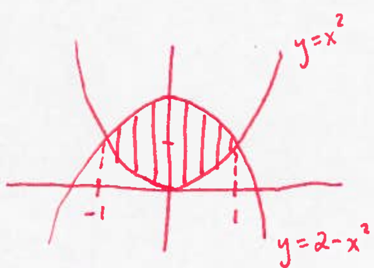
Math 241 X8 **Name:** Solutions

Quiz # 5

October 17, 2013 No electronic devices or interpersonal communication allowed. Show work to get credit.

1) Compute $\iint_R x^2 y \, dA$ where R is the region bounded by the curves $x^2 + y = 2$ and $y = x^2$.

$y = 2 - x^2$



intersection?

$x^2 = 2 - x^2$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

$$\int_{-1}^{1} \int_{x^2}^{2-x^2} x^2 y \, dy \, dx$$

$$= \int_{-1}^{1} \left[\tfrac{1}{2} x^2 y^2 \right]_{y=x^2}^{2-x^2} dx$$

$$= \tfrac{1}{2} \int_{-1}^{1} \left(x^2(2-x^2)^2 - x^2(x^2)^2 \right) dx$$

$$= \tfrac{1}{2} \int_{-1}^{1} \left(x^2(4 - 4x^2 + x^4) - x^2(x^4) \right) dx$$

$$= \tfrac{1}{2} \int_{-1}^{1} (4x^2 - 4x^4) \, dx$$

$$= \tfrac{1}{2} \left(\tfrac{4}{3} x^3 \Big|_{-1}^{1} - \tfrac{4}{5} x^5 \Big|_{-1}^{1} \right)$$

$$= \tfrac{1}{2} \left(\tfrac{8}{3} - \tfrac{8}{5} \right)$$

$$= \tfrac{4}{3} - \tfrac{4}{5} \quad \left(= \tfrac{8}{15} \right).$$

2) Compute $\iint_R x^2\, dA$ where R is the region bounded by the ellipse $4x^2 + y^2 = 1$.

Gauss-Green: Take $m = 0$, $n = \frac{1}{3}x^3$. Then

$$\iint_R x^2\, dA = \iint_R (\partial_x n - \partial_y m)\, dA = \int_{\text{bdry } R} m\,dx + n\,dy = \int_{\text{bdry } R} \tfrac{1}{3}x^3\, dy.$$

Parametrize bdry of R:

$$x = \tfrac{1}{2}\cos t \qquad t \in [0, 2\pi)$$
$$y = \sin t$$

$$= \int_0^{2\pi} \tfrac{1}{3}\left(\tfrac{1}{2}\cos t\right)^3 (\cos t\, dt)$$

$$= \frac{1}{24}\int_0^{2\pi} \cos^4 t\, dt$$

$$= \frac{1}{24}\int_0^{2\pi} \left(\tfrac{1}{2}(1+\cos(2t))\right)^2 dt$$

$$= \frac{1}{96}\int_0^{2\pi} \left(1 + 2\cos(2t) + \cos^2(2t)\right) dt$$

$$= \frac{1}{96}\int_0^{2\pi} \left(1 + 2\cos(2t) + \tfrac{1}{2} + \tfrac{1}{2}\cos(4t)\right) dt$$

$$= \frac{1}{96}\left(2\pi + 0 + \pi + 0\right)$$

$$= \frac{\pi}{32}.$$