Name: _Solutions_

The following are roughly the instructions for the real exam.

- **READ THE FOLLOWING DIRECTIONS!**

- **Do NOT open the exam until instructed to do so.**

- You have two hours to complete this exam. When you are told to stop writing, do it or you will lose all points on the page(s) you write on.

- You may not communicate with other students during this test.

- Keep your eyes on your own paper.

- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctor.

- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).

- Any case of cheating will be taken extremely seriously.

- Show all your work and explain your answers when appropriate.

- Before turning in your exam, check to make certain you've answered all the questions.

1. Write the following in radical form.

   (a) $x^{1/3} = \sqrt[3]{x}$

   (b) $3y^{3/5} = 3\sqrt[5]{y^3}$

   (c) $m^{-1/2} = \dfrac{1}{\sqrt{m}}$

   (d) $(x+y)^{3/2} = \sqrt{(x+y)^3}$

2. Write the following in exponential form.

   (a) $\sqrt[4]{x^3} = x^{3/4}$

   (b) $\sqrt{2x} = (2x)^{1/2}$

3. Simplify the following. (Leave no negative exponents, but do leave answers in exponent form.)

   (a) $(-3x)^2 = 9x^2$

   (b) $-3x^2 = -3x^2$

   (c) $-(3x)^2 = -9x^2$

   (d) $8x^{-1/3} = \dfrac{8}{x^{1/3}}$

   (e) $\left(\dfrac{6xy^{-3}}{16x^4y^2}\right)^{-2/3} = \left(\dfrac{3}{8x^3y^5}\right)^{-2/3} = \left(\dfrac{8x^3y^5}{3}\right)^{2/3}$

   $= \dfrac{8^{2/3}\,x^2\,y^{10/3}}{3^{2/3}} = \boxed{\dfrac{4x^2y^{10/3}}{3^{2/3}}}$

4. Simplify the following. (Leave no negative exponents, and do leave answers in radical form.)

(a) $\sqrt[4]{81}$ $= 3$

(b) $\sqrt{80}$ $= \sqrt{16 \cdot 5}$ $= 4\sqrt{5}$

(c) $\sqrt[3]{16x^3y^{14}}$ $= \sqrt[3]{8 \cdot 2 \cdot x^3 \cdot y^{12} \cdot y^2}$

$= 2xy^4 \cdot \sqrt[3]{2y^2}$

(d) $\sqrt[4]{x^{-6}}$ $= \dfrac{1}{\sqrt[4]{x^6}}$ $= \dfrac{1}{x^{6/4}}$ $= \dfrac{1}{x^{3/2}}$ $= \dfrac{1}{\sqrt{x^3}}$ $= \dfrac{1}{x\sqrt{x}}$

(e) $3\sqrt{75} - 2\sqrt{108}$ $= 3(5\sqrt{3}) - 2(6\sqrt{3}) = 15\sqrt{3} - 12\sqrt{3} = 3\sqrt{3}$

(f) $(\sqrt{2} + \sqrt{3y})^2$ $= (\sqrt{2} + \sqrt{3y})(\sqrt{2} + \sqrt{3y})$ $= 2 + 2\sqrt{6y} + 3y$

5. Find all solutions to the following equations.

(a) $\left(-\dfrac{1}{10}x\right)=\dfrac{1}{2} \cdot -10$

$-10 \cdot$

$$\boxed{x = -5}$$

(b) $7(2x-3)-3(3x-2)=5(x+3)$) expand

$14x - 21 - 9x + 6 = 5x + 15$) simplify

$5x - 15 = 5x + 15$) $-5x$

$-15 = 15$ ← contradiction

$$\boxed{\text{No solutions!}}$$

(c) $7(2x-3)-3(x-2)=5(x+3)$) expand

$14x - 21 - 3x + 6 = 5x + 15$) simplify

$11x - 15 = 5x + 15$) $-5x, +15$

$6x = 30$) $\div 6$

$$\boxed{x = 5}$$

(d) $7(2x-3)-3(3x-2)=5(x-3)$) expand

$14x - 21 - 9x + 6 = 5x - 15$) simplify

$5x - 15 = 5x - 15$ Identity

$$\boxed{\text{Every real } x \text{ is a solution!}}$$

$12 \cdot$

(e) $\left(\dfrac{1}{6}x+\dfrac{1}{3}x-\dfrac{1}{4}\right)=\left(\dfrac{2}{3}x-1\right) \cdot 12$

) $\cdot 12$, distribute

$2x + 4x - 3 = 8x - 12$) simplify

$6x - 3 = 8x - 12$) $-6x, +12$

$9 = 2x$) $\div 2$

$$\boxed{\dfrac{9}{2} = x}$$

6. Solve the following literal equations for the indicated variables.

   (a) $V = \pi r^2 \dfrac{h}{3}$, for h.

   $\cdot \dfrac{3}{\pi r^2}$ on each side $\leadsto$ $\boxed{\dfrac{3V}{\pi r^2} = h}$

   (b) $I_1 + I_2 = \dfrac{V_1}{R_1} + \dfrac{V_2}{R_2}$, for R_1.

   $\qquad -\dfrac{V_2}{R_2}$,

   $\dfrac{V_1}{R_1} = I_1 + I_2 - \dfrac{V_2}{R_2}$

   common denominator on RHS

   $\dfrac{V_1}{R_1} = \dfrac{I_1 R_2 + I_2 R_2 - V_2}{R_2}$

   invert

   $\dfrac{R_1}{V_1} = \dfrac{R_2}{I_1 R_2 + I_2 R_2 - V_2}$ $\quad \cdot V_1$

   $\boxed{R_1 = \dfrac{V_1 R_2}{I_1 R_2 + I_2 R_2 - V_2}}$

7. Find an expression that represents each of the following.

   (a) The value, in cents, of dimes and quarters, where d is the number of dimes and there are three times as many dimes as quarters.

