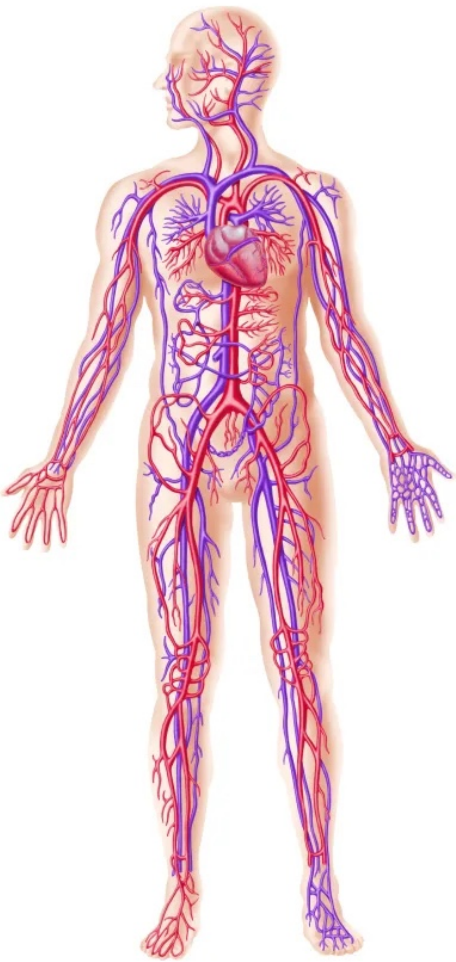
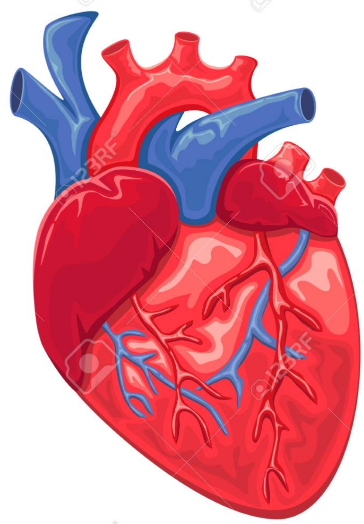
Fluid Mechanics

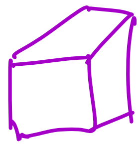


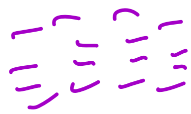


Fluid Mechanics

Introduction

Fluid

Normal force →  solid (more) — Eng. mechanics, SOM

force →  liquids, gases — Fluid mechanics

shear stress

deformation } solids

Properties of fluids

① Density : $\frac{\text{mass of the fluid}}{\text{volume}} = \frac{m}{\forall}$ $\underline{Kg/m^3}$ "ρ" density Kg/m^3

ρ_w water is a std. fluid

$\rho_w = 1000\ Kg/m^3$

② Weight density / Specific weight

$\gamma = \frac{\text{weight of the fluid}}{\text{volume}} = N/m^3$ $\gamma_w = 9810\ N/m^3 = 9.810\ KN/m^3$

③ Specific volume $v = \frac{\text{volume}}{\text{mass}} = \frac{1}{\rho} \Rightarrow m^3/Kg$

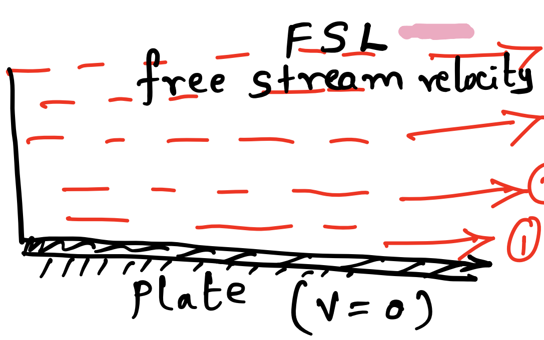
④ Specific gravity [S G] : $\frac{\rho \text{ of the fluid}}{\rho \text{ of the std fluid}} = \frac{\gamma \text{ of the fluid}}{\gamma \text{ of the std. fluid.}}$

$\gamma_w = 9810\ N/m^3 = 9.810\ kN/m^3$

$$SG_{fluid} = \frac{\rho_f}{1000} \text{ or } \frac{\gamma_f}{9810} \quad \frac{Kg/m^3}{Kg/m^3} \text{ or } \frac{N/m^3}{N/m^3}$$

5 Viscosity [Dynamic]

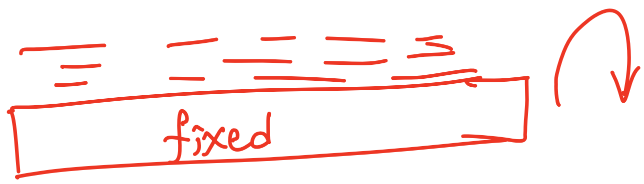
Resistance offered b fluid for movement one layer to other

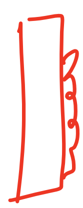


u — max
3 — less max
2 — less 3
1 — less
0

layers

Kinematic } Viscosity
Dynamic





FSL

→ u + 3du → free stream veloci-

→ u + 2du

→ u + du } viscosity

→ u

Viscosity

Dynamic viscosity → Poise

Kinematic viscosity → Stoke

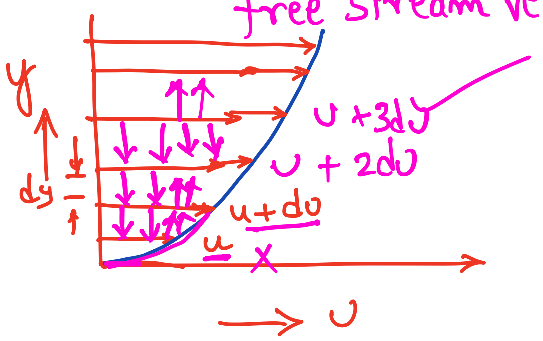
Dynamic viscosity

Newton's law of viscosity

"shear stress acting on the fluid is proportional velocity gradient."

τ shear stress $= 0$ static fluid

free stream velocity U/V



$\tau \propto \frac{du}{dy}$ Newton's law of viscosity

$\tau = \mu \frac{du}{dy}$ 'μ' dynamic viscosity

$\mu = \frac{\tau}{\frac{du}{dy}} = Ns/m^2$

Poise

$\mu = \frac{\tau}{\frac{du}{dy}}$ $\boxed{Ns/m^2 \text{ or Poise}}$

$1\ Ns/m^2 = 10$ poise

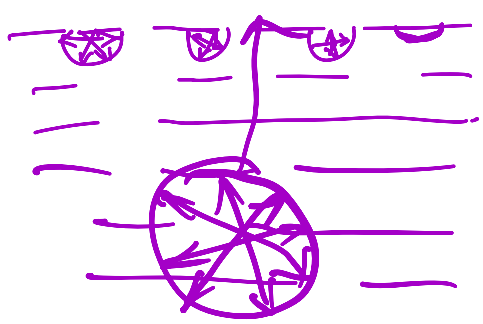
Viscosity $\rightarrow$ dynamic viscosity (μ)

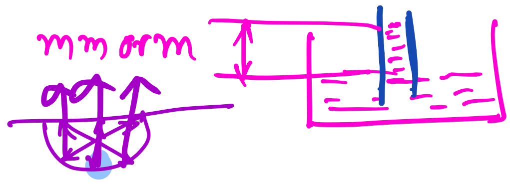
Kinematic viscosity $\rightarrow$ $\nu = \dfrac{\mu}{\rho} = \dfrac{\text{dynamic viscosity}}{\text{density of the fluid}} = \boxed{m^2/s}$ or stoke

$\boxed{1 \text{ stoke} = 10^{-4}\, m^2/s}$

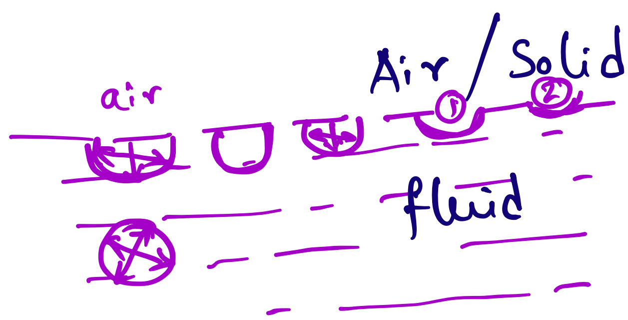
Capillarity $\rightarrow$: Raise or fall the fluid compared adjacent level of fluid in a small glass tube. mm or m of fluid

Surface tension $\rightarrow$: N/m





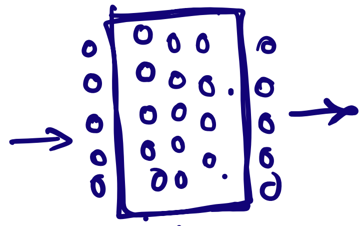
Tensile force acting on the surface of the fluid., N/m



Concept of continuum

ρ, μ, γ, SG

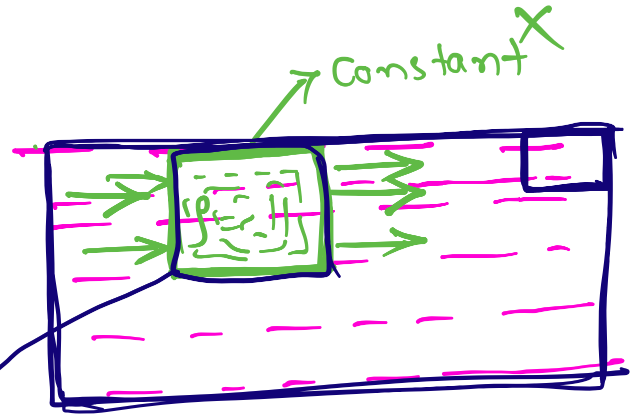
$$\rho = \frac{\text{mass}}{\text{volume}}\,\text{fluid} = \frac{dm}{dV} = \frac{m}{V}$$



t, dm

$t+\Delta t = \boxed{dm + \Delta dm}$

$$\rho = \frac{dm}{dV} \checkmark$$



$$\rho = \frac{dm}{dV}$$

$$\rho = \frac{m}{V} \times$$

$$\rho = \frac{dm}{dV} \checkmark$$

γ

SG, μ, ν

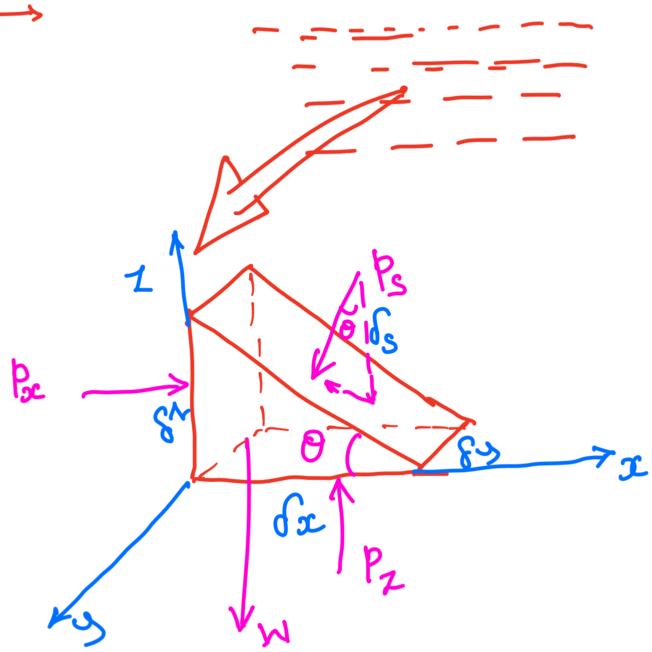
$$\rho = \frac{dm}{dV} = \frac{m}{V}$$

Volume of fluid

↓

mass of the fluid

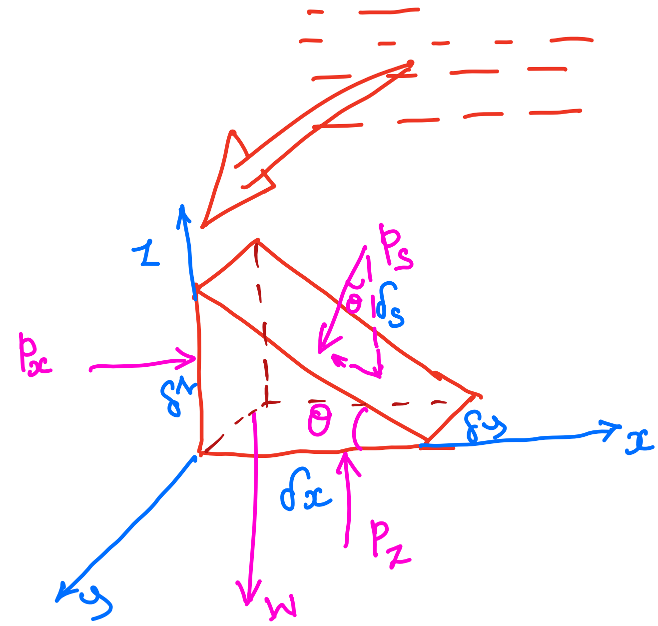
Pascal's law



$F = ma$

Statement: Pressure at a point is independent of direction as long as there are no shear stresses present.

Pressure is uniformly distributed.
Pressure is equal in all directions



$$\Sigma F = ma$$

$$\Sigma F_x = ma_x, \quad \Sigma F_z = ma_z$$

$$\Sigma F_x = P_x \delta y \delta z - P_s \sin\theta \, \delta y \, \delta s = m a_x \quad \text{---- (1)}$$

$$\Sigma F_z = P_z \delta x \delta y - P_s \cos\theta \, \delta y \, \delta s - W = m a_z \quad \text{---- (2)}$$

eqn (1)

$$P_x \delta y \delta z - P_s \frac{\delta z}{\cancel{\delta s}} \delta y \cancel{\delta s} = \underset{\text{density}}{\rho} \underbrace{\frac{\delta x \delta y \delta z}{2}}_{\to \text{volume}} a_x$$

$$[P_x - P_s] \cancel{\delta y \delta z} = \rho \frac{\delta x \cancel{\delta y \delta z}}{2} a_x$$

$$P_x - P_s = \rho \delta x \frac{a_x}{2} \quad \text{-------- (3)}$$

From (2)

$$P_z \delta x \delta y - P_s \frac{\delta x}{\cancel{\delta s}} \delta y \cancel{\delta s} - \rho g \frac{\delta x \delta y \delta z}{2} = \rho \frac{\delta x \delta y \delta z}{2} a_z$$

$$[P_z - P_s]\,\delta x\,\delta y = \rho\,\frac{\delta x\,\delta y\,\delta z}{2}\,a_z + \rho g\,\frac{\delta x\,\delta y\,\delta z}{2}$$

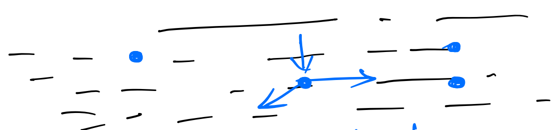
$$[P_z - P_s]\,\cancel{\delta x\,\delta y} = \rho\,\frac{\cancel{\delta x\,\delta y}\,\delta z}{2}\,[a_z + g]$$

$$[P_z - P_s] = \rho\,\frac{\delta z}{2}[a_z + g] \quad \text{-------} \quad (4)$$

For a point, δx, δy & δz limits tending to zero

$$\left.\begin{array}{l} P_x - P_s = 0 \\ P_z - P_s = 0 \end{array}\right\} \qquad P_x = P_s = P_z$$

Hence Pascal's law is proved.

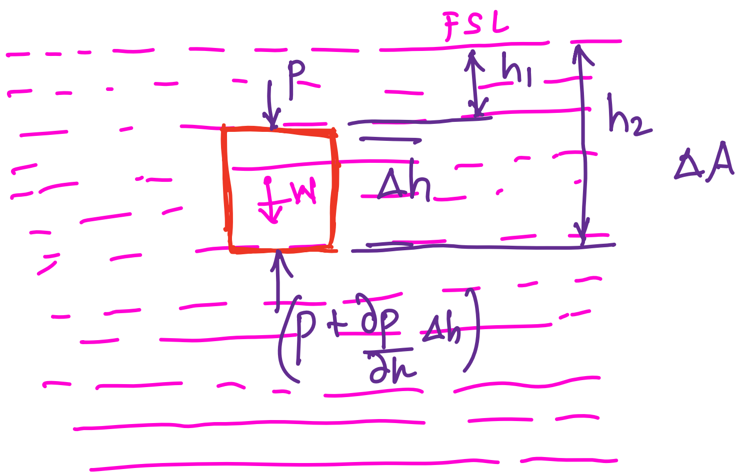


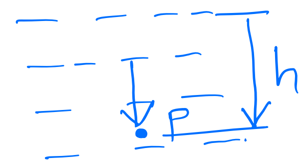
Pascal's law

Hydrostatic law

Pressure acting in vertical downward direction is equal to specific weight of the fluid.

$$\frac{P}{h} = \gamma$$





$$\frac{P}{h} = \gamma = \rho g$$

$$P = \rho g h$$
$$= \gamma h$$

$$\Sigma F = 0, \quad p\Delta A - \left(p + \frac{\partial p}{\partial h}\Delta h\right)\Delta A + W = 0$$

$$p\cancel{\Delta A} - p\cancel{\Delta A} - \frac{\partial p}{\partial h}\Delta h \Delta A + \rho g \Delta A \Delta h = 0$$

$$\frac{\partial p}{\partial h}\cancel{\Delta h \Delta A} = \rho g \cancel{\Delta A \Delta h}$$

$$\frac{\partial p}{\partial h} = \rho g = \gamma$$

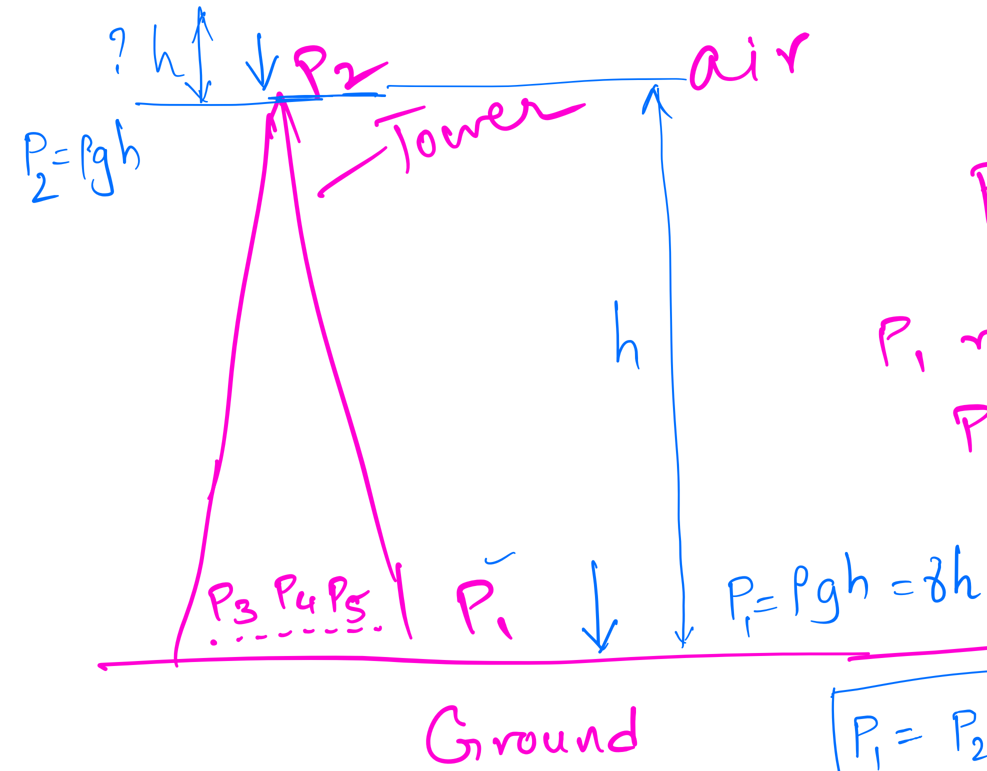
$$\partial P = \rho g \partial h$$

$$P = \rho g h$$

$$\boxed{\frac{P}{h} = \rho g = \gamma}$$

$$P = \rho g h = \gamma h$$

Hence hydrostatic law is proved.

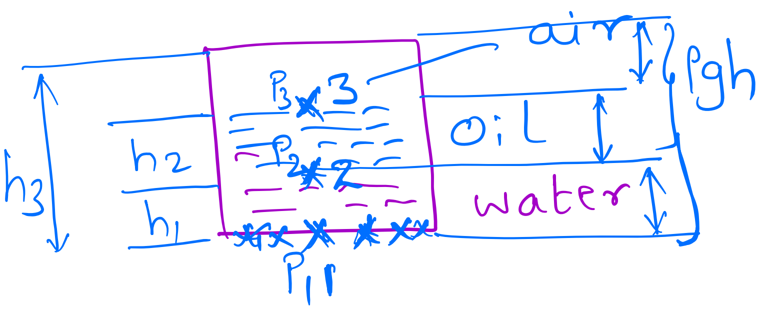


$P_1 > P_2$

P_1 related

$P_3, P_4, P_5 \ldots$

$$\boxed{P_1 = P_2 + \rho g h}$$



$P_{bottom} =$

$$P_3 = \underbrace{\rho g h}_{air}\ {}_{air}$$

$$P_2 = \underbrace{\underbrace{\rho g h}_{air}\ {}_{air} + \underbrace{\rho g h}_{oil}\ {}_{oil}}_{\rho g h}$$

$$P_1 = \rho g h$$

$$P_1 = (\rho g h)_{air} + (\rho g h)_{oil} + (\rho g h)_{water}$$

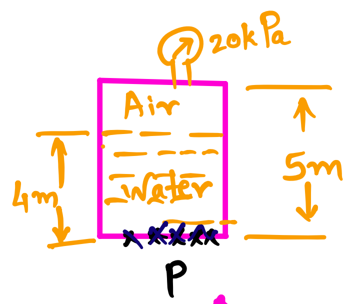
2.2 A closed, 5-m-tall tank is filled with water to a depth of 4 m. The top portion of the tank is filled with air which, as indicated by a pressure gage at the top of the tank, is at a pressure of 20 kPa. Determine the pressure that the water exerts on the bottom of the tank.

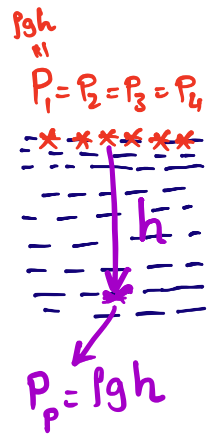
5 m tank height

4 m water

1 m air — 20 kPa = 20×10^3 Pa

P_{bottom}

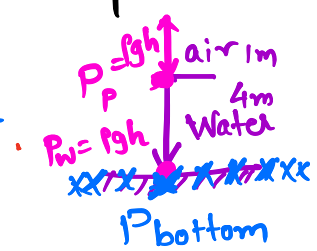




$P = \rho g h = \gamma h$

$P_{bottom} = P_{water} + P_{air}$

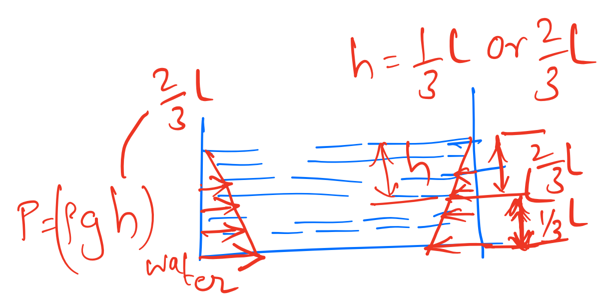
$P_{bottom} = (\rho g h)_{water} + (\rho g h)_{air}$

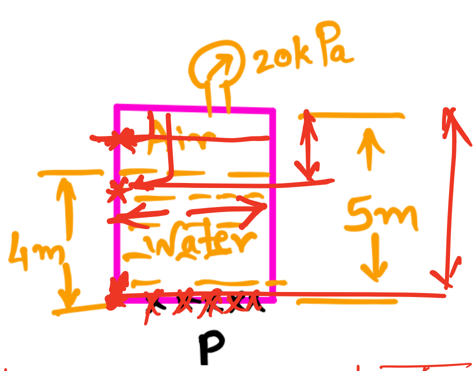


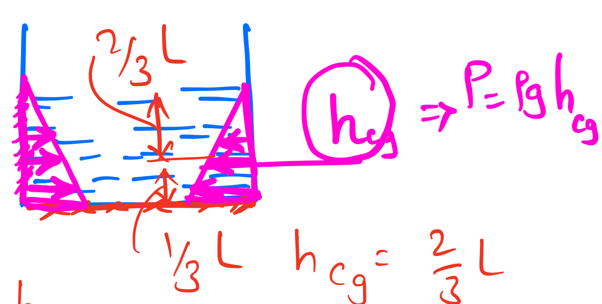
$$P_{bottom} = \frac{1000\ kg/m^3 \times 9.81\ m/s^2 \times 4\ m}{1000} + \boxed{20\ kPa}$$

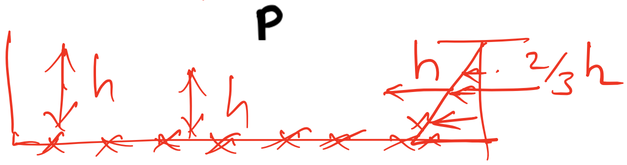
$P_{bottom} =$ 39.244 kPa + 20 kPa

$$\boxed{P_{bottom} = 59.24\ kPa}$$

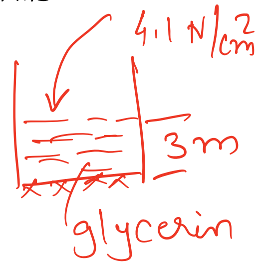








2.3 A closed tank is partially filled with glycerin. If the air pressure in the tank is 4.1 N/cm2 and the depth of glycerin is 3 m, what is the pressure at the bottom of the tank? Take specific weight of glycerin =12.4 kN/m3



Data

$$P_{air} = 4.1\ N/cm^2 = \frac{4.1 \times 100 \times 100}{1000} = 41\ kN/m^2$$

KPa

[Take $\gamma_{gly} = 12.4\ kN/m^3$

$$P_{bot.} = P_{air} + P_{gly.}$$

$$= 41 + (\rho g h)_{gly}$$

$$= 41 + 12.4 \times 3$$

$$\boxed{P = 78.2\ kN/m^2}$$

2.4 Blood pressure is usually given as a ratio of the maximum pressure (systolic pressure) to the minimum pressure (diastolic pressure). As shown in Video V2.3, such pressures are commonly measured with a mercury manometer. A typical value for this ratio fora human would be 120/70, where the pressures are in mm Hg. (a) What would these pressures be in pascals? (b) If your car tire was inflated to 120 mm Hg, would it be sufficient for normal driving?

$\gamma_{Hg} = 133\ kN/m^3$

Systolic Pr. = 120 mm of Hg

Diastolic Pr = 70 mmgHg

$P = \rho g h$

$\frac{P}{\rho g} = h$ (Pr. head) 'm'

$P = \rho g h$

$Pa = kg/m^3 \times m/s^2 \times m$

120 mm Hg

70 mm Hg

P

Pa / KPa

$\left(\frac{P}{\rho g}\right)_{sys} =$ 120 mm of Hg $= h_{sys}$

$\left(\frac{P}{\rho g}\right)_{dias} =$ 70 mm of Hg $= h_{dias}$

$\left(\frac{P}{\rho g}\right)_{sys} = \frac{120 \text{ m of Hg}}{1000} = \gamma \rightarrow Hg$

$P = ?$

$\gamma_{Hg} = 133\ KN/m^3$

$P = (\rho g)_{Hg} \times \frac{120}{1000} = 15.9\ KPa$

$P = 133 \times \frac{120}{1000}$ KPa = KPa

P = Pa

$P_{70} = 9.31\ KPa$

$P_{car} = \rho g h \quad [h = 120 \text{ mm Hg}]$

$P_{car} =$ ______ $\underline{\underline{kPa}}$

$P_{car(prac)} = 35\,Psi = \underline{kPa}$

compare

$P = \rho g h \quad N/m^2$

$\frac{P}{\rho g} = h \Rightarrow \frac{mm}{m}$ of fluid

$= mm\ Hg$

$\underline{\underline{P}} = \rho g h$

2.5. An unknown immiscible liquid seeps into the bottom of an open oil tank. Some measurements indicate that the depth of the unknown liquid is 1.5 m and the depth of the oil (specific weight = 8.5 kN/m3) floating on top is 5.0 m. A pressure gage connected to the bottom of the tank reads 65 kPa. What is the specific gravity of the unknown liquid?

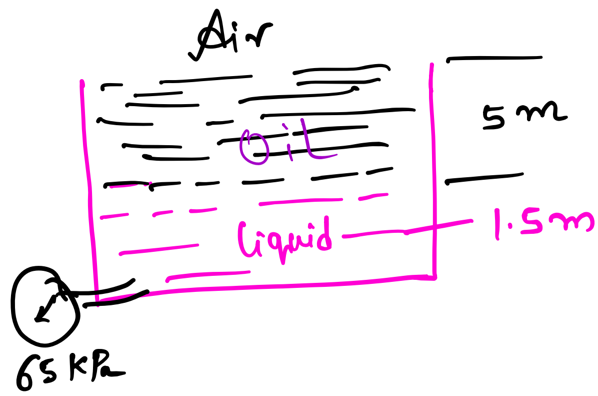
Data

$h_{unknow} = 1.5 m$

$h_{oil} = 5.0 m$

$\gamma_{oil} = 8.5 KN/m^3$

$P_{gag} = 65 KPa$



$$P_{gag} = P_{oil} + P_{liq}$$

$$65 KPa = (\rho g h)_{oil} + (\rho g h)_{liq}$$

$$= (8.5 \times (5))_{oil} + (\rho g \, 1.5 m)_{liq}$$

$$\boxed{\rho g = \gamma = 15 KN/m^3}$$

$$SG = \frac{\gamma_{liq}}{\gamma_w} = \frac{15}{9.810} = 1.53$$

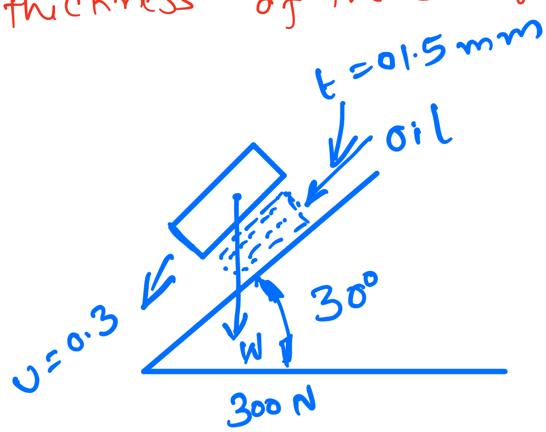
2.6 Bathyscaphes are capable of submerging to great depths in the ocean. What is the pressure at a depth of 5 km, assuming that seawater has a constant specific weight of 10.1 kN/m^3?

$$P = \rho g h$$

$$P = 10.1 \times (5 \times 1000)$$

$$P = 50500 \ kN/m^2 \ \text{or} \ 50500 \ kPa$$

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined plane with angle of inclination 30° as shown in fig. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of the oil film is 1.5 mm



Data

Area of the plate $a = 0.8 \times 0.8 = 0.64\,m^2$

Angle of the plate $\theta = 30°$

weight of the plate $W = 300\,N$

Velocity of the plate $u = 0.3\,m/s$

Thickness of oil film $t = 1.5\,mm = 1.5 \times 10^{-3}\,m$

Shear stress $\tau = \dfrac{F}{a} = \dfrac{W \sin 30°}{a} = \dfrac{300 \times \sin 30°}{0.64}$

$$\tau = 234.3\,N/m^2$$

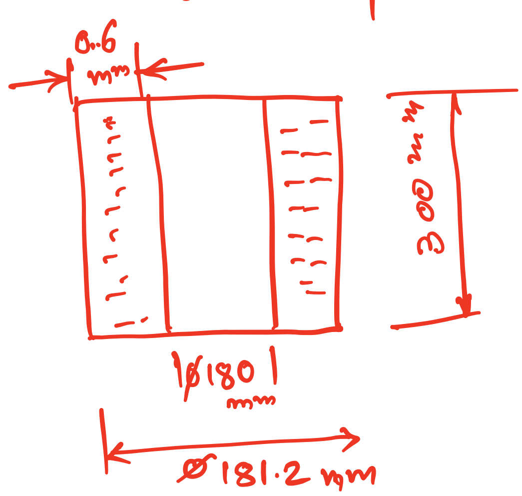
But $\tau = \mu \dfrac{du}{dy} = \mu \dfrac{du}{t}$

$$234.3 = \mu \frac{0.3}{1.5\times10^{-3}}$$

$\mu = 1.171\ Ns/m^2$ or

$$\boxed{\mu = 11.71 \text{ poise}}$$

A vertical cylinder of diameter 180mm rotates concentrically inside another cylinder of diameter 181.2mm. Both the cylinders are 300mm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. Determine the viscosity of the fluid if a torque of 20 Nm is required to rotate the inner cylinder at 120 rpm.



