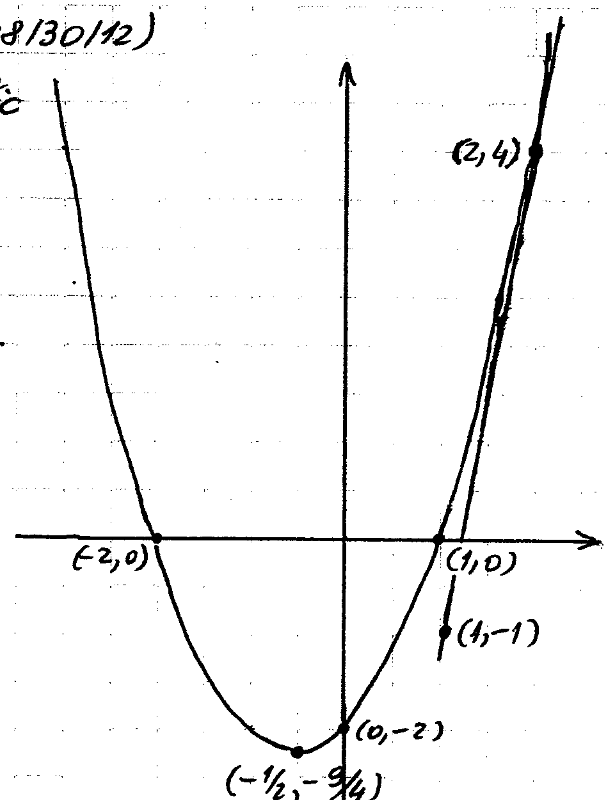
WORKSHEET 2 (08/30/12)



① (a) $y = f(x) = x^2 + x - 2$ is a quadratic function with positive leading coefficient (=1). Its min value is attained at $x = -\frac{1}{2}$ and $f(-\frac{1}{2}) = -\frac{9}{4}$.

x intercepts: $f(x) = 0 \iff x \in \{-2, 1\}$

y intercept: $f(0) = -2$

(b) Tangent line equation when $x = 2$, $y = f(2) = 4$: $f'(x) = 2x+1$

slope $m = f'(2) = 5$, equation:

$$m = \frac{y-4}{x-2} = 5 \iff y = 5x - 6.$$

(c) For a convenient parameterization of the tangent line from (b) take $x - 2 = t$, so $y = y(t) = 4 + 5t$ and

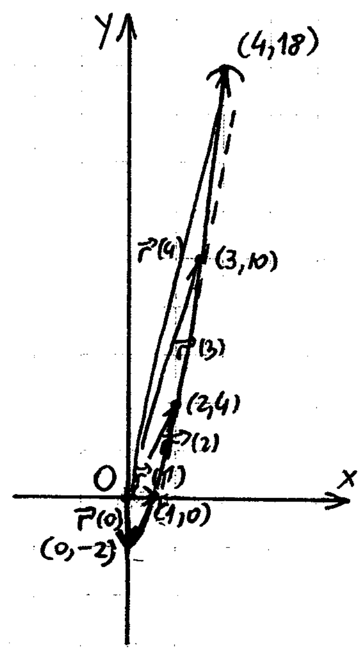
$$\vec{v}(t) = \langle 2+t, 4+5t \rangle = \langle 2,4 \rangle + t\langle 1,5 \rangle \quad ; \quad \vec{v}(-1) = \langle 1,-1 \rangle.$$

② This is a proper subset of the graph in ①(a). Here $\vec{r}(t) = \langle t, t^2+t-2 \rangle$.

