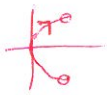
Math 251
Quiz 9 November 21, 2016

Name:

By handing in this quiz you assert that you understand and have followed IIT's guidelines for academic integrity.

Consider the vector field $\vec{F}(x,y) = \langle 2xy+1, x^2+\pi\cos(\pi y)\rangle$ and the curve C that is the part of the parabola $x = y^2$ oriented from $(1,-1)$ to $(1,1)$.



(1) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ DIRECTLY.

Parametrize C: $x = t^2$, $y = t$, $t \in [-1,1]$

$$\int_{-1}^{1} \langle 2(t^2)(t)+1,\ (t^2)^2+\pi\cos(\pi t)\rangle \cdot \langle 2t, 1\rangle\, dt$$

$$= \int_{-1}^{1} (4t^4+2t) + (t^4+\pi\cos(\pi t))\, dt$$

$$= \int_{-1}^{1} 5t^4+2t+\pi\cos(\pi t)\, dt = \left[t^5+t^2+\sin(\pi t)\right]_{-1}^{1} = 2+0+0$$

(2) $\vec{F}$ is conservative; find a potential function for it.

$f_x = 2xy+1 \Rightarrow f = x^2y + x + g(y)$

$f_y = x^2 + 0 + g'(y) = x^2 + \pi\cos(\pi y)$

$\Rightarrow g'(y) = \pi\cos(\pi y) \Rightarrow g = \sin(\pi y) + K$

$f(x,y) = x^2y + x + \sin(\pi y)$ is a potential function

(3) Verify your answer to part (1) using the Fundamental Theorem of Line Integrals.

$$\int_C \vec{F}\cdot d\vec{r} = f(1,1) - f(1,-1)$$

$$= (1+1+0) - (-1+1+0) = 2$$