

Math 241 X8**Name(s):****Homework 8 supplement**

This is a written homework supplement to the homework for Unit 8: Sources, Sinks, Swirls, and Singularities.

- (1) Let $\mathbf{F}(x, y) = (e^x \sin y, e^x \cos y)$. Compute $\operatorname{div} \mathbf{F}$ and $\operatorname{rot} \mathbf{F}$. What do these tell you about the net flow of \mathbf{F} across/along closed curves?

- (2) Consider the rectangle C with vertices at $(-1, -2)$, $(5, -2)$, $(5, 2)$, and $(-1, 2)$. Measure the net flow of $\mathbf{F}(x, y) = (x^2 + 2y^2, x^2 - 2y^2)$ across C . Is it inside to outside or outside to inside? (Hint: you could parametrize that curve, but you'd rather not.)

- (3) Consider the same rectangle and the same \mathbf{F} from problem (2). Measure the net flow of \mathbf{F} along C . Is it clockwise or counterclockwise? (Again, find a way to avoid parametrizing the curve.)

- (4) Classify the points of the plane as sources, sinks, or neither, for the vector field

$$\mathbf{G}(x, y) = \left\langle \frac{x^2 - x + y^2}{x^2 + y^2}, \frac{x^2 + 2y + y^2}{x^2 + y^2} \right\rangle.$$

(It may help to manipulate some algebra before jumping into derivatives. Don't forget to check singularities separately.)