QUIZ 2: CHAPTER 2 FEBRUARY 21

Name: _____

- All answers should be fully justified.
- Complete this quiz without any aids, including the text or your peers.

(1) Consider the statement "if G is connected and every vertex in G is even, then G is Eulerian." (The domain for G is the set of all graphs.)

In each proof technique, write down what you assume to be true and what you then must show in order to prove the statement. *[You are not expected to know what the statement means, and therefore are not asked to complete a proof the statement.]*

(a) **Direct proof**

Assume that ...

G is (an arbitrary graph that is) connected and every vertex in G is even.

Our goal is ...

show G is Eulerian

(b) **Proof of the contrapositive**

Assume that ...

G (is an arbitrary graph that) is not Eulerian.

Our goal is ...

show that G is not connected OR some vertex of G is odd.

(c) **Proof by contradiction**

Assume that ...

G (is an arbitrary graph ~~=~~ that) is connected, every vertex of G is even, and G is not Eulerian.

Our goal is ...

to derive a contradiction.

(2) Prove that the sum of a rational number and an irrational number is always irrational.

(In other words: $\forall x \forall y \, (x \in \mathbb{Q} \wedge y \notin \mathbb{Q} \rightarrow x+y \notin \mathbb{Q})$.)

Proof (by contradiction):

Suppose $x, y \in \mathbb{R}$, $x \in \mathbb{Q}$, $y \notin \mathbb{Q}$, and $x+y \in \mathbb{Q}$.

Then $x = \frac{a}{b}$, $x+y = \frac{c}{d}$ for some $a, b, c, d \in \mathbb{Z}$, $b, d \neq 0$.

Hence $y = x+y-x = \frac{c}{d} - \frac{a}{b} = \frac{bc-ad}{bd}$,

and $bc-ad$, $bd \in \mathbb{Z}$, $bd \neq 0$,

i.e. $y \in \mathbb{Q}$, contradicting our assumption. $\square$