Worksheet 14 March 9, 2011

1. There are two ways a point can by a *critical point* for a curve; what are they?

2. Make 6 sketches: for each of the two kinds of critical points, sketch situations where such a critical point is (1) a maximum, (2) a minimum, and (3) neither.

3. My Halloween costume often involves LEDs wired so that I can activate them by touching my fingers together. To avoid frying all the bulbs, I need some resistors. If two resistors with resistances R_1 and R_2 are wired in parallel, then the total resistance R_t is given by $\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2}$. If I use a device with variable resistance (not so useful for my costume, but much more useful for temperature controllers), then all three resistances may be functions of time. If R_1 is increasing at a rate of 0.3 ohms/second, R_2 is increasing at 0.2 ohms/second, and $R_1 = 80$ ohms, $R_2 = 100$ ohms, then what is R_t and how fast is it changing?

4. In section 3.10 of your textbook, linear approximations are discussed. Linear approximations are just another way of talking about tangent lines: tangent lines are the best way to approximate a function by a linear function (which is nice because linear functions are much simpler than most other functions). Compute the linear approximation to $\sin x$ at $x = 0$. (Remark: this should be an easy problem, but the answer is *very* commonly used in various fields of science.)

5. We might want to allow somewhat more complicated functions as approximations, say quadratics. You might call a parabola the *tangent parabola* to a curve $y = f(x)$ at a point $x = a$ if its value, derivative, and also second derivative match those of the function at the point a.

   (a) The generic equation of a quadratic is $ax^2 + bx + c$ for some constants a, b, c. What are the value of this function, its first derivative, and its second derivative at $x = 0$?

   (b) Use this information to find the equation of the tangent parabola to $\sin x$ at $x = 0$.

   (c) Find the equation of the tangent parabola to $x^3 + 2x^2 - 4$ at $x = 0$.

   (By the way, the above idea is the foundation of what I think is the most interesting part of Calc2.)

6. An object with weight w is attached to a rope. The rope is pulled with an angle of θ from the horizontal, dragging the object along the floor. If the floor has a *coefficient of friction* μ, then the magnitude F of the force required to keep the object moving is given by

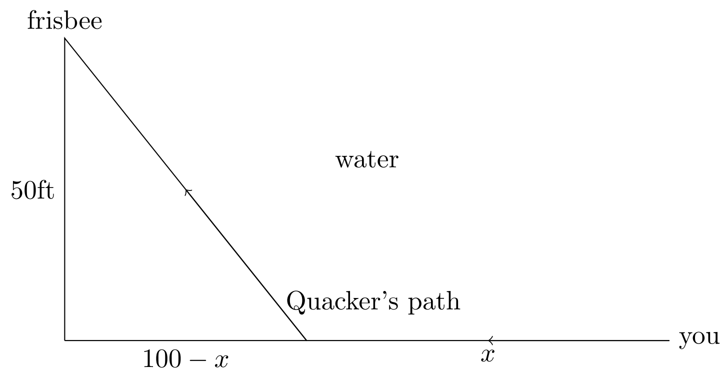
$$F = \frac{\mu w}{\mu \sin\theta + \cos\theta}.$$

   For what angle of θ is F minimized? (That is, what angle should you pull at to minimize the work you do?)

7. For what value of k does the equation $e^{2x} = k\sqrt{x}$ have exactly one solution? (For some insight, consider the graphs of the functions on either side of the equation; for differing values of k, how many solutions *can* the equation have?)

8. Find the rectangle of maximum area that can be inscribed in the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$. (You can assume that such a rectangle has sides parallel to the axes. Make use of symmetry.)

9. There seems to be some evidence that dogs know calculus. Let's say you're throwing a frisbee for your dog Quackers near the beach. To make things simple, let's say you're both standing right where the water reaches the land. You throw your frisbee into the water, 100ft along the beach and 50ft into the water. Quackers would like to retrieve it in the shortest amount of time possible. He might run straight through the water, but he swims slowly: only 10ft/s, whereas he can run 20ft/s. Calculate the minimum amount of time needed for Quackers to get to the frisbee. (Hint: Say Quackers runs x feet along the beach before jumping in the water, then swims diagonally $100 - x$ ft parallel to the beachfront and 50 ft perpendicular, as in the diagram below. Then how much time does he spend on the beach, and how much time in the water?) In some limited experiments, dogs have actually chosen paths remarkably similar to the optimal one.



10. Make a fairly good sketch of the graph of $f(x) = (x-8)^{10}(x+3)$ using the function and its first two derivatives. In doing this, you should list the intervals on which it is increasing, decreasing, concave up, and concave down, and list any local extrema. (Suggestion: make your vertical scaling very large.) What are the global minimum and global maximum on $[-3, 0]$? On $[0, 3]$?