Math 251 **Name(s):**

PaperAssign 5

Workshop (in-class)

October 13, 2017

1. More double integrals over general regions

There's one last property we need to introduce. It is analogous to the property from Calc1 that

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx.$$

Theorem. *If R_1 and R_2 are disjoint (i.e., they share no points in common), then*

$$\iint_{R_1 \cup R_2} f(x,y)\,dA = \iint_{R_1} f(x,y)\,dA + \iint_{R_2} f(x,y)\,dA.$$

(1.1) Let R be the triangle with vertices $(0,0),(0,1),(1,2)$. Set up $\iint_R x\,dA$ in both orders. Evaluate them both.

(1.2) Let R be the trapezoid with vertices $(\pm 1,1),(\pm 2,2)$. Set up $\iint_R y\,dA$ in both orders. Evaluate the easier.

(1.3) Same trapezoid, now evaluate $\iint_R x\,dA$.

(1.4) Evaluate the following by reversing the order of integration.

$$\int_1^2 \int_{-\sqrt{y/2}}^{-\sqrt{y-1}} y^2\,dx\,dy + \int_1^2 \int_{\sqrt{y-1}}^{\sqrt{y/2}} y^2\,dx\,dy + \int_0^1 \int_{-\sqrt{y/2}}^{\sqrt{y/2}} y^2\,dx\,dy.$$

2. Double integrals with polar coordinates

(2.1) What does the equation $r = k$ (where k is a constant) represent in polar coordinates? What about $\theta = k$?

(2.2) By selecting several values of k in the previous question, we get "polar graph paper," a new grid on the plane. Draw such a grid.

(2.3) Now, the goal is to be able to compute $\iint_R f(x,y)\,dA$ using polar coordinates, when R is easier to describe in polar than in Cartesian coordinates. The bounds on the integrals should be easy (that's why we want to use polar), and the integrand can be transformed just by substituting $x = r\cos\theta$ and $y = r\sin\theta$. But what about dA? Before, it was the area of a small rectangle in the grid; so now, it needs to represent the area of one of your small grid regions.

Fixing a value of r and θ, and thinking of dr and $d\theta$ as small changes to those inputs, we get one of the small grid regions. Find its area, in terms of $r, \theta, dr, d\theta$.

(2.4) Consider the integral $\iint_R y\,dA$, where R is the region bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ and the x-axis, where $y \geq 0$. Sketch the region. How many integrals would you need to compute it in the order $dx\,dy$? How many with $dy\,dx$? Set up and compute the integral in polar coordinates instead.