CHAPTER 1 SECTION 1.9 (Part II)

Sums and differences of radicals

Like radicals

We have learned that in addition and subtraction of algebraic expressions, we can only combine like terms. This same idea applies when we are dealing with radicals. **We can add or subtract only like radicals.** Like radicals are radicals having the same index and the same radicand. For example, $3\sqrt{5x}$ and $-2\sqrt{5x}$ are like radicals, but $5\sqrt{7x}$ and $7\sqrt{5x}$ are not, because the radicands are not the same.

Addition and subtraction involving square roots

Addition and subtraction of radicals follow the same procedure as addition and subtraction of algebraic expressions. That is, *once we have determined that we have like radicals, the operations of addition and subtraction are performed only with the numerical coefficients.*

Examples

Perform the indicated operations and simplify. Assume that all variables represent nonnegative real numbers.

1. $5\sqrt{2} + 3\sqrt{2} = (5 + 3)\sqrt{2} = 8\sqrt{2}$ Apply the distributive property

2. $12\sqrt{3} - \sqrt{3} = (12 - 1)\sqrt{3} = 11\sqrt{3}$

3. $2\sqrt{a} + 3\sqrt{a} = 5\sqrt{a}$

Consider the example

$$\sqrt{27} + 4\sqrt{3}$$

It appears that the indicated addition cannot be performed since we do not have like radicals. However we should have observed that the $\sqrt{27}$ can be simplified as

$$\sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$$

Our problem then becomes

$$\sqrt{27} + 4\sqrt{3} = 3\sqrt{3} + 4\sqrt{3} = 7\sqrt{3}$$

and we are able to add the like radicals. Therefore, *whenever we are working with radicals, we must be certain that all radicals are in simplest form.*

To combine like radicals

1. Perform any simplification within the terms.
2. Use the distributive property to combine terms that have like radicals.

Examples

Perform the indicated operations. Assume that all variables represent nonnegative real numbers.

1. $\sqrt{45} + \sqrt{20} = \sqrt{9 \cdot 5} + \sqrt{4 \cdot 5}$ Factor $45 = 9 \cdot 5$ and $20 = 4 \cdot 5$

$= 3\sqrt{5} + 2\sqrt{5}$ $\sqrt{4} = 2$; $\sqrt{9} = 3$

$= 5\sqrt{5}$ Add coefficients

2. $\sqrt{32} + 5\sqrt{8} = \sqrt{16 \cdot 2} + 5\sqrt{4 \cdot 2}$ Factor $32 = 16 \cdot 2$; $8 = 4 \cdot 2$

$= 4\sqrt{2} + 5 \cdot 2\sqrt{2}$ $\sqrt{16} = 4$ and $\sqrt{4} = 2$

$= 4\sqrt{2} + 10\sqrt{2}$ Multiply $5 \cdot 2 = 10$

$= 14\sqrt{2}$ Add coefficients

3. $3\sqrt{3a} - \sqrt{12a} + 5\sqrt{48a}$

$= 3\sqrt{3a} - \sqrt{4 \cdot 3a} + 5\sqrt{16 \cdot 3a}$ Factor $12 = 4 \cdot 3$; $48 = 16 \cdot 3$

$= 3\sqrt{3a} - 2\sqrt{3a} + 5 \cdot 4\sqrt{3a}$ $\sqrt{4} = 2$; $\sqrt{16} = 4$

$= 3\sqrt{3a} - 2\sqrt{3a} + 20\sqrt{3a}$ $5 \cdot 4 = 20$

$= 21\sqrt{3a}$ Combine coefficients

Addition and subtraction involving nth roots

Addition and subtraction of radicals other than square roots follow the same procedure as addition and subtraction of expressions containing square roots. That is, *once we have determined that we have like radicals, the operations of addition and subtraction are performed only with the numerical coefficients.*

Examples

Perform the indicated operations and simplify. Assume that all variables represent nonnegative real numbers.

