

1. Consider the ellipsoid with implicit equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

- (a) Parameterize this ellipsoid.
- (b) Set up, but do not evaluate, a double integral that computes its surface area.

2. Let

$$\mathbf{r}(u, v) = \langle (2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u \rangle,$$

where  $0 \leq u \leq 2\pi$  and  $0 \leq v \leq 2\pi$ .

- (a) Sketch the surface parameterized by this function.
- (b) Compute its surface area.

3. Consider the surface integral

$$\iint_{\Sigma} z \, dS$$

where  $\Sigma$  is the surface with sides  $S_1$  given by the cylinder  $x^2 + y^2 = 1$ ,  $S_2$  given by the unit disk in the  $xy$ -plane, and  $S_3$  given by the plane  $z = x + 1$ . Evaluate this integral as follows:

- (a) Parameterize  $S_1$  using  $(\theta, z)$  coordinates.
- (b) Evaluate the integral over the surface  $S_2$  without parameterizing.
- (c) Parameterize  $S_3$  in (Des)cartesian coordinates and evaluate the resulting integral using polar coordinates.

4. Let  $C$  be the circle in the plane with equation  $x^2 + y^2 - 2x = 0$ .

- (a) Parameterize  $C$  as follows. For each choice of a slope  $t$ , consider the line  $L_t$  whose equation is  $y = tx$ . Then the intersection  $L_t \cap C$  of  $L_t$  and  $C$  contains two points, one of which is  $(0, 0)$ . Find the other point of intersection, and call its  $x$ - and  $y$ -coordinates  $x(t)$  and  $y(t)$ . Compute a formula for  $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ . Check your answer with your TA.
- (b) Suppose that  $t = \frac{p}{q}$  is a rational number. Show that  $x(p/q)$  and  $y(p/q)$  are also rational numbers. Explain how, by clearing denominators in  $x(p/q) - 1$  and  $y(p/q)$ , you can find a triple of integers  $U, V$ , and  $W$  for which  $U^2 + V^2 = W^2$ .

- (c) Compute  $\int_C \frac{1}{2} \langle -y, x \rangle \cdot d\mathbf{r}$  using your parameterization above.