Math 241, Sections BL1 and BL2

Quiz # 4 BDL

October 18, 2012

Solve both exercises. Show work to get credit.

1) [5pts.] Evaluate the line integral

$$\int_C \sin x \, dx + \cos y \, dy,$$

where C consists of the top half of the circle $x^2 + y^2 = 16$ from $(4, 0)$ to $(-4, 0)$ followed by the line segment from $(-4, 0)$ to $(-5, 5)$. (Note: One of the properties of conservative vector fields may simplify your calculation.)

The note suggests that the vector field is conservative, so let's try to find a potential function f. We must have

$$f_x = \sin x \qquad f_y = \cos y$$

$$\Rightarrow f = -\cos x + g(y)$$

$$\Rightarrow f_y = 0 + g'(y) = \cos y$$

$$\Rightarrow g(y) = \sin y + c,$$ so $f(x,y) = \sin y - \cos x$ is a potential function (taking $c=0$).

Then FTLI $\Rightarrow$ $$\int_C \sin x\, dx + \cos y\, dy = f(-5,5) - f(4,0)$$

$$= \boxed{\sin(5) - \cos(-5) + \cos(4)}.$$

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By the definition:

C_1: $r(t) = \langle 4\cos t, 4\sin t\rangle$, $t \in [0, \pi]$

$r'(t) = \langle -4\sin t, 4\cos t\rangle$

C_2: $r(t) = \langle -t, 5(t-4)\rangle$, $t \in [4,5]$

$r'(t) = \langle -1, 5\rangle$

$$\int_{C_1} \sin x\,dx + \cos y\,dy$$

$$= \int_0^{\pi} \big(\sin(4\cos t)(-4\sin t) + \cos(4\sin t)(4\cos t)\big)\,dt$$

$u = 4\cos t$, $du = -4\sin t\,dt$; $u = 4\sin t$, $du = 4\cos t\,dt$

$$= \int_4^{-4} \sin u\,du + \int_0^0 \cos u\,du$$

$$= -\cos u \Big|_4^{-4}$$

$$= -\cos(-4) + \cos(4)$$

$$\int_{C_2} \sin x\,dx + \cos y\,dy$$

$$= \int_4^5 \big(\sin(-t)(-1) + \cos(5t-20)(5)\big)\,dt$$

$$= \Big[-\cos(-t) + \sin(5t-20)\Big]_4^5$$

$$= \big(-\cos(-5) + \sin(5)\big) - \big(-\cos(-4) + \sin(0)\big)$$

$$= -\cos(-5) + \sin(5) + \cos(-4)$$

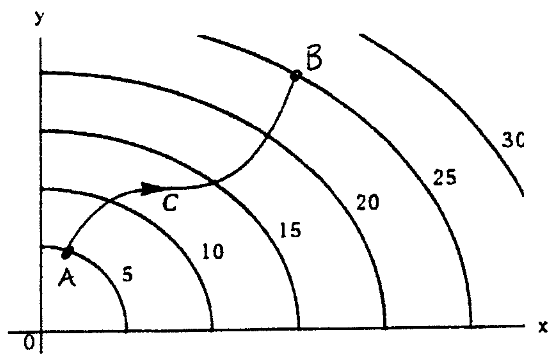
$$\int_C = \int_{C_1} + \int_{C_2} = \boxed{\cos(4) - \cos(-5) + \sin(5)}.$$

2) [5pts.] (a) The figure below shows a curve C and a contour map of a function f whose gradient is continuous. Find

$$\int_C (\nabla f) \cdot d\vec{r}. \quad = f(B) - f(A)$$

$$= 25 - 5$$

$$= \boxed{20}.$$



(b) A table of values of a function f with continuous gradient is given. Find

$$\int_C (\nabla f) \cdot d\vec{r},$$

where C has the parametric equations

$$x = t^4 + 1, \quad y = t^3 + t, \quad t \in [0, 1].$$

x\y	0	1	2
0	1	6	4
1	1	5	9
2	6	3	9

$$= f(x(1), y(1)) - f(x(0), y(0))$$

$$= f(2, 2) - f(1, 0)$$

$$= 9 - 1$$

$$= \boxed{8}.$$