

# Chapter 1 Real Numbers

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## Section 1.1 Sets of Real Numbers

The **natural numbers** are sometimes called the counting numbers; they are:

1, 2, 3, 4, 5, 6, ...

The **whole numbers** are:

0, 1, 2, 3, 4, 5, 6, ...

The **integers** are the whole numbers and their opposites; they are:

..., -3, -2, -1, 0, 1, 2, 3, ...

**Rational numbers** are any numbers that can be written as the ratio of two integers. (the second being nonzero)

Note: The decimal form of a rational number is a terminating or repeating decimal.

**Irrational numbers** are real numbers that are not rational !!

### Examples of Rational Numbers:

0,  $10^7$ ,  $\frac{4}{3}$ , 1.0125,  
1.045,  $-\frac{8}{5}$ , -0.7,  
 $\sqrt{4}$ ,  $\sqrt[3]{-8}$

### Examples of Irrational Numbers:

$\sqrt{2}$ ,  $\sqrt{6}$ ,  $\pi$ ,  $e$ ,  $-\sqrt{5}$

Note: All integers are also rational numbers. e.g.  $6 = \frac{6}{1}$

Note: Every real number is either rational or irrational.

Are there any numbers that are **not real numbers**??  $\sqrt{-1}$ ,  $\frac{5}{0}$ ,  $\frac{0}{0}$ ,  $\infty$ ,  $\sqrt[4]{-2}$

Example Consider the this set  $S = \{6, \sqrt{6}, \sqrt{36}, -\sqrt{36}, \sqrt{-36}, -\frac{5}{6}, \frac{0}{6}, \frac{6}{0}, 0.\bar{6}, 0.676776777 \dots\}$

List all elements of S which are

- |                       |   |             |              |                |                |                     |             |                     |
|-----------------------|---|-------------|--------------|----------------|----------------|---------------------|-------------|---------------------|
| a) Natural numbers    | 6 | $\sqrt{36}$ |              |                |                |                     |             |                     |
| b) Whole numbers      | 6 | $\sqrt{36}$ |              |                |                |                     |             |                     |
| c) Integers           | 6 | $\sqrt{36}$ | $-\sqrt{36}$ |                |                |                     |             |                     |
| d) Rational numbers   | 6 | $\sqrt{36}$ | $-\sqrt{36}$ | $-\frac{5}{6}$ | $\frac{0}{6}$  | $0.\bar{6}$         |             |                     |
| e) Irrational numbers |   | $\sqrt{6}$  |              |                |                | $0.676776777 \dots$ |             |                     |
| f) Real numbers       | 6 | $\sqrt{6}$  | $\sqrt{36}$  | $-\sqrt{36}$   | $-\frac{5}{6}$ | $\frac{0}{6}$       | $0.\bar{6}$ | $0.676776777 \dots$ |

### Properties of Real Numbers

The **commutative** properties have to do with \_\_\_\_\_.

Commutative Property of Addition:  $a + b = b + a$

Commutative Property of Multiplication:  $a \cdot b = b \cdot a$

The **associative** properties have to do with \_\_\_\_\_.

Associative Property of Addition:  $(a + b) + c = a + (b + c)$

Associative Property of Multiplication:  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

The **Distributive** Property of Multiplication over Addition:

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

**Additive Identity Property:**

$$a + 0 = a$$

Note: 0 is called the additive identity.

**Multiplicative Identity Property:**

$$a \cdot 1 = a$$

Note: 1 is called the multiplicative identity.

**Additive Inverse Property:**

$$a + (-a) = 0$$

Note: The additive inverse of  $a$  is  $-a$ .

**Multiplicative Inverse Property:**

$$a \cdot \frac{1}{a} = 1 \quad \text{if } a \neq 0$$

Note: The multiplicative inverse of  $a$  is  $a^{-1} = \frac{1}{a}$ .

Example: The multiplicative inverse of  $\frac{3}{4}$  is  $\frac{4}{3}$ .

Identify the property illustrated:

- a)  $2 + (y + 0) = (2 + y) + 0$       Assoc. +
- b)  $2 + (y + 0) = 2 + y$       + Identity
- c)  $2 + (y + 0) = 2 + (0 + y)$       + comm.
- d)  $5(2x - 3) = 10x - 15$       distributivity
- e)  $\frac{x+y}{3} \cdot \frac{3}{x+y} = 1$       mult. inverses
- f)  $(x - 1)(x + 1) = (x + 1)(x - 1)$       mult. comm.
- g)  $2a + 3a = (2 + 3)a$       dist.

## Section 1.1 Exercises

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For exercises #1 - #6 consider this set of numbers:  $S = \left\{ -\frac{3}{5}, -3, \frac{6}{3}, -\sqrt{3}, \sqrt{-3}, 3.6, 3.\bar{4}, 0, \frac{\sqrt{2}}{0}, \frac{\pi}{6} \right\}$

Which elements of  $S$  are . . .

1. . . natural numbers?
  2. . . whole numbers?
  3. . . integers?
  4. . . rational numbers?
  5. . . irrational numbers?
  6. . . real numbers?
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7. State three numbers that are integers but are not natural numbers.
  8. State three numbers that are real numbers but are not rational numbers.
  9. State three numbers that are rational numbers but are not integers.
  10. State three numbers that are both integers and rational numbers.
  11. State three numbers that are both whole numbers and integers.

(#12 - #16) State the additive inverse of each number:

12. 5

13.  $-m$

14.  $\frac{1}{2}$

15.  $\sqrt{7}$

16.  $a - b$

(#17 - #21) State the multiplicative inverse of each number:

17.  $-2$

18.  $\frac{2}{3}$

19.  $\frac{1}{4}$

20.  $k$

21.  $x + y$

(#22 - #27) Name the property illustrated by each of the following:

22.  $3 \cdot 5 + 3 \cdot 9 = 3(5 + 9)$

23.  $2(x + 3) = (x + 3) \cdot 2$

24.  $1(x + 3) = x + 3$

25.  $1(x + 3) = 1(3 + x)$

26.  $2(3x) = (2 \cdot 3)x$

27.  $\frac{5}{3} + -\frac{5}{3} = 0$

1.  $\frac{6}{3}$

3.  $-3, \frac{6}{3}, 0$

5.  $-\sqrt{3}, \frac{\pi}{6}$

7. Answers will vary.

9. Answers will vary.

11. Answers will vary.

13.  $m$

15.  $-\sqrt{7}$

17.  $\frac{1}{2}$

19.  $-4$

21.  $\frac{1}{x+y}$

23. Commutative Property of Multiplication

25. Commutative Property of Addition

27. Additive Inverse Property