Chapter 1

Section 1.1

Number Systems

In this section we introduce the number systems that we will work with in the remainder of this text.

The Natural Numbers

We begin with a definition of the *natural numbers*, or the *counting numbers*.

Definition 1. *The set of* natural numbers *is the set*

$$\mathbb{N} = \{1, 2, 3, \ldots\}. \tag{2}$$

The notation in **equation (2)**[2] 's read "$\mathbb{N}$ is the set whose members are 1, 2, 3, and so on." The ellipsis (the three dots) at the end in **equation (2)** is a mathematician's way of saying "et-cetera." We list just enough numbers to establish a recognizable pattern, then write "and so on," assuming that a pattern has been sufficiently established so that the reader can intuit the rest of the numbers in the set. Thus, the next few numbers in the set $\mathbb{N}$ are 4, 5, 6, 7, "and so on."

Note that there are an infinite number of natural numbers. Other examples of natural numbers are 578,736 and 55,617,778. The set $\mathbb{N}$ of natural numbers is unbounded; i.e., there is no largest natural number. For any natural number you choose, adding one to your choice produces a larger natural number.

For any natural number n, we call m a *divisor* or *factor* of n if there is another natural number k so that $n = mk$. For example, 4 is a divisor of 12 (because $12 = 4 \times 3$), but 5 is not. In like manner, 6 is a divisor of 12 (because $12 = 6 \times 2$), but 8 is not.

We next define a very special subset of the natural numbers.

Definition 3. *If the only divisors of a natural number p are 1 and itself, then p is said to be* prime.

For example, because its only divisors are 1 and itself, 11 is a prime number. On the other hand, 14 is not prime (it has divisors other than 1 and itself, i.e., 2 and 7). In like manner, each of the natural numbers 2, 3, 5, 7, 11, 13, 17, and 19 is prime. Note that 2 is the only even natural number that is prime.[3]

If a natural number other than 1 is not prime, then we say that it is *composite*. Note that any natural number (except 1) falls into one of two classes; it is either prime, or it is composite.

Although the natural number 1 has only 1 and itself as divisors, mathematicians, particularly number theorists, don't consider 1 to be prime. There are good reasons for this, but that might take us too far afield. For now, just note that 1 is not a prime number. Any number that is prime has exactly two factors, namely itself and 1.

We can factor the composite number 36 as a product of prime factors, namely

$$36 = 2 \times 2 \times 3 \times 3.$$

Other than rearranging the factors, this is the only way that we can express 36 as a product of prime factors.

Theorem 4. The *Fundamental Theorem of Arithmetic* says that every natural number has a *unique prime factorization.*

No matter how you begin the factorization process, all roads lead to the same prime factorization. For example, consider two different approaches for obtaining the prime factorization of 72.

$$\begin{aligned} 72 &= 8 \times 9 \\ &= (4 \times 2) \times (3 \times 3) \\ &= 2 \times 2 \times 2 \times 3 \times 3 \end{aligned} \qquad \begin{aligned} 72 &= 4 \times 18 \\ &= (2 \times 2) \times (2 \times 9) \\ &= 2 \times 2 \times 2 \times 3 \times 3 \end{aligned}$$

In each case, the result is the same, $72 = 2 \times 2 \times 2 \times 3 \times 3$.

Zero

The use of zero as a placeholder and as a number has a rich and storied history. The ancient Babylonians recorded their work on clay tablets, pressing into the soft clay with a stylus. Consequently, tablets from as early as 1700 BC exist today in museums around the world. A photo of the famous Plimpton_322 is shown in **Figure 1**, where the markings are considered by some to be Pythagorean triples, or the measures of the sides of right triangles.

Figure 1. Plimpton_322

The people of this ancient culture had a sexagesimal (base 60) numbering system that survived without the use of zero as a placeholder for over 1000 years. In the early Babylonian system, the numbers 216 and 2106 had identical recordings on the clay tablets of the authors. One could only tell the difference between the two numbers based upon the context in which they were used. Somewhere around the year 400 BC, the Babylonians started using two wedge symbols to denote a zero as a placeholder (some tablets show a single or a double-hook for this placeholder).

