Math 251 **Name(s)*:**

PaperAssign 1

Homework

Due September 6, 2017

*This homework is an exception to the general policy on group work: you may turn in one submission for up to four students. Otherwise, the usual policy applies: you may talk about the problems outside of the group, but what you write in your submission must be done solely within your group.

There are two problems; you may turn in this sheet or just work on lined paper, but be neat in any case.

(1) Consider the plane $5x + 2y - 3z = 11$. Find a parametric (vector) equation for the plane. **Explain your method** (there are at least three good methods).

We need a point & two non-parallel direction vectors.

Method I. Find three points, either by inspection

or $x=y=0 \Rightarrow z=-\frac{11}{3}$

$x=z=0 \Rightarrow y=\frac{11}{2}$

$y=z=0 \Rightarrow x=\frac{11}{5}$

Take two vectors joining them:

$\vec{u}=(0,\frac{11}{2},0)-(0,0,\frac{-11}{3})=\langle 0,\frac{11}{2},\frac{11}{3}\rangle$

$\vec{v}=(\frac{11}{5},0,0)-(0,0,-\frac{11}{3})=\langle \frac{11}{5},0,\frac{11}{3}\rangle$

Non-parallel? ✓

$\vec{r}(s,t)=\langle \frac{11}{5},0,0\rangle+s\langle 0,\frac{11}{2},\frac{11}{3}\rangle+t\langle \frac{11}{5},0,\frac{11}{3}\rangle$

Method II. Find one point, and two direction vectors by inspection, (need $\langle a,b,c\rangle\cdot\langle 5,2,-3\rangle=0$)

e.g. $\vec{u}=\langle 1,-1,1\rangle$

$\vec{v}=\langle -2,5,0\rangle$

Non-parallel ✓

$\vec{r}(s,t)=\langle \frac{11}{5},0,0\rangle+s\langle 1,-1,1\rangle+t\langle -2,5,0\rangle$

Method III. Abuse the cross product.

$\vec{u}=\vec{n}\times\hat{i}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\5&2&-3\\1&0&0\end{vmatrix}=\langle 0,-3,-2\rangle$

$\vec{v}=\vec{n}\times\vec{u}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\5&2&-3\\0&-3&-2\end{vmatrix}=\langle -13,10,-15\rangle$

Automatically non-parallel; in fact, necessarily perpendicular!

$\vec{r}(s,t)=\langle \frac{11}{5},0,0\rangle+s\langle 0,-3,-2\rangle+t\langle -13,10,-15\rangle$

(2) Verify that the line $\mathbf{r}(t) = \langle 2, -1, 0\rangle + t\langle 3, 2, 5\rangle$ is parallel to the plane $4x - y - 2z = 17$. Then, find an equation of the line that is perpendicular to the given line, parallel to the given plane, and passes through $(2, -1, 0)$. **Explain** as you go.

The line is // to the plane $\Leftrightarrow$ the line's direction is $\perp$ to the plane's normal.

$$\Leftrightarrow \langle 3,2,5\rangle \cdot \langle 4,-1,-2\rangle = 0$$

$$12 - 2 - 10 \stackrel{\checkmark}{=} 0.$$

The direction vector $\vec{v}$ of the new line needs to be

1) $\vec{v} \perp \langle 3,2,5\rangle$
2) $\vec{v} \perp \langle 4,-1,-2\rangle$

so $\vec{v} = \langle 3,2,5\rangle \times \langle 4,-1,-2\rangle$ will work

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 5 \\ 4 & -1 & -2 \end{vmatrix} = \langle 1, 26, -11\rangle$$

$$\boxed{\ell(t) = \langle 2,-1,0\rangle + t\langle 1, 26, -11\rangle.}$$