Math 241, Sections BL1 and BL2

Quiz # 2 BDL

September 25, 2012

Solve both exercises. Show work to get credit.

1) [5pts.] Find the linearization of $\boxed{f(x,y) = 7 + e^{-xy}\cos y}$ at $\boxed{(\pi, 0)}$

Solution: We compute

$$\begin{aligned} f(\pi,0) &= 7 + e^0\cos(0) = 8, \\ f_x(\pi,0) &= -ye^{-xy}\cos y\big|_{(\pi,0)} = 0, \\ f_y(\pi,0) &= -xe^{-xy}\cos y - e^{-xy}\sin y\big|_{(\pi,0)} = -\pi. \end{aligned}$$

Hence the linearization is given by

$$L(x,y) = 8 + 0(x-\pi) - \pi(y-0) = 8 - \pi y.$$

2) [5pts.] Use the equation $\boxed{x - z = 3\arctan(yz)}$ to find $\boxed{\dfrac{\partial z}{\partial x}}$ and $\boxed{\dfrac{\partial z}{\partial y}}$

Solution: Define $F(x,y,z) = x - z - 3\arctan(yz)$. Our discussion on the Implicit Function Theorem tells us that

$$\begin{aligned} \frac{\partial z}{\partial x} &= -\frac{F_x}{F_z} \\ &= -\frac{1}{-1 - \frac{3y}{1+(yz)^2}}, \\ \frac{\partial z}{\partial y} &= -\frac{F_y}{F_z} \\ &= -\frac{\frac{-3z}{1+(yz)^2}}{-1 - \frac{3y}{1+(yz)^2}}. \end{aligned}$$

You need not simplify, but this last expression is pretty ugly and easy to clean up:

$$\frac{\partial z}{\partial y} = \frac{-3z}{1 + 3y + y^2z^2}.$$