# Worksheet 5 February 7, 2011

1. Suppose Ben Roethlisberger's position is given by $p(t) = 2t + 5$. What is his velocity at time $t = 3$?
2. Suppose his position is given by a function $f(t)$. How can you estimate his velocity at time $t = 3$?
3. Let's say Ben has position function $t^2$. Write an explicit formula that gives Ben's average velocity in the time interval $[3, 3 + h]$ for any $h \neq 0$ (do you need to treat positive and negative $h$ differently?). An old paradox says that velocity *at a given moment* cannot really exist, since velocity is defined by distance per time, and these are both zero in an instant. Explain how you might give meaning to Ben's "instantaneous velocity" at $t = 3$ given the formula you have derived.

Recall the special limit $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

4. Do the limits $\lim_{x \to 0^+} \ln \sin x$ or $\lim_{x \to 0^+} \ln x$ exist? What about $\lim_{x \to 0^+} (\ln \sin x - \ln x)$?
5. Determine $\lim_{x \to \infty} x \sin \frac{1}{x}$.
6. Determine the limits $\lim_{x \to 0} \frac{\sin(3x)}{x}$ and $\lim_{x \to 0} \frac{\tan(3x)}{\tan(-2x)}$.
7. The foundation of special relativity is that light seems to move at the same speed regardless of who's watching. ("Special" here just means no acceleration). For instance, normally if I'm standing still but you are in a vehicle, when you throw an object forward it will seem to you to be moving slower than it will seem to me (I see the ball moving at a speed equal to the *sum* of your velocity and the ball's velocity relative to you). Relativity says that this is not true for high speeds: if you turn on your headlights, the light will move forward at a speed that looks to you to be the same speed as it looks to me. This has all sorts of weird effects on physics.
   (a) The mass of an object with velocity $v$ is given by $m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$, where $c$ is the speed of light. First, interpret $m_0$ in this equation. Assuming $m_0 > 0$ (which should be reasonable given your interpretation), determine $\lim_{v \to c^-} m(v)$. Interpret this result in terms of physics.
   (b) The length of an object with velocity $v$ is given by $L(v) = L_0 \sqrt{1 - v^2/c^2}$. Repeat the questions from (a).
8. Suppose Han Solo is caught in a trash compactor. The walls approach each other, starting at positions +10 dm and -10 dm and moving toward position 0 at constant speed 1 dm/s. Han moves back and forth between the walls according the the equation $x(t) = (10 - t) \sin t$. Where is Han when $t$ is near 10? Why? (Hint: what are the position functions for the walls?) Make a general statement about limits; in particular, what do you need to know about the position functions described above for your conclusion to be accurate?

   (Don't worry; R2D2 does his job in less than 10s.)
9. Suppose that sodium pentobarbitol will anesthetize a dog when its bloodstream contains at least 45 mg of sodium pentobarbitol per kilogram of body weight of the dog. Suppose also that sodium pentobarbitol is eliminated exponentially from a dog's bloodstream, with a half-life of 5 hours. What size dose should be administered to anesthetize a 50 kg dog for 1 hour? (Hint: start by writing down

a formula for the function that gives the amount of the drug in the dog's bloodstream as a function of time.)

10. The following expressions are definitely *not* numbers. But they make sense in terms of limits; evaluate those that can be evaluated, and indicate why the others are indeterminate (that is, find two functions that give that limit form, but which have different actual limits). $c$ is a nonzero real number. (Be careful about signs!)

$$\infty - \infty \qquad \infty - c \qquad \frac{c}{0} \qquad 0 \cdot \infty \qquad c \cdot \infty \qquad \frac{c}{\infty} \qquad \frac{0}{\infty} \qquad \frac{\infty}{0} \qquad \frac{\infty}{\infty}$$

11. We define the *greatest integer function* (most mathematicians like this term), or *floor function* (most computer scientists prefer this term), by $[[x]] = \lfloor x \rfloor =$ the greatest integer $\leq x$. (Note: $\lfloor -3.5 \rfloor \neq -3$)

    (a) Plot the graph of $\lfloor \cos x \rfloor$ on $[-2\pi, 2\pi]$. Based on the graph, find $\lim_{x \to 0} \lfloor \cos x \rfloor$. What is $\lfloor \cos 0 \rfloor$?

    (b) Evaluate $\lim_{x \to 0} (\lfloor x \rfloor + \lfloor -x \rfloor)$.

12. Is it true that you were once exactly three feet tall? Explain using your height as a function of time since birth.