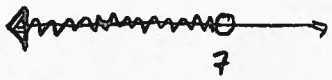
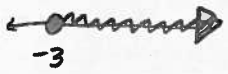
Name: Solutions

The following are roughly the instructions for the real exam.

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have two hours to complete this exam. When you are told to stop writing, do it or you will lose all points on the page(s) you write on.
- You may not communicate with other students during this test.
- Keep your eyes on your own paper.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctor.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers when appropriate.
- Before turning in your exam, check to make certain you've answered all the questions.

1. Complete the following table.

inequality	interval	graph
$x < 7$	$(-\infty, 7)$	
$x \geq -3$	$[-3, \infty)$	
$-4 < x \leq 6$	$(-4, 6]$	

2. Solve the inequality $-3 < 3x + 1 \leq 7$.

$-1 \quad -1 \quad -1$

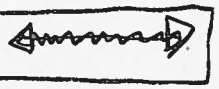
$-4 < 3x \leq 6$

$-\frac{4}{3} < x \leq 2$

3. Solve the following inequalities. Give your answers in each of the following forms: **1.** a simplified inequality, **2.** interval notation, **3.** a graph on the real line.

(a) $3x + 2 \leq 3x + 7$

$-3x \quad -3x$

$2 \leq 7 \qquad (-\infty, \infty)$ 

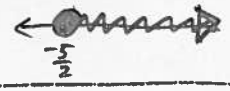
Always true

("unconditional inequality")

(b) $3x + 2 \leq 5x + 7$

$-3x \ -7 \quad -3x \ -7$

$-5 \leq 2x$

$-\frac{5}{2} \leq x \qquad \left[-\frac{5}{2}, \infty\right)$ 

$-\frac{5}{2}$

(c) $5x + 2 \leq 5x - 2$

$-5x \quad -5x$

$2 \leq -2 \qquad \varnothing$ 

Never true

("contradiction")

4. Find all solutions to the following equations.

(a) $x^2 + 3x + 2 = 0$

$(x+1)(x+2) = 0$

$x+1 = 0$ OR $x+2 = 0$

$x = -1$ OR $x = -2$

(b) $x^2 + 3x - 7 = 0$

$$x = \frac{-3 \pm \sqrt{9+28}}{2} = \frac{-3 \pm \sqrt{37}}{2} = x$$

(c) $(x+1)^2 = 4$

$x+1 = \pm\sqrt{4} = \pm 2$

$x = -1 \pm 2$

$x = -3$ OR $x = 1$

5. The following equation has a small integer solution. Find it, then find all solutions.

$$x^3 + 2x^2 - 1 = 0$$

$x=0$: $0+0-1 \stackrel{?}{=} 0$ No

$x=1$: $1+2-1 \stackrel{?}{=} 0$ No

$x=-1$: $-1+2-1 \stackrel{?}{=} 0$ Yes!

So -1 is a solution,

hence $x-(-1)$ is a factor of x^3+2x^2-1.

$$\begin{array}{r} x^2+x-1 \\ x+1 \overline{)\, x^3+2x^2+0x-1} \\ -(x^3+x^2) \\ \hline x^2+0x-1 \\ -(x^2+x) \\ \hline -x-1 \\ -(-x-1) \\ \hline 0 \end{array}$$

i.e.

$$x^3+2x^2-1 = (x+1)(x^2+x-1)$$

$$x^2+x-1=0 \iff x = \frac{-1\pm\sqrt{1+4}}{2} = \frac{-1\pm\sqrt{5}}{2}$$

$$\boxed{x=-1 \quad \text{OR} \quad x=\frac{-1\pm\sqrt{5}}{2}}$$

6. For each of the following equations, find (and clearly label) the x- and y-intercepts, plot at least five points, then sketch the plot.