1. $\sqrt[3]{5} + 6\sqrt[3]{5} = (1 + 6)\sqrt[3]{5} = 7\sqrt[3]{5}$ Combine coefficients

2. $4\sqrt[3]{81} - \sqrt[3]{24} = 4\sqrt[3]{27 \cdot 3} - \sqrt[3]{8 \cdot 3}$ Factor $81 = 27 \cdot 3$; $24 = 8 \cdot 3$

$= 4 \cdot 3\sqrt[3]{3} - 2\sqrt[3]{3}$ $\sqrt[3]{27} = 3$; $\sqrt[3]{8} = 2$

$= 12\sqrt[3]{3} - 2\sqrt[3]{3}$ $4 \cdot 3 = 12$

$= 10\sqrt[3]{3}$ Subtract coefficients

3. $\sqrt[3]{16x^2y} + \sqrt[3]{54x^2y} = \sqrt[3]{8 \cdot 2x^2y} + \sqrt[3]{27 \cdot 2x^2y}$ Factor $16 = 8 \cdot 2$; $54 = 27 \cdot 2$

$= 2\sqrt[3]{2x^2y} + 3\sqrt[3]{2x^2y}$ $\sqrt[3]{8} = 2$; $\sqrt[3]{27} = 3$

$= 5\sqrt[3]{2x^2y}$ Add coefficients

Exercises

Perform the indicated operations and simplify. Assume that all variables represent nonnegative real numbers. See examples

Examples $3\sqrt{6} + 2\sqrt{6} - \sqrt{6}$

Solutions $= (3 + 2 - 1)\sqrt{6}$ Distributive property

$= 4\sqrt{6}$ Combine coefficients

$5\sqrt{2} + \sqrt{18}$

$= 5\sqrt{2} + \sqrt{9 \cdot 2}$ Factor $18 = 9 \cdot 2$

$= 5\sqrt{2} + 3\sqrt{2}$ $\sqrt{9} = 3$

$= (5 + 3)\sqrt{2}$ Distributive property

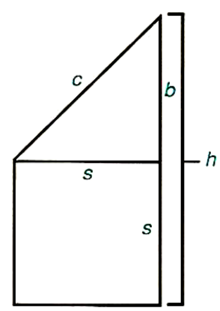
$= 8\sqrt{2}$ Add coefficients

1. $5\sqrt{3} + 4\sqrt{3}$
2. $8\sqrt{7} - 2\sqrt{7}$
3. $6\sqrt{5} + 4\sqrt{5}$
4. $9\sqrt{6} - 6\sqrt{6}$
5. $2\sqrt{3} + 7\sqrt{3} - 3\sqrt{3}$
6. $5\sqrt{5} - 4\sqrt{5} + 6\sqrt{5}$
7. $\sqrt{7} + 5\sqrt{7} - 3\sqrt{7}$
8. $2\sqrt{10} + 11\sqrt{10} - 9\sqrt{10}$
9. $\sqrt{a} + 2\sqrt{a}$
10. $3\sqrt{x} + 4\sqrt{x}$
11. $5\sqrt{a} - 4\sqrt{a} + 7\sqrt{a}$
12. $6\sqrt{y} - \sqrt{y} + 4\sqrt{y}$
13. $5\sqrt{xy} + 2\sqrt{xy}$
14. $3\sqrt{x} + 2\sqrt{y} - \sqrt{x}$
15. $5\sqrt{a} + 2\sqrt{ab} + 3\sqrt{a}$
16. $\sqrt{ab} + 2\sqrt{ab} + 3\sqrt{a}$
17. $5\sqrt{xy} - \sqrt{xy} + 3\sqrt{y}$
18. $\sqrt{20} + 3\sqrt{5}$
19. $\sqrt{8} + 5\sqrt{2}$
20. $\sqrt{12} + \sqrt{75}$
21. $\sqrt{48} - \sqrt{27}$
22. $2\sqrt{3} + 3\sqrt{12}$
23. $5\sqrt{7} + 4\sqrt{63}$
24. $5\sqrt{3} + \sqrt{27} - \sqrt{12}$
25. $4\sqrt{2} - \sqrt{8} + \sqrt{50}$
26. $\sqrt{75} - 4\sqrt{3} + 2\sqrt{27}$
27. $\sqrt{12} + \sqrt{18} + \sqrt{50}$