Data

$d_{inner} = 180\ mm = 0.18\ mm$

$d_{outer} = 181.2\ mm = 0.181\ m$

$L = 0.3m$

$N = 120\ rpm$

$T = 20\ Nm$

$$u = \frac{\pi d N}{60} = \frac{\pi d_{inner} N}{60} = \frac{\pi \times 0.18 \times 120}{60} = 1.13\ m/s$$

$$\tau = \mu \frac{du}{dy} = \mu \frac{1.13}{0.0006m}$$

$\left[dy = \frac{0.1812 - 0.180}{2} \right.$

$dy = 0.0006\ m$

$\tau = 1883.3\mu$

$$\tau = \frac{F}{a} \Rightarrow F = \tau a = 1883.3\mu \cdot \times 0.1696$$

$\left[a = \pi d L \right.$

$= \pi\ 0.180 \times 0.3$

$= 0.1696\ m^2$

$F = 319.4\mu$

But

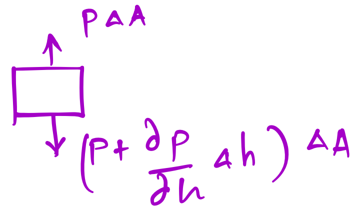
$$T = F \times \frac{d}{2}$$

$$20 = 319.4\mu \left(\frac{0.18}{2}\right)$$

$\mu = 0.696\ Ns/m^2$ or 6.96 poise

Derive the expression for hydrostatic pressures in the case of liquids and gases. (For gases use $P = \rho R T$)

For liquids



Hydrostatic law derivation

$$\frac{P}{h} = \gamma$$

For gases,

$$P = \rho R T, \quad \rho = \frac{P}{RT}$$

but $$\frac{\partial P}{\partial h} = \rho g$$

So $$\frac{\partial P}{\partial h} = \frac{P}{RT} g$$

$$\frac{\partial p}{p} = \frac{g\,\partial h}{RT}$$

$$\int \frac{\partial p}{p} = \frac{g}{RT}\int \partial h$$

$$\boxed{\ln P = \frac{g}{RT} h + C}$$

or

$$P_2 - P_1 = e^{\frac{g}{RT} h} \cdot C$$

Develop an expression for the pressure variation in a liquid in which the specific weight increases with depth h, as $\gamma = Kh^{-2} + \gamma_0$, where K is a constant and γ_0 is the specific weight at the free surface.

$$\frac{\partial p}{\partial h} = \gamma$$

$$\frac{\partial p}{\partial h} = Kh^{-2} + \gamma_0$$

$$\partial p = K h^{-2} \partial h + \gamma_0 \partial h$$

$$\int_1^2 \partial p = K \int_1^2 h^{-2} \partial h + \gamma_0 \int_1^2 \partial h$$

$$\boxed{P_2 - P_1 = -K\left[\frac{1}{h_2} - \frac{1}{h_1}\right] + \gamma_0 \left[h_2 - h_1\right]}$$

2.9

Develop an expression for the pressure variation in a liquid in which the specific weight increases with depth, h, as $\gamma = kh + \gamma_0$ where K is a constant and γ_0 is the specific weight at the free surface.

$$\gamma = kh + \gamma_0$$

$$\frac{dp}{dh} = \gamma$$

$$dp = \gamma dh$$

$$dp = (kh + \gamma_0)\, dh$$

$$\int_0^p dp = \int_0^h (kh + \gamma_0)\, dh$$

$$\boxed{P = \frac{kh^2}{2} + \gamma_0 h}$$

The specific weight of water in the ocean may be calculated by using a relation $\gamma = \gamma_0 + 8k\sqrt{h}$, set up an expression for the pressure at any point 'H' meters below the surface and work out the pressure at a depth of 1.5 km, $k = 8.5 \times 10^{-5}$ and $\gamma_0 = 10.3\ kN/m^3$.

$$\frac{dp}{dh} = \gamma$$

$$dp = \gamma\, dh$$

$$\int dp = \int \gamma\, dh$$

$$\int dp = \int_0^H (\gamma_0 + 8k h^{1/2})\, dh$$

$$\Delta p = \left[\gamma_0 h + 8k h^{3/2} \cdot \frac{2}{3}\right]_0^H$$

$$\boxed{\Delta p = \gamma_0 H + \frac{16}{3} K H^{3/2}}$$

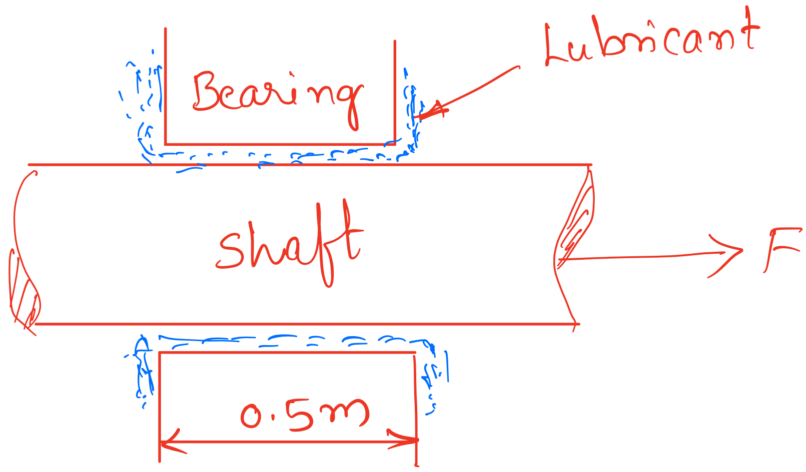
Now

$$\Delta p = \gamma_0 H + \frac{16}{3} K H^{3/2}$$

$$\Delta p = 10.3 \times 10^3 \times 1500 + \frac{16}{3} \cdot 8.5 \times 10^{-5} \times (1500)^{3/2}$$

$$\boxed{\Delta p = 15.45 \text{ MPa}}$$

A 30 mm diameter shaft is pulled through a cylindrical bearing as shown in fig. The lubricant that fills the 0.2 mm gap b/w the shaft and the bearing is an oil having a kinematic viscosity of 8×10^{-4} m²/s and a specific gravity of 0.80. Determine the force F required to pull the shaft at a velocity of 5 m/s. Assume the velocity distribution in the gap is linear.



$$\nu = \frac{\mu}{\rho}$$

$$\mu = \nu \cdot \rho = 8 \times 10^{-4} \cdot (0.80 \times 1000)$$

$$\mu = 0.64 \ \text{Kg/m.s}$$

$$\frac{m^2}{s} \times \frac{Kg}{m^3}$$

$$= Kg/ms$$

$$\tau = \mu \frac{du}{dy}$$

$$= 0.64 \times \frac{5}{0.2 \times 10^{-3}}$$

$$\boxed{\tau = 16000 \ N/m^2}$$

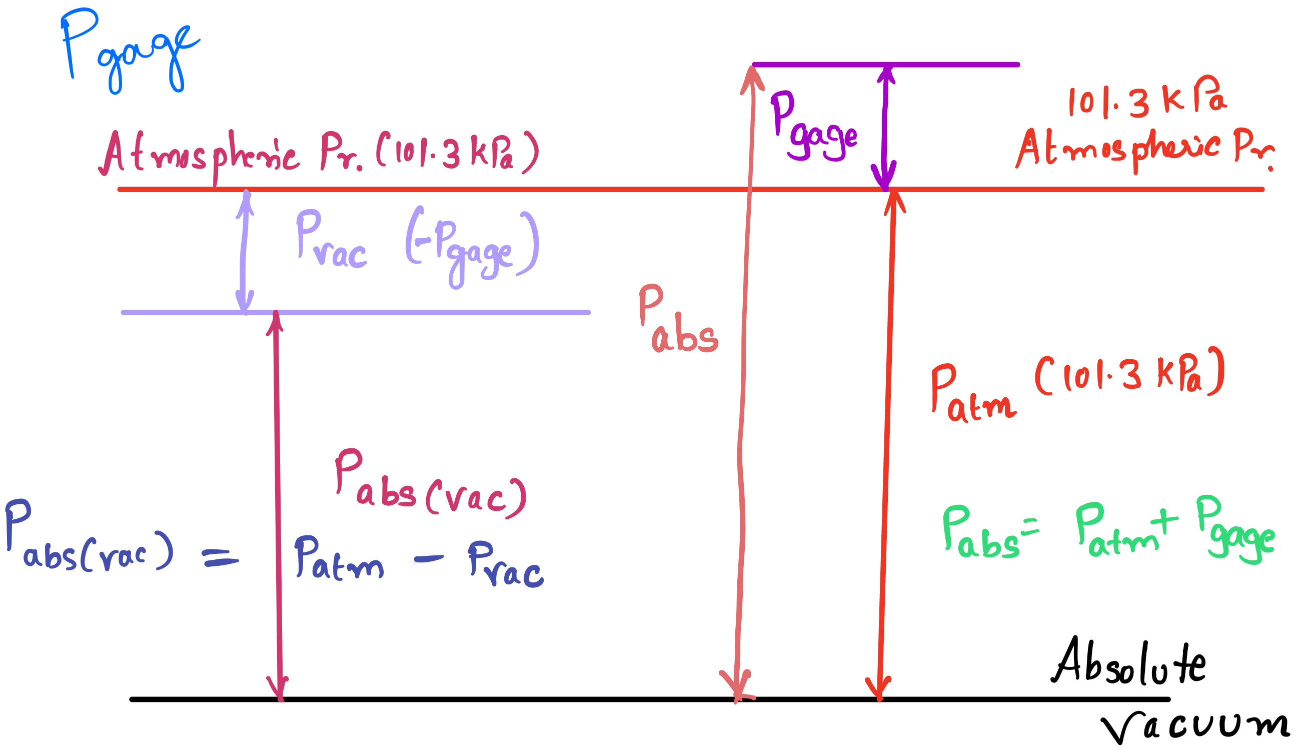
$$F = \tau A$$

$$= 16000 \times \overbrace{\pi \times 0.03 \times 0.5}^{\pi d L}$$

$$\boxed{F = 754\ N}$$

Measurement of Pressure

Pressure measuring devices $\rightarrow$ Gage pressure



Pressure measuring devices

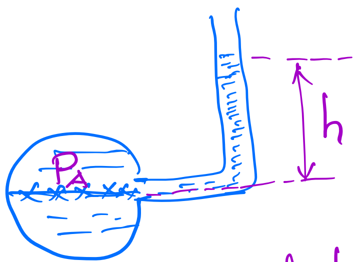
- Manometer
  1. Piezometer
  2. Barometer
  3. U-tube manometer
  4. Differential manometer
  5. Inclined manometer
- Mechanical gauges
  - Bourdon's pressure gauge
  - Diaphragm pressure gauge

Manometer

These are pressure measuring devices.

* U- tube manometer
* Differential manometer
* Inclined manometer.

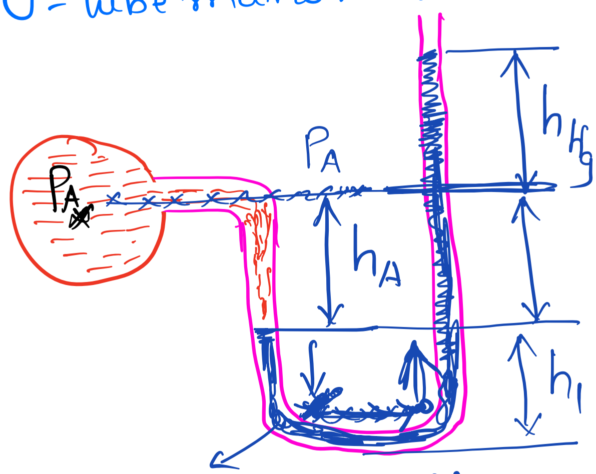
Piezometer



$$P_A = \rho g h$$

Manometer

U-tube manometer



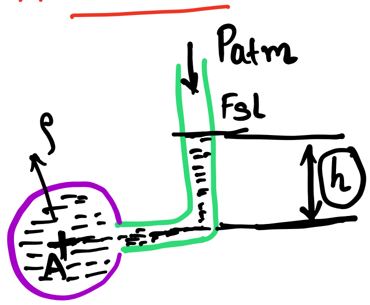
Manometric fluid (Hg)

$$\rho_{Hg} g h_{Hg} + \rho_{Hg} g h_A - \rho_A g h_A = P_A$$

$$P_A = \rho_{Hg} g h_A + \rho_{Hg} g h_{Hg} - \rho_A g h_A$$

$$P_A + \rho_A g h_A + \cancel{\rho_{Hg} g h_1} - \cancel{\rho_{Hg} g h_1} - \rho_{Hg} g h_A - \rho_{Hg} g h_{Hg} = 0$$

① Piezometer

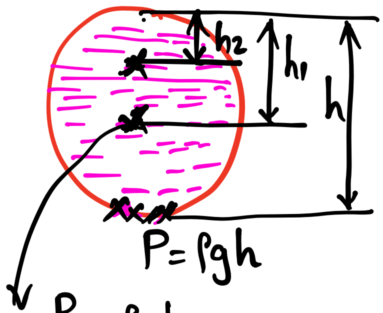


$P = \rho g h$

Referance

$\underbrace{P_A = \rho g h}_{\text{gage}} + \cancel{P_{atm}}$

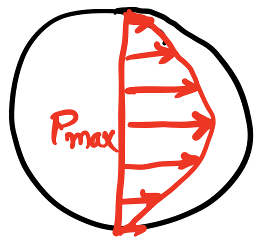
$\boxed{P_A = \rho g h}$



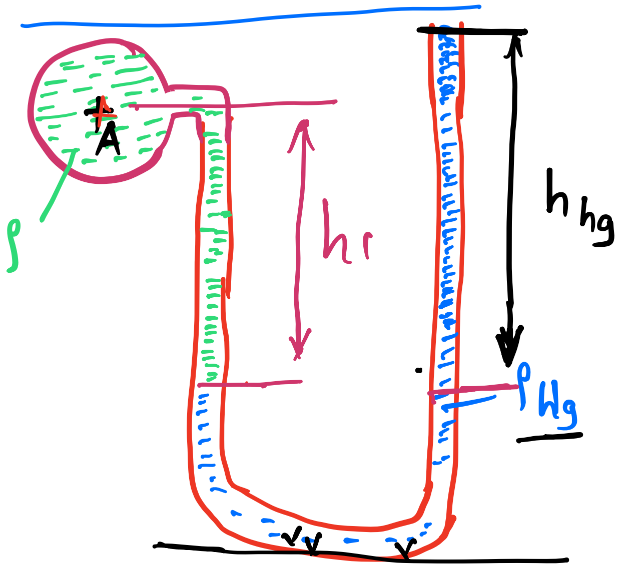
$P = \rho g h$

$P_1 = \rho g h_1$

$P_2 = \rho g h_2$



U-tube manometer



$P_A + \rho g h + \cancel{\rho g h_1} - \cancel{\rho g h_1} - \rho g h_{Hg} = 0$

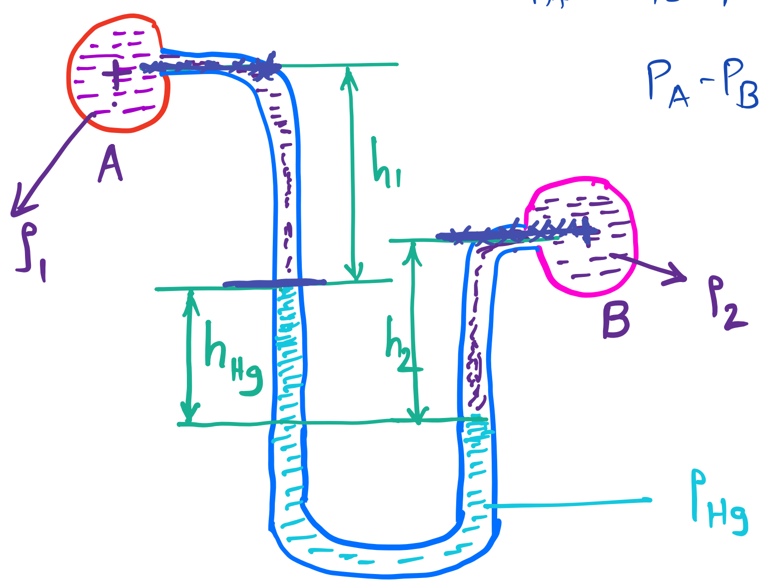
$\boxed{P_A = \rho g h_{Hg} - \rho g h}$

$\boxed{+\rho g h_{hg} - \rho g h = P_A}$

② Differential manometer

$$P_A + \rho_1 g h_1 + \rho_{Hg} g h_{Hg} - \rho_2 g h_2 - P_B = 0$$

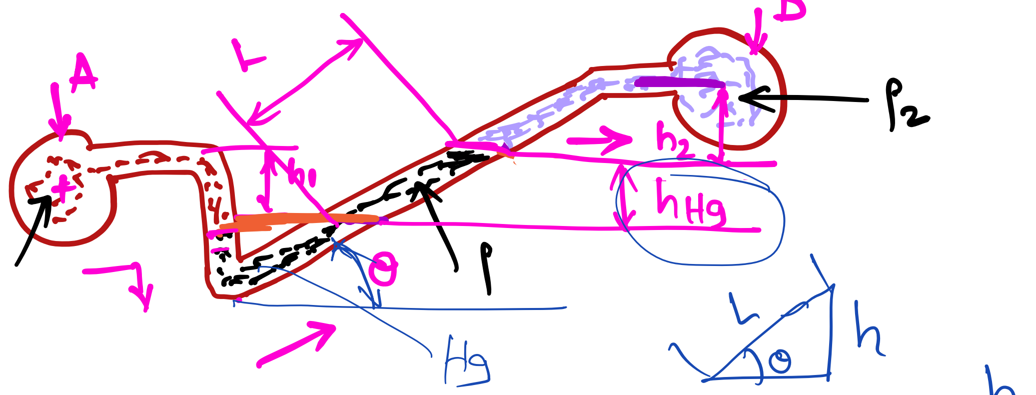
$$P_A - P_B = \rho_2 g h_2 - \rho_1 g h_1 - \rho_{Hg} g h_{Hg}$$



$$+P_A + \rho_1 g h_1 + \rho_{Hg} g h_{Hg} + \cancel{\rho_{Hg} g h_{Hg}} - \cancel{\rho_{Hg} g h_{Hg}} - \rho_2 g h_2 - P_B = 0$$

$$\boxed{P_A - P_B = \rho_2 g h_2 - \rho_1 g h_1 - \rho_{Hg} g h_{Hg}}$$

Inclined manometer



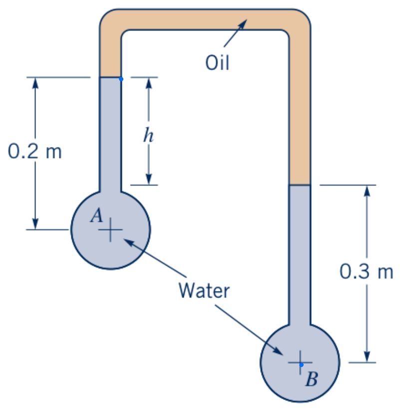
$$\sin\theta = \frac{h}{L}$$

$$P_A + \rho_1 g h_1 - \rho_{Hg} g h_{Hg} - \rho_2 g h_2 - P_B = 0$$

$$\boxed{h_{Hg} = L\sin\theta}$$

$$P_A - P_B = \rho_2 g h_2 + \rho_{Hg} g h_{Hg} - \rho_1 g h_1$$

2.53 The inverted U-tube manometer of Fig. P2.53 contains oil ($SG = 0.9$) and water as shown. The pressure differential between pipes A and B, $p_A - p_B$, is -5 kPa. Determine the differential reading h.



$$P_A - (\rho g h)_{water} + (\rho g h)_{oil} + (\rho g h)_{water} - P_B = 0$$

$$P_A - P_B - (9.81 \times 0.2) + ((0.9 \times 9.810) h) + (9.810 \times 0.3) = 0$$

$$-5 - (9.81 \times 0.2) + (0.9 \times 9.810) h + (9.810 \times 0.3) = 0$$

$$h = +0.452 \text{ m}$$

$$P_A - \rho_w g h_{0.2m} + \rho_{oil} g h + \rho_w g h_{0.3m} = P_B$$

$$SG_{oil} = \frac{\rho_{oil}}{\rho_w}$$

$$SG_{oil} \times \rho_w = \rho_{oil}$$

$$\rho_{oil} = \frac{0.9 \times 1000}{1000}$$

2.32 For the inclined-tube manometer of Fig. P2.32 the pressure in pipe *A* is 4.1 kPa. The fluid in both pipes *A* and *B* is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe *B* corresponding to the differential reading shown?

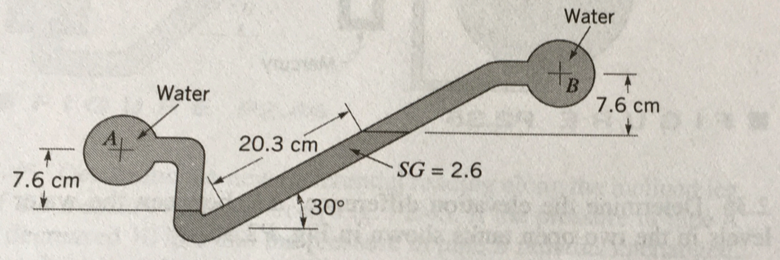


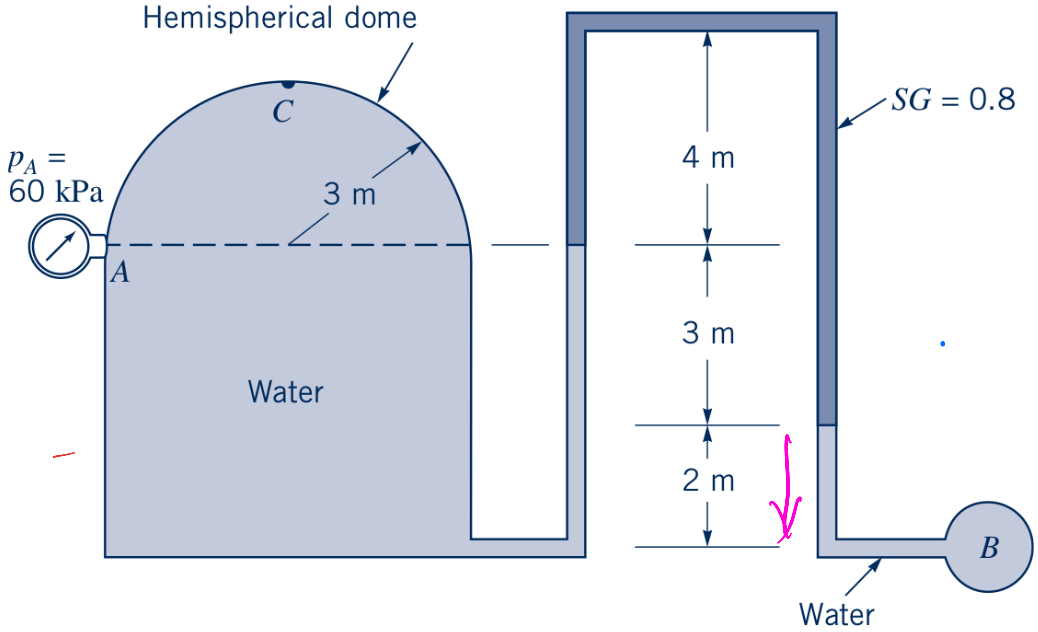
FIGURE P2.32

$$P_A + (\rho g h)_{\substack{water \\ 7.6cm}} - (\rho g h)_{\substack{water \\ (L=20.3cm)}} - (\rho g h)_{\substack{water \\ (7.6cm)}} = P_B$$

$$4.1 + 9.81 \times \frac{7.6}{100} - (2.6 \times 9.81) \times \frac{\sin 30^\circ \times 20.3}{100} - 9.81 \times \frac{7.6}{100} = P_B$$

$$\boxed{P_B = 1.51 \, kPa}$$

2.39 A closed cylindrical tank filled with water has a hemispherical dome and is connected to an inverted piping system as shown in Fig. P2.39. The liquid in the top part of the piping system has a specific gravity of 0.8, and the remaining parts of the system are filled with water. If the pressure gage reading at *A* is 60 kPa, determine (**a**) the pressure in pipe *B*, and (**b**) the pressure head, in millimeters of mercury, at the top of the dome (point *C*).



$$P_A + \cancel{(\rho g h)}_{\substack{water \\ 5m}} - \cancel{(\rho g h)}_{\substack{water \\ 5m}} + (\rho g h)_{\substack{SG\,0.8 \\ 3m}}$$

$$+ (\rho g h)_{\substack{water \\ 2m}} = P_B$$

$$60 kPa + \frac{0.8 \times 1000}{1000} \times 9.81 \times 3 + 9.81 \times 2 = P_B$$

$$\boxed{P_B = 103.3 kPa}$$

$$\boxed{P_C = P_A - \rho_w g h_w}$$

$$P_C = 60\,kPa - 9.81 \times 3$$

$$\boxed{P_C = 30.5\,kPa}$$

$$P_C = 30.5\,kPa$$

$$\boxed{SG_{hg} = 13.6}$$

$$P_C = \rho_{hg}\, g\, h_{hg} = \rho_w\, g\, h_w$$

$$30.5\,kPa = \underline{13.6} \times \underline{9.81} \times \underline{h_{hg}}$$

$$h_{hg} = 0.229\,m$$

$$h_{hg} = 229\,mm$$

$P_c = 30.5\,kPa$

$$P_c = 30.5\,kPa = (\rho g h)_{liquid} = (\rho g h)_{water}$$
$$= (\rho g h)_{Hg}$$

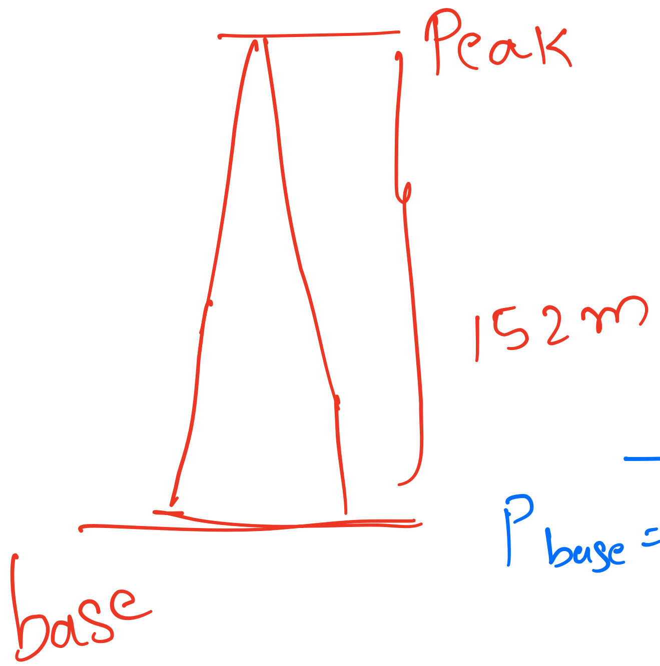
$$P_c = 30.5\,kPa = (\rho g h)_{Hg}$$

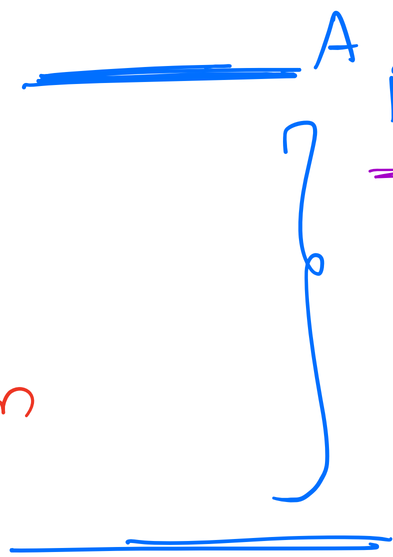
$$30.5\,kPa = 13.6 \times 9.81 \times h_{hg}$$

$$\boxed{h_{hg} = 0.229\ m}$$

$$\boxed{h_{hg} = 229\ mm}$$

2.25 On a given day, a barometer at the base of the Washington Monument reads 76 cm . of mercury. What would the barometer reading be when you carry it up to the observation deck 152 m above the base of the monument?





$P_A = P_{base} - \rho g h$

$P_{base} = 101.3\ kPa$

$\frac{P_{base}}{\rho g} = 76\,cm$ of hg

$P_{base} = \rho_{hg}\, g\, h_{hg} = 13.6 \times 9.81 \times \frac{76}{100}$

$P_{base} = 101.3\ kPa$

$\rho_{air} = 1.2\ kg/m^3$

$$P_{top} = P_{base} - \rho g h_{152\,m}$$

$$= 101.1 - \frac{1.2 \times 9.81}{1000} \times 152$$

$$P_{top} = 101.3 - 1.79$$

$$P_{top} = 99.5\ KPa$$

Cengel

3–12 The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig. P3–12. Determine the gage pressure of air in the tank if $h_1 = 0.4$ m, $h_2 = 0.6$ m, and $h_3 = 0.8$ m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.

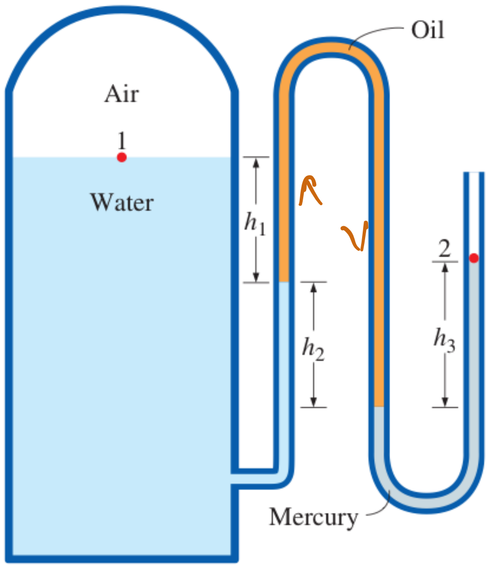


FIGURE P3–12

$$P_{air} + \rho_{water} g(h_1+h_2) - \rho_{water} g h_2 - \cancel{(\rho_{oil} g h_1)} + \cancel{(\rho_{oil} g h_1)}$$

$$+ (\rho_{oil} g h_2) - (\rho_{mercury} g h_3) = 0$$

$$P_{air} + 9.81\times(0.4+0.6) - 9.81\times(0.6) + \frac{850\times 9.81}{1000}(0.6) - \frac{13600\times 9.81}{1000}(0.8) = 0$$

$$P_{air} = 97.80\,\text{KPa}$$

At two points P and Q in a fluid flow system, the following gauge readings were obtained at P, gage reading is $200\,kN/m^2$ and at Q, gage reading is $-40\,kN/m^2$. Expressure the pressure in
(i) Meters of water, (ii) Absolute pressure
(iii) Meters of oil (specific gravity 0.77),
(iv) Meters of mercury and (4) meters of CCL_4 (s.g. 2.15).

