Math 241 X8 **Name:** Solutions

Quiz # 6

October 24, 2013 No electronic devices or interpersonal communication allowed. Show work to get credit.

(1) [12pts] Find all sources and sinks of $\mathbf{F}(x,y) = \left\langle \frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2} \right\rangle$.

$$\operatorname{div} \vec{F} = \nabla \cdot \vec{F} = \partial_x\left(\frac{y}{x^2+y^2}\right) + \partial_y\left(\frac{-x}{x^2+y^2}\right)$$

$$= \frac{-2xy}{(x^2+y^2)^2} + \frac{2xy}{(x^2+y^2)^2} = 0.$$

So no sources or sinks except perhaps at the singularity at $(0,0)$.

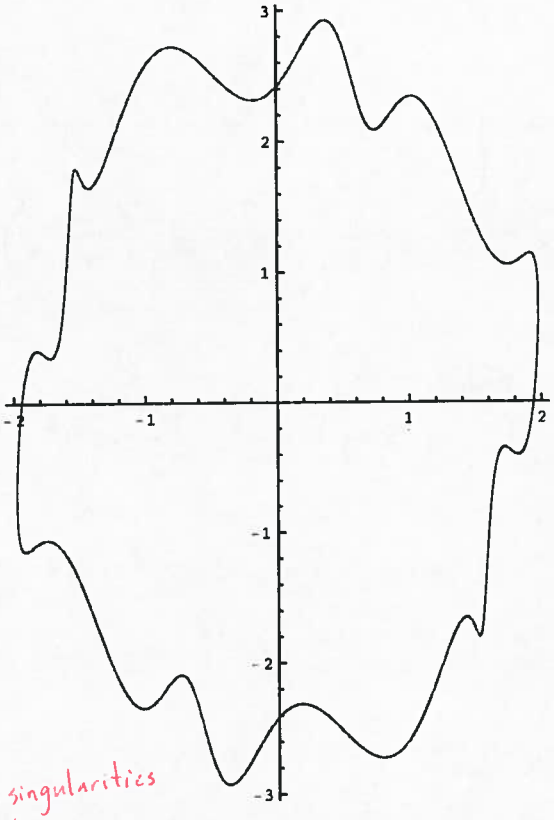
flow across unit circle

$$= \int_0^{2\pi} \left\langle \frac{r\sin t}{r^2}, \frac{-r\cos t}{r^2} \right\rangle \cdot \langle \underbrace{r\cos t}_{dy}, \underbrace{r\sin t}_{-dx} \rangle \, dt$$

$$= \int_0^{2\pi} 0 \, dt = 0.$$

The singularity is also neither a source nor sink.

(2) [8pts] Consider the vector field $\mathbf{F}(x,y) = \langle 3x + 2y, -4x + 9y \rangle$, and the curve C shown below. The area of the region bounded by C is 3π. Compute the flow of $\mathbf{F}$ along C. Which direction is it?



no singularities

flow along C $= \iint_{\text{inside } C} \text{rot } \vec{F} \, dA$

$= \iint_{\text{inside } C} (\partial_x(-4x+9y) - \partial_y(3x+2y)) \, dA$

$= \iint_{\text{inside } C} (-4-2) \, dA$

$= -6 \iint_{\text{inside } C} 1 \, dA = -6 \cdot \text{Area}(\text{inside } C) = -18\pi.$

Flow is clockwise.