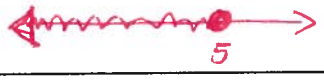
Name: _____

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 12:45pm to complete this exam. When you are told to stop writing, do it or you will lose all points on the page(s) you write on.
- You may not communicate with other students during this test.
- Keep your eyes on your own paper.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctor.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers when appropriate.
- Before turning in your exam, check to make certain you've answered all the questions.

Question	Points	Score
1	9	
2	6	
3	12	
4	15	
5	5	
6	6	
7	10	
8	10	
9	0	
10	6	
11	9	
12	10	
13	16	
14	10	
15	14	
Total:	138	

1. (9 points) Complete the following table.

inequality	interval	graph
$x > -2$	$(-2, \infty)$	
$x \leq 5$	$(-\infty, 5]$	
$1 \leq x < 4$	$[1, 4)$	

2. (6 points) Solve the inequality $1 < 3 - 2x \leq 7$. Write your answer in each of the following forms: **1.** a simplified inequality, **2.** interval notation, **3.** a graph on the real line.

$1 < 3 - 2x \leq 7$ -3

$-2 < -2x \leq 4$ $\div -2$

$1 > x \geq -2$

$[-2, 1)$

3. (12 points) Solve the following inequalities. Give your answers in each of the following forms: **1.** a simplified inequality, **2.** interval notation.

(a) $4x + 3 \leq 4x + 5$

$-4x$

$3 \leq 5$

$(-\infty, \infty)$

(b) $4x + 3 \leq 3x + 5$

$-3x$

$x + 3 \leq 5$

-3

$x \leq 2$

$(-\infty, 2]$

(c) $3x + 5 \leq 3x + 4$

$-3x$

$5 \leq 4$

$\emptyset$

4. (15 points) Find all solutions to the following equations using the indicated method.

(a) $x^2 - x - 6 = 0$ by factoring

$(x-3)(x+2) = 0$

$x = 3$ OR $x = -2$

(b) $2x^2 - 8x - 3 = 0$ by completing the square

$2x^2 - 8x = 3$

$x^2 - 4x = \frac{3}{2}$

$(x-2)^2 = \frac{3}{2} + 4 = \frac{3}{2} + \frac{8}{2} = \frac{11}{2}$

$x - 2 = \pm\sqrt{\frac{11}{2}}$

$x = 2 \pm \sqrt{\frac{11}{2}}$

(c) $x^2 + 3x - 5 = 0$ using the quadratic formula

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)} = \frac{-3 \pm \sqrt{29}}{2}$$

5. (5 points) Find all solutions to $x^3 + 2x^2 + x = 0$.

Factor: $x(x^2+2x+1) = 0$

$x(x+1)^2 = 0$

$x = 0$ OR $x+1 = 0$

$x = -1$

6. (6 points) Use the discriminant b^2-4ac to find the number of solutions to each of the following. (You do not have to find those solutions.)

(a) $x^2 + 2x - 5 = 0$

$2^2-4(1)(-5) = 24 > 0 \Rightarrow$ two solutions

(b) $x^2 - 2x + 5 = 0$

$(-2)^2-4(1)(5) = -20 < 0 \Rightarrow$ no solutions

(c) $x^2 + 6x + 9 = 0$

$6^2-4(1)(9) = 0 \Rightarrow$ one solution

7. (10 points) For the equation $y = 4|x + 1| - 2$, find (and clearly label) the x- and y-intercepts, plot at least five points, then sketch the plot.

