

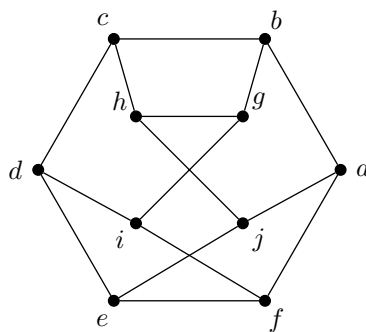
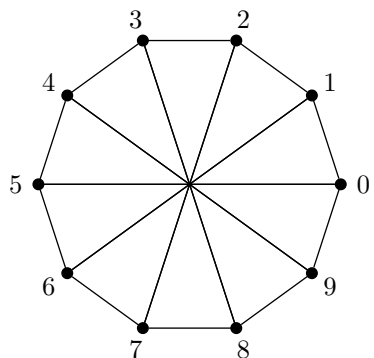
Name: \_\_\_\_\_

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have seventy-five (75) minutes to complete this exam. When you are told to stop writing, do it or you will lose all points on the page(s) you write on.
- You may not communicate with other students during this test.
- Keep your eyes on your own paper.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctor.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers when appropriate.
- Before turning in your exam, check to make certain you've answered all the questions.

Question:	1	2	3	4	5	6	7	Total
Points:	12	12	12	12	12	20	40	120
Score:								

## Short answer

1. (12 points) Determine whether the following two graphs are isomorphic. (Give an isomorphism or a short argument why they are not isomorphic.)



2. (12 points) Decompose  $K_5$  into one copy of each of the four trees below.



## Algorithms

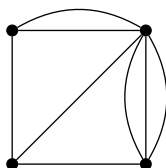
Give brief justifications for your answers (but not necessarily full proofs; a computation with each step clearly written may suffice).

3. (12 points) Determine whether the following sequences are graphic (the degree sequence of a simple graph).

(a) 6 5 5 5 2 2 2 1 1

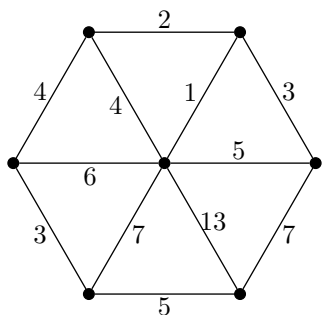
(b) 6 5 5 5 2 2 2 1

4. (12 points) How many spanning trees does the following graph have?

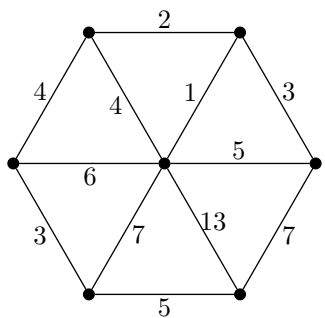


5. (12 points) *Weighted graph algorithms*

- (a) Use either Kruskal's or Prim's algorithm to find a minimum spanning tree in the following weighted graph.



- (b) Use Dijkstra's algorithm to find the minimum distances from the central vertex to each other vertex.



**Proofs**

6. (20 points) *Digraphs*

(a) Prove that if  $D$  is a digraph with  $\delta^+(D) \geq 1$ , then  $D$  has a (directed) cycle.

(b) Use the statement in part (a) to prove that if  $D$  is a digraph with  $d^+(v) = d^-(v)$  at every vertex  $v$ , then  $D$  decomposes into (directed) cycles.

7. (40 points) Give complete careful proofs of **2 of the following 3** statements.

- (i) If  $G$  is disconnected, then  $\overline{G}$  is connected.
- (ii) A graph is a tree if and only if it is loopless and has exactly one spanning tree.
- (iii) If  $G$  is even (i.e., every vertex degree is even), then  $G$  has no cut edge.

(The statements are repeated here for your convenience.)

- (i) If  $G$  is disconnected, then  $\overline{G}$  is connected.
- (ii) A graph is a tree if and only if it is loopless and has exactly one spanning tree.
- (iii) If  $G$  is even (i.e., every vertex degree is even), then  $G$  has no cut edge.

**Scratch Paper - Do Not Remove**