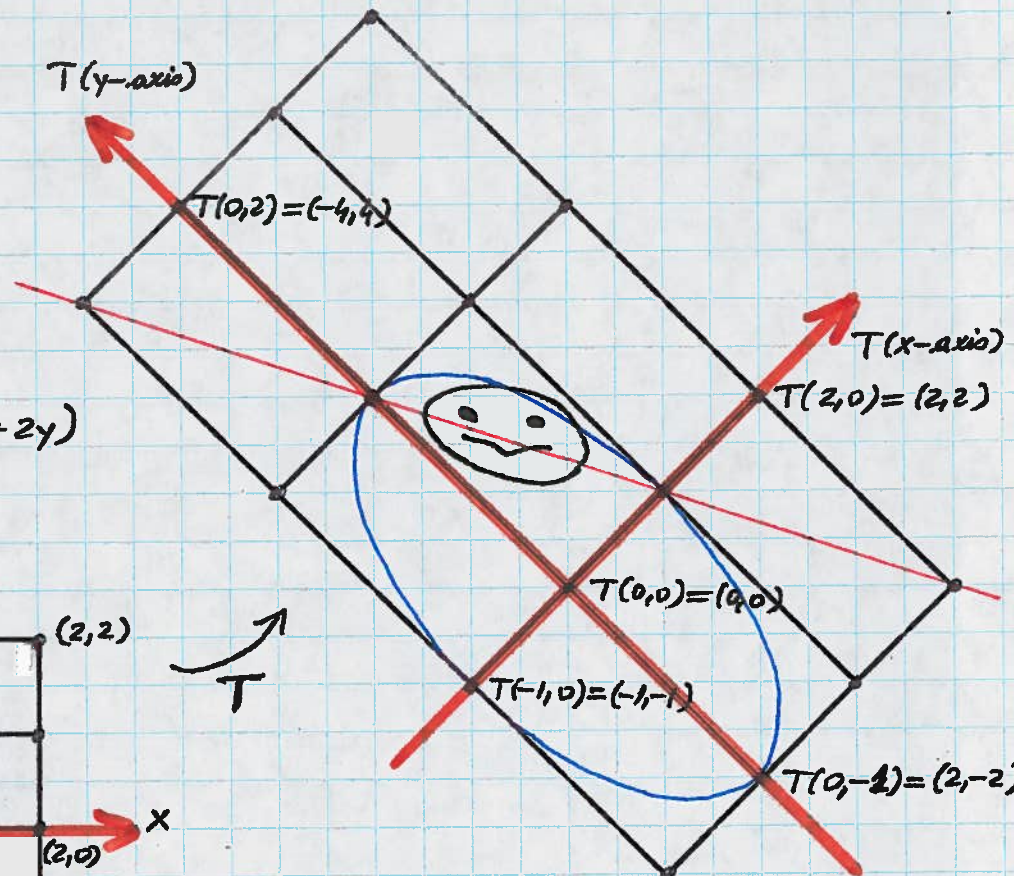
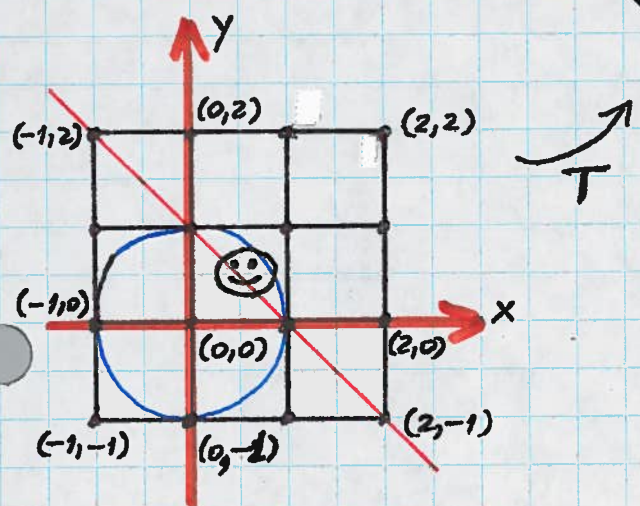
WORKSHEET SOLUTIONS (11/06/12)



$T(x,y) = (x-2y, x+2y)$



$T(x,0) = (x,x) \Longrightarrow T(\text{x-axis}) = \text{the line } y = x$

$T(0,y) = (-2y, 2y) \Longrightarrow T(\text{y-axis}) = \text{the line } y = -x.$

$T(x, 1-x) = (x-2(1-x), x+2(1-x)) = (3x-2, -x+2) = (-2,2) + x\,(3,-1)$

the line of direction $(3,-1)$ through $(-2,2)$.