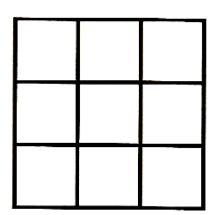
28. $\sqrt{63} - \sqrt{28} + \sqrt{24}$
29. $\sqrt{50a} + \sqrt{8a}$
30. $\sqrt{32a} - \sqrt{18a}$
31. $3\sqrt{9x} - 5\sqrt{4x}$
32. $2\sqrt{4x^2y} + 3\sqrt{25x^2y}$
33. $2\sqrt{8a} + 4\sqrt{50a} - 7\sqrt{2a}$
34. $3\sqrt{48b} - 2\sqrt{12b} + \sqrt{3b}$
35. $\sqrt{50a} + 3\sqrt{12a} - \sqrt{18a}$
36. $4\sqrt{25x^2y} + 3\sqrt{81x^2y} - 2\sqrt{2y}$

37. We can find the height, h, of the given figure by finding b from the formula $b = \sqrt{c^2 - s^2}$. If $c = 10$ units and $s = 6$ units, find h.

38. Use exercise 37 to find the height of the figure if $c = 13$ feet and $s = 5$ feet.



39. The figure is made up of 9 equal squares in which each square has an area of 7.29 square units. What are the dimensions of the figure?



Perform the indicated operations and simplify. Assume that all variables represent nonnegative real numbers.

40. $3\sqrt[3]{4} + 5\sqrt[3]{4}$

41. $7\sqrt[5]{2} - 4\sqrt[5]{2} + 3\sqrt[5]{2}$

42. $9\sqrt[4]{3} + 6\sqrt[4]{3} + 2\sqrt[4]{3}$

43. $\sqrt[3]{16} + \sqrt[3]{54}$

44. $\sqrt[3]{24} - \sqrt[3]{81}$

45. $\sqrt[3]{81} + 2\sqrt[3]{250}$

46. $\sqrt[3]{8a^2} + \sqrt[3]{27a^2}$

47. $\sqrt[4]{16x^3} + \sqrt[4]{81x^3}$

48. $\sqrt[4]{625a} - \sqrt[4]{81a}$

49. $\sqrt[3]{64x^2y} - \sqrt[3]{27x^2y}$

50. $\sqrt[3]{x^6y} + 2x^2\sqrt[3]{y}$

51. $3a\sqrt[3]{b^2} - \sqrt[3]{a^3b^2}$

52. $\sqrt[3]{16a^2b} + \sqrt[3]{54a^2b}$

Further operations with radicals

Multiplication of radical expressions

In Part I we learned the procedure for multiplying two radicals. We now combine those ideas along with the *distributive property,* $a(b + c) = ab + ac$, to perform multiplication of radical expressions containing more than one term.

Examples

Perform the indicated operations and simplify. Assume that all variables represent nonnegative real numbers.

1. $\sqrt{3}(3 + \sqrt{3}) = \sqrt{3} \cdot 3 + \sqrt{3}\sqrt{3}$ — Distributive property
$= 3\sqrt{3} + \sqrt{9}$ — $\sqrt{3}\sqrt{3} = \sqrt{9}$
$= 3\sqrt{3} + 3$ — $\sqrt{9} = 3$

2. $\sqrt{3}(\sqrt{6} + \sqrt{21}) = \sqrt{3}\sqrt{6} + \sqrt{3}\sqrt{21}$ — Distributive property
$= \sqrt{18} + \sqrt{63}$ — Multiply radicands
$= \sqrt{9 \cdot 2} + \sqrt{9 \cdot 7}$ — Factor $18 = 9 \cdot 2$; $63 = 9 \cdot 7$
$= 3\sqrt{2} + 3\sqrt{7}$ — Simplify radicals

3. $(\sqrt{2} + \sqrt{3})(\sqrt{2} + 5\sqrt{3})$

Note In this example, we are multiplying two binomials. Therefore, as we did in chapter 3, we will *multiply each term in the first parentheses by each term in the second parentheses.*

$= \sqrt{2}\sqrt{2} + \sqrt{2} \cdot 5\sqrt{3} + \sqrt{3}\sqrt{2} + \sqrt{3} \cdot 5\sqrt{3}$ — Distributive property
$= \sqrt{4} + 5\sqrt{6} + \sqrt{6} + 5 \cdot \sqrt{9}$ — Multiply radicands
$= 2 + 5\sqrt{6} + \sqrt{6} + 5 \cdot 3$ — $\sqrt{4} = 2$, $\sqrt{9} = 3$
$= 2 + 5\sqrt{6} + \sqrt{6} + 15$
$= 17 + 6\sqrt{6}$ — Combine like terms

