Worksheet 6 February 9, 2011

1. Compute $\lim_{x\to 4} \frac{x^2-2x-3}{x-3}$.

2. Compute $\lim_{x\to 3} \frac{x^2-2x-3}{x-3}$.

3. Compute $\lim_{x\to 3} \frac{x^2-2x-2}{x-3}$.

4. The following expressions are definitely *not* numbers. But they make sense in terms of limits; evaluate those that can be evaluated, and indicate why the others are indeterminate (that is, find two functions that give that limit form, but which have different actual limits). c is a nonzero real number.

   $$\infty-\infty \qquad \infty-c \qquad \frac{c}{0} \qquad \frac{0}{0} \qquad 0\cdot\infty \qquad c\cdot\infty \qquad \frac{c}{\infty} \qquad \frac{0}{\infty} \qquad \frac{\infty}{0} \qquad \frac{\infty}{\infty}$$

5. What is the definition of a function being *continuous* at a point? The definition involves a single equation, but this equation has three inherent conditions; what are they? Draw three graphs of functions: each function should fail exactly one of these three conditions.

6. Is it true that you were once exactly three feet tall? Explain using your height as a function of time since birth.

7. Show that at some point in your life so far, your weight in pounds was equal to your height in inches. (Hint: remember that the difference of continuous functions is continuous.)

8. In the game Sunday, Green Bay ended with 31 points; is it true that they must have had 15 points at some time in the game? Why is this different than the previous two problems? (Less-math question: is it possible they had 15 points at some time? Is it likely?)

9. Determine the value of $\lim_{x\to 0} x^2 \sin\left(\frac{x^{23}-\arcsin x+\ln|x|}{\pi x^3}\right)$.

10. We define the *greatest integer function*, or *floor function*, by $\lfloor x\rfloor =$ the greatest integer $\leq x$. (Note: $\lfloor -3.5\rfloor \neq -3$. Also, your book uses the notation $[[x]]$.)

    (a) Plot the graph of $\lfloor \cos x\rfloor$ on $[-\pi,\pi]$. Based on the graph, find $\lim_{x\to 0}\lfloor \cos x\rfloor$. What is $\lfloor \cos 0\rfloor$?

    (b) Evaluate $\lim_{x\to 0}(\lfloor x\rfloor + \lfloor -x\rfloor)$.

11. Determine the following limits. You should use algebraic tricks to put the expressions into forms that are not indeterminate.

    (a) $\lim_{x\to\infty} \frac{x^3-2x^2+\pi x-1}{5x^4+2x^3-ex+7}$

    (b) $\lim_{x\to\infty} \frac{x^3-2x^2+\pi x-1}{7x^2-1}$

    (c) $\lim_{x\to\infty} \frac{x^3-2x^2+\pi x-1}{3x^3-22x+\ln(2)}$

(d) $\lim_{x\to 9} \dfrac{x-9}{\sqrt{2x+7}-5}$

(e) $\lim_{x\to 9} \dfrac{x-10}{\sqrt{2x+7}-5}$

12. Suppose a state's income tax code states that tax liability is 12% on the first \$20,000 of taxable earnings and 16% on the remainder. Write down a piecewise defined function $T(x)$ that captures this situation. Check that $\lim_{x\to 0^+} T(x) = 0$ and $\lim_{x\to 20000} T(x)$ exists. Why is it important that these two facts be true?

13. Use the Intermediate Value Theorem (and appropriate guesses) to estimate the root of $x^3 - x - 1$, accurate to one decimal place. (This will take a bit of computation; after your initial "large-scale" guesses, use $x^3 - x = x(x^2 - 1)$ to make it slightly easier.)