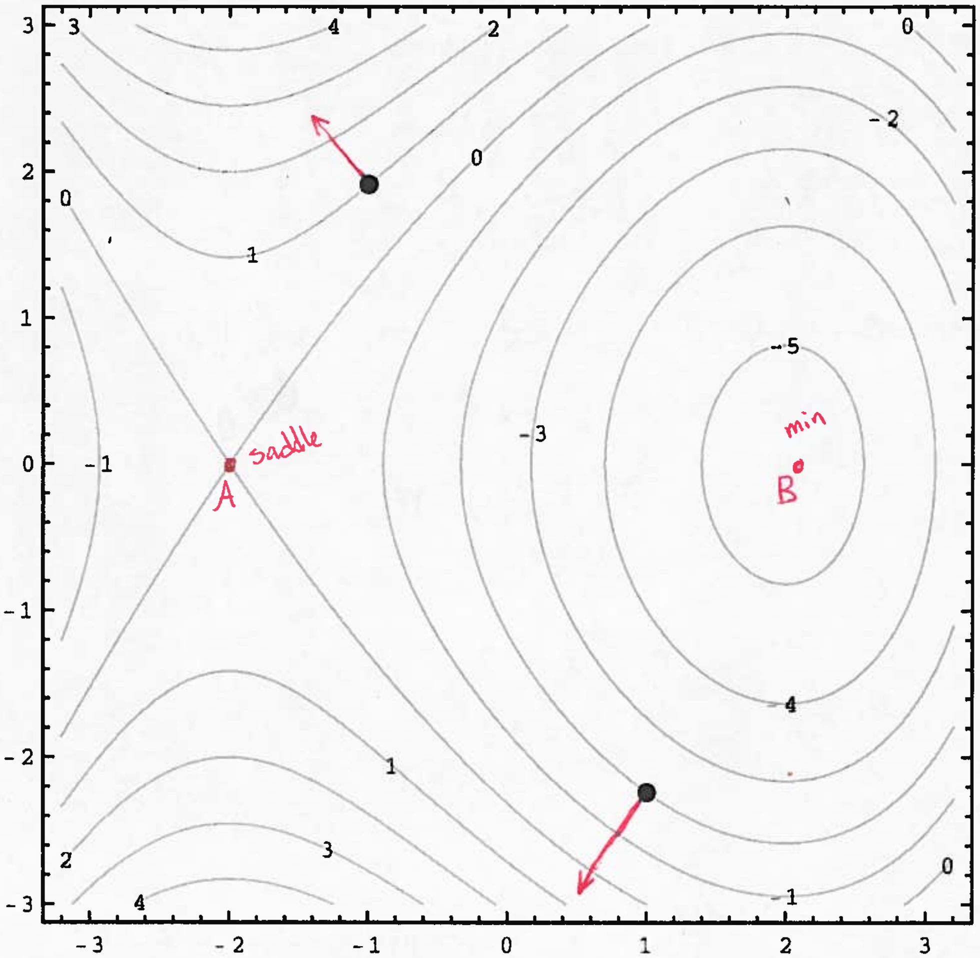
Math 241 X8 **Name:** Solutions

Quiz # 3

September 24, 2013 No electronic devices or interpersonal communication allowed. Show work to get credit.

Let $f(x,y) = \frac{1}{2}y^2 + \frac{1}{6}x^3 - 2x - \frac{8}{3}$. A contour map for f is shown below, with integer-valued level curves.



1) [4pts.] At each of the two points shown, sketch in the gradient vector of f with its tail at that point.

2) [5pts.] Mark the (approximate) locations of the critical points of f. Classify them (as local max/min/saddle). How do you know?

Geometric: in the y-direction, A is a min & in the x-direction it is a max

the level curves decrease as they center in on B.

· 2^{nd} Deriv Test: $\begin{cases} \partial_x f = \frac{1}{2}x^2 - 2 \\ \partial_y f = y \end{cases}$ $D = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} x & 0 \\ 0 & 1 \end{vmatrix} = x$

$\nabla f = \langle 0,0 \rangle$

$\Leftrightarrow x = \pm 2$ & $y = 0$ $D(A) \approx -2 < 0$ $D(B) \approx 2 > 0$.

3) [8pts.] Let $f(x,y) = 3x + 4y$. Find the maximum and minimum values of f on the region $x^2 + 2y^2 \leq 17$.

Let $g(x,y) = x^2 + 2y^2$.

Interior: $\nabla f = \langle 3,4 \rangle \neq \langle 0,0 \rangle$, so no crit. pts.

Boundary: Use Lagrange multipliers:

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 17 \end{cases} \Rightarrow \begin{cases} 3 = 2\lambda x & (1) \\ 4 = 4\lambda y & (2) \\ x^2 + 2y^2 = 17 & (3) \end{cases}$$

$(2) \div (1)$: $\frac{4}{3} = 2\frac{y}{x}$ or $x = 0$ ↯ (1)

$$\Rightarrow y = \frac{2}{3}x \quad (4)$$

$$\overset{(3)}{\Rightarrow} x^2 + 2\left(\frac{2}{3}x\right)^2 = 17$$

$$\Rightarrow \frac{17}{9}x^2 = 17$$

$$\Rightarrow x = \pm 3$$

$$\overset{(4)}{\Rightarrow} y = \pm 2$$

(same sign)

$f(-3,-2) = -17$

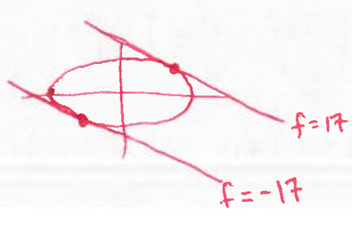
$f(3,2) = 17$

(Region is closed & bounded, so max & min exist, so)

min of -17 @ $(-3,-2)$

max of 17 @ $(3,2)$

(Note:



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