HW1 §1.1 – 1.5

1) Let w: "walks like a duck",
t: "talks like a duck",
d: "is a duck".

Then (i): $(w \wedge t) \to d$

(ii): $(\neg w \wedge \neg t) \to \neg d$

(iii): $d \to (w \wedge t)$

(iv): $\neg w \vee \neg t \vee d$

[This could all be done with a truth table.]

(i) & (iv) are equivalent,
by the Conditional Law & DeMorgan:

$$(w \wedge t) \to d \equiv \neg (w \wedge t) \vee d \equiv \neg w \vee \neg t \vee d$$

The others are both different:
if w: F, t: F, d: T
then (i) T, (ii) F, (iii) F,

Finally, (ii) & (iii) are different from each other:
if w: F, t: T, d: T
then (ii) T but (iii) F.

2) a)

p	q	r	...	$p \to (q \vee r)$	$(p \to q) \vee (p \to r)$
T	T	T		T	T
T	T	F		T	T
T	F	T		T	T
T	F	F		F	F
F	T	T		T	T
F	T	F		T	T
F	F	T		T	T
F	F	F		T	T

b) $(p \to q) \vee (p \to r) \equiv (\neg p \vee q) \vee (\neg p \vee r)$ Conditional Law

$\equiv (\neg p \vee \neg p) \vee (q \vee r)$ Associativity & Commutativity

$\equiv \neg p \vee (q \vee r)$ Idempotency

$\equiv p \to (q \vee r)$ Conditional Law

3) No. If p:F & r:F then $p \to (q \to r)$ is T
(it doesn't matter what q is) $(p \to q) \to r$ is F

4) $p \vee \neg q \leftrightarrow r \vee q \equiv (p \vee \neg q \to r \vee q) \wedge (r \vee q \to p \vee \neg q)$ Biconditional Law

$\equiv \big(\neg(p \vee \neg q) \vee r \vee q\big) \wedge \big(\neg(r \vee q) \vee p \vee \neg q\big)$ Conditional Law

$\equiv \big((\neg p \wedge q) \vee r \vee q\big) \wedge \big((\neg r \wedge \neg q) \vee p \vee \neg q\big)$ DeMorgan, double-negative

$\equiv \big(q \vee (q \wedge \neg p) \vee r\big) \wedge \big(\neg q \vee (\neg q \wedge \neg r) \vee p\big)$ Commutativity

$\equiv (q \vee r) \wedge (\neg q \vee p)$ Absorption

$\equiv \neg(\neg q \wedge \neg r) \wedge (\neg(q \wedge \neg p))$ DeMorgan, double-negative

[Other (more complicated?) expressions work here.]

Every proposition can be written this way,
by using the conditional laws to remove $\leftrightarrow$ then $\to$,
then DeMorgan to remove $\vee$ (at the expense of many parentheses & negations).

5) p: Cole is honest
q: Dot is honest

statement given is
$q \to \neg p$

p	q	$q \to \neg p$
T	T	F
T	F	T
F	T	T
F	F	T

In these rows p:T, i.e. Cole is Honest, so this statement $(q \to \neg p)$ must be T. Thus the first row is impossible.

In these rows, Cole is a Liar, so his statement must be False; thus these two rows are also impossible.

Cole is Honest
& Dot is a Liar