WORKSHOP 11: §6.1-6.2   FEBRUARY 16, 2017

Name: ______________________________

(1) Write an algorithm in pseudocode that tests whether an input sequence $(a_1, \ldots, a_n)$ contains the same number twice, in the following two situations:
   (a) The sequence is sorted in nondecreasing order.
   (b) The sequence is not necessarily sorted.

(2) Let $f(n) = 5n^3 - 7n^2 + n + 2$.
   (a) Prove that f is $O(n^3)$. (Using $n_0 = 1$ will work; what c is needed?)
   (b) Prove that f is $\Omega(n^3)$. (You could again use $n_0 = 1$, but it might be easier to use $n_0 = 2$.)
   (c) Prove that f is NOT $\Omega(n^4)$.

(3) Prove that if $f = O(g)$ and $g = O(h)$, then $f = O(h)$. Here's a sketch:

   Proof. We use a direct proof. Let f, g, h be arbitrary functions $(\mathbb{N} \to \mathbb{N})$ such that $f = O(g)$ and $g = O(h)$. Since $f = O(g)$, there are positive constants n_0 and c such that for any $n \geq n_0$, **???**. Since $g = O(h)$, there are positive constants m_0 and d such that **?????**. Therefore, for $n \geq$**???**, both $f(n) \leq$ **??** and $g(n) \leq$ **??**, and so $f(n) \leq$ **??????????** $\leq$ (**??**)$h(n)$. □

(4) An important measure of the difference between two equal-length strands of DNA is the number of nucleotides in which the DNA differ. This is calculated by lining up the first strand $(a_1, a_2, \ldots, a_n)$ against the second strand $(b_1, b_2, \ldots, b_n)$ (where all values are A, G, C, or T), and counting the indices i for which $a_i \neq b_i$.
   Write an algorithm in pseudocode using a loop and an If-statement that computes the number of nucleotides in which two equal-length strands of DNA differ.

(5) Prove the statements in zyBook's Figure 6.2.2: let f, g, and h be functions from $\mathbb{R}^+$ to $\mathbb{R}^+$.
   (a) If $f = O(h)$ AND $g = O(h)$, then $f + g = O(h)$.
   (b) If $f = \Omega(h)$ OR $g = \Omega(h)$, then $f + g = \Omega(h)$.
   (c) If $f = O(g)$ and c is a constant greater than 0, then $c \cdot f = O(g)$.
   (d) If $f = \Omega(g)$ and c is a constant greater than 0, then $c \cdot f = \Omega(g)$.
   Use the above to prove that the following are true also.
   (e) If $f = \Theta(h)$ and $g = \Theta(h)$, then $f + g = \Theta(h)$.
   (f) If $f = \Theta(g)$ and c is a constant greater than 0, then $c \cdot f = \Theta(g)$.