$P = 200\,kN/m^2 \qquad Q = -40\,kN/m^2$

(i) water

$$P = \rho g h$$

$$h = \frac{P}{\rho g} \Rightarrow h_P = \frac{200}{9.81} = 20.39\,m$$

$$h_Q = \frac{-40}{9.81} = -4.08\,m$$

(ii) Absolute pressure

$$P_{abs(P)} = P_P + P_o = 200 + 101.3$$
$$= 301.3 \text{ kN/m}^3$$

$$P_{abs(Q)} = P_Q + P_o = -40 + 101.3$$
$$= 61.3 \text{ kN/m}^3$$

(iii) Oil

$$sg = 0.77 \Rightarrow \rho = 77 \text{ kg/m}^3$$

$$h_P = \frac{P}{\rho g} = \frac{200}{0.77 \times 9.81} = 26.48 \text{ m of oil}$$

$$h_Q = \frac{P}{\rho g} = \frac{-40}{0.77 \times 9.81} = -5.29 \text{ m of oil}$$

(iv) Mercury

$$h_P = \frac{P}{\rho g} = \frac{200}{133} = 1.5 \text{ m of Hg}$$

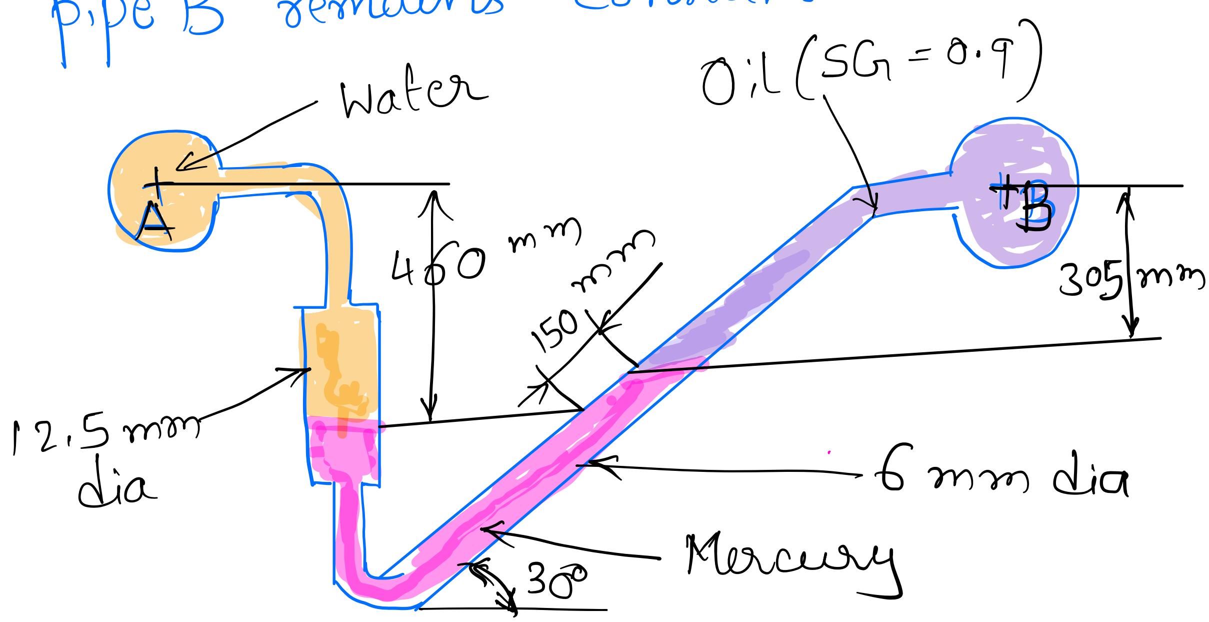
$$h_Q = \frac{P}{\rho g} = \frac{-40}{133} = -0.3 \text{ of Hg}$$

(v) CCl_4

$$h_P = \frac{P}{\rho g} = \frac{200}{2.15 \times 9.81} = 9.48 \text{ m of } CCl_4$$

$$h_Q = \frac{P}{\rho g} = \frac{-40}{2.15 \times 9.81} = -1.89 \text{ m of } CCl_4$$

Determine the change in the elevation of the mercury in the left leg of the manometer (as shown in fig) as a result of an increase in pressure of 35 kPa in pipe A while the pressure in pipe B remains constant.



(i) $P_A + (\rho g h)_{water} - (\rho g h)_{Hg} - (\rho g h)_{oil} - P_B = 0$

$P_A + \rho_w g \times (0.46) - \rho_{Hg} g \; 0.15 \sin 30^\circ$

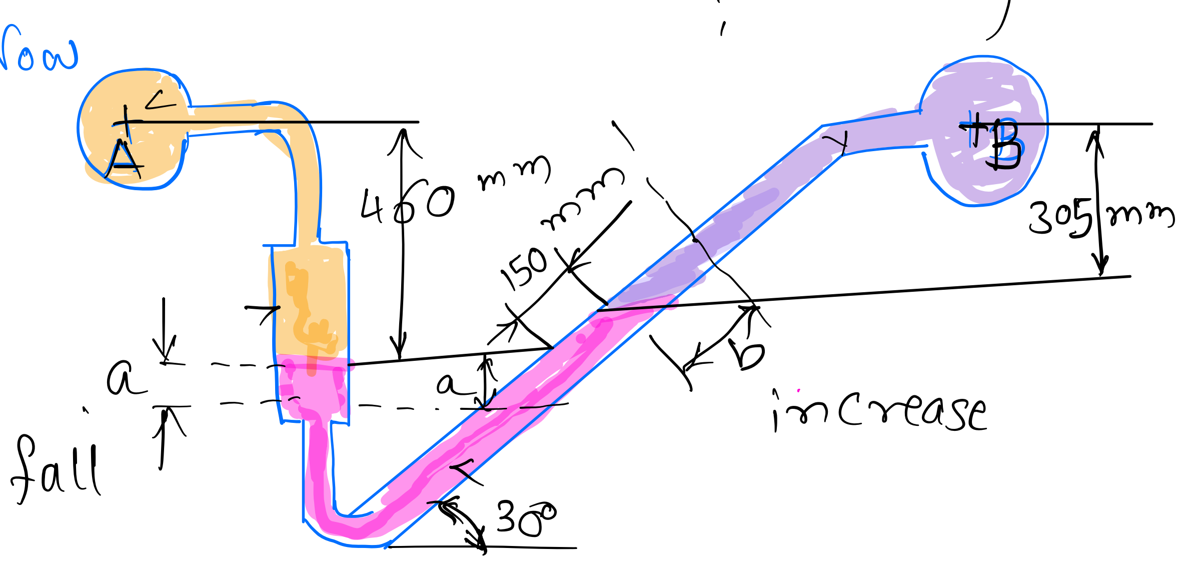
$- \rho_{oil} g \times 0.305 - P_B = 0$

$P_A - P_B = -1000 \times 9.81 \times 0.46 + 13600 \times 9.81 \times 0.15 \sin 30^\circ + 900 \times 9.81 \times 0.305$

$\boxed{P_A - P_B = 8186.4 \ N/m^2}$

(ii) If P_A increases by 35 kPa

Now



$$P_A + 35000 + (\rho g h)_w - (\rho g h)_{Hg} - (\rho g h)_{oil} - P_B = 0$$

$$P_A + 35000 + (1000 \cdot 9.81 \times (0.46 + a)) - 13600 \times 9.81 \times$$
$$(a + b\sin 30^\circ) - 900 \times 9.81 (0.305 - b\sin 30^\circ)$$
$$- P_B = 0$$

From fig $A_1 \cdot a = A_2 \cdot b$

$$a = \frac{(0.006)^2}{(0.0125)^2} b$$

$$b = 4.34\, a$$

$$P_A + 35000 + 9810 \times (0.46 + a) - 13600 \times 9.81 \times$$
$$(a + 4.34a \sin 30^\circ) - 900 \times 9.81 (0.305 -$$
$$(4.34a \cdot \sin 30^\circ) - P_B = 0$$

$$P_A - P_B = 8186.4 \, N/m^2$$

So $$a = \frac{35000}{393959.79}$$

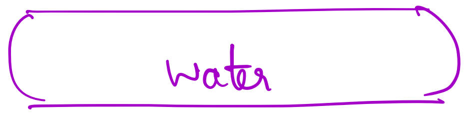
$$\boxed{a = 0.089 m}$$

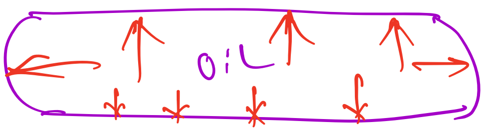
$$b = 4.34 a$$

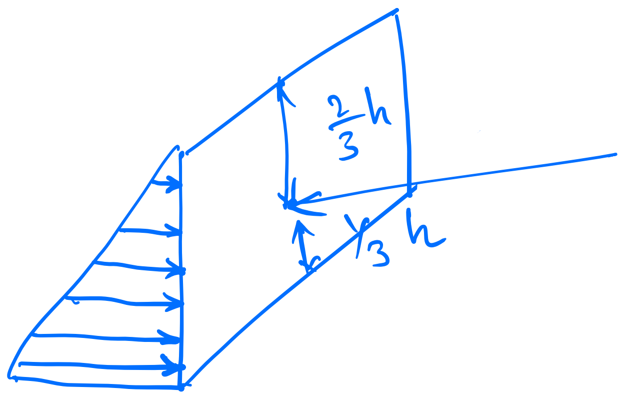
$$b = 4.34 \times 0.089$$

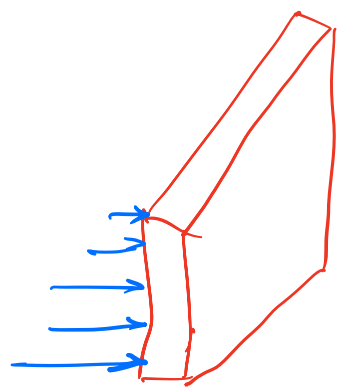
$$\boxed{b = 0.38 m}$$

Unit - 2









Unit - 2

Fluid Statics

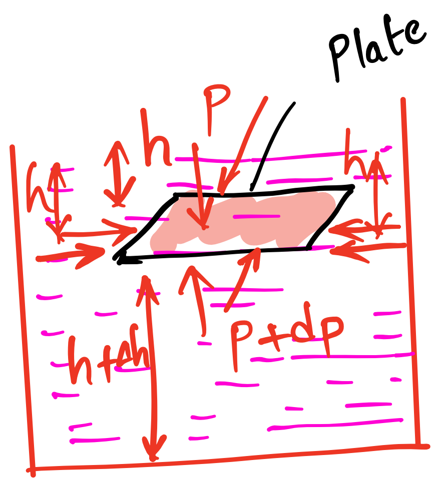
1. Pressure force
2. Gravity force

Eg:

Dams
Gates - flood gates
Tanks

$$\boxed{P = \rho g h}$$

$$W = \gamma \forall = \rho g \forall$$

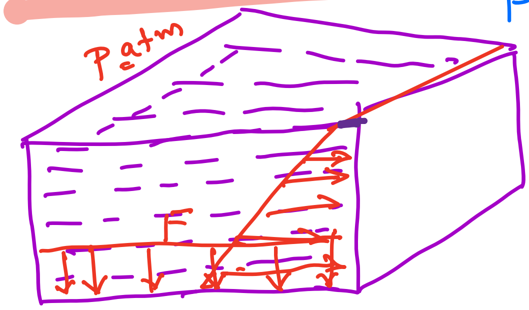


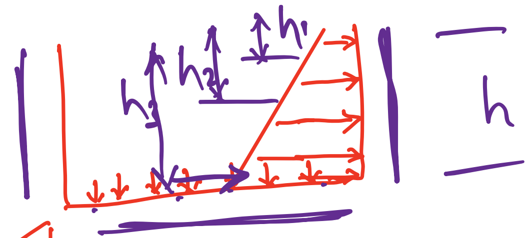
Unit 2

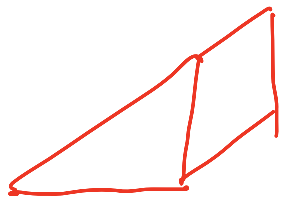
Hydrostatic forces on the surface

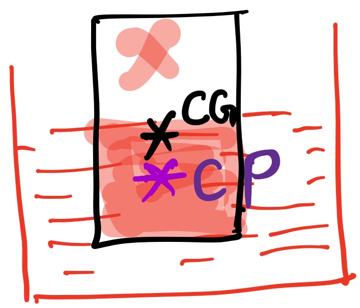
Total pressure force : Force exerted by fluid on the immersed surface.

Centre of presure : It is a point where total pressure force acts.

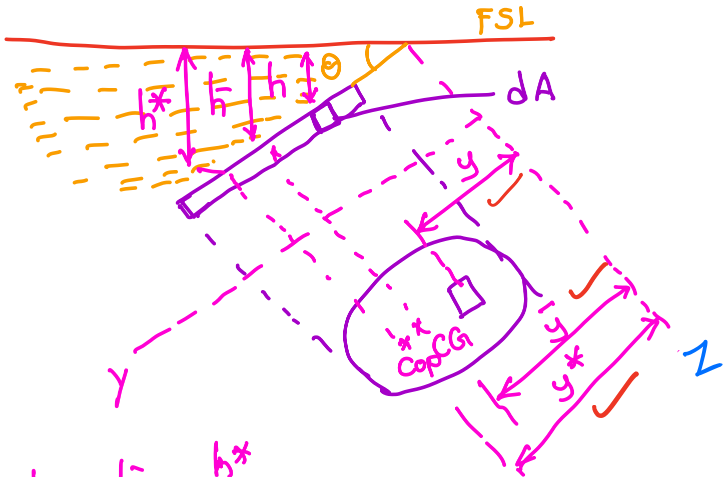








Total pressure force and centre of pressure on an inclined surface submerged in fluid.



$$\sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$$

Total pressure force on elemental area dA is

$$dF = p\,dA = \rho g h\,dA = \gamma h\,dA$$

Total pressure force for entire area 'A'

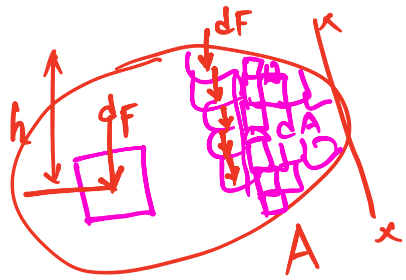
$$\int dF = \int \gamma h\,dA$$

$$F = \gamma \int h\,dA = \gamma \bar{h} A = \rho g A \bar{h}$$

$$\boxed{F = \rho g \bar{h} A = \gamma \bar{h} A}$$ or $\rho g \bar{y} \sin\theta A$

Centre of Pressure h^*

Principle of moments



$$dF\,xy = \rho g\,dA\,h\,y = \gamma\,h\,dA\,y$$

$$\int dF\,xy = \int \gamma\,h\,dA\,y$$

$$F\,xy^* = \int \gamma\,h\,dA\,y = \int \gamma\,y\sin\theta\,dA\,y = \gamma\sin\theta\int y^2\,dA$$

$$F\,xy^* = \gamma\sin\theta\,I_o$$

$$\left[\int y^2\,dA = I_o\right.$$

$$y^* = \frac{\gamma\sin\theta}{F}\,I_o$$

$$I_o = I_G + A\bar{y}^2$$

$$= \frac{\cancel{\gamma}\sin\theta}{\cancel{\gamma}\bar{h}A}\left[I_G + A\bar{y}^2\right]$$

$$\bar{y} = \frac{\bar{h}}{\sin\theta}$$

$$= \frac{\sin\theta}{A\bar{h}}\left[I_G + A\frac{\bar{h}^2}{\sin^2\theta}\right]$$

$$y^* = \frac{I_G\sin\theta}{A\bar{h}} + \frac{\bar{h}}{\sin\theta}$$

$$\frac{h^*}{\sin\theta} = \frac{I_G\sin\theta}{A\bar{h}} + \frac{\bar{h}}{\sin\theta}$$

$$\boxed{h^* = \frac{I_G\sin^2\theta}{A\bar{h}} + \bar{h}}$$

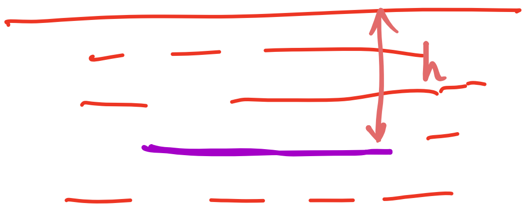
$$\boxed{h^* = y^*\sin\theta}$$

Vertical surface submerged in fluid.

$$F = \rho g A\bar{h} = \gamma A\bar{h}$$

$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

Horizontal surface ✓



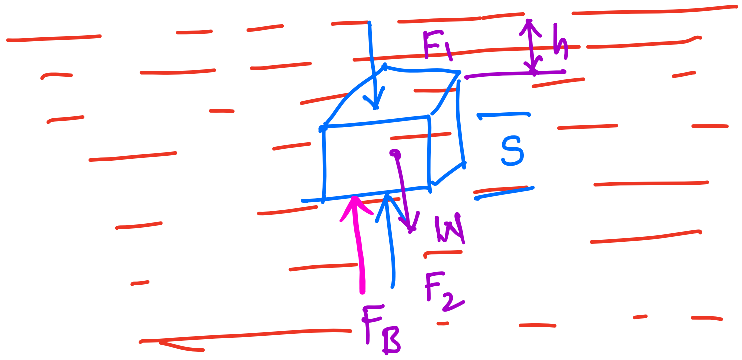
$$F = \rho g A\bar{h} = \gamma A\bar{h}$$

$$h^* = \bar{h} = CG$$

$$\bar{h} = h_{cg}$$

Body submerged in fluid

$\forall_{body} = A \cdot S$



A – c/s area

Buoyancy

$F_1 = \rho g A h$

$F_2 = \rho g A (h+S)$

$F_B = F_1 \sim F_2$

$= \rho g A h \sim \rho g A (h+S)$

$= \rho g A S = \gamma (AS) = \gamma \forall_{body}$

$F_B = \gamma V_{body} = W_{body}$

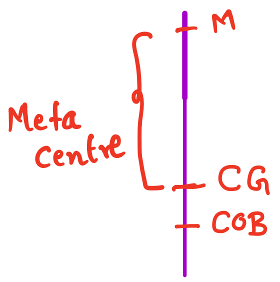
$F_B = \gamma \forall_{body} = \gamma \forall_{body}$

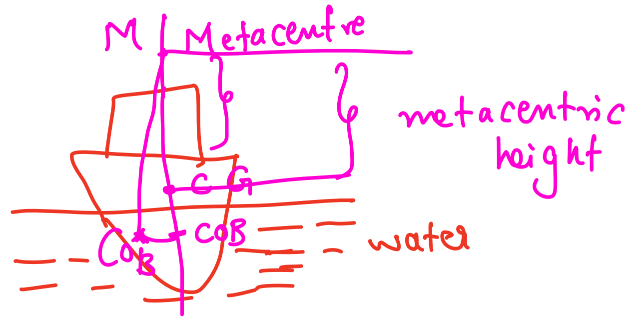
$\underline{F_B = \gamma \forall_{body} = \gamma \forall_{body}}$

$F_B = \gamma_{fluid} \forall_{\substack{fluid \\ displaced \\ by\ the \\ body}} = \gamma_{body} \forall_{body}$

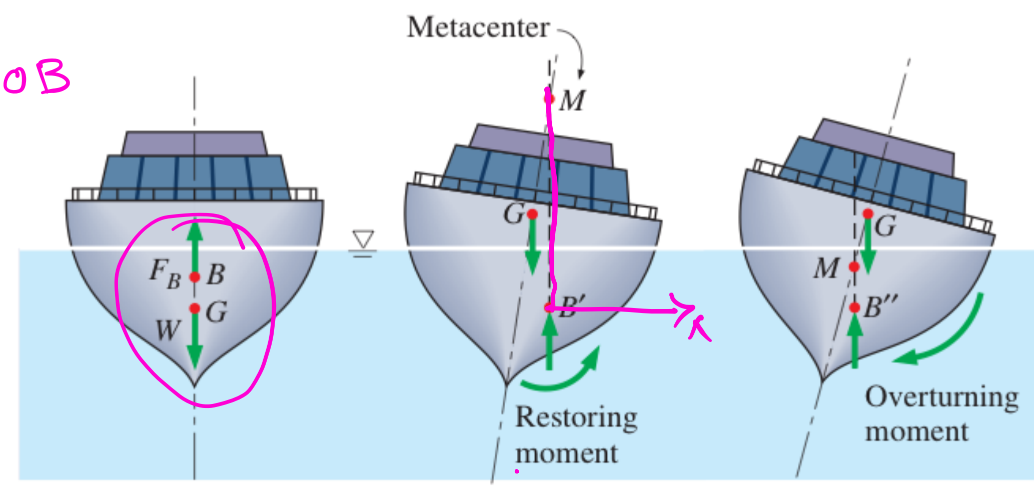
$F_B = W_{\substack{fluid \\ displaced}} = W_{body}$

$\boxed{F_B \text{ acts at centre of buoyancy (point)}}$ COB





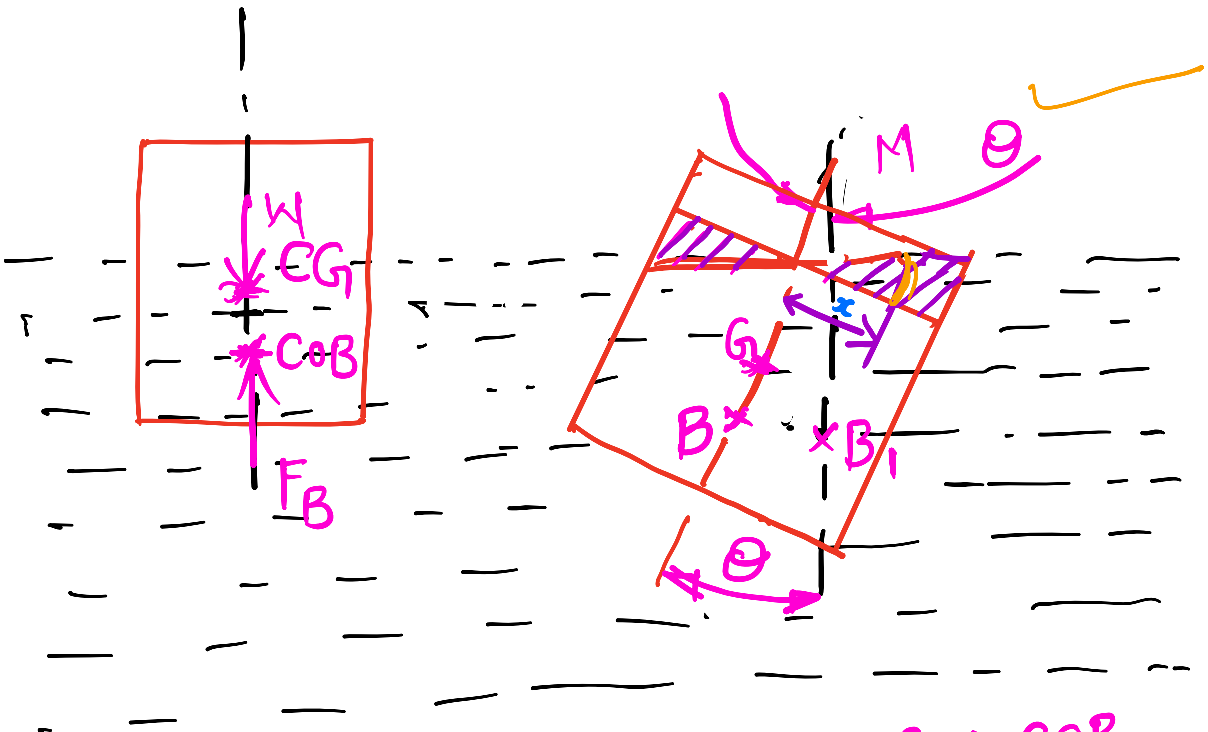
B - COB



(a) Stable (b) Stable (c) Unstable

Metacentric height → GM

Determination of metacentric height by analytical method



B → COB

G → CG

$$\delta F_B = \gamma \delta \forall$$
$$= \gamma x \ \delta A$$
$$\delta F_B \, x = \gamma \, x\theta \, \delta A \cdot x$$
$$\delta F_B \, x = \gamma x^2 \theta \, \delta A$$
$$\int \delta F_B \, x = \int \gamma x^2 \theta \, \delta A$$

$$F_B \cdot BB_1 = \gamma\theta \int x^2 \, dA$$

$$F_B \cdot BB_1 = \gamma\theta I$$

$$\cancel{\gamma}\forall \cdot BB_1 = \cancel{\gamma}\theta I$$

$$BB_1 = \frac{I\theta}{\forall}$$ [since θ is very small $\sin\theta = \theta$

$$BM \cdot \cancel{\sin\theta} = \frac{I\cancel{\theta}}{\forall}$$

$$BG + GM = \frac{I}{\forall}$$

$$\boxed{GM = \frac{I}{\forall} - BG}$$

Formulae

$$F = \gamma A \bar{h} = \rho g A \bar{h} \qquad \bar{h} = h_{CG}$$

$$h^* = \frac{I_G \sin^2\theta}{A h_{CG}} + h_{CG} \rightarrow \theta$$

$$\sin\theta = \frac{h^*}{y^*} = \frac{\bar{h}}{\bar{y}} = \frac{h_{cg}}{y_{CG}}$$

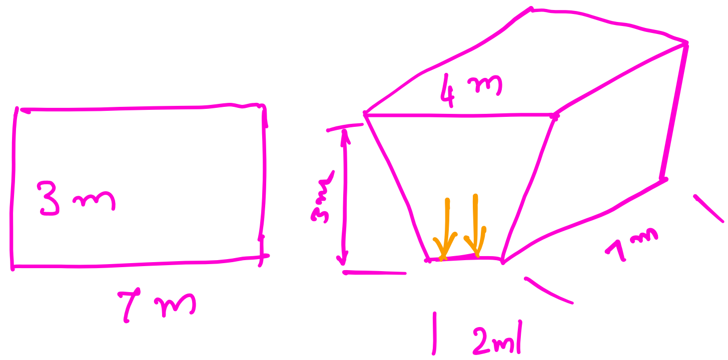
$$F_B = W_{fluid} = W_{body}$$

$$= \gamma \forall_{\substack{fluid \\ displaced}} = \gamma \forall_{body\ in\ fluid}$$

$$GM = \frac{I}{\forall} - BG$$

P2.50 A vat filled with oil (SG = 0.85) is 7 m long and 3 m deep and has a trapezoidal cross section 2 m wide at the bottom and 4 m wide at the top. Compute (*a*) the weight of oil in the vat, (*b*) the force on the vat bottom, and (*c*) the force on the trapezoidal end panel.

$P_{2.50}$ FM White



Volume of oil in the vat is $3\,m \times 7\,m \left(\frac{4m + 2m}{2}\right)$

$= 63\,m^3$

(a) weight of the oil in the vat is

$W_{oil} = \gamma_{oil} Vol._{oil} = (0.85)(9810\ N/m^3)(63\,m^3)$

$\boxed{W_{oil} = 525.254\,kN}$

$W_{body} = W_{oil}$

$\gamma_{body} \forall_{body}$ $\quad \gamma_{oil} \forall_{oil}$

(b) The force on the vat bottom surface

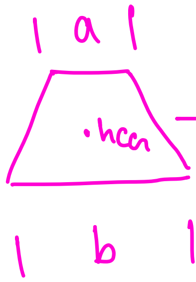
$F_{bot} = \gamma_{oil} h_{cg} A_{bot} = (0.85)(9810)(3)(2 \times 7)$

$\boxed{F_{bot} \simeq 350\ kN}$

(c) The force on the trapezoidal end panel is

$F_{side} = \gamma_{oil} h_{cg} A_{side} = 0.85(9810)(\ \ )(\ \ )$

h_{CG} of trapezoid surface is



$$h_{CG} = \frac{h}{3}\left(\frac{b+2a}{b+a}\right)$$

$$= \frac{3}{3}\left(\frac{4+2\times 2}{4+2}\right) = 1.33\,m \text{ from top}$$

problem

Surface area of trapezoid $= A = \frac{1}{2}(a+b)h$

$$= \frac{1}{2}(2+4)\,3$$

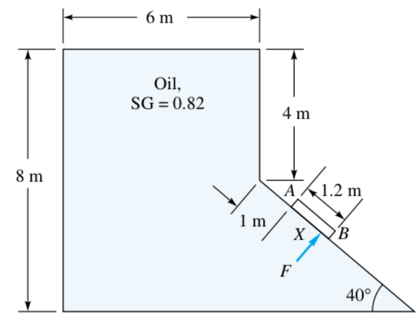
$$= 9\,m^2$$

$$F_{side} = \gamma_{oil}\,\bar{h}\,A_{side} = (0.85\times 9810)(1.33)(9)$$

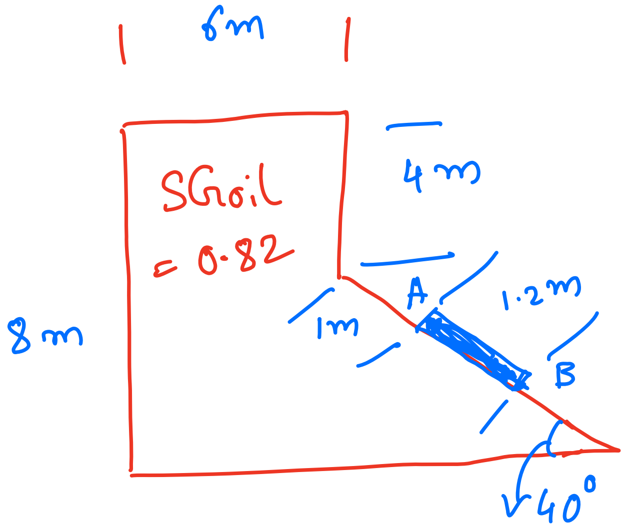
$$\boxed{F_{side} = 99811\ N}$$

2.51

P2.51 Gate *AB* in Fig. P2.51 is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric pressure, compute the force *F* on the gate and its center-of-pressure position *X*.



P2.51



$$h_{CG} = 4 + (1 + 0.6)\sin 40^{\circ}$$

$$= 5.028\ m$$

$$F_{AB} = \gamma_{oil}\, h_{CG}\, A_{gate} = (0.82 \times 9810)(5.028)(1.2 \times 0.8)$$

$$\boxed{F_{AB} = 37881\ N}$$

$$h^* = \frac{I_G \sin^2\theta}{A\, h_{CG}} + h_{CG} = \frac{\frac{bd^3}{12}\sin^2 40}{(1.2 \times 0.8)\, 5.028} + 5.028 m$$

$$= \frac{\frac{0.8 \times 1.2^3}{12}\sin^2 40}{4.82} = 5.03 m$$

$$h^* = \frac{I_G \sin^2\theta}{A \bar{h}} + \bar{h} = y^* \sin\theta$$

or $$y^* = \frac{I_G \sin\theta}{A \bar{h}} + \bar{h} \sin\theta$$

$$y^* = \frac{\frac{0.8 \times 1.2^3}{12} \times \sin 40^\circ}{4.82} + 5.028 \sin 40^\circ$$

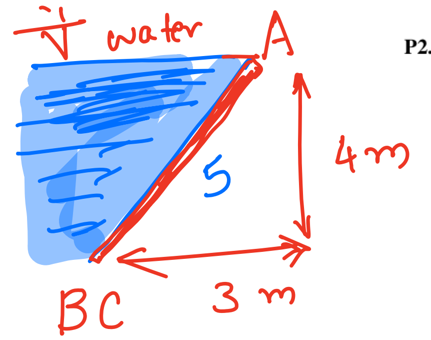
$$y^* = 3.24\ m$$

$$\frac{h^*}{y^*} = \frac{\bar{h}}{\bar{y}} = \frac{h}{y}$$

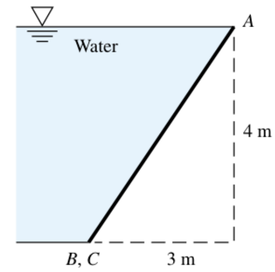
$$\bar{y} = \frac{\bar{h}\, y^*}{h^*} = \frac{5.028 \times 3.24}{5.03}$$

$$\bar{y} = 3.22m$$

2.53



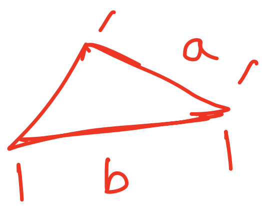
P2.53 Panel *ABC* in the slanted side of a water tank is an isosceles triangle with the vertex at *A* and the base *BC* = 2 m, as in Fig. P2.53. Find the water force on the panel and its line of action.



