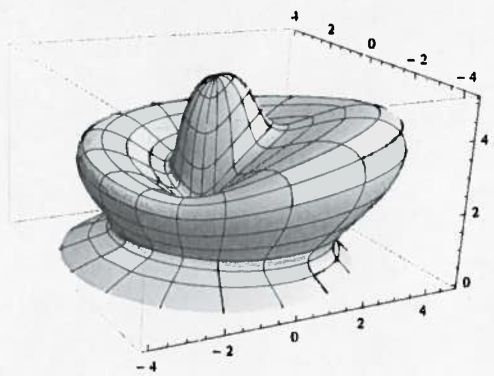
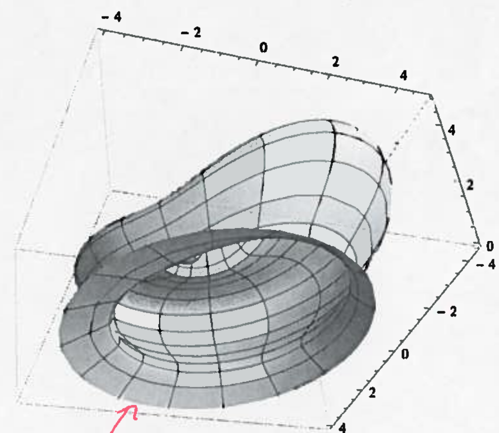
Math 241 X8 **Name:**

Quiz # 8

November 20, 2013 No electronic devices or interpersonal communication allowed. Show work to get credit.

(1) A surface R is given in spherical coordinates as $\rho = 4 + \sin(2\varphi)(e^{-\sin\theta}\cos\theta) + \cos(7\varphi)$. Below are two pictures of R. Find the net flow of $\mathbf{F}(x,y,z) = \langle xz + e^y, -yz - \sin z, 1\rangle$ across R. Big Hint: don't do this directly.





Boundary is where $\varphi = \frac{\pi}{2}$:

$\rho = 4 + 0 + 0 = 4$, $\theta \in [0, 2\pi]$

is a circle in xy-plane of radius 4.

$$\operatorname{div}\vec{F} = \nabla\cdot\vec{F} = (z) + (-z) + (0) = 0.$$

& No singularities, so we may use a substitute surface.

The disk of radius 4 in xy plane, D, works. D: $x = r\cos\theta$, $y = r\sin\theta$, $z = 0$

$$\vec{dS} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \cos\theta & \sin\theta & 0 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = \langle 0,0,r\rangle$$

$$\text{net flow across } D = \iint_D F\cdot\vec{dS} = \int_0^{2\pi}\int_0^4 \langle \cdots, \cdots, 1\rangle\cdot\langle 0,0,r\rangle\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^4 r\, dr\, d\theta = 16\pi \quad \text{upward,}$$

so net flow across R is also 16π upward/outward.

(For the solid region, net flow is zero; for the disk, 16π goes inward to the 3D region, so 16π must exit through R.)