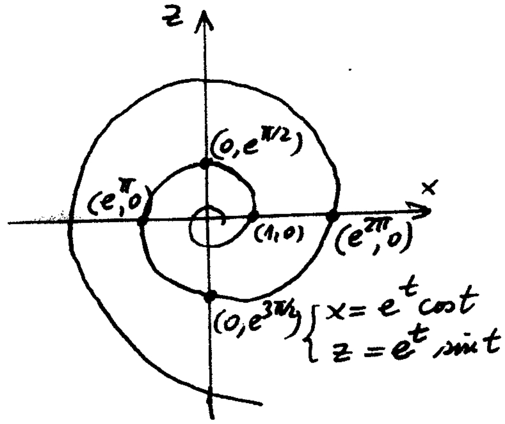
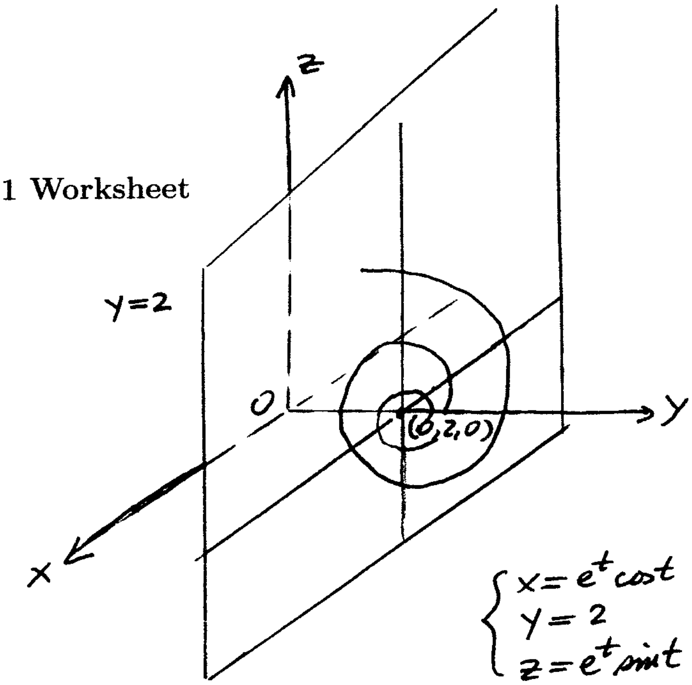
Solutions to the 10/11 Worksheet

1) (a)





(b) $\vec{r}\,'(t) = \langle e^t(\cos t - \sin t), 0, e^t(\sin t + \cos t)\rangle$.
$|\vec{r}\,'(t)| = e^t\sqrt{(\cos t - \sin t)^2 + (\sin t + \cos t)^2} = \sqrt{2}\,e^t$.
$s(t) = \displaystyle\int_0^t |\vec{r}\,'(u)|\,du = \sqrt{2}\int_0^t e^u\,du = \sqrt{2}\,(e^t - 1)$.

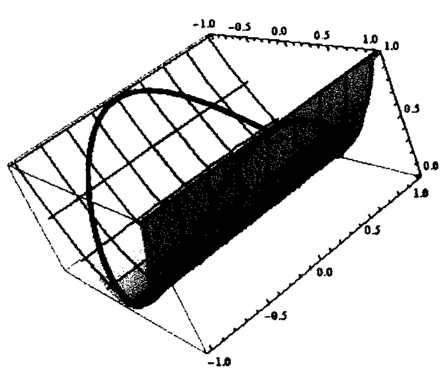
(c) $\sqrt{2}\,(e^t - 1) = s \iff e^t = 1 + \frac{s}{\sqrt{2}} \iff t = t(s) = \ln\left(1 + \frac{s}{\sqrt{2}}\right)$.

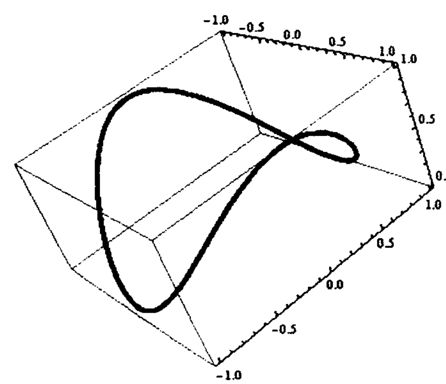
(d) + (e) We must have $s = \displaystyle\int_0^s |\vec{f}\,'(u)|\,du$ for every s. Differentiating with respect to s and applying FTC we find $1 = |\vec{f}\,'(s)|$. A choice for $h(s)$ that works is $h(s) = t(s) = \ln\left(1 + \frac{s}{\sqrt{2}}\right)$.

2) (a) The circle $x^2 + y^2 = 1$ can be parameterized as $x = \sin t$, $y = \cos t$, which is inserted into $z = x^2$ to yield

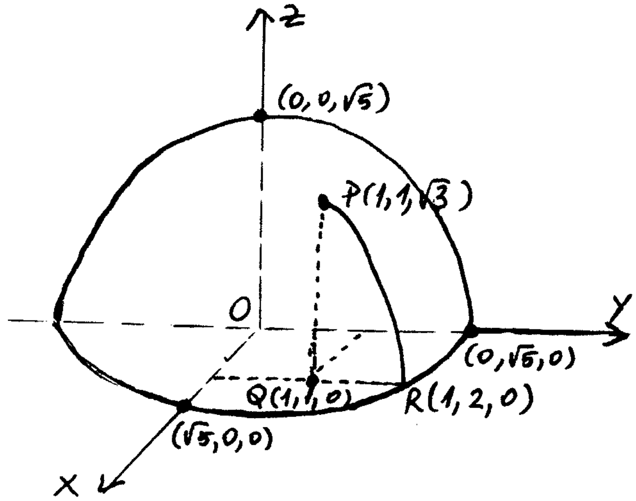
$$\vec{r}(t) = \langle \sin t, \cos t, \sin^2 t\rangle.$$

(b)





3) (a)



$\left.\begin{array}{l} 1^2+1^2+(\sqrt{3})^2=0 \\ \sqrt{3}>0 \end{array}\right\} \Rightarrow P$ lies on the half-sphere

(b) $z=\sqrt{5-x^2-y^2}$ with $(x,y)\in D=\{(x,y): x^2+y^2\le 5\}$.

(c) + (d) If the ant is going down in the vertical direction PQ (thus leaving the sphere): $\vec{r}_0=\langle 1,1,\sqrt{3}\rangle$. Direction vector: $\vec{v}=\langle 0,0,-1\rangle$. Parametrization of ant's path:

$$\vec{r}(t)=\vec{r}_0+t\vec{v}=\langle 1,1,\sqrt{3}-t\rangle.$$

The ant is at t when $t=0$ and it hits the xy-plane at $t=\sqrt{3}$.

If the ant is moving downward on the sphere on a trajectory that is parallel to the yz-plane (the circular segment PR), then x must stay constant and we can take $x=1$, $y=t$, thus $z=\sqrt{5-1^2-t^2}=\sqrt{4-t^2}$ and $\vec{r}(t)=\langle 1,t,\sqrt{4-t^2}\,\rangle$. In this case $t\ge 1$ and the inequality $4-t^2$ gives $t\in[1,2]$. The initial time here is$t=1$ and the final time $t=2$.

4) The Chain Rule gives $\vec{f}\,'(s)=h'(s)\vec{r}\,'(h(s))$ hence $|\vec{f}\,'(s)|=|h'(s)|\,|\vec{r}\,'(h(s))|$.