Worksheet 12 March 2, 2011

1. Let $f(x) = x^{2/3}$ and $g(x) = x^3$. Show that the composition of these functions in either order is differentiable at $x = 0$ but that f is not differentiable at $x = 0$. Does this contradict the chain rule? Explain.
2. Suppose that $g(0) = 0$ and $g'(0) = 2$. What is the derivative of $g(g(g(x)))$ at $x = 0$?
3. Find the derivative of $\sec^2 x - \tan^2 x$. (Do this the straightforward way first; after you get an answer, simplify, then try to realize why this problem could have been done last chapter.)
4. We'll now compute the derivative of x^x. Can you use the power rule? Exponential derivative rule? So we need a trick: let $y = x^x$, take logarithms, then differentiate. (By the way, what's the domain of this function? Of its derivative?)
5. Find all points on the curve defined by $(x^2 + y^2)^2 = x^2 - y^2$ where the tangent line is horizontal or vertical (there are four and two of these, respectively).
6. Find the derivatives of the following functions.
   (a) $\sec^3(\sqrt{\cos x})$
   (b) $\sqrt{t^3 + \sin t} \cdot \sin(\sqrt{e^t + 1})$
   (c) $\dfrac{u^3 + \sqrt[3]{u}}{\sin(\cos u)}$
   (d) $\sqrt{\dfrac{\tan x}{\sec x}}$
   (e) $\sqrt{\dfrac{\tan x}{\ln x}}$
7. Solve the differential equation $\dfrac{dy}{dx} = ky$, $y(0) = 5$.
8. Compute $\dfrac{dy}{dx}$ when $\sqrt{x} + \sqrt{y} = 1$.
9. Let $f(t) = e^{at}$, where a is some constant. Determine $f^{(n)}(t)$, the nth derivative of f.
10. Let $g(t) = \cos(at)$; what is $g^{(n)}(t)$? (Hint: you'll need some cases.)
11. Compute all the derivatives of $P(x) = x^5 + x^4 + x^3 + x^2 + x + 1$.
12. If $f(x) = x^n$, what are $f^{(n)}(x)$ and $f^{(n+1)}(x)$?
13. A metal disk is heating in the sun. As it heats, it expands. If the radius is expanding at a rate of 1cm/hr, how fast is the area increasing? (Hint: start with a formula relating radius and area; note that both of these are functions of time in this situation, so implicit differentiation is more complicated this time.)