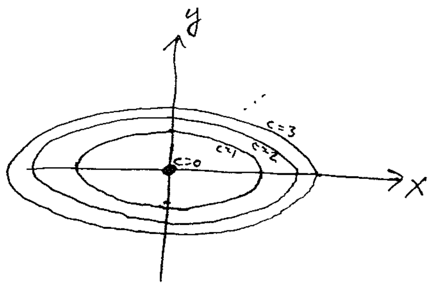
Worksheet Tuesday Sept. 11 Solutions

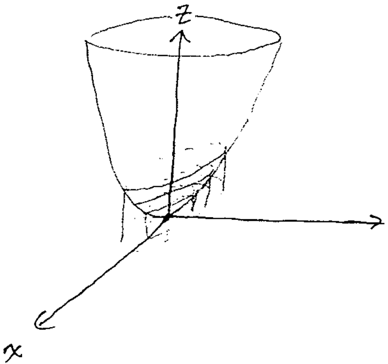
1) $x^2 + 3y^2 = c$ is an ellipse when $c > 0$,

the point $(0,0)$ when $c = 0$,

$\emptyset$ when $c < 0$.

The graph of f is a paraboloid (with elliptic cross-sections).



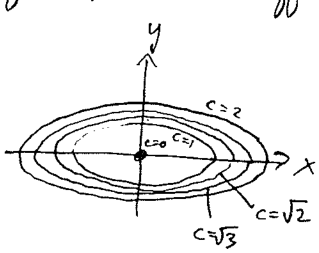


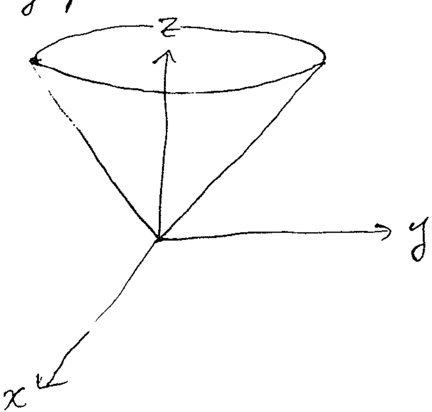
2) $\sqrt{x^2+3y^2} = c$ if $c<0$, $\emptyset$;

$\Rightarrow x^2 + 3y^2 = c^2$ if $c = 0$, $(x,y) = (0,0)$;

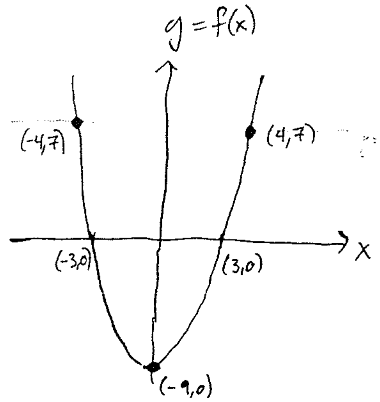
if $c > 0$, get an ellipse.

These are the same cross-sections in (1), but they correspond to different values of c, i.e. different heights in the graph.





3) graph(f):



Let $c = 7$: $x^2 - 9 = 7$

$\iff x^2 = 16$

$\iff x = \pm 4$

The level set corresponding to $c = 7$ is the pair of points -4, 4.

4) $x^2+2y^2+5z^2-3=c$

$\iff x^2+2y^2+5z^2=c+3.$

if $c+3>0$, this is an ellipsoid

" $c+3=0$, " " the point $(x,y,z)=(0,0,0)$

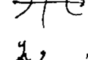
" $c+3<0$, " " $\emptyset$

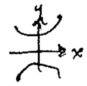
The graph is $3+1=4$-dimensional.

5) $x^2-y^2+10=c$

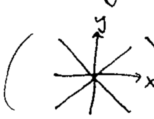
$\iff x^2-y^2=c-10$

if $c-10\neq 0$, this is a hyperbola

(if $c-10>0$, opens ,

if $c-10<0$, opens );

if $c-10=0$, this is the pair of lines $y=x$ & $y=-x$

()

The graph of f is a hyperbolic paraboloid.

6,7) Think about these a bit; we'll answer them in the next couple of weeks ...