MATH 454
HOMEWORK 5 DUE FEBRUARY 22

Name: ______________________________

- Refer to the syllabus regarding allowed collaboration on this homework assignment.
- Refer to other homework instructions and suggestions posted in Blackboard.
- All answers must be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

Turn in four of the following problems to be graded.

(1) (1.4.25)
 (a) Prove that every connected graph has an orientation in which the number of vertices with odd outdegree is at most 1.
 (b) Use part (a) to prove that a simple connected graph with an even number of edges has a decomposition into copies of P_3.

(2) (1.4.29) Suppose that G is a graph and D is an orientation of G that is strongly connected. Prove that if G has an odd cycle C, then D also has an odd cycle. *(Hint: consider each pair of consecutive vertices on C; what does strong connectivity of D give you? You may use the result of Exercise 1.4.4.)*

(3) (1.4.38) Prove that there is an n-vertex tournament in which every vertex is a king except for $n \in \{2, 4\}$.

(4) (2.1.13) Prove that every connected graph with diameter d has an independent set of size $\lceil (1+d)/2 \rceil$.

(5) Prove that for every graph G, $\mathrm{rad}(G) \le \mathrm{diam}(G) \le 2\,\mathrm{rad}(G)$. (See Exercise 2.1.47.)

(6) Prove that the number of leaves in a tree T (with at least two vertices) is equal to

$$2 + \sum_{\substack{v \in V(T): \\ d(v) \ge 2}} (d(v) - 2).$$