

Name: _____

- Please refer to the syllabus regarding allowed collaboration on this homework assignment.
- All answers should be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

- (1) Consider the following domain of letters and numbers:

5, A, C, 2, E, 7, 4, B

Let $\text{Even}(x)$, $\text{Odd}(x)$, $\text{Prime}(x)$, and $\text{Vowel}(x)$ be predicates that are true precisely when the object x is an even number, odd number, prime number, or vowel (aeiou), respectively. Consider also the predicate $\text{RightOf}(x, y)$ that is true precisely when object x is (strictly) to the right of object y in the list. Determine which of the following are true.

- (a) $\exists x (\text{Even}(x) \wedge \text{RightOf}(C, x))$
 - (b) $\forall x (\text{Odd}(x) \rightarrow \text{Prime}(x))$
 - (c) $\exists x (\text{Odd}(x) \rightarrow \neg \text{Prime}(x))$
 - (d) $\exists y \forall x (\text{Vowel}(y) \wedge (\text{Even}(x) \rightarrow \text{RightOf}(x, y)))$
- (2) Use the domain of students at IIT, and let $F(x, y)$: “ x is a friend of y ” and $C(x, y)$: “ x and y take some course together.” Assume that no one is their own friend. Translate the following into formal logic. (You may need to rephrase the statement first.)
- (a) Lucy and Sally are friends but not classmates.
 - (b) Everyone has a friend.
 - (c) Some student is friends with everyone.
 - (d) Alice takes a course only if she has a friend also taking it.
 - (e) Bob has no friends in any of his classes.
 - (f) Any two friends of Cindy are friends of each other.
 - (g) Doug is not taking any courses. (*Assume that every class has at least two students enrolled.*)
- (3) For each predicate below, determine whether $\forall x \exists y P(x, y)$ is true or false and whether $\exists y \forall x P(x, y)$ is true or false. The domain is \mathbb{R} .
- (a) $x + y = 7$
 - (b) $x + y = y + x$
 - (c) $x^2 - 2xy + y^2 = 0$
 - (d) $(x - 3)(y + 5) = 0$
- (4) Write the negation of the following.
- (a) $\exists x \forall y x + y = y$ (*give your answer in formal logic without the symbol \neg*)
 - (b) Every graph is either connected or its complement is connected. (*No, you don't need to know what that means; you should still be able to give its negation.*)
- (5) Complete the Challenge Activity from §1.7.

“In America, someone steals a car every fifteen seconds. We have to find that person and stop them.”