Thursday, October 4   ⁕⁕   *Constrained min/max via Lagrange multipliers.*

1. Let C be the curve in $\mathbb{R}^2$ given by $x^3 + y^3 = 16$.

    (a) Sketch the curve C.

    (b) Is C bounded?

    (c) Is C closed?

2. Consider the function $f(x, y) = e^{xy}$ on C.

    (a) Is f continuous? What does the Extreme Value Theorem tell you about the existence of global min and max of f on C?

    (b) Use Lagrange multipliers to determine both the min and max values of f on C.

3. Consider the surface S given by $z^2 = x^2 + y^2$

    (a) Sketch S.

    (b) Use Lagrange multipliers to find the points on S that are closest to $(4, 2, 0)$.

4. For the function shown on the back of the sheet, use the level curves to find the locations and types (min/max/saddle) for all the critical points of the function:

$$f(x, y) = 3x - x^3 - 2y^2 + y^4$$

Use the formula for f and the 2$^{\text{nd}}$-derivative test to check your answer.

5. If the length of the diagonal of a rectangular box must be L, what is the largest possible volume?