(a) $x + (y+1)^2 = 3$

x-int: $(2,0)$

$x + (0+1)^2 = 3$

$x + 1 = 3$

$x = 2$

y-int: $(0, -1-\sqrt{3})$ & $(0, -1+\sqrt{3})$

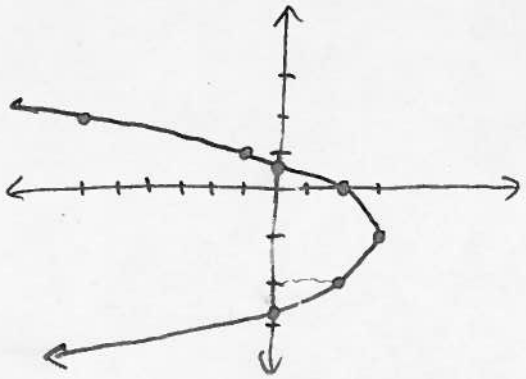
$0 + (y+1)^2 = 3$

$y+1 = \pm\sqrt{3}$

$y = -1 \pm \sqrt{3}$

$x = 3 - (y+1)^2$

x	y
2	-2
3	-1
2	0
-1	1
-6	2



(b) $y = 2|x-1| - 3$

x-int:

$0 = 2|x-1| - 3$

$3 = 2|x-1|$

$\frac{3}{2} = |x-1|$

$\pm\frac{3}{2} = x - 1$

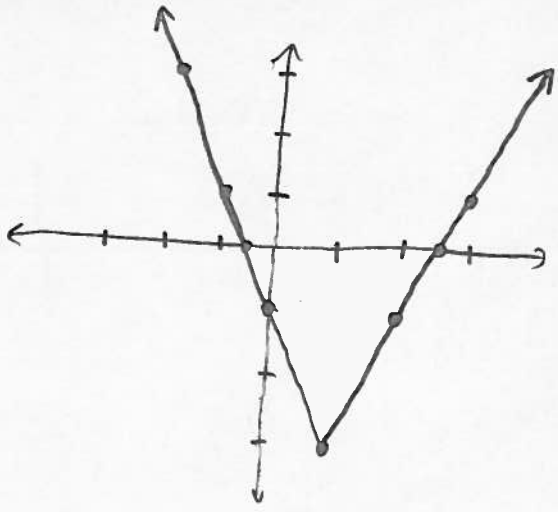
$1 \pm \frac{3}{2} = x$

$(\frac{5}{2}, 0)$ & $(-\frac{1}{2}, 0)$

y-int: $(0, -1)$

$y = 2|0-1| - 3 = 2 - 3 = -1$

x	y
-2	3
-1	1
0	-1
1	-3
2	-1
3	1



7. Find a point on the graph of $y = 2x$ that is the same distance from $(3, 0)$ as from $(0, 4)$.

$\hookrightarrow (a, 2a)$

$$\sqrt{(a-3)^2 + (2a-0)^2} = \sqrt{(a-0)^2 + (2a-4)^2}$$

$$(a-3)^2 + (2a)^2 = a^2 + (2a-4)^2$$

$$a^2 - 6a + 9 + 4a^2 = a^2 + 4a^2 - 16a + 16$$

$$-6a + 9 = -16a + 16$$

$$+16a \quad -9 \qquad +16a \quad -9$$

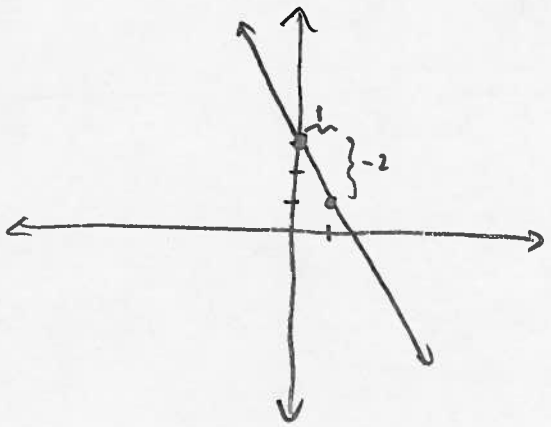
$$10a = 7$$

$$a = \frac{7}{10}$$

$$\boxed{\left(\frac{7}{10}, \frac{7}{5}\right)}$$

8. Identify the slope of the line $y = 3 - 2x$ and use this to graph the line.

slope $= -2$ y-int: $(0, 3)$



9. Find the equation of the line that is perpendicular to the line from (8) and passes through the point $(3, -1)$.

slope $= +\frac{1}{2}$

$y - (-1) = \frac{1}{2}(x - 3)$

$y + 1 = \frac{1}{2}(x - 3)$ point-slope

OR

$y = \frac{1}{2}x - \frac{5}{2}$ slope-intercept

OR

$x - 2y = 5$ standard

10. The speed of a ball thrown straight up is given by $v = 200 - 32t$, where v is the speed measured in meters per second and t is the time in seconds after the ball is thrown. **Identify and interpret** the slope of this linear equation.

slope $= -32$

means every second the speed decreases by $32 \frac{m}{s}$

11. Find equations for each of the following lines.

(a) with y-intercept $(0, 2)$ and slope -3

$y = -3x + 2$

(b) with x-intercept $(-1, 0)$ and slope $\frac{1}{2}$

$y - 0 = \frac{1}{2}(x + 1)$

$y = \frac{1}{2}x + \frac{1}{2}$

(c) parallel to the line $x - y = 3$ and passing through $(1, 3)$

