Math 241, Sections BL1 and BL2

Quiz # 4 BDD

October 18, 2012

Solve both exercises. Show work to get credit.

1) [5pts.] Find the work done by the force field

$$\vec{F}(x,y) = x\vec{i} + (y+1)\vec{j}$$

in moving an object along an arch of the cycloid

$$\vec{r}(t) = (t - \sin t)\vec{i} + (1 - \cos t)\vec{j}, \quad t \in [0, 2\pi].$$

Let's try to find a potential function for F; we need

$f_x = x \qquad f_y = y+1$

$\Rightarrow f = \frac{1}{2}x^2 + g(y)$

$\Rightarrow f_y = 0 + g'(y) = y+1$

$\Rightarrow g(y) = \frac{1}{2}y^2 + y \Rightarrow f(x,y) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + y.$

So FTLI $\Rightarrow W = \int_C F \cdot d\vec{r} = f(r(2\pi)) - f(r(0))$

$= f(2\pi, 0) - f(0,0)$

$= \left(\frac{1}{2}(2\pi)^2 + 0\right) - (0)$

$= \boxed{2\pi^2.}$

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$$\vec{F}(x,y) = x\vec{i} + (y+1)\vec{j}$$

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$$\vec{r}(t) = (t - \sin t)\vec{i} + (1 - \cos t)\vec{j}, \quad t \in [0, 2\pi].$$

$$r'(t) = (1 - \cos t)\vec{i} + (\sin t)\vec{j}$$

Or, from the definition,

$$W = \int_C F \cdot d\vec{r} = \int_0^{2\pi} \langle t - \sin t,\ 2 - \cos t \rangle \cdot \langle 1 - \cos t,\ \sin t \rangle \, dt$$

$$= \int_0^{2\pi} \Big((t - \sin t)(1 - \cos t) + (2 - \cos t)(\sin t) \Big) \, dt$$

$$= \int_0^{2\pi} \left(t - t\cos t - \sin t + \sin t \cos t + 2\sin t - \sin t \cos t \right) dt$$

$$= \int_0^{2\pi} \left(t - t\cos t + \sin t \right) dt$$

$$= \left[\tfrac{1}{2}t^2 - \cos t \right]_0^{2\pi} - \int_0^{2\pi} t \cos t \, dt \qquad \begin{array}{ll} u = t & dv = \cos t\, dt \\ du = dt & v = \sin t \end{array}$$

$$= 2\pi^2 - 1 - (0 - 1) - \left(t \sin t \Big|_0^{2\pi} - \int_0^{2\pi} \sin t \, dt \right)$$

$$= 2\pi^2 - \left(0 + \cos t \Big|_0^{2\pi} \right)$$

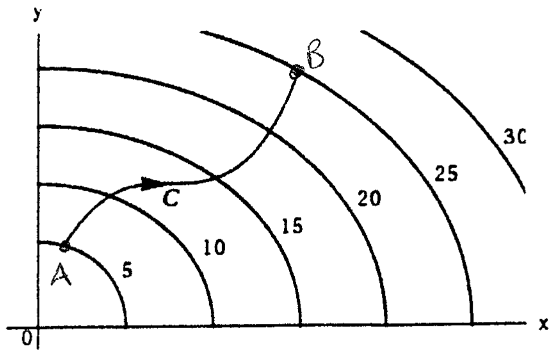
$$= \boxed{2\pi^2}$$

2) [5pts.] (a) The figure below shows a curve C and a contour map of a function f whose gradient is continuous. Find

$$\int_C (\nabla f) \cdot d\vec{r}. \quad = f(B) - f(A)$$

$$= 25 - 5$$

$$= \boxed{20}$$



(b) A table of values of a function f with continuous gradient is given. Find

$$\int_C (\nabla f) \cdot d\vec{r},$$

where C has the parametric equations

$$x = t^4 + 1, \quad y = t^3 + t, \quad t \in [0, 1].$$

$x \backslash y$	0	1	2
0	1	6	4
1	1	5	9
2	6	3	9

$$= f(x(1), y(1)) - f(x(0), y(0))$$

$$= f(2, 2) - f(1, 0)$$

$$= 9 - 1$$

$$= \boxed{8}$$