SECTION 1.5 ALGEBAIC EXPRESSIONS & POLYNOMIALS(PART I)

Algebraic Terminology

A **variable** is a symbol that represents an unspecified number. A variable is able to take on any of the different values that it represents. In the relationship

$$y = 2x$$

y and x are variables since they both can assume various numerical values.

A constant is a symbol that does not change its value. In the relationship

$$y = 2x$$

2 is a constant. A number is a constant. If a symbol represents only one value, that symbol is a constant.

Any meaningful collection of variables, constants, grouping symbols, and signs of operations is called an algebraic expression. Examples of algebraic expressions would be

$$5xy, \quad \frac{xy}{z}, \quad 2\ell + 2w, \quad \frac{x^2 - 1}{x^2 + 1}, \quad 3x^2 + 2x - 1, \quad 5(a + 2b).$$

In an algebraic expression, terms are any constants, variables, or products or quotients of these. *Terms are separated by plus or minus signs.*

Example 1

Determine the number of terms in the algebraic expression.

The plus and minus signs separate the algebraic expression into three terms

1. $5x^2 + 2x - 1$ (1st, 2nd, 3rd) — There are three terms

2. $x^2 + y^2$ (1st, 2nd) — There are two terms

3. $4x^5y^2z^4$ (1st) — There is one term

4. $a^2 + \frac{b + c^2}{d}$ (1st, 2nd) — There are two terms since the fraction bar forms a grouping. Observe that the second term has two terms in the numerator

In the expression $5xy$, *each factor or grouping of factors is called the* **coefficient** *of the remaining factors.* That is, 5 is the coefficient of xy; x is the coefficient of $5y$; $5x$ is the coefficient of y; and so on. The 5 is called the **numerical coefficient,** and it tells us how many xy's we have in the expression.

Since we often talk about the numerical coefficients of a term, we will eliminate the word "numerical" and just say "coefficient." It will be understood that we are referring to the numerical coefficient. If no numerical coefficient appears in a term, the coefficient is *understood* to be 1.

Example 2

The algebraic expression $6x - 3y + z$ is thought of as the sum of terms $6x + (-3y) + z$, therefore 6 is the coefficient of x, -3 is the coefficient of y, and 1 is understood to be the coefficient of z.

▶ ***Quick check*** What are the coefficients in the algebraic expression $a^2 - 2a + 4b$?

Polynomials

A special kind of algebraic expression is a **polynomial.** The following are characteristics of a polynomial.

1. It has real number coefficients.
2. All variables in a polynomial are raised to only **whole** number powers.
3. The operations performed by the variables are limited to addition, subtraction, and multiplication.

A polynomial that contains just one term is called a **monomial;** a polynomial that contains two terms is called a **binomial;** and a polynomial that contains three terms is called a **trinomial.** Any polynomial that contains more than one term is called a **multinomial,** but no special names are given to polynomials that contain more than three terms.

Example 3

Determine if each of the following algebraic expressions is a polynomial. If it is a polynomial, what name best describes it? If it is not a polynomial, state why it is not.

1. x, $4x$, 3, and $5x^2y$ are monomials.

2. $3x + 1$, $x + y$, and $81W^2 - 9T^2$ are binomials.

3. $5x^3 + 2y - 1$ and $z^2 + 9z - 10$ are trinomials.

4. $6x^3 - 2x^2 + 4x + 1$ is a polynomial of 4 terms.

5. $\frac{4}{x+2}$ is not a polynomial since it contains a variable in the denominator.

Note We should simplify any expression, before identifying it. Also, in an expression, the combining of all of the constant terms is understood to be a single term. For example, $x + 3 + \pi$ is thought of as $x + (3 + \pi)$ and is a binomial.

▶ ***Quick check*** Determine if each is a polynomial. If it is, what name best describes it? If it is not, state why it is not.

$5x^2y + 2z$; $\quad 5x^2y + \frac{2}{z}$ ■

Another way that we identify different types of polynomials is by the degree of the polynomial. *The* **degree** *of a polynomial in one variable is the greatest exponent of that variable in any one term.*

Example 4

Determine the degree of the polynomial.

1. $5x^3$ — Third degree because the exponent of x is 3

2. $x^4 - 2x^3 + 3x - 5$ — Fourth degree because the greatest exponent of x in any one term is 4

Note In example 2, the polynomial has been arranged in *descending powers* of the variable. This is the form that we will use when we write polynomials in one variable.

3. $4y^5 - 7y^2 + 3$ — Fifth degree because the greatest exponent of y in any one term is 5 ■

Algebraic Notation

Many problems that we encounter will be stated verbally. These will need to be translated into algebraic expressions. While there is no standard procedure for changing a verbal phrase into an algebraic expression, the following guidelines should be of use.

