Name: ______________________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 10:50am to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

1. Consider the parametric equations $x = t^2$, $y = \sin(\pi t)$.
   (a) Find the slope(s) of the tangent line(s) to the curve at the point(s) where $x = 9$.

   (b) What are the minimum and maximum values taken by x and y in this curve?

2. Let $\vec{u} = \langle 2, 6 \rangle$ and $\vec{v} = \langle -2, -1 \rangle$. Compute and plot the following together with u and v.

   (a) $\vec{u} + \vec{v}$

   (b) $2\vec{v}$

   (c) the angle between $\vec{u}$ and $\vec{v}$

   (d) the push/projection of $\vec{u}$ in the direction of $\vec{v}$

   (e) If $\vec{u}$ and $\vec{v}$ live in the plane of this paper, and you consider the paper in the 3D classroom, then which direction does $\vec{u} \times \vec{v}$ point?

3. Consider the lines $\ell_1(t) = (0, 1, 3) + t(-2, -2, -10)$ and $\ell_2(t) = (-5, 2, 3) + t(-9, 9, 30)$. Are they parallel, perpendicular, or neither? Do they intersect or not? If they intersect or are parallel, find an xyz-equation of the plane containing them. Otherwise find the distance between them.

4. Consider the lines $\ell_1(t) = (0, 1, 3) + t(2, 1, 5)$ from above and $\ell_3(t) = (3, 0, 1) + t(-2, -1, 1)$. Are they parallel, perpendicular, or neither? Do they intersect or not? If they intersect or are parallel, find an xyz-equation of the plane containing them. Otherwise find the distance between them.

5. Consider the two planes given by equations

$$3x - y + z = 4$$
$$2x + y - 2z = 6.$$

Find an equation of the line that is the intersection of these planes.

6. Consider the two planes given by equations

$$\begin{aligned} 3x + 12y - 3z &= 1 \\ 2x + 8y - 2z &= 7. \end{aligned}$$

Find the distance between them.

7. Suppose a particle moves in the plane, with position at time t given by (t^2, t^3) at time t.

   (a) Find the velocity at time $t = 9$.

   (b) Find the acceleration at time $t = 9$.

   (c) Find the tangential component of acceleration (i.e. the push of acceleration in the direction tangential to motion) at time $t = 9$.

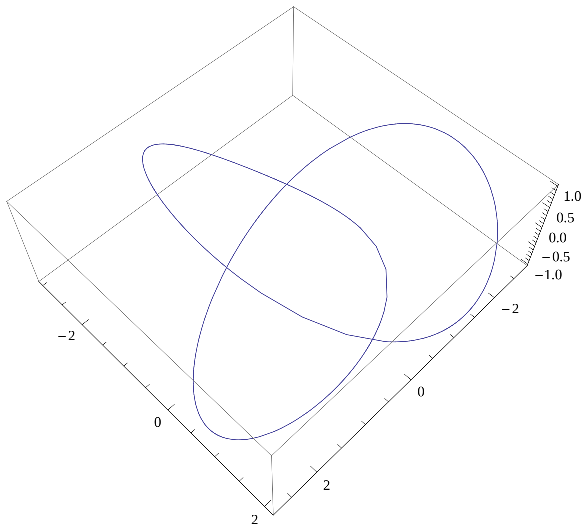
   (d) Find the normal component of acceleration (i.e. the push of acceleration in the direction perpendicular to motion.

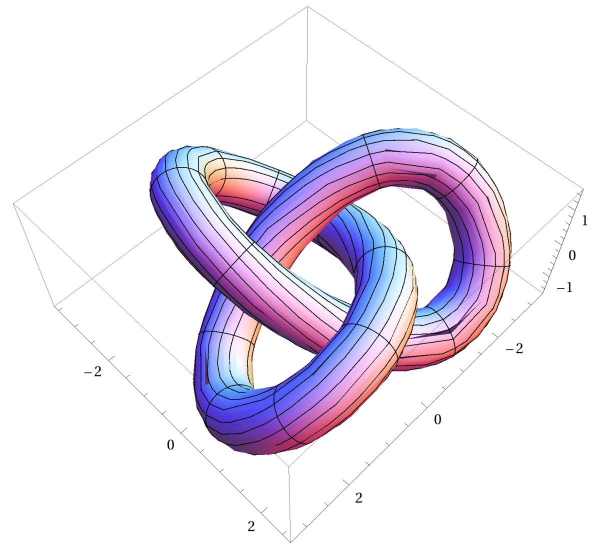
   (e) What do you know about how the speed of the particle is changing at $t = 9$?

