null hypothesis. The conclusion would be that there is strong evidence to suggest Filter Agent 1 is more effective at reducing impurity than Filter Agent 2.

Hypothesis Testing, Exercise 7.2

F-Test for Variances

Null Hypothesis (H0): The population variances of income for males and females are equal.

Alternative Hypothesis (H1): The population variances of income for males and females are not equal.

F-test results	Males	Females
Count	60	60
Mean Income	54.53	46.85
Variance	205.09	223.36

The resulting F-statistic is 0.918 and the two-tailed p-value is 0.716.

Since the p-value (0.716) is greater than the standard alpha level of 0.05, we do not reject the null hypothesis. The data is consistent with the assumption of equal population variance. Consequently we will use the t-test assuming equal variances to test the hypotheses.

T-Test (Assuming Equal Variances)

The research question is whether male income exceeds female income, so I will use a one-tailed test

Null Hypothesis (H0): The population mean income for males is less than or equal to the mean income for females ($\mu_{males} \leq \mu_{females}$).

Alternative Hypothesis (H1): The population mean income for males is greater than the mean income for females (μmales>μfemales).

T-test results	Value
Difference in Means	7.68
Degrees of Freedom (df)	118
t-statistic	2.583
p-value (one-tail)	0.0055

The one-tailed p-value is 0.0055. This value is less than the 5% significance level ($p < 0.05$) and even the 1% level ($p < 0.01$).

Therefore, I reject the null hypothesis. The results are statistically significant: data suggests that the population mean income for males exceeds that of females. On average, the male income in the sample was $7,680 higher than the female income (assuming income is in thousands).

Assumptions of the Analysis

The validity of this independent samples t-test rests on a couple of assumptions:

1. Independence of Samples: This is assumed to be true based on the data being collected from different individuals but there is possibility that they could correspond to business units that involve some form of monetary function between accounts.
2. Normality: I assume the income data for both the male and female populations are approximately normally distributed for the sake of the exercise but they could be cherrypicked to mislead.

There was another assumption that was validated as part of the test:

1. Equality of Variances: This was validated by the F-test where the p-value of 0.716 was not significant, indicating the data are consistent with this assumption.