**Discussion 26 – Thursday, November 18th**

**1.** The function

$$\Phi(s,t) = ((2+\cos s)\cos t, (2+\cos s)\sin t, \sin s), 0 \le s,t \le 2\pi$$

is a parametrization of a torus. (This torus is the circle in $xz$-plane of radius 1 centered at (2,0,0) rotated about the $z$-axis.)
(a) Show that the point $P\left(\frac{5\sqrt{2}}{4}, \frac{5\sqrt{2}}{4}, \frac{\sqrt{3}}{2}\right)$ lies on the torus.
(b) Find a normal vector to the torus at the point $P$.
(c) Find an equation for the tangent plane to the torus at the point $P$.

**2.** Again, consider the parametrization of the torus.
(a) Show that the normal vector $\Phi_s \times \Phi_t = -(2+\cos s)(\cos s \cos t, \cos s \sin t, \sin s)$.
(b) Compute $||\Phi_s \times \Phi_t||$. Why does this show that there is a well defined tangent plane to every point on the torus?
(c) Compute the surface area of the torus.

**3.** Consider the parametrization $\Phi(s,t) = (a\cosh s \cos t, a\cosh s \sin t, a\sinh s)$, $0 \le s \le 2\pi$, $0 \le t \le \pi$ and $a$ is a fixed positive constant. Identify this quadratic surface by name and by finding a Cartesian equation $F(x,y,z) = c$. (Hint: $\cosh^2 x - \sinh^2 x = 1$.)

**4.** Let $S$ be the hemisphere $x^2+y^2+z^2 = 1$, $z \ge 0$.
(a) Find a parametrization $\Phi(s,t)$ of the surface $S$. (Hint: What change of variables can be used to quickly describe this surface?)
(b) Compute the normal vector $\Phi_s \times \Phi_t$ associated with your parametrization. Does your answer make sense?
(c) Compute the integral $\iint_S yz\, dS$.

**5.** Let the surface $S$ be the part of the plane $2x+y+z = 4$ in the first octant.
(a) Find a parametrization $\Phi(s,t)$ of the surface $S$. Note: You must always state the domain of the function $\Phi$ in order to have a complete parametrization.
(b) Compute the normal vector $\Phi_s \times \Phi_t$ associated with your parametrization. Does your answer make sense?
(c) Find the area of the part of the plane $2x+y+z = 4$ in the first octant.