4. $(3 - \sqrt{2})(3 + \sqrt{2}) = 9 + 3\sqrt{2} - 3\sqrt{2} - \sqrt{4}$ — Distributive property
$= 9 + 3\sqrt{2} - 3\sqrt{2} - 2$ — Simplify radicals
$= 9 - 2$ — Combine like terms
$= 7$ — Subtract

We observe that when we simplified, there were no longer any radicals in the answer.

5. $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$
$= \sqrt{a}\sqrt{a} + \sqrt{a}\sqrt{b} - \sqrt{b}\sqrt{a} - \sqrt{b}\sqrt{b}$ — Distributive property
$= \sqrt{a^2} + \sqrt{ab} - \sqrt{ab} - \sqrt{b^2}$ — Multiply radicands
$= a + \sqrt{ab} - \sqrt{ab} - b$ — $\sqrt{a^2} = a$ and $\sqrt{b^2} = b$
$= a - b$ — Combine like terms

6. $(\sqrt{3} + 2\sqrt{2})^2$

$= (\sqrt{3} + 2\sqrt{2})(\sqrt{3} + 2\sqrt{2})$	
$= \sqrt{3}\sqrt{3} + \sqrt{3} \cdot 2\sqrt{2} + \sqrt{3} \cdot 2\sqrt{2} + 2\sqrt{2} \cdot 2\sqrt{2}$	Distributive property
$= \sqrt{9} + 2\sqrt{6} + 2\sqrt{6} + 4\sqrt{4}$	Multiply radicands
$= 3 + 2\sqrt{6} + 2\sqrt{6} + 4 \cdot 2$	Simplify radicals
$= 3 + 2\sqrt{6} + 2\sqrt{6} + 8$	Multiply
$= 11 + 4\sqrt{6}$	Combine like terms

Conjugate factors

The type of factors that we are multiplying in examples 4 and 5 are called **conjugate factors.** The conjugate is used to rationalize the denominator of a fraction when the denominator contains two terms where one or both terms contain a square root. The idea of conjugate factors is derived from the factorization of the difference of two squares. When multiplying conjugate factors, we can simply write our answer as the square of the second term subtracted from the square of the first term.

In examples 4 and 5, we could have performed the multiplication as follows:

4. $(3 - \sqrt{2})(3 + \sqrt{2}) = (3)^2 - (\sqrt{2})^2 = 9 - 2 = 7;$
5. $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b.$

To determine what the conjugate of a given factor is, we write the original factor and change the sign of the second term.

Examples

Form the conjugates of the given expressions.

1. $\sqrt{7} + 2$ — The conjugate is $\sqrt{7} - 2$.

2. $\sqrt{11} - \sqrt{6}$ — The conjugate is $\sqrt{11} + \sqrt{6}$.

3. $-5 - 2\sqrt{3}$ — The conjugate is $-5 + 2\sqrt{3}$.

4. $\sqrt{a} + \sqrt{b}$ — The conjugate is $\sqrt{a} - \sqrt{b}$.

Rationalizing the denominator

If we wish to rationalize the denominator of the fraction

$$\frac{1}{3-\sqrt{2}}$$

we recall from example that when we multiplied $3-\sqrt{2}$ by $3+\sqrt{2}$, there were no radicals left in our product. This result is precisely what we want to occur in our denominator. Therefore, to rationalize this fraction, we apply the fundamental principle of fractions and multiply the numerator and the denominator by $3+\sqrt{2}$, the conjugate of the denominator.

$$\frac{1}{3-\sqrt{2}} = \frac{1}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}}$$ $3+\sqrt{2}$ is the conjugate of the denominator

$$= \frac{1(3+\sqrt{2})}{(3)^2-(\sqrt{2})^2}$$ (first term)^2 − (second term)^2

$$= \frac{3+\sqrt{2}}{9-2}$$ No radicals remain in the denominator

$$= \frac{3+\sqrt{2}}{7}$$ Denominator is rationalized

Examples

Rationalize the denominators.

