MATH 454
HOMEWORK 6 DUE MARCH 1

Name: ____________________

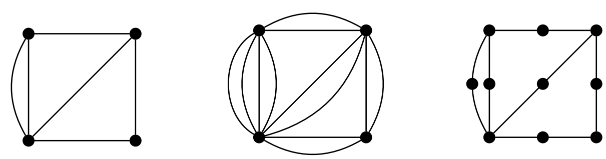
- Refer to the syllabus regarding allowed collaboration on this homework assignment.
- Refer to other homework instructions and suggestions posted in Blackboard.
- All answers must be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

Turn in four of the following problems to be graded.

(1) (2.2.23) Assume for this problem that the Graceful Tree Conjecture is true. Let T be a tree with m edges. Show that K_{2m} decomposes into copies of T. *(Hint: modify the construction that decomposes K_{2m-1} into copies of a tree with $m-1$ edges.)*

(2) (2.2.10, first part) Find $\tau(K_{2,n})$.

(3) (2.2.12) From a graph G, let $G^{(k)}$ be obtained by replacing each edge by k copies of that edge; and let $G^{1/k}$ be obtained by replacing each edge uv by a u,v-path of length k through $k-1$ new vertices. Determine $\tau(G^{(k)})$ and $\tau(G^{1/k})$ in terms of $\tau(G)$, k, $n(G)$, and $e(G)$. *(Below are examples of these constructions.)*



G $\quad$ $G^{(2)}$ $\quad$ $G^{1/2}$

(4) (2.3.10) **Prim's Algorithm** grows a spanning tree from a given vertex of a connected weighted graph G, iteratively adding the cheapest edge from a vertex already reached to a vertex not yet reached, finishing when all the vertices of G have been reached. Prove that Prim's Algorithm produces a minimum-weight spanning tree of G.

(5) (2.3.12) *Minimum spanning path.* In a weighted complete graph, iteratively select the edge of least weight such that the edges selected so far form a disjoint union of paths. After $n-1$ steps, the result is a spanning path. This greedy algorithm *does not* always produce a minimum-weight spanning path; provide an infinite family of weighted complete graphs for which the algorithm produces a suboptimal spanning path.

(6) (2.3.16) Four people must cross a canyon at night on a fragile bridge. At most two people can be on the bridge at once. Crossing requires carrying a flashlight, and there is only one flashlight (which can only cross by being carried). Alone, the four people cross in 10, 5, 2, 1 minutes, respectively. When two cross together, they move at the speed of the slower person. In 18 minutes, a flash flood coming down the canyon will wash away the bridge. Can the four people get across in time? Describe how the answer can be found using graph theory.