The ancient Greeks were well aware of the Babylonian positional system, but most of the emphasis of Greek mathematics was geometrical, so the use of zero as a placeholder was not as important. However, there is some evidence that the Greeks used a symbol resembling a large omicron in some of their astronomical tables.

It was not until about 650 AD that the use of zero as a number began to creep into the mathematics of India. Brahmagupta (598-670?), in his work *Brahmasphutasiddhanta*, was one of the first recorded mathematicians who attempted arithmetic operations with the number zero. Still, he didn't quite know what to do with division by zero when he wrote

> *Positive or negative numbers when divided by zero is a fraction with zero as denominator.*

Note that he states that the result of division by zero is a fraction with zero in the denominator. Not very informative. Nearly 200 years later, Mahavira (800-870) didn't do much better when he wrote

> *A number remains unchanged when divided by zero.*

It seems that the Indian mathematicians could not admit that division by zero was impossible.

The Mayan culture (250-900 AD) had a base 20 positional system and a symbol they used as a zero placeholder. The work of the Indian mathematicians spread into the Arabic and Islamic world and was improved upon. This work eventually made its way to the far east and also into Europe. Still, as late as the 1500s European mathematicians were still not using zero as a number on a regular basis. It was not until the 1600s that the use of zero as a number became widespread.

Of course, today we know that adding zero to a number leaves that number unchanged and that division by zero is meaningless,[4] but as we struggle with these concepts, we should keep in mind how long it took humanity to come to grips with this powerful abstraction (zero as a number).

If we add the number zero to the set of natural numbers, we have a new set of numbers which are called the *whole numbers*.

Definition 5. *The set of* whole numbers *is the set*

$$\mathbb{W} = \{0, 1, 2, 3, \ldots\}.$$

The Integers

Today, much as we take for granted the fact that there exists a number zero, denoted by 0, such that

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[4] It makes no sense to ask how many groups of zero are in five. Thus, 5/0 is undefined.

$$a + 0 = a \tag{6}$$

for any whole number a, we similarly take for granted that for any whole number a there exists a unique number $-a$, called the "negative" or "opposite" of a, so that

$$a + (-a) = 0. \tag{7}$$

In a natural way, or so it seems to modern-day mathematicians, this easily introduces the concept of a *negative* number. However, history teaches us that the concept of negative numbers was not embraced wholeheartedly by mathematicians until somewhere around the 17th century.

In his work *Arithmetica* (c. 250 AD?), the Greek mathematician Diophantus (c. 200-284 AD?), who some call the "Father of Algebra," described the equation $4 = 4x + 20$ as "absurd," for how could one talk about an answer less than nothing? Girolamo Cardano (1501-1576), in his seminal work *Ars Magna* (c. 1545 AD) referred to negative numbers as "numeri ficti," while the German mathematician Michael Stifel (1487-1567) referred to them as "numeri absurdi." John Napier (1550-1617) (the creator of logarithms) called negative numbers "defectivi," and Rene Descartes (1596-1650) (the creator of analytic geometry) labeled negative solutions of algebraic equations as "false roots."

On the other hand, there were mathematicians whose treatment of negative numbers resembled somewhat our modern notions of the properties held by negative numbers. The Indian mathematician Brahmagupta, whose work with zero we've already mentioned, described arithmetical rules in terms of fortunes (positive number) and debts (negative numbers). Indeed, in his work *Brahmasphutasiddhanta*, he writes "a fortune subtracted from zero is a debt," which in modern notation would resemble $0 - 4 = -4$. Further, "a debt subtracted from zero is a fortune," which resonates as $0 - (-4) = 4$. Further, Brahmagupta describes rules for multiplication and division of positive and negative numbers:

- *The product or quotient of two fortunes is one fortune.*
- *The product or quotient of two debts is one fortune.*
- *The product or quotient of a debt and a fortune is a debt.*
- *The product or quotient of a fortune and a debt is a debt.*

In modern-day use we might say that "like signs give a positive answer," while "unlike signs give a negative answer." Modern examples of Brahmagupta's first two rules are $(5)(4) = 20$ and $(-5)(-4) = 20$, while examples of the latter two are $(-5)(4) = -20$ and $(5)(-4) = -20$. The rules are similar for division.