8. Give parametric equations for the unit circle in the plane $3x - y + z = 4$ centered at $(1, 0, 1)$.

9. Below is shown a curve (the trefoil knot) and a "fattening" of it into a closed tube. The curve can be parametrized by $\vec{r}(t)$ for $t \in [0, 2\pi)$. The tube consists of circles of radius 0.5 centered on the curve that lie in planes that cut the curve perpendicularly.

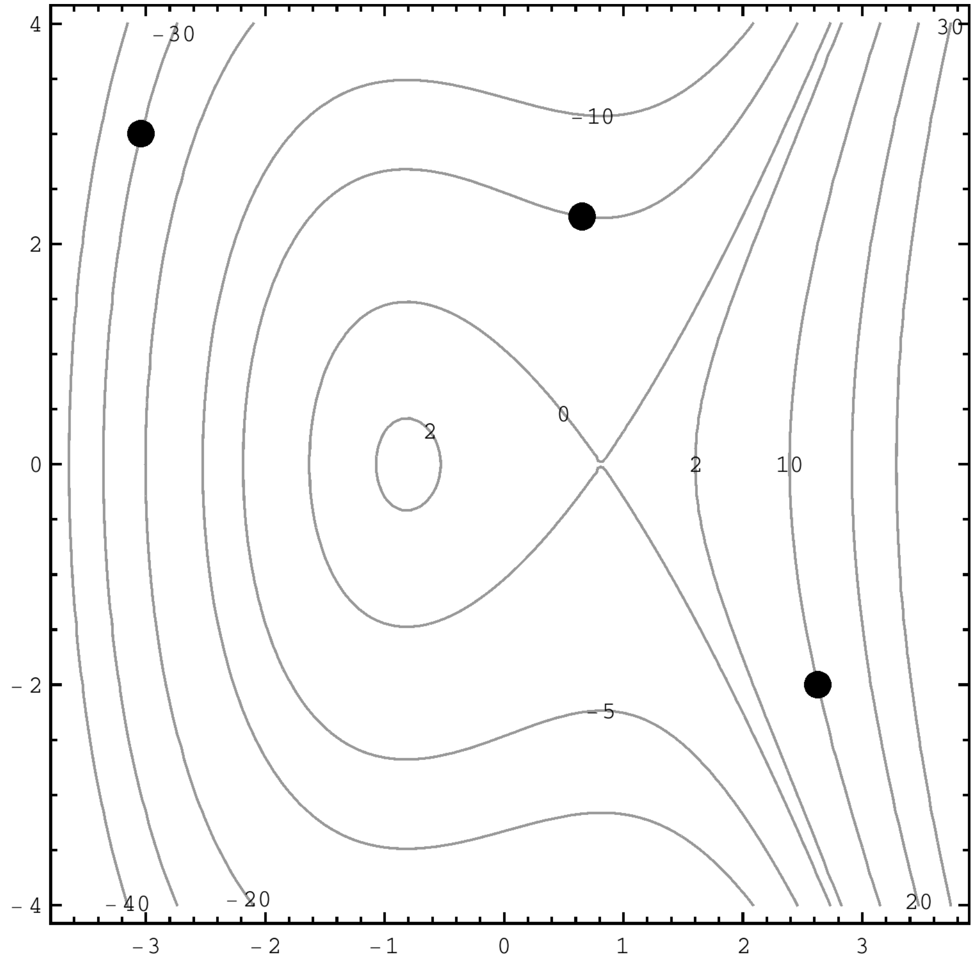
   Without doing computatations, give parametric equations for the tube. You should define any objects you use in terms of $\vec{r}(t)$.





