Math 251

Quiz 10 November 30, 2016

Name:

By handing in this quiz you assert that you understand and have followed IIT's guidelines for academic integrity.

(1) Use Green's Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x,y) = \langle \underbrace{x^4 + y}_{P}, \underbrace{4x - e^y}_{Q} \rangle$ and C is the unit circle centered at the origin.

$$= \iint_{\substack{\text{inside}\\ C}} Q_x - P_y \, dA = \iint_{\substack{\text{inside}\\ C}} 4 - 1 \, dA = 3 \cdot \text{Area}(\text{inside } C)$$

$$= 3 \cdot \pi (1)^2$$

$$= \boxed{3\pi}$$

(2) Compute $\iint_\Sigma x^2 z^2 \, dS$, where Σ is the portion of the cone $z = \sqrt{x^2+y^2}$ with $z \le 1$.

$x = u$

$y = v$

$z = \sqrt{u^2+v^2}$

$\vec{r}_u = \langle 1, 0, \frac{u}{\sqrt{u^2+v^2}} \rangle$

$\vec{r}_v = \langle 0, 1, \frac{v}{\sqrt{u^2+v^2}} \rangle$

$\vec{dS} = \langle -\frac{u}{\sqrt{\cdots}}, -\frac{v}{\sqrt{\cdots}}, 1 \rangle$

$dS = \sqrt{\frac{u^2}{u^2+v^2} + \frac{v^2}{u^2+v^2} + 1} = \sqrt{2}$

$$\iint_\Sigma x^2 z^2 \, dS$$

$$= \iint_{u^2+v^2 \le 1} u^2(u^2+v^2)\sqrt{2} \, du \, dv$$

$$= \sqrt{2} \int_0^{2\pi} \int_0^1 r^2 \cos^2\theta \cdot r^2 \cdot r \, dr \, d\theta$$

$$= \sqrt{2} \cdot \frac{1}{6} \cdot \pi$$

OR

$x = r\cos\theta$

$y = r\sin\theta$

$z = r$

$\vec{r}_r = \langle \cos\theta, \sin\theta, 1 \rangle$

$\vec{r}_\theta = \langle -r\sin\theta, r\cos\theta, 0 \rangle$

$\vec{dS} = \langle -r\cos\theta, -r\sin\theta, r \rangle$

$dS = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta + r^2}$

$= r\sqrt{2}$

$$\iint_\Sigma x^2 z^2 \, dS = \int_0^{2\pi}\int_0^1 r^2\cos^2\theta \cdot r^2 \cdot r\sqrt{2} \, dr \, d\theta$$

$$= \cdots$$