In any event, if we begin with the set of natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$, add zero, then add the negative of each natural number, we obtain the set of *integers*.

Definition 8. *The set of* **integers** *is the set*

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}. \tag{9}$$

The letter $\mathbb{Z}$ comes from the word *Zahl*, which is a German word for "number."

It is important to note that an *integer* is a "whole" number, either positive, negative, or zero. Thus, $-11\,456$, -57, 0, 235, and $41\,234\,576$ are integers, but the numbers $-2/5$, 0.125, $\sqrt{2}$ and π are not. We'll have more to say about the classification of the latter numbers in the sections that follow.

Rational Numbers

You might have noticed that every natural number is also a whole number. That is, every number in the set $\mathbb{N} = \{1, 2, 3, \ldots\}$ is also a number in the set $\mathbb{W} = \{0, 1, 2, 3, \ldots\}$. Mathematicians say that "$\mathbb{N}$ is a subset of $\mathbb{W}$," meaning that each member of the set $\mathbb{N}$ is also a member of the set $\mathbb{W}$. In a similar vein, each whole number is also an integer, so the set $\mathbb{W}$ is a subset of the set $\mathbb{Z} = \{\ldots, -2, -2, -1, 0, 1, 2, 3, \ldots\}$.

We will now add fractions to our growing set of numbers. Fractions have been used since ancient times. They were well known and used by the ancient Babylonians and Egyptians.

In modern times, we use the phrase *rational number* to describe any number that is the *ratio* of two integers. We will denote the set of rational numbers with the letter $\mathbb{Q}$.

Definition 10. *The set of* **rational numbers** *is the set*

$$\mathbb{Q} = \left\{ \frac{m}{n} : m, n \text{ are integers}, \ n \neq 0 \right\}. \tag{11}$$

This notation is read "the set of all ratios m/n, such that m and n are integers, and n is not 0." The restriction on n is required because division by zero is undefined.

Clearly, numbers such as $-221/31$, $-8/9$, and $447/119$, being the ratios of two integers, are rational numbers (fractions). However, if we think of the integer 6 as the ratio $6/1$ (or alternately, as $24/4$, $-48/-8$, etc.), then we note that 6 is also a rational number. In this way, any integer can be thought of as a rational number (e.g., $12 = 12/1$, $-13 = -13/1$, etc.). Therefore, the set $\mathbb{Z}$ of integers is a subset of the set $\mathbb{Q}$ of rational numbers.

But wait, there is more. Any decimal that terminates is also a rational number. For example,

$$0.25 = \frac{25}{100}, \qquad 0.125 = \frac{125}{1000}, \qquad \text{and} \ -7.6642 = -\frac{76642}{10000}.$$

The process for converting a terminating decimal to a fraction is clear; count the number of decimal places, then write 1 followed by that number of zeros for the denominator.

For example, in 7.638 there are three decimal places, so place the number over 1000, as in

$$\frac{7638}{1000}.$$

But wait, there is still more, for any decimal that repeats can also be expressed as the ratio of two integers. Consider, for example, the repeating decimal

$$0.0\overline{21} = 0.0212121\ldots.$$

Note that the sequence of integers under the "repeating bar" are repeated over and over indefinitely. Further, in the case of $0.0\overline{21}$, there are precisely two digits[5] under the repeating bar. Thus, if we let $x = 0.0\overline{21}$, then

$$x = 0.0212121\ldots,$$

and multiplying by 100 moves the decimal two places to the right.

