Worksheet 15 March 14 (3/14), 2011

1. Find the derivative of $\pi^{\pi \log_\pi(\sqrt[\pi]{x^\pi+\pi})}$ with respect to x.

2. Define $\Pi(r)$ to be the ratio of circumference to diameter of a circle with radius r. In regular old Euclidean geometry, this is a constant function. There are two major other types of geometry: spherical and hyperbolic. In a certain hyperbolic geometry, this function is *not* constant; in fact it is given by
$$\Pi(r) = \pi \frac{e^r - e^{-r}}{2r}$$
(where π is old faithful Euclidean π). Find the maximum and minimum values of Π in this space (if they exist). (Hint: you won't be able to explicitly solve; simplify down to a point where you have an exponential equal to other things, and think about what those other things can be.) What kind of circles achieve those extrema?

3. Determine $\displaystyle\lim_{x\to\pi^-} \frac{\tan x}{(x-\pi)^2}$

4. Find $\displaystyle\lim_{x\to 0} \frac{\sin(\pi x) - \pi x}{x^3}$.

5. Compute $\displaystyle\lim_{x\to 0^+} (\pi x)\ln(\pi x)$

6. Use the previous problem to evaluate $\displaystyle\lim_{x\to 0^+} (\pi x)^{\pi x}$.

7. Compute $\displaystyle\lim_{x\to\infty} \frac{\sqrt{\pi x + 1}}{\sqrt{\pi x - 1}}$.

8. Suppose we are given constants a, b, c, d such that
$$\lim_{x\to 0} \frac{ax^2 + \sin bx + \sin cx + \sin dx}{3x^2 + 5x^4 + 7x^6} = \pi.$$
Find the value of $a + b + c + d$.

9. Find the point on the curve $y = x^2$ that is closest to the point $(2\pi^3, 1/2)$. (Suggestion: minimizing the square of the distance also minimizes the distance.)

10. Suppose you have 10ft of wire and need to divide it into two pieces: one to form a square, and the other to form a circle. Find the maximum and minimum possible total amount of area enclosed by both shapes.

11. Show that the volume of a pizza, in the shape of a right circular cylinder of radius z and height a, is $pizza$. (Okay, that isn't calculus, and it is a joke. But you'll need it for the next problem.)

12. Suppose you have enough material to make a can (which should be in the shape of a right circular cylinder) with surface area 2π. What is the maximum volume such a can can enclose?

13. Suppose my sole goal in life is to maximize the amount of pie I eat. The obvious solution is to spend all my time eating delicious pie, but it is well known that this will shorten my life span. Suppose my life span can be modeled as a function of the amount of pie per year I consume, say $\ell(p) = 75 - p^2$. What amound of pie per year should I consume to maximize my total consumption, and what is this maximum?

14. Find the rectangle of maximum area that can be inscribed in the ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{25}=1$. (You can assume that such a rectangle has sides parallel to the axes. Make use of symmetry.)