Math 251 **Name(s):**

PaperAssign 7

Workshop (in-class)

November 27, 2017

Theorem. *If* $\mathbf{F}$ *is conservative, then* $\operatorname{curl}\mathbf{F} = \mathbf{0}$.

Theorem. *If* $\operatorname{curl}\mathbf{F} = \mathbf{0}$ *and* $\mathbf{F}$ *is defined on an open and simply connected region, then* $\mathbf{F}$ *is conservative.*

Theorem. *If* $\mathbf{F}$ *is conservative, then every line integral of* $\mathbf{F}$ *along a closed curve is zero.*

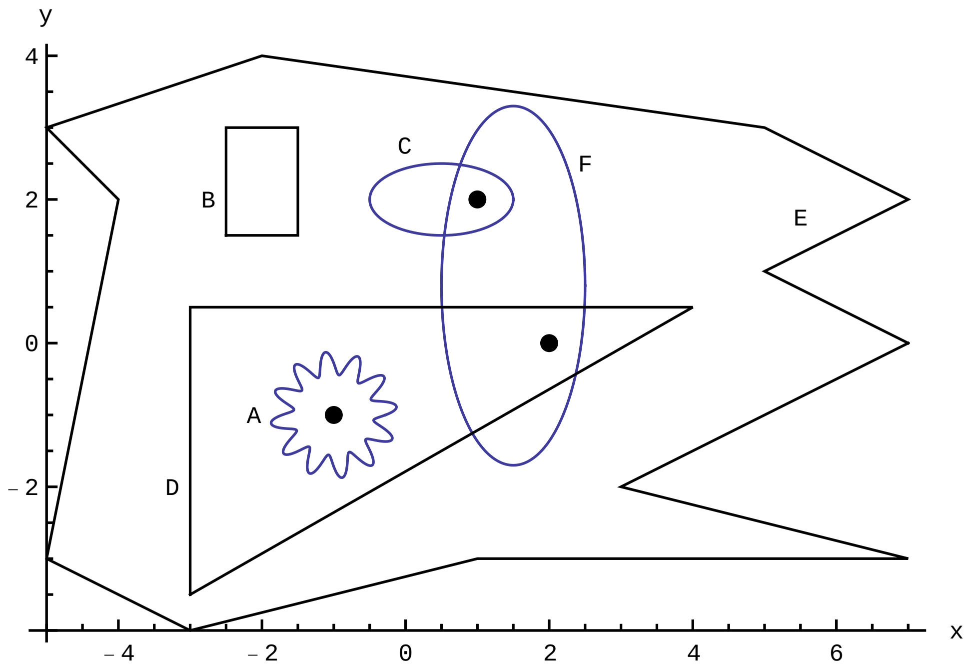
(1) Consider the following vector fields:

$$\mathbf{F}(x,y) = \langle y, -x\rangle, \qquad \mathbf{G}(x,y) = \frac{1}{\sqrt{x^2+y^2}}\langle y,-x\rangle, \qquad \mathbf{H}(x,y) = \frac{1}{x^2+y^2}\langle y,-x\rangle$$

- (a) Sketch the three fields.
- (b) From your sketches, can you determine whether any of the three are conservative?
- (c) Compute the curl of each vector field.
- (d) You should now be able to conclude that two of the fields are *not* conservative. Which two? What about the third one?
- (e) Compute $\oint_C \mathbf{H}\cdot d\mathbf{r}$ where C is the unit circle, directly.
- (f) Does (1e) contradict (1d)? Why?
- (g) Consider $f(x,y) = \arctan(x/y)$. Find ∇f. Does this contradict your earlier work? Why?
- (h) Can you use (1g) to find the work done by $\mathbf{H}$ in moving a particle from $(1,-1)$ to $(2,7)$ along a straight line? What about from $(-1,1)$ to $(2,7)$ along a straight line? What about from $(-1,1)$ to $(2,7)$ along the piece of the parabola $y = 2x^2 - 1$?

(2) Consider now $\mathbf{F} = \dfrac{1}{\sqrt{x^2+y^2}}\langle x, y\rangle$. Repeat the steps above for this field. Is it conservative? If so, find a potential function.

(3) Suppose the vector field $\mathbf{F}(x, y)$ has the property that $\operatorname{curl} \mathbf{F} = \mathbf{0}$ everywhere it is defined, but $\mathbf{F}$ is not defined at the three points $(-1, -1)$, $(1, 2)$, and $(2, 0)$. Below are shown these three points together with several curves.



You are given that

$$\oint_D \mathbf{F} \cdot d\mathbf{r} = -1, \qquad \oint_E \mathbf{F} \cdot d\mathbf{r} = 3, \quad \text{and} \quad \oint_F \mathbf{F} \cdot d\mathbf{r} = 9.$$

(a) Find the line integral of $\mathbf{F}$ along each of A, B, C.

(b) Draw another simple closed curve G for which the line integral of $\mathbf{F}$ along G is a different value from any of those above.