Math 251

Quiz 6 October 19, 2016

Name:

By handing in this quiz you assert that you understand and have followed IIT's guidelines for academic integrity.

(1) True or false: every double-integral is positive, because it represents volume. signed volume

(2) True or false: for every R, f, and g, $\iint_R f \cdot g \, dA = \left(\iint_R f \, dA\right) \cdot \left(\iint_R g \, dA\right)$. For example, if $f = g = 1$, then the Left side is Area(R) & the right side is $[\text{Area}(R)]^2$.

(3) Evaluate $\iint_R (xy^2 + 2) \, dA$ where R is the disk $x^2 + y^2 \leq 4$.

(Hint: no calculations are necessary.)

$$= \iint_R xy^2 \, dA + \iint_R 2 \, dA$$

$$= 0 + 2 \cdot \text{Area}(R) = 8\pi$$

(xy^2 is odd w.r.t. x, & R is symmetric)

OR

$$= \int_0^{2\pi}\int_0^2 \left(r^3 \cos\theta \sin^2\theta + 2\right) r \, dr \, d\theta$$

$$= \int_0^{2\pi} \frac{1}{5} r^5 \cos\theta \sin^2\theta + r^2 \Big|_0^2 \, d\theta$$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

$$= \frac{32}{5} \frac{\sin^3\theta}{3}\Big|_0^{2\pi} + 8\pi = 8\pi$$

OR

$$= \int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (xy^2 + 2) \, dx \, dy = \int_{-2}^{2} \frac{1}{2}x^2y^2 + 2x \Big|_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dy$$

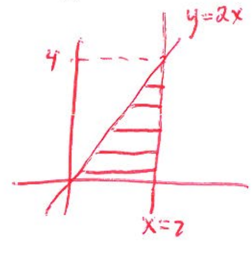
$$= \int_{-2}^{2} \Big(\underbrace{\tfrac{1}{2}y^2\left((4-y^2) - (4-y^2)\right)}_{0} + 4\sqrt{4-y^2}\Big) dy$$

$$= 4\int_{-2}^{2} \sqrt{4-y^2} \, dy = 4 \cdot \text{Area}(\text{half-disk}) = 4(2\pi) = 8\pi$$

$z = \sqrt{4-y^2}$

OR $y = 2\sin t$: $4\int_{-\pi/2}^{\pi/2} 2\cos t \cdot 2\cos t \, dt = 8\int_{-\pi/2}^{\pi/2} (1 + \cos(2t)) \, dt$ $\,//\,$

(4) Change the order of integration to find $\int_0^4 \int_{y/2}^2 e^{-x^2} \, dx \, dy$.



$x = \frac{y}{2} \longleftrightarrow y = 2x$

$$= \int_0^2 \int_0^{2x} e^{-x^2} \, dy \, dx$$

$$= \int_0^2 2xe^{-x^2} \, dx = -e^{-x^2}\Big|_0^2 = -e^{-4} + 1$$

$u = -x^2$

$du = -2x \, dx$