Math 241, Sections BL1 and BL2

Quiz # 3   BDD

October 11, 2012

Solve both exercises. Show work to get credit.

1) [5pts.] Use Lagrange multipliers to find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane

$$x + 9y + 8z = 27.$$

Solution: We wish to maximize $f(x, y, z) = xyz$ subject to the constraint $g(x, y, z) = 27$, where $g(x, y, z) = x + 9y + 8z$. The method of Lagrange multipliers requires us to solve the system of equations

$$\begin{aligned} yz &= \lambda(1) \\ xz &= \lambda(9) \\ xy &= \lambda(8) \\ x + 9y + 8z &= 27. \end{aligned}$$

The problem allows us to assume each of x, y, z is strictly greater than zero. Equations 1 and 2 then imply (since $z > 0$) that $x = 9y$. Equations 1 and 3 (since $y > 0$) imply that $x = 8z$. Thus the last equation gives $3x = 27$, so $x = 9$; then $y = 1$ and $z = 9/8$. In the context of the problem, it is obvious that this must be a maximum.

2) [5pts.] Find the length of the curve

$$\vec{r}(t) = \left\langle 6t, t^2, \frac{t^3}{9} \right\rangle, \quad 0 \le t \le 1.$$

Solution: The length of the curve is

$$\begin{aligned}
\int_0^1 |\mathbf{r}'(t)|\, dt &= \int_0^1 \sqrt{(6)^2 + (2t)^2 + \left(\frac{1}{3}t^2\right)^2}\, dt \\
&= \int_0^1 \sqrt{36 + 4t^2 + \frac{1}{9}t^4}\, dt \\
&= \int_0^1 \sqrt{\left(6 + \frac{1}{3}t^2\right)^2}\, dt \\
&= \int_0^1 \left(6 + \frac{1}{3}t^2\right)\, dt \\
&= \left[6t + \frac{1}{9}t^3\right]_0^1 \\
&= 6 + \frac{1}{9}.
\end{aligned}$$