HW 11

1) a) $P(10,3) = 10\cdot 9\cdot 8 = \frac{10!}{7!}$

b) $P(21,3)\cdot\binom{18}{7} = \binom{21}{7}\cdot P(14,3) = \frac{21!}{7!\,11!}$

c) $P(13,3)\cdot\binom{18}{7} = \frac{13!\,18!}{10!\,7!\,11!}$

d) $P(21,3)\cdot\binom{18}{7} - 2\cdot 19\cdot\binom{18}{7}$

2) a) Choose $a=b=1$ in the Binomial Theorem: $\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} = (a+b)^n$

$$\sum_{k=0}^{n}\binom{n}{k} = \sum_{k=0}^{n}\binom{n}{k}1\cdot 1 = (1+1)^n = 2^n.$$

b) Count the binary strings of length n.

On one hand, we already know there are 2^n [choose 0/1 independently n times]

On the other, there are $\binom{n}{k}$ strings with exactly k 1's (choose the locations), and every string has some number k of 1's, $0\le k\le n$. So the number of strings is $\sum_{k=0}^{n}\binom{n}{k}$.

c) Base Case: $n=0$. $\sum_{k=0}^{0}\binom{0}{k} = \binom{0}{0} = 1 = 2^0$.

Ind. Hyp. Assume for some n, $\sum_{k=0}^{n}\binom{n}{k} = 2^n$.

Ind. Step.
$$\sum_{k=0}^{n+1}\binom{n+1}{k} \underset{\text{Pascal}}{=} \sum_{k=0}^{n+1}\left[\binom{n}{k-1}+\binom{n}{k}\right] = \sum_{k=0}^{n+1}\binom{n}{k-1} + \sum_{k=0}^{n+1}\binom{n}{k} \overset{j=k-1}{=} \sum_{j=-1}^{n}\binom{n}{j} + \sum_{k=0}^{n+1}\binom{n}{k}$$

$$= \binom{n}{-1} + \sum_{j=0}^{n}\binom{n}{j} + \sum_{k=0}^{n}\binom{n}{k} + \binom{n}{n+1} \qquad \text{splitting off first/last terms}$$

$$= 0 + 2^n + 2^n + 0 \qquad \text{I.H.}$$

$$= 2^{n+1}.$$

So by PMI, $\square$

3) a)

x	1	2	3	4	5
$f(x)$	u	u	u	u	u

is counted, but is not surjective.

b)

x	1	2	3	4	5
$g(x)$	v	w	u	u	u

is not counted, but is surjective.

c)

x	1	2	3	4	5
$h(x)$	u	u	u	v	w

is not counted, but is surjective.

d)

x	1	2	3	4	5
$h(x)$	u	u	u	v	w

is counted more than once:

e.g., once as $x_u = 1,\ x_v = 4,\ x_w = 5$

and again as $x_u = 2,\ x_v = 4,\ x_w = 5$