4) Lagrange's method as part of a solution

Here's the problem under consideration. A stove burner is in the shape of a disk of radius 4. The burner heats unevenly; in fact, the temperature at the point {x, y} on the burner is given by

temp[x, y] = 40/(x^2 + (y + 1)^2 + 1) - 30/((x - 1)^2 + (y - 1)^2 + 1) - 0.1 (x^2 + y^2)^2 + 120

where the center of the burner is located at the origin. We want to know where the hottest and coldest spots are.

```
Clear[temp, x, y]
temp[x_, y_] =
 40 / (x^2 + (y + 1) ^2 + 1) - 30 / ((x - 1) ^2 + (y - 1) ^2 + 1) - 0.1 (x^2 + y^2) ^2 + 120
```

$$120 - \frac{30}{1 + (-1 + x)^2 + (-1 + y)^2} - 0.1\left(x^2 + y^2\right)^2 + \frac{40}{1 + x^2 + (1 + y)^2}$$

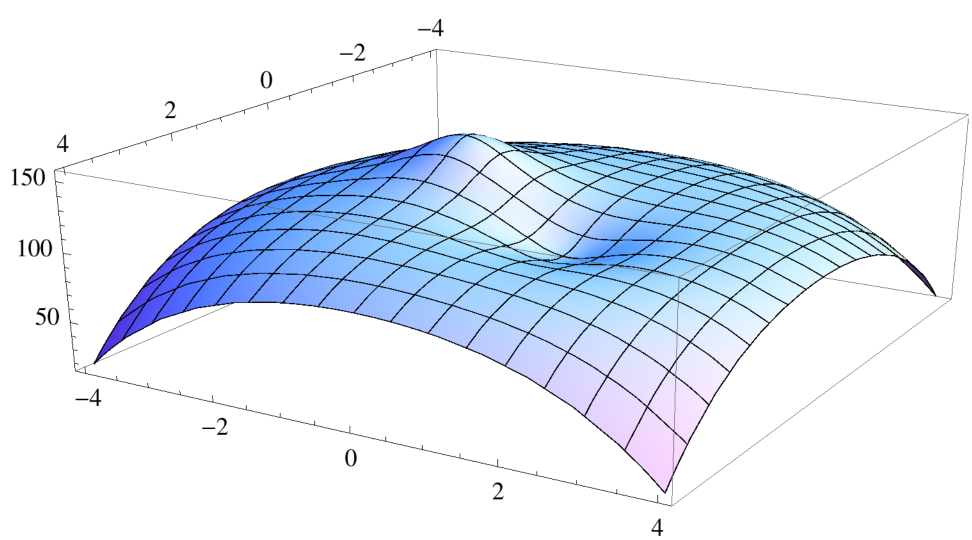
☐ **a)**

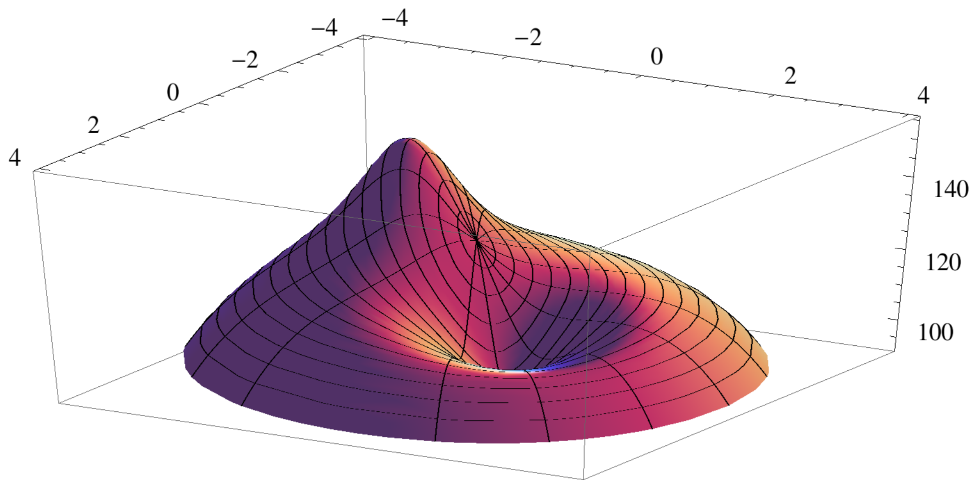
Find all the critical points for the temperature function (as a function on all of $\mathbb{R}^2$).
(Use NSolve, or Mathematica will probably timeout. You' ll get lots of complex points, which you should ignore; you should find three real critical points.)
What is the temperature at these points? Are these points on the burner?

☐ **b)**

Here is a plot of the temperature function, shown first on a big square, then over just the burner.

```
Plot3D[temp[x, y], {x, -4, 4}, {y, -4, 4}, AspectRatio → 0.5, ViewPoint → CMView]
ParametricPlot3D[{s Cos[t], s Sin[t], temp[s Cos[t], s Sin[t]]},
 {s, 0, 4}, {t, 0, 2 Pi}, AspectRatio → 0.5, ViewPoint → CMView + {0, 0, 15}]
```





As you can hopefully see, the minimum temperature on the burner doesn' t occur at any of the critical points. It happens on the boundary of the burner. Use Lagrange' s method to find the maximum and minimum temperatures on the boundary (circle) of the burner. Then say what the maximum and minimum temperatures on the whole burner are, and where they occur.