Tuesday, September 11 ** *Multivariate functions: Introduction*

1. Consider the function $f(x, y) = x^2 + 3y^2$. Describe its level sets. Describe its graph.

2. Consider the function $f(x, y) = \sqrt{x^2 + 3y^2}$. Describe its level sets. How does this differ from (1)?

3. Consider the function $f(x) = x^2 - 9$. Describe its level sets. Sketch its graph, and highlight one nonempty level set.

4. Consider the function $f(x, y, z) = x^2 + 2y^2 + 5z^2 - 3$. Describe its level sets. How many dimensions does its graph occupy?

5. Consider the function $f(x, y) = x^2 - y^2 + 10$. Describe its level sets. Graph several of these level sets. Describe the graph of f.

6. Suppose we want to find local maxima and minima of a multivariate function. What must the level sets look like near such points?

7. Imagine you're given a contour map of a function $f(x, y)$. If $f(x, y)$ represents the altitude at the point (x, y), and you're standing on one of the level curves, which direction is the flattest walk? Which direction is the steepest walk?