Math 241 X8 **Name:** Solutions

Quiz # 7

November 14, 2013 No electronic devices or interpersonal communication allowed. Show work to get credit.

(1) [10pts] Find the volume of the region R inside (above) the cone $z = \sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2=9$.

R in spherical: $0 \le \rho \le 3$

$0 \le \varphi \le \frac{\pi}{4}$

$0 \le \theta \le 2\pi$

You might remember this from the HW; otherwise,

Cone: $z = \sqrt{x^2+y^2}$

$\Rightarrow \rho\cos\varphi = \rho\sin\varphi$

$\Rightarrow 1 = \tan\varphi \Rightarrow \varphi = \frac{\pi}{4}$.

$$\text{Volume} = \iiint_R 1 \, dx\,dy\,dz$$

$$= \int_0^{2\pi}\int_0^{\pi/4}\int_0^3 1\cdot \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} 9\sin\varphi \, d\varphi\, d\theta$$

$$= 2\pi \cdot 9 \cdot (-\cos\varphi)\Big|_0^{\pi/4}$$

$$= 18\pi\left(1 - \frac{\sqrt{2}}{2}\right).$$

Oops; density is negative
at some points in S...

(2) [10pts] Let S be the region inside the parallelepiped bounded by the planes

$$x + 2y + 3z = 1, \qquad x + 2y + 3z = 4,$$
$$2x - 4y - 6z = -3, \qquad 2x - 4y - 6z = 2,$$
$$2x + 4y - 6z = 0, \qquad 2x + 4y - 6z = 7.$$

A solid in the shape of S has density at each point given by $(3x - 2y - 3z)$ kg/m^3. Find the mass of this solid.

Let $u = x+2y+3z \qquad 1 \le u \le 4$

$v = 2x-4y-6z \qquad -3 \le v \le 2$

$w = 2x+4y-6z \qquad 0 \le w \le 7$

Method I

$\frac{1}{4}(2u+v) = x$

$-\frac{1}{12}(w-2u) = z$

$\frac{1}{8}(w-v) = y$

$$J = \begin{array}{c|ccc|} & x & y & z \\ \partial_u & 1/2 & 0 & 1/6 \\ \partial_v & 1/4 & -1/8 & 0 \\ \partial_w & 0 & 1/8 & -1/12 \end{array} = \frac{1}{2}\left(\frac{1}{96}\right) - 0 + \frac{1}{6}\left(\frac{1}{32}\right) = \frac{1}{96}.$$

Method II

$$V_{uvw}(x,y,z) = \begin{array}{c|ccc|} & u & v & w \\ \partial_x & 1 & 2 & 2 \\ \partial_y & 2 & -4 & 4 \\ \partial_z & 3 & -6 & -6 \end{array}$$

$= 1(24+24) - 2(-12-12) + 2(-12+12)$

$= 96$

$\Rightarrow V_{xyz}(u,v,w) = \frac{1}{96}$;

and $3x-2y-3z = u+v$
by inspection.

$$\text{Mass} = \iiint_S (3x-2y-3z)\,dx\,dy\,dz$$

$$= \int_0^7 \int_{-3}^2 \int_1^4 \left(\frac{3}{4}(2u+v) - \frac{1}{4}(w-v) + \frac{1}{4}(w-2u)\right)\frac{1}{96}\,du\,dv\,dw$$

$$= \int_0^7 \int_{-3}^2 \int_1^4 \frac{1}{96}(u+v)\,du\,dv\,dw$$

$$= \frac{1}{96} \cdot 7 \cdot \int_{-3}^2 \left[\frac{1}{2}u^2 + uv\right]_{u=1}^4 dv = \frac{7}{96}\int_{-3}^2 \left(\frac{15}{2} + 3v\right) dv = \frac{7}{96}\left[\frac{15}{2}v + \frac{3}{2}v^2\right]_{-3}^2$$

$$= \frac{7}{96}\left(\frac{75}{2} - \frac{15}{2}\right) \text{ kg}. \quad \left(= \frac{35}{16} \text{ kg}\right)$$