Worksheet 25 April 25, 2011

1. Use Newton's method with the function $x^4 - 3x^2 + 1$ with starting point $x = 1$. You should iterate the method twice. (If you are handy with a programmable calculator, consider writing a quick program to iterate Newton's method several times. By the way, you're approximating the value of the (conjugate) golden ratio here.)

2. Approximate $\sqrt{80}$ using the linearization of an appropriate function at an appropriate point.

3. What is an easy-to-differentiate function that is zero at $\sqrt{80}$? Use Newton's method on this function to approximate $\sqrt{80}$. Use only one iteration.

4. Explain why the result from the previous two problems isn't really that surprising. More generally, how is the process of approximating a function's value by a linear approximation related to the process of approximating a root of a function with Newton's method?

5. Compute the integral $\int \cos^3(x)\, dx$.

6. Let's now compute the integral $\int \sec x\, dx$ in a way different from lecture. (Recall that in lecture he used $\sec x = \cos x / \cos^2 x$, then broke a fraction down, then integrated term by term.) This time we'll not rewrite secant, but instead just multiply top and bottom by $\sec x + \tan x$. Now let u be the denominator, and see where that takes you.

7. Compute the volume of the "cap" of a radius R sphere with height h. (This is useful if, for example, you want to partially submerge a sphere in liquid and need to know what volume you're displacing.)

8. Consider the region bounded below by the x-axis and above by the graph of $y = \lfloor x \rfloor$, for $0 \le x \le 3.5$. The region is now rotated about the following line; compute the resulting solid's volume.

   (a) the x-axis

   (b) the y-axis

   (c) the line $x = 4$

   (d) the line $y = -3$

9. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal areas.

10. Calculus is a fundamental part of probability when dealing with situations that have a continuum (or really just infinitely many) possible outcomes. For instance, age or height of people, lifetime of a machine, or time between passing cars all have infinitely many possible outcomes. We compute probabilities of these events by integrating a *probability density function*.

    (a) For example, suppose the amount of time you spend in a line at the DMV has a density function $f(t) = 4t - 4t^3$, where t is measured in hours and $0 \le t \le 1$ (for other times the probability is zero). To find the probability that you wait between 1/4 hour and 1/2 hour, you need to calculate $\int_{1/4}^{1/2} f(t)\, dt$. Do so.

(b) What should be the probability that you wait at least 0 minutes? Compute the probability that you wait between 0 and 1 hour to verify that this is true for the given density function. (Remember that we're assuming it's impossible to wait more than an hour; perhaps you just give up and leave after an hour.)

(c) Calculus also gives us a way to compute the *expected* waiting time. To do this we just compute $\int t \cdot f(t)\, dt$. Do so, integrating from 0 to 1.