Math 241 X8 **Name:** Solutions

Quiz # 4

October 10, 2013 No electronic devices or interpersonal communication allowed. Show work to get credit.

1) Compute directly the net flow of $\vec{F}(x, y) = \langle 2e^{4x^2}, e^{y^2} \rangle$ along the line segment going from $(2, -4)$ to $(-1, 2)$.

line segment can be parametrized as $\ell(t) = (1-t)\langle 2,-4\rangle + t\langle -1,2\rangle,\ t \in [0,1]$

$$\Rightarrow x(t) = 2-3t$$
$$y(t) = -4+6t$$

$$\text{flow along} = \int_C \vec{F} \cdot \langle dx, dy \rangle$$
$$= \int_0^1 \langle 2e^{4(2-3t)^2}, e^{(-4+6t)^2} \rangle \cdot \langle -3, 6 \rangle \, dt$$
$$= \int_0^1 \left(-6e^{4(4-12t+9t^2)} + 6e^{(16-48t+36t^2)}\right) dt$$
$$= \int_0^1 0 \, dt$$
$$= \boxed{0.}$$

2) Find the flow of $\vec{F}$ along the curve $y = 2 + 6\cos(\pi x/2)$, running x from -1 to 2.

$(-1, 2)$ $(2,-4)$

$\partial_x(e^{y^2}) = 0 = \partial_y(2e^{4x^2})$, and $\vec{F}$ has no singularities,

so $\vec{F}$ is conservative.

Since conservative vector fields have path-independent flow-along integrals,

this flow is the same as in (1), but reversed

$$-0 = \boxed{0}$$

3) Find a potential function for $\vec{G}(x,y) = \langle 2x^3e^y + \sin x, \frac{1}{2}x^4e^y + y^2 + 1\rangle$.

If $\nabla f = \vec{G}$, then

$$\partial_x f = 2x^3e^y + \sin x$$

$$\Rightarrow \quad f = \frac{1}{2}x^4e^y - \cos x + h(y)$$

$$\Downarrow$$

$$\frac{1}{2}x^4e^y + y^2 + 1 = \partial_y f = \frac{1}{2}x^4e^y - 0 + h'(y)$$

$$\Rightarrow \quad h'(y) = y^2 + 1$$

$$\Rightarrow \quad h(y) = \frac{1}{3}y^3 + y + C$$

So $\boxed{f(x,y) = \frac{1}{2}x^4e^y - \cos x + \frac{1}{3}y^3 + y}$ is a potential function for $\vec{G}$.

4) Find the net flow of $\vec{G}$ along the circle $x^2 + y^2 = 5$.

By (3), $\vec{G}$ is conservative, (and no singularities) so the flow along a closed curve is $\boxed{0}$.