Math 415 ADG **Name:** Solution

Quiz # 7

April 4, 2014

No notes, electronic devices, or interpersonal communication allowed. Show work to get credit. Use the methods from this class.

Find the least-squares solution to the equation

$$\underbrace{\begin{bmatrix} 1 & 3 \\ 0 & -2 \\ 2 & 1 \end{bmatrix}}_{A} \mathbf{x} = \underbrace{\begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}}_{b}.$$

Least-squares solution is same as solution $\hat{x}$ to

$$A^TA\hat{x} = A^Tb.$$

$$A^TA = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 5 & 14 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 14 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 5 & 5 & 0 \\ 5 & 14 & 14 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cc|c} 5 & 5 & 0 \\ 0 & -9 & 14 \end{array}\right] \xrightarrow{R_1 \div 5} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -9 & 14 \end{array}\right]$$

$$\begin{cases} x_1 + x_2 = 0 \\ -9x_2 = 14 \end{cases} \Rightarrow x_2 = -\frac{14}{9} \Rightarrow x_1 = +\frac{14}{9}$$

$$\begin{bmatrix} 14/9 \\ -14/9 \end{bmatrix}$$