Worksheet 8 February 16, 2011

1. Are there any functions that are both even and odd?
2. Can even functions have inverses? Can odd functions have inverses? Are their inverses necessarily even or odd?
3. Suppose $f(x)$ is even; is $f'(x)$ necessarily even or odd?
4. Suppose $f(x)$ is odd; is $f'(x)$ necessarily even or odd?
5. Sketch the graph of $f(\theta) = \tan(2\theta + \pi)$. Then sketch $\dfrac{df}{d\theta}(\theta)$.
6. Suppose $\tan\theta = 0.8$ and $\pi < \theta < 2\pi$. What is $\cos\theta$? Compute this in two ways, and note in what ways they are similar:
   (i) by drawing and labeling an appropriate right triangle; and
   (ii) by using a pythagorean identity.
7. Is there a real number that is one less than its fourth power?
8. Consider the function
$$f(t) = \begin{cases} 2t+5 & \text{if } t < 1 \\ t^2 & \text{if } t \geq 1 \end{cases}$$
   (a) What is the derivative of $2x + 5$ at $x = 1$?
   (b) What is the derivative of x^2 at $x = 1$?
   (c) Is f differentiable at $t = 1$? (Be careful! You want to know if the limit of the difference quotient exists; compute the left and right limits separately, by hand.)
9. Compute the derivative of $\sin\theta$ at $\theta = \pi$.
10. Find all n such that $e^{3\ln(n^2-1)} = 0$. (Careful!)
11. Find all $m > 0$ such that $m^m = m$.
12. Find all k such that $2^k = 5^{k-1}$.
13. Let $f(y) = \dfrac{1}{\sqrt{4-y}}$, $g(x) = \sqrt{x}$. What is the domain of $f \circ g$?
14. What is the domain of the function $\sqrt{1+\sqrt{t}}$?
15. Compute $\lim\limits_{x\to\infty} \sqrt{4x-3} - x$
16. Define $\chi(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$
    Show that $\lim\limits_{x\to 0} \chi(x) = 0$. (Hint: this would be true even if "rational" and "irrational" were replaced with any property and its negation.)