QUIZ 1: CHAPTER 1 FEBRUARY 9

Name: Solutions

- All answers should be fully justified.
- Complete this quiz without any aids, including the text or your peers.

(1) Prove that $p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$ using the table of common logical equivalences.

$$\begin{aligned} p \to (q \wedge r) &\equiv \neg p \vee (q \wedge r) && \text{Conditional} \\ &\equiv (\neg p \vee q) \wedge (\neg p \vee r) && \text{Distributive} \\ &\equiv (p \to q) \wedge (p \to r) && \text{Conditional } (\times 2) \end{aligned}$$

(2) Prove that $(p \wedge q) \to r \not\equiv (p \to r) \wedge (q \to r)$.

$p$: T

$q$: F

$r$: F

make $\underbrace{(\underbrace{p \wedge q}_{F}) \to r}_{\substack{T \\ \text{(vacuously)}}}$ but $\underbrace{(\underbrace{p \to r}_{F}) \wedge (\underbrace{q \to r}_{T})}_{F}$

(3) Determine whether each of the following statements are true or false in each of the given domains. Give brief justifications.

| Domain | $\exists x\ \forall y\ x \leq y$ | $\forall y\ \exists x\ x \leq y$ |
|---|---|---|
| $\mathbb{N}$ $=\{0,1,2,\cdots\}$ | T w/ x=0, $\forall y$ $0 \leq y$ is true in $\mathbb{N}$ | T Given y, picking x=y satisfies $x \leq y$ |
| $\mathbb{Z}$ | F no matter what x is, some integers are less. (There is no "smallest integer.") | T same |

(4) Complete the proof below that the following argument is valid.

$$\begin{array}{lr} \forall x\ (P(x) \to Q(x)) & (a) \\ \exists x\ \neg Q(x) & (b) \\ \hline \therefore\ \exists x\ \neg P(x) & \end{array}$$

| | |
|---|---|
| 1. $\exists x\ \neg Q(x)$ | Hypothesis (b) |
| 2. ($c$ is a particular element)$\wedge\neg Q(c)$ | Existential Instantiation, (1) |
| 3. $\neg Q(c)$ | Simplification, (2) |
| 4. $\forall x\ (P(x) \to Q(x))$ | Hypothesis (a) |
| 5. $c$ is a particular element | Simplification, (2) |
| 6. $P(c) \to Q(c)$ | Universal Instantiation, (4) & (5) |
| 7. $\neg P(c)$ | Modus Tollens using statement numbers ... (3) & (6) |
| 8. $\exists x\ \neg P(x)$ | Existential Generalization, (5) & (7) |