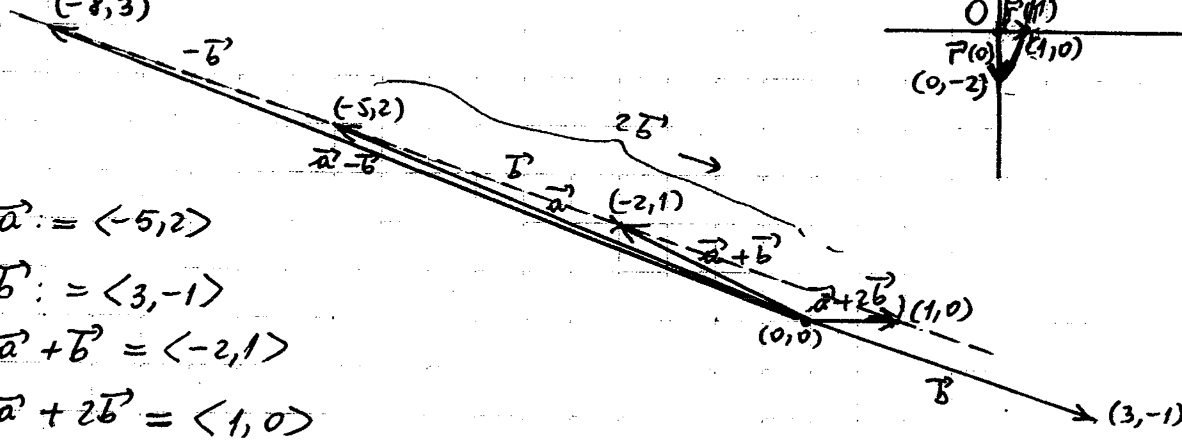
(c) $\vec{r}\,'(t) = \langle 1, 2t+1 \rangle$

$\vec{r}\,'(2) = \langle 1,5 \rangle$ represents a direction for the tangent line through (2,4)

(d) $s = |\vec{r}\,'(2)| = \sqrt{26}$



③



$\vec{a} := \langle -5, 2 \rangle$

$\vec{b} := \langle 3, -1 \rangle$

$\vec{a} + \vec{b} = \langle -2, 1 \rangle$

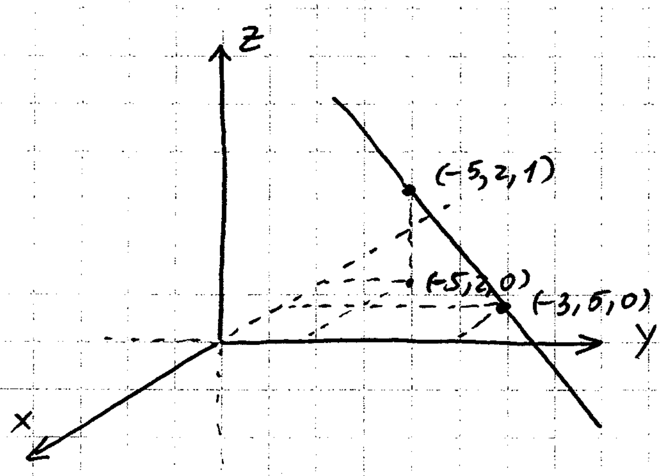
$\vec{a} + 2\vec{b} = \langle 1, 0 \rangle$

$\vec{a} - \vec{b} = \langle -8, 3 \rangle.$

(e) The straight line determined by the points $(-5,2)$ and $(-2,1)$.

④ (a) $\vec{\ell}(t) = \langle -5+2t, 2+3t, 1-t\rangle = \underbrace{\langle -5,2,1\rangle}_{\vec{p}} + t\underbrace{\langle 2,3,-1\rangle}_{\vec{v}}$.

(b) The straight line determined by the points $(-5,2,1)$ and $(-3,5,0)$



(c) All vectors with tail at $(-5,2,1) = \vec{\ell}(0)$ and head on the line from (b) are of the form $t\vec{v}$, so $\vec{v}$ describes the direction of this line

⑤ $\vec{a} = (-\sqrt{3}, 0, -1, 0)$, $\vec{b} = (1, 1, 0, 1)$

$\vec{a} - \vec{b} = (-\sqrt{3}-1, 0-1, -1-0, 0-1) = (-\sqrt{3}-1, -1, -1, -1)$

(a) $|\vec{a}-\vec{b}| = \sqrt{(-\sqrt{3}-1)^2+(-1)^2+(-1)^2+(-1)^2} = \sqrt{4+2\sqrt{3}+3} = \sqrt{7+2\sqrt{3}}$

(b) $\vec{a}\cdot\vec{b} = -\sqrt{3}+0+0+0 = -\sqrt{3}$

$|\vec{a}| = \sqrt{3+1} = 2$; $|\vec{b}| = \sqrt{1+1+1} = \sqrt{3}$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \dfrac{-\sqrt{3}}{2\sqrt{3}} = -\dfrac{1}{2} \Longrightarrow \theta = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$.