# Math 415 ADG

**Name:** Solutions

### Quiz # 2

February 7, 2014  No notes, electronic devices, or interpersonal communication allowed. Show work to get credit. Use the methods from this class.

Is $\begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix}$ in the span of $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 7 \\ -2 \end{bmatrix}$ ?

$\left(\right.$ "in the span of" means "is a linear combination of", so we want to know if there are scalars $c_1$ & $c_2$ such that

$$\begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 7 \\ -2 \end{bmatrix},$$

i.e. $\begin{cases} 1c_1 + 1c_2 = 2 \\ 1c_1 + 7c_2 = -4 \\ 1c_1 - 2c_2 = 5 \end{cases}$ $\left.\right)$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & 7 & -4 \\ 1 & -2 & 5 \end{array}\right] \xrightarrow[R_3 := R_3 - R_1]{R_2 := R_2 - R_1} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 6 & -6 \\ 0 & -3 & 3 \end{array}\right]$$

$$\xrightarrow{R_3 := R_3 + \frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 6 & -6 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow{R_2 := \frac{1}{6}R_2} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 := R_1 - R_2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

This is in echelon form, and is enough to tell us that the system is consistent, hence

$\boxed{\text{Yes, } \begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix} \text{ is in } \mathrm{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ -2 \end{bmatrix} \right\}.}$

$\left(\right.$ from the reduced echelon form $\rightarrow$ , we see that $c_1 = 3$ & $c_2 = -1$, i.e.

$$\begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - 1 \begin{bmatrix} 1 \\ 7 \\ -2 \end{bmatrix},$$

which easily confirms that $\left.\right)$