P2.53

$$h_{ca} \text{ or } \bar{h} = \frac{2}{3}\times 4 = 2.67\,m$$

$$F_{ABC} = \gamma h_{ca} A = 9810 \times 2.67 \times A$$



$$= \quad Area = \frac{b}{4}\sqrt{4a^2 - b^2}$$

$$= \frac{2}{4}\sqrt{4\times 5^2 - 2^2}$$

$$= 4.89 \approx 4.9\,m^2$$

$$F_{ABC} = 9810 \times 2.67\,(4.9)$$

$$\boxed{F_{ABC} = 128\,kN}$$

$$I_{xx} = \frac{bh^3}{36} = \frac{2\times 4^3}{36} = 3.5\ m^4$$

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

$$h^* = \frac{3.5 \sin^2 53^\circ}{4.9\times 2.67} + 2.67$$

$$\boxed{h^* = 2.84\ m}$$

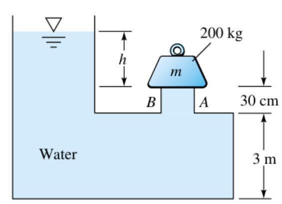
2.58

$$F = \gamma h^* A_{gate}$$

$$200\times 9.81 = 9810\, h_{CG}\, \frac{\pi}{4}(0.8)^2$$

$$\boxed{h = 0.397\ m \simeq 0.40\ m}$$

P2.58 In Fig. P2.58, the cover gate *AB* closes a circular opening 80 cm in diameter. The gate is held closed by a 200-kg mass as shown. Assume standard gravity at 20°C. At what water level *h* will the gate be dislodged? Neglect the weight of the gate.



P2.58

2.99 An open tank has a vertical partition and on one side contains gasoline with a density $\rho = 700 \text{ kg/m}^3$ at a depth of 4 m, as shown in Fig. P2.99. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open?

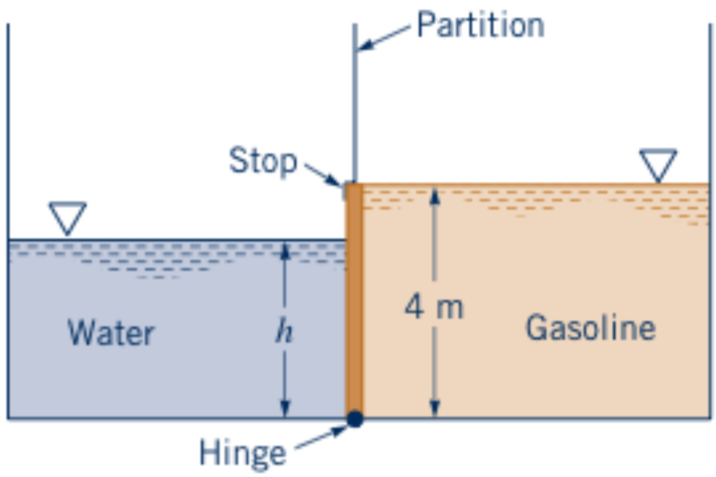


Figure P2.99

$\Sigma M_{hinge} = 0$

$\left(F \times \frac{h}{3}\right)_{water} = \left(F \times \frac{4}{3}\right)_{gas}$

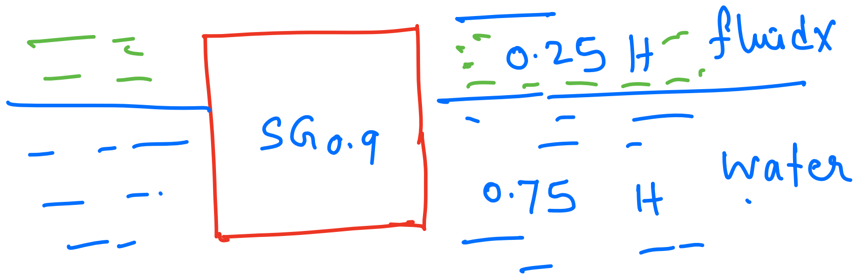
$\rho g A h_{cg} \times \frac{h}{3} = \rho g A h_{cg} \times \frac{4}{3}$

$9.81 \times (2 \times h)\left(\frac{h}{2}\right) \times \frac{h}{3} = \left(\frac{700 \times 9.81}{1000}\right)(4 \times 2)\left(\frac{4}{2}\right)\left(\frac{4}{3}\right)$

$h = 3.55 \text{ m}$

P.103

P2.103 A solid block, of specific gravity 0.9, floats such that 75 percent of its volume is in water and 25 percent of its volume is in fluid X, which is layered above the water. What is the specific gravity of fluid X?



$$\gamma_{block} V_{block} = \gamma_w Vol_{water} + \gamma_x Vol_x$$

$$0.9 \times 9810 \, V_{block} = 9810 \times 0.75 \, Vol_{block} + \gamma_x \, 0.25 \, V_{block}$$

$$0.9 \times 9810 \, \cancel{V_{block}} = \cancel{V_{block}} \left[9810 \times 0.75 + \gamma_x \, 0.25 \right]$$

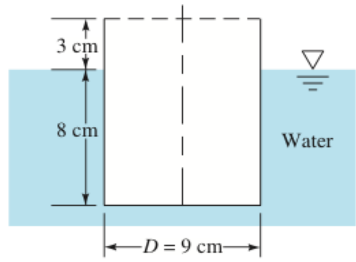
$$\gamma_x = \frac{0.9 \times 9810 - 9810 \times 0.75}{0.25}$$

$$\gamma_x = 5886 \ N/m^3$$

$$\boxed{SG_x = 0.6}$$

P_{104}

P2.104 The can in Fig. P2.104 floats in the position shown. What is its weight in N?



P2.104

$$W = \gamma \forall_{displaced} = 9810 \times \frac{\pi}{4}(0.09)^2(0.08)$$

$$\boxed{W = 5.0N}$$

P_{108}

$$F_B = F_{B_{al}} = F_{Brass}$$

$$\gamma Vol_{displac} = \gamma Vol_{al} = \gamma Vol_{brass}$$

fluid

$$2.7 \times 9810 \forall_{al} = 8.5 \times 9810 \forall_{brass}$$

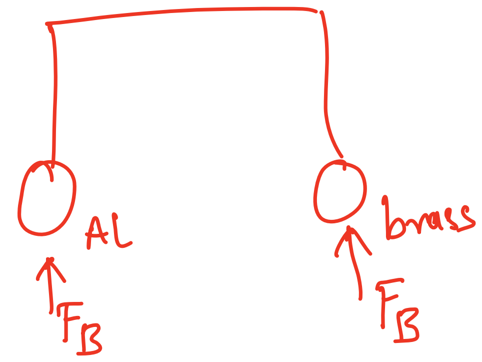
$$2.7 \times 9810 \times \frac{4}{3}\pi r^3 = 8.5 \times 9810 \times \frac{4}{3}\pi r^3$$

$$2.7 \times \cancel{9810} \times \cancel{\frac{4}{3}\pi}(0.035)^3 = 8.5 \times \cancel{9810} \times \cancel{\frac{4}{3}\pi} r^3$$

$$r^3 = \frac{2.7 \times 0.035^3}{8.5}$$

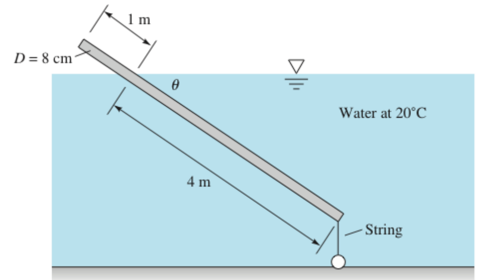
$$\boxed{r = 2.38cm}$$

b

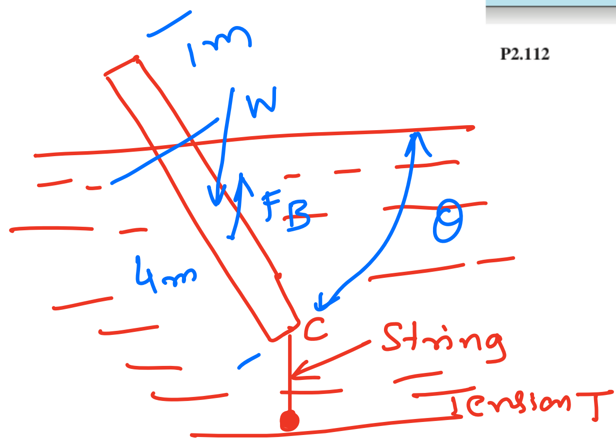


P112

P2.112 The uniform 5-m-long round wooden rod in Fig. P2.112 is tied to the bottom by a string. Determine (*a*) the tension in the string and (*b*) the specific gravity of the wood. Is it possible for the given information to determine the inclination angle θ? Explain.



P2.112



$$\Sigma M_C = 0 = W(2.5\sin\theta) - F_B(2\sin\theta)$$

$$F_B = \gamma(\text{Vol})_{\text{displaced}} = 9810 \times \frac{\pi}{4}(0.08)^2(4\,m)$$

$$= 197\,N$$

So $W(2.5\,\cancel{\sin\theta}) = 197(2\,\cancel{\sin\theta})$

$$\underline{W = 157.6\,N}$$

$$W = \gamma_{wood} V_{displaced} = \gamma_{fluid} V_{fluid\ displaced}$$

$$157.6 = \gamma_{wood} \frac{\pi}{4} \times 0.08^2 \times 5$$

$$\gamma_{wood} = 6270.704\ N$$

$$SG_{wood} = \frac{\gamma_{wood}}{\gamma_{water}} = \frac{6270.7}{9810}$$

$$\boxed{SG_{wood} = 0.64}$$

Summation of vertical forces

$$T = F_B - W = 197 - 157.6$$

$$\boxed{T = 39.4\ N}$$

2.97
Munson

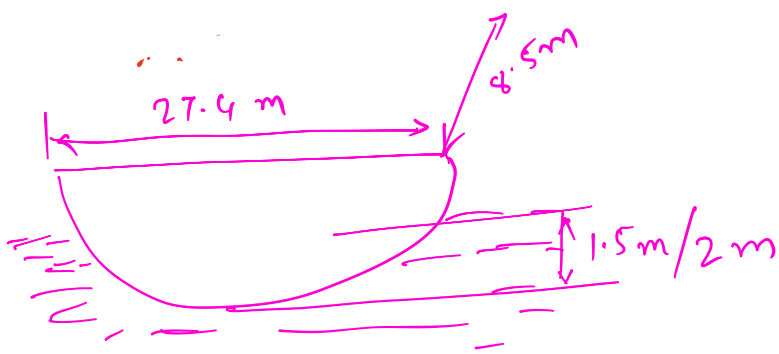
A freshly cut log floats with $1/4^{th}$ of its volume protruding above the water surface, determine the specific weight of the log.

$\gamma_{log} = 7.35 \ kN/m^3$

2.98 Mun A river barge, whose cross section is approximately rectangular, carries a load of grain. The barge is 8.5 m wide and 27.4 m long. When unloaded, its draft is 1.5 m and with the load of the grain the draft is 2 m. Determine (a) the unloaded weight of the barge, and (b) the weight of the grain

I $W_b = 3427\ \text{KN}$

II $W_g = 1142.37\ \text{KN}$



$F_B = W_{barge} = W_{fluid\ displaced}$

$= W_{barge} = \gamma V_{fluid\ displaced} = 9.81 \times (27.4 \times 8.5 \times 1.5\,m)$

$W_{barge} = 3427\ \text{KN}$

II

$$F_B = W_{barge} = W_{\substack{fluid \\ displaced}}$$

$$= \left(W_{barge} + W_{grain}\right) = \gamma \, Vol_{\substack{fluid \\ displaced}}$$

$$= 3427 + W_{grain} = 9.81 \times (27.4 \times 8.5 \times 2)$$

$$\boxed{W_{grain} = 1142.38 \text{ kN}}$$

2.107 How much extra water does a 654 kN canoe displace compared to an ultra light weight 169 kN Kevlar canoe of the same size carrying the same load?

I $Vol_w = 66.6 m^3$ II Extra water $= 49.44 m^3$

2.108 An iceberg (specific gravity 0.917) floats in the ocean (sp. gr. 1.025). What percent of the volume of the iceberg is under water?

Ans

$$\frac{V_{sub}}{V_{ice}} = 0.895$$

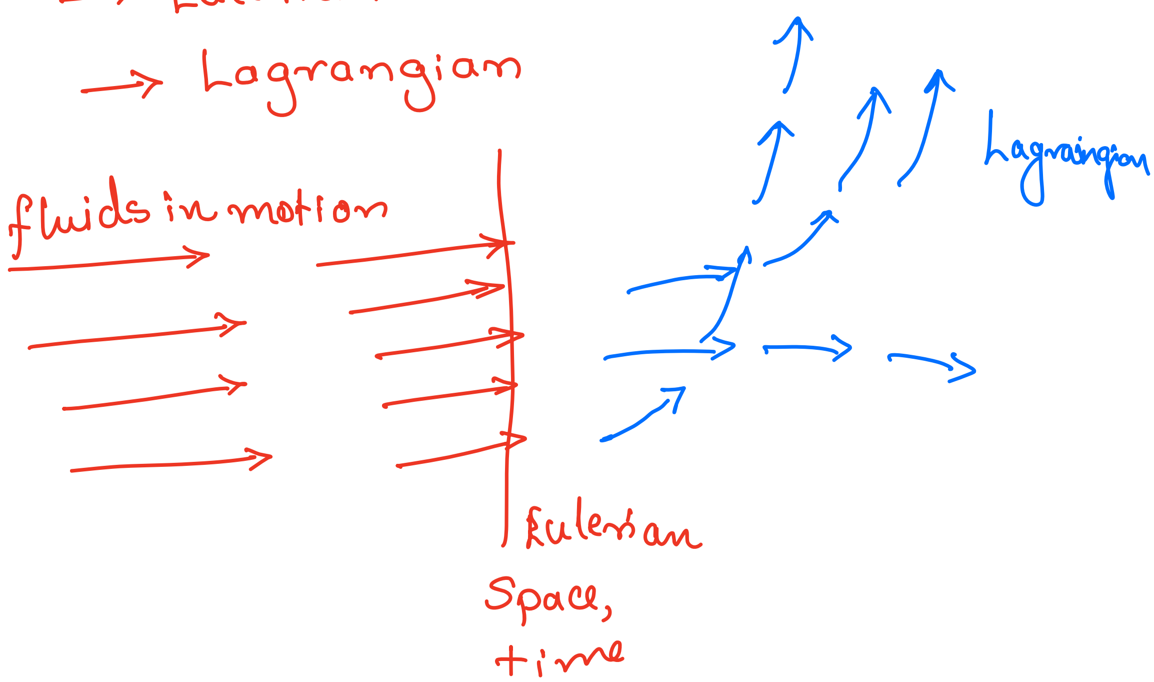
2.138 When the Tucurui Dam was constructed in northern Brazil, the lake that was created covered a large forest of valuable hardwood trees. It was found that even after 15 years underwater the trees were perfectly preserved and underwater logging was started. During the logging process a tree is selected, trimmed, and anchored with ropes to prevent it from shooting to the surface like a missile when cut. Assume that a typical large tree can be approximated as a truncated cone with a base diameter of 8 ft, a top diameter of 2 ft, and a height of 100 ft. Determine the resultant vertical force that the ropes must resist when the completely sub- merged tree is cut. The specific gravity of the wood is approximately 0.6.

Fluid Kinematics

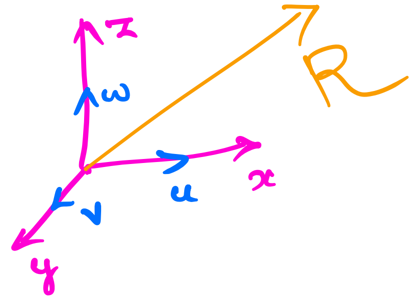
Study of fluids in motion without considering the forces causing the motion.

That means only pressure and gravity forces are considered.

Two approaches
→ Eulerian
→ Lagrangian



$$v = \underbrace{\overset{u}{\boxed{\frac{dx}{dt}}}, \overset{v}{\boxed{\frac{dy}{dt}}}, \overset{w}{\boxed{\frac{dz}{dt}}}}_{\text{velocity components}}$$



$$V = \sqrt{u^2 + v^2 + w^2}$$

acceleration

$$a = \frac{dV}{dt}$$

$$V = f(x, y, z, t)$$

$$\frac{dx}{dt} = u$$

$$\frac{dy}{dt} = v$$

$$\frac{dz}{dt} = w$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\overset{u}{\boxed{\frac{dx}{dt}}} + \frac{\partial u}{\partial y}\overset{v}{\boxed{\frac{dy}{dt}}} + \frac{\partial u}{\partial z}\cdot\overset{w}{\boxed{\frac{dz}{dt}}} + \frac{\partial u}{\partial t}\cdot\frac{\cancel{dt}}{\cancel{dt}}$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial x}\boxed{\frac{dx}{dt}} + \frac{\partial v}{\partial y}\cdot\boxed{\frac{dy}{dt}} + \frac{\partial v}{\partial z}\cdot\boxed{\frac{dz}{dt}} + \frac{\partial v}{\partial t}\cdot\frac{\cancel{dt}}{\cancel{dt}}$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial x}\boxed{\frac{dx}{dt}} + \frac{\partial w}{\partial y}\boxed{\frac{dy}{dt}} + \frac{\partial w}{\partial z}\cdot\boxed{\frac{dz}{dt}} + \frac{\partial w}{\partial t}\cdot\frac{\cancel{dt}}{\cancel{dt}}$$

$$a_x = u\cdot\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\cdot\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\cdot\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$\frac{D}{Dt}$$

Convective accln.

local accln.

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$v = f(x, y, z, t), \quad a = f(v, t), \quad v = f(u, v, w, t)$$

$$\vec{a}_x = \frac{d\vec{v}}{dt} = \frac{\partial u}{\partial x}\overset{u}{\left(\frac{dx}{dt}\right)} + \frac{\partial u}{\partial y}\overset{v}{\left(\frac{dy}{dt}\right)} + \frac{\partial u}{\partial z}\overset{w}{\left(\frac{dz}{dt}\right)} + \frac{\partial u}{\partial t}\cancel{\frac{dt}{dt}}$$

$$\vec{a}_y = \frac{\partial v}{\partial x}\overset{u}{\left(\frac{dx}{dt}\right)} + \frac{\partial v}{\partial y}\overset{v}{\left(\frac{dy}{dt}\right)} + \frac{\partial v}{\partial z}\overset{w}{\left(\frac{dz}{dt}\right)} + \frac{\partial v}{\partial t}\cancel{\frac{dt}{dt}}$$

$$a_z = \frac{\partial w}{\partial x}\overset{u}{\left(\frac{dx}{dt}\right)} + \frac{\partial w}{\partial y}\overset{v}{\left(\frac{dx}{dt}\right)} + \frac{\partial w}{\partial z}\overset{w}{\left(\frac{dz}{dt}\right)} + \frac{\partial w}{\partial t}\cancel{\frac{dt}{dt}}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$\vec{a} = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Types of flows

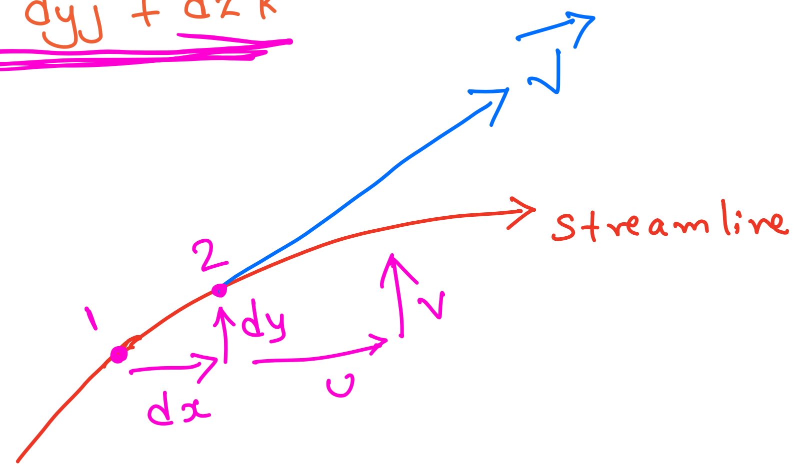
1. Steady (constant) and unsteady (variable → 't' time) flows
2. Viscous and inviscid flows
3. Internal and external flows
4. Compressible and incompressible flows
5. Uniform (velocity constant) and Non uniform (variable) flows
6. One, two and three dimensional flows

Stream line
Stream tube
Streak line
Path line
Time line

Stream line

A stream line is a curve that is tangent to instantaneous velocity vector

$$\vec{dr} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$



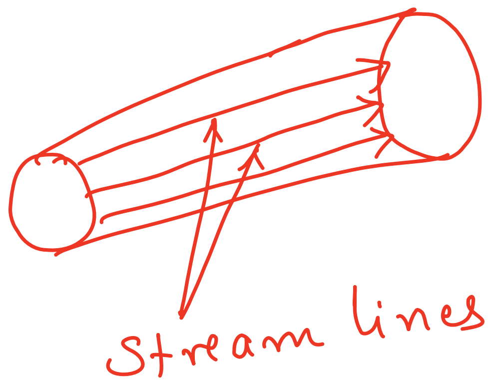
$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

From similar triangles on a 2D plane

$$\boxed{\frac{dx}{u} = \frac{dy}{v} \quad \text{or} \quad \begin{aligned} u\,dy &= v\,dx \\ v\,dx &= u\,dy \\ v\,dx - u\,dy &= 0 \end{aligned}}$$

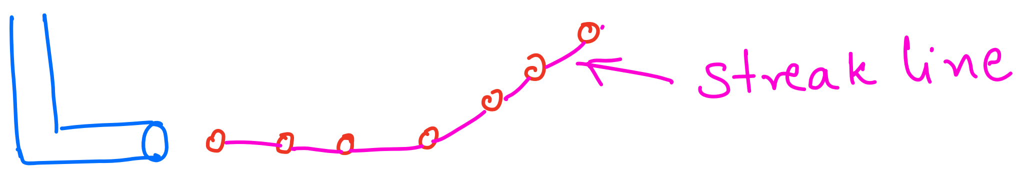
Stream tube

It is a bunch of stream lines



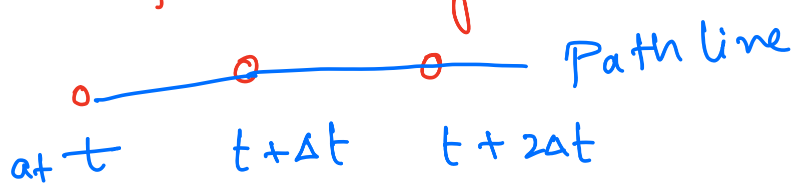
Streak line

It is locus of fluid particles passing through a common point sequentially.



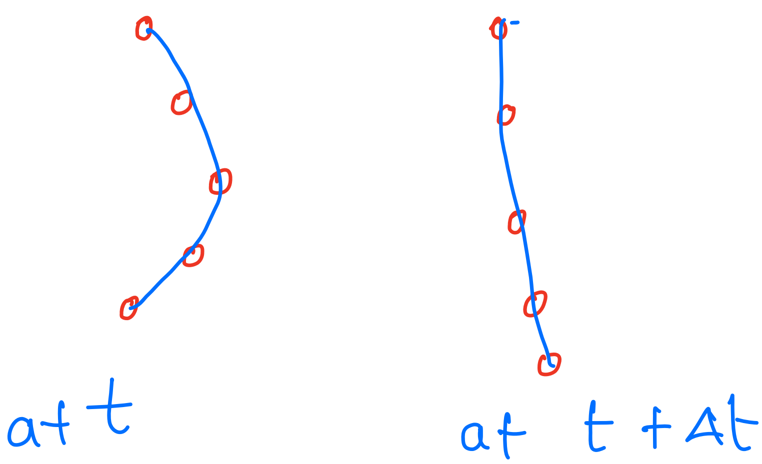
Path line

Path followed by a fluid particle



Time line

Line joining the fluid particles at a particular time instant.



Note :

Under steady state conditions

Stream line
Streak line
Time line

are the same

Mass flow rate $\dot{m} = \frac{dm}{dt}$

$\Rightarrow \frac{\rho \forall}{t} = \rho a \frac{L}{t} = \rho a V$, $\quad \frac{L}{t} \rightarrow V$

$\dot{m} = \rho a V$

Rotation and Vorticity

Rotation $\vec{\omega}$ (omega)

Vorticity ζ (zeta) or Ω (omega)

Vorticity = 2 x angular velocity

Ω or $\zeta = 2 \times \vec{\omega}$

Vorticity is also curl V

$$\Omega \text{ or } \zeta = \nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \text{curl } V$$

$$\Omega \text{ or } \zeta = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

$$= \underbrace{\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)}_{\Omega_x}\hat{i} + \underbrace{\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)}_{\Omega_y}\hat{j} + \underbrace{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}_{\Omega_z}\hat{k}$$

So $\Omega_x = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$

$\Omega_y = \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$

$\Omega_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$

$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$

$\omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$

$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$

Fluid Dynamics

Forces causing the motion of fluid particles are considered.

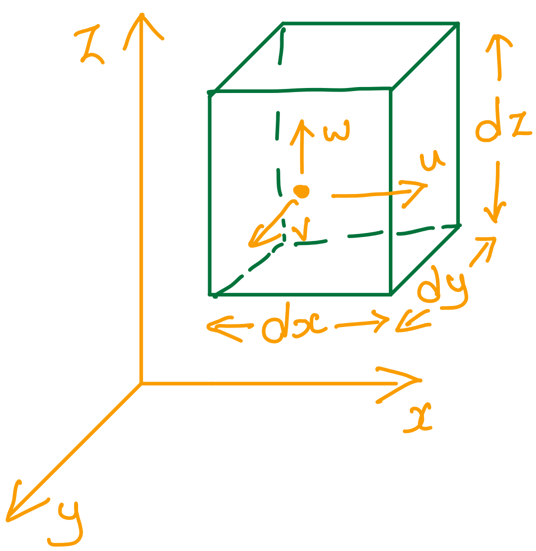
Gravity
Pressure forces
Tension
Compressibity

$$F = ma$$

Gravity force
Pressure force — Euler's eqn
Viscous force — Navier's stokes
Turbulent force — Reynold's eqn
Compressibility force
Surface tension force

Continuity equation

Continuity equation is based on principle of conservation of mass law



$\rho = \frac{m}{\forall}$

$\rho = \frac{m}{a \times L}$

$\frac{m}{t} = \frac{\rho a L}{t}$ V

Rate of mass flow through right face in the x direction is given by Taylor's expansion series. That is

$$\int_{x+\frac{dx}{2}} \rho V dA = \int_{x+\frac{dx}{2}} \frac{\rho \forall}{t} = \left[\rho u + \frac{\partial(\rho u)}{\partial x} \cdot \frac{dx}{2} + \frac{\partial^2(\rho u)}{\partial x^2} \cdot \left(\frac{dx}{2}\right)^2 \cdot \frac{1}{2!} + \cdots\right] dy dz$$

Neglecting higher order term

$$\int_{x+\frac{dx}{2}} \frac{\rho \forall}{t} = \left[\rho u + \frac{\partial}{\partial x}(\rho u) \frac{dx}{2}\right] dy dz$$

Similarly rate of mass flow through left face in the x direction is

$$\int_{x-\frac{dx}{2}} \frac{m}{t} = \left[\rho u - \frac{\partial(\rho u)}{\partial x}\frac{dx}{2}\right] dy\,dz$$ by Taylor's series neglecting higher order terms

Net rate of mass flow in the x direction is

$$= \left[\left[\cancel{\rho u} + \frac{\partial(\rho u)}{\partial x}\frac{dx}{2}\right] - \left[\cancel{\rho u} - \frac{\partial(\rho u)}{\partial x}\frac{dx}{2}\right]\right] dy\,dz$$

$$= \frac{\partial(\rho u)}{\partial x}\,dx\,dy\,dz$$

Similarly rate of mass flow in the y-direction is

$$= \frac{\partial(\rho v)}{\partial y}\,dx\,dy\,dz$$

Similarly rate of mass flow in the z-direction is

$$= \frac{\partial(\rho w)}{\partial z}\,dx\,dy\,dz$$

Net change in mass flow rate is given by

$$= \frac{\partial}{\partial x}(\rho u)\,dx\,dy\,dz + \frac{\partial}{\partial y}(\rho v)\,dx\,dy\,dz + \frac{\partial}{\partial z}(\rho w)\,dx\,dy\,dz$$

$$= \left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] dx\,dy\,dz$$

But ρ is a function of $\rho(x, y, z, t)$ and using Conservation of mass principle.

$$\int \frac{m}{t} = \int \frac{\rho \cancel{V}}{t} = \left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] \cancel{dx\,dy\,dz} = 0$$

So $$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

This is Continuity equation for compressible fluids [gases] flow.