1. $$\frac{2}{\sqrt{7}+2} = \frac{2}{\sqrt{7}+2} \quad \frac{\sqrt{7}-2}{\sqrt{7}-2}$$ Multiply by the conjugate of the denominator

$$= \frac{2(\sqrt{7}-2)}{(\sqrt{7})^2-(2)^2}$$ $(x+y)(x-y) = x^2-y^2$

$$= \frac{2\sqrt{7}-4}{7-4}$$ Simplify in numerator and denominator

$$= \frac{2\sqrt{7}-4}{3}$$ Subtract in denominator

2. $$\frac{5}{\sqrt{11}-\sqrt{6}} = \frac{5}{\sqrt{11}-\sqrt{6}} \cdot \frac{\sqrt{11}+\sqrt{6}}{\sqrt{11}+\sqrt{6}}$$ Multiply by the conjugate of the denominator

$$= \frac{5(\sqrt{11}+\sqrt{6})}{(\sqrt{11})^2-(\sqrt{6})^2}$$ $(x+y)(x-y) = x^2-y^2$

$$= \frac{5(\sqrt{11}+\sqrt{6})}{11-6}$$ Simplify radicals

$$= \frac{5(\sqrt{11}+\sqrt{6})}{5}$$ Subtract in denominator

$$= \sqrt{11}+\sqrt{6}$$ Reduce (by 5) to lowest terms

3. $$\frac{\sqrt{3}}{5-2\sqrt{3}} = \frac{\sqrt{3}}{5-2\sqrt{3}} \cdot \frac{5+2\sqrt{3}}{5+2\sqrt{3}}$$ Multiply by the conjugate of the denominator

$$= \frac{\sqrt{3}(5+2\sqrt{3})}{(5)^2-(2\sqrt{3})^2}$$ $(x+y)(x-y) = x^2 - y^2$

$$= \frac{5\sqrt{3}+2\sqrt{9}}{5^2-2^2(\sqrt{3})^2}$$ Simplify radicals

$$= \frac{5\sqrt{3}+2\cdot 3}{25-4\cdot 3}$$ Simplify radicals

$$= \frac{5\sqrt{3}+6}{25-12}$$ Perform operations

$$= \frac{5\sqrt{3}+6}{13}$$ Subtract in denominator

EXERCISES

Perform the indicated operations and simplify. Assume that all variables represent positive real numbers. See example

Examples	$2(\sqrt{3}+\sqrt{5})$		$\sqrt{2}(\sqrt{14}+\sqrt{6})$	
Solutions	$=2\sqrt{3}+2\sqrt{5}$	Distributive property	$=\sqrt{2}\sqrt{14}+\sqrt{2}\sqrt{6}$	Distributive property
			$=\sqrt{28}+\sqrt{12}$	Product property
			$=\sqrt{4\cdot 7}+\sqrt{4\cdot 3}$	Factor $28=4\cdot 7$; $12=4\cdot 3$
			$=2\sqrt{7}+2\sqrt{3}$	$\sqrt{4}=2$

Example $(\sqrt{2}+\sqrt{3})(\sqrt{2}-2\sqrt{3})$

Solution	$=\sqrt{2}\sqrt{2}-\sqrt{2}\cdot 2\sqrt{3}+\sqrt{3}\sqrt{2}-\sqrt{3}\cdot 2\sqrt{3}$	Distributive property
	$=2-2\sqrt{6}+\sqrt{6}-2\cdot 3$	$\sqrt{2}\sqrt{2}=2$; $\sqrt{3}\sqrt{3}=3$
	$=2-\sqrt{6}-6$	Combine like radicals
	$=-4-\sqrt{6}$	$2-6=-4$

