Name: ____________________

- **READ THE FOLLOWING DIRECTIONS!**
- **Do NOT open the exam until instructed to do so.**
- You have 75 minutes to complete this exam. When you are told to stop writing, do it or you will lose all points on the page you write on.
- You may not communicate with other students during this test.
- No written materials of any kind are allowed. No scratch paper is allowed except as given by the proctors.
- No phones, calculators, or any other electronic devices are allowed for any reason, including checking the time (a simple wristwatch is fine).
- Any case of cheating will be taken extremely seriously.
- Show all your work and explain your answers.
- Before turning in your exam, check to make certain you've answered all the questions.

1. Let $\mathbf{u} = \langle 2, 6\rangle$ and $\mathbf{v} = \langle -2, -1\rangle$. Compute and plot the following together with $\mathbf{u}$ and $\mathbf{v}$.
   (a) $\mathbf{u} + \mathbf{v}$

   (b) $2\mathbf{v}$

   (c) the angle between $\mathbf{u}$ and $\mathbf{v}$

   (d) the projection of $\mathbf{u}$ in the direction of $\mathbf{v}$

   (e) If $\mathbf{u}$ and $\mathbf{v}$ live in the plane of this paper, and you consider the paper in the 3D classroom, then which direction does $\mathbf{u} \times \mathbf{v}$ point?

2. Consider the lines $\ell_1(t) = (0,1,3)+t(-2,-2,-10)$ and $\ell_2(t) = (-5,2,3)+t(-9,9,30)$. Are they parallel, perpendicular, or neither? Do they intersect or not? If they intersect or are parallel, find an xyz-equation of the plane containing them. Otherwise find the distance between them.

3. Consider the lines $\ell_1(t) = (0,1,3) + t(2,1,5)$ and $\ell_2(t) = (3,0,1) + t(-2,-1,1)$. Are they parallel, perpendicular, or neither? Do they intersect or not? If they intersect or are parallel, find an xyz-equation of the plane containing them. Otherwise find the distance between them.

4. Consider the two planes given by equations

$$\begin{aligned} 3x - y + z &= 4 \\ 2x + y - 2z &= 6. \end{aligned}$$

Find an equation of the line that is the intersection of these planes.

5. Consider the two planes given by equations

$$\begin{aligned} 3x + 12y - 3z &= 1 \\ 2x + 8y - 2z &= 7. \end{aligned}$$

Find the distance between them.

6. Consider the triangle with vertices $(1, 0, 5)$, $(2, -1, 2)$, and $(3, 1, -1)$.
   (a) Find its area.

   (b) Is it acute, right, or obtuse?

7. Identify by name the following surfaces, and sketch them.

   (a) $z = (x-1)^2 + 4y^2$

   (b) $(z-1)^2 = x + 4y^2$

   (c) $\dfrac{x^2}{4} - \dfrac{y^2}{9} + \dfrac{z^2}{25} = -1$

   (d) $x^2 + y^2 - 2x + 4y = z^2 - 3$

   (e) $1 = xz$

   (f) $1 = x + z$

8. Suppose a particle moves in the plane, with position at time t given by (t^2, t^3) at time t.

   (a) Find the velocity at time $t = 9$.

   (b) Find the acceleration at time $t = 9$.

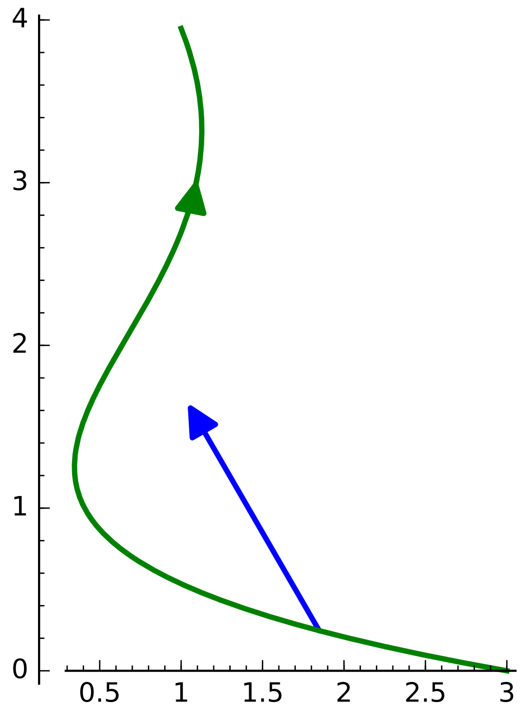
   (c) Find the tangential component of acceleration at time $t = 9$.

   (d) Find the normal component of acceleration at time $t = 9$.

   (e) What do you know about how the speed of the particle is changing at $t = 9$?

9. Parametrize the intersection of the surfaces $z = 4x^2 + y^2$ and $y = x^2$.

10. Here is a plot of a particle's position (in green). In blue is the particle's acceleration at a particular time (with its tail located at the particle's position at that time).



(a) Is the particle speeding up or slowing down at that time? Why?

(b) If you were given the position function $\mathbf{r}(t)$ and that t ran from 1 to 3, then give a formula for the distance traveled. Estimate the distance traveled.

