Worksheet April 11, 2014

1. Apply Gram-Schmidt to the (ordered) basis

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ -14 \\ -2 \\ 2 \end{bmatrix} \right\}$$

in order to find an orthonormal basis for the same vector space.

2. If I just ask you for an orthonormal basis for $\mathrm{Col}\left(\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 5 \\ -1 & -2 & 1 & 0 \end{bmatrix} \right)$ (without specifying how to do it), it's too easy if you just stop and think. Why?

3. Find the QR decomposition of $A = \begin{bmatrix} 1 & 2 & 6 \\ 0 & 1 & -14 \\ -1 & 2 & -2 \\ 2 & 3 & 2 \end{bmatrix}$.

4. Find the least-squares solution to $\begin{bmatrix} 1 & 2 & 6 \\ 0 & 1 & -14 \\ -1 & 2 & -2 \\ 2 & 3 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. (Use your answers to the previous questions.)

5. What's the projection of the zero vector onto a nonzero vector? What's the projection of a nonzero vector onto the zero vector?

6. Consider the following 2-player game. You and your friend take turns entering a number into an $n \times n$ array that is initially empty; you start. When all the spots are filled, you win if the resulting matrix has an inverse, otherwise your friend wins. If n is even, who "should" win? (There's a theorem that says, given a game like this with no hidden information and equal moves available to each player and no draws, one of the players has a strategy by which they can always win. I'm asking you to find such a strategy for one of the players.)