CHAPTER 2 SECTION 2.1

Solving Linear Equations

The process consists of forming equivalent equations until we have our equation in the form of $x = n$. The properties that we will use are the following:

1. We can add or subtract the same number in both members of the equation.
2. We can multiply or divide both members of the equation by the same nonzero number.

If we use these two properties, making sure that both members of the equation are treated in exactly the same manner as we apply each of the properties, we will be forming equivalent equations.

Procedure for solving a linear equation

Using these properties, there are four basic steps to solve a linear equation. We shall now apply the properties to the equation $6(x + 1) = 4x + 10$.

Solving a linear equation

$$6(x + 1) = 4x + 10$$

Step 1 *Simplify each member of the equation.* Perform all indicated addition, subtraction, multiplication, and division. Remove all grouping symbols. In our example, step 1 would be to carry out the indicated multiplication in the left member as follows:

$$\begin{aligned} 6(x + 1) &= 4x + 10 \\ 6x + 6 &= 4x + 10 \end{aligned}$$

Step 2 *Use the addition and subtraction property of equality to form an equivalent equation where all the terms involving the unknown are in one member of the equation.* By subtracting $4x$ from *both* members of the equation, we have

$$\begin{aligned} 6x + 6 &= 4x + 10 \\ 6x - 4x + 6 &= 4x - 4x + 10 \\ 2x + 6 &= 10 \end{aligned}$$

Step 3 *Use the addition and subtraction property of equality to form an equivalent equation where all the terms not involving the unknown are in the other member of the equation.* Subtracting 6 from *both* members of the equation, we have

$$2x + 6 = 10$$
$$2x + 6 - 6 = 10 - 6$$
$$2x = 4$$

Step 4 *Use the multiplication and division property of equality to form an equivalent equation where the coefficient of the unknown is 1.* That is, $x = n$. By dividing *both* members of the equation by 2, we have

$$2x = 4$$
$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

The solution set is denoted by $\{2\}$.

Step 5 To check the solution, we substitute the solution in place of the unknown in the original equation. If we get a true statement, we say that the solution "satisfies" the equation.

In the equation $6(x + 1) = 4x + 10$, we found that $x = 2$. We can check the solution by substituting 2 in place of x in the original equation.

$$6[(2) + 1] = 4(2) + 10 \qquad \text{Substitute}$$
$$6[3] = 8 + 10 \qquad \text{Order of operations}$$
$$18 = 18 \quad \text{(True)} \qquad \text{Solution checks}$$

We see that $x = 2$ satisfies the equation.

Examples

Find the solution set and check.

1. $6y + 5 - 7y = 10 - 2y + 3$

$$5 - y = 13 - 2y \qquad \text{Simplify each member by combining like terms}$$
$$5 - y + 2y = 13 - 2y + 2y \qquad \text{Add } 2y \text{ to both members}$$
$$5 + y = 13$$
$$5 + y - 5 = 13 - 5 \qquad \text{Subtract 5 from both members}$$
$$y = 8$$

Check:

$$6(8) + 5 - 7(8) = 10 - 2(8) + 3 \qquad \text{Substitute}$$
$$48 + 5 - 56 = 10 - 16 + 3 \qquad \text{Order of operations}$$
$$53 - 56 = -6 + 3$$
$$-3 = -3 \quad \text{(True)} \qquad \text{Solution checks}$$

The solution set is $\{8\}$.

2. $8y + 5 - 7y = 10 - 2y + 3$

$$
\begin{aligned}
5 + y &= 13 - 2y && \text{Combine like terms}\\
5 + y + 2y &= 13 - 2y + 2y && \text{Add } 2y\\
5 + 3y &= 13 && \text{Combine like terms}\\
5 + 3y - 5 &= 13 - 5 && \text{Subtract 5}\\
3y &= 8 && \text{Combine like terms}\\
\frac{3y}{3} &= \frac{8}{3} && \text{Divide by 3}\\
y &= \frac{8}{3}
\end{aligned}
$$

Check:

$$
\begin{aligned}
8\left(\frac{8}{3}\right) + 5 - 7\left(\frac{8}{3}\right) &= 10 - 2\left(\frac{8}{3}\right) + 3 && \text{Substitute } \frac{8}{3} \text{ for } y\\
\frac{64}{3} + \frac{15}{3} - \frac{56}{3} &= \frac{30}{3} - \frac{16}{3} + \frac{9}{3} && \text{Multiply, change to common denominator}\\
\frac{64 + 15 - 56}{3} &= \frac{30 - 16 + 9}{3} && \text{Add and subtract in numerators}\\
\frac{23}{3} &= \frac{23}{3} \quad \text{(True)} && \text{Solution checks}
\end{aligned}
$$

The solution set is $\left\{\frac{8}{3}\right\}$.

