

# WORKSHEET 1      JANUARY 19, 2011

## 1. *breaking bad habits*

Which of the following statements are false? Talk amongst your group, giving counterexamples or examples to defend your choices. If a false statement can be made true by a simple edit, do so:

- (a)  $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$
- (b) If  $x \geq 0$ , then  $\sqrt{x} \leq x$ .
- (c) For all  $x$ ,  $\sin(\arcsin(x)) = x$ .
- (d) For all  $x$ ,  $\arcsin(\sin(x)) = x$ .
- (e) For all  $x$ ,  $-x < 0$ .
- (f) If  $a < b$ , then  $2a < a + b < 2b$ .
- (g) If  $a < b$ , then  $a^2 < ab < b^2$ .
- (h)  $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$
- (i)  $\log(a \cdot b) = \log a \cdot \log b$
- (j)  $\log(a \cdot b) = \log a + \log b$
- (k)  $\log(a+b) = \log a + \log b$
- (l)  $\log(a+b) = \log a \cdot \log b$
- (m)  $(e^x)^y = (e^y)^x$
- (n)  $e^x \cdot e^y = e^{x+y}$
- (o)  $(x-9)(x+2) = 0$  implies that either  $x = 9$  or  $x = -2$ .
- (p)  $x(x-1)(x+3) = 0$  implies that either  $x = 1$  or  $x = -3$ .
- (q)  $(a+b)^2 = a^2 + b^2$

## 2. *motivating calculus*

Suppose a squirrel moves along a meterstick, and you decide to record his location at several points in time:

time	0	1	2	3	4	5	6	7
position	0	4	7	30	30	30	32	80

(time is in seconds, position in cm along the meterstick.)

- (a) Plot the data. Fill in your guess at the squirrel's position during the entire interval  $[0, 7]$ .
- (b) Do the data above give position as a function of time? Does your guess at actual position? Is this always the case?
- (c) Do the data above give time as a function of position? Does your guess at actual position? Is this always the case?
- (d) What is the average velocity of the squirrel during his travels?
- (e) Estimate the squirrel's velocity at time 1, time 2, and time 4. How confident are you with each of these calculations? Why?
- (f) What information would help you get better estimates for these velocities?
- (g) How does these speeds (both actual speeds and your estimates) relate to your graph?

- (h) Is it possible that the squirrel maintained a constant speed throughout the interval  $[0, 7]$ ? (Be careful! You might want to think about what constant speed means on your graph. Either answer is acceptable with an appropriate argument.)

**3. functions**

- (a) What does it mean for a function  $g(x)$  to be the *inverse* of  $f(x)$ ? When does a function have an inverse?
- (b) Consider the function that takes as input a name from a student in this class and outputs his/her UIN. Is the function 1-1? Does it have an inverse?
- (c) Consider the function that takes as input a name from a student in this class and outputs his/her first initial. Is this function 1-1? Does it have an inverse? What if you restrict the domain to just those students at your table?
- (d) Recall that when a function is given to you by an algebraic formula, the domain is assumed to be the set of all (real) numbers so that the formula makes sense. What are the domains of the following functions? What are their ranges?
- (i)  $\sqrt{1+x^2}$
  - (ii)  $\sqrt{1+x}$
  - (iii)  $\tan x$
  - (iv)  $\frac{1+x}{1+x}$
  - (v)  $\sqrt{1-x^2}$
  - (vi)  $\sqrt{1-x}\sqrt{x-1}$