$$100x = 2.12121\ldots$$

If we align these two results

$$\begin{aligned} 100x &= 2.12121\ldots \\ -x &= 0.02121\ldots \end{aligned}$$

and subtract, then the result is

$$\begin{aligned} 99x &= 2.1 \\ x &= \frac{2.1}{99}. \end{aligned}$$

However, this last result is not a ratio of two integers. This is easily rectified by multiplying both numerator and denominator by 10.

$$x = \frac{21}{990}$$

We can reduce this last result by dividing both numerator and denominator by 3. Thus, $0.0\overline{21} = 7/330$, being the ratio of two integers, is a rational number.

Let's look at another example.

▶ **Example 12.** *Show that $0.\overline{621}$ is a rational number.*

In this case, there are three digits under the repeating bar. If we let $x = 0.\overline{621}$, then multiply by 1000 (three zeros), this will move the decimal three places to the right.

$$\begin{aligned} 1000x &= 621.621621\ldots \\ x &= 0.621621\ldots \end{aligned}$$

Subtracting,

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[5] The singletons 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are called *digits*.

$$
\begin{aligned}
999x &= 621 \\
x &= \frac{621}{999}.
\end{aligned}
$$

Dividing numerator and denominator by 27 (or first by 9 then by 3), we find that $0.\overline{621} = 23/37$. Thus, $0.\overline{621}$, being the ratio of two integers, is a rational number.

At this point, it is natural to wonder, "Are all numbers rational?" Or, "Are there other types of numbers we haven't discussed as yet?" Let's investigate further.

The Irrational Numbers

If a number is not rational, mathematicians say that it is *irrational*.

Definition 13. *Any number that cannot be expressed as a ratio of two integers is called an* **irrational number**.

Mathematicians have struggled with the concept of irrational numbers throughout history. Pictured in **Figure 2** is an ancient Babylonian artifact called *The Square Root of Two Tablet*.

Figure 2. The *Square Root of Two Tablet.*

There is an ancient fable that tells of a disciple of Pythagoras who provided a geometrical proof of the irrationality of $\sqrt{2}$. However, the Pythagoreans believed in the absoluteness of numbers, and could not abide the thought of numbers that were not rational. As a punishment, Pythagoras sentenced his disciple to death by drowning, or so the story goes.

But what about $\sqrt{2}$? Is it rational or not? A classic proof, known in the time of Euclid (the "Father of Geometry," c. 300 BC), uses *proof by contradiction*. Let us assume that $\sqrt{2}$ is indeed rational, which means that $\sqrt{2}$ can be expressed as the ratio of two integers p and q as follows.

$$\sqrt{2} = \frac{p}{q}$$

Square both sides,

$$2 = \frac{p^2}{q^2},$$

then clear the equation of fractions by multiplying both sides by q^2.

$$p^2 = 2q^2 \tag{14}$$

Now p and q each have their own unique prime factorizations. Both p^2 and q^2 have an *even* number of factors in their prime factorizations.[6] But this contradicts equation 14, because the left side would have an even number of factors in its prime factorization, while the right side would have an odd number of factors in its prime factorization (there's one extra 2 on the right side).

Therefore, our assumption that $\sqrt{2}$ was rational is false. Thus, $\sqrt{2}$ is irrational.

There are many other examples of irrational numbers. For example, π is an irrational number, as is the number e, which we will encounter when we study exponential functions. Decimals that neither repeat nor terminate, such as

$$0.1411411141114\ldots,$$

are also irrational. Proofs of the irrationality of such numbers are beyond the scope of this course, but if you decide on a career in mathematics, you will someday look closely at these proofs. Suffice it to say, there are a lot of irrational numbers out there. Indeed, there are many more irrational numbers than there are rational numbers.

The Real Numbers

If we take all of the numbers that we have discussed in this section, the natural numbers, the whole numbers, the integers, the rational numbers, and the irrational numbers, and lump them all into one giant set of numbers, then we have what is known as the set of *real numbers.* We will use the letter $\mathbb{R}$ to denote the set of all real numbers.