11. Compute the curvature of a circle of radius r.

12. Consider the vector function $\mathbf{r}(t) = \langle \cos t,\ \sin t,\ \ln(\cos t)\rangle$ ($t \in [-\pi/2, \pi/2]$). Find $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$ at the point $(1, 0, 0)$. Find an equation for the tangent line to the curve at $(1, 0, 0)$.

13. Find the following limits if they exist.

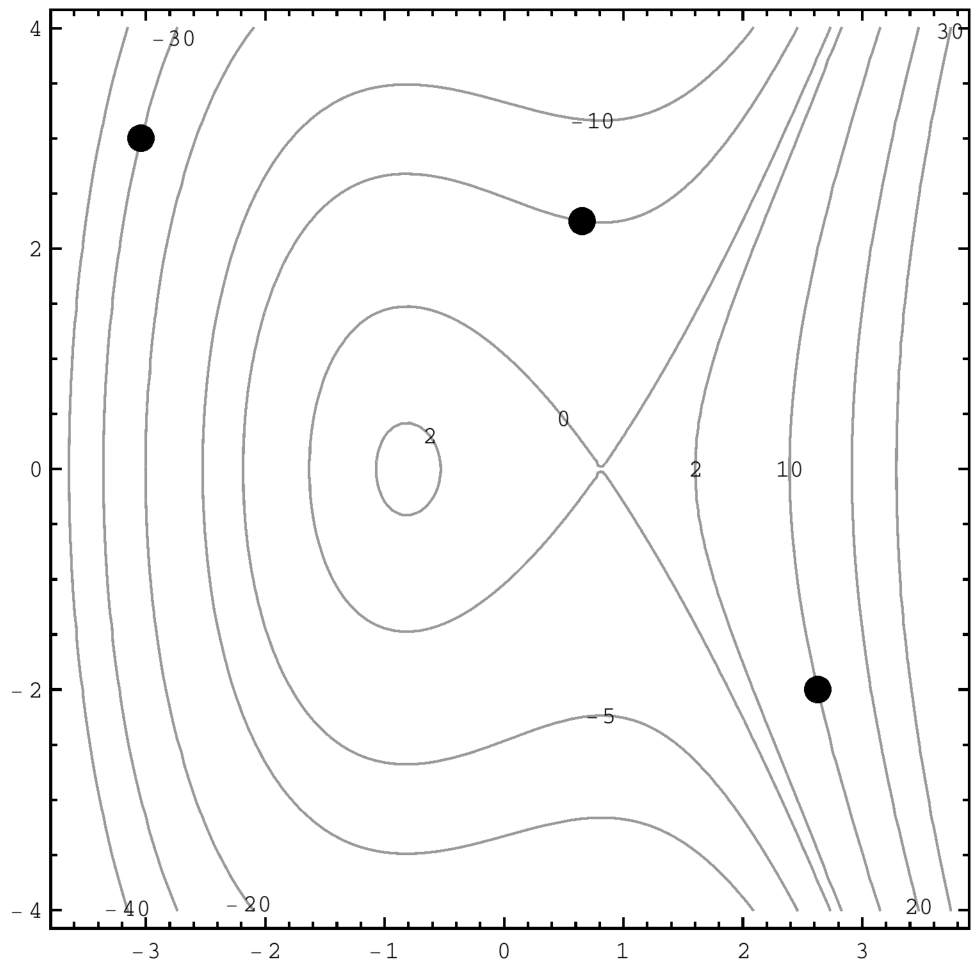
(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2ye^{-1/y^2}}{x^2+y^2}$

(d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$

14. Below is a plot of several level curves of a function $f(x, y)$ (they are NOT at equally-spaced heights). At the indicated points, sketch in the gradient vectors. At the lower-right point, is f_{xy} positive, negative, or zero? At the upper-central point, estimate f_x and f_y. Find the (approximate) locations of the critical points of f, then classify them.



15. Find the linearization of $f(x, y) = x^2 e^y$ based at the point $(1, 0)$.

16. Approximate $f(1.1, 0.1)$ using the linearization above.

17. Suppose $f(x, y, z)$, $x(s, t)$, $y(s, t)$, and $z(s, t)$ are all differentiable everywhere. You are given the following:

$x(1,2) = 3$	$y(1,2) = 4$	$z(1,2) = 5$
$x_t(1,2) = -1$	$y_t(1,2) = -2$	$z_t(1,2) = -3$
$x_s(1,2) = 7$	$y_s(1,2) = 8$	$z_s(1,2) = 9$
$f(-1,-2,-3) = 11$	$f(0,0,0) = 12$	$f(3,4,5) = 10$
$f_x(-1,-2,-3) = -7$	$f_x(0,0,0) = 0$	$f_x(3,4,5) = -4$
$f_y(-1,-2,-3) = -8$	$f_y(0,0,0) = -9$	$f_y(3,4,5) = -5$
$f_z(-1,-2,-3) = -10$	$f_z(0,0,0) = -11$	$f_z(3,4,5) = -6$

Compute $\dfrac{\partial f}{\partial t}$ at $(s, t) = (1, 2)$.

18. Find and classify the local extrema of $f(x, y) = x^3 + y^3 - 3x^2 - 3y^2 - 9x$.

19. (a) Find the maximum and minimum values of $f(x, y) = xy$ on the disk $x^2 + y^2 \leq 4$.

(b) Draw the disk together with any critical points you found and the level curves of f corresponding to the values at those critical points.