Theorem 1. $\chi(G) \leq \chi_\ell(G)$.

Theorem 2. If $H \subseteq G$, then $\chi(H) \leq \chi(G)$.
Hence $\chi(G) \geq \omega(G)$.

Theorem 3. $\chi(G) \geq n(G)/\alpha(G)$.

Theorem 4 (greedy bounds)**.** $\chi_\ell(G) \leq \Delta(G) + 1$.
If G has degree sequence $d_1, \ldots, d_n$, then $\chi_\ell(G) \leq 1 + \max_i \min\{d_i, i-1\}$.
$\chi_\ell(G) \leq 1 + \max_{H \subseteq G} \delta(H)$.

Theorem 5. If D is an orientation of G with longest path length $l(D)$, then $\chi(G) \leq 1 + l(D)$. Furthermore, equality is achieved for some orientation.

Theorem 6 (Brooks)**.** If G is neither complete nor an odd cycle, then $\chi_\ell(G) \leq \Delta(G)$.

Theorem 7 (Turán)**.** The balanced complete k-partite graph has the most edges among K_{k+1}-free simple graphs.

Theorem 8. G is k-critical if and only if it is connected, has $\chi(G) = k$, and $\chi(G - e) \leq k - 1$ for every $e \in E(G)$.

Theorem 9. The Mycielskian of G is $(k+1)$-chromatic if G is k-chromatic. It is triangle free if G is. It is color-critical if G is.

Theorem 10. $\chi(G + H) = \max\{\chi(G), \chi(H)\}$. $\chi(G \vee H) = \chi(G) + \chi(H)$. $\chi(G \,\square\, H) = \max\{\chi(G), \chi(H)\}$.
$G \vee H$ is color-critical if both G and H are.

Theorem 11. M is a maximum matching if and only if there are no M-augmenting paths.

Theorem 12 (Hall)**.** An X, Y-bigraph has a matching saturating X if and only if for every $S \subseteq X$, $|S| \leq |N(S)|$.

Theorem 13. Every nontrivial regular bigraph has a perfect matching.

Theorem 14 (weak duality)**.** Always $\alpha'(G) \leq \beta(G)$.

Theorem 15 (König-Egerváry)**.** If G is bipartite, then $\alpha'(G) = \beta(G)$.

Theorem 16 (Tutte)**.** G has a perfect matching if and only if for every $S \subseteq V(G)$, $|S| \geq o(G - S)$.

Theorem 17. Every 3-regular graph without a cut edge has a perfect matching.

Theorem 18. Every nontrivial even regular graph has a 2-factor.