MATH 002 EXAM 2 (FORM B)

– SOLUTIONS –

Name:

1) (6pts each) Simplify the following exponential expressions. Finals answers should only contain **positive** exponents.

a) $(18^{-4}\cdot 7^{3})(6^{2}\cdot 14^{-2})$

$$= (2\cdot 3^2)^{-4}\cdot 7^3\cdot (2\cdot 3)^2\,(2\cdot 7)^{-2}$$

$$= 2^{-4}\,3^{-8}\,7^3\cdot 2^2\cdot 3^2\,2^{-2}\,7^{-2} = (2^{-4}\,2^2\,2^{-2})(3^{-8}\,3^2)(7^3\cdot 7^{-2})$$

$$= 2^{-4+2-2}\,3^{-8+2}\,7^{3-2} = 2^{-4}\,3^{-6}\,7$$

$$= \boxed{\frac{7}{2^4\,3^6}}$$

b) $-\frac{1}{3}x^{-4}y^{2}\cdot -\frac{12x^{4}}{y^0}\cdot -\frac{1}{2}x^{2}y^{-2} = \left(-\frac{1}{\cancel{3}}\cdot(-\cancel{12})\cdot\left(-\frac{1}{\cancel{2}}\right)\right)(x^{-4}x^{4}x^{2})(y^{2}y^{-2})$

$$= (-2)\,x^{-4+4+2}\,y^{2-2}$$

$$= (-2)\,x^2\,y^0 = \boxed{(-2)\,x^2}$$

$\boxed{y^0 = 1}$

c) $\dfrac{xy^{2}\cdot -x^{4}y^{-1}}{(2x^{-2}y^{-2})^{3}} = \dfrac{(x\cdot(-x^4))\,(y^2\cdot y^{-1})}{2^3\,x^{-6}\,y^{-6}} = \dfrac{-\,x^5\,y^1}{2^3\,x^6\,y^{-6}}$

$$= -\,\frac{x^{5-6}\,y^{1-(-6)}}{2^3}$$

$$= -\,\frac{x^{-1}\,y^7}{2^3} = \boxed{-\,\frac{y^7}{2^3\,x}}$$

d) $\left(\dfrac{2xy}{(x^3y^2\cdot 2x^2y^3)^{-4}}\right)^{-2} = \left(\dfrac{2xy}{(2x^5y^5)^{-4}}\right)^{-2} = \left(\dfrac{(2x^5y^5)^{-4}}{2xy}\right)^{+2}$

$$= \left(\frac{2^{-4}x^{-20}y^{-20}}{2\,xy}\right)^{2} = \left(2^{-4+1}\,x^{-20-1}\,y^{-20-1}\right)^{2}$$

$$= \left(2^{-3}x^{-21}y^{-21}\right)^{2} = 2^{-6}x^{-42}y^{-42}$$

$$= \boxed{\frac{1}{2^6x^{42}y^{42}}}$$

e) $\dfrac{(-1)^{27}+(-1+2)^{32}-1}{3^2\cdot 7}+\dfrac{200^0(2^3-(-1)^2)}{7^2} = \dfrac{-1+(1)^{32}-1}{63}+\dfrac{1\cdot(8-(+1))}{49}$

$$= \frac{-1+\cancel{1}-\cancel{1}}{63}+\frac{7}{49}$$

$$= \frac{-1}{63}+\frac{1}{7} \qquad CD=63$$

$$= \frac{-1}{63}+\frac{9}{63}=\boxed{\frac{8}{63}}$$

2) (3pts each) First decide whether each expression given below is a polynomial expression or not. If it is not polynomial explain all reasons why not. If it is a polynomial, give the number of term, the coefficients and the degree of the polynomial.

a) $\frac{2}{5}x^3+\frac{4}{3}x-2$ Polynomial, 3 terms, Coefficients: $\frac{2}{5}, \frac{4}{3}, -2$

Degree $=3$

b) $3^{-1}xy^3-\frac{2}{\sqrt{2}}xy^2-y$ Polynomial, 3 terms, Coefficients $= 3^{-1}, -\frac{2}{\sqrt{2}}, -1$

Degree $=4$

c) $-x^4+3xy^4+1$ Polynomial, 3 terms, Coefficients: $-1, 3, 1$

Degree $= 5$

3) (1.5pt each) T/F. If False either explain why or give the correct answer.

