Math 415 ADG **Name:** Solution

Quiz # 4

February 28, 2014

No notes, electronic devices, or interpersonal communication allowed. Show work to get credit. Use the methods from this class.

Determine whether the following vectors are linearly independent.

$$\begin{array}{ccccc} \begin{bmatrix} 5 \\ -4 \\ -1 \end{bmatrix}, & & \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, & & \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \\ v_1 & & v_2 & & v_3 \end{array}$$

$\Leftrightarrow$ no nontrivial solutions to $x_1 v_1 + x_2 v_2 + x_3 v_3 = \vec{0}$

$$x_1 v_1 + x_2 v_2 + x_3 v_3 = [v_1\ v_2\ v_3]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 5 & 1 & -1 & 0 \\ -4 & 2 & 2 & 0 \\ -1 & -3 & -1 & 0 \end{array}\right] \xrightarrow[R_3 \times = -1]{R_1 += R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ -4 & 2 & 2 & 0 \\ 1 & 3 & 1 & 0 \end{array}\right] \xrightarrow{R_3 -= R_1} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ -4 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

not in echelon form, but already we know there are fewer pivots than columns.

$$\xrightarrow{R_2 += 4R_1} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 14 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

echelon form

$$\Leftrightarrow \quad x_2 = -\frac{6}{14}x_3 = -\frac{3}{7}x_3$$

$$x_1 = -3x_2 - x_3 = \frac{2}{7}x_3$$

So, e.g., $2v_1 - 3v_2 + 7v_3 = \vec{0}$

taking $x_3 = 7$

This system of equations has a free variable, and has the trivial solution, hence has ∞ly many solutions.

Hence $\{v_1, v_2, v_3\}$ is NOT linearly independent. (It is linearly dependent.)

$$\dim \underset{A}{\underbrace{\text{Nul}\begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 1 & 2 & 0 & 2 & 8 \\ 2 & 4 & -3 & 10 & 9 \\ 3 & 6 & 0 & 6 & 9 \end{bmatrix}}}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & -4 & 8 & 0 \\ 1 & 2 & 0 & 2 & 8 & 0 \\ 2 & 4 & -3 & 10 & 9 & 0 \\ 3 & 6 & 0 & 6 & 9 & 0 \end{array}\right] \xrightarrow[\substack{R_3 -= 2R_1 \\ R_4 -= 3R_1}]{R_2 -= R_1} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & -4 & 8 & 0 \\ 0 & 0 & -3 & 6 & 0 & 0 \\ 0 & 0 & -9 & 18 & -7 & 0 \\ 0 & 0 & -9 & 18 & -15 & 0 \end{array}\right]$$

$$\xrightarrow[R_4 -= 3R_2]{\substack{R_1 += R_2 \\ R_3 -= 3R_2}} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 2 & 8 & 0 \\ 0 & 0 & -3 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & -7 & 0 \\ 0 & 0 & 0 & 0 & -15 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 \div= -7]{R_2 \div= -3} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 2 & 8 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -15 & 0 \end{array}\right]$$

$$\xrightarrow[R_4 += 15R_3]{R_1 -= 8R_3} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \longleftrightarrow \begin{array}{l} x_1 = -2x_2 - 2x_4 \\ x_2 \text{ free} \\ x_3 = 2x_4 \\ x_4 \text{ free} \\ x_5 = 0 \end{array}$$

$$\text{So } \text{Nul}\, A = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} x_4 \;:\; x_2, x_4 \in \mathbb{R} \right\}$$

$$= \text{Span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\},$$

and $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ is linearly independent,

so $\dim \text{Nul}\, A = 2$.