Math 241, Sections BL1 and BL2

Quiz # 5

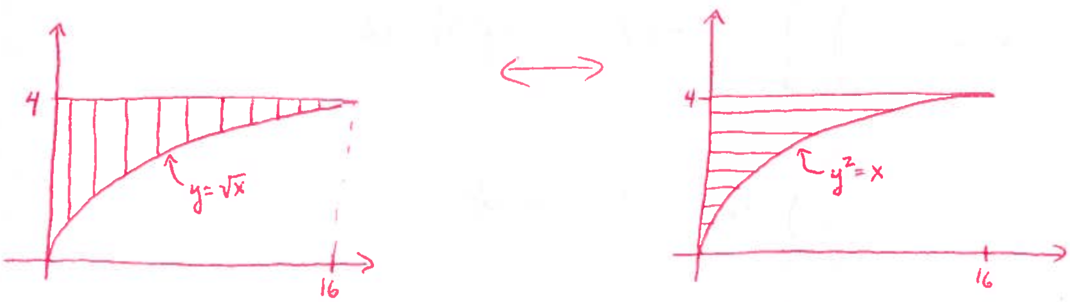
November 1, 2012

Solve both exercises. Show work to get credit.

1) [5pts.] Evaluate the following integral by reversing the order of integration:

$$\int_0^{16}\int_{\sqrt{x}}^{4} \frac{1}{y^3+1}\,dy\,dx.$$

$\sqrt{x} \le y \le 4, \quad 0 \le x \le 16$



$0 \le x \le y^2, \quad 0 \le y \le 4$

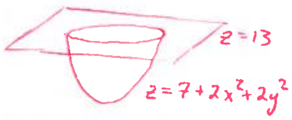
$$\int_0^4\int_0^{y^2} \frac{1}{y^3+1}\,dx\,dy = \int_0^4 \frac{y^2}{y^3+1}\,dy \qquad \begin{array}{l} u = y^3+1 \\ du = 3y^2\,dy \end{array}$$

$$= \frac{1}{3}\int_1^{65} \frac{du}{u}$$

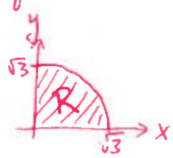
$$= \frac{1}{3}\left(\ln 65 - \ln 1\right)$$

$$= \frac{1}{3}\ln 65 .$$

2) [5pts.] Use polar coordinates to find the volume of the solid bounded by the paraboloid $z = 7 + 2x^2 + 2y^2$ and the plane $z = 13$ in the first octant.



"Shadow" of solid in xy-plane is the quarter-disk



$13 = 7 + 2x^2 + 2y^2$

$\Leftrightarrow x^2 + y^2 = 3$

In polar, this is the region $0 \le r \le \sqrt{3}$

$0 \le \theta \le \frac{\pi}{2}$.

$$\text{Volume} = \iint_R \left(13 - (7 + 2x^2 + 2y^2)\right) dA$$

$$= \int_0^{\pi/2} \int_0^{\sqrt{3}} (6 - 2r^2)\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \left[3r^2 - \frac{1}{2}r^4\right]_0^{\sqrt{3}} d\theta$$

$$= \frac{9}{2}\left[\theta\right]_0^{\pi/2}$$

$$= \frac{9\pi}{4}.$$