

MATH 454
HOMEWORK 3 DUE FEBRUARY 8

Name: _____

- Refer to the syllabus regarding allowed collaboration on this homework assignment.
- Refer to other homework instructions and suggestions posted in Blackboard.
- All answers must be fully justified.
- Your homework should be neatly written on additional paper; you may attach this cover page if you would like to keep the questions attached to the answers.

Turn in four of the following problems to be graded. You *must* choose at least one of the hypercube problems.

Solve at least one of the following two hypercube problems.

- Q1 (1.3.26) Count the 6-cycles in Q_3 . Prove that every 6-cycle in Q_k ($k \geq 3$) lies in exactly one 3-dimensional subcube. Use this to count the 6-cycles in Q_k for $k \geq 3$.
- Q2 (1.3.27) Given $k \in \mathbb{N}$, let G be the subgraph of Q_{2k+1} induced by vertices in which the number of ones is k or $k + 1$. Prove that G is regular, and compute $n(G)$, $e(G)$, and the girth of G .
- P1 (a) (1.3.3) Let u and v be adjacent vertices in a simple graph G . Prove that uv belongs to at least $d(u) + d(v) - n(G)$ triangles of G .
- (b) (1.3.41) Prove that if G is an n -vertex simple graph with $\Delta(G) = \lceil n/2 \rceil$ and $\delta(G) = \lfloor n/2 \rfloor - 1$, then G is connected.
- P2 (1.3.9) In a league with two divisions of 13 teams each, determine whether it is possible to schedule a season with each team playing nine games against teams within its division and four games against teams in the other division. (Clearly state your graph theoretic model for this problem.)
- P3 (1.3.17) Let G be a graph with at least two vertices. Prove or disprove:
- (a) Deleting a vertex of degree $\Delta(G)$ cannot increase the average degree.
- (b) Deleting a vertex of degree $\delta(G)$ cannot decrease the average degree.
- P4 (1.3.32) Prove that the number of simple even graphs with vertex set $[n]$ is $2^{\binom{n-1}{2}}$. (Hint: find a bijection with the set of simple graphs with vertex set $[n - 1]$.)