Math 251
Quiz 5 September 28, 2016

Name:

By handing in this quiz you assert that you understand and have followed IIT's guidelines for academic integrity.

(1) Find the limit, if it exists: $\lim_{(x,y)\to(0,0)} \dfrac{x^3 e^y}{x^2+y^2}$

$$\lim_{r\to 0^+} \frac{r^3\cos^3\theta\, e^{r\sin\theta}}{r^2} = \lim_{r\to 0^+} \underbrace{r}_{\to 0}\ \underbrace{\cos^3\theta}_{\text{bounded}}\ \underbrace{e^{r\sin\theta}}_{\text{bounded},\ \to 1} = 0$$

[You should probably have tried some path-limits first; they will all come out as zero.]

(2) Find an equation of the tangent plane to the surface $z = x^2 e^y$ at the point $(2, 0, 4)$.

Let $f(x,y) = x^2 e^y$. $f(2,0) = 4$

$f_x = 2xe^y$ $f_x(2,0) = 4$

$f_y = x^2 e^y$ $f_y(2,0) = 4$

Tangent plane: $z = f(2,0) + f_x(2,0)(x-2) + f_y(2,0)(y-0)$

$$\boxed{z = 4 + 4(x-2) + 4(y)}$$

OR, from §14.6, let $g(x,y,z) = x^2e^y - z$.

We want a tangent plane to the level surface $g(x,y,z) = 0$.

∇g serves as a normal vector!

$\nabla g = \langle 2xe^y, x^2e^y, -1\rangle$, $\nabla g(2,0,4) = \langle 4, 4, -1\rangle$

Tangent plane: $\boxed{4(x-2) + 4(y-0) - 1(z-4) = 0}$