a) $(81)^{-3/4}=(-3)^{-3}$ False because $81=3^4$

$81^{-3/4}=(3^4)^{-3/4}=3^{4\cdot(-\frac{3}{4})}=3^{-3}$

$\neq(-3)^{-3}$

b) $\left(\frac{x^{-1}}{y^{-3}}\right)^{-2}=\left(\frac{y^3}{x}\right)^2$ False because $\left(\frac{x^{-1}}{y^{-3}}\right)^{-2}=\left(\frac{y^{-3}}{x^{-1}}\right)^{+2}=\left(\frac{x}{y^3}\right)^2\neq\left(\frac{y^3}{x}\right)^2$

c) $(x+7)^0=\frac{1}{(x+7)}$ False because $(x+7)^0=1$ but $\frac{1}{x+7}\neq 1$

d) $3^{11}\cdot(-3)^7=-3^{18}$ True because $(-3)^7=-3^7$

$3^{11}\cdot-3^7=-3^{11+7}=-3^{18}$

4) (6pts each) Perform the indicated operations for the following polynomials and simplify

a) $(\frac{x^2}{2}+\frac{x}{3}-5)-(\frac{2}{3}x^2-x+2)$ $= \left(\frac{x^2}{2}-\frac{2x^2}{3}\right)+\left(\frac{x}{3}+x\right)+(-5-2)$

$= \left(\frac{3x^2}{6}-\frac{4x^2}{6}\right)+\left(\frac{x+3x}{3}\right)+(-7)$

$= \boxed{-\frac{x^2}{6}+\frac{4x}{3}-7} = \boxed{-\frac{1}{6}x^2+\frac{4}{3}x-7}$

b) $(x+5)(x^2-5x+25)$ $= (x+5)x^2+(x+5)(-5x)+(x+5)25$

$= x^3+\cancel{5x^2}-\cancel{5x^2}-\cancel{25x}+\cancel{25x}+125$

$= \boxed{x^3+125}$

c) $(3x-5y)^2$ $= (3x)^2-2(3x)(5y)+(5y)^2$

$= \boxed{9x^2-30xy+25y^2}$

d) $(x^3-8)\div(x-2)$

$$x^2+2x+4$$

$$x-2\,\overline{\big)\,\cancel{x^3}+\_\_+\_\_-8}$$

$$-\quad(-)\cancel{x^3}\overset{(+)}{-}2x^2$$

$$2\cancel{x^2}+\_\_-8$$

$$-\quad(-)\cancel{2x^2}\overset{(+)}{-}4x$$

$$4x-8$$

$$-\quad(-)4x\overset{(+)}{-}8$$

$$0$$

$$4y^2+8y+9$$

e) $5y-6\,\overline{\big)\,20y^3+16y^2-3y-54}$

$$-\quad(-)\cancel{20y^3}\overset{(+)}{-}24y^2$$

$$40y^2-3y-54$$

$$-\quad(-)\cancel{40y^2}\overset{(+)}{-}48y$$

$$45y-54$$

$$-\quad(-)45y\overset{(+)}{-}54$$

$$0$$

5) **(5pts each)** Factor out the GCF of the following polynomial. (Just the GCF)

a) $24b^2x^3 - 56b^2x^2 + 40b^2x^6 = 8b^2x^2(3x^6 - 7 + 5x^4)$

b) $(5x-2y)(3x) + (2y-5x) = (5x-2y)(3x) - (5x-2y)$

$= \boxed{(5x-2y)(3x-1)}$

6) **(5pts each)** If possible factor out **completely** the following polynomials. If it is **not** possible write "Prime" or "Cannot be factored". **(Hint: Factor out the GCF first!!!)**

a) $v^2 + 7v - vw - 7w = (v^2 - vw) + (7v - 7w)$

$= v(v-w) + 7(v-w)$

$= \boxed{(v-w)(v+7)}$

b) $x^2 + 18x - 36$

$P \cdot Q = -36$

$P + Q = 18$

$(+36)(-1) = -36$	$36 - 1 = 35$ X
$18\ (-2) = -36$	$18 - 2 = 16$ X
$12\ (-3) = -36$	$12 - 3 = 9$ X
$9\ (-4) = -36$	$9 - 4 = 5$ X
$6 \cdot (-6) = -36$	$6 - 6 = 0$ X

This polynomial is $\boxed{\text{prime}}$

5) **(5pts each)** Factor out the GCF of the following polynomial. (Just the GCF)

a) $24b^2x^8 - 56b^2x^2 + 40b^2x^6$ $= \boxed{8b^2x^2(3x^6 - 7 + 5x^4)}$

b) $(5x-2y)(3x)+(2y-5x)$ $= (5x-2y)(3x) - (5x-2y)$

$= \boxed{(5x-2y)(3x-1)}$

6) **(5pts each)** If possible factor out **completely** the following polynomials. If it is **not** possible write "Prime" or "Cannot be factored". **(Hint: Factor out the GCF first!!!)**

a) $v^2 + 7v - vw - 7w$ $= (v^2 - vw) + (7v - 7w)$

$= v(v-w) + 7(v-w)$

$= \boxed{(v-w)(v+7)}$

b) $-3x^2 \cancel{-} 48x + 108$ $= -3(x^2 \cancel{+} 16x - 36)$

$= \boxed{-3(x+18)(x-2)}$

$PQ = -36$

$P + Q = 16$

$P = 18$

$Q = -2$