1. $3(\sqrt{2}+\sqrt{3})$	**2.** $5(2\sqrt{6}+\sqrt{2})$	**3.** $\sqrt{2}(\sqrt{3}+\sqrt{7})$
4. $\sqrt{5}(\sqrt{7}-\sqrt{3})$	$\boxed{\mathbf{5.}}$ $3\sqrt{2}(2\sqrt{3}-\sqrt{11})$	**6.** $\sqrt{6}(\sqrt{2}+\sqrt{3})$
7. $\sqrt{5}(\sqrt{15}-\sqrt{10})$	**8.** $\sqrt{14}(\sqrt{21}+\sqrt{10})$	**9.** $2\sqrt{7}(\sqrt{35}-3\sqrt{14})$
10. $\sqrt{a}(\sqrt{ab}+\sqrt{b})$	**11.** $\sqrt{a}(3\sqrt{a}+\sqrt{b})$	**12.** $(5+\sqrt{3})(4-\sqrt{3})$
13. $(3+\sqrt{2})(4+\sqrt{2})$	**14.** $(5-\sqrt{5})(5-\sqrt{5})$	$\boxed{\mathbf{15.}}$ $(3-4\sqrt{a})(4-3\sqrt{a})$
16. $(7+2\sqrt{y})(6+5\sqrt{y})$	$\boxed{\mathbf{17.}}$ $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$	**18.** $(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})$
19. $(2+\sqrt{6})(2-\sqrt{6})$	**20.** $(5-\sqrt{3})(5+\sqrt{3})$	$\boxed{\mathbf{21.}}$ $(2+\sqrt{5})^2$
22. $(3-\sqrt{7})^2$	**23.** $(\sqrt{x}+\sqrt{y})^2$	**24.** $(\sqrt{a}-\sqrt{b})^2$
25. $(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})$	$\boxed{\mathbf{26.}}$ $(2\sqrt{a}-\sqrt{b})(2\sqrt{a}+\sqrt{b})$	**27.** $(x\sqrt{y}+\sqrt{z})(x\sqrt{y}-\sqrt{z})$
28. $(a\sqrt{b}+c)(a\sqrt{b}-c)$	**29.** $(2\sqrt{x}+y)^2$	**30.** $(3\sqrt{a}+\sqrt{b})^2$

Form the conjugate of the given expressions. See example

Example $6-3\sqrt{2}$

Solution $6+3\sqrt{2}$ First term remains the same, change the sign of the second term

31. $11-\sqrt{3}$	**32.** $-5\sqrt{7}-\sqrt{2}$	**33.** $\sqrt{a}+3\sqrt{b}$	**34.** $a\sqrt{b}-\sqrt{c}$

Simplify the following expressions, leaving all denominators rationalized. Assume that all variables represent positive real numbers and that no denominator is equal to zero. See example

Examples $\frac{3}{7+\sqrt{2}}$		$\frac{2}{\sqrt{11}-3}$	
Solutions $=\frac{3}{7+\sqrt{2}}\cdot\frac{7-\sqrt{2}}{7-\sqrt{2}}$	Multiply by the conjugate	$=\frac{2}{\sqrt{11}-3}\cdot\frac{\sqrt{11}+3}{\sqrt{11}+3}$	Multiply by the conjugate
$=\frac{3(7-\sqrt{2})}{(7)^2-(\sqrt{2})^2}$	$(x+y)(x-y)=x^2-y^2$	$=\frac{2(\sqrt{11}+3)}{(\sqrt{11})^2-(3)^2}$	$(x+y)(x-y)=x^2-y^2$
$=\frac{3(7-\sqrt{2})}{49-2}$	Simplify denominator	$=\frac{2(\sqrt{11}+3)}{11-9}$	Simplify denominator
$=\frac{3(7-\sqrt{2})}{47}$	Simplify denominator	$=\frac{2(\sqrt{11}+3)}{2}$	Simplify denominator
$=\frac{21-3\sqrt{2}}{47}$	Multiply in numerator	$=\sqrt{11}+3$	Reduce fraction

35. $\frac{1}{\sqrt{2}+3}$ **36.** $\frac{1}{\sqrt{3}-2}$ **37.** $\frac{7}{2+\sqrt{7}}$ $\boxed{\textbf{38.}}$ $\frac{6}{3-\sqrt{6}}$

39. $\frac{3}{\sqrt{6}-\sqrt{3}}$ **40.** $\frac{1}{\sqrt{a}+b}$ **41.** $\frac{3}{2\sqrt{3}-\sqrt{5}}$ **42.** $\frac{4}{2\sqrt{3}-\sqrt{6}}$

$\boxed{\textbf{43.}}$ $\frac{1+\sqrt{5}}{1-\sqrt{5}}$ **44.** $\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$ **45.** $\frac{\sqrt{a}+b}{\sqrt{a}-b}$