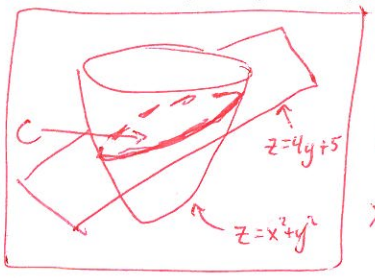
Math 251, section 01
Quiz 3 September 14, 2016

Name:

By handing in this quiz you assert that you understand and have followed IIT's guidelines for academic integrity.

(1) Let C be the curve that is the intersection of the surfaces $z = x^2 + y^2$ and $z = 4y + 5$. Sketch C. Find a parametrization of C.

(Hint for the parametrization: eliminate z first, then choose x and y, then find z.)



$x^2+y^2 = 4y+5$

$x^2+y^2-4y+4 = 5+4$

$x^2+(y-2)^2 = 9$

$x = 3\cos t$

$y = 2+3\sin t$

$z = 4y+5 = 13+12\sin t$

$t \in [0, 2\pi]$

(2) Consider the vector function $\mathbf{r}(t) = \langle t\cos t,\ t,\ t\sin t\rangle$.

(a) Find $\mathbf{r}'(t)$.

$= \langle \cos t - t\sin t,\ 1,\ \sin t + t\cos t\rangle$

(b) Find an equation for the tangent line to $\mathbf{r}$ at $t = \pi/2$.

$\ell(t) = \vec{r}\left(\frac{\pi}{2}\right) + t\cdot\vec{r}'\left(\frac{\pi}{2}\right)$

$= \left\langle 0, \frac{\pi}{2}, \frac{\pi}{2}\right\rangle + t\left\langle -\frac{\pi}{2}, 1, 1\right\rangle$

(c) Set up, but do not evaluate, an integral that gives the arc length of $\mathbf{r}(t)$ from $t = 0$ to $t = \pi$. (Your final answer should not involve any vectors, only a Calc1&2 integral.)

$= \int_0^{\pi} |r'(t)|\,dt = \int_0^{\pi} \sqrt{(\cos t - t\sin t)^2 + 1^2 + (\sin t + t\cos t)^2}\,dt$

$\left(= \int_0^{\pi} \sqrt{2+t^2}\,dt \approx 6.95 \right)$