3. $4(5x - 2) + 7 = 5(3x + 1)$

$$
\begin{aligned}
20x - 8 + 7 &= 15x + 5 && \text{Distributive property}\\
20x - 1 &= 15x + 5 && \text{Combine like terms}\\
20x - 15x - 1 &= 15x - 15x + 5 && \text{Subtract } 15x\\
5x - 1 &= 5\\
5x - 1 + 1 &= 5 + 1 && \text{Add 1}\\
5x &= 6\\
\frac{5x}{5} &= \frac{6}{5} && \text{Divide by 5}\\
x &= \frac{6}{5}
\end{aligned}
$$

Check:

$$
\begin{aligned}
4\left[5\left(\frac{6}{5}\right) - 2\right] + 7 &= 5\left[3\left(\frac{6}{5}\right) + 1\right] && \text{Substitute } \frac{6}{5} \text{ for } x\\
4[6 - 2] + 7 &= 5\left[\frac{18}{5} + 1\right] && \text{Order of operations}\\
4[4] + 7 &= 5\left[\frac{18}{5} + \frac{5}{5}\right]\\
16 + 7 &= 5\left[\frac{23}{5}\right]\\
23 &= 23 \quad \text{(True)} && \text{Solution checks}
\end{aligned}
$$

The solution set is $\left\{\frac{6}{5}\right\}$.

At this point, we will no longer show the check of our solution, but you should realize that a check of your solution is an important final step.

The following equations contain several fractions. When this occurs, it is usually easier to *clear the equation of all fractions.* We do this by multiplying both members of the equation by the least common denominator of all the fractions. Clearing all fractions is considered a means of simplifying the equation and will be done as a first step when necessary.

Examples

Find the solution set.

1. $\frac{1}{4}x + 2 = \frac{1}{2}$

$4\left(\frac{1}{4}x + 2\right) = 4\left(\frac{1}{2}\right)$ — The least common denominator of the fractions is 4, multiply both members by 4

$4\left(\frac{1}{4}x\right) + 4(2) = 4\left(\frac{1}{2}\right)$ — Simplify (distributive property)

$x + 8 = 2$ — All fractions have been cleared

$x + 8 - 8 = 2 - 8$ — Subtract 8

$x = -6$

The solution set is $\{-6\}$.

2. $\frac{5}{6}x - \frac{2}{3} = \frac{3}{4}x + 2$

$12\left(\frac{5}{6}x - \frac{2}{3}\right) = 12\left(\frac{3}{4}x + 2\right)$ — The least common denominator of the fractions is 12; multiply by 12

$12\left(\frac{5}{6}x\right) - 12\left(\frac{2}{3}\right) = 12\left(\frac{3}{4}x\right) + 12(2)$ — Simplify, distributive property

$10x - 8 = 9x + 24$

$10x - 9x - 8 = 9x - 9x + 24$ — Subtract $9x$

$x - 8 = 24$

$x - 8 + 8 = 24 + 8$ — Add 8

$x = 32$

The solution set is $\{32\}$.

More Examples:

1. $\dfrac{x-3}{4} = \dfrac{x}{8}$

We can see that the LCD of the denominators 4 and 8 is 8.

$\dfrac{8}{1} \cdot \dfrac{(x-3)}{4} = \dfrac{8}{1} \cdot \dfrac{x}{8}$	Multiply each member by 8
$2(x-3) = x$	Reduce in each member
$2x - 6 = x$	Multiply as indicated
$x - 6 = 0$	Subtract x from each member
$x = 6$	Add 6 to each member

The solution set is $\{6\}$. If we wish to check our work, we substitute 6 for x in the original equation to obtain the equivalent equation.

$\dfrac{6-3}{4} = \dfrac{6}{8}$	Replace x with 6 in the original equation
$\dfrac{3}{4} = \dfrac{3}{4}$	(True)

2. $\dfrac{t}{4} - \dfrac{t-4}{5} = \dfrac{7}{10}$

The LCD of the denominators 4, 5, and 10 is 20.

$\dfrac{20}{1} \cdot \dfrac{t}{4} - \dfrac{20}{1} \cdot \dfrac{t-4}{5} = \dfrac{20}{1} \cdot \dfrac{7}{10}$	Multiply each term by the LCD 20
$5t - 4(t-4) = 2 \cdot 7$	Reduce each term
$5t - 4t + 16 = 14$	Multiply as indicated
$t + 16 = 14$	Combine like terms
$t = -2$	Subtract 16 from each member

The solution set is $\{-2\}$. Check your answer by replacing t with -2 in the original equation.

Note A common error that is made when multiplying $-4(t-4)$ is to get $-4t - 16$. Do not forget that you are using the distributive property to multiply -4 times each term in the group $(t-4)$. The correct result is $-4t + 16$.