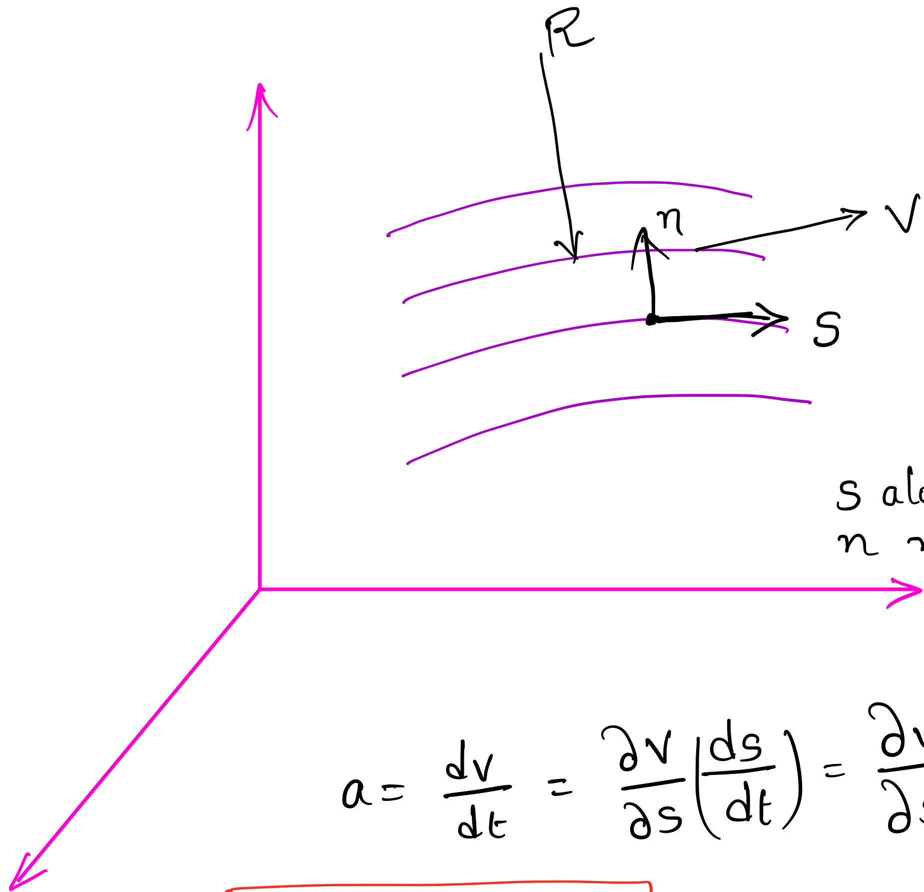
For incompressible flows ρ is constant

So $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

For steady flow conditions $\frac{\partial \rho}{\partial t}$ is constant

so $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Streamlines



s along stream line
n normal to stream line

$$a = \frac{dv}{dt} = \frac{\partial v}{\partial s}\left(\frac{ds}{dt}\right) = \frac{\partial v}{\partial s} V$$

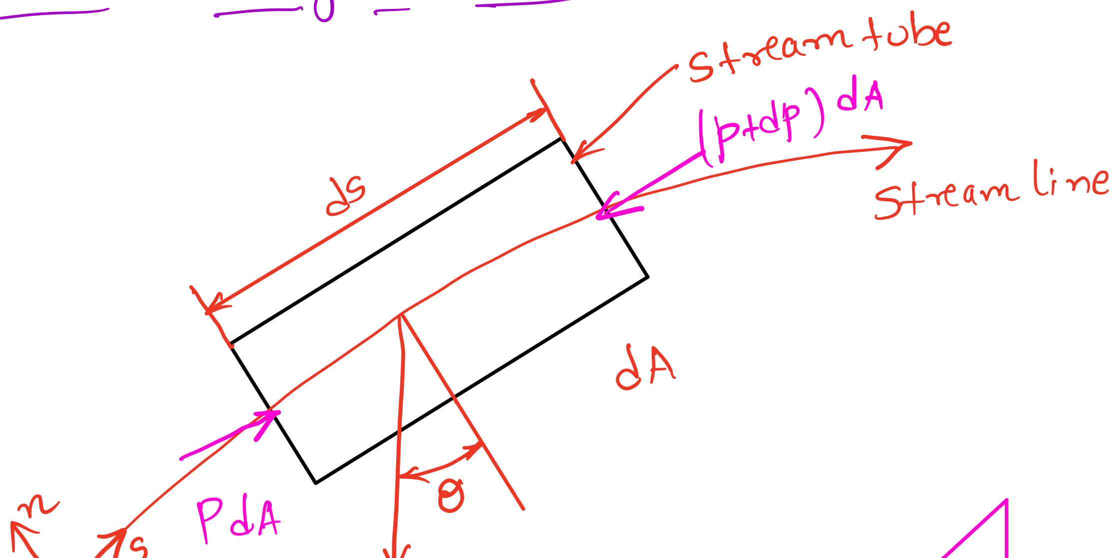
$$\boxed{\begin{aligned} a_s &= V \frac{\partial v}{\partial s} \\ a_n &= \frac{V^2}{R} \end{aligned}}$$

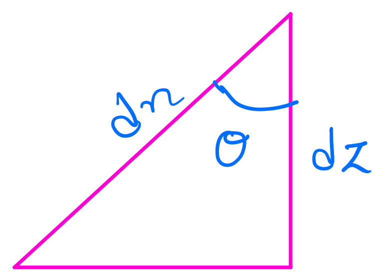
F = ma along and normal to stream line

$F = ma$ or Newton's second law along a stream line

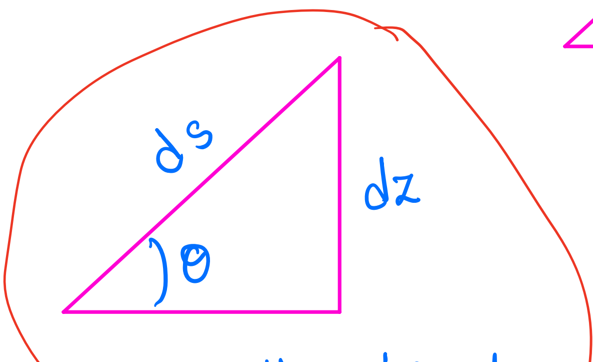
$F = ma$ normal to stream line

F = ma along a stream line





Normal to stream line



Along the stream line

Assumptions

1. Fluid is incompressible, steady, irrotational, non-viscous and continous.
2. Only gravity and pressure forces are considered.

As per $F = ma$

$$p\,dA - (p+dp)\,dA - dW\sin\theta = m \cdot V \frac{dv}{ds}$$

Here $m = \rho \forall = \rho\, dA\, ds$

$dW = mg = \rho\, dA\, ds \cdot g$

$\sin\theta = \dfrac{dz}{ds}$

Substituting,

$$\cancel{p\,dA} - \cancel{p\,dA} - dp\,dA - \rho\, dA\, \cancel{ds} \cdot g \left(\frac{dz}{\cancel{ds}}\right) = \rho\, dA\, \cancel{ds} \cdot V \frac{dv}{\cancel{ds}}$$

$$-dp\,\cancel{dA} - \rho g\, \cancel{dA}\, dz = \rho\, \cancel{dA}\, V \cdot dV$$

$$-dp - \rho g\, dz = \rho V \cdot dV$$

Dividing by ρ and rearranging,

$$\boxed{\frac{dp}{\rho} + g\,dz + V\,dV = 0}$$

This equation is known as Euler's equation of motion

Integrating,

$$\int \frac{dp}{\rho} + \int v\,dV + \int g\,dz =$$

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{Constant}$$

This equation is known as Bernoulli equation and is valued for assumptions considered.

Other forms of this equation are

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \cdots\cdots$$

or

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \cdots\cdots$$

or

$$P + \rho\frac{V^2}{2} + \rho g z = \text{Constant}$$

where p is the static pressure

$\frac{\rho V^2}{2}$ is the dynamic pressure

$\rho g z$ is the hydrostatic pressure

* The sum of the static, dynamic and hydrostatic pressures is called total pressure.

The sum of static and dynamic pressure is called stagnation pressure.

i,e $$p + \frac{\rho V^2}{2} = P_{stag}$$

$\frac{P}{\rho}$ is pressure energy

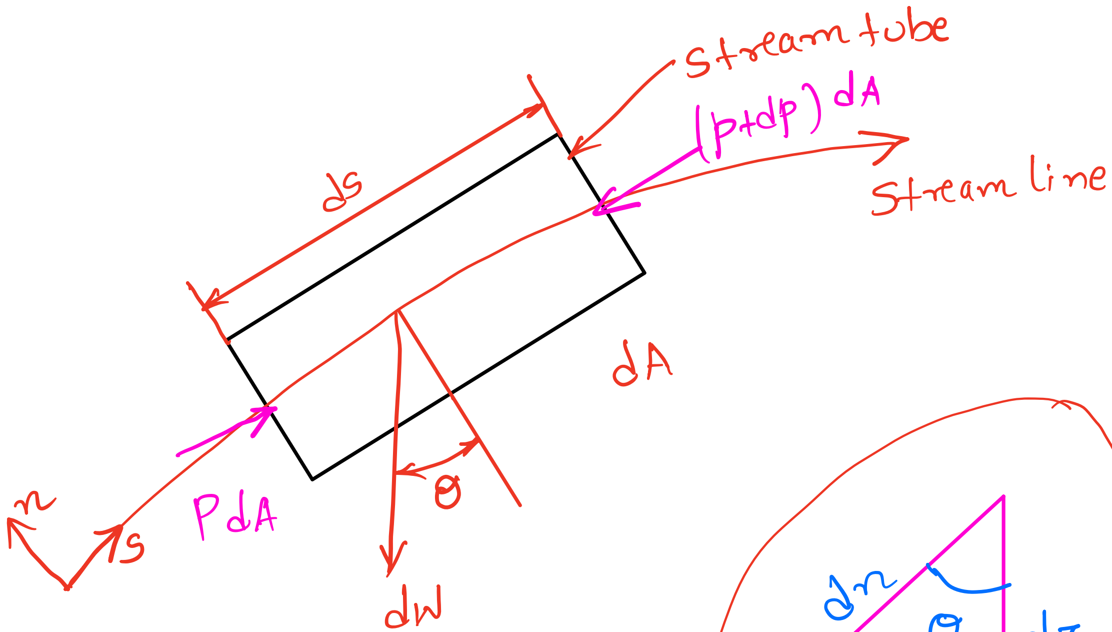
$\frac{V^2}{2}$ is kinetic energy

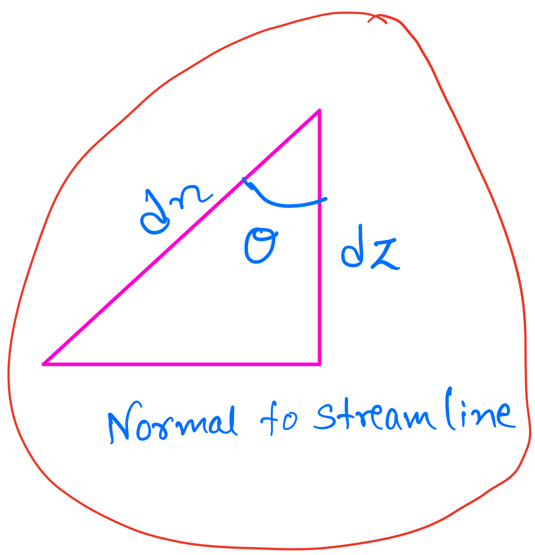
gz is potential energy

F = ma normal (across) the stream line

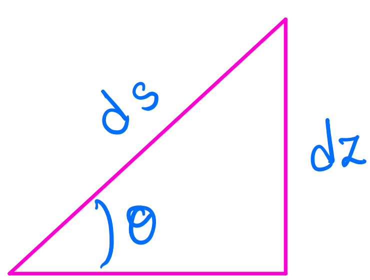
Assumptions

1. Fluid is incompressible, steady, irrotational, non-viscous and continous.
2. Only gravity and pressure forces are considered.





Normal to stream line



Along the streamline

As per $F = ma$

$$p\,dA - (p+dp)\,dA - dW\cos\theta = m \cdot \frac{V^2}{R}$$

Here $m = \rho \forall = \rho\, dA\, dn$

$dW = mg = \rho\, dA\, dn \cdot g$

$\cos\theta = \frac{dz}{dn}$

$$\cancel{p\,dA} - \cancel{p\,dA} - dp\,dA - \rho\, dA\, dn\, g \cdot \frac{dz}{dn} = \rho\, dA\, dn\, \frac{V^2}{R}$$

$$dp\,\cancel{dA} + \rho\, \cancel{dA}\, \cancel{dn}\, g\, \frac{dz}{\cancel{dn}} + \rho\, \cancel{dA}\, dn\, \frac{V^2}{R} = 0$$

$$dp + \rho g\, dz + \rho\, dn\, \frac{V^2}{R} = 0$$

Dividing by ρ

$$\frac{dp}{\rho} + g\,dz + \frac{V^2}{R}\,dn = 0$$

Integrating,

$$\int \frac{dp}{\rho} + \int g\,dz + \int \frac{V^2}{R}\,dn = 0$$

$$\boxed{\frac{p}{\rho} + gz + \int \frac{V^2}{R}\,dn = \text{constant}}$$

Applications of Bernoulli's expression

1. Venturi meter
2. Orificemeter
3. Pitot tube

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \text{Constant}$$

Discharge Q / Flow rate

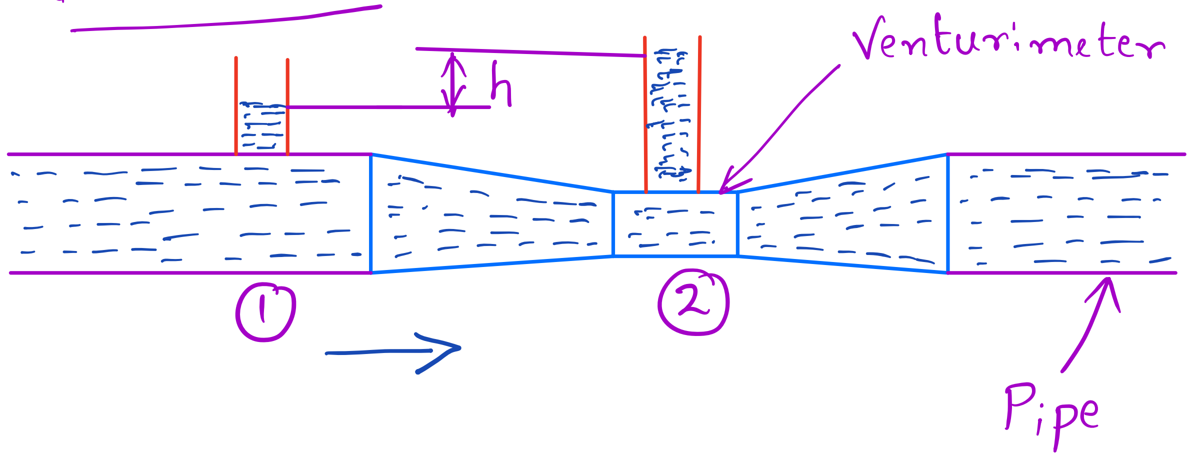
Q = area of crossection × velocity of flow

$$Q = aV$$

$$Q = a \times \frac{L}{t} = \frac{\text{Volume}}{t}$$

$$Q = a_1 V_1 = a_2 V_2 \ldots \text{—}$$

Expression for discharge through the Venturimeter



Consider a venturimeter fitted to a horizontal pipe carrying fluid (water) as shown in the above figure.

Let d_1 and d_2 be the diameter of the pipe and throat at section 1 and 2 respectively.

Similarly P_1, V_1, a_1 and P_2, V_2, a_2 are pressure, velocity and area of cross-sectional at section 1 and 2 respectively.

Applying Bernoulli's expression between section 1 and 2, we have,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

Since pipe is horizontal, $z_1 = z_2$

So, $\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$

But, $\frac{P_1 - P_2}{\rho g} = h$, pressure head

So $$h = \frac{V_2^2 - V_1^2}{2g} \quad \text{-------------- (1)}$$

Now, applying Continuity equation $Q_1 = Q_2$,

$$a_1 V_1 = a_2 V_2$$

$$V_1 = \frac{a_2 V_2}{a_1}$$

Substituting the value of V_1 in equation (1),

$$h = \frac{V_2^2 - \left(\frac{a_2 V_2}{a_1}\right)^2}{2g}$$

$$h = \frac{V_2^2 - \frac{a_2^2 V_2^2}{a_1^2}}{2g} = \frac{V_2^2 \left[1 - \frac{a_2^2}{a_1^2}\right]}{2g}$$

or

$$2gh = V_2^2\left[1 - \frac{a_2^2}{a_1^2}\right]$$

or

$$V_2 = \frac{\sqrt{2gh}}{\sqrt{\frac{a_1^2 - a_2^2}{a_1^2}}} = \frac{a_1\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

Now discharge equation $Q = a_1 V_1 = a_2 V_2$

$$Q = a_2 V_2 = a_2\left(\frac{a_1\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}\right)$$

$$Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

or

$$Q_{th} = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

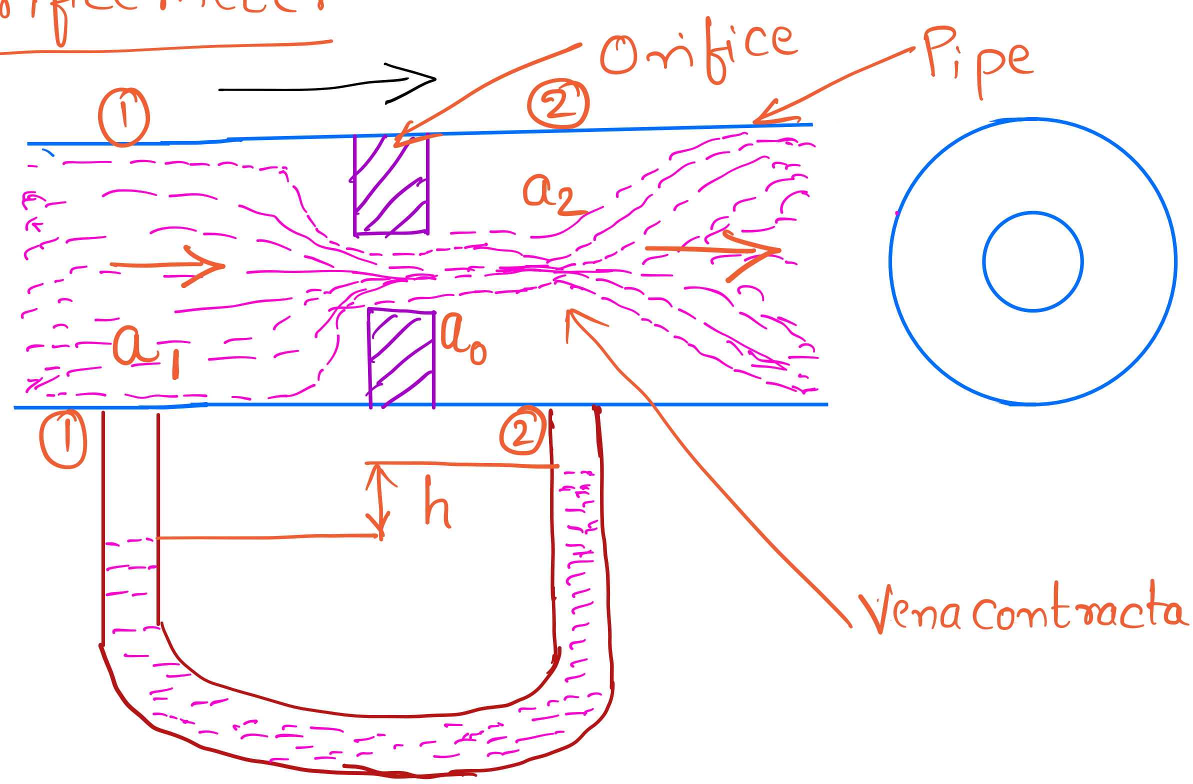
$$Q_{actual} = C_d \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

where C_d is co-efficient of discharge

This is the expression for discharge through the venturimeter.

The value of C_d is always less than 1.0.

Expression for discharge through the Orificemeter



Consider a flow of fluid in a horizontal pipe, a orificemeter is connected to the pipe as shown in the above figure.

Consider section 1-1 and 2-2 as shown in the figure. Section 1-1 is considered at the pipe, where one of the limbs of the manometer is connected. Section 2-2 is considered at the venacontracta region, where other limb of the manometer is connected. Manometer carrys same fluid that flows in the pipe. Let h be its reading.

P_1, V_1, a_1 and p_2, V_2, a_2 are the pressure, velocity and area of cross-section at section ① and ② respectively.

Applying Bernoulli's equation between section ① and ②,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

But $\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = h$

So, $$h = \frac{V_2^2 - V_1^2}{2g} \quad \text{-------- ①}$$

Using continuity equation,

$$Q = a_1 V_1 = a_2 V_2$$

but here $a_2 = C_c a_0$
where a_0 is area of orifice
C_c is co-efficient of contraction.

So $$a_1 V_1 = C_c a_0 V_2$$

So $V_1 = \dfrac{C_c a_0 V_2}{a_1}$

Substituting the value of V_1 in equation (1),

$$h = \frac{V_2^2 - V_1^2}{2g}$$

$$h = \frac{V_2^2 - \dfrac{C_c^2 a_0^2 V_2^2}{a_1^2}}{2g} = \frac{V_2^2\left[a_1^2 - C_c^2 a_0^2\right]}{2g\, a_1^2}$$

$$\frac{2g\, a_1^2 h}{a_1^2 - C_c^2 a_0^2} = V_2^2$$

or

$$V_2 = \frac{\sqrt{2gh}\; a_1}{\sqrt{a_1^2 - C_c^2 a_0^2}}$$

So $Q = a_2 V_2 = C_c a_0 V_2 = \dfrac{C_c\, a_0 \cdot \sqrt{2gh}\; a_1}{\sqrt{a_1^2 - C_c^2 a_0^2}}$

Rearranging,

$$Q = \frac{C_c\, a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - C_c^2 a_0^2}}$$

But C_d, co-efficient of discharge is related to C_c, co-efficient of contraction by

$$C_d = C_c \frac{\sqrt{a_1^2 - a_0^2}}{\sqrt{a_1^2 - C_c^2 a_0^2}}$$

So

$$Q = \frac{C_c\, a_0\, a_1 \sqrt{2gh}}{\sqrt{a_1^2 - C_c^2 a_0^2}} = \frac{C_d \cancel{\sqrt{a_1^2 - C_c^2 a_0^2}}}{\sqrt{a_1^2 - a_0^2}} \times \frac{a_0\, a_1 \sqrt{2gh}}{\cancel{\sqrt{a_1^2 - C_c^2 a_0^2}}}$$

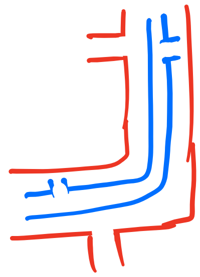
$$Q = C_d \frac{a_0\, a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

or

$$Q_{act} = \frac{C_d\, a_0\, a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

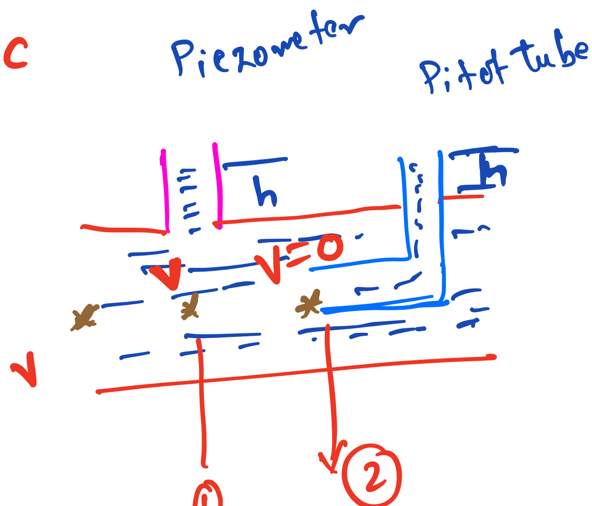
This is the equation for discharge through orificemeter.

Pitot tube $\rightarrow$ Expression for velocity of the flow.



Static pressure
Stagnation pressure $\Big\}$ velocity

$$\underbrace{\frac{P_1}{\rho g}}_{\text{static pr.}} + \overbrace{\frac{V_1^2}{2g}}^{\text{dynamic pr}} + X_1 = C$$



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1}^{\,0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}^{\,0}$$

$$\frac{P_1}{\rho g} + \frac{V^2}{2g} = \frac{P_2}{\rho g} + 0$$

$$\frac{V^2}{2g} = \frac{P_2 - P_1}{\rho g} = h$$

$$\frac{V^2}{2g} = h \qquad V = \sqrt{2gh} = \sqrt{2g\left(\frac{P_2 - P_1}{\rho g}\right)}$$

$$V = \sqrt{2gh} = \sqrt{2g\left(\frac{P_2 - P_1}{\rho g}\right)}$$

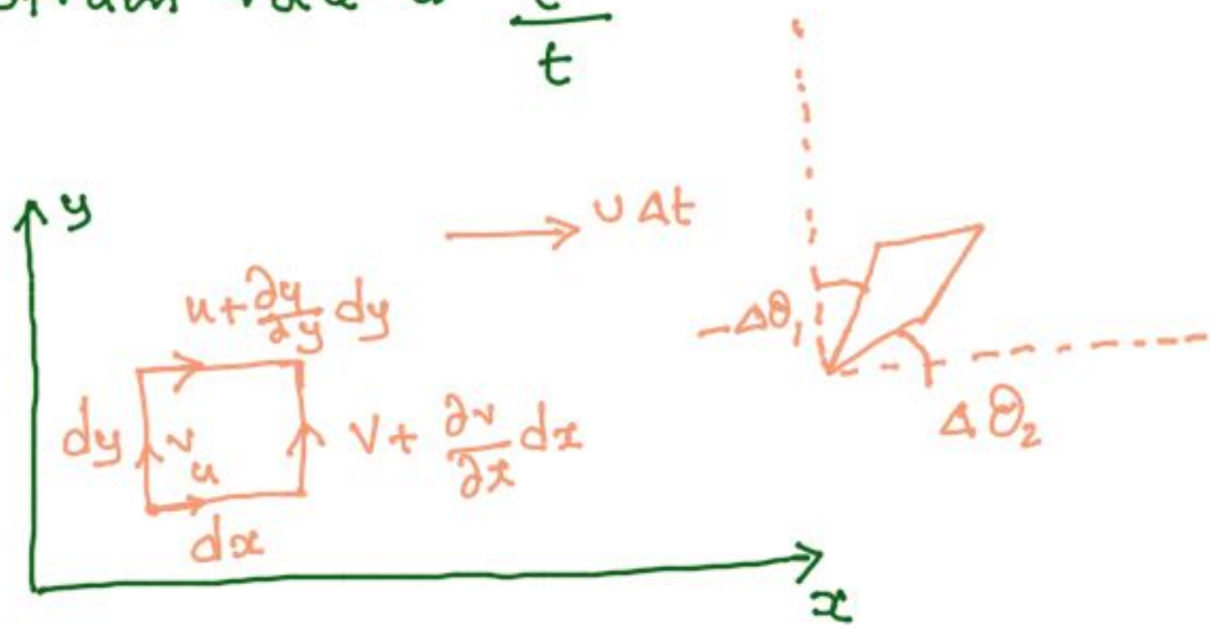
This is the equation for velocity of flow.

Strain rate

$$\frac{\sigma}{\epsilon} = E$$

$$\epsilon = \frac{\sigma}{E}$$

Strain rate is $\frac{\epsilon}{t}$



$$-\Delta\theta_1 = \frac{\partial u}{\partial y} \Delta t$$

$$\frac{d\theta_1}{dt} = -\frac{\partial u}{\partial y}$$

$$\frac{d\theta_2}{dt} = \frac{\partial v}{\partial x}$$

Strain $= \Delta\theta_2 - \Delta\theta_1$

Strain rate

$$\frac{d(\text{Strain})}{dt} = \epsilon_{xy} = \frac{d\Delta\theta_2}{dt} - \frac{d\Delta\theta_1}{dt}$$

$$\epsilon_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Similarly

$$\epsilon_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

and

$$\epsilon_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

Navier-Stokes Equations

Cartesian co-ordinates

Normal stresses

$$\sigma_{xx} = -p + 2\mu \frac{\partial u}{\partial x}$$

$$\sigma_{yy} = -p + 2\mu \frac{\partial v}{\partial y}$$

$$\sigma_{zz} = -p + 2\mu \frac{\partial w}{\partial z}$$

Shearing stresses

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\tau_{yz} = \tau_{zy} = \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

$$\tau_{zx} = \tau_{xz} = \mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)$$

Using continuity equation and Solving

x direction

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x +$$

$$\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

y direction

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

z direction

$$\rho\left(\frac{\partial w}{\partial t} + \mu\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

6.2 The velocity in a certain two-dimensional flow field is given by the equation

$$\mathbf{V} = 2xt\hat{\mathbf{i}} - 2yt\hat{\mathbf{j}}$$

where the velocity is in m/s when x, y, and t are in meters and seconds, respectively. Determine expressions for the local and convective components of acceleration in the x and y directions. What is the magnitude and direction of the velocity and the acceleration at the point $x = y = 0.6$ m at the time $t = 0$?

conv. acln

local

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$\vec{a} = \sqrt{\vec{a}_x^2 + \vec{a}_y^2 + \vec{a}_z^2}$$

local accln

convective accln.

$$\frac{D}{Dt} = \quad \begin{aligned} a_x &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \\ a_y &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \\ a_z &= \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \end{aligned}$$

material derivative

$$V = u\hat{i} + v\hat{j} + w\hat{k}$$

$$V = (2xt)\hat{i} - (2yt)\hat{j}$$

$u \qquad v$

$$\frac{\partial u}{\partial x} = 2t$$

$$\frac{\partial u}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} = 0$$

$$\frac{\partial v}{\partial y} = -2t$$

$$\frac{\partial u}{\partial t} = 2x$$

$$\frac{\partial v}{\partial y} = -2y$$

$a_x =$

$a_y =$

$$a_x = 4xt^2 + 2x$$

$$a_y = 4yt^2 - 2y$$

Local acceleration components are

$$a_x = 2x, \quad a_y = -2y$$

Convective acceleration components are

$$a_x = 4xt^2 \qquad a_y = 4yt^2$$

$$V = \sqrt{u^2 + v^2} = \sqrt{(2xt)^2 + (-2yt)^2}$$

$$V = \sqrt{(2 \times 0.6 \times 0)^2 + (2 \times 0.6 \times 0)^2}$$

$$V = 0 \, m/s$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(4xt^2 + 2x)^2 + (4yt^2 - 2y)^2}$$

$$a = \sqrt{(4 \cdot \cancel{0.6 \cdot 0^2}^{\,0} + 2 \times 0.6)^2 + (4 \times \cancel{0.6 \times 0^2}^{\,0} - 2 \times 0.6)^2}$$

$$a = \sqrt{1.44 + 1.44}$$

$$a = 1.69 \, m/s^2$$

6.3 The velocity in a certain flow field is given by the equation

$$\mathbf{V} = x\hat{\mathbf{i}} + x^2 z\hat{\mathbf{j}} + yz\hat{\mathbf{k}}$$

Determine the expressions for the three rectangular components of acceleration.

$a_x = ?$

$a_y = ?$

$a_z = ?$

6.3 $V = x\hat{i} + x^2 z\hat{j} + yz\hat{k}$

$u = x$

$v = x^2 z$

$\omega = yz$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= x(1) + x^2 z(0) + yz(0) + (0)$$

$$\boxed{a_x = x}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$= x(2xz) + x^2 z(0) + yz(x^2) + 0$$

$$\boxed{a_y = 2x^2 z + x^2 yz}$$

$$a_z = u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} + \omega\frac{\partial \omega}{\partial z} + \frac{\partial \omega}{\partial t}$$

$$a_z = x(0) + x^2 z(z) + yz(y) + 0$$

$$\boxed{a_z = x^2 z^2 + y^2 z}$$

The velocity along the centreline of a nozzle of length l is given by $V = 2t\left(1-\frac{x}{2l}\right)^2$ where V is velocity in m/s, t is time in seconds from commencement of flow, x is the distance from inlet to nozzle. Calculate the convective and local accln and total acceleration at $t = 6s$, $x = 1m$ and $l = 1.6m$

The velocity along the centre line is

$$V = 2t\left(1-\frac{x}{2l}\right)^2$$

Local acceleration $= \frac{\partial V}{\partial t}$

$$= 2\left[1-\frac{x}{2l}\right]^2$$

At $t = 6s$, $l = 1.6m$, $x = 1\,m$

$$\frac{\partial V}{\partial t} = 2\left[1-\frac{1}{2\times 1.6}\right]^2$$

$$\boxed{\frac{\partial V}{\partial t} = 0.945\ m/s^2}$$

Convective acceleration

$$= V\frac{\partial V}{\partial x}$$

$$= 2t\left[1-\frac{x}{2l}\right]^2 \times 2t \times 2\left(1-\frac{x}{2l}\right)\left(-\frac{1}{2l}\right)$$

$$= -\frac{4t^2}{L}\left(1-\frac{x}{2L}\right)^3$$

At $t=6s$, $x=1m$ and $L=1.6m$

convective acceleration $= -\frac{4\times 6^2}{1.6}\left(1-\frac{1}{2\times 1.6}\right)^3$

$$\boxed{\text{Convective accl}^n = -29.24 \ m/s^2}$$

Total acceleration = Local acceleration + convective acceleration

$$= 0.945 + (-29.24)$$

$$\boxed{\text{Total accl}^n = -28.295 \ m/s^2}$$

The velocity field in a fluid flow medium is given by

$$\vec{V} = 3xy^2\hat{i} + 2xy\hat{j} + (2zy + 3t)\hat{k}$$

Find the magnitude and directions of

(1) translational velocity

(2) rotational velocity

(3) Vorticity

At (1, 2, 1) and at time $t = 3$

(1) Translational velocity vector at (1, 2, 1) and at $t = 3$ can be written as

$$\vec{V} = 3(1)2^2\hat{i} + 2\cdot1\cdot2\hat{j} + (2\cdot1\cdot2 + 3\cdot3)\hat{k}$$

$$\vec{V} = 12\hat{i} + 4\hat{j} + 13\hat{k} = 18.13\,m/s$$

So here $u = 12$, $v = 4$ and $w = 13$.

