Workshop 10 September 29, 2011

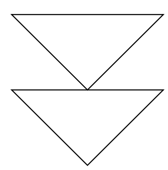
1. Suppose you have a flat surface that is submerged (completely) underwater in such a way that there is a horizontal axis of symmetry to the surface. Now rotate the surface so that it lies flat (the center of the surface should remain at the same depth). We'll prove that the hydrostatic force on the surface is the same in these two situations. (Some of you wanted to use this fact to avoid computing an integral from last time; this is good, but you should also be comfortable using the integral.)

   (a) The point here is that the statement above should be true for *any* shape with symmetry about a horizontal axis. Set the origin at the center of the object, and say the center of the object is at depth d. Let $\ell(y)$ be a function that gives the cross-sectional length of the surface at height y. Let h be the distance from the center to the top of the surface. Use these bits of notation to set up an integral representing the hydrostatic force on the surface.

   (b) Now split the integral into two parts. Recognize one of the integrals as being zero by symmetry of the integral, and recognize the other integral as some geometric quantity.

   (c) Show that the answer is equivalent to the hydrostatic force on the surface if it is laid flat.

2. Can you use the previous problem to easily find the hydrostatic force on a plate shown below?



3. Can you use symmetry to easily find the center of mass of the above object (assuming constant density)?

4. Remember our torus, generated by taking a circle with center $(c, 0)$ and radius r and rotating about the y-axis? The Theorem of Pappus in your text has a natural surface area/arc length version:

   > Let C be a curve in the plane that lies entirely on one side of a line ℓ in the plane. If C is rotated about ℓ, then the surface area of the resulting surface is the product of the arc length of C and the distance d travelled by the centroid of C.

   Use this to find the surface area of the torus (again).

5. Consider the region bounded by $y = \sec x$, $x = -1/2$, $x = 1/2$, and the x-axis. Draw this region. Find its center of mass (again assuming constant density).

6. Now consider the region bounded by $y = \sec x$, $x = -\pi/2 + \epsilon_1$, $x = \pi/2 - \epsilon_2$, and the x-axis, for some small positive ϵ_1 and ϵ_2. Find its center of mass. When ϵ_1 and ϵ_2 tend to 0 what happens? Is the center of mass even in the region? How do you make sense of "center of mass" in this case?

7. As functions of real numbers, argue that neither $c(x) = \sin(\pi x)$ nor $d(x) = \cos(\pi x)$ converge as $x \to \infty$. Show however, that as sequences (i.e. functions whose domain is just positive integers), $c(n) = \sin(\pi n)$ converges, while $d(n) = \cos(\pi n)$ does not. Do you think the sequence $a(n) = \sin(n)$ converges?