Math 251 **Name(s):**

PaperAssign 4

Workshop (in-class)

September 25, 2017

Consider the function

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(1) Find $\lim_{(x,y)\to(0,0)} f(x,y)$.

(2) Where is f continuous?

(3) Compute $f_y(x,y)$ when $(x,y) \neq (0,0)$.

(4) Compute $f_y(0,0)$ as follows:
   (a) Fix $x = 0$; what is $f(0,y)$? (It's defined piecewise!)
   (b) So, what is $f_y(0,y)$? (Does this match your answer in (3)?)
   (c) So, what is $f_y(0,0)$?

(5) Compute $f_x(0,0)$ in the same way.

(6) What is the (Cartesian) equation of the tangent plane to f at $(0,0)$?

(7) Consider the slice along the line $y = x$: let $g(x) = f(x,x)$. What is $g'(0)$?

(8) From (6) and (7), we can see that the tangent line to the slice along $y = x$ **does not** lie on the tangent plane. This implies that the function is not differentiable at $(0,0)$.

   By Theorem 8 in the textbook's section 14.4 (the sufficient condition for differentiability that was presented in class), it must be the case that one of f_x and f_y is not continuous at $(0,0)$. Verify that f_y is discontinuous at $(0,0)$ using (3) and (4).