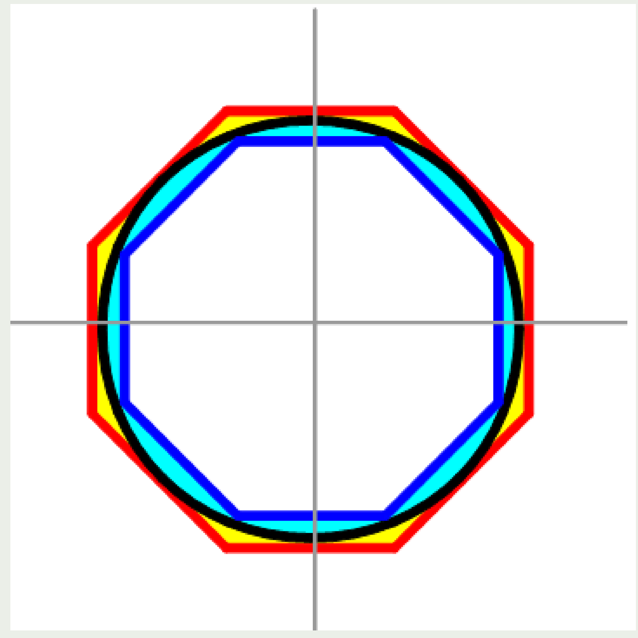
Pursuing Polygons

Archimedes' Legacy

Archimedes used the perimeters of inscribed and circumscribed polygons of up to 96 sides to find upper and lower bounds for π. Using 96 sides, he showed that



$$3 + \frac{10}{71} < \pi < 3 + \frac{1}{7}.$$

In the following millennium, improvements were obtained by simply using the method on polygons with more sides.

Willebrord Snell van Royen

The first big improvement in the method came from William Snell in 1621. Snell suggested that the perimeter of the inscribed (inside) polygon of n sides converges to π twice as fast as the perimeter of the circumscribed (outside) polygon. Although this insight wouldn't be proven until Christiaan Huygens verified it in 1654, Snell used it to obtain 7 digits of π using a 96 sided polygon.

Huygens, Wallis, & the End for Polygons

In the 1650's Christiaan Huygens improved Snell's theorems and, using only a triangle, matched the accuracy of Archimedes's π approximation!
While Huygens was improving polygon methods in the Netherlands, English mathematician John Wallis introduced another infinite product approximation to the world! He greatly improved on previous work by only using rational numbers.


C. Huygens (L) and J. Wallis (R)

Gregory, Leibniz, & an Inefficient Method

In the 1670's James Gregory and Gottfried Wilhelm Leibniz use the arctangent series at $x = 1$ to approximate π. Although interesting, this formula would need 300 terms calculated to produce 2 digits of π.

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 4\left(\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \dots\right)$$

A Stress on Efficiency

In the 1720's mathematicians stressed finding more efficient calculations for π. Newton for example discovered two more infinite series for π, one of which yielded 3.1416 after only 4 terms. In 1748 Euler found an arctangent formula which converges significantly faster than the Gregory-Leibniz series.

Figure: Leonhard Euler

Progress 1699-1873

Year	Name	Digits
1699	Abraham Sharp	72 (71 correct)
1701	John Machin	100
1719	Thomas de Lagny	127 (112 correct)
1794	Georg Vega	140
1837	J.F. Callet	152
1847	Thomas Clausen	248
1853	William Rutherford	440
1873	Shanks	707 (527 correct)