CHAPTER 2 SECTION 2.2

Solving literal equations and formulas

Literal equations

Equations that contain two or more variables are called **literal equations.** In a literal equation, we generally solve the equation for one variable in terms of the remaining variables and constants. *The procedure for solving a literal equation is the same as the procedure for solving linear equations.*

Formulas

A formula is a mathematical equation that states the relationship between two or more physical conditions. Consider the formula $d = rt$, which expresses the fact that distance (d) is equal to rate (r) multiplied by the time (t). If we knew how far it was between two cities (d) and we wanted to travel this distance in a certain amount of time (t), then the equation could be solved for the necessary rate (r) to achieve this.

$$d = rt \qquad \text{Divide each member by } t$$
$$\frac{d}{t} = r$$

The equation is now solved for r in terms of d and t. If the distance and rate were known, the equation could be solved for time as follows:

$$d = rt \qquad \text{Divide each member by } r$$
$$\frac{d}{r} = t$$

The equation is now solved for t in terms of d and r.

We observe from this example that we may solve a literal equation or formula for a specified variable. The following list is a restatement of the procedure for solving linear equations that we will now apply to literal equations.

Solving a literal equation or a formula

Step 1 Simplify each member of the equation.

Step 2 Collect all the terms with the variable for which we are solving in one member of the equation. (Addition and subtraction property)

Step 3 Remove any term that is being added to or subtracted from the variable for which we are solving. (Addition and subtraction property)

Step 4 Divide each member of the equation by the coefficient of the variable for which we are solving. (Multiplication and division property)

Examples

Solve for the specified variable.

1. The volume of a rectangular solid is found by multiplying length (ℓ) times width (w) times height (h), $V = \ell wh$. Solve the equation for h.

$V = \ell wh$	Original equation
$V = (\ell w)h$	Coefficient of h is ℓw
$\dfrac{V}{\ell w} = \dfrac{\ell wh}{\ell w}$	Divide by ℓw
$\dfrac{V}{\ell w} = h$	Equation is solved for h in terms of V, ℓ, and w
$h = \dfrac{V}{\ell w}$	Symmetric property

2. The simple interest (I) earned on the principal (P) over a time period (t) at an interest rate (r) is given by $I = Prt$. Solve for r.

$I = Prt$	Original equation
$I = (Pt)r$	Pt is the coefficient of r
$\dfrac{I}{Pt} = \dfrac{Ptr}{Pt}$	Divide by Pt
$\dfrac{I}{Pt} = r$	Equation is solved for r in terms of I, P, and t
$r = \dfrac{I}{Pt}$	Symmetric property

3. If we know the temperature in degrees Fahrenheit (F), the temperature in degrees Celsius (C) can be found by the equation $C = \frac{5}{9}(F - 32)$. Solve the formula for F.

$C = \dfrac{5}{9}(F - 32)$	
$9C = 9 \cdot \dfrac{5}{9}(F - 32)$	Clear the fraction
$9C = 5(F - 32)$	Apply the distributive property
$9C = 5F - 160$	
$9C + 160 = 5F$	Add 160
$\dfrac{9C + 160}{5} = F$	Divide by 5
$F = \dfrac{9C + 160}{5}$	Symmetric property

If the temperature is given in degrees Celsius, we use this form of the equation to determine the temperature in degrees Fahrenheit.

Note Although we have not stated any restrictions on the variables, it is understood that the values that the variables can take on must be such that no denominator is ever zero. That is, in example 1, $\ell \neq 0$ and $w \neq 0$; in example 2, $P \neq 0$, $t \neq 0$.

Whether we are solving for x in a linear equation or a literal equation, the procedure is the same.

Linear equation	**Literal equation**	
$5(x + 1) = 2x + 7$	$5(x + y) = 2x + 7y$	Original equation
$5x + 5 = 2x + 7$	$5x + 5y = 2x + 7y$	Simplify (distributive property)
$3x + 5 = 7$	$3x + 5y = 7y$	All x's in one member
$3x = 2$	$3x = 2y$	Terms not containing x in the other member
$x = \frac{2}{3}$	$x = \frac{2y}{3}$	Divide by the coefficient

In the linear equation, we have a solution for x, and in the literal equation, we have solved for x in terms of y.

Exercises

Solve for the specified variable. See example 2–7 A.

Example $P = 2\ell + 2w$, for ℓ

Solution $P - 2w = 2\ell$ Subtract $2w$

$\dfrac{P - 2w}{2} = \ell$ Divide by 2

$\ell = \dfrac{P - 2w}{2}$ Symmetric property

1. $V = \ell wh$, for w
2. $V = \ell wh$, for ℓ
3. $I = Prt$, for P
4. $I = Prt$, for t
5. $F = ma$, for m
6. $E = IR$, for R
7. $K = PV$, for V
8. $E = mc^2$, for m
9. $W = I^2R$, for R
10. $A = \ell w$, for w
11. $P = 2\ell + 2w$, for w
12. $C = \pi D$, for π
13. $P = a + b + c$, for a
14. $A = \dfrac{1}{2}bh$, for b
15. $ay - 3 = by + c$, for a
16. $ay - 3 = by + c$, for b
17. $V = k + gt$, for k
18. $V = k + gt$, for t
19. $A = \dfrac{1}{2}h(b + c)$, for b
20. $A = \dfrac{1}{2}h(b + c)$, for h
21. $\ell = a + (n - 1)d$, for a
22. $\ell = a + (n - 1)d$, for d
23. $A = P(1 + r)$, for P
24. $\ell = a + (n - 1)d$, for n
25. $T = 2f + g$, for f
26. $i = \dfrac{prm}{12}$, for r
27. $D = dq + R$, for q
28. $M = -P(\ell - x)$, for x
29. $R = W - b(2c + b)$, for c
30. $F = k\dfrac{m_1m_2}{d^2}$, for k
31. $A = P(1 + rt)$, for r
32. $A = P(1 + rt)$, for P
33. $V = r^2(a - b)$, for a
34. $P = n(P_2 - P_1) - c$, for P_2
35. $3x - y = 4x + 5y$, for x
36. $3x - y = 4x + 5y$, for y
37. $2S = 2vt - gt^2$, for g
38. $ax + by = c$, for y

39. The distance s that a body projected downward with an initial velocity of v will fall in t seconds because of the force of gravity is given by $s = \frac{1}{2}gt^2 + vt$. Solve for g.

40. Solve the formula in exercise 39 for v.

41. The net profit P on sales of n identical tape decks is given by $P = n(S - C) - e$, where S is the selling price, C is the cost to the dealer, and e is the operating expense. Solve for S.

42. Solve the formula in exercise 41 for C.

43. Solve the formula in exercise 41 for e.