

## Math 241 X8

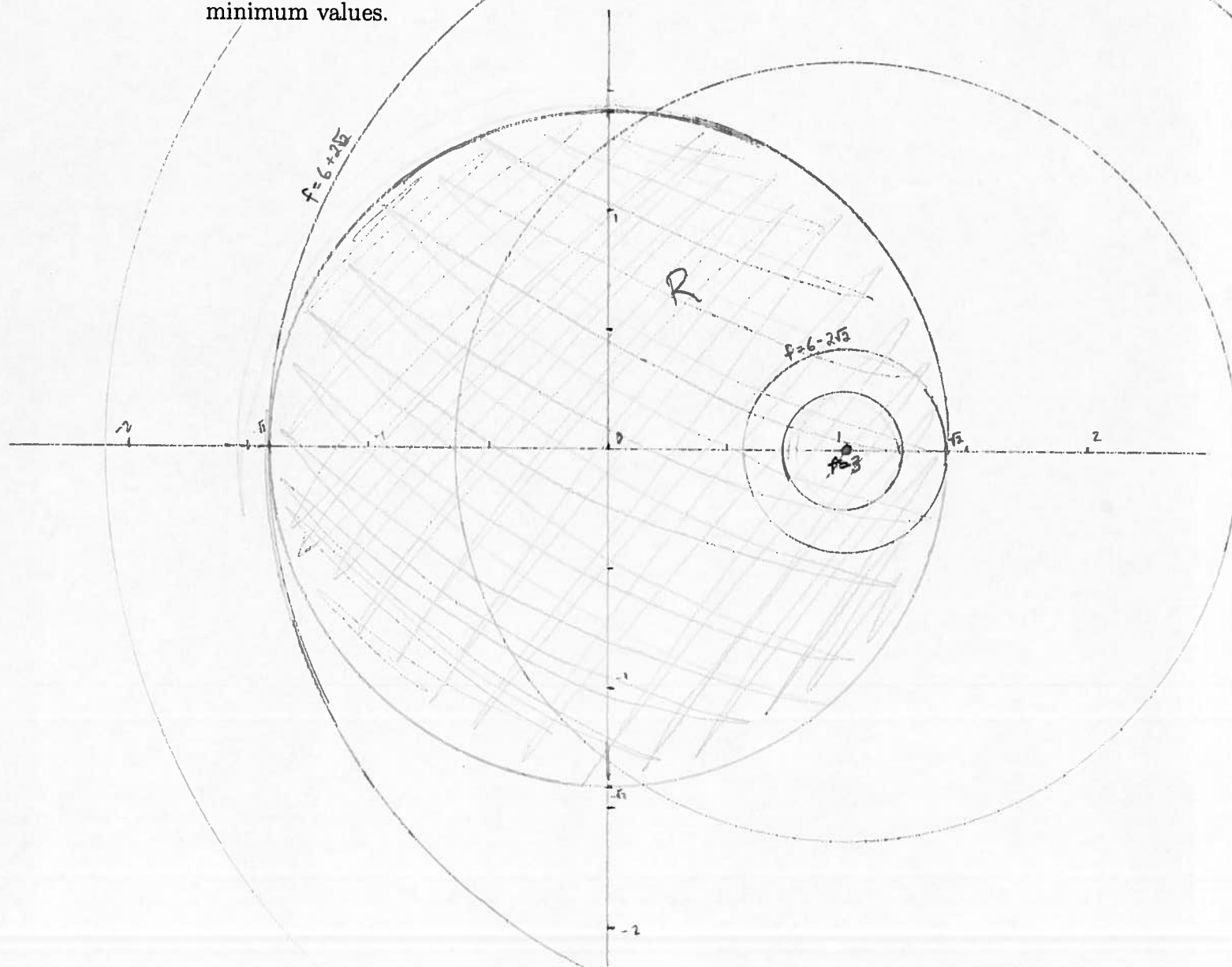
Name(s):

### Homework 4 supplement

This is a written homework supplement to the homework for Unit 4: the Gradient.

Suppose we want to find the minimum and maximum values of  $f(x, y) = (x-1)^2 + y^2 + 3$  on the region  $R$  that is the disk  $x^2 + y^2 \leq 2$ .

- (1) Start by sketching the region in question.
- (2) Add to that sketch the sketch of several of the level curves of  $f$ . (Make sure to have level curves that have different relationships to  $R$ . Don't worry about the value  $f$  takes at these curves yet.)
- (3) Find the maximum and minimum values of  $f$  on  $R$ . (You may want to complete electronic homework 4.4 before this.)
- (4) On your graph, label the level curves of  $f$  that correspond to these maximum and minimum values.



$(x^2+y^2 < 2)$   
For the interior of  $R$ , find where  $\nabla f = \langle 0, 0 \rangle$ :

$$\nabla f = \langle 2(x-1), 2y \rangle = \langle 0, 0 \rangle \Leftrightarrow x=1 \text{ \& } y=0$$

$$f(1,0) = 3$$

For the boundary of  $R$  ( $\underbrace{x^2+y^2}_{g(x,y)} = 2$ ), use Lagrange multipliers:

$$\begin{aligned} \nabla f &= \lambda \nabla g \\ g &= 2 \end{aligned}$$

$$\begin{cases} 2(x-1) = 2\lambda x & \textcircled{1} \\ 2y = 2\lambda y & \textcircled{2} \\ x^2+y^2 = 2 & \textcircled{3} \end{cases} \Rightarrow (\lambda-1)y=0 \Rightarrow \lambda=1 \text{ or } y=0$$

$\textcircled{1} \downarrow$   
 $-1=0$

$\textcircled{2} \downarrow$   
 $x = \pm\sqrt{2}$

$$f(-\sqrt{2}, 0) = (-\sqrt{2}-1)^2 + 3 = 6 + 2\sqrt{2} \approx 8.8$$

$$f(\sqrt{2}, 0) = (\sqrt{2}-1)^2 + 3 = 6 - 2\sqrt{2} \approx 3.2$$

So min is 3, @ (1,0)

max is  $6+2\sqrt{2}$ , @  $(-\sqrt{2}, 0)$ .