

**Tuesday, August 28**    \*\*    *A review of some important calculus topics*

1. Chain Rule:

- (a) Let  $h(t) = \sin(\cos(\tan t))$ . Find the derivative with respect to  $t$ .
- (b) Let  $s(x) = \sqrt[4]{x}$  where  $x(t) = \ln(f(t))$  and  $f(t)$  is a differentiable function. Find  $\frac{ds}{dt}$ .

2. Parameterized curves:

- (a) Describe and sketch the curve given parametrically by

$$\begin{cases} x = 5 \sin(3t) \\ y = 3 \cos(3t) \end{cases} \quad \text{for } 0 \leq t < \frac{2\pi}{3}.$$

What happens if we instead allow  $t$  to vary between 0 and  $2\pi$ ?

- (b) Set up, but **do not evaluate** an integral that calculates the arc length of the curve described in part (a).
- (c) Consider the equation  $x^2 + y^2 = 16$ . Graph the set of solutions of this equation in  $\mathbb{R}^2$  and find a parameterization that traverses the curve once counterclockwise.

3. 1st and 2nd Derivative Tests:

- (a) Use the 2nd Derivative Test to classify the critical numbers of the function  $f(x) = x^4 - 8x^2 + 10$ .
- (b) Use the 1st Derivative Test and find the extrema of  $h(s) = s^4 + 4s^3 - 1$ .
- (c) Explain why the 2nd Derivative test is unable to classify all the critical numbers of  $h(s) = s^4 + 4s^3 - 1$ .

4. Consider the function  $f(x) = x^2 e^{-x}$ .

- (a) Find the best linear approximation to  $f$  at  $x = 0$ .
- (b) Compute the second-order Taylor polynomial at  $x = 0$ .
- (c) Explain how the second-order Taylor polynomial at  $x = 0$  demonstrates that  $f$  must have a local minimum at  $x = 0$ .

5. Consider the integral  $\int_0^{\sqrt{3\pi}} 2x \cos(x^2) dx$ .

- (a) Sketch the area in the  $xy$ -plane that is implicitly defined by this integral.
- (b) To evaluate, you will need to perform a substitution. Choose a proper  $u = f(x)$  and rewrite the integral in terms of  $u$ . Sketch the area in the  $uv$ -plane that is implicitly defined by this integral.

- (c) Evaluate the integral  $\int_0^{\sqrt{3\pi}} 2x \cos(x^2) dx$ .