Math 251, section 01
Quiz 4 September 21, 2016

Name:

By handing in this quiz you assert that you understand and have followed IIT's guidelines for academic integrity.

(1) Suppose the position of a particle at time t is given by $\langle (1+t)^{3/2},\ t-t^2,\ e^{-t}\rangle$.

(a) Find the velocity of the particle at time 0.

$\vec v(t) = \langle \frac32(1+t)^{1/2}, 1-2t, -e^{-t}\rangle \qquad v(0) = \langle \frac32, 1, -1\rangle$

(b) Find the acceleration of the particle at time 0.

$\vec a(t) = \langle \frac34(1+t)^{-1/2}, -2, e^{-t}\rangle \qquad a(0) = \langle \frac34, -2, 1\rangle$

(c) Find the tangential component of acceleration at time 0.

$\vec a_T(0) = \text{proj}_{\vec v(0)}\vec a(0) = \frac{\vec v(0)\cdot\vec a(0)}{|\vec v(0)|^2}\vec v(0)$

$= \frac{\frac98 - 2 - 1}{\frac94 + 1 + 1}\langle \frac32, 1, -1\rangle = -\frac{15}{34}\langle \frac32, 1, -1\rangle$

OR

$a_T = \frac{v\cdot a}{|v|} = \frac{\frac98 - 2 - 1}{\sqrt{\frac94+1+1}} = -\frac{15}{4\sqrt{17}}$

(d) Find the normal component of acceleration at time 0.

$\vec a_N(0) = \vec a(0) - \vec a_T(0)$

$= \langle \frac34 + \frac{45}{68}, -2 + \frac{15}{34}, +1 - \frac{15}{34}\rangle$

$= \langle \frac{24}{17}, -\frac{53}{34}, +\frac{19}{34}\rangle$

OR

$a_N(0) = \frac{|v(0)\times a(0)|}{|v(0)|} = \frac{|\langle -1, -\frac94, -\frac{15}{4}\rangle|}{\sqrt{17/4}}$

$= \frac{\frac14|\langle -4, -9, -15\rangle|}{\frac12\sqrt{17}} = \frac{\sqrt{322}}{2\sqrt{17}} = \frac{\sqrt{161}}{\sqrt{34}}$

(e) At time 0, is the particle speeding up or <u>slowing down</u>? How do you know?

$\vec a_T(0)$ is in the opposite direction as $\vec v(0)$

— OR —

$a_T(0) < 0$

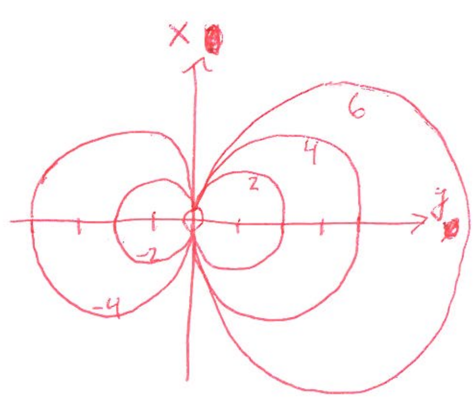
(2) Consider the function $f(x,y) = \dfrac{x^2+y^2}{y}$. Sketch the contour plot for f. (You should include at least five level curves, but you can choose them; the most convenient ones will occur at heights that are even integers.)

$\frac{x^2+y^2}{y} = k$

$x^2+y^2 = ky$

$x^2+y^2-ky+\left(\frac k2\right)^2 = 0 + \left(\frac k2\right)^2$

$x^2 + \left(y-\frac k2\right)^2 = \left(\frac k2\right)^2$



(Drew it sideways; sorry)