

# WORKSHOP 1: §1.1-5      JANUARY 12, 2017

Name: \_\_\_\_\_

- (1) Determine the truth value of  $p \wedge \neg q \vee r$  given the truth values  $p: T, q: F, r: F$ . (Make sure to use the correct order of operations.)
- (2) Consider the statement “If today is Thanksgiving, then tomorrow is Friday.”
  - (a) Is this a proposition?
  - (b) Is it true or false?
  - (c) What is its converse? Is that true or false?
  - (d) What is its inverse? Is that true or false?
  - (e) What is its contrapositive? Is that true or false?
  - (f) What is its negation? Is that true or false?
- (3) Why is the proposition “if Ben owns a unicorn, then  $0 = 1$ ” true?
- (4) Let  $h$  be “John is healthy,”  $w$  be “John is wealthy,” and  $s$  be “John is wise.” Rewrite each of the following using the symbols  $\neg, \wedge, \vee, \rightarrow, h, w, s$ :
  - (a) John is healthy and wealthy but not wise.
  - (b) John is not wealthy but he is healthy and wise.
  - (c) John is neither healthy, wealthy, nor wise.
  - (d) John is neither wealthy nor wise, but he is healthy.
  - (e) John is wealthy, but he is not both healthy and wise.
  - (f) John is wise only if he is healthy.
  - (g) John’s being healthy and wise is sufficient to make him wealthy.

- (5) (a) Complete the following truth table.

$p$	$q$	$r$	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

- (b) Is the last proposition above a tautology, a contradiction, or neither?
  - (c) What does that say about the propositions  $p \wedge (q \vee r)$  and  $(p \wedge q) \vee (p \wedge r)$ ?  
*(You’ve just finished proving one of §1.5’s Laws of Logic.)*
- (6) In our order of operations, we didn’t specify whether  $p \vee q \vee r$  should mean  $(p \vee q) \vee r$  or  $p \vee (q \vee r)$ . Why not?
- (7) (Smullyan’s Knights and Knaves, adapted) An island consists of a tribe of Honest people who always tell the truth and a tribe of Liars who always lie. You meet there Alice and Bob. Alice says to you “Bob is Honest,” and Bob tells you “Alice and I are from opposite tribes.” Can you determine which tribes Alice and Bob are from?

You can do this using a truth table and cleverly chosen propositions. Let  $p$  be the proposition “Alice is Honest” and  $q$  be the proposition “Bob is Honest.” The key idea is that the truth value of  $p$  is the same as the truth value of Alice’s statement (if Alice is Honest, then her statement is true; if Alice is a Liar, then her statement is false). When you complete an appropriate truth table, that fact (and the analogous fact about Bob) will rule out some of the possible rows.

A linguistics professor was lecturing to his class one day. "In English," he said, "a double negative forms a positive. In some languages though, such as Russian, a double negative is still a negative.

"However," he pointed out, "there is no language wherein a double positive can form a negative."

A voice from the back of the room piped up, "Yeah, right."