Worksheet April 4, 2014

1. Sketch a picture showing the meaning of $\text{proj}_V w$. (What kind of objects are V and w?)

2. Explain how the picture from (1) relates to least-squares solutions to $Ax = b$. (How do V, w relate to A, b?)

3. Consider the equation $Ax = b$, the least-squares equation $A\widehat{x} = \widehat{b}$, and the corresponding "normal equation" $A^T A\widehat{x} = A^T b$.

   (a) If $Ax = b$ has a unique solution, what can you say about solutions to the normal equation?

   (b) If $Ax = b$ has infinitely many solutions, what can you say about solutions to the normal equation?

   (c) If $Ax = b$ has no solution, then does the normal equation? If so, describe them.

4. Suppose I try to find the projection of $v = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ onto $W = \text{Col}\left(\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\right)$ as follows:

   Let $w_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $w_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Then

   $$\begin{aligned} \text{proj}_W v &= \text{proj}_{\text{Span}(w_1)} v + \text{proj}_{\text{Span}(w_2)} v \\ &= \text{proj}_{w_1} v + \text{proj}_{w_2} v \\ &= \frac{w_1 \cdot v}{w_1 \cdot w_1} w_1 + \frac{w_2 \cdot v}{w_2 \cdot w_2} w_2 \\ &= 4w_1 + 2.5w_2 \\ &= \begin{bmatrix} 6.5 \\ 2.5 \\ 0 \end{bmatrix}. \end{aligned}$$

   What's another name for W? Sketch a picture of v and $\text{proj}_W v$; you should notice that something doesn't look right. What went wrong? Fix it.

5. You track the energy output of a certain reaction at 1 second time intervals, and get the following data.

t	0	1	2	3	4	5	6	7	9
e	2	1.4	1.3	50	1	1.2	1.2	1.5	1.9

Sketch the data. Discuss time $t = 3$. After deciding how to deal with $t = 3$, decide on an appropriate fitting function (linear, quadratic, exponential), and find the best-fit function of your chosen type. (Set up the computation as a matrix equation by hand. You may use a calculator or computer to solve the matrix equation.)