Worksheet 16 March 16, 2011

1. Suppose my sole goal in life is to maximize the number of cheap cheeseburgers I eat. The obvious solution is to spend all my time eating cheeseburgers, but it is well known that this will shorten my lifespan. Suppose my future lifespan can be modeled as a function of the amount of cheesy meaty goodness per week I consume, say $\ell(z) = 80z/(1+z^4)$. Here ℓ is for 'l'ifespan and is measured in years, and z is for chee'z'burger and is measured in burgers per week.

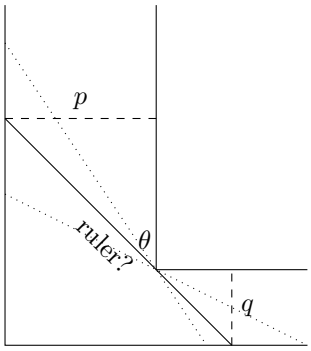
   (a) How many cheeseburgers per week should I eat to maximize my future lifespan? If I do this, how many cheeseburgers will I eat over the course of my future lifetime?

   (b) How many cheeseburgers per week should I consume to maximize my total consumption of cheeseburger? What is this maximum?

   (For the record, $3^{3/4}/4 \approx 0.57$ and $\sqrt{3}/4 \approx 0.43$.)

2. Find $\frac{dy}{dx}$ given that $x^{\arcsin y} = y^{\arcsin x}$. (Please don't try to simplify anything.)

3. Suppose you want to carry a very long ruler through your hallway (to measure stuff?). Unfortunately along the way you need to maneuver around a corner: coming into the corner the hallway is p meters wide, and leaving the corner the hallway is q meters wide. What is the maximum length of the ruler that will make it through the corner? [Note: you should assume that you don't tilt the ruler vertically (maybe it's triggered to explosives, or maybe we'll adjust the length of the ruler to account for the maximum such tilt, but either way you should ignore it for this problem and stay two-dimensional).] Hint: use the following diagram.



   Solve this in general if you like (i.e. in terms of p, q), or consider the special case case of $p = \sqrt{27}$, $q = 1$.

4. What is WRONG about the following calculation:

$$\lim_{x\to 0^+} \frac{e^x - 1}{x^2} \overset{(L)}{=} \lim_{x\to 0^+} \frac{e^x}{2x} \overset{(L)}{=} \lim_{x\to 0^+} \frac{e^x}{2} = \frac{1}{2}.$$

   Fix the calculation to find the real value of $\lim_{x\to 0^+} \frac{e^x - 1}{x^2}$.

5. Make a fairly good sketch of the graph of $f(x) = (x-8)^{10}(x+3)$ using the function and its first two derivatives. In doing this, you should list the intervals on which it is increasing, decreasing,

concave up, and concave down, and list any local extrema. (Suggestion: don't choose your vertical scaling until after you've found the function values of interest; it will need to be very large.) What are the global minimum and global maximum on $[-3, 0]$? On $[0, 3]$?

6. Show that the volume of a pizza, in the shape of a right circular cylinder of radius z and height a, is $pizza$. (Okay, that isn't calculus, and it is a joke. But you'll need the formula you used here for the next problem.)

7. Suppose you have enough material to make a can (which should be in the shape of a right circular cylinder) with surface area 2π. What is the maximum volume such a can can enclose?

8. Find the point on the curve $y = x^2$ that is closest to the point $(2\pi^3, 1/2)$. How far apart are they? (Suggestion: minimizing the square of the distance also minimizes the distance.)

9. Suppose you have 10ft of wire and need to divide it into two pieces: one to form a square, and the other to form a circle. Find the maximum and minimum possible total amount of area enclosed by both shapes.

10. Find the rectangle of maximum area that can be inscribed in the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$. (You can assume that such a rectangle has sides parallel to the axes. Make use of symmetry.)