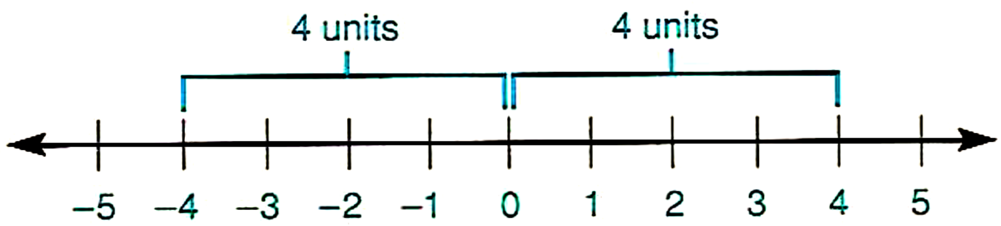
Section 1.4 Absolute Value

Geometric Definition of Absolute Value

As we study the number line, we observe a very useful property called symmetry. The numbers are symmetrical with respect to the origin. That is, if we go four units to the right of 0, we come to the number 4. If we go four units to the lest of 0 we come to the opposite of 4 which is −4



The **absolute value of a number** a, denoted **|a|**, is the distance from a to 0 on the number line. Absolute value speaks to the question of "how far," and not "which way." The phrase how far implies length, and length is always a nonnegative (zero or positive) quantity. Thus, the absolute value of a number is a nonnegative number. This is shown in the following examples:

Example 1

Evaluate the following expressions.

1. $|-7| = 7$ **2.** $|3| = 3$ **3.** $|0| = 0$

4. $\left|\frac{2}{3}\right| = \frac{2}{3}$ **5.** $|-3.7| = 3.7$ **6.** $-|-6| = -6$

Note The absolute value of a number is *never* negative; that is, $|x| \geq 0$, for every $x \in R$, however the absolute value bars are only applied to the symbol contained within them. The − sign in front of the absolute value bars is not affected by the absolute value bars. Example 6 would be read "the opposite of the absolute value of −6," and the answer is −6.

► ***Quick check*** Evaluate the following expressions.
$|-3|$; $|12|$; $-|-4|$

Algebraic Definition of Absolute Value

The **absolute value** of a number a is

$$|a| = \begin{cases} a & \text{if } a \geq \mathbf{0} \\ -a & \text{if } a < \mathbf{0} \end{cases}$$

The algebraic definition takes into account the fact that the number a could be either positive or zero (≥0) or negative (<0)

1) If the number **a** is positive or zero (≥ 0), the first part of the definition applies. The first part of the definition tells us that if the number enclosed in the absolute bars is a nonnegative number, the absolute value of the number is the number itself.
2) If the number **a** is negative (< 0), the second part of the definition applies. The second part of the definition tells us that if the number enclosed within the absolute value bars is a negative number, the absolute value of the number is the opposite of the number. The opposite of a negative number is a positive number.

Example 2

1) $|8|$
   The number enclosed within the absolute value bars is a nonnegative number so the first part of the definition applies. This part says that the absolute value of 8 is 8 itself.
   $|8|=8$.

2) $|-3|$
   The number enclosed within absolute value bars is a negative number so the second part of the definition applies. This part says that the absolute value of −3 is the opposite of −3, which is − (−3). By the double-negative property, − (−3) = 3.
   $|-3|=3$.

3) $-|-52|$
   The number enclosed within absolute value bars is a negative number so the second part of the definition applies. This part says that the absolute value of −52 is the opposite of −52, which is − (−52). By the double-negative property, − (−52) = (52). But in this problem we have an extra negative outside of the absolute value symbol. When finding the ultimate solution we have to take into account and follow the order of operations to write : $\mathbf{-|-52| = -(52) = -52}$

Exercises:

Evaluate the following expressions. See the example first.

Examples	$\lvert -3 \rvert$	$\lvert 12 \rvert$	$-\lvert -4 \rvert$
Solutions	3 -3 is 3 units from the origin	12 12 is 12 units from the origin	-4 -4 is 4 units from the origin. The problem is to find the opposite of the absolute value

48. $\lvert 0 \rvert$ **49.** $\lvert 2 \rvert$ **50.** $\lvert 8 \rvert$ **51.** $\lvert -5 \rvert$ **52.** $\lvert -7 \rvert$

53. $\lvert 4 \rvert$ **54.** $\left\lvert \frac{2}{3} \right\rvert$ **55.** $\left\lvert -\frac{1}{2} \right\rvert$ **56.** $\left\lvert -\frac{3}{4} \right\rvert$ **57.** $\left\lvert 1\frac{1}{2} \right\rvert$

58. $-\left\lvert -2\frac{3}{4} \right\rvert$ **59.** $-\left\lvert \frac{5}{8} \right\rvert$ **60.** $-\lvert -2 \rvert$ **61.** $-\lvert 6 \rvert$

Replace the ? with the proper inequality symbol, $<$ or $>$. See examples 1–3 C and D.

Examples $\lvert -6 \rvert \ ? \ \lvert -3 \rvert$ $\lvert 4 \rvert \ ? \ \lvert -7 \rvert$

Solutions

$6 \ ? \ 3$ $\lvert -6 \rvert$ is 6 and $\lvert -3 \rvert$ is 3

$6 > 3$ 6 is to the right of 3 on the number line

then $\lvert -6 \rvert > \lvert -3 \rvert$

$4 \ ? \ 7$ $\lvert 4 \rvert$ is 4 and $\lvert -7 \rvert$ is 7

$4 < 7$ 4 is to the left of 7 on the number line

then $\lvert 4 \rvert < \lvert -7 \rvert$

62. $\lvert -2 \rvert \ ? \ \lvert -4 \rvert$ **63.** $\lvert 5 \rvert \ ? \ \lvert -7 \rvert$ **64.** $\lvert -3 \rvert \ ? \ \lvert -4 \rvert$ **65.** $\lvert 0 \rvert \ ? \ \lvert -2 \rvert$

66. $\lvert -3 \rvert \ ? \ \lvert 4 \rvert$ **67.** $\lvert -8 \rvert \ ? \ \lvert -5 \rvert$ **68.** $\lvert -9 \rvert \ ? \ \lvert 7 \rvert$ **69.** $\lvert -6 \rvert \ ? \ \lvert -2 \rvert$

70. $\lvert -5 \rvert \ ? \ 3$ **71.** $7 \ ? \ \lvert -2 \rvert$ **72.** $4 \ ? \ \lvert -8 \rvert$ **73.** $\lvert -4 \rvert \ ? \ 6$

Use absolute value to write each of the following. See figure 1–7.

Example The distance between -8 and 0

Solution $\lvert -8 \rvert$ The absolute value of a number is the distance from the number to the origin.

74. The distance between 14 and 0

75. The distance between -27 and 0

76. The distance between 18 and 0

77. The distance between -9 and 0

78. The distance between -19 and 0