Math 241 X8 **Name:** Solutions

Quiz # 2

September 17, 2013 No electronic devices or interpersonal communication allowed. Show work to get credit.

1) [5pts.] Compute $\langle 1,2,3\rangle \times \langle 3,2,-1\rangle$.

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & -1 \end{vmatrix} = (2(-1)-2(3))\hat{\imath} - (1(-1)-3(3))\hat{\jmath} + (1(2)-3(2))\hat{k}$$

$$= -8\hat{\imath} + 10\hat{\jmath} - 4\hat{k} = \langle -8, 10, -4\rangle$$

2) [5pts.] Are the planes $3x-2y+5z = 7$ and $-6x+4y+10z = 30$ parallel, perpendicular, or neither? If they intersect, find an equation of the line of their intersection; otherwise, find the (minimum) distance between them.

$\vec{n}_1 = \langle 3, -2, 5\rangle$ $\vec{n}_2 = \langle -6, 4, 10\rangle$

$\vec{n}_1 \neq c\vec{n}_2$ for any c $\Rightarrow$ not parallel $\Rightarrow$ intersect

$\vec{n}_1 \cdot \vec{n}_2 = -18 - 8 + 50 = 24 \neq 0 \Rightarrow$ not perpendicular

For line, need

point &

try $x=0$: $\begin{cases} -2y+5z=7 & (1) \\ 4y+10z=30 & (2) \end{cases}$

$2(1)+(2)$: $20z = 44$

$z = 2.2$

$y = 2$

direction

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -2 & 5 \\ -6 & 4 & 10 \end{vmatrix}$$

$= \cdots = \langle -40, -60, 0\rangle$

$$\ell(t) = \langle 0, 2, 2.2\rangle + t \cdot \langle -40, -60, 0\rangle$$

3) A closed curve in the plane is parametrized by $\langle x(t), y(t)\rangle$ and is traced out counter-clockwise as t advances from 0 to 8. In terms of $x(t)$ and $y(t)$, find each of the following for each t (i.e., as functions of t):

(a) [2pts.] a tangent vector to the curve;

(b) [4pts.] a unit tangent vector to the curve;

(c) [4pts.] an outward-pointing normal vector to the curve.

(The curve is "nice enough": no self-intersections, continuous, no corners, etc. "Closed" means it starts and ends at the same point.)

(a) $\langle x'(t), y'(t)\rangle$

(b) $$\frac{\langle x'(t), y'(t)\rangle}{\sqrt{(x'(t))^2 + (y'(t))^2}}$$

(c) $\langle y'(t), -x'(t)\rangle$