Math 415 ADG   Name: *Solution*

Quiz # 8

April 11, 2014

No notes, electronic devices, or interpersonal communication allowed. **Show work to get credit.** Use the methods from this class.

Apply Gram-Schmidt to the basis $\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix} \right\}$.

$\underbrace{\hphantom{x}}_{a_1} \quad \underbrace{\hphantom{x}}_{a_2} \quad \underbrace{\hphantom{x}}_{a_3}$

$q_1 = a_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

$q_2 = a_2 - \text{proj}_{q_1} a_2 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} - \dfrac{(2+0+3)}{(4+0+1)} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

$q_3 = a_3 - \text{proj}_{q_1} a_3 - \text{proj}_{q_2} a_3 = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix} - \dfrac{(6+0-1)}{(4+0+1)} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} - \dfrac{(-3-7-2)}{(1+1+4)} \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$

$\left(\{q_1, q_2, q_3\} \text{ is orthogonal; need orthonormal} \right)$

$$\left\{ \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \ \frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \ \frac{1}{\sqrt{30}} \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix} \right\}$$