(2) Rotational velocity vector

$$\vec{\omega} = \frac{1}{2}(\nabla \times \vec{V}) = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\vec{\omega} = \frac{\hat{i}}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + \frac{\hat{j}}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \frac{\hat{k}}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \hat{i}\,\frac{1}{2}\left[\frac{\partial}{\partial y}(2zy+3t) - \frac{\partial}{\partial z}(2xy)\right] + \hat{j}\cdot\frac{1}{2}\left[\frac{\partial}{\partial z}(3xy^2) - \frac{\partial}{\partial x}(2zy+3t)\right]$$

$$+ \hat{k}\cdot\frac{1}{2}\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial y}(3xy^2)\right]$$

$$\vec{\omega} = z\hat{i} + 0\hat{j} + (y - 3xy)\hat{k}$$

At $(1,2,1)$ and $t=3$

$$\vec{\omega} = 1\hat{i} + (2 - 3\cdot1\cdot2)\hat{k}$$

$$\boxed{\vec{\omega} = 1\hat{i} - 4\hat{k}} = 4.123 \text{ rad/s}$$

That is $\omega_x = 1$ unit, $\omega_y = 0$ unit and $\omega_z = -4$ units

(3) Vorticity

$$\vec{\Omega} = 2\vec{\omega}$$

$$= 2(1\hat{i} - 4\hat{k})$$

$$\vec{\Omega} = 2\hat{i} - 8\hat{k} = 8.246 \text{ rad/s}$$

In an incompressible flow, the velocity vector is given by
$$V = (6xt + yz^2)\hat{i} + (3t + xy^2)\hat{j} + (xy - 2xyz - 6tz)\hat{k}$$
(i) Verify whether continuity equation is satisfied.
(ii) Determine the acceleration at point (2, 2, 2) and at t=2.0

$$V = (6xt + yz^2)\hat{i} + (3t + xy^2)\hat{j} + (xy - 2xyz - 6tz)\hat{k}$$

$$U = 6xt + yz^2$$
$$V = 3t + xy^2$$
$$w = xy - 2xyz - 6tz$$

$$\frac{\partial u}{\partial x} = 6t, \quad \frac{\partial v}{\partial y} = 2xy \quad \frac{\partial w}{\partial z} = -2xy - 6t$$

So $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \rightarrow$ continuity equation.

$$\cancel{6t} + \cancel{2xy} - \cancel{2xy} - \cancel{6t} = 0$$

$$0 = 0$$

So continuity equation is satisfied

(ii) Acceleration $a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_x = 6x + (6xt + yz^2)6t + (3t + xy^2)z^2 + (xy - 2xyz - 6tz)2yz.$$

At point (2, 2, 2) and at $t = 2$.

$$a_x = 6\cdot 2 + (6\cdot 2\cdot 2 + 2\cdot 2^2)(6\cdot 2) + (3\cdot 2 + 2\cdot 2^2)2^2 + 2\cdot 2 -$$
$$2\cdot 2\cdot 2\cdot 2 - 6\cdot 2(2\cdot 2\cdot 2)$$

$$\boxed{a_x = 164 \text{ units}}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$= 3 + (6xt + yz^2)y^2 + (3t + xy^2)(2xy) + xy - 2xyz - 6tz(0)$$

At (2, 2, 2), and $t = 2.0$

$$= 3(6\cdot 2\cdot 2 + 2\cdot 2^2)2^2 + (3\cdot 2 + 2\cdot 2^2)(2\cdot 2\cdot 2)$$

$$\boxed{a_y = 243 \text{ units}}$$

$$a_z = \frac{\partial w}{\partial t} + u\cdot\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$a_z = -6z + (6xt + yz^2)(y - 2yz) + (3t + xy^2)(x - 2xz)$$
$$+ (xy - 2xyz - 6tz)(-2xy - 6t)$$

At $(2,2,2)$ and at $t = 2.0$

$$a_z = -6.2 + (6.2.2 + 2.2^2)(2 - 2.2.2) + (3.2 + 2.2^2)(2.2.2)$$
$$+ (2.2 - 2.2.2 - 6.2.2)(-2.2.2 - 6.2)$$

$a_z = 432$ units

So $a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$a = 164\hat{i} + 243\hat{j} + 432\hat{k}$

$a = \sqrt{164^2 + 243^2 + 432^2}$

$\boxed{a = 522 \text{ units}}$

Problems

3.2 Air flows steadily along a streamline from point (1) to point (2) with negligible viscous effects. The following conditions are measured: At point (1) $z_1 = 2$ m and $p_1 = 0$ kPa; at point (2) $z_2 = 10$ m, $p_2 = 20$ N/m^2, and $V_2 = 0$. Determine the velocity at point (1).

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$0 + \frac{V_1^2}{2 \times 9.81} + 2 = \frac{20}{1.22 \times 9.81} + 0 + 10$$

$$\rho_{air} = 1.225 \text{ kg/m}^3$$

$$\boxed{V_1 = 13.7 \text{ m/s}}$$

3.19 At a given point on a horizontal streamline in flowing air, the static pressure is −14 kPa (i.e., a vacuum) and the velocity is 46 m/s. Determine the pressure at a stagnation point on that streamline.

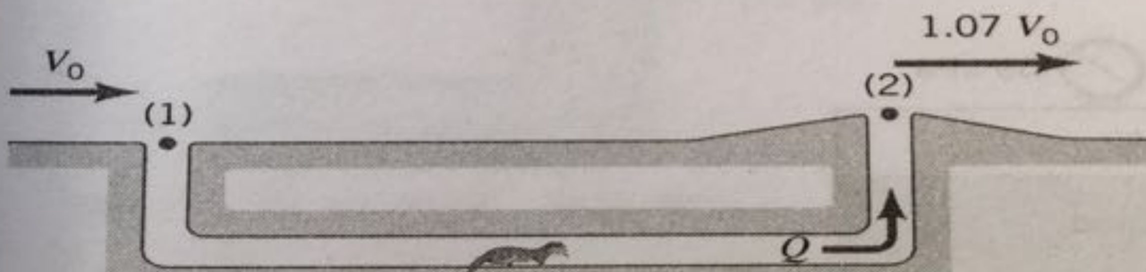
$$P_{stag} = P_{static} + \frac{1}{2}\rho V^2$$

$$= -14\,kPa + \frac{1}{2}\left(\frac{1.2}{1000}\right)46^2$$

$$= -12.7\,kPa$$

$$\rho_{air} = 1.23\ kg/m^3$$

3.22 Some animals have learned to take advantage of Bernoulli effect without having read a fluid mechanics book. For example, a typical prairie dog burrow contains two entrances—a flat front door, and a mounded back door as shown in Fig. P3.22. When the wind blows with velocity V_0 across the front door, the average velocity across the back door is greater than V_0 because of the mound. Assume the air velocity across the back door is $1.07 V_0$. For a wind velocity of 6 m/s, what pressure differences, $p_1 - p_2$, are generated to provide a fresh air flow within the burrow?



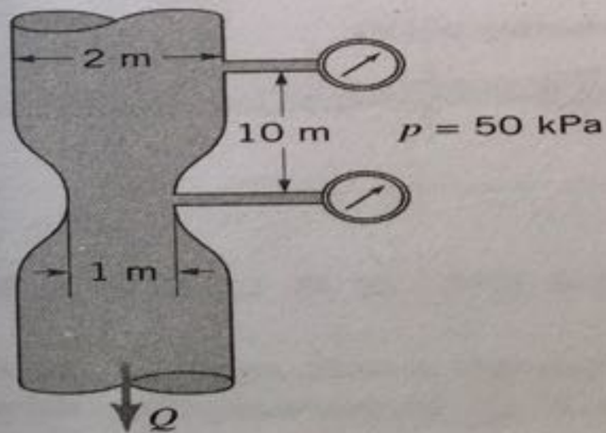
$$p_1 + \frac{\rho V_1^2}{2} + \gamma z_1 = p_2 + \frac{\rho V_2^2}{2} + \gamma z_2$$

$$p_1 + \frac{\rho V_0^2}{2} = p_2 + \frac{\rho (1.07 V_0)^2}{2}$$

$$p_1 - p_2 = \frac{\rho (1.07 V_0)^2}{2} - \frac{\rho V_0^2}{2}$$

$$\boxed{p_1 - p_2 = 3.12\ N/m^2} \quad \boxed{3.18\ N/m^2}$$

3.47 Water (assumed inviscid and incompressible) flows steadily in the vertical variable-area pipe shown in Fig. P3.47. Determine the flowrate if the pressure in each of the gages reads 50 kPa.



■ FIGURE P3.47

$$\cancel{P_1} + \frac{\rho V_1^2}{2} + \gamma z_1 = \cancel{P_2} + \frac{\rho V_2^2}{2} + \gamma z_2$$

$$P_1 = P_2 = 50 \text{ kPa}$$

$$\frac{\rho v_1^2}{2} - \frac{\rho V_2^2}{2} = \gamma (z_2 - z_1)$$

$$\frac{\cancel{\rho}}{2}(V_1^2 - V_2^2) = \cancel{\rho} g \overbrace{(z_2 - z_1)}^{10}$$

$$\frac{V_1^2 - V_2^2}{2} = 98.1$$

But $a_1 V_1 = a_2 V_2$

$$\frac{\cancel{\pi} d_1^2}{\cancel{4}} V_1 = \frac{\cancel{\pi} d_2^2}{\cancel{4}} V_2$$

$$V_2 = \frac{d_1^2}{d_2^2} V_1$$

$$V_2 = \frac{1^2}{2^2} V_1$$

$$\boxed{V_2 = \frac{V_1}{4}}$$

So in $\frac{V_1^2 - V_2^2}{2} = 98.1$

$$\frac{V_1^2 - \frac{V_1^2}{4^2}}{2} = 98.1$$

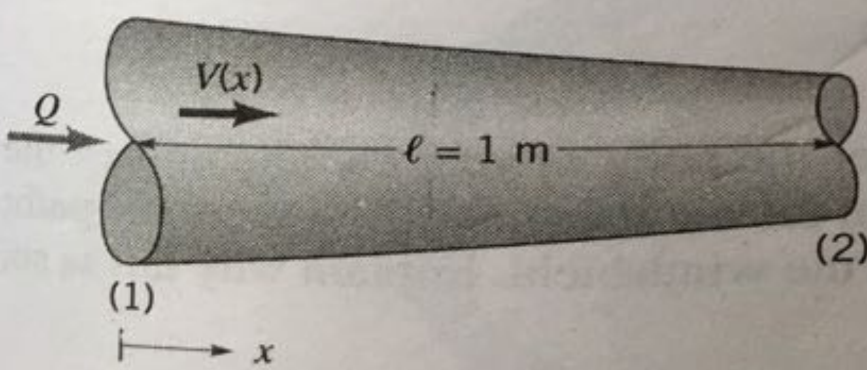
$$\frac{V_1^2 (16 - 1)}{16 \times 2} = 98.1$$

$$\boxed{V_1 = 14.4 \text{ m/s}}$$

$$Q = a_1 V_1 = \frac{\pi d_1^2}{4} \times 14.4$$

$$\boxed{Q = 11.3 \text{ m}^3/\text{s}}$$

3.3 Water flows steadily through the variable area horizontal pipe shown in Fig. P3.3. The centerline velocity is given by $\mathbf{V} = 3(1 + 3x)\,\hat{\mathbf{i}}$ m/s, where x is in meters. Viscous effects are neglected. **(a)** Determine the pressure gradient, $\partial p/\partial x$, (as a function of x) needed to produce this flow. **(b)** If the pressure at section (1) is 345 kPa, determine the pressure at (2) by (i) integration of the pressure gradient obtained in **(a)**, (ii) application of the Bernoulli equation.



■ FIGURE P3.3

$$V = 3(1+3x)\hat{i}$$

$$p + \frac{1}{2}\rho V^2 = \text{Constant}$$

$$\frac{\partial p}{\partial x} = -\rho V \frac{dV}{dx}$$

$$= -\rho\, 3(1+3x)(9)$$

$$\frac{\partial p}{\partial x} = -1000 \times 3(1+3x)\,9$$

$$\boxed{\frac{\partial p}{\partial x} = -27000\,(1+3x)}$$

$$\int_1^2 \frac{\partial p}{\partial x} = \int_{x_1}^{x_2} -27000\,(1+3x)\,dx$$

$$\int_{345\,kPa}^{P_2} \partial p = -\int_0^{1m} \frac{27000}{1000}(1+3x)\,dx$$

$$P_2 - 345 = -\frac{27000}{1000}\left[x + \frac{3x^2}{2}\right]_0^{1m}$$

$$\boxed{P_2 = +277.5\ kPa}$$

By Bernoulli's eqn

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$V_1 = 3(1+3x) = 3(1+0)$$

$$V_1 = 3\ m/s$$

$$V_2 = 3(1+3x) = 3(1+3\times \cancel{0.9})\quad 1m$$

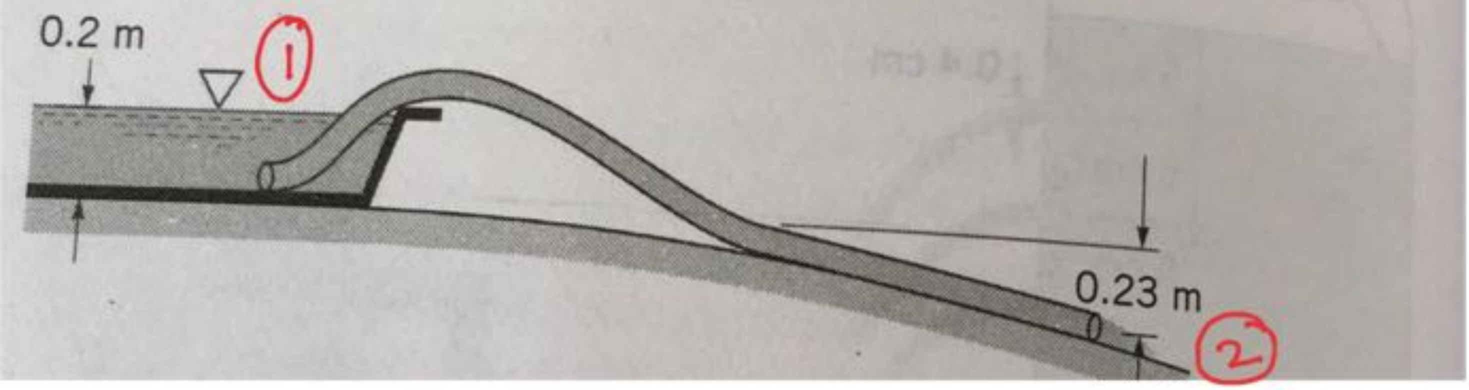
$$V_2 = \cancel{11.1\ m/s}\quad 12\ m/s$$

$$\frac{345}{9.810} + \frac{v_1^2}{2g} + 0 = \frac{P_2}{9.81} + \frac{v_2^2}{2g} + 0$$

$$\frac{345}{9.81} + \frac{3^2}{2 \times 9.81} = \frac{P_2}{9.81} + \frac{\cancel{11.1}^2 \; 12^2}{2 \times 9.81}$$

$$\boxed{P_2 = +277.5 \ kN/m^2}$$

3.59 A smooth plastic, 10-m-long garden hose with an inside diameter of 20 mm is used to drain a wading pool as is shown in Fig. P3.59. If viscous effects are neglected, what is the flowrate from the pool?



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$P_1 = P_2 = 0$

$V_1 = 0$

$V_2 = ?$

$$\cancel{\frac{P_1}{\rho g}}^{0} + \cancel{\frac{V_1^2}{2g}}^{0} + z_1 = \cancel{\frac{P_2}{\rho g}}^{0} + \frac{V_2^2}{2g} + z_2$$

$$\frac{V_2^2}{2g} = z_1 - z_2$$

$$V_2 = \sqrt{2g(z_2 - z_1)} = \sqrt{2 \times 9.81\,((0.23 + 0.2) - 0)}$$

$\because z_2 = 0.23 + 0.2, \quad z_1 = 0$

$V_2 = 2.9\ m/s$ $\boxed{\text{So } Q = a_2 V_2 = 9.11 \times 10^{-4}\ m^3/s}$

3.60 Water exits a pipe as a free jet and flows to a height h above the exit plane as shown in Fig. P3.60. The flow is steady, incompressible, and frictionless. **(a)** Determine the height h. **(b)** Determine the velocity and pressure at section (1).

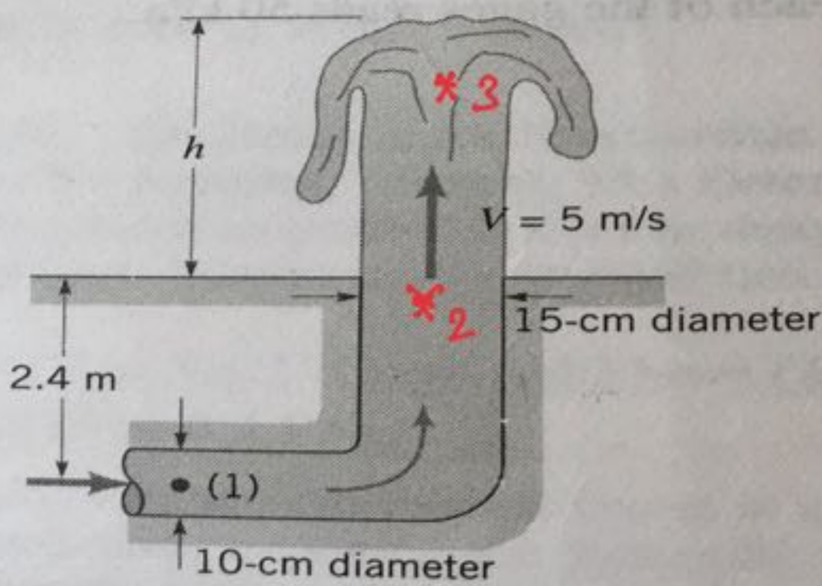


FIGURE P3.60

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + z_3$$

Considering 2 & 3

Here $P_2 = P_3 = 0$

$V_2 = 5\ m/s$

$V_3 = 0$

$$\cancel{\frac{P_2}{\rho g}}^{0} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{P_3}{\rho g}}^{0} + \cancel{\frac{V_3^2}{2g}}^{0} + z_3$$

$$\frac{25}{2\times 9.81} = z_3 - z_2$$ But $z_2 = 0$, $z_3 = h$

$1.274 = h$

$$\boxed{h = 1.274 m}$$

Similarly $a_1 v_1 = a_2 v_2$

but $v_2 = 5 m/s$

$$\frac{\pi}{4} d_1^2 v_1 = \frac{\pi}{4} d_2^2 v_2$$

$$\frac{\pi}{4} 0.1^2 v_1 = \frac{\pi}{4} 0.15^2 . 5$$

$$\boxed{v_1 = 11.25 m/s}$$

Bernoulli's eqn b/w ① & ②

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

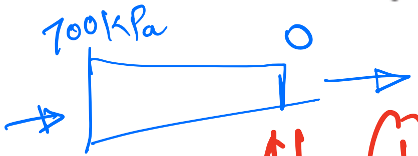
$P_2 = 0$

$$\frac{P_1}{\rho g} + \frac{11.25^2}{2 \times 9.81} + 0 = 0 + \frac{5^2}{2 \times 9.81} + 2.4$$

$$\boxed{P_1 = -27.3 \, kPa}$$

Cengel

5–42 In a hydroelectric power plant, water enters the turbine nozzles at 800 kPa absolute with a low velocity. If the nozzle outlets are exposed to atmospheric pressure of 100 kPa, determine the maximum velocity to which water can be accelerated by the nozzles before striking the turbine blades.



At ① 800 KPa absolute = (800 − 100) = 700 KPa (gage)

low velocity

② 100 KPa = (100 − 100) = 0 KPa

V = ?

low velocity

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1}^{\,0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}^{\,0}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} \quad \text{or} \quad V_2^2 = \frac{2(P_1 - P_2)}{\rho}$$

$$V_2 = \sqrt{2\left(\frac{P_1 - P_2}{\rho}\right)} = \sqrt{\frac{2(700 - 0)}{1000}}$$

$$V_2 = 1.18 \text{ m/s}$$

Cengel

5–43 A Pitot-static probe is used to measure the speed of an aircraft flying at 3000 m. If the differential pressure reading is 3 kPa, determine the speed of the aircraft. Take

$\rho_{air} = 0.909 \ Kg/m^3$

$$V = \sqrt{\frac{2(P - P_2)}{\rho}} = \sqrt{\frac{2 \times 3}{0.909}}$$

$V = 2.56 \ m/s$

A horizontal venturimeter with inlet diameter 20cm and throat diameter 10cm is used to measure the flow of oil of sp. gravity 0.8. The discharge of oil through venturimeter is 60 Lt/s. Find the reading of the oil-mercury differential manometer. Take $C_d = 0.98$

$$Q_{act} = C_d \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$Q = 60 \text{ Lt/s}$
$= 0.06 \, m^3/s$

$a_1 = \frac{\pi d^2}{4} = \frac{\pi (0.2)^2}{4}$

$a_1 = 0.031 \, m^2$

$a_2 = \frac{\pi d^2}{4} = \frac{\pi \, 0.1^2}{4}$

$a_2 = 0.0078 \, m^2$

$$Q_{act} = \frac{0.98 \times 0.031 \times 0.0078 \sqrt{2gh}}{\sqrt{0.031^2 - 0.0078^2}}$$

$Q_{act} \rightarrow 0.06$

$h = 2.89 \, m$

If heavier/lighter liquid in manometer

① heavier fluid in pipe.

$$h = x\left[\frac{SG_m}{SG_h} - 1\right]$$

2. lighter fluid in pipe

$$h = x\left[1 - \frac{SG_m}{SG_L}\right]$$

$$h = x\left[\frac{SG_m}{SG_{oil}} - 1\right]$$

$$2.9\,m = x\left[\frac{13.6}{0.8} - 1\right]$$

$$\boxed{x = 0.181\,m}$$

Unit - 4

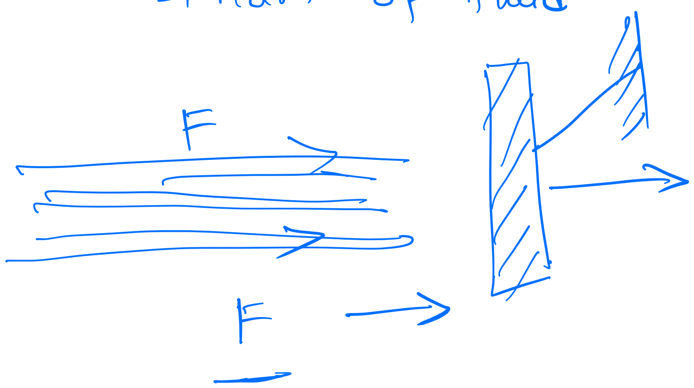
Impact of jets

Viscous flow through pipes

External flows

Jet

Stream of fluid

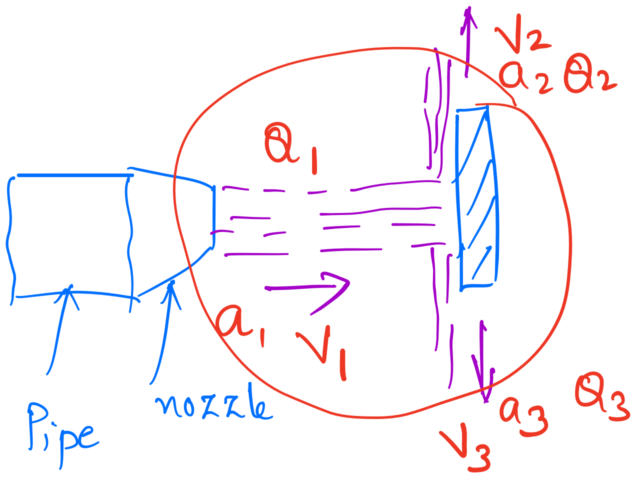


$$F = ma$$

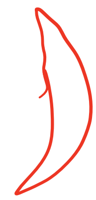
$$= m \cdot \frac{v}{t} = \rho a v \cdot \frac{v}{t} = \frac{\rho a v^2}{}$$

$$= \frac{m}{t} v$$

Impact of Jets



$a_1 = a_2 + a_3$



$$Q_1 = Q_2 + Q_3$$

$$\rho a_1 v_1 = \rho a_2 v_2 + \rho a_3 v_3$$

$$F_x = \frac{\text{change of momentum}}{\text{time}} \text{ in x axis}$$

$$F_x = m a_x$$

$$F_x = \frac{mv}{t}$$

$$F_x = \frac{m}{t} v$$

$$F_x = \rho a V . V$$

$$F_x = \rho a V \left[\text{Intial velocity} - \text{Final velocity} \right]$$

Consideration

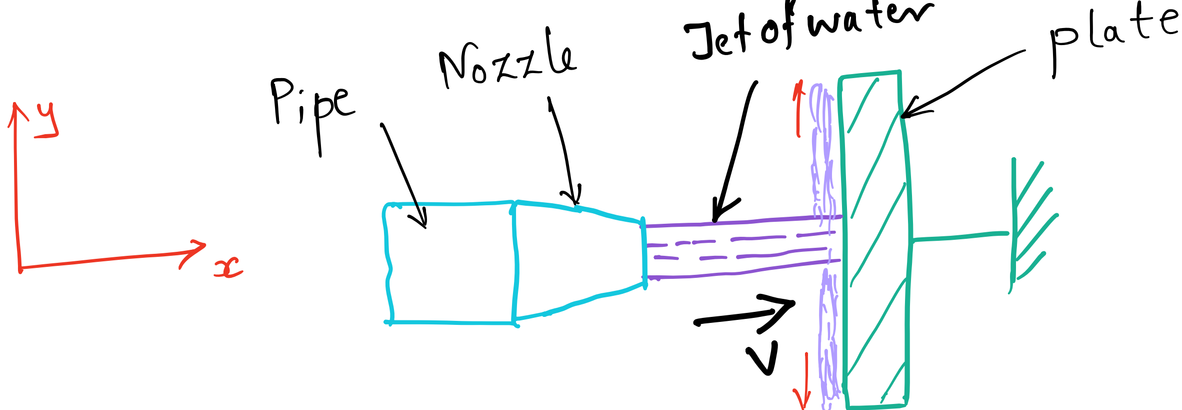
Incompressible, stead, inviscid.

Impact of Jets

Stationary plates

Stationary vertical plate

Force exerted by a jet of water on a stationary fixed plate



Consider a jet of water coming out from a nozzle of diameter 'd', strikes a fixed plate with velocity 'v' as shown in the figure.

Let 'a' be the area of cross-section of the jet.

From the momentum equation,

Force exerted by the jet on the plate = rate of change of momentum

$$F = \frac{\text{Initial momentum} - \text{Final momentum}}{\text{time}}$$

$$F_x = \frac{\text{mass}}{\text{time}}\left[\text{Initial velocity} - \text{final velocity}\right]$$

$$F_x = \rho a v\,[v - 0]$$

$$\boxed{F_x = \rho a v^2}$$

Similarly

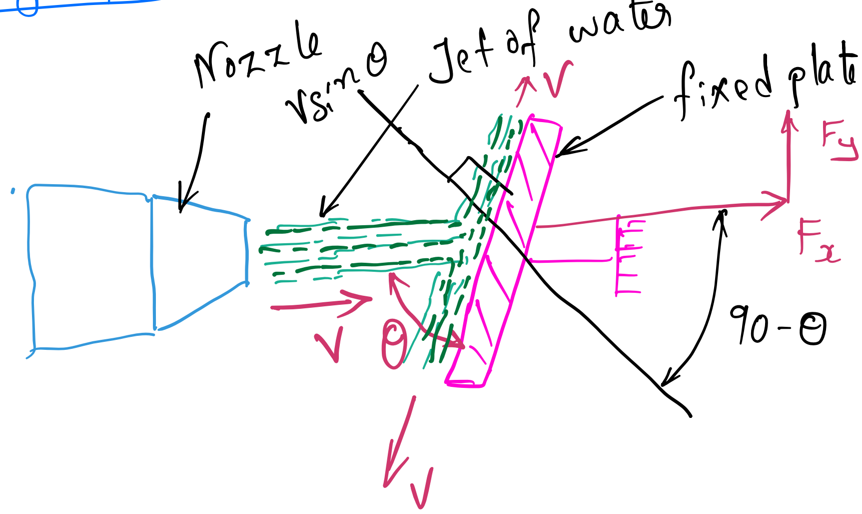
$$F_y = \frac{\text{mass}}{\text{time}}\left[\text{Initial velocity} - \text{final velocity}\right]$$

$$F_y = \rho a v\,[0 - v]$$

$$\boxed{F_y = -\rho a v^2}$$

Stationary Inclined plate

Force exerted by a jet of water on an inclined stationary plate.



$$F_x = F_n\cos(90^\circ-\theta) = F_n\sin\theta$$

$$F_y = F_n\sin(90^\circ-\theta) = F_n\cos\theta$$

F_n is the normal force

Now

Force in x-dirn. is

$$F_x = F_n\sin\theta$$

$$F_x = \frac{m}{t}\left[\text{Initial velocity} - \text{Final velocity}\right]\sin\theta$$

$$F_x = \rho a V\left[V\sin\theta - 0\right]\sin\theta$$

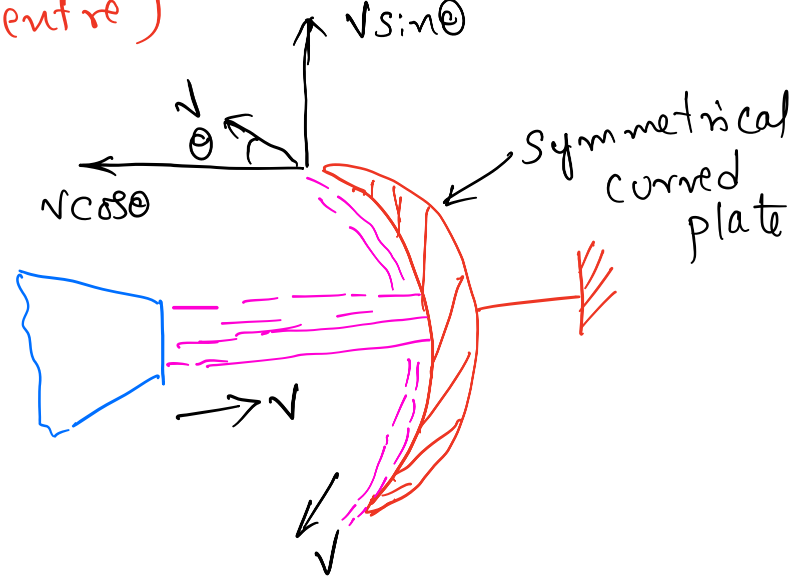
$$\boxed{F_x = \rho a V^2\sin^2\theta}$$

$$F_y = \rho a V\left[0 + V\sin\theta\right]\cos\theta$$

$$\boxed{F_y = \rho a V^2\sin\theta\cos\theta}$$

Stationary Curved plate

Force exerted by a jet on a stationary curved plate (at the centre)



$$F_x = \frac{\text{mass}}{\text{time}}\left[\text{Initial velocity} - \text{final velocity}\right]$$

$$F_x = \rho a v\left[V - (-V\cos\theta)\right]$$

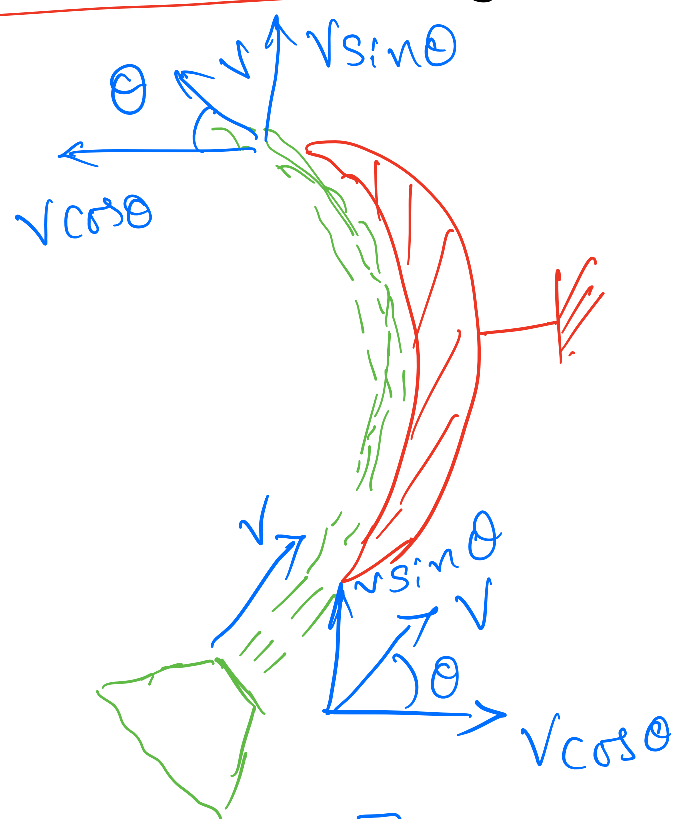
$$\boxed{F_x = \rho a v^2\left[1 + \cos\theta\right]}$$

$$F_y = \rho a v\left[0 - V\sin\theta\right]$$

$$\boxed{F_y = -\rho a V^2 \sin\theta}$$

Stationary curved plate

Force exerted by the jet on a fixed curved plate at one end tangentially



$$F_x = \rho a v\,[V\cos\theta - (-V\cos\theta)]$$

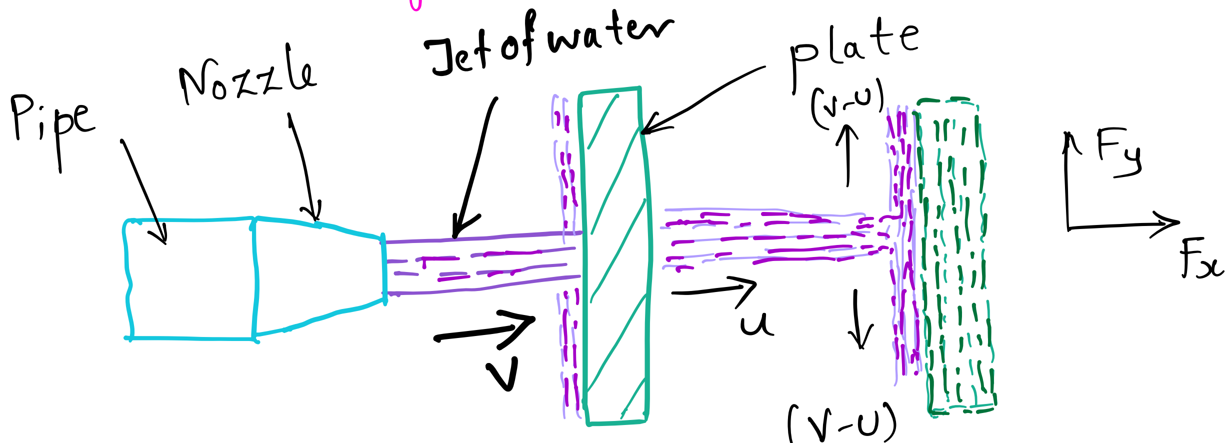
$$F_x = 2\rho a V^2\cos\theta$$

$$F_y = \rho a v\,[\cancel{V\sin\theta} - \cancel{V\sin\theta}]$$

$$F_y = 0$$

Moving plates

Force exerted by a jet on a moving vertical plate



Consider a jet of water from a nozzle of diameter 'd' strikes a movable plate. As a result the plate moves with u m/s away from the jet. Let v m/s be the velocity of the jet of water.

