Worksheet 3 January 26, 2011

1. Without performing any calculations, how many points on the line $y = 6x+22$ have their y-coordinate equal to twice their x-coordinate? Now find all such points.
2. Same question, but now with the line $y = 2x - 13$.
3. In how many points can a line intersect the curve given by $x^2 + 4x + y^2 - 8y - 5 = 0$? Find equations for a line of each type.
4. Same question, but with the curve $x^2 + 2x + y^2 - 10y + 26 = 0$. (Hint: there's something strange going on here...)
5. In problems 3 and 4, are there lines of each type that have slope π?
6. Sketch the graphs of the following functions:
   (a) $y = \sin x$
   (b) $y = \sin(2x)$
   (c) $y = 4\sin(2x)$
   (d) $y = 4\sin(2x - \pi/3)$
   (e) $y = 4\sin(2x - \pi/3) - 5$
7. If $|\cos\theta| = 2/7$ and $0 < \theta < \pi/2$, find $\sin\theta$ and $\tan\theta$. What if $\pi/2 < \theta < \pi$? $\pi < \theta < 3\pi/2$?
8. Simplify $\sin(\arccos(-1/2))$.
9. Sketch the graphs of
   (a) $\tan x$
   (b) $\arctan x$
   (c) $\cos x$
   (d) $\arccos x$
   (e) $\sqrt[3]{x}$

   Your graphs of $\arctan x$ and $\sqrt[3]{x}$ might look similar. There should be one or two major differences though; can you name them?
10. How many parabolas in the plane pass through the points $(1, 3)$ and $(2, 4)$? Are there any such parabolas that also go through the origin? Through $(0, 4)$? Through $(0, 2)$? Through $(1, 1)$? Find equations for them if they exist.
11. How many points do you think are needed to completely determine a polynomial of degree 3? Degree 4?
12. Prove the Pythagorean identities
    (a) $\sin^2 x + \cos^2 x = 1$
    (b) $\tan^2 x + 1 = \sec^2 x$

(c) $\cot^2 x + 1 = \csc^2 x$

13. Compile among your group a list of trig facts that you know.

14. (a) Rewrite the equation $\log_b x = y$ in terms of an exponential function.
    (b) Explain why $b^n \cdot b^m = b^{n+m}$ makes sense when n, m are positive integers.
    (c) Now translate the property in (b) into a property of logarithms.
    (d) Given that $\log(2) \approx 0.30103$, approximate $\log(64000)$.