Name: ______________________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 10:50am to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.
- Make sure to check whether you are asked to *set up* or to *evaluate* integrals.

Some possibly useful formulas:

$$\cos^2 t = \frac{1}{2}(1+\cos(2t))$$
$$\sin^2 t = \frac{1}{2}(1-\cos(2t))$$
$$\sin(2t) = 2\sin(t)\cos(t)$$

Question:	1	2	3	4	Total
Points:	25	25	35	15	100
Score:					

1. (25 points) Compute the flow of $\mathbf{F}(x, y, z) = \langle yz^2 + e^z + x, ze^z + x + y, xe^x + xy + z \rangle$ across the surface R that is the boundary of the solid cube D given by $-1 \le x \le 1$, $-1 \le y \le 1$, $-1 \le z \le 1$. Which direction is it?

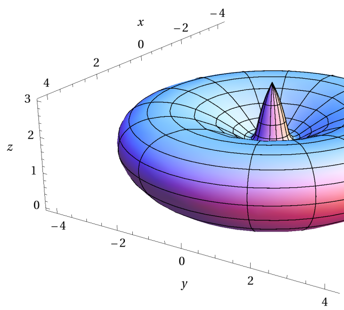
2. (25 points) Find the volume contained between the hemispheres $z = \sqrt{4 - x^2 - y^2}$ and $z = \sqrt{1 - x^2 - y^2}$ and the cone $z = \sqrt{x^2 + y^2}$.

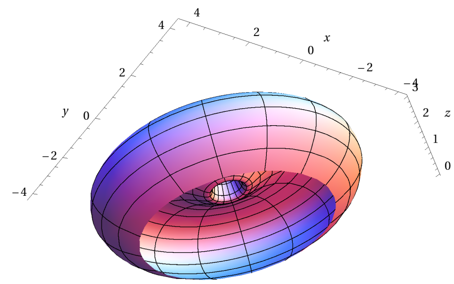
3. (35 points) Consider the surface R that is the part of the cone $z = \sqrt{x^2 + y^2}$ with $z \leq 1$. Let $\mathbf{F}(x, y, z) = \langle yz, -xz, 1 \rangle$.

   (a) Directly compute $\iint_R \operatorname{curl}(\mathbf{F}) \cdot \mathbf{dS}$. Use a downward/outward normal vector.

   (b) Check your answer to (a) using Stokes's Theorem.

4. (15 points) Let $\mathbf{F}(x,y,z) = \langle x, -y+z, y^2 \rangle$. Set up an integral (that you could feed into Mathematica) to compute the flow of $\mathbf{F}$ across the surface R shown below. R can be described in spherical coordinates by $\rho = 1.5(2 - \sin(4\varphi))$ (θ free), $0 \le \varphi \le \pi/2$. (In Mathematica's spherical notation, that's $r[s\_, t\_] = 1.5(2 - Sin[4s])$, $0 \le s \le \pi/2$.) Carefully explain your work. (Hint: don't work too hard.)





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