Math 415 ADG   Name: Solution
Quiz # 10
April 25, 2014

No notes, electronic devices, or interpersonal communication allowed. **Show work to get credit.** Use the methods from this class.

Find matrices P and D such that $PDP^{-1} = \begin{bmatrix} 2 & 1 & 4 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

eigenvalues are $2, -1 \ (*2)$ $\begin{bmatrix} \text{triangular matrices' e-val's} \\ \text{are their diagonal entries} \end{bmatrix}$

$E_2 = \text{Nul} \begin{bmatrix} 0 & 1 & 4 \\ & -3 & 0 \\ & & -3 \end{bmatrix} \overset{R_3 += 3R_3}{=} \text{Nul} \begin{bmatrix} 0 & 1 & 4 \\ 0 & 0 & 12 \\ 0 & 0 & -3 \end{bmatrix} \overset{R_2 \cdot \frac{1}{12} R_2}{\underset{R_3 += 3R_2}{=}} \text{Nul} \begin{bmatrix} 0 & 1 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \overset{R_1 -= 4R_2}{=} \text{Nul} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{pmatrix} x_2 = 0 \\ x_3 = 0 \\ x_1 \text{ free} \end{pmatrix} = \text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

$E_{-1} = \text{Nul} \begin{bmatrix} 3 & 1 & 4 \\ 0 & 0 & 0 \\ & & 0 \end{bmatrix} = \text{Span} \left\{ \begin{bmatrix} -1/3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4/3 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\begin{pmatrix} x_1 = \frac{1}{3}(-x_2 - 4x_3) \\ x_2, x_3 \text{ free} \end{pmatrix}$

So, e.g., $P = \begin{bmatrix} -1/3 & -4/3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ & $D = \begin{bmatrix} -1 & & \\ & -1 & \\ & & 2 \end{bmatrix}$ work.