**Math 241 C8** **Name:**

**Quiz # 6**

April 10, 2013

No electronic devices, notes, or interpersonal communication allowed.

Show work to get credit.

(1) [10pts] Find the volume of the region bounded by the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$.

Volume $= \iiint 1 \, dx\,dy\,dz$.

Use spherical coordinates;

volume conversion is $\rho^2 \sin\varphi$

"Inside" cone $\Rightarrow 0 \leq \varphi \leq \frac{\pi}{4}$

from HW, or $z = \sqrt{x^2+y^2}$

$\rightarrow \rho\cos\varphi = \sqrt{\rho^2 \sin^2\varphi \cos^2\theta + \rho^2 \sin^2\varphi s}$

$\Rightarrow \rho\cos\varphi = \rho\sin\varphi$

$\Rightarrow \cos\varphi = \sin\varphi$

$\Rightarrow \varphi = \frac{\pi}{4}$

"Inside" sphere $\Rightarrow 0 \leq \rho \leq 2$

$0 \leq \theta \leq 2\pi$


So volume $= \int_0^{2\pi} \int_0^{\pi/4} \int_0^{2} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$

$= \frac{8}{3} \int_0^{2\pi} \int_0^{\pi/4} \sin\varphi \, d\varphi \, d\theta$

$= \boxed{\frac{8}{3}\left(1 - \frac{\sqrt{2}}{2}\right)(2\pi)}$

(2) [10pts] Rewrite ~~Compute~~ $\iiint_R z\,dx\,dy\,dz$ as an integral over a box (i.e. with constant bounds), where $R$ is the region bounded by the planes $z = 2x-1$, $z = 2x+3$, $y = x$, $y = x+1$, $y = 2x+2$, and $y = 2x+5$.

Let

$$u = z - 2x \qquad u \in [-1, 3]$$
$$v = y - x \qquad v \in [0, 1]$$
$$w = y - 2x \qquad w \in [2, 5]$$

Then

$$v - w = x$$
$$-w + 2v = y$$
$$z = u + 2x = u + v - w$$

$$J = \begin{array}{cc} & \begin{array}{ccc} u & v & w \end{array} \\ \begin{array}{c} x \\ y \\ z \end{array} & \begin{vmatrix} 0 & 1 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & -1 \end{vmatrix} \end{array} = 0 - 0 + 1 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = 1.$$

expand along column 1

So

$$\iiint_R z\,dx\,dy\,dz = \boxed{\int_2^5 \int_0^1 \int_{-1}^3 (u+v-w)(1)\,du\,dv\,dw.}$$