Worksheet 5 February 2, 2011

1. Suppose a particle's position is given by $p(t) = 3t + 5$. What is its velocity at time $t = 3$?
2. Suppose a particle's position is given by $p(t) = t^2$. How can you estimate its velocity at time $t = 3$?
3. Suppose a particle's position is given by a function $f(t)$. How can you estimate its velocity at time $t = 3$?
4. Let's go back to our particle with position function t^2. Write an explicit formula that gives the average velocity for the particle in the interval $[3, 3 + h]$ for any $h \neq 0$ (do you need to treat positive and negative h differently?). An old paradox says that velocity *at a given moment* cannot really exist, since velocity is defined by distance per time, and these are both zero in an instant. Explain how you might interpret "instantaneous velocity" at $t = 3$ given the formula you have derived.
5. Determine the value of $\log_2(3)\log_3(4)\log_4(5)\ldots\log_{31}(32)$. (Hint: try a simpler but similar problem first.)
6. Do the limits $\lim_{x\to 0} \ln \sin x$ or $\lim_{x\to 0} \ln x$ exist? What about $\lim_{x\to 0} \ln \sin x - \ln x$?
7. Determine $\lim_{x\to\infty} x \sin \frac{1}{x}$.
8. The foundation of special relativity is that light seems to move at the same speed regardless of who's watching. This assumes no acceleration. For instance, normally if I'm standing still but you are in a vehicle, when you throw an object forward it will seem to you to be moving slower than it will seem to me (I see the ball moving at a speed equal to the *sum* of your velocity and the ball's velocity relative to you). Relativity says that this is not true for light: if you turn on your headlights, the light will move forward at a speed that looks to you to be the same speed as it looks to me. This has all sorts of weird effects on physics.
   (a) The mass of an object with velocity v is given by $m(v) = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$, where c is the speed of light. First, interpret m_0 in this equation. Assuming $m_0 > 0$ (which should be reasonably given your interpretation), determine $\lim_{v\to\infty} m(v)$. Interpret this result in terms of physics.
   (b) The length of an object with velocity v is given by $L(v) = L_0\sqrt{1 - v^2/c^2}$. Repeat the questions from (a).
9. The following data was actually collected about the population of brewers' yeast:

Time (hr)	0	1.5	9	10	18	23	25.5	27	34	38	42	45.5	47
Volume	0.4	1.6	6.2	8.9	11	12.5	12.6	12.9	13.3	12.8	12.9	12.9	12.7

   Plot this data. Comment on the shape that you see. Can you say anything about limits?
10. The following expressions are definitely *not* numbers. But they make sense in terms of limits; evaluate those that can be evaluated, and indicate why the others are indeterminate (that is, find two functions that give that limit form, but which have different actual limits). c is a nonzero real number.

$$\infty - \infty \qquad \infty - c \qquad \frac{c}{0} \qquad 0 \cdot \infty \qquad c \cdot \infty \qquad \frac{c}{\infty} \qquad \frac{0}{\infty} \qquad \frac{\infty}{0} \qquad \frac{\infty}{\infty}$$

11. Plot the solution to the following inequalities on a number line:

(a) $|x| < 3$

(b) $|x - 2| < 3$

(c) $|x + 4| < 7$

For constants c, r, interpret the set of solutions to $|x - c| < r$. What about $0 < |x - c| < r$?

12. We define the *greatest integer function*, or *floor function*, by $\lfloor x \rfloor =$ the greatest integer $\leq x$. (Note: $\lfloor -3.5 \rfloor \neq -3$!)

    (a) Plot the graph of $\lfloor \cos x \rfloor$ on $[-2\pi, 2\pi]$. Based on the graph, find $\lim_{x \to 0} \lfloor \cos x \rfloor$. What is $\lfloor \cos 0 \rfloor$?

    (b) Evaluate $\lim_{x \to 0} (\lfloor x \rfloor + \lfloor -x \rfloor)$.