Definition 15.

$$\mathbb{R} = \{x : \ x \text{ is a real number}\}.$$

This notation is read "the set of all x such that x is a real number." The set of real numbers $\mathbb{R}$ encompasses all of the numbers that we will encounter in this course.

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[6] For example, if $p = 2 \times 3 \times 3 \times 5$, then $p^2 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$, which has an even number of factors.

Exercises

In **Exercises 1-8**, find the prime factorization of the given natural number.

1. 80

2. 108

3. 180

4. 160

5. 128

6. 192

7. 32

8. 72

In **Exercises 9-16**, convert the given decimal to a fraction.

9. 0.648

10. 0.62

11. 0.240

12. 0.90

13. 0.14

14. 0.760

15. 0.888

16. 0.104

In **Exercises 17-24**, convert the given repeating decimal to a fraction.

17. $0.\overline{27}$

18. $0.\overline{171}$

19. $0.\overline{24}$

20. $0.\overline{882}$

21. $0.\overline{84}$

22. $0.\overline{384}$

23. $0.\overline{63}$

24. $0.\overline{60}$

25. Prove that $\sqrt{3}$ is irrational.

26. Prove that $\sqrt{5}$ is irrational.

In **Exercises 27-30**, copy the given table onto your homework paper. In each row, place a check mark in each column that is appropriate. That is, if the number at the start of the row is rational, place a check mark in the rational column. *Note: Most (but not all) rows will have more than one check mark.*

27.

	$\mathbb{N}$	$\mathbb{W}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
0					
-2					
$-2/3$					
0.15					
$0.\overline{2}$					
$\sqrt{5}$					

28.

	$\mathbb{N}$	$\mathbb{W}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
$10/2$					
π					
-6					
$0.\overline{9}$					
$\sqrt{2}$					
0.37					

29.

	$\mathbb{N}$	$\mathbb{W}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
$-4/3$					
12					
0					
$\sqrt{11}$					
$1.\overline{3}$					
$6/2$					

30.

	$\mathbb{N}$	$\mathbb{W}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
$-3/5$					
$\sqrt{10}$					
1.625					
$10/2$					
$0/5$					
11					

In Exercises 31-42, consider the given statement and determine whether it is true or false. Write a sentence explaining your answer. In particular, if the statement is false, try to give an example that contradicts the statement.

31. All natural numbers are whole numbers.

32. All whole numbers are rational numbers.

33. All rational numbers are integers.

34. All rational numbers are whole numbers.

35. Some natural numbers are irrational.

36. Some whole numbers are irrational.

37. Some real numbers are irrational.

38. All integers are real numbers.

39. All integers are rational numbers.

40. No rational numbers are natural numbers.

41. No real numbers are integers.

42. All whole numbers are natural numbers.

Properties of the Real Numbers

- The Closure Properties
- The Commutative Properties
- The Associative Properties
- The Distributive Properties
- The Identity Properties
- The Inverse Properties

Property

A **property** of a collection of objects is a characteristic that describes the collection. We shall now examine some of the properties of the collection of real numbers. The properties we will examine are expressed in terms of addition and multiplication.

The Closure Properties

The Closure Properties

If a and b are real numbers, then $a + b$ is a unique real number, and $a \cdot b$ is a unique real number.

For example, 3 and 11 are real numbers; $3 + 11 = 14$ and $3 \cdot 11 = 33$, and both 14 and 33 are real numbers. Although this property seems obvious, some collections are not closed under certain operations. For example,

Example

The real numbers are not closed under division since, although 5 and 0 are real numbers, $5/0$ and $0/0$ are not real numbers.

Example

The natural numbers are not closed under subtraction since, although 8 is a natural number, $8 - 8$ is not. ($8 - 8 = 0$ and 0 is not a natural number.)

The Commutative Properties

Let a and b represent real numbers.