x-ints: $0 = 4|x+1| - 2$

$2 = 4|x+1|$

$\frac{1}{2} = \frac{2}{4} = |x+1|$

$x+1 = \pm\frac{1}{2}$

$x = -1 \pm \frac{1}{2} = \begin{cases} -\frac{3}{2} \\ \text{OR} \\ -\frac{1}{2} \end{cases}$

x-ints: $(-\frac{1}{2}, 0)$ & $(-\frac{3}{2}, 0)$

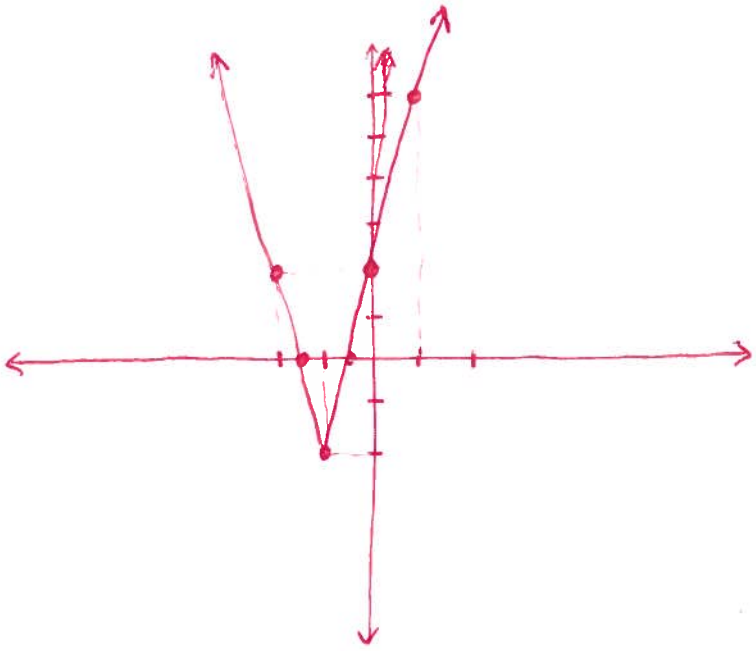
y-int: $y = 4|0+1| - 2$

$= 4 \cdot 1 - 2$

$= 2$

y-int: $(0, 2)$

x	y
-2	2
-1	-2
0	2
1	6



8. (10 points) Use the distance formula to test whether the points $(1,3)$, $(-2,1)$, and $(4,-2)$ form the vertices of a right triangle.

A B C

$|AB| = \sqrt{(1-3)^2+(-2-1)^2} = \sqrt{4+9} = \sqrt{13}$

$|AC| = \sqrt{(-2-3)^2+(4-1)^2} = \sqrt{25+9} = \sqrt{34}$

$|BC| = \sqrt{(-2-1)^2+(4-(-2))^2} = \sqrt{9+36} = \sqrt{45}$ ← largest

Right triangle $\iff \sqrt{13}^2 + \sqrt{34}^2 = \sqrt{45}^2$

$13 + 34 \stackrel{?}{=} 45$

$47 \stackrel{?}{=} 45$

No, ABC is not a right triangle.

9. Bonus: check the same thing using what you know about slopes of lines. Make sure you say how you know whether the triangle is a right triangle.

$\text{slope}(AB) = \frac{1-3}{-2-1} = \frac{-2}{-3} = \frac{2}{3}$

$\text{slope}(AC) = \frac{-2-3}{4-1} = \frac{-5}{3}$

$\text{slope}(BC) = \frac{-2-1}{4-(-2)} = \frac{-3}{6} = -\frac{1}{2}$

Right triangle $\iff$ two of the edges are perpendicular, i.e. have opposite reciprocal slopes.

No

10. (6 points) Find the midpoint of the line segment joining $(-3, 1)$ to $(4, -5)$.

