Worksheet 20 April 6, 2011

1. Use properties of the integral together with the FTC to evaluate the derivative with respect to x of

   (a) $\displaystyle\int_1^x e^{t^4}\,dt$

   (b) $\displaystyle\int_{-4}^{x^2} e^{t^4}\,dt$

   (c) $\displaystyle\int_x^{-2} e^{t^4}\,dt$

   (d) $\displaystyle\int_{x^2}^{x^3} e^{t^4}\,dt$

   (Hint: there isn't a nice way to write down a formula for an antiderivative of e^{t^4}, so don't try.)

2. Suppose you know the value of the integral $\displaystyle\int_a^b f(x)\,dx = I$. By considering graphs, evaluate $\displaystyle\int_{a/2}^{b/2} f(2x)\,dx$. Check that you get the same answer by substitution.

3. We saw before the antiderivative of $2x/(1+x^2)$. Compute this again using the formalisms of substitution.

4. Compute the following integrals:

   (a) $\displaystyle\int (2x+7)^{22}\,dx$

   (b) $\displaystyle\int_1^4 \frac{2x^3}{1+x^4}\,dx$

   (c) $\displaystyle\int_1^4 \frac{2x}{1+x^4}\,dx$

   (d) $\displaystyle\int_0^{\pi^2} t\sin(t^2)\,dt$

   (e) $\displaystyle\int \frac{1}{1+16x^2}\,dx$

   (f) $\displaystyle\int_{-1}^1 xe^{-x^2}\,dx$

   (g) $\displaystyle\int \frac{1}{x\ln x}\,dx$

5. Define $A(x)$ to be the area beneath the graph of $f(t) = \lfloor t \rfloor$ between $t = 0$ and $t = x$. Find an explicit formula for $A(x)$ for $0 \le x \le 6$. (Your formula will need to be piecewise.) Where is this function continuous? Where is it differentiable, and what is $A'(x)$ where it's defined? Notice in particular that $A'(x)$ is *not* just $f(x)$; why doesn't this contradict the FTC?

6. Compute $\lim_{n\to\infty} \frac{\sqrt[3]{1}+\sqrt[3]{2}+\sqrt[3]{3}+\cdots+\sqrt[3]{n}}{n^{4/3}}$ by evaluating an integral of the form $\int_0^1 f(x)\,dx$.

7. Not every function is integrable. Suppose we want to compute $\int_0^1 \chi(x)\,dx$, where

$$\chi(x) = \begin{cases} 0 & \text{if } x \text{ irrational} \\ 1 & \text{if } x \text{ rational.} \end{cases}$$

By choosing appropriate sample points x_i^*, show that the Riemann sums can always be made to be 0, but can also be made to be 1.