MATH 454
HOMEWORK 7 DUE MARCH 22

Name: ________________________________

- Refer to the syllabus regarding allowed collaboration on this homework assignment.
- Refer to other homework instructions and suggestions posted in Blackboard.
- All answers must be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

Turn in four of the following problems to be graded.

(1) (5.1.19) *spot the error in a false proof; see textbook for statement*
(2) (5.1.20) Let G be a graph whose odd cycles are pairwise intersecting, i.e. every two odd cycles have a common vertex. Prove that $\chi(G) \leq 5$.
(3) (5.1.24) Let G be any 20-regular 360-vertex graph with the following properties: the vertices are evenly spaced around a circle; vertices separated by 1 or 2 degrees (along the circle) are nonadjacent, while vertices separated by 3, 4, 5, or 6 degrees are adjacent. (Vertices separated by more than 6 degrees may or may not be adjacent, subject to the property that G is 20-regular.) Prove that $\chi(G) \leq 19$. *(Remark: Brooks's Theorem gives* $\chi(G) \leq 20$*. Hint: color the vertices successively along the circle.)*
(4) (5.1.34) For all k, construct a tree T_k with maximum degree k and an ordering σ of $V(T_k)$ such that the greedy coloring relative to σ uses $k+1$ colors. *(Hint: use induction on* k*, and construct the tree and the ordering together.)*
(5) (5.1.31) Prove that G is m-colorable if and only if $G \,\square\, K_m$ has an independent set of size $|V(G)|$.
(6) (8.4.21) Prove that $K_{k,m}$ is k-choosable if and only if $m < k^k$.