Name: ______________________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have 75 minutes to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

Question:	1	2	3	4	5	6	Total
Points:	15	18	14	18	15	10	90
Score:							

1. (15 points) Consider the function

$$f(x,y) = \begin{cases} \dfrac{7x^3y}{2x^4+y^4} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Where is f continuous?

2. Let $f(x, y) = x^2 + xy^2 + 2y^2$.

   (a) (10 points) Compute each of the following:

      i. f_x

      ii. f_y

      iii. f_{xx}

      iv. f_{xy}

      v. f_{yy}

   (b) (8 points) Find and classify the critical points of f.

3. At right is the temperature T (in Celsius) of the surface of Planet X at coordinates (x, y) at several points.

   A rover is traveling along the path $x(t) = 2t^2$, $y(t) = t^3 - t$, where t is the time in hours since it landed.

T: $y \backslash x$	0	1	2	3	4
3	2	4	5	6	2
2	3	5	6	8	3
1	1	4	3	7	3
0	3	2	4	6	3
−1	6	8	5	4	1

(a) (6 points) Estimate $T_x(2, 0)$ and $T_y(2, 0)$.
Show enough work that I know that you know what you are doing.

(b) (4 points) Write the Chain Rule for computing $\dfrac{dT}{dt}$.

(c) (4 points) Estimate $\dfrac{dT}{dt}$ one hour into the rover's trip. What are the units?

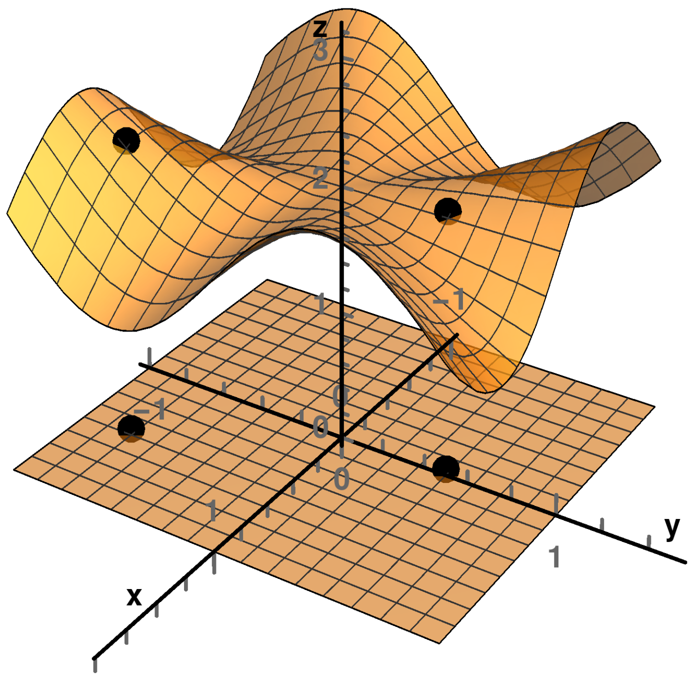
4. Let $f(x, y) = (x - 1)^2 + y^2$, and let D be the solid ellipse defined by $9x^2 + 4y^2 \leq 36$.

   (a) (5 points) Does the Extreme Value Theorem imply here that f attains a maximum and a minimum on D? (Which of the hypotheses are satisfied here?)

   (b) (5 points) Sketch D together with a contour plot of f, and use this to estimate the locations of the maximum and minimum of f on D (if they exist). (Indicate these locations in your drawing.)

   (c) (8 points) Find the precise maximum and minimum of f on D, or say that they do not exist.

5. Below is shown the graph of a function f, the points $P = (0.5, -0.75)$ and $Q = (0, 0.5)$ in the xy-plane, and the lifts of those points to the graph.



(a) (6 points) Sketch $\nabla f(Q)$ (in the xy-plane, in the picture above).

(b) (9 points) Circle below the sign of each partial derivative. Briefly justify each.

$f_{xx}(P)$: $+$ $-$ 0

$f_{yy}(P)$: $+$ $-$ 0

$f_{xy}(Q)$: $+$ $-$ 0

6. (10 points) Circle 'True' or 'False' and give a brief justification.

(a) True False If $\nabla f(a,b)$ exists, then f is differentiable at (a,b).

(b) True False If f is continuous, then the tangent plane to the graph of $f(x,y)$ at a point $(a,b,f(a,b))$ must contain all the tangent lines to the graph at $(a,b,f(a,b))$.

(c) True False If f is differentiable and has a local maximum at (a,b), then $\nabla f(a,b) = \mathbf{0}$.

(d) True False If f is differentiable at (a,b), then it is continuous at (a,b).

(e) True False If f is continuous at (a,b), then it is differentiable at (a,b).

Scratch Paper - Do Not Remove

Scratch Paper - you may remove this if you find it convenient

Scratch Paper - you may remove this if you find it convenient