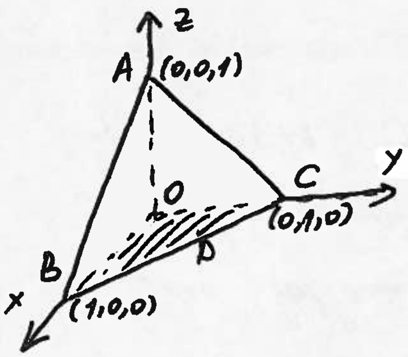
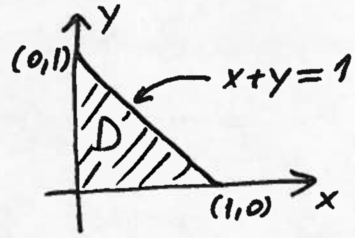
WORKSHEET SOLUTIONS (11/13/12)

① (a) The x, y, respectively z-intercepts of the plane $x+y+z=1$ are $(x,y,z)=(1,0,0)$, $(x,y,z)=(0,1,0)$, respectively $(0,0,1)$.



(b)



$$D=\{(x,y)\mid x,y\geq 0,\ x+y\leq 1\}$$
$$=\{(x,y)\mid 0\leq x\leq 1,\ 0\leq y\leq 1-x\}.$$

Taking $x=u$, $y=v$ we find $z=1-u-v$ and

$$\vec{r}:D\longrightarrow\mathbb{R}^3,\quad \vec{r}(u,v)=(u,v,1-u-v)$$

(c) $\vec{r}_u=\langle 1,0,-1\rangle$, $\quad\vec{r}_v=\langle 0,1,-1\rangle$

$$\vec{r}_u\times\vec{r}_v=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&0&-1\\0&1&-1\end{vmatrix}=\vec{i}+\vec{j}+\vec{k}\quad;\quad|\vec{r}_u\times\vec{r}_v|=\sqrt{3}$$

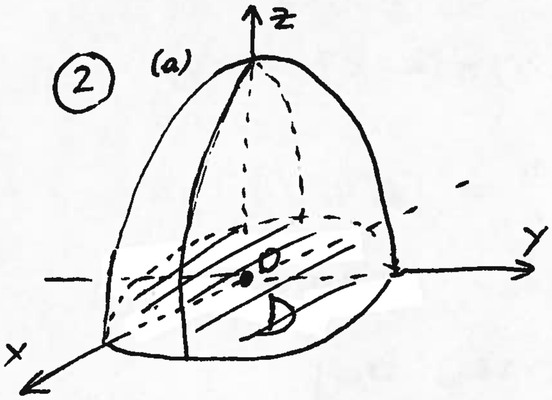
$$\text{Area}(S)=\iint_D\sqrt{3}\,dA=\sqrt{3}\ \text{Area}(D)=\sqrt{3}\cdot\frac{1}{2}=\frac{\sqrt{3}}{2}$$

(d) $\vec{AB}=\langle 1,0,-1\rangle$, $\quad\vec{AC}=\langle 0,1,-1\rangle$

$$\vec{AB}\times\vec{AC}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&0&-1\\0&1&-1\end{vmatrix}=\vec{i}+\vec{j}+\vec{k}$$

$$\text{Area}(S)=\text{Area}(\triangle ABC)=\frac{1}{2}|\vec{AB}\times\vec{AC}|=\frac{1}{2}\sqrt{3}$$

② (a)



(b) Since $z=1-x^2-y^2$,

$$z\geq 0\iff x^2+y^2\leq 1.$$

$\vec{r}:D\longrightarrow\mathbb{R}^3$ where

$D=\{(x,y)\mid x^2+y^2\leq 1\}$, $\quad x=u,\ y=v$

$$\vec{r}(u,v)=(u,v,1-u^2-v^2)$$

Alternatively one could parameterize using polar coordinates as:

$\vec{r} : [0,1] \times [0,2\pi] \longrightarrow \mathbb{R}^3$, $\quad \vec{r}(r,\theta) = (x,y,z)$ with

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 1 - r^2. \end{cases}$$

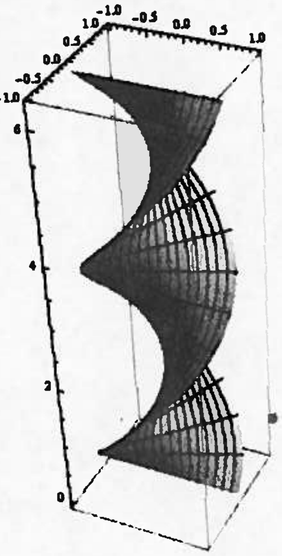
3 (a) $\vec{r}(L) = \{\vec{r}(0,v) \mid v \in [0,2\pi]\} = \{(0,0,v) \mid v \in [0,2\pi]\}$

is a vertical segment of length 2π through $(0,0,0)$.

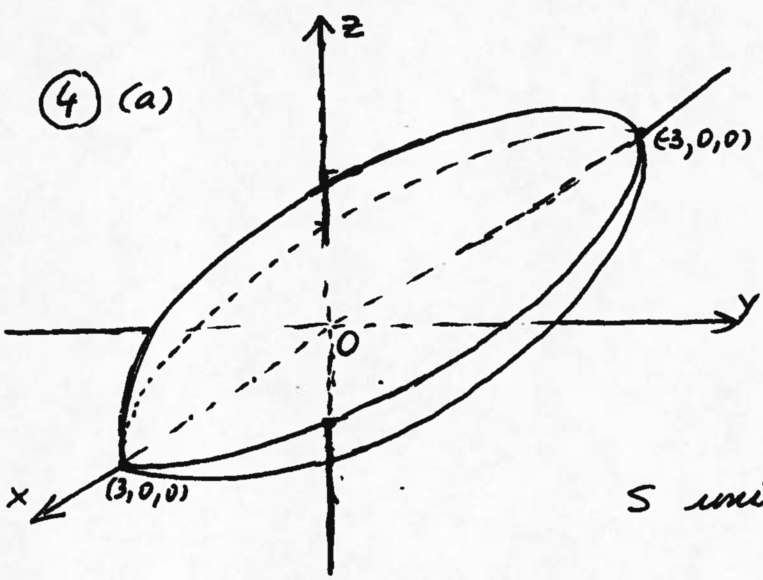
(b) $\vec{r}\{(-1,v) \mid v \in [0,2\pi]\} = \{(-\cos v, -\sin v, v) \mid v \in [0,2\pi]\}$ is a helix

$\vec{r}\{(1,v) \mid v \in [0,2\pi]\} = \{(\cos v, \sin v, v) \mid v \in [0,2\pi]\}$ is also a helix, with the opposite orientation.

(c) S looks like a "ribbon".



4 (a)



S unit sphere $x^2+y^2+z^2=1$

(b) $[0,2\pi] \times [0,\pi] \longrightarrow S \longrightarrow E : \left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 + z^2 = 1$

$(\theta,\varphi) \longrightarrow (\sin\varphi\cos\theta, \sin\varphi\sin\theta, \cos\varphi)$

$(x,y,z) \longrightarrow (3x, 2y, z)$

$\vec{r} : [0,2\pi] \times [0,\pi] \longrightarrow E$

$\vec{r}(\theta,\varphi) = (x,y,z)$ with $\begin{cases} x = 3\sin\varphi\cos\theta \\ y = 2\sin\varphi\sin\theta \\ z = \cos\varphi. \end{cases}$