

**Tuesday, September 4**    \*\*    *Projections, distances, and planes.*

1. Let  $\mathbf{a} = \mathbf{i} + \mathbf{j}$  and  $\mathbf{b} = 2\mathbf{i} - \mathbf{j}$ .

- (a) Calculate  $\text{proj}_{\mathbf{b}}\mathbf{a}$  and draw a picture of it together with  $\mathbf{a}$  and  $\mathbf{b}$ .
- (b) The orthogonal complement of the vector  $\mathbf{a}$  with respect to  $\mathbf{b}$  is defined by

$$\text{orth}_{\mathbf{b}}\mathbf{a} = \mathbf{a} - \text{proj}_{\mathbf{b}}\mathbf{a}.$$

Calculate  $\text{orth}_{\mathbf{b}}\mathbf{a}$  and draw two copies of it in your picture from part (a), one based at  $\mathbf{0}$  and the other at  $\text{proj}_{\mathbf{b}}\mathbf{a}$ .

- (c) Check that  $\text{orth}_{\mathbf{b}}\mathbf{a}$  calculated in (b) is orthogonal to  $\text{proj}_{\mathbf{b}}\mathbf{a}$  calculated in (a).
  - (d) Find the distance of the point  $(1, 1)$  from the line  $(x, y) = t(2, -1)$ . Hint: relate this to your picture.
2. Let  $\mathbf{a}$  and  $\mathbf{b}$  be vectors in  $\mathbb{R}^n$ . Use the definitions of  $\text{proj}_{\mathbf{b}}\mathbf{a}$  and  $\text{orth}_{\mathbf{b}}\mathbf{a}$  to show that  $\text{orth}_{\mathbf{b}}\mathbf{a}$  is always orthogonal to  $\text{proj}_{\mathbf{b}}\mathbf{a}$ .
3. Find the distance between the point  $P(3, 4, -1)$  and the line  $\mathbf{l}(t) = (2, 3, -2) + t(1, -1, 1)$ . Hint: Consider a vector starting at some point on the line and ending at  $P$ , and connect this to what you learned in Problem 1.
4. Consider the equation of the plane  $x + 2y + 3z = 12$ .
- (a) Find a normal vector to the plane. (Just look at the equation!)
  - (b) Find where the  $x$ ,  $y$ , and  $z$ -axes intersect the plane. Using this information, sketch the portion of the plane in the first octant where  $x \geq 0, y \geq 0, z \geq 0$ .
  - (c) Using the points in part (b), find two non-parallel vectors that are parallel to the plane.
  - (d) Using part (c) and the cross product, find another normal vector to the plane. Show that this vector is parallel to the vector from part (a).
  - (e) Using the new normal vector and one of the points from (b), find an alternative equation for the plane. Compare this new equation to  $x + 2y + 3z = 12$ . How are these two equations related? Is it clear that they describe the same set of points  $(x, y, z)$  in  $\mathbb{R}^3$ ?
5. *The Triangle Inequality.* Let  $\mathbf{a}$  and  $\mathbf{b}$  be any vectors in  $\mathbb{R}^n$ . The triangle inequality states that  $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$ .
- (a) Give a geometric interpretation of the triangle inequality. (E.g. draw a picture in  $\mathbb{R}^2$  or  $\mathbb{R}^3$  that represents this inequality.)
  - (b) Use what we know about the dot product to explain why  $|\mathbf{a} \cdot \mathbf{b}| \leq |\mathbf{a}||\mathbf{b}|$ . This is called the Cauchy-Schwarz inequality.
  - (c) Use part (b) to justify the triangle inequality. Hint: Start with the fact that  $|\mathbf{a} + \mathbf{b}|^2 = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b})$  and then use properties of the dot product and the Cauchy-Schwarz inequality to manipulate the right-hand side into looking like  $|\mathbf{a}|^2 + |\mathbf{b}|^2$ .