The Commutative Properties

COMMUTATIVE PROPERTY OF ADDITION	COMMUTATIVE PROPERTY OF MULTIPLICATION
$a + b = b + a$	$a \cdot b = b \cdot a$

The commutative properties tell us that two numbers can be added or multiplied in any order without affecting the result.

The following are examples of the commutative properties.

Example

$3+4=4+3$ Both equal 7.

Example

$5+x=x+5$ Both represent the same sum.

Example

$4\cdot 8=8\cdot 4$ Both equal 32.

Example

$y7=7y$ Both represent the same product.

Example

$5\,(a+1)=(a+1)\,5$ Both represent the same product.

Example

$(x+4)\,(y+2)=(y+2)\,(x+4)$ Both represent the same product.

The Associative Properties

Let a, b, and c represent real numbers.

The Associative Properties

ASSOCIATIVE PROPERTY OF ADDITION	ASSOCIATIVE PROPERTY OF MULTIPLICATION
$(a+b)+c=a+(b+c)$	$(ab)\,c=a\,(bc)$

The associative properties tell us that we may group together the quantities as we please without affecting the result.

The following examples show how the associative properties can be used.

Example

$$\begin{aligned}(2+6)+1 &= 2+(6+1)\\ 8+1 &= 2+7\\ 9 &= 9 \qquad \text{Both equal 9.}\end{aligned}$$

Example

$(3+x)+17=3+(x+17)$ Both represent the same sum.

Example

$$\begin{aligned}(2\cdot 3)\cdot 5 &= 2\cdot(3\cdot 5)\\ 6\cdot 5 &= 2\cdot 15\\ 30 &= 30 \qquad \text{Both equal 30.}\end{aligned}$$

Example

$(9y)\,4 = 9\,(y4)$ Both represent the same product.

Example

Simplify (rearrange into a simpler form): $5x6b8ac4$.

According to the commutative property of multiplication, we can make a series of consecutive switches and get all the numbers together and all the letters together.

$5 \cdot 6 \cdot 8 \cdot 4 \cdot x \cdot b \cdot a \cdot c$

$960xbac$ Multiply the numbers.

$960abcx$ By convention, we will, when possible, write all letters in alphabetical order.

The Distributive Properties

When we were first introduced to multiplication we saw that it was developed as a description for repeated addition.

$4+4+4 = 3 \cdot 4$

Notice that there are three 4's, that is, 4 appears 3 **times**. Hence, 3 times 4.

We know that algebra is generalized arithmetic. We can now make an important generalization.

When a number a is added repeatedly n times, we have

$$\underbrace{a+a+a+\cdots+a}_{a \text{ appears } n \text{ times}}$$

Then, using multiplication as a description for repeated addition, we can replace

$\underbrace{a+a+a+\cdots+a}_{n \text{ times}}$ with na

For example:

Example

$x+x+x+x$ can be written as $4x$ since x is repeatedly added 4 times.

$x+x+x+x = 4x$

Example

$r+r$ can be written as $2r$ since r is repeatedly added 2 times.

$r+r = 2r$

The distributive property involves both multiplication and addition. Let's rewrite $4\,(a+b)$. We proceed by reading $4\,(a+b)$ as a multiplication: 4 times the quantity $(a+b)$. This directs us to write

$$\begin{aligned} 4\,(a+b) &= (a+b)+(a+b)+(a+b)+(a+b) \\ &= a+b+a+b+a+b+a+b \end{aligned}$$

Now we use the commutative property of addition to collect all the a's together and all the b's together.

$$4\,(a+b) = \underbrace{a+a+a+a}_{4a\text{'s}} + \underbrace{b+b+b+b}_{4b\text{'s}}$$

Now, using multiplication as a description for repeated addition, we have

$$4\,(a+b) = 4a+4b$$

We have **distributed** the 4 over the sum to both a and b.

$$4(a+b) = 4a+4b$$

The Distributive Property

$a\,(b+c) = a \cdot b + a \cdot c \quad (b+c)\; a = a \cdot b + a \cdot c$

The distributive property is useful when we cannot or do not wish to perform operations inside parentheses.

