HOMEWORK 4: §2.2-2.5 DUE FEBRUARY 9

Name: _____

- Please refer to the syllabus regarding allowed collaboration on this homework assignment.
- All answers should be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

When n is a multiple of d, we write $d \mid n$ (pronounced "d divides n").
(Note that $\mid$ is different from $/$; "$d \mid n$" is a proposition, whereas "d/n" and "n/d" are numbers.)

(1) Find (with proof) all integers n such that $0 \mid n$.
(2) Find (with proof) all integers n such that $n \mid 0$.

Prove the following statements.

(3) For all integers a, b, c: if $a \mid b$ and $b \mid c$, then $a \mid c$.
(4) For all integers a, b, c: if $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.
(5) For all integers a, b: if ab is odd, then a and b are both odd. *(Use the WLOG technique.)*
(6) The square of any integer has the form $4k$ or $4k + 1$ for some integer k.
(7) The sum of a rational number and an irrational number is irrational.
(8) Every composite number n has a factor d with $1 < d \leq \sqrt{n}$.

Challenge: is it possible to make exactly \$3 using exactly 50 coins that are all pennies, dimes, and quarters? (Some of the homework problems may help, but are not necessary. Too hard? Try a warmup: is it possible to make \$3.14 using any combination of nickels, dimes, and quarters?)

"A mathematician is a device for turning coffee into theorems."
–Alfréd Rényi