WORKSHEET 2 JANUARY 24, 2011

1. A rope is tied snugly around the equator of the earth. Twenty meters of extra rope is now added to the old rope. The new rope is now held in a circular shape centered about the earth. Which of the following can now walk underneath the rope without touching it: an amoeba, an ant, or you? Guess, then make a calculation. Do you need any information about the size of the earth?
2. Is the function $f(x) = x^2$ one-to-one? Does it have an inverse? Is $g(x) = \sqrt{x}$ an inverse?
3. Is the function $f(x) = x^3$ one-to-one? Does it have an inverse? Is $g(x) = \sqrt[3]{x}$ an inverse?
4. Is the function $f(x) = 1/x$ one-to-one? Does it have an inverse?
5. What is the domain of $f(x) = \sqrt{9 - |x|}$? Range? Sketch its graph.
6. Rewrite each of the following functions as compositions of several elementary functions. Then find their domains and ranges.
   (a) $\sqrt{1 + x^2}$
   (b) $\sin^2 x - 2 \sin x - 3$
   (c) $\cos \sqrt{x^2 - 2x}$
7. How many lines in the plane pass through the points $(1, 3)$ and $(5, 1)$? Find an equation for such a line ℓ. Are there any lines through the origin that are parallel or perpendicular to ℓ? Find equations if they exist.
8. How many parabolas in the plane pass through the points $(1, 3)$ and $(2, 4)$? Are there any such parabolas that also go through the origin? Through $(0, 4)$? Through $(0, 2)$? Through $(1, 1)$? Find equations for them if they exist.
9. How many points do you think are needed to completely determine a polynomial of degree 3? Degree 4?
10. Without performing any calculations, how many points on the line $y = 6x + 22$ have their y-coordinate equal to twice their x-coordinate? Now find all such points.
11. Same question, but now with the line $y = 2x - 13$.
12. In how many points can a line intersect the curve given by $x^2+4x+y^2-8y-5 = 0$? Find equations for a line of each type.
13. Same question, but with the curve $x^2 + 2x + y^2 - 10y + 26 = 0$. (Hint: there's something strange going on here...)
14. Sketch the graphs of the following functions:
    (a) $y = \sin x$
    (b) $y = \sin(x - \pi/3)$
    (c) $y = 4 \sin(x - \pi/3)$
    (d) $y = 4 \sin(2x - \pi/3)$
    (e) $y = 4 \sin(2x - \pi/3) - 5$
15. The earth is now tightly gift-wrapped. (Assume the earth is spherical.) Four hundred square meters of paper are added to the wrapping, and the new material is formed into a spherical shape centered about the earth. Now what can fit

underneath the paper: an amoeba, an ant, or you? Do you need any information about the size of the earth? (The earth's radius is about 6 million meters.)

16. If $\cos\theta = 2/7$ and $0 < \theta < \pi/2$, find $\sin\theta$ and $\tan\theta$. What if $\pi/2 < \theta < \pi$? $\pi < \theta < 3\pi/2$?
17. Simplify $\sin(\arccos(-1/2))$.
18. Sketch the graphs of
    (a) $\tan x$
    (b) $\arctan x$
    (c) $\cos x$
    (d) $\arccos x$
    (e) $\sqrt[3]{x}$
    Your graphs of $\arctan x$ and $\sqrt{x}$ might look similar. There should be one or two major differences though; can you name them?
19. Prove the Pythagorean identities
    (a) $\sin^2 x + \cos^2 x = 1$
    (b) $\tan^2 x + 1 = \sec^2 x$
    (c) $\cot^2 x + 1 = \csc^2 x$
20. Compile among your group a list of trig facts that you know.
21. (a) Rewrite the equation $\log_b x = y$ in terms of an exponential function.
    (b) Explain why $b^n \cdot b^m = b^{n+m}$ makes sense when n, m are positive integers.
    (c) Now translate the property in (b) into a property of logarithms.
    (d) Given that $\log(2) \approx 0.30103$, approximate $\log(64000)$.