The circle C given by $x^2+y^2=1$ can be parameterized as

$x = \cos t$, $y = \sin t$, $t \in [0, 2\pi]$.

$$T(\cos t, \sin t) = (\underbrace{\cos t - 2\sin t}_{u}, \underbrace{\cos t + 2\sin t}_{v})$$

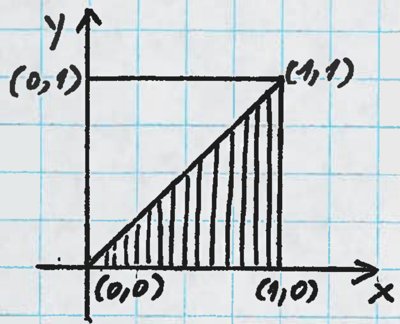
Eliminate t:
$$\left.\begin{array}{l} u+v = 2\cos t \iff \cos t = \dfrac{u+v}{2} \\ u-v = -4\sin t \iff -\sin t = \dfrac{u-v}{4} \end{array}\right\}$$

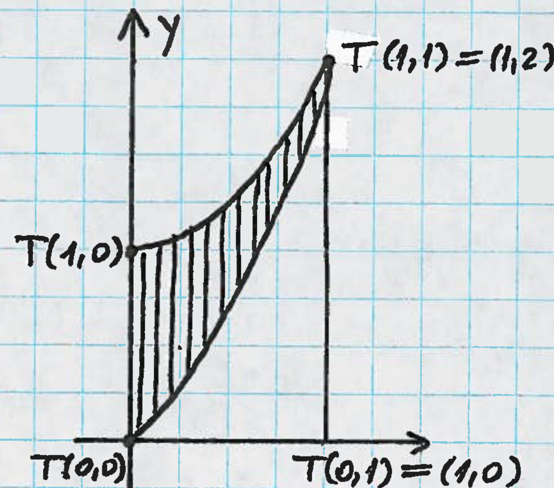
$$\cos^2 t + \sin^2 t = 1 \iff \left(\frac{u+v}{2}\right)^2 + \left(\frac{v-u}{4}\right)^2 = 1$$

$$\iff 4(u+v)^2 + (v-u)^2 = 16 \iff 5u^2 + 6uv + 5v^2 = 16$$

ellipse with center at $(0,0)$ through points $(2,-2), (1,1), (-2,2), (-1,1)$.

② $T(x,y) = (y, x(1+y^2))$





$T(x,0) = (0,x)$

$T(1,y) = (y, 1+y^2)$

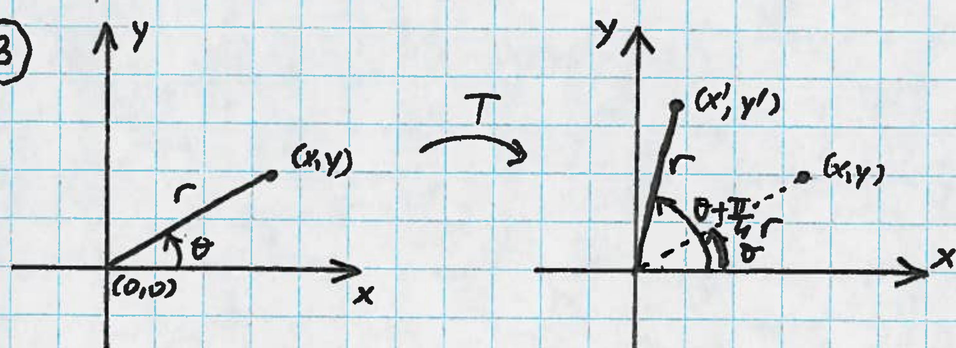
$T(x,1) = (1, 2x)$

$T(0,y) = (y, 0)$

$T(x,x) = (x, x+x^3)$

$x, y \in [0,1]$

③



(a) In polar coordinates $(r,\theta) \longrightarrow (r, \theta + \frac{\pi}{4})$

(b) $(x,y) \longrightarrow (r,\theta) \longrightarrow (r, \theta+\frac{\pi}{4}) \longrightarrow (x' = r\cos(\theta+\frac{\pi}{4}), y' = r\sin(\theta+\frac{\pi}{4}))$

$$\cos\left(\theta+\frac{\pi}{4}\right) = \cos\theta\cos\frac{\pi}{4} - \sin\theta\sin\frac{\pi}{4} = \frac{\cos\theta - \sin\theta}{\sqrt{2}} \Longrightarrow r\cos\left(\theta+\frac{\pi}{4}\right) = \frac{x-y}{\sqrt{2}}$$

$$\sin\left(\theta+\frac{\pi}{4}\right) = \sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4} = \frac{\cos\theta+\sin\theta}{\sqrt{2}} \longrightarrow r\sin\left(\theta+\frac{\pi}{4}\right) = \frac{x+y}{\sqrt{2}}$$

$$T(x,y) = \left(\frac{x-y}{\sqrt{2}}, \frac{x+y}{\sqrt{2}}\right).$$