Math 241 C8 **Name:**

Quiz # 4

March 5, 2013

No electronic devices, notes, or interpersonal communication allowed.

Show work to get credit.

(1) [10pts] Use an appropriate path integral to find the flow of $\mathbf{F}(x,y) = (xy, -x)$ along the $\underbrace{\text{part of the parabola } y = x^2 \text{ from } (0,0) \text{ to } (2,4)}_{C}$.

$$\int_C F \cdot \langle dx, dy \rangle \qquad C: \begin{array}{l} x = t \\ y = t^2 \end{array} \quad t \in [0,2]$$

$$= \int_0^2 \langle (t)(t^2), -(t) \rangle \cdot \langle 1, 2t \rangle \, dt$$

$$= \int_0^2 (t^3 - 2t^2) \, dt$$

$$= \left[\frac{1}{4} t^4 - \frac{2}{3} t^3 \right]_0^2$$

$$= \left(4 - \frac{16}{3}\right) - 0$$

$$= -\frac{4}{3}$$

(2) Consider the vector field $\mathbf{F}(x, y) = (y, x + e^y)$.

(a) [4pts] Find a potential function for $\mathbf{F}$. (Remember, that means an f such that $\nabla f = \mathbf{F}$.)

If $\nabla f = \vec{F}$, then $\partial_x f = y$ ① and $\partial_y f = x + e^y$ ②

① $\Rightarrow f(x,y) = xy + g(y)$ for some g

② $\Rightarrow \partial_y f = x + g'(y) = x + e^y$

$\Rightarrow g'(y) = e^y$

$\Rightarrow g(y) = e^y \; (+C)$

So $f(x,y) = xy + e^y$ is a potential function for $\vec{F}$.

(b) [2pts] Find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the unit circle, counterclockwise.

(a) shows $\vec{F}$ is a gradient field,

and C is a closed curve,

so $\int_C \vec{F} \cdot \vec{dr} = 0$.

(c) [4pts] Find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the first-quadrant part of the unit circle going from $(0, 1)$ to $(1, 0)$.

The fundamental theorem of path integrals, together with (a), gives

$$\int_C \vec{F} \cdot \vec{dr} = f(1,0) - f(0,1)$$

$$= 1 - e.$$