1. Read the problem carefully, determining useful prior knowledge. Note what information is given and what information we are asked to find.
2. Let some letter represent one of the unknowns. Then express any other unknowns in terms of it.
3. Use the given conditions in the problem and the unknowns from step 2 to write an algebraic expression.

When translating verbal phrases into equations, we should be looking for phrases that involve the basic operations of addition, subtraction, multiplication, and division. Table 2–1 shows some examples of phrases that are commonly encountered. We will let x represent the unknown number.

Table 2–1

Phrase	Algebraic expression
Addition	
6 more than a number the sum of a number and 6 6 plus a number a number increased by 6 6 added to a number	$x + 6$
Subtraction	
6 less than a number a number diminished by 6 the difference of a number and 6 a number minus 6 a number less 6 a number decreased by 6 6 subtracted from a number a number reduced by 6	$x - 6$
Multiplication	
a number multiplied by 6 6 times a number the product of a number and 6	$6x$
Division	
a number divided by 6 the quotient of a number and 6 $\frac{1}{6}$ of a number	$\frac{x}{6}$

Example 5

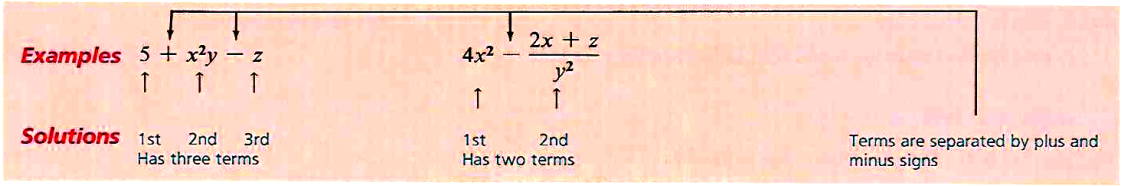
Write an algebraic expression for each.

1. The product of a and b $\quad a \cdot b$
2. The sum of a and 4 $\quad a + 4$
3. x decreased by 9 $\quad x - 9$
4. y divided by 3 $\quad \frac{y}{3}$
5. A number increased by 6 $\quad n + 6$
6. Two times a number and that product decreased by 5 $\quad 2n - 5$
7. A number divided by 3 and that quotient increased by 2 $\quad \frac{n}{3} + 2$
8. Twice the sum of x and 4 $\quad 2(x + 4)$

▶ ***Quick check*** Write an algebraic expression for the product of x and y. Write an algebraic expression for a number increased by 6. ■

Exercises 1.5(PART I)

Specify the number of terms in each expression.



1. $3x + 4y$ **2.** $5xyz$ **3.** $4x^2 + 3x - 1$ **4.** $x^3 - 4x + 7$

5. $\frac{6x}{5}$ **6.** $\frac{x}{3}$ **7.** $8xy + \frac{5y}{2} - 6x$ **8.** $\frac{15x^2 + y}{8}$

9. $5x^3 + (3x^2 - 4)$ **10.** $x^2 + a^2(y^2 - z)$ **11.** $(x + y + z)$ **12.** 7

13. $a^2(b + c) - x^2(y + z)$ **14.** $x^2 + \frac{y - z}{a} + c$

Determine the numerical coefficients of the following algebraic expressions.

Example $a^2 - 2a + 4b$

Solution 1 is understood to be the coefficient of a^2, -2 is the coefficient of a, 4 is the coefficient of b.

15. $5x^2 + x - 4z$ **16.** $a^2b + 4ab^2 - ab$ **17.** $x - y - 3z$

18. $3x^4 - x^2 + x^2$ **19.** $-2a - b + c$

Determine if each of the following algebraic expressions is a polynomial. If it is a polynomial, what name best describes it? If it is not a polynomial, state why it is not.

Examples $5x^2y + 2z$ $5x^2y + \frac{2}{z}$

Solutions It is a polynomial. Since there are two terms, it is a binomial. Not a polynomial because a variable is used as a divisor (appears in the denominator)

20. $ax^2 + bx + c$ **21.** $mx + b$ **22.** $5x^2 + 2x$ **23.** $y + \frac{1}{x}$

24. $\frac{a + b}{5} - c$ **25.** $\frac{a + b}{c} + d$ **26.** $4x^5 - 7x^3 + 3x - 2$ **27.** $9x^6 + 2x^2 + 4$

Write an algebraic expression for each of the following.

Examples The product of x and y A number increased by 6

Solutions $x \cdot y$ Let x represent the number; hence $x + 6$

28. The sum of a and b

29. 3 times a, subtracted from b

30. 7 less than x

31. 5 more than y

32. The sum of x and y, divided by z

33. x times the sum of y and z

34. a decreased by 5

35. a decreased by b

36. $\frac{1}{2}$ of x, decreased by 2 times x

37. A number decreased by 12

38. A number added to 4

39. 3 times a number and that product increased by 1

40. A number divided by 5

41. 2 times the sum of a number and 4

42. A number decreased by 6 and that difference divided by 11