Worksheet 11 February 28, 2011

1. Find a, b so that the parabola $y = ax^2 + bx$ has a tangent line at $(1, 1)$ with equation $y = 3x - 2$.

2. Find an equation for the tangent line to the curve $y = \tan x$ at the point $x = 0$. Try to do the same at $x = \pi/2$; what goes wrong?

3. Find an equation for the tangent line to the function $f \circ g$ at $x = 3$ if you are given $f(3) = 2$, $g(3) = -5$, $f'(3) = 1$, $g'(3) = 8$.

4. There are two lines that are tangent to the parabola $y = x^2$ and pass through the point $(3, 0)$. One is the x-axis; find the equation for the other line.

5. Are there any lines tangent to the parabola $y = x^2$ that pass through the point $(1, 5)$? Decide this algebraically, then explain geometrically.

6. You could (painfully) compute the following limits directly. Instead, recognize them as being the derivative of a function, and use this function together with your shortcut derivative rules to evaluate them.

   (a) $\lim_{h \to 0} \dfrac{\sqrt{1+h} - 1}{h}$

   (b) $\lim_{x \to 1} \dfrac{x^{214} - 1}{x - 1}$

   (c) $\lim_{h \to 0} \dfrac{(3+h)^4 - 81}{h}$

7. Let $f(x) = x^{2/3}$ and $g(x) = x^3$. Show that the composition of these functions in either order is differentiable at $x = 0$ but that f is not differentiable at $x = 0$. Does this contradict the chain rule? Explain.

8. Suppose that $g(0) = 0$ and $g'(0) = 2$. What is the derivative of $g(g(g(x)))$ at $x = 0$?

9. Find the derivative of $\sec^2 x - \tan^2 x$. (Do this the straightforward way first; after you get an answer, simplify, then try to realize why this problem could have been done last chapter.)

10. We'll now compute the derivative of x^x. Can you use the power rule? Exponential derivative rule? So we need a trick: let $y = x^x$, take logarithms, then differentiate.

11. Find all points on the curve defined by $(x^2 + y^2)^2 = x^2 - y^2$ where the tangent line is horizontal or vertical (there are four and two of these, respectively).

12. Find the derivatives of the following functions.

    (a) $\sqrt{t^3 + \sin t} \cdot \sin(\sqrt{t^2 + 1})$

    (b) $\dfrac{u^3 + \sqrt[3]{u}}{\sin(\cos u)}$

    (c) $\sqrt{\dfrac{\tan x}{\sec x}}$

(d) $\sqrt{\dfrac{\tan x}{\ln x}}$

(e) $\sec^3(\sqrt{\cos x})$

13. Compute $\dfrac{dy}{dx}$ when $\sqrt{x} + \sqrt{y} = 1$.

14. Let $f(t) = e^{at}$, where a is some constant. Determine $f^{(n)}(t)$, the nth derivative of f.

15. Let $g(t) = \cos(at)$; what is $g^{(n)}(t)$? (Hint: you'll need some cases.)

16. Compute all the derivatives of $P(x) = x^5 + x^4 + x^3 + x^2 + x + 1$.