Math 241 C8 **Name:**

Quiz # 5

March 12, 2013

No electronic devices, notes, or interpersonal communication allowed.

Show work to get credit.

(1) [10pts] Find all sources and sinks of $\mathbf{F}(x,y) = \left\langle \frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2} \right\rangle$.

$$\operatorname{div} \vec{F} = \frac{-2xy}{(x^2+y^2)^2} + \frac{2xy}{(x^2+y^2)^2} = 0 \text{ except at } (0,0).$$

$$\int_{\substack{\text{unit}\\\text{circle}}} \vec{F} \cdot \langle dy, -dx \rangle \qquad \begin{array}{l} x = \cos t \\ y = \sin t \end{array} \quad t \in [0, 2\pi]$$

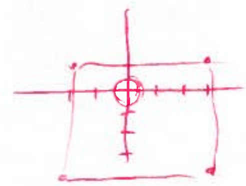
$$= \int_0^{2\pi} \left\langle \frac{\sin t}{1}, \frac{-\cos t}{1} \right\rangle \cdot \langle \cos t, \sin t \rangle \, dt$$

$$= \int_0^{2\pi} 0 \, dt$$

$$= 0.$$

So no points are sources or sinks.

(2) [10pts] Find the net flow of $\mathbf{F}(x,y) = \left\langle \dfrac{y}{x^2+y^2}, \dfrac{-x}{x^2+y^2} \right\rangle$ along the rectangle with vertices at $(-2,1)$, $(3,1)$, $(3,-4)$, and $(-2,-4)$.



$$\overset{\text{rot}}{\cancel{\text{curl}}}\ \vec{F} = \underbrace{\frac{x^2-y^2}{(x^2+y^2)^2}}_{}\overset{\partial_x n}{} - \overset{\partial_y m}{\frac{x^2-y^2}{(x^2+y^2)^2}} = 0$$

except at $(0,0)$.

So net flow along the rectangle equals net flow along the unit circle,

$$\int_{\substack{\text{unit}\\\text{circle}}} \vec{F} \cdot \langle dx, dy \rangle$$

$$= \int_0^{2\pi} \langle \sin t, -\cos t \rangle \cdot \langle -\sin t, \cos t \rangle \, dt$$

$$= \int_0^{2\pi} (-\sin^2 t - \cos^2 t) \, dt$$

$$= \int_0^{2\pi} -1 \, dt$$

$$= -2\pi,$$

i.e. net flow is 2π clockwise.

Note: You cannot apply Gauss-Green $\left(\text{"} = \iint \overset{\text{rot}}{\cancel{\text{curl}}}\ \vec{F}\, dx\,dy \text{"}\right)$ because of the singularity at $(0,0)$.