Use the distributive property to rewrite each of the following quantities.

Example

$2(5+7) = 2\cdot 5 + 2\cdot 7$ **Both equal 24.**

Example

$6(x+3) = 6\cdot x + 6\cdot 3$ **Both represent the same number.**

$= 6x + 18$

Example

$(z+5)y = zy + 5y = yz + 5y$

The Identity Properties

Additive Identity

The number 0 is called the **additive identity** since when it is added to any real number, it preserves the identity of that number. Zero is the only additive identity.

For example, $6+0=6$.

Multiplicative Identity

The number 1 is called the **multiplicative identity** since when it multiplies any real number, it preserves the identity of that number. One is the only multiplicative identity.

For example $6\cdot 1 = 6$.

We summarize the identity properties as follows.

ADDITIVE IDENTITY PROPERTY	MULTIPLICATIVE IDENTITY PROPERTY
If a is a real number, then	If a is a real number, then
$a+0=a$ and $0+a=a$	$a\cdot 1=a$ and $1\cdot a=a$

The Inverse Properties

Additive Inverses

When two numbers are added together and the result is the additive identity, 0, the numbers are called **additive inverses** of each other. For example, when 3 is added to -3 the result is 0, that is, $3+(-3)=0$. The numbers 3 and -3 are additive inverses of each other.

Multiplicative Inverses

When two numbers are multiplied together and the result is the multiplicative identity, 1, the numbers are called **multiplicative inverses** of each other. For example, when 6 and $\frac{1}{6}$ are multiplied together, the result is 1, that is, $6\cdot\frac{1}{6}=1$. The numbers 6 and $\frac{1}{6}$ are multiplicative inverses of each other.

We summarize the inverse properties as follows.

The Inverse Properties

1. If a is any real number, then there is a unique real number $-a$, such that

   $a+(-a)=0$ and $-a+a=0$

   The numbers a and $-a$ are called **additive inverses** of each other.
2. If a is any nonzero real number, then there is a unique real number $\frac{1}{a}$ such that

   $a\cdot\frac{1}{a}=1$ and $\frac{1}{a}\cdot a=1$

   The numbers a and $\frac{1}{a}$ are called **multiplicative inverses** of each other.

EXERCISES

In problems **(1)-(8)** fill in the ( ) to make each statement true. Also write what property of real numbers you have used.

1) $m + 12 = 12 + (\quad)$

2) $x + (5 + y) = (\quad) + y$

3) $6a = a(\quad)$

4) $(9 + 2) + 5 = 9 + (\quad)$

5) $(11a)\,6 = 11(\quad)$

6) $4(k - 5) = (\quad)\,4$

7) $(9a - 1)(\quad) = (2b + 7)(9a - 1)$

8) $[(7m - 2)(m + 3)](m + 4) = (7m - 2)\left[(\quad)(\quad)\right]$

9) Simplify each of the following quantities.

a) $3a7y9d$

b) $6b8acz4 \cdot 5$

c) $4p6qr3\,(a + b)$

Chapter 1

Use the commutative property of addition and multiplication to write expressions for an equal number for the problems **(10)-(16).** You need not perform any calculations.

10) $(8+a)(x+6)$

11) $(x+16)(a+7)$

12) $(x+y)(x-y)$

13) $0.06m$

14) 8☆

15) $k(10a-b)$

16) □ · [U+25CB]

For the problems **(17)-(26)**, use the distributive property to expand the quantities.

17) $m(u+a)$

18) $k(j+1)$

19) $z(x+9w)$

20) $(1+d)e$

21) $(8+2f)g$

22) $15x(2y+3z)$

23) $(a+6)(x+y)$

24) $(x+10)(a+b+c)$

25) $1(a+16)$

26) $2z_t(L_m+8k)$