10. Find the maximum and minimum values of $f(x, y) = xy$ on the disk $x^2 + y^2 \leq 4$.

11. Below is a plot of several level curves of a function $f(x, y)$. At the indicated points, sketch in the gradient vectors. Find the (approximate) locations of the critical points of f, then classify them.

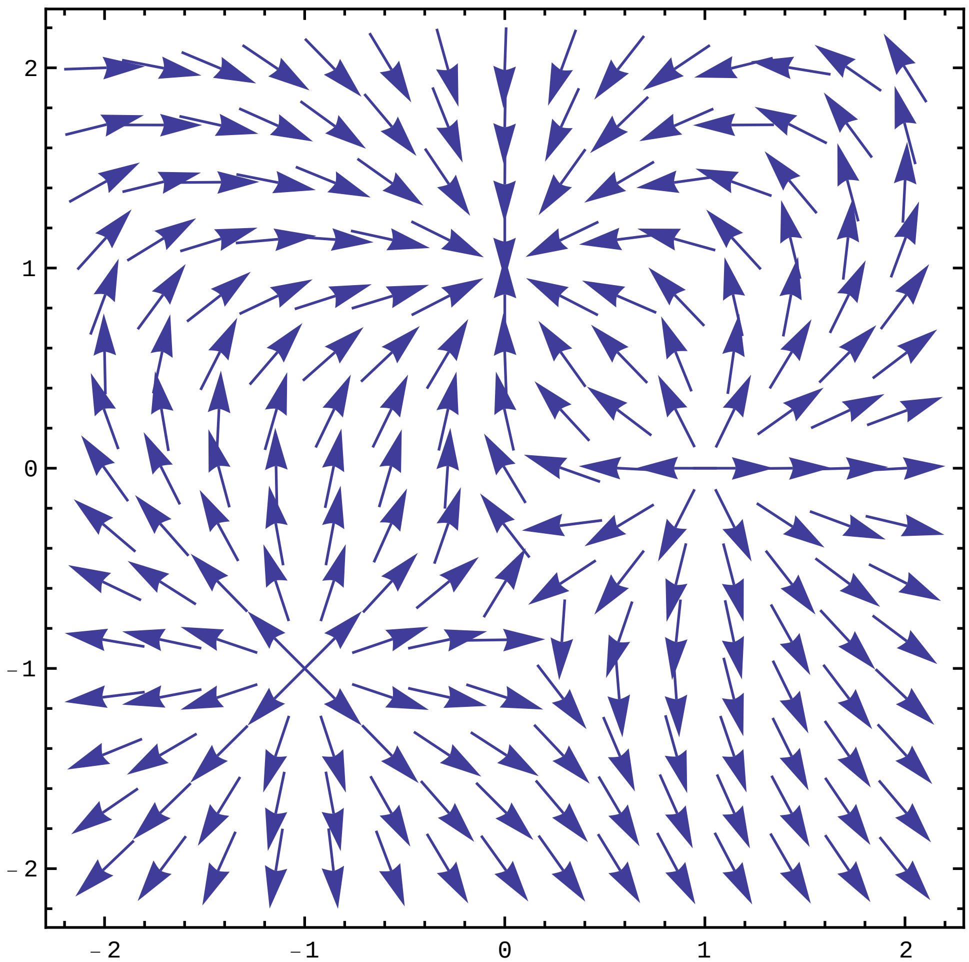


12. Consider $F(x, y) = \langle 1, y \rangle$.
    (a) Draw enough vectors from F to get the feel for what it looks like.

    (b) Add to the plot a few trajectories.
    (c) Give the differential equations that define the trajectories of F.

    (d) Solve the system of differential equations to find the trajectory that passes through $(2, 1)$.

13. Here's a plot of the gradient field $F = \nabla f$ for some function f. What do the trajectories in the field tell you about f?



14. Here are the tangential and normal components of some F on a curve C. What do they suggest about the net flow of F along and across C?