MATH 454
HOMEWORK 11 DUE APRIL 26*

Name: ______________________________

- Refer to the syllabus regarding allowed collaboration on this homework assignment.
- Refer to other homework instructions and suggestions posted in Blackboard.
- All answers must be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

Turn in four of the following problems to be graded.

(1) (6.1.14) Prove or disprove: for every $n \in \mathbb{N}$, there is a simple connected 4-regular planar graph with at least n vertices.

(2) (6.2.5) Determine the maximum number of edges in a planar subgraph of the Petersen graph.

(3) (6.2.7) *(A graph is* outerplanar *if it has a planar drawing in which all vertices lie on the unbounded face.)* Use Kuratowski's Theorem to prove that G is outerplanar if and only if it has no subgraph that is a subdivision of K_4 or $K_{2,3}$.

(4) (6.3.14, "only if") Prove that every 3-colorable plane triangulation is Eulerian. *(Hint: consider the dual)*

(5) (7.1.26) Let G be a regular graph with a cut-vertex. Prove that $\chi'(G) > \Delta(G)$.

(6) (7.2.8) On a chessboard, a knight can move from one square to another that differs by 1 in one coordinate and by 2 in the other coordinate. A *knight's tour* is a traversal of the board by a knight in which each square is visited exactly once, except that the knight returns to its starting square. Prove that no $4 \times n$ chessboard has a knight's tour. *(Hint: find a set of vertices in the corresponding graph that violates the necessary condition for a Hamiltonian cycle. There is also an alternative proof using a very early result of ours in a clever way.)*

(7) (7.3.7) Let G be a plane triangulation.
   (a) Prove that the dual G^* has a 2-factor.
   (b) Use part (a) to prove that the vertices of G can be 2-colored (not properly) so that every face has vertices of both colors. *(Hint: use the idea in the proof of Theorem 7.3.2.)*