HOMEWORK 5: CHAPTER 3 DUE FEBRUARY 16

Name: _____

- Please refer to the syllabus regarding allowed collaboration on this homework assignment.
- All answers should be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

(1) Short exercises:
   (a) What is $\big|\{1,2,5\} \cup \{1,5,7\}\big|$?
   (b) What is $\big|\{1,2,5\} \cap \{1,5,7\}\big|$?
   (c) What is $\big|\{1,2,5\} \times \{1,5,7\}\big|$?
   (d) What is $\mathcal{P}(\varnothing \times \{1,2,3\})$?

(2) If $A \subseteq B$, then what is $A \cup B$? What is $A \cap B$? Prove one of your answers (whichever you prefer).

(3) Prove that if $A \subseteq B$, then $A \times C \subseteq B \times C$.

(4) Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

(5) Prove or disprove the following.
   (a) For all sets A, B, if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
   (b) For all sets A, B, $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.
   (c) For all sets A, B, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.
   (d) For all sets A, B, $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$.

(6) Suppose A, B are sets such that $A - B$, $B - A$, and $A \cap B$ are all nonempty. Prove that $A - B$, $B - A$, $A \cap B$ form a partition of $A \cup B$.

"Consider the set of all sets that have not yet been considered.
Oops, now it's empty..."