

Name: Solutions

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have until 12:45pm to complete this exam. When you are told to stop writing, do it or you will lose all points on the page(s) you write on.
- You may not communicate with other students during this test.
- Keep your eyes on your own paper.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctor.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers when appropriate.
- Before turning in your exam, check to make certain you've answered all the questions.

Question	Points	Score
1	13	
2	8	
3	4	
4	4	
5	12	
6	3	
7	6	
8	12	
9	8	
10	8	
11	12	
12	12	
13	0	
Total:	102	

1. (13 points) Simplify each of the following. Do not leave negative exponents in your answer.

$$(a) (-2x)^{-2} = \frac{1}{(-2x)^2} = \frac{1}{4x^2}$$

$$(b) -(2x)^{-2} = \frac{-1}{(2x)^2} = -\frac{1}{4x^2}$$

$$(c) -2x^{-2} = \frac{-2}{x^2}$$

$$(d) (4x^6)^{3/2} = 4^{3/2} \cdot x^{6 \cdot \frac{3}{2}} = 8x^9$$

2. (8 points) Simplify each of the following. Do not leave negative exponents in your answer.

(a) $(x^3 y^{1/2})(3xy^{3/2})$

$$= 3x^4 y^2$$

(b) $\left(\frac{8xy^{-2}}{18x^3y^{-5}}\right)^{3/2} = \left(\frac{4y^3}{9x^2}\right)^{3/2} = \frac{8y^{9/2}}{27x^3}$

3. (4 points) Rewrite the following in radical form.

(a) $2x^{1/2} = 2\sqrt{x}$

(b) $y^{7/3} = \sqrt[3]{y^7} \text{ or } \sqrt[3]{y}^7$

4. (4 points) Rewrite the following in exponential form.

(a) $3\sqrt{5x^7} = 3(5x^7)^{1/2}$

(b) $\sqrt[5]{x^3} = x^{3/5}$

5. (12 points) Simplify the following.

(a) $\sqrt[4]{16x^{21}}$

$$= \sqrt[4]{16} \cdot \sqrt[4]{x^{20}} \cdot \sqrt[4]{x}$$

$$= 2 x^5 \cdot \sqrt[4]{x}$$

(b) $\sqrt{\frac{27y^{14}}{12x^4}} = \sqrt{\frac{9y^{14}}{4x^4}} = \frac{3y^7}{2x^2}$

(c) $\sqrt{27} + 3\sqrt{6} - \sqrt{24} + 2\sqrt{75}$

$$= \sqrt{9 \cdot 3} + 3\sqrt{6} - \sqrt{4 \cdot 6} + 2\sqrt{25 \cdot 3}$$

$$= 3\sqrt{3} + 3\sqrt{6} - 2\sqrt{6} + 10\sqrt{3}$$

$$= 13\sqrt{3} + \sqrt{6}$$

6. (3 points) What does it mean for a number to be a *solution* to an equation (with one variable)?

When the number is substituted in for the variable,
the equation is true.

7. (6 points) Find all solutions to the following equations.

(a) $3x + 5 = 11$

$$3x = 6$$

$$x = 2$$

(b) $6x + 7 = 12$

$$6x = 5$$

$$x = \frac{5}{6}$$

8. (12 points) Find all solutions to the following equations.

(a) $2(5x + 3) - 3(3x - 1) = x + 7$

$$10x + 6 - 9x + 3 = x + 7$$

$$x + 9 = x + 7$$

No solutions

(b) $2(5x + 3) - 3(3x - 1) = 2x + 7$

$$x + 9 = 2x + 7$$

$$2 = x$$

(c) $2(5x + 3) - 3(3x - 1) = x + 9$

$$x + 9 = x + 9$$

Every real number is a solution

9. (8 points) Find all solutions to the following equations.

(a) $\left(\frac{1}{2}x + \frac{4}{3} = \frac{1}{3}x - \frac{1}{6} \right) * 6$

$$3x + 8 = 2x - 1$$

$$\boxed{x = -9}$$

(b) $\left(2x - 3 = \frac{7x + 2}{6} \right) * 6$

$$12x - 18 = 7x + 2$$

$$5x = 20$$

$$\boxed{x = 4}$$

10. (8 points) Solve the following equations for the variable indicated.

(a) $ax + by = c$, for y

$$by = c - ax$$
$$\boxed{y = \frac{c - ax}{b}}$$

(b) $\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$, for a

$$\frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\frac{1}{a} = \frac{b - c}{bc}$$

$$\boxed{a = \frac{bc}{b - c}}$$

11. (12 points) Find expressions that represent the following.

(a) The value, measured in dollars, of a collection of n nickels and q quarters.

$$0.05n + 0.25q \quad (\text{dollars})$$

(b) The thickness of an iceberg after y years if it starts 325 meters thick and loses 0.5 meters of thickness per year.

$$325 - 0.5y \quad (\text{meters})$$

(c) The perimeter of a rectangle whose width is 7 inches less than five times its length.
Your expression should be in terms of only its length ℓ .

$$\hookrightarrow w = 5\ell - 7$$

$$2w + 2\ell$$

$$= 2(5\ell - 7) + 2\ell$$

$$= \boxed{12\ell - 14} \quad (\text{inches})$$

12. (12 points) Write an appropriate equation (or system of equations) to represent the following scenarios. Then solve those equations to answer the questions.

- (a) When 100 is added to a number, the result is 32 less than 3 times the number. Find the number.

Let x be the number.

$$100 + x = 3x - 32$$

$$132 = 2x$$

$$66 = x$$

- (b) A car and a truck leave two towns 900 miles apart and head toward each other. The car travels 65mph and the truck 55mph. How long will it take the two to meet?

$$\begin{array}{lll} d_c = r_c t_c & r_c = 65 & d_c + d_t = 900 \\ d_t = r_t t_t & r_t = 55 & t_c = t_t =: t \end{array}$$
$$65t + 55t = 900$$

$$120t = 900$$

$$t = \frac{900}{120} = \frac{90}{12} = \frac{30}{4} = \frac{15}{2} = 7.5 \text{ hours}$$

13. (Bonus). A car and a motorcycle leave Champaign at noon heading north on I-57. The car drives 8mph faster than the motorcycle. At what time are the vehicles 20 miles apart?

$$\begin{aligned}d_c &= r_c t_c & r_c &= r_m + 8 \\d_m &= r_m t_m & t_c &= t_m =: t\end{aligned}$$

$$d_c - d_m = 20$$

$$r_c t - r_m t = 20$$

$$(r_m + 8)t - r_m t = 20$$

$$8t = 20$$

$$t = \frac{20}{8} = \frac{5}{2} = 2.5 \text{ hours}$$

$$\boxed{2:30 \text{ pm}}$$

Scratch Paper - Do Not Remove