Name: Solutions

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 12:45pm to complete this exam. When you are told to stop writing, do it or you will lose all points on the page(s) you write on.
- You may not communicate with other students during this test.
- Keep your eyes on your own paper.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctor.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers when appropriate.
- Before turning in your exam, check to make certain you've answered all the questions.

Question	Points	Score
1	12	
2	10	
3	6	
4	4	
5	5	
6	10	
7	5	
8	6	
9	12	
10	12	
11	12	
12	12	
13	20	
14	8	
15	16	
16	0	
Total:	150	

1. (12 points) Give an example of each of the following, or say that no such example exists.
   (a) a real number that is not rational π (or e, $\sqrt{2}$, $\sqrt{3}$, $0.0100100010000 1\cdots$, etc.)
   (b) an integer that is not a whole number -1 (or -2, -3, ...)
   (c) an irrational number that is not real none ("irrational" = real & not rational)
   (d) an integer that is not rational none (an integer n can be written $\frac{n}{1}$, which is obviously rational)
   (e) an expression that is not a real number $\frac{1}{0}$ (or $\sqrt{-1}$, etc.)
   (f) a rational number that is not an integer $\frac{1}{2}$ (or $\frac{3}{5}$, 0.1, etc.)

2. (10 points) Which properties of real numbers are demonstrated by each of the following equalities?
   (a) $1(0+x) = 1x$ 0 is the additive identity
   (b) $1(\pi + x) = (1\pi) + (1x)$ Distributivity
   (c) $1(\pi x) = \pi x$ 1 is the multiplicative identity
   (d) $1(\pi x) = (1\pi)x$ Associativity of multiplication
   (e) $1(\pi x) = 1(x \cdot \pi)$ Commutativity of multiplication

3. (6 points) Simplify $-2^2\left(5\left(3-2^2\right)+1\right)^2$.

$$= -2^2\left(5(3-4)+1\right)^2$$
$$= -2^2\left(5(-1)+1\right)^2$$
$$= -2^2\left(-5+1\right)^2$$
$$= -2^2\left(-4\right)^2$$
$$= -2^2 \cdot 16$$
$$= -4 \cdot 16 = -64$$

4. (4 points) Simplify $\left|\sqrt{13}-5\right|$. (Give an exact answer.)

$\sqrt{13} < 4$, so $\sqrt{13} - 5 < 0$, so

$$\left|\sqrt{13}-5\right| = -\left(\sqrt{13}-5\right)$$
$$= -\sqrt{13} + 5.$$

5. (5 points) Convert $\frac{7}{15}$ to decimal.

$$
\begin{array}{r}
0.466\ldots \\
15\,\overline{)\,7.000} \\
\underline{6\,0} \\
1\,0\,0 \\
\underline{9\,0} \\
100
\end{array}
$$

$0.4\overline{6}$

6. Convert each of the following to a fraction in least terms.

(a) (5 points) $0.\overline{042}$

$$x = 0.\overline{042}$$
$$1000x = 42.\overline{042}$$

$$\Rightarrow \quad 999x = 42$$

$$x = \frac{42}{999} \overset{\div 3}{\underset{\div 3}{=}} \boxed{\frac{14}{333}}$$

$14 = 2 \cdot 7$, neither 2 nor 7 divide 333 so this is in least terms

(b) (5 points) 0.012

$$= \frac{12}{1000} \overset{\div 4}{\underset{\div 4}{=}} \boxed{\frac{3}{250}}$$

7. (5 points) Find the least common multiple and greatest common divisor of the numbers 100 and 140. You may leave answers as products. Be sure to clearly label which is the lcm and which is the gcd!

$100 = 2^2 \cdot 5^2$ $\qquad$ $\text{lcm} = 2^2 \cdot 5^2 \cdot 7 = 700$

$140 = 2^2 \cdot 5 \cdot 7$ $\qquad$ $\text{gcd} = 2^2 \cdot 5 = 20$

8. (6 points) Find the least common multiple of the polynomials

$$4x^3y(x+2) \qquad \text{and} \qquad 6xy^2(x+2)(y-1).$$

Bonus: find the greatest common divisor also. (Again, clearly specify which is which.)

$\text{lcm} = 12x^3y^2(x+2)(y-1)$

$\text{gcd} = 2xy(x+2)$

9. (12 points) Simplify the following. (Write them as one fraction in lowest terms.)

(a) $\frac{1}{18} - \frac{11}{60}$

$$= \frac{1}{2\cdot 3^2}\cdot\frac{2\cdot 5}{2\cdot 5} - \frac{11}{2^2\cdot 3\cdot 5}\cdot\frac{3}{3}$$

$18 = 2\cdot 3^2$

$60 = 2^2\cdot 3\cdot 5$

$$= \frac{10}{2^2\cdot 3^2\cdot 5} - \frac{33}{2^2\cdot 3^2\cdot 5}$$

$\text{lcm} = 2^2\cdot 3^2\cdot 5$

$$= \frac{10-33}{2^2\cdot 3^2\cdot 5}$$

$$= \frac{-23}{2^2\cdot 3^2\cdot 5}$$ this is reduced

$$= -\frac{23}{180}$$

(b) $\frac{35}{18} \div \frac{7}{60}$

$$= \frac{35}{18}\cdot\frac{60}{7} = \frac{5\cdot\cancel{7}}{\cancel{6}\cdot 3}\cdot\frac{\cancel{6}\cdot 10}{\cancel{7}}$$

$$= \frac{50}{3}$$

10. (12 points) Which of the following are polynomials? Write "polynomial" or "not".