F = rate of change of momentum

$$F = \frac{\text{mass}}{\text{time}}\left[\text{Initial velocity} - \text{Final velocity}\right]$$

$$F_x = \rho a(v-u)\left[(v-u) - 0\right]$$

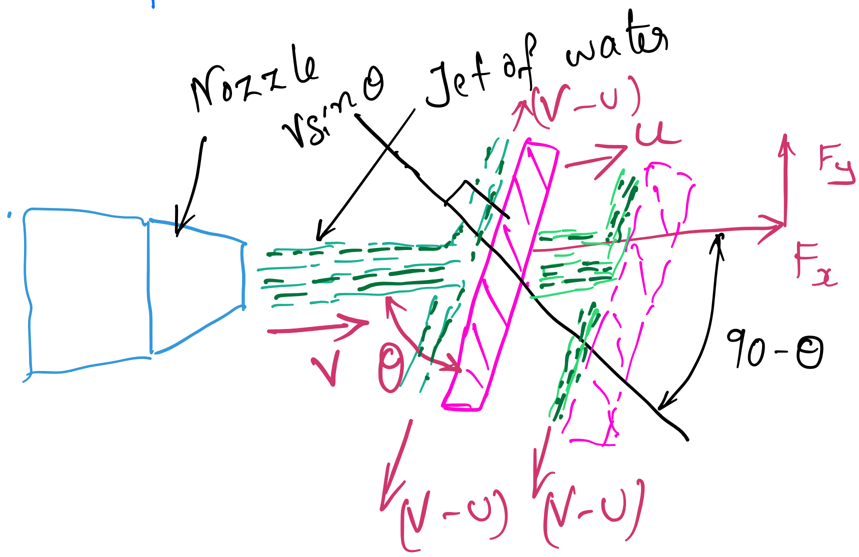
$$\boxed{F_x = \rho a (v-u)^2}$$

$$F_y = \rho a(v-u)\left(0 - (v-u)\right)$$

$$\boxed{F_y = -\rho a (v-u)^2}$$

Movable Inclined plate

Force exerted by a jet of water on an inclined movable plate



$F_x = F_n \cos(90° - \theta) = F_n \sin\theta$

$F_y = F_n \sin(90° - \theta) = F_n \cos\theta$

F_n is the normal force

Now

Force in x-dirn. is

$F_x = F_n \sin\theta$

$F_x = \frac{m}{t}\left[\text{Initial velocity} - \text{Final velocity}\right]\sin\theta$

$F_x = \rho a (V-U)\left[(V-U)\sin\theta - 0\right]\sin\theta$

$$\boxed{F_x = \rho a (V-U)^2 \sin^2\theta}$$

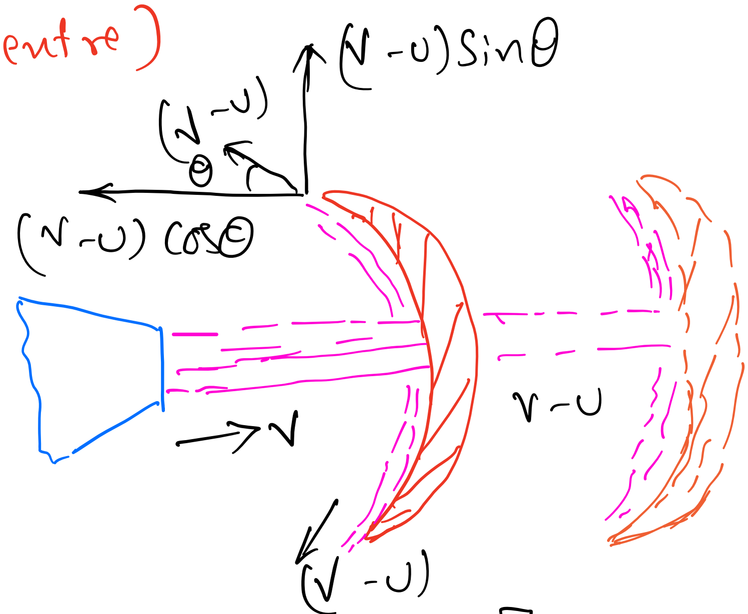
$$F_y = F_n \cos\theta$$

$$F_y = \rho a (v-u)\left[0 + (v-u)\sin\theta\right]\cos\theta$$

$$F_y = \rho a (v-u)^2 \sin\theta \cos\theta$$

Moving Curved plate

Force exerted by a jet on a moving curved plate (at the centre)



$$F_x = \frac{\text{mass}}{\text{time}}\left[\text{Initial velocity} - \text{final velocity}\right]$$

$$F_x = \rho a(V-U)\left[(V-U) - \left(-(V-U)\cos\theta\right)\right]$$

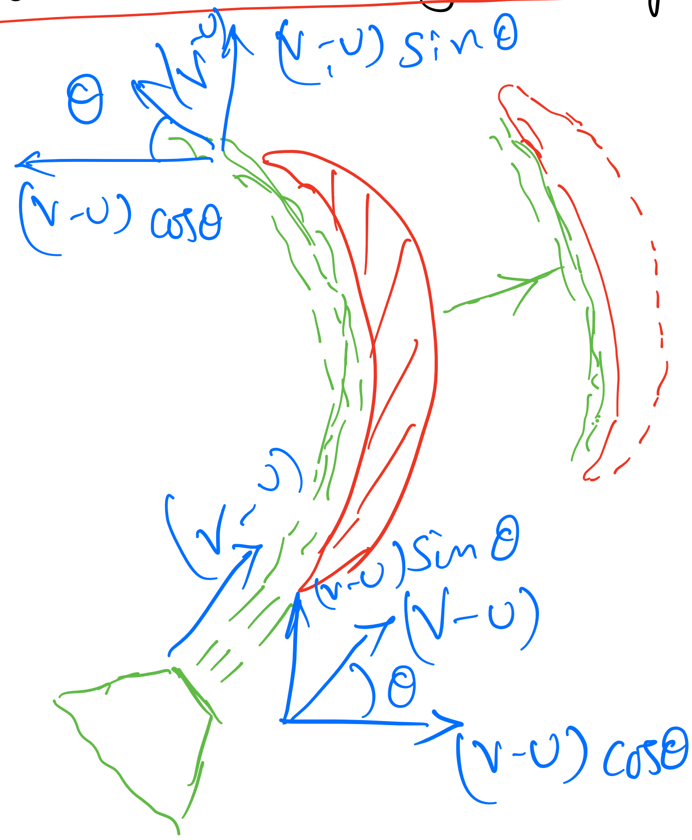
$$\boxed{F_x = \rho a (V-U)^2\left[1+\cos\theta\right]}$$

$$F_y = \rho a(V-U)\left[0 - (V-U)\sin\theta\right]$$

$$\boxed{F_y = -\rho a(V-U)^2\sin\theta}$$

Moving curved plate

Force exerted by the jet on a movable curved plate at one end tangentially



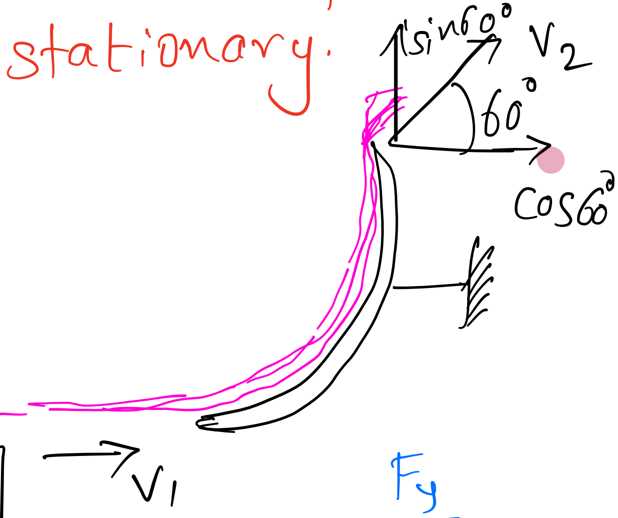
$$F_x = \rho a (V-U)\left[(V-U)\cos\theta - \left(-(V-U)\cos\theta\right)\right]$$

$$\boxed{F_x = 2\rho a (V-U)^2 \cos\theta}$$

$$F_y = \rho a (V-U)\left[(V-U)\cancel{\sin}\theta - (V-U)\cancel{\sin}\theta\right]$$

$$\boxed{F_y = 0}$$

A jet of water from a nozzle is deflected through 60° from its original direction by a curved plate which it enters tagentially without shock with a velocity of 30 m/s and leaves with a velocity of 25 m/s. If the discharge from the nozzle is 0.8 kg/s. Calculate the magnitude and direction of resultant force, if the vane is stationary.



$\dot{m} = 0.8\,kg/s$

$V_1 = 30\,m/s, \quad V_2 = 25\,m/s$

$F_x = \dot{m}\left[V_1 - V_2\cos 60^\circ\right]$

$F_x = 0.8\left[30 - 25\cos 60^\circ\right]$

$F_x = 14\,N$

$F_y = \dot{m}\left[0 - V_2 \sin 60^\circ\right]$

$F_y = 0.8\left[0 - 12.5\right]$

$F_y = -17.32\,N$

$\tan\theta = \dfrac{F_y}{F_x}$

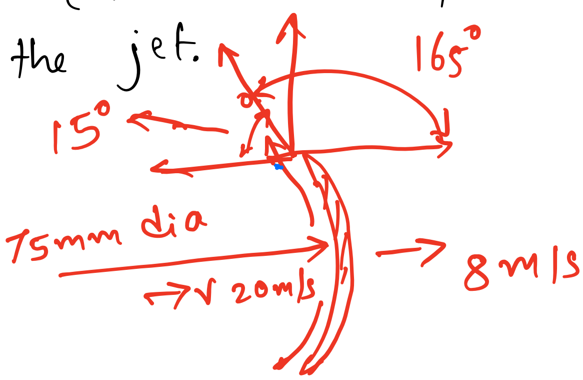
$\boxed{\theta = -51^\circ}$

Resultant $F = \sqrt{F_x^2 + F_y^2}$

$F = \sqrt{14^2 + (-17.32)^2}$

$\boxed{F = 22.27\,N}$

A jet of water of diameter 75mm strikes a curved plate at its centre with a velocity of 20m/s. The curved plate is moving with a velocity of 8 m/s in the direction of the jet. The jet is deflected through an angle of 165°. Find (i) the force exerted by the jet on the plate (ii) Work done per second and (iii) Efficiency of the jet.



$\theta = 180^\circ - 165^\circ = 15^\circ$

$F_x = \rho a (V-U) \left[(V-U) - (-(V-U)\cos\theta) \right]$

$F_x = \rho a (V-U)^2 \left[1 + \cos\theta \right]$

$F_x = 1000 \times \frac{\pi}{4} 0.075^2 \left[20 - 8 \right]^2 \times \left[1 + \cos 15^\circ \right]$

$\boxed{F_x = 1.25 \text{ kN}}$

$$\frac{\text{Power of the jet}}{(\text{workdone}/\text{sec})} = F_x \times U$$

$$= 1.25 \times 8$$

$$= 10 \text{ KW}$$

$$\text{Efficiency of the jet } \eta = \frac{\text{workdone}/\text{sec}}{\text{Kinetic energy}/\text{sec}}$$

$$\eta = \frac{1.250 \times 8}{\frac{1}{2} m V^2} = \frac{1.250 \times 8}{\frac{1}{2} (\rho a V) V^2} = \frac{1.250 \times 8}{\frac{1}{2} \times 1000 \times 0.0044 \times 20 \times 20^2}$$

$$\boxed{\eta = 56.4\%}$$

Energy losses in the flow through pipes

Energy losses

Major energy losses

Due to Friction in pipes

Calculated by

1. Darcy-Weisbach formula
2. Chezy's formula

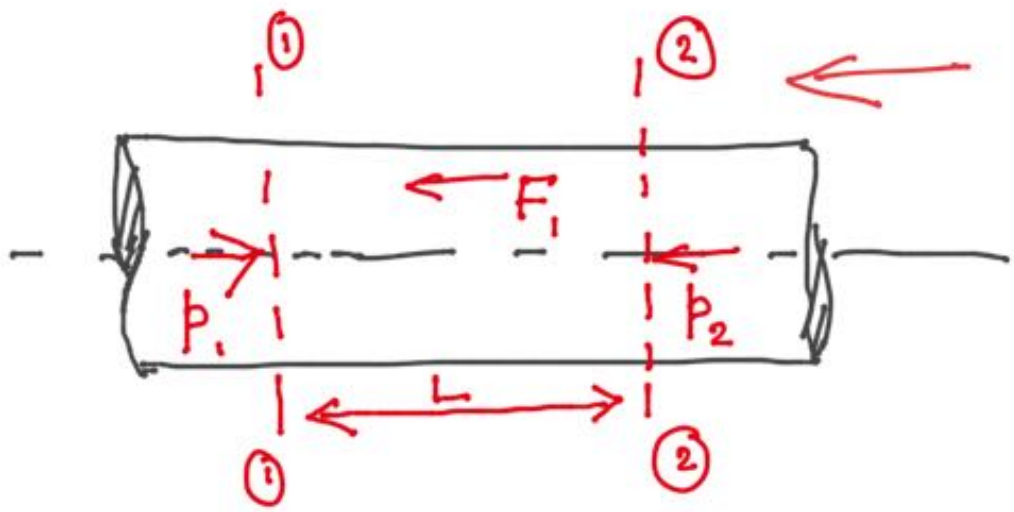
Minor energy losses

Due to change in cross-sectional area

Eg:- Bend
pipe fittings
sudden expansion
sudden contraction
Obstruction, etc

Individually by applying Bernoulli's expression

Expression for Darcy - Weisbach formula



F_1 is viscous force

Applying Bernoulli's eqn. b/w section ①-① and ②-②,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

Bernoulli's eqn for real fluids

$h_f \rightarrow$ head loss due to friction

Since pipe is horizontal and uniform diameter

$z_1 = z_2$ and $v_1 = v_2$

So,
$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f$$

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = h_f$$

$$h_f = \frac{P_1 - P_2}{\rho g}$$

$$\boxed{\rho g h_f = P_1 - P_2}$$

Resolving the forces in x-dirn./horizontal dirn., we have

$$p_1 a - p_2 a - F_1 = 0$$

but $F_1 = f' \pi d L v^2$

$$p_1 a - p_2 a - f' \pi d L v^2 = 0$$

$$p_1 - p_2 = f' \frac{\pi d L}{a} v^2$$

But $\frac{\pi d L}{a} = \frac{\cancel{\pi} \cancel{d} L}{\frac{\cancel{\pi} d^{\cancel{2}}}{4}} = \frac{4L}{d}$

So

$$\underline{p_1 - p_2} = \frac{f' 4 L}{d} v^2$$

$$\rho g h_f = \frac{f' 4 L v^2}{d}$$

$$\rho g h_f = \frac{f' 4 L V^2}{d}$$

$$h_f = \frac{f' 4 L V^2}{\rho g d}$$

but $\frac{f'}{\rho} = \frac{f}{2}$, $f \rightarrow$ co-efficient of friction

$$h_f = \frac{f}{2} \frac{4 L V^2}{g d}$$

rearranging,

$$\boxed{h_f = \frac{4 f L v^2}{2 g d}}$$

4f is friction factor

This is Darcy's-Weisbach formula

<u>Chezy's formula</u>

$$h_f = \frac{f' 4 L V^2}{\rho g d} = \frac{f' L V^2}{\rho g \frac{d}{4}}$$

$\frac{d}{4}$ is called hydraulic mean depth 'm

$$h_f = \frac{f' 4 L V^2}{\rho g d} = \frac{f' L V^2}{\rho g \frac{d}{4}}$$

So $$h_f = \frac{f' L V^2}{\rho g m}$$

rearranging

$$V^2 = \frac{h_f \rho g m}{f' L} = \frac{h_f}{L} \times \frac{\rho g}{f'} \times m$$

$$V = \underbrace{\sqrt{\frac{h_f}{L}}}_{i} \times \underbrace{\sqrt{\frac{\rho g}{f'}}}_{C} \times \sqrt{m}$$

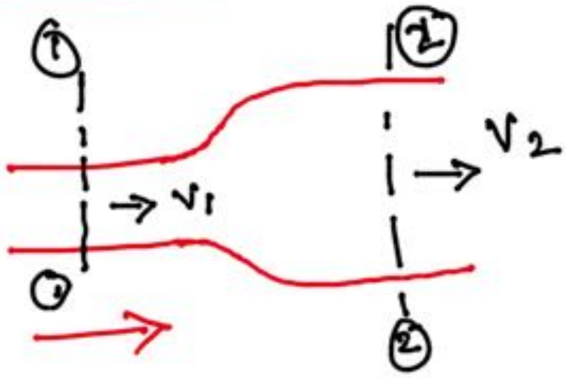
Here $\sqrt{\frac{\rho g}{f'}} = C$, a constant known as Chezy's constant

$\frac{h_f}{L}$ is head loss per unit length, 'i'

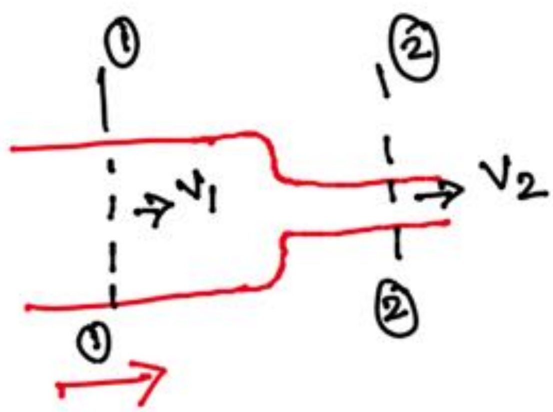
So $\boxed{V = C\sqrt{mi}}$ Chezy's formula

Minor losses

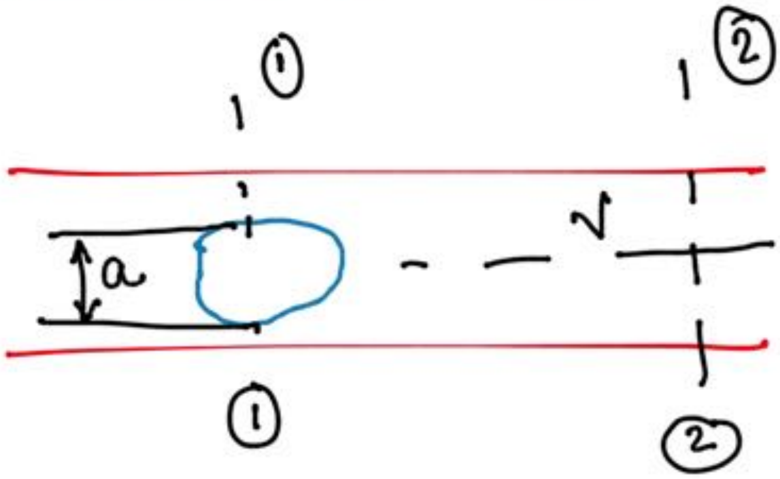
Sudden enlargement



Sudden contraction



Obstruction in a pipe



Minor losses

1. Head loss due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

2. Head loss due to sudden contraction

$$h_c = 0.5\frac{V_2^2}{2g} = \left[\frac{1}{C_c} - 1\right]\frac{V_2^2}{2g}$$

3. Head loss at the entrance of the pipe

$$h_i = \frac{0.5V^2}{2g}$$

4. Head loss at the exit of the pipe

$$h_{exit} = \frac{V^2}{2g}$$

5. Head loss due to an obstruction in a pipe.

$$h_{obs} = \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]$$

6. Head loss due to bend in pipe

$$h_b = \frac{kv^2}{2g}$$

7. Head Loss due to various pipe fittings

$$h_{fittings} = \frac{kv^2}{2g}$$

Problems

An oil of specific gravity 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 lt/s. Find the head loss due to friction and power required to maintain the flow for a length of 1000 m. Take $\nu = 0.29$ stokes.

$$V = \frac{Q}{A} = \frac{0.5}{\frac{\pi}{4}d^2} = \frac{0.5}{\frac{\pi}{4}\times 0.3^2} = 7.073\ m/s$$

Reynold's no. $R_e = \frac{V d}{\nu} = \frac{7.073\times 0.3}{0.29\times 10^{-4}} = 7.316\times 10^4$

$$\boxed{R_e = \frac{\rho V D}{\mu} \qquad R_e = \frac{VD}{\mu/\rho} = \frac{VD}{\nu}}$$

$$f = \frac{0.079}{R_e^{1/4}} = \frac{0.079}{(7.316\times 10^4)^{1/4}} = 0.0048$$

$$h_f = \frac{4fLV^2}{2gd} = \frac{4\times 0.0048\times 1000\times 7.073^2}{2\times 9.81\times 0.3} = 163.18\ m$$

Power required $P = \rho g Q h_f$

$$P = (0.7\times 1000)\,9.81\times\left(\frac{500}{1000}\ m^3/s\right)\times 163.18$$

$$P = 560.28\ kW$$

R_e

$$f = \frac{16}{R_e} \text{ for } R_e < 2000$$

$$f = \frac{0.079}{R_e^{1/4}} \text{ for } R_e \text{ b/w } 4000 \text{ to } 10^6$$

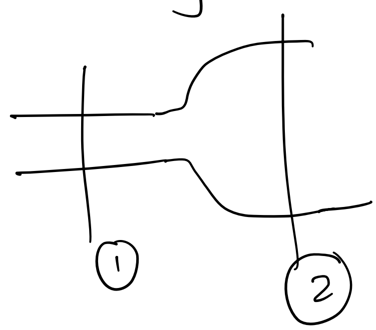
At a sudden enlargement of a water main from 240mm to 480mm diameter, the hydraulic gradient rises by 10mm. Estimate the rate of flow

$$\text{Hydraulic gradient} = \left(z_2 + \frac{P_2}{\rho g}\right) - \left(z_1 + \frac{P_1}{\rho g}\right)$$

$$= 10\,mm = 0.01\,m$$

Head loss due to sudden enlargement =

$$= h_e = \frac{(V_1 - V_2)^2}{2g}$$



$$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} 0.240^2$$

$$a_2 = \frac{\pi}{4} 0.480^2$$

$$V_1 = \frac{a_2}{a_1} V_2 = 4V_2$$

$$\text{So } h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Bernoulli's exp.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e$$

$$\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \frac{9V_2^2}{2g}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \underbrace{\left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right)}_{0.01\,m}$$

$$\boxed{V_2 = 0.181\ m/s}$$

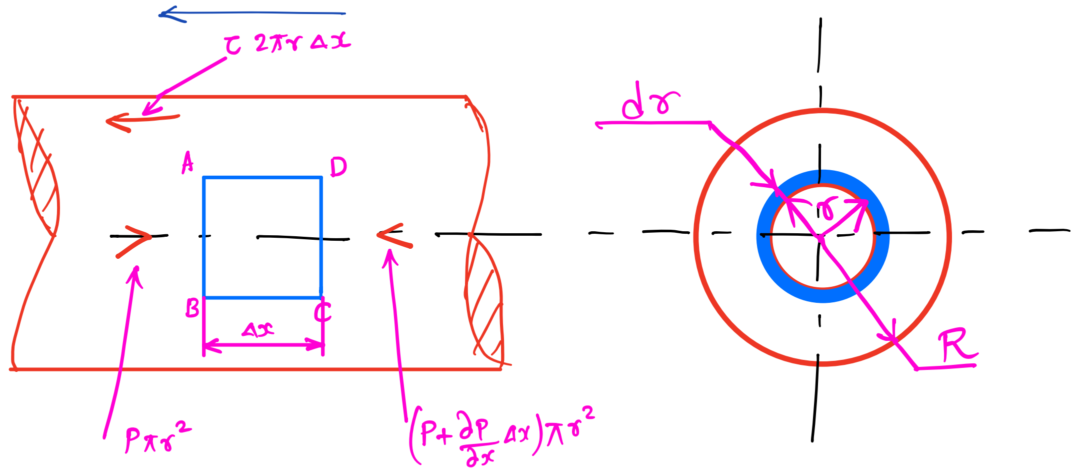
$$Q = a_2 V_2 = \frac{\pi}{4} 0.48^2 \times 0.181$$

$$\boxed{Q = 32.75\ lt/s = 0.032\ m^3/s}$$

Hagen Poiseuille Equation

or

Flow of the viscous fluid through a circular pipe



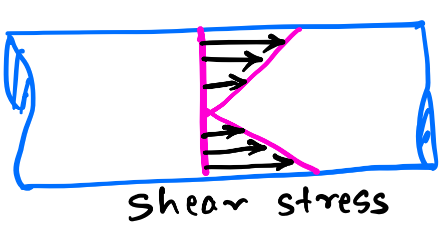
$\Sigma F_x = 0$

So,

$$\cancel{p\pi r^2} - \left(\cancel{p} + \frac{\partial p}{\partial x}\Delta x\right)\cancel{\pi r^2} - \tau\, 2\pi r\, \Delta x = 0$$

$$-\frac{\partial p}{\partial x}\cancel{\Delta x}\,\cancel{\pi}\,\cancel{r^2} = \tau\, 2\cancel{\pi}\cancel{r}\,\cancel{\Delta x}$$

$$\boxed{\tau = -\frac{\partial p}{\partial x}\cdot\frac{r}{2}} \quad \text{--- --- --- } (1)$$



Shear stress

Velocity

τ is also $= \mu \frac{du}{dy}$

But here $y = R - r$

$dy = -dr$

So $\tau = \mu \frac{du}{-dr}$

From equation 1

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2} = -\mu \frac{du}{dr}$$

$$\frac{du}{dr} = \frac{\partial p}{\partial x}\frac{r}{2\mu}$$

Integrating w.r.t r

$$\boxed{u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C} \quad \text{--- --- (2)}$$

To find the value of constant 'C'.

Boundary conditions

At $r = R$, $u = 0$

So, $u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C$, becomes

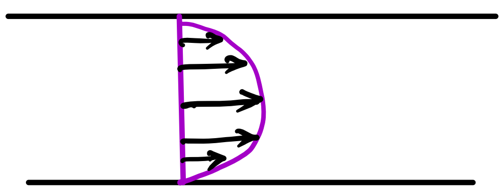
$$0 = \frac{1}{4\mu}\frac{\partial p}{\partial x}R^2 + C$$

$$C = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2$$

Substituting C in equation 2, we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x}R^2$$

$$\boxed{u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right]}$$



Velocity distribution

To find the average velocity U_{avg}

The average velocity is obtained by dividing the discharge of the fluid across the sectional area in which it is flowing.

$$dQ = \text{Velocity at radius} \times \text{area of ring element}$$

$$dQ = u \times 2\pi r\, dr$$

$$dQ = -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] 2\pi r\, dr$$

$$Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] 2\pi r\, dr$$

$$Q = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \int_0^R (R^2 - r^2)\, r\, dr$$

$$Q = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$Q = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$\boxed{Q = \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4}$$

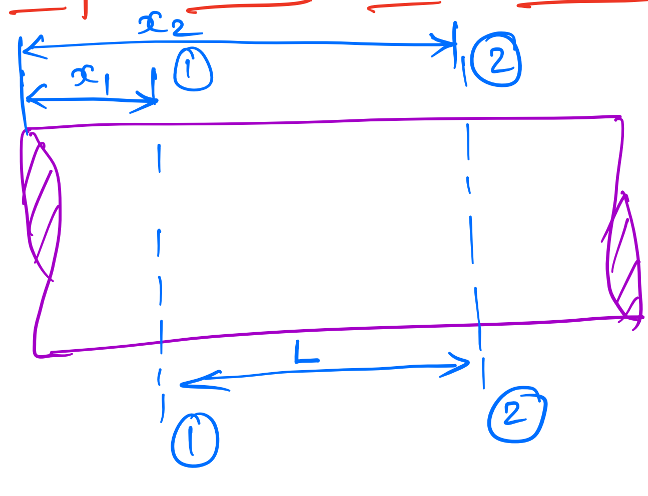
$$u_{avg} = \bar{U} = \frac{Q}{\text{area}}$$

$$\bar{U} = \frac{\frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4}{\pi R^2}$$

$$\frac{U_{max}}{\bar{U}} = \frac{-\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)R^2}{-\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)R^2}$$

$$\boxed{\frac{U_{max}}{\bar{U}} = 2.0}$$

Pressure drop across the section



Using $\bar{U}$ equation

$$\bar{U} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

$$-\frac{\partial p}{\partial x} = 8\mu\frac{\bar{U}}{R^2}$$

$$-\int_2^1 \partial p = 8\int_2^1 \mu\frac{\bar{U}}{R^2}\,dx$$

$$-(P_1 - P_2) = 8\mu\frac{\bar{U}}{R^2}\left[x_1 - x_2\right]$$

$$P_1 - P_2 = 8\mu\frac{\bar{U}}{R^2}\left[x_2 - x_1\right]$$

But $x_2 - x_1 = L$

$$P_1 - P_2 = 8\mu\frac{\bar{U}}{R^2}L$$

$$P_1 - P_2 = 8\mu\frac{\bar{U}L}{(d/2)^2} = \frac{32\mu\bar{U}L}{d^2}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{32\mu\bar{U}L}{\rho g d^2}$$

So $$\boxed{h_f = \frac{P_1 - P_2}{\rho g} = \frac{32 \mu \bar{U} L}{\rho g d^2}}$$

This equation is known as Hagen-Poiseuille's equation.

