Math 415 ADG **Name:** Solution

Quiz # 6

March 14, 2014

No notes, electronic devices, or interpersonal communication allowed. Show work to get credit. Use the methods from this class.

Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation satisfying

$$T\left(\begin{bmatrix}2\\0\\0\end{bmatrix}\right) = \begin{bmatrix}8\\2\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}9\\2\end{bmatrix}, \quad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = \begin{bmatrix}4\\3\end{bmatrix}.$$

Consider the bases $\mathcal{B} = \left\{ \overset{v_1}{\begin{bmatrix}1\\0\\0\end{bmatrix}}, \overset{v_2}{\begin{bmatrix}-1\\1\\0\end{bmatrix}}, \overset{v_3}{\begin{bmatrix}0\\0\\1\end{bmatrix}} \right\}$ of $\mathbb{R}^3$ and $\mathcal{C} = \left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}4\\1\end{bmatrix} \right\}$ of $\mathbb{R}^2$.

Determine $[T]_{\mathcal{B},\mathcal{C}}$. Include your calculations! (You needn't show work in solving easy systems of linear equations.)

$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = T\left(\frac{1}{2}\begin{bmatrix}2\\0\\0\end{bmatrix}\right) = \frac{1}{2}T\left(\begin{bmatrix}2\\0\\0\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}8\\2\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix} = 0\begin{bmatrix}1\\0\end{bmatrix} + 1\begin{bmatrix}4\\1\end{bmatrix}$, so $[T(v_1)]_{\mathcal{C}} = \begin{bmatrix}0\\1\end{bmatrix}$;

$T\left(\begin{bmatrix}-1\\1\\0\end{bmatrix}\right) = T\left(-\frac{1}{2}\begin{bmatrix}2\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = -\frac{1}{2}\begin{bmatrix}8\\2\end{bmatrix} + 1\begin{bmatrix}9\\2\end{bmatrix} = \begin{bmatrix}5\\1\end{bmatrix} = 1\begin{bmatrix}1\\0\end{bmatrix} + 1\begin{bmatrix}4\\1\end{bmatrix}$, so $[T(v_2)]_{\mathcal{C}} = \begin{bmatrix}1\\1\end{bmatrix}$; an

$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = T\left(-1\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\1\end{bmatrix}\right) = -1\begin{bmatrix}9\\2\end{bmatrix} + 1\begin{bmatrix}4\\3\end{bmatrix} = \begin{bmatrix}-5\\1\end{bmatrix} = -9\begin{bmatrix}1\\0\end{bmatrix} + 1\begin{bmatrix}4\\1\end{bmatrix}$, so $[T(v_3)]_{\mathcal{C}} = \begin{bmatrix}-9\\1\end{bmatrix}$.

So $[T]_{\mathcal{B},\mathcal{C}} = \begin{bmatrix}0 & 1 & -9\\1 & 1 & 1\end{bmatrix}$.