   $d = 3q$ $\qquad$ value $= 10d + 25q$

   $\qquad\qquad\qquad = 10(3q) + 25q$

   $\qquad\qquad\qquad = 30q + 25q$

   $\qquad\qquad\qquad = \boxed{55q}$

   (b) The perimeter of a rectangle whose length ℓ is 3in more than its width.

   $\ell = \overset{w}{w + 3}$

   Want to eliminate w, not ℓ!

   $w = \ell - 3$

   $P = 2w + 2\ell$

   $\quad = 2(\ell - 3) + 2\ell$

   $\quad = \boxed{4\ell - 6}$
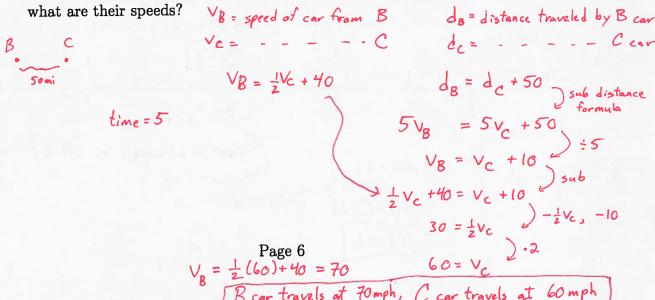
8. Set up an equation for each of the following problems. Then solve that equation.

   (a) Two cars leave Champaign heading in opposite directions at constant speeds. One travels 5mph faster than the other. If after 2 hours the cars are 300 miles apart, what are their speeds?

$$v_2 = v_1 + 5$$

$$d_1 + d_2 = 300$$

$$(2v_1) + (2v_2) = 300$$

$$\boxed{2v_1 + 2(v_1 + 5) = 300}$$

$$\text{distance} = \text{speed} \cdot \text{time}$$

Let d_1, d_2 be the distance traveled by car 1 & car 2 (respectively).

Let v_1 be speed of car 1.

$$2v_1 + 2v_1 + 10 = 300$$
$$4v_1 = 300$$
$$v_1 = \boxed{75 \text{ mph}}$$
$$v_2 = v_1 + 5 = 80 \text{mph}$$

   (b) One car leaves St. Louis and another leaves Chicago, both at noon. (The distance between STL and CHI is about 300 miles.) They drive toward one another along I-55, one going 8mph faster than the other. They pass each other at 3pm. What are their speeds?

$$\text{time} = 3 \text{ hrs} \qquad v_2 = v_1 + 8$$

$$d_1 + d_2 = 300$$

$$3v_1 + 3v_2 = 300 \qquad \text{or, } \div 3 \text{ first}$$

$$3v_1 + 3(v_1 + 8) = 300 \qquad v_1 + (v_1 + 8) = 100$$

$$6v_1 + 24 = 300 \qquad\qquad 2v_1 = 92$$

$$6v_1 = 276 \qquad\qquad\qquad v_1 = 46$$

$$v_1 = 46 \qquad v_2 = 54$$

$$\boxed{46 \text{ mph} \ \& \ 54 \text{ mph}}$$

   (c) One car leaves Champaign and another leaves Bloomington, both heading east along I-74, at the same time. (Bloomington is about 50 miles west of Champaign.) The car leaving Bloomington drives 40mph faster than half the speed of the one leaving Champaign. If the Bloomington car passes the Champaign car 5 hours later, then what are their speeds?

v_B = speed of car from B
v_C = - - - - - - · C
d_B = distance traveled by B car
d_C = - - - - - - C car

B · ⌣ C
50 mi

$$v_B = \tfrac{1}{2}v_C + 40 \qquad\qquad d_B = d_C + 50 \ \Big\} \text{sub distance formula}$$

$$\text{time} = 5 \qquad\qquad\qquad 5v_B = 5v_C + 50 \ \Big\} \div 5$$

$$v_B = v_C + 10 \ \Big\} \text{sub}$$

$$\tfrac{1}{2}v_C + 40 = v_C + 10 \ \Big\} -\tfrac{1}{2}v_C, -10$$

$$30 = \tfrac{1}{2}v_C \ \Big\} \cdot 2$$

$$v_B = \tfrac{1}{2}(60) + 40 = 70 \qquad\qquad 60 = v_C$$

$$\boxed{B \text{ car travels at } 70 \text{mph}, \ C \text{ car travels at } 60 \text{mph}}$$

9. You are asked to put together a special task force.

   (a) The hardware (office supplies, computers, maintenance, etc.) will cost you $5000/month, and each employee you hire on will cost $4000/month. Give an expression that represents the cost per month if you hire w employees.

$$5000 + 4000w$$

   (b) You receive internal funding of $52000/month. How many employees can you hire?

$$5000 + 4000w = 52000$$
$$4000w = 47000$$
$$w = \frac{47}{4} = 11.75$$

Can hire 11 employees.

   (c) You are awarded an additional external grant of $18000/month. How many employees can you hire now?

$$5000 + 4000w = 52000 + 18000$$
$$5000 + 4000w = 70000$$
$$4000w = 65000$$
$$w = \frac{65}{4} = 16.25$$

(Note that 18000 is only enough to hire 4 employees by itself; but we had leftovers from our $52,000 !)

Can hire 16 employees.