$\left(\frac{1}{2}(-3+4), \frac{1}{2}(1+-5)\right)$

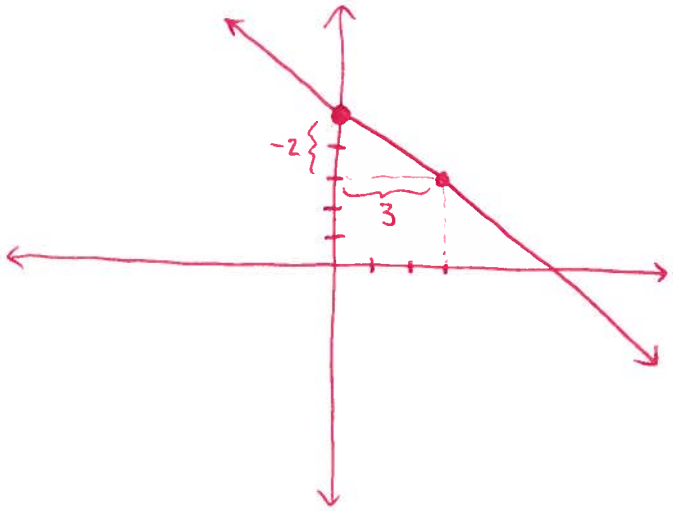
$= \left(\frac{1}{2}, -2\right)$

11. Consider the line $y = 5 - \frac{2}{3}x$.

(a) (5 points) Identify the slope, and use this to graph the line.

slope $= -\frac{2}{3}$

y-int $= (0, 5)$



(b) (4 points) Find the equation of a line that is perpendicular to the given line and passes through the point $(-2, 1)$.

slope $= +\frac{3}{2}$

$y - 1 = \frac{3}{2}(x - (-2))$

$\boxed{y - 1 = \frac{3}{2}(x + 2)}$

12. (10 points) Suppose the population of yeast in a petri dish at noon is 1.8 billion, and at 2pm is 4.6 billion. Let y denote the population of yeast in billions and x denote the number of hours past noon. (Assume that the population growth of the yeast is linear.)

(a) Write the information above as two points (x, y).

$(0, 1.8)$, $(2, 4.6)$

(b) Find the equation of a line containing those two points.

$$\text{slope} = \frac{4.6 - 1.8}{2 - 0} = \frac{2.8}{2} = 1.4$$

y-int $= (0, 1.8)$

$$\boxed{y = 1.4x + 1.8}$$

(c) Identify and interpret the slope of this linear equation (in the context of the original problem).

$\text{slope} = \frac{1.4}{1}$ (rise / run)

Every hour, the population of yeast grows by 1.4 billion.

13. (16 points) Find equations for each of the following lines.

(a) with y-intercept $(0, -1)$ and slope -3

$y = -3x - 1$

(b) with x-intercept $(4, 0)$ and slope $\frac{1}{2}$

$y - 0 = \frac{1}{2}(x - 4)$

$\boxed{y = \frac{1}{2}(x - 4)}$

(c) parallel to the line $x - 2y = 7$ and passing through $(1, 3)$

$-2y = 7 - x$

$y = -\frac{7}{2} + \boxed{\frac{1}{2}}x$ ← slope

$\boxed{y - 3 = \frac{1}{2}(x - 1)}$

(d) that is vertical and passes through $(3, 5)$

$x = 3$

14. (8 points) Which of the following are functions?

(a) Assign to each person in this class their height. Function

(b) Assign to each height (as a whole number of inches) the person in the world with that height. Not

(c) $f(x) = \pm x^2$ Not

(d) $f(x) = \sqrt{x}$ Function

15. (14 points) Let $g(x) = 4x - x^2$. Find and simplify the following.

(a) $g(-5) = 4(-5) - (-5)^2 = -20 - 25 = -45$

(b) $-g(5) = -\left(4(5) - (5)^2\right) = -\left(20 - 25\right) = -(-5) = 5$

(c) $g(2x) = 4(2x) - (2x)^2 = 8x - 4x^2$

(d) $2g(x) = 2\left(4x - x^2\right) = 8x - 2x^2$

(e) $g(x+h) = 4(x+h) - (x+h)^2 = 4x + 4h - x^2 - 2xh - h^2$

(f) $g(x) + g(h) = (4x - x^2) + (4h - h^2) = 4x + 4h - x^2 - h^2$

(g) $\dfrac{g(x+h) - g(x)}{h} = \dfrac{(\quad) - (4x - x^2)}{h} = \dfrac{4h - 2xh - h^2}{h} = 4 - 2x - h.$

Scratch Paper - Do Not Remove