Theorem 1. If G is simple, then $\kappa(G) \le \kappa'(G) \le \delta(G)$.

Theorem 2. If G is 3-regular and simple, then $\kappa(G) = \kappa'(G)$.

Theorem 3. If $n(G) \ge 3$, then the following are equivalent:

A) G is connected and has no cut-vertex.

B) For all $x, y \in V(G)$, there are two internally disjoint x, y-paths in G.

C) For all $x, y \in V(G)$, there is a cycle containing both x and y.

D) $\delta(G) \ge 1$, and every pair of edges in G lie in a common cycle.

Theorem 4 (Menger)**.** Let G be a graph or digraph.
(Local) For every pair x, y of distinct vertices, $\kappa(x, y) = \lambda'(x, y)$. If $xy \notin E(G)$, then $\kappa(x, y) = \lambda(x, y)$.
(Global) $\kappa(G) = \min_{x,y} \lambda(x, y)$ and $\kappa'(G) = \min_{x,y} \lambda'(x, y)$.

Theorem 5. The Ford-Fulkerson algorithm finds either a flow-augmenting path or a source/sink cut with capacity equal to the value of the flow.

Corollary 6 (Max-flow=min-cut)**.** In a network, the maximum value of a feasible flow equals the minimum capacity of a source/sink cut.

Corollary 7 (Integrality)**.** If all capacities in a network are integers, then there is a maximum flow with integral flow on each edge. Futhermore, some such flow can be partitioned into flows of unit value along source-sink paths.

Theorem 8. In a plane graph G, $2e = \sum \ell(f_i)$.

Theorem 9. For a plane graph G, the following are equivalent:

A) G is bipartite.

B) Every face of G has even length.

C) The dual graph G^* is Eulerian.

Theorem 10 (Euler's Formula)**.** For a connected plane graph, $n(G) - e(G) + f(G) = 2$.

Theorem 11. If G is simple, planar, and $n(G) \ge 3$, then $e(G) \le 3n(G) - 6$.
If also G is triangle-free, then $e(G) \le 2n(G) - 4$.

Theorem 12 (Kuratowski)**.** A graph is planar if and only if it contains no subdivision of K_5 or $K_{3,3}$.

Theorem 13 (Four color theorem)**.** Every planar graph is 4-colorable.

Theorem 14. Always $\chi'(G) \ge \Delta(G)$.

Theorem 15 (König)**.** If G is bipartite, then $\chi'(G) = \Delta(G)$.

Theorem 16 (Vizing)**.** If G is simple, then $\chi'(G) \le \Delta(G) + 1$.

Theorem 17. If G is Hamiltonian, then for every nonempty $S \subseteq V(G)$, $|S| \ge c(G - S)$.

Theorem 18 (Dirac, Ore)**.** If G is simple and $\delta(G) \ge n/2$, then G is Hamiltonian.
If G is simple and for every $xy \notin E(G)$, $d(x) + d(y) \ge n$, then G is Hamiltonian.

Theorem 19 (Tait)**.** Let G be a simple 2-edge-connected 3-regular plane graph. G is 3-edge-colorable if and only if G is 4-face-colorable.