Math 251 **Name(s):**

PaperAssign 6

Workshop (in-class)

October 27, 2017

(1) The equation $17x^2 + 10xy + 13y^2 + 2x - 6y = 0$, being a degree 2 polynomial equation in x and y, must be one of the conic sections. Find out which by using the change of coordinates $u = x - 3y$, $v = 2x + y$. [Since this is a linear transformation, it preserves the type of conic section.]

(2) Compute the volume inside the ellipsoid $\dfrac{x^2}{4} + \dfrac{y^2}{9} + z^2 = 1$. Start by using an appropriate *linear* transformation.

(3) Compute the volume inside the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$ and in the positive octant. *(Hint: a change of variables, $u = \sqrt{x}, \ldots$.)*

(4) Evaluate $\iint_R x^2\, dA$, where R is the solid triangle with vertices $(0,0)$, $(1,0)$, and $(1,\sqrt{3})$:
  (a) with horizontal slices
  (b) with vertical slices
  (c) with polar coordinates

(5) Compute $\iiint_E y^2 z^2\, dV$, where E is bounded by $x = 1 - y^2 - z^2$ and $x = 0$.

(6) Compute $\iiint_E z\, dV$, where E is bounded by $y = 0$, $z = 0$, $x + y = 2$, $y^2 + z^2 = 1$, and lies in the positive octant.

(7) Compute the volume of above $z = x^2 + y^2$ and below $z = \sqrt{x^2 + y^2}$.

(8) Compute $\iint_R e^{x+y}\, dA$, where R is the region defined by $|x| + |y| \leq 1$:
  (a) with horizontal or vertical slices
  (b) using a change of coordinates

(9) Find the centroid (the center of mass, assuming constant density) for the solid between $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{3x^2 + 3y^2}$ and below $z = 4$. *(The integration is reasonably nice in both cylindrical and spherical coordinates.)*

(10) Compute $\iiint_E \frac{z}{y}\, dV$, where E is the region bounded by the surfaces $xyz = 1$, $xyz = 4$, $xy = 1$, $xy = 3$, $z = x + 5$, and $z = x + 7$. *(Remark: you have to be pretty careful about whether the Jacobian is positive or not.)*