3. $\dfrac{5}{3a} + \dfrac{4}{9} = \dfrac{5}{12a}$

We determine that the LCD of the denominators is $36a$.

$36a \cdot \dfrac{5}{3a} + 36a \cdot \dfrac{4}{9} = 36a \cdot \dfrac{5}{12a} \quad (a \neq 0)$	Multiply each term by the LCD $36a$
$12 \cdot 5 + 4a \cdot 4 = 3 \cdot 5$	Reduce each term
$60 + 16a = 15$	Multiply as indicated
$16a = -45$	Subtract 60 from each member
$a = -\dfrac{45}{16}$	Divide each member by 16

The solution set is $\left\{-\dfrac{45}{16}\right\}$. Check your solution.

Exercises

Find the solution set of the following equations, and check the solution.

1. $2x = 4$
2. $3x = 11$
3. $5x = -10$
4. $-2x = 8$
5. $\frac{x}{2} = 18$
6. $\frac{x}{4} = 24$
7. $\frac{3x}{2} = 8$
8. $\frac{5x}{3} = 18$
9. $x + 7 = 11$
10. $x - 4 = 9$
11. $x + 5 = 5$
12. $x - 4 = -4$
13. $3x + 1 = 10$
14. $5x - 2 = 13$
15. $4x + 7 = 7$
16. $6x + 2 = 2$
17. $5x + 2x = x + 6$
18. $2x + (3x - 1) = 4 - x$
19. $2x + 3x - 6x = 4x - 8$
20. $\frac{x}{2} + 7 = 14$
21. $5 - \frac{3x}{5} = 11$
22. $\frac{5x + 2x}{6} = 10$
23. $\frac{1}{2}x + 3 = \frac{3}{4}$
24. $\frac{1}{5}x - 1 = \frac{7}{10}$
25. $\frac{1}{3}x + 2 = \frac{1}{2}x - 1$

26. $\frac{1}{4}x - 3 = \frac{1}{8}x + 1$
27. $\frac{2}{3}x + 5 = \frac{3}{4}$
28. $\frac{3}{5}x - 3 = \frac{3}{10}$
29. $\frac{3}{8}x + \frac{1}{2} = \frac{1}{4}x + 2$
30. $\frac{7}{12}x + 1 = \frac{2}{3}x - 1$

$\boxed{31.}$ $3(2x - 1) = 4x + 3$

32. $5(7x - 3) = 30x + 11$
33. $12x - 8 = 5x + 2$
34. $3(2x + 5) = 4(x - 3)$

$\boxed{35.}$ $8 - 2(3x + 4) = 5x - 16$

36. $(3x + 2) - (2x - 5) = 7$
37. $(7x - 6) - (4 - 3x) = 27$
38. $2(x - 4) - 3(5 - 2x) = 16$
39. $3(2x + 3) = 5 - 4(x - 2)$
40. $6(3x - 2) = 7(x - 3) - 2$
41. $2(x + 5) = 16$
42. $6 = 2(2x - 1)$
43. $2x - (3 - x) = 0$
44. $3(7 - 2x) = 30 - 7(x + 1)$

More Exercises

1. $\frac{y}{4} = \frac{2}{3}$

$\boxed{2.}$ $\frac{4x}{5} - \frac{2}{3} = 4$

3. $\frac{p}{6} = \frac{7}{9}$

4. $\frac{a}{3} + \frac{5}{2} = 6$

5. $\frac{z}{8} + 3 = \frac{1}{4}$

$\boxed{6.}$ $\frac{3R}{4} - 5 = \frac{5}{6}$

7. $\frac{3a}{6} + \frac{2a}{5} = 1$

8. $\frac{5x}{8} - \frac{x}{12} = 3$

9. $\frac{2x+1}{7} - \frac{2x-3}{14} = 1$

10. $\frac{3a+1}{9} + \frac{1}{12} = \frac{2a-1}{3}$

11. $\frac{3}{2x} = \frac{4}{5} + \frac{2}{x}$

12. $\frac{4}{b} - \frac{7}{3b} = \frac{2}{5}$

13. $\frac{2}{3R} + \frac{3}{2R} + \frac{1}{R} = 4$

14. $\frac{3}{w} - \frac{6}{5w} + \frac{1}{2w} = 5$

$\boxed{15.}$ $\frac{4}{6y} + 5 = \frac{1}{9y} + 2$

16. $\frac{16}{5a} - 1 = 5 + \frac{3}{4a}$

17. $\frac{5}{3b} - \frac{1}{2} = \frac{7}{6b}$

18. $3 - \frac{5}{9x} = \frac{4}{6x}$

19. $\frac{3p+2}{7p} - 3 = \frac{p}{14p}$

20. $\frac{5-x}{8x} = \frac{2x+5}{6x}$

21. $\frac{a-4}{3a} = \frac{2a-1}{4a}$