

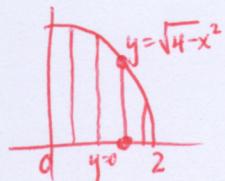
Math 251

Quiz 7 October 26, 2016

Name:

By handing in this quiz you assert that you understand and have followed IIT's guidelines for academic integrity.

(1) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2} dy dx$.

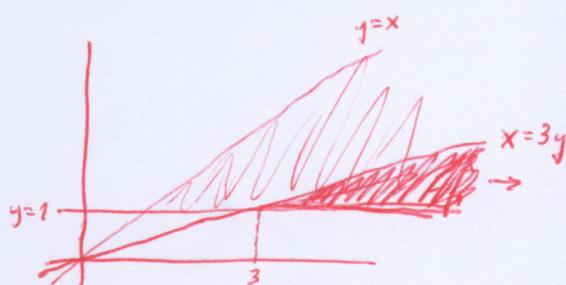


$$\begin{aligned}
 &= \int_0^{\pi/2} \int_0^2 e^{-r^2} r dr d\theta \\
 &\quad u = -r^2 \\
 &\quad du = -2r dr \\
 &= \frac{\pi}{2} \cdot \frac{-1}{2} \left[e^{-r^2} \right]_0^2 \\
 &= -\frac{\pi}{4} (e^{-4} - 1) = \frac{\pi}{4} (1 - e^{-4})
 \end{aligned}$$

(2) Consider random variables X, Y with joint probability distribution

$$\rho(x, y) = \begin{cases} 1/(x^2 y) & \text{if } x \geq y \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Compute $P(X \geq 3Y)$. (It is $\iint_R \rho(x, y) dA$, where R is the region where $x \geq 3y$. Now set up this integral and compute it.)



$$\begin{aligned}
 &= \int_1^\infty \int_{3y}^\infty \frac{1}{x^2 y} dx dy = \int_1^\infty -\frac{1}{xy} \Big|_{x=3y}^{x=\infty} dy \\
 &= \int_1^\infty \left(0 + \frac{1}{3y^2} \right) dy = \left[-\frac{1}{3y} \right]_1^\infty = 0 + \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 &\int_1^\infty \int_1^{x/3} \frac{1}{x^2 y} dy dx = \int_1^\infty \frac{1}{x^2} \ln y \Big|_1^{x/3} dx \\
 &= \int_1^\infty \left(\frac{1}{x^2} \ln \left(\frac{x}{3} \right) - 0 \right) dx \\
 &\quad u = \ln \left(\frac{x}{3} \right) \quad dv = \frac{1}{x^2} dx \\
 &\quad du = \frac{1}{x} dx \quad v = -\frac{1}{x} \\
 &= \left[-\frac{1}{x} \ln \left(\frac{x}{3} \right) \right]_1^\infty + \int_1^\infty \frac{1}{x^2} dx \\
 &= \cancel{\infty} \\
 &\quad \lim_{x \rightarrow \infty} -\frac{1}{x} \ln \left(\frac{x}{3} \right) + \frac{1}{3} \ln(1) + \left[-\frac{1}{x} \right]_1^\infty \\
 &= 0 + 0 + 0 + \frac{1}{3}
 \end{aligned}$$