

Math 241 X8

Name(s):

Homework 10 supplement

This is a written homework supplement to the homework for Units 10 and 11: 3D Transformations and Spherical Coordinates.

- (1) For each of the 6 ways to slice up the region (6 orders of integration), write a triple integral that represents the volume of the region bounded by the surfaces $y = x^2$, $z = 0$, and $y + 2z = 4$. (At least try to sketch the region by hand first; you may use Mathematica to make sure you're thinking of the correct region.)

- (2) Sometimes you can get away with less work than the “standard approach” to transformations. Consider the 3D region that is bounded by the planes $x + y + 2z = -1$, $x + y + 2z = 5$, $x - y - 2z = -3$, $x - y - 2z = 6$, and the surfaces $\sinh(x) - y + 2z = -1$ and $\sinh(x) - y + 2z = 2$. You should agree that this begs for the transformation

$$u = x + y + 2z, \quad v = x - y - 2z, \quad w = \sinh(x) - y + 2z.$$

Now the usual next step is to solve for x, y, z in terms of u, v, w , but *we don't have to do this here*. Instead, compute the volume conversion factor $V_{uvw}(x, y, z)$. Then with a little bit of common sense, conclude what $V_{xyz}(u, v, w)$ is. With that in hand, you're ready to complete the computation: do it. (The derivative of $\sinh(x)$ is $\cosh(x)$.)

- (3) Quickie: compute $\iiint_R z \, dV$ where R is the portion of the unit ball in the first octant.