$\hookrightarrow y = x - 3$

slope $= 1$ $\quad y - 3 = 1(x - 1)$ OR $y = x + 2$

(d) with the same x-intercept as $x - y = 5$ and the same y-intercept as $x + 2y = -1$

x-int: $x = 5$, $(5, 0)$ $\qquad$ y-int: $2y = -1 \Rightarrow y = -\frac{1}{2}$, $(0, -\frac{1}{2})$

slope $= \frac{0 - (-\frac{1}{2})}{5 - 0} = \frac{1}{10}$ $\qquad y = \frac{1}{10}x - \frac{1}{2}$

(e) that is horizontal and passes through $(3, 5)$

$y = 5$

(f) the perpendicular bisector of the segment joining $(1, 2)$ to $(3, 8)$ (the perpendicular bisector is perpendicular to the segment and passes through its midpoint)

midpoint $= \left(\frac{1}{2}(1+3), \frac{1}{2}(2+8)\right) = (2, 5)$

slope of segment $= \frac{8-2}{3-1} = \frac{6}{2} = 3$

slope of our line $= -\frac{1}{3}$

$y - 5 = -\frac{1}{3}(x - 2)$

12. Which of the following are functions?

(a) Assign to each person in this class their birthdate. Yes

~~(b) Assign to each date the person in this class with that date as their birthdate.~~ *

(c) Assign to each date the person in the world with that date as their birthdate. No (inputs have many outputs)

(d) $f(x) = x^2$ Yes

(e) $f(x) = \sqrt{x}$ Yes (with domain nonnegative real #s)

(f) $f(x) = \pm\sqrt{x}$ No (inputs get two outputs)

13. Let $g(x) = x^2 - x$. Find and simplify the following.

(a) $g(-5) = (-5)^2 - (-5) = 25 + 25 = 50$

(b) $-g(5) = -(5^2 - 5) = -(25 - 5) = -20$

(c) $g(2x) = (2x)^2 - (2x) = 4x^2 - 2x$

(d) $2g(x) = 2(x^2 - x) = 2x^2 - 2x$

(e) $g(x+h) = (x+h)^2 - (x+h) = (x^2 + 2xh + h^2) - (x+h)$
$= x^2 + 2xh + h^2 - x - h$

(f) $g(x) + g(h) = x^2 - x + h^2 - h$

(g) $\dfrac{g(x+h) - g(x)}{h} = \dfrac{(\quad) - (x^2 - x)}{h} = \dfrac{2xh + h^2 - h}{h} = 2x - 1 + h$ if $h \neq 0$

* I've decided I don't like this question.
The best answer is probably "no: some inputs have no output",
but if the domain is properly restricted (to birthdates of people in this class),
then the answer is "probably yes" since with only seven of us it is unlikely that any two of us share a birthdate