HOMEWORK 1: §1.1-1.5 DUE JANUARY 19

Name: _____

- Please refer to the syllabus regarding allowed collaboration on this homework assignment.
- All answers should be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

(1) Which of the following propositions are logically equivalent? *(For example, your answer may be "(i) and (ii) are equivalent and (iii) and (iv) are equivalent, but they are not all equivalent" or "(i) and (ii) and (iii) are equivalent, but (iv) is different" or "none are equivalent.")*
   (i) If it walks like a duck and it talks like a duck, then it is a duck.
   (ii) If it does not walk like a duck and it does not talk like a duck, then it is not a duck.
   (iii) If it is a duck, then it walks like a duck and it talks like a duck.
   (iv) It does not walk like a duck, or it does not talk like a duck, or it is a duck.

(2) Consider the equivalence $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$.
   (a) Prove the equivalence using a truth table.
   (b) Prove the equivalence using step-by-step replacement via the "Laws of propositional logic" (Table 1.5.1 in §1.5). *(Hint: first replace all occurrences of $\to$ using the Conditional Identity.)*

(3) Are the propositions $p \to (q \to r)$ and $(p \to q) \to r$ logically equivalent? (In other words, is there an associative law for "$\to$"?)

(4) Rewrite the proposition $p \vee \neg q \leftrightarrow r \vee q$ using only the operations $\neg$ and $\wedge$. (That is, find another proposition that is logically equivalent to the given one, but uses only the symbols $(,), p, q, r, \neg, \wedge$. Use Table 1.5.1.) Are there any compound propositions that cannot be rewritten this way (using just the propositional variables involved and the operations $\neg$ and $\wedge$)?

(5) Back to the island of the Honest and the Liars. Cole and Dot approach you. Cole says "If Dot is Honest, then I am a Liar." Can you determine which tribes the two are from?

(6) Complete the Challenge Activities from sections 1.2, 1.3, and 1.5. *(Of course, there is no writing component to this part.)*

...

"Then you should say what you mean," the March Hare went on.

"I do," Alice hastily replied; "at least–at least I mean what I say–that's the same thing, you know."

"Not the same thing a bit!" said the Hatter. "You might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"

"You might just as well say," added the March Hare, "that 'I like what I get' is the same thing as 'I get what I like'!"

"You might just as well say," added the Dormouse, who seemed to be talking in his sleep, "that 'I breathe when I sleep' is the same thing as 'I sleep when I breathe'!"

–*Alice in Wonderland*