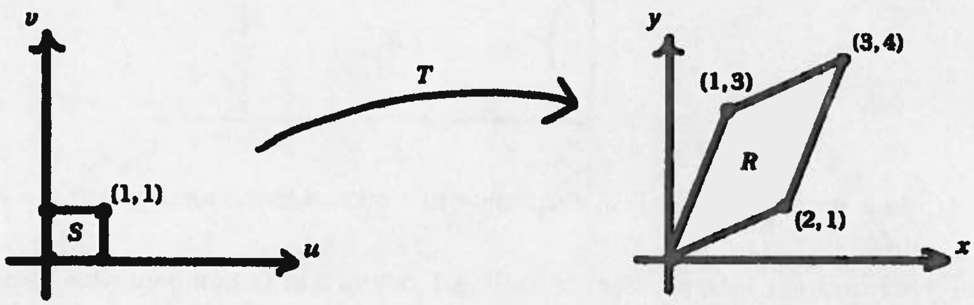
Thursday November 8 ** *Changing coordinates*

1. Consider the region R in $\mathbb{R}^2$ shown below at right. In this problem, you will do a change of coordinates to evaluate:

$$\iint_R x - 2y\, dA$$



   (a) Find a linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ which takes the unit square S to R.
   Write you answer both as a matrix $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ and as $T(u,v) = (au+bv,\ cu+dv)$, and check your answer with the instructor.

   (b) Compute $\iint_R x - 2y\, dA$ by relating it to an integral over S and evaluating that. Check your answer with the instructor.

2. Another simple type of transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a translation, which has the general form $T(u,v) = (u+a, v+b)$ for a fixed a and b.

   (a) If T is a translation, what is its Jacobian matrix? How does it distort area?

   (b) Consider the region $S = \{u^2 + v^2 \leq 1\}$ in $\mathbb{R}^2$ with coordinates (u,v), and the region $R = \{(x-2)^2 + (y-1)^2 \leq 1\}$ in $\mathbb{R}^2$ with coordinates (x,y).
   Make separate sketches of S and R.

   (c) Find a translation T where $T(S) = R$.

   (d) Use T to reduce

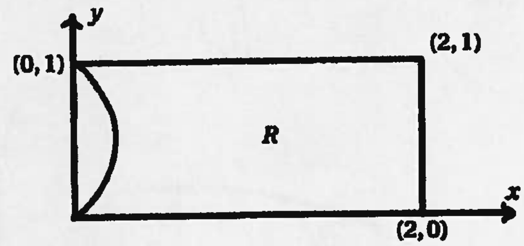
$$\iint_R x\, dA$$

   to an integral over S, and then evaluate that new integral using polar coordinates.

   (e) Check your answer in (d) with the instructor.

Problems 3 and 4 on the back.

3. Consider the region R shown below. Here the curved left side is given by $x = y - y^2$. In this problem, you will find a transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ which takes the unit square $S = [0,1] \times [0,1]$ to R.



(a) As a warm up, find a transformation that takes S to the rectangle $[0,2] \times [0,1]$ which contains R.

(b) Returning to the problem of finding T taking S to R, come up with formulas for $T(u,0)$, $T(u,1)$, $T(0,v)$, and $T(1,v)$. Hint: For three of these, use your answer in part (a).

(c) Now extend your answer in (b) to the needed transformation T. Hint: Try "filling in" between $T(0,v)$ and $T(1,v)$ with a straight line.

(d) Compute the area of R in two ways, once using T to change coordinates and once directly.

4. If you get this far, evaluate the integrals in Problems 1 and 2 directly, without doing a change of coordinates. It's a fun-filled task...