(a) $x^3 + \sqrt{2}x + \pi$ polynomial

(b) $x^3 + \sqrt{2x} + \pi$ not ($\sqrt{x}$ is not allowed)

(c) $x^2 + \sqrt{\pi}$ polynomial

(d) $x - \dfrac{1}{x}$ not ($\frac{1}{x}$ is not allowed)

11. (12 points) Expand and simplify the following. (Write them in standard form.)

(a) $(x^3 - 3x + 7) - (3x^2 - 5x + 6)$

$= x^3 - 3x^2 + 2x + 1$

(b) $(x^2 - x + 1)(x - 3)$

$= x^3 - 4x^2 + 4x - 3$

12. (12 points) Compute the following. (Write the answer as a polynomial plus a proper rational expression.)

(a) $(x^3 + 6x + 7) \div (x + 1)$

$$\begin{array}{r} x^2 - x + 7 \\ x+1 \overline{\big) x^3 + 0x^2 + 6x + 7} \\ -(x^3 + x^2) \\ \hline -x^2 + 6x + 7 \\ -(-x^2 - x) \\ \hline 7x + 7 \\ -(7x + 7) \\ \hline 0 \end{array}$$

$\boxed{x^2 - x + 7}$

(b) $(x^3 - x^2 + 5x - 35) \div (x - 3)$

$$\begin{array}{r} x^2 + 2x + 11 \\ x-3 \overline{\big) x^3 - x^2 + 5x - 35} \\ -(x^3 - 3x^2) \\ \hline 2x^2 + 5x - 35 \\ -(2x^2 - 6x) \\ \hline 11x - 35 \\ -(11x - 33) \\ \hline -2 \end{array}$$

$\boxed{x^2 + 2x + 11 - \frac{2}{x-3}}$

13. (20 points) Completely factor each of the following polynomials (over the integers).

(a) $a^2 - 7a + 10$

$A \cdot B = 10$
$A + B = -7$ $\Rightarrow -2, -5$

$\boxed{(a-2)(a-5)}$

(b) $4x^2 + 10x + 6$

$= 2(2x^2 + 5x + 3)$

$A \cdot B = 6$
$A + B = 5$ $\Rightarrow 2, 3$

$= 2(2x^2 + 2x + 3x + 3)$

$= 2(2x(x+1) + 3(x+1))$

$= \boxed{2(2x+3)(x+1)}$

(c) $8x^3 + 27$

$= (2x)^3 + (3)^3$

$= (2x+3)((2x)^2 - (2x)(3) + (3)^2)$

$= \boxed{(2x+3)(4x^2 - 6x + 9)}$

prime

(d) $x^4 - 16$

$= (x^2+4)(x^2-4)$

$= \boxed{(x^2+4)(x+2)(x-2)}$

prime

14. (8 points) It is true that $(2x^3 + 3x^2 - 11x - 6) \div (2x + 1) = (x^2 + x - 6)$. Use this fact to help completely factor the polynomial $2x^3 + 3x^2 - 11x - 6$.

$2x^3+3x^2-11x-6 = (2x+1)(x^2+x-6)$ from given fact

$= \boxed{(2x+1)(x+3)(x-2)}$

$A \cdot B = -6$, $A+B = 1$ $\Rightarrow +3, -2$

15. (16 points) Simplify the following. (Assume all expressions are nonzero.)

(a) $\dfrac{m+2}{m-4} \div \dfrac{5m^3+40}{m^2-m-12}$

$A \cdot B = -12$
$A + B = -1$ $\Rightarrow -4, +3$

$5m^3+40$
$= 5(m^3+8)$
$= 5(m+2)(m^2+2m+4)$

prime

$$= \frac{m+2}{m-4} \cdot \frac{m^2-m-12}{5m^3+40}$$

$$= \frac{\cancel{m+2}}{\cancel{m-4}} \cdot \frac{(\cancel{m-4})(m+3)}{5(\cancel{m+2})(m^2+2m+4)}$$

$$= \frac{m+3}{5(m^2+2m+4)}$$

(b) $\dfrac{20}{y^2-2y-24} - \dfrac{8}{y^2+y-12}$

$(y-6)(y+4)$ $(y+4)(y-3)$

$LCM = (y-6)(y+4)(y-3)$

$$= \frac{20}{(y-6)(y+4)} \cdot \frac{y-3}{y-3} - \frac{8}{(y+4)(y-3)} \cdot \frac{y-6}{y-6}$$

$$= \frac{20y-60}{(y-6)(y+4)(y-3)} - \frac{8y-48}{(y-6)(y+4)(y-3)}$$

$$= \frac{12y-12}{LCM}$$

$$= \boxed{\frac{12(y-1)}{(y-6)(y+4)(y-3)}}$$ reduced

16. Bonus: which of the following are properties of the real numbers? For each, either briefly justify why it is true (using known properties) or give an example to show that it is false.

(a) $a \div (b + c) = (a \div b) + (a \div c)$

False $a = b = c = 1$: Left $= \frac{1}{2}$
Right $= 2$

(b) $(a + b) \div c = (a \div c) + (b \div c)$

True : Left $= \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} =$ Right

(c) $a - (b + c) = (a - b) + c$

False : $a = b = c = 1$: Left $= -1$
Right $= 1$

(d) $a \div (b \div c) = (a \div b) \div c$

False $a = 4$, $b = c = 2$; Left $= 4$
Right $= 1$

(e) $a - (b + c) = (a - b) + (a - c)$

False $a = b = c = 1$: Left $= -1$
Right $= 0$

Scratch Paper - Do Not Remove