Problems

An oil of viscosity $0.1\,Ns/m^2$ and relative density 0.9 is flowing through a circular pipe of diameter 50mm and length 300m. Find the pressure drop in a length of 300m and also the shear stress at the pipe wall. Also find the max. velocity and velocity at 4mm from the wall. Also find the power required. (Take discharge as 3.5 lt/s)

$\mu = 0.1\,Ns/m^2$

$\rho = 0.9 \times 1000 = 900\,kg/m^3$

$d = 0.05\,m,\; R = 0.025\,m$

$Q = 3.5\,lt/s = \frac{3.5}{1000} = 0.0035\,m^3/s$

To find

$p_1 - p_2$

u_{max}

$u_{@\,4mm}$

Power

$$R_e = \frac{\rho \bar{V} d}{\mu} = \frac{900 \times \left(\frac{Q}{a}\right) \times .05}{0.1}$$

$R_e = 801.9$

$R_e < 2000$, so flow is viscous laminar

$$\bar{v} = Q/a = \frac{0.0035}{\pi/4\, d^2} = 1.782\ m/s$$

$$p_1 - p_2 = \frac{32 \mu \bar{v} l}{d^2} = \frac{32 \times 0.1 \times 1.782 \times 300}{0.05^2}$$

$$\boxed{p_1 - p_2 = 684\ kN/m^2}$$

τ_0

$$\tau_0 = -\frac{\partial p}{\partial x} \frac{r}{2}$$

Shear stress at the pipewall $r = R$

$$\tau_0 = -\frac{\partial p}{\partial x} \frac{R}{2}$$

$$-\frac{\partial p}{\partial x} = -\frac{(p_2 - p_1)}{x_2 - x_1} = \frac{p_1 - p_2}{L} = \frac{684}{300} = 2.28\ kN/m^3$$

$$\tau_0 = 2.28 \times \frac{0.025}{2}$$

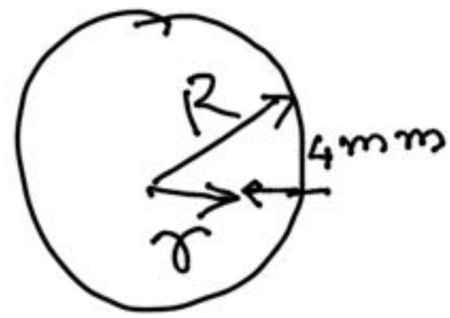
$$\boxed{\tau_0 = 0.028\ kN/m^2}$$

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$U_{max} = \frac{1}{4 \times 0.1} \times 2.28 \times 0.025^2$$

$$\boxed{U_{max} = 3.564 \, m/s}$$

$U_{@4mm}$



$$r = R - 4$$

$$= 0.025 - 0.004$$

$$\boxed{r = 0.021 m}$$

$$U_{@4mm} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] = \frac{1}{4 \times 0.1} \times 2.28 \times \left[0.025^2 - 0.021^2\right]$$

$$\boxed{U_{@4mm} = 1.04 \, m/s}$$

Power

$$P = \rho g Q h_f$$

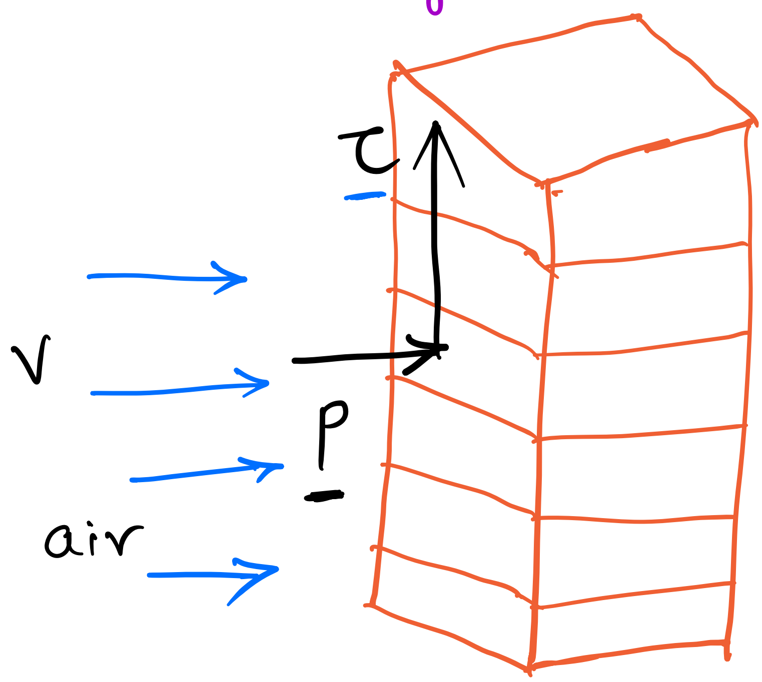
$$P = \cancel{900} \times \cancel{9.81} \times 0.0035 \times \frac{p_1 - p_2}{\cancel{\rho g}}$$

$$P = 0.0035 \times 684$$

$$\boxed{P = 2.4 \, KW}$$

External Flow

Drag and Lift

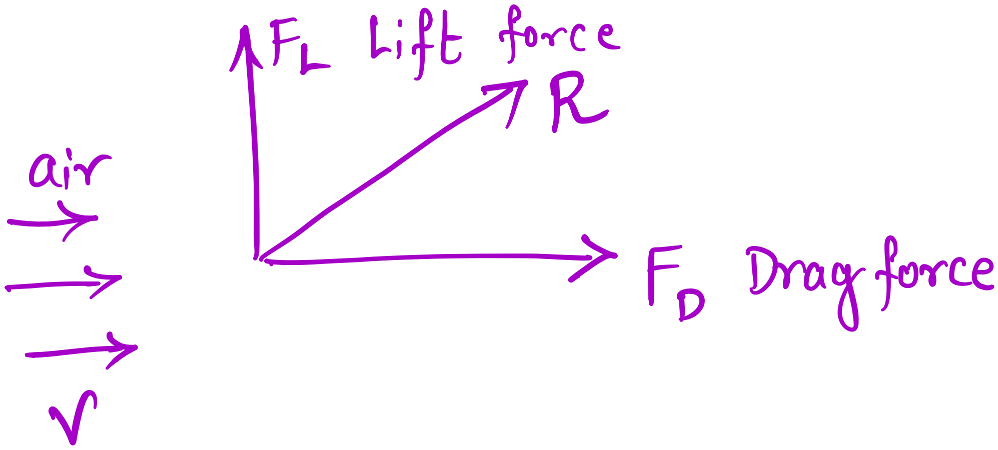


Air blowing over building, trees, plant, automobiles, aircrafts.
↓
External flow

Resultant force of pressure and shear forces acting on the entire surface of the body.

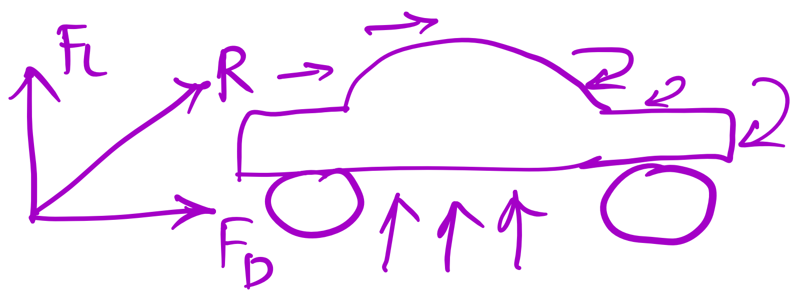
Component of the resultant pressure and shear forces acting in the flow direction is called as the drag force or drag.

Component of the resultant pressure and shear forces acting normal to the flow direction is called as Lift force or Lift.



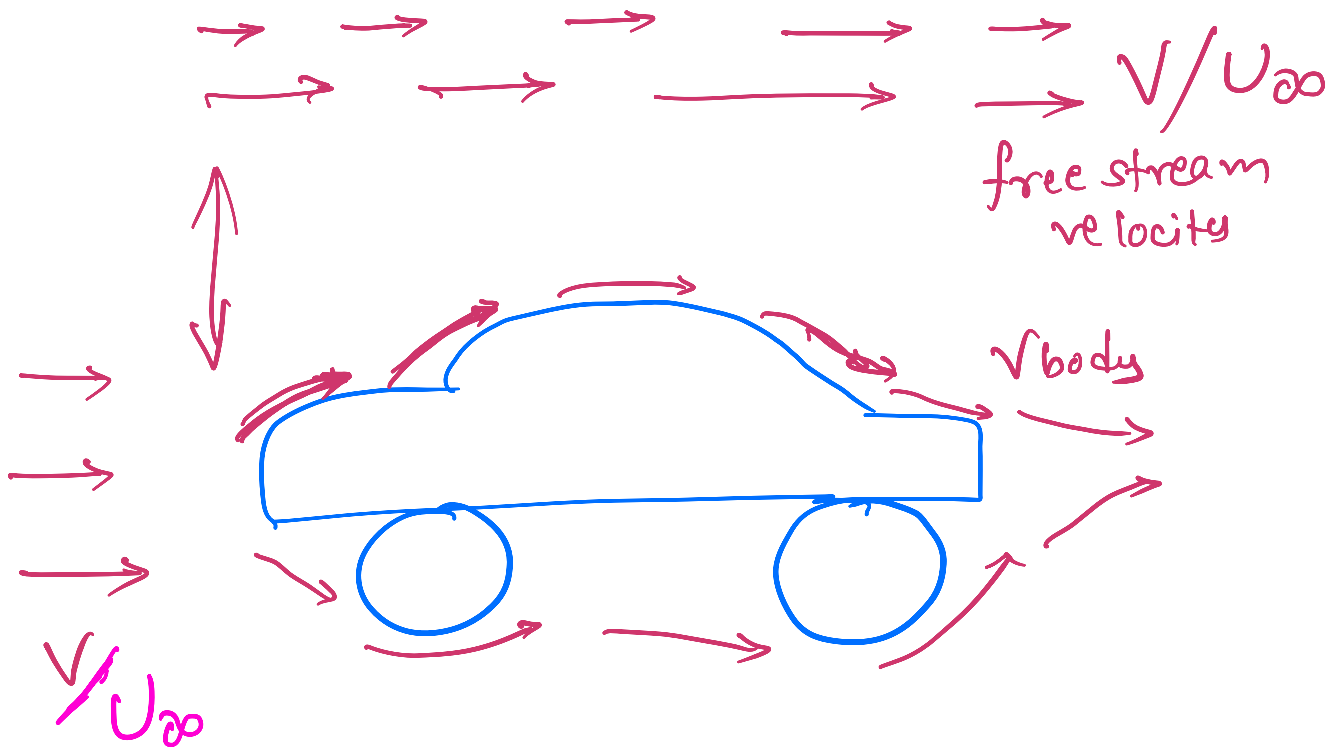
Lift force will move the body in that direction.

Drag force will resist the flow of the body.



Lift (Pressure and shear forces)

Drag (Pressure and shear forces)



Free stream velocity: The velocity of the fluid approaching a body.

Incompressible flows:

Flow over building / automobiles / trees / ($Ma < 0.3$)

Compressible flows:

Flow over aircrafts (high speeds), rockets, missiles.

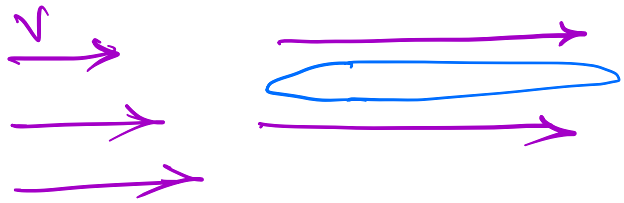
(blunt body)

Bluff body → building, trees

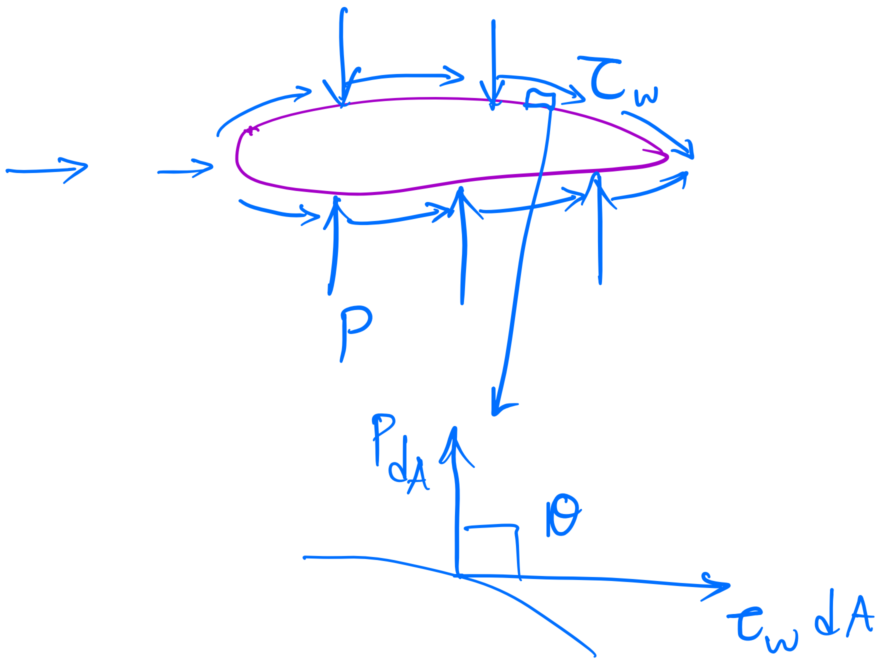
(block the flow of air)

Stream line body → aircrafts.

(body is having a shape align with stream lines of the flow)



Drag and Lift



$$dF_D = -PdA\cos\theta + \tau_w dA\sin\theta$$

$$dF_L = -PdA\sin\theta - \tau_w dA\cos\theta$$

Drag force on body $F_D = \int_A dF_D = \int_A -PdA\cos\theta + \tau_w dA\sin\theta$

Lift force on body $F_L = \int_A dF_L = -\int_A PdA\sin\theta + \tau_w dA\cos\theta$

F_D and F_L depends on density ρ of the fluid, upstream velocity V, size, shape and orientation of the body.

Drag co-efficient $\quad C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$

Lift co-efficient $\quad C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A}$

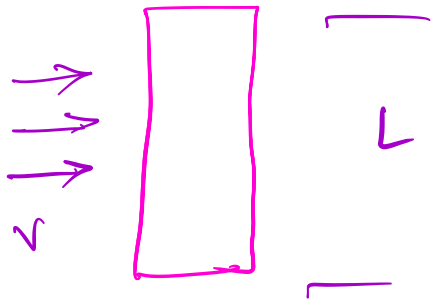
where A is the frontal area (plane area normal to the direction of flow).

$\frac{1}{2}\rho V^2 \longrightarrow$ dynamic pressure.

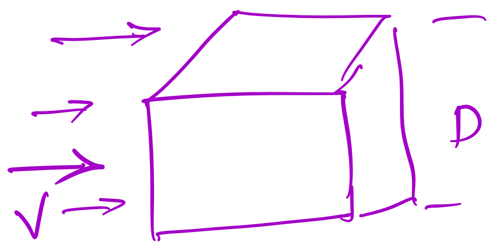
$A \rightarrow$ frontal area of different solids

A

Cylinder

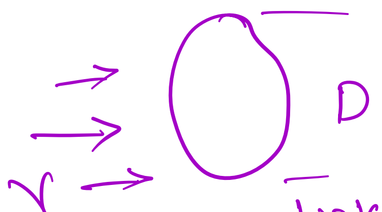


$A = LD$



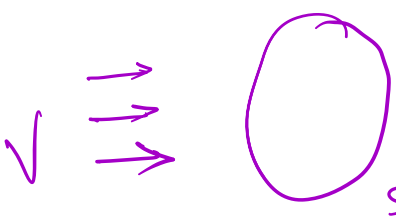
Cube

$A = D \times D$



disk

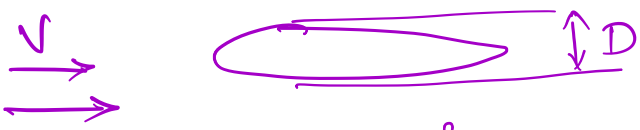
$A = \frac{\pi d^2}{4}$



sphere

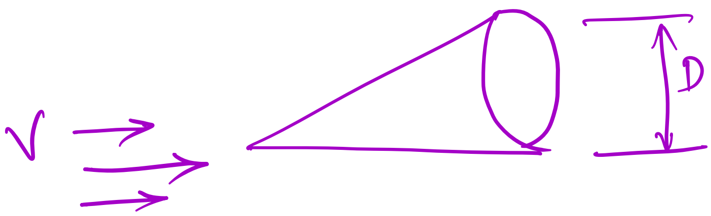
$A = \frac{\pi d^2}{4}$

Stream lined body



$A = \pi d^2/4$

Cone



$A = \pi D$

Friction and Pressure drag

$$\text{Drag force} = \text{Pressure force} + \text{Shear force}$$

$$F_D = P\,dA + \tau_w\,dA$$

The part of drag that is due directly to wall shear stress τ_w is called the skin friction drag or friction drag

The part of drag that is due to pressure p is called the pressure drag.

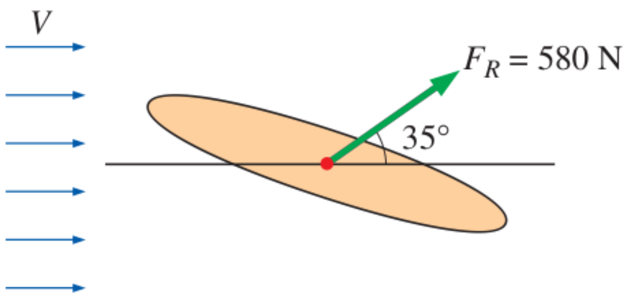
$$C_{D,\text{friction}} = \frac{F_{D,\text{friction}}}{\frac{1}{2}\rho V^2 A}$$

$$C_{D,\text{pressure}} = \frac{F_{D,\text{pressure}}}{\frac{1}{2}\rho V^2 A}$$

$$C_D = C_{D,\text{friction}} + C_{D,\text{pressure}}$$

$$F_D = F_{D,\text{friction}} + F_{D,\text{pressure}}$$

11–22 The resultant of the pressure and wall shear forces acting on a body is measured to be 580 N, making 35° with the direction of flow. Determine the drag and the lift forces acting on the body.



Drag force $F_D = F_R \cos\theta$

$F_D = 580 \cdot \cos 35^\circ$

$\boxed{F_D = 475\ \text{N}}$

Lift force $F_L = F_R \sin\theta$

$F_L = 580 \cdot \sin 35^\circ$

$\boxed{F_L = 333\ \text{N}}$

11–21 The drag coefficient of a car at the design conditions of 1 atm, 25°C, and 90 km/h is to be determined experimentally in a large wind tunnel in a full-scale test. The height and width of the car are 1.25 m and 1.65 m, respectively. If the horizontal force acting on the car is measured to be 220 N, determine the total drag coefficient of this car. *Answer:* 0.29

Assume density of air = 1.164 kg/m³.

Drag force $F_D = C_D A \frac{\rho V^2}{2}$

$$C_D = \frac{2 F_D}{\rho A V^2}$$

$V = 90 \text{ km/hr}$

$V = \frac{90 \times 1000}{3600} \text{ m/s}$

$V = 25 \text{ m/s}$

$$C_D = \frac{2 \times 220}{1.164 \times (1.25 \times 1.65) \times 25^2} = 0.29$$

$$\boxed{C_D = 0.29}$$

11–24 A car is moving at a constant velocity of 110 km/h. Determine the upstream velocity to be used in fluid flow analysis if (*a*) the air is calm, (*b*) wind is blowing against the direction of motion of the car at 30 km/h, and (*c*) wind is blowing in the same direction of motion of the car at 30 km/h.

a. The air is calm

$V = V_{car} = 110 \text{ km/h}$

b. wind is blowing against the direction of the car at 30 km/hr

$V = V_{car} + V_{wind} = 110 + 30$

$V = 140 \text{ km/h}$

c. wind is blowing in the same direction of the car at 30 km/h

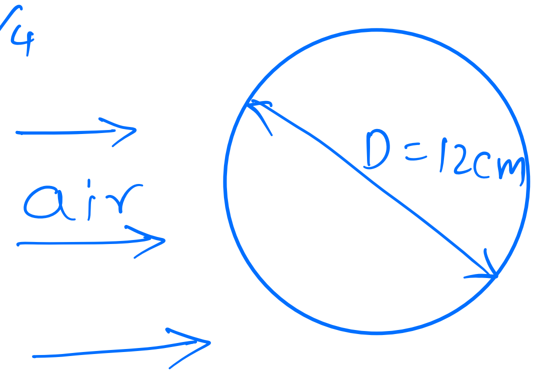
$V = V_{car} - V_{wind} = 110 - 30$

$V = 80 \text{ km/h}$

11–23 During a high Reynolds number experiment, the total drag force acting on a spherical body of diameter $D = 12$ cm subjected to airflow at 1 atm and 5°C is measured to be 5.2 N. The pressure drag acting on the body is calculated by integrating the pressure distribution (measured by the use of pressure sensors throughout the surface) to be 4.9 N. Determine the friction drag coefficient of the sphere. *Answer:* 0.0115

Take $\rho_{air} = 1.269\ kg/m^3$ $\nu = 1.382 \times 10^{-5}\ m^2/s$

$C_D = 0.2$, $A = \pi d^2/4$



$$F_D = F_{D\,friction} + F_{D\,pressure}$$

$$F_{D\,friction} = F_D - F_{D.\,pressure} = 5.2 - 4.9$$

$$F_{D.friction} = 0.3\ N$$

$$F_D = C_D A \frac{\rho V^2}{2} \quad \text{and} \quad F_{D.friction} = C_{D.friction} \frac{A V^2}{2}$$

$$\frac{F_{D.friction}}{F_D} = \frac{C_{D.friction}\,\cancel{A}\,\cancel{\frac{V^2}{2}}}{C_D\,\cancel{A}\,\rho\,\cancel{\frac{V^2}{2}}}$$

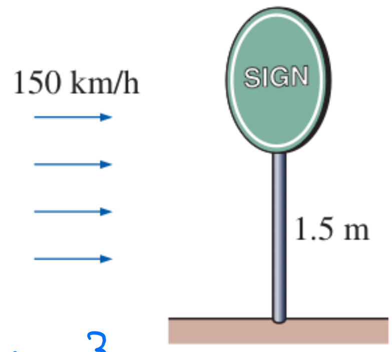
$$C_{D.friction} = \frac{C_D \cdot F_{D.friction}}{F_D}$$

$$= \frac{0.2 \cdot 0.3}{5.2}$$

$$C_{D.friction} = 0.0115$$

11–27 A circular sign has a diameter of 50 cm and is subjected to normal winds up to 150 km/h at 10°C and 100 kPa. Determine the drag force acting on the sign. Also determine the bending moment at the bottom of its pole whose height from the ground to the bottom of the sign is 1.5 m. Disregard the drag on the pole. Take $\rho_{air} = 1.231\ kg/m^3$

$C_D = 1.1$



$$V = 150\ km/hr = \frac{150\times10^3}{3600} = 41.67\ m^2/s$$

$$\text{Frontal area } A = \frac{\pi d^2}{4} = \frac{\pi\ 0.5^2}{4} = 0.20\ m^2$$

$$F_D = C_D A \frac{\rho V^2}{2} = \frac{1.1 \times 0.20 \times 1.231 \times (41.67)^2}{2}$$

$$\boxed{F_D = 235\ N}$$

Bending moment at the bottom of the pole is

$$M_{bottom} = F_D \times L = 235\,(0.25 + 1.5)$$

$$\boxed{M_{bottom} = 411\ Nm}$$

11–29 Advertisement signs are commonly carried by taxicabs for additional income, but they also increase the fuel cost. Consider a sign that consists of a 0.30-m-high, 0.9-m-wide, and 0.9-m-long rectangular block mounted on top of a taxicab such that the sign has a frontal area of 0.3 m by 0.9 m from all four sides. Determine the increase in the annual fuel cost of this taxicab due to this sign. Assume the taxicab is driven 60,000 km a year at an average speed of 50 km/h and the overall efficiency of the engine is 28 percent. Take the density, unit price, and heating value of gasoline to be 0.72 kg/L, \$1.10/L, and 42,000 kJ/kg, respectively, and the density of air to be 1.25 kg/m³. Take $C_D = 2.2$

$$F_D = C_D A \frac{\rho V^2}{2}$$

$$= 2.2 \times (0.3 \times 0.9) \frac{1.25 \times}{2} \left(\frac{50 \times 1000}{3600}\right)$$

$$\boxed{F_D = 71.61 \text{N}}$$

Work done to overcome this drag force is (F_D x distance)

$$W_{drag} = F_D \times L$$

$$= 71.61 \times 60{,}000 \text{ km/year}$$

$$= 4.3 \times 10^6 \text{ kJ/year}$$

Energy input required is

$$E_{in} = \frac{W_{drag}}{\eta_{car}} = \frac{4.30 \times 10^6 \text{ kJ/year}}{0.28}$$

$$E_{in} = 1.54 \times 10^7 \text{ kJ/year}$$

The amount and cost of fuel is

Amount of fuel (Volume of fuel) $= \dfrac{m_{fuel}}{\rho_{fuel}} = \dfrac{E_{in}/HV}{\rho_{fuel}}$

$$\text{Volume of fuel} = \frac{(1.54 \times 10^7 \text{ kJ/year}) \cdot 42000 \text{ kJ/kg}}{0.72 \text{ kg/L}}$$

$$\boxed{\text{Volume of fuel} = 509 \text{ L/year}}$$

$$\text{Cost} = \text{Volume of fuel} \times \text{Unitcost}$$

$$= 509 \text{ L/year} \times 1.10/\text{L}$$

$$\boxed{\text{Cost} = \$560 \text{ /year}}$$

Increase in annual cost due to this sign is \$560/year.

Dimensional Analysis

Mathematical tool for solving complex experimental problems.

Methods

1. Rayleigh's
2. Buckingham π method
3. Matrix-tensor method
4. Bridgham's method

Dimensions

Measurable

1. Fundamental dimension [Primary]
2. Derived dimensions

Fundamental dimensions

- Mass (M) / Force (F)
- Length (L)
- Time (T)
- Temperature (θ)

Derived dimensions

Area $L \times L = L^2$

$V = \frac{L}{T} = LT^{-1}$

$\rho = ML^{-3}$

Diemsional homogenity

① Area = Length x bredth

$L^2 = L \times L$

$\boxed{L^2 = L^2}$ ✓

② Density

$\rho = \frac{m}{Volume}$

$\boxed{ML^{-3} = \frac{M}{L^3} = ML^{-3}}$ ✓

③ Pressure = Force $\cancel{\times}$ $\cancel{Volume}$ Area

= 1

$ML^{-1}T^{-2} = MLT^{-2} \cancel{\times} L^2$

$ML^{-1}T^{-2} = ML^{-1}T^{-2}$

Quantity	Dimensions
Mass	M
Length	L
Time	T
Area	L^2
Volume	L^3
Velocity	LT^{-1}
Angular velocity	T^{-1}
Acceleration	LT^{-2}
Angular acceleration	T^{-2}
Discharge	L^3T^{-1}
Gravity	LT^{-2}
Dynamic viscosity	$ML^{-1}T^{-1}$
Kinematic viscosity	L^2T^{-1}
Force	MLT^{-2}
Density	ML^{-3}
Specific weight	$ML^{-2}T^{-2}$
Pressure	$ML^{-1}T^{-2}$
Work	ML^2T^{-2}
Power	ML^2T^{-3}

Rayleigh's method

Q depends on pressure drop Δp, fluid density ρ, pipe diameter D, and orifice diameter d.

Mathematically $Q = f(\Delta p, \rho, D, d)$

$$Q = C[\Delta p^a, \rho^b, D^c, d^d]$$

where C is dimensionless term and a, b, c, and d are arbitary power to be evaluated.

Dimensions

Q	Δp	ρ	D	d
L^3T^{-1}	$ML^{-1}T^{-2}$	ML^{-3}	L	L

No. of fundamental dimensions 'm' = 3

$$Q = C[\Delta p^a, \rho^b, D^c, d^d] \quad \text{------} \quad (A)$$

$$L^3T^{-1} = C[(ML^{-1}T^{-2})^a, (ML^{-3})^b, L^c, L^d]$$

$$M \rightarrow 0 = a + b \quad \text{------} \quad (1)$$

$$L \rightarrow 3 = -a - 3b + c + d \quad \text{------} \quad (2)$$

$$T \rightarrow -1 = -2a \quad \text{------} \quad (3)$$

Orifice diameter is said to be an important parameter. So power of orifice diameter (d) is taken as referance.

From eqn (3),

$$a = \frac{1}{2}$$

From eqn (1)

$$b = -a = -\frac{1}{2}$$

$$b = -\frac{1}{2}$$

From eqn (2)

$$c = +3 + a + 3b - d$$

$$c = 3 + \frac{1}{2} - 3\frac{1}{2} - d$$

$$c = 2 - d$$

Using eqn (A)

$$Q = C\left[\Delta p^{a},\ \rho^{b},\ D^{c},\ d^{d}\right]$$

$$Q = C\left[\Delta p^{\frac{1}{2}},\ \rho^{-\frac{1}{2}}\ D^{2-d},\ d^{d}\right]$$

$$Q = C\left[\Delta p^{\frac{1}{2}},\ \rho^{-\frac{1}{2}}\ D^{2},\ D^{-d},\ d^{d}\right]$$

$$\boxed{Q = D^{2}\sqrt{\frac{\Delta p}{\rho}}\ \phi\left[\frac{d}{D}\right]}$$

Rayleigh's method

Find an expression for drag force on smooth sphere of diameter D, moving with a uniform velocity V in a fluid density ρ and dynamic viscosity μ.

'F' → drag force ✓ → dependent variable → 1

D → diameter
V → velocity
ρ → density
μ → viscosity

Independent variable → 4

Mathematically

$$F = f(D, V, \rho, \mu)$$

$$F = C\left[D^a, V^b, \rho^c, \mu^d\right]$$

where C is a non-dimensional number
a, b, c & d are powers of D, V, ρ & μ respectively to be determined.

Dimensions

$F = MLT^{-2}$

$D = L$

$V = LT^{-1}$

$\rho = ML^{-3}$

$\mu = ML^{-1}T^{-1}$

No. of fundamental dimensions
'm' = 3

$$MLT^{-2} = \left[L^a, (LT^{-1})^b, (ML^{-3})^c (ML^{-1}T^{-1})^d\right]$$

$M \rightarrow 1 = c + d$

$L \rightarrow 1 = a + b - 3c - d$

$T \rightarrow -2 = -b - d$

Assuming μ to play a vital role among independent variables. So power of μ is 'd', express a, b, c values in terms d.

Using $1 = c + d$

$$\boxed{\begin{aligned} c &= -d + 1 \\ b &= 2 - d \end{aligned}}$$

$$1 = a + b - 3c - d$$

$$a = 1 - b + 3c + d$$

$$a = 1 - (2 - d) + 3(1 - d) + d$$

$$\boxed{a = 2 - d}$$

$$F = C\left[D^{a},\ V^{b},\ \rho^{c},\ \mu^{d}\right]$$

$$F = C\left[D^{2-d},\ V^{2-d},\ \rho^{1-d},\ \mu^{d}\right]$$

$$F = C\left[D^{2} D^{-d},\ V^{2},\ V^{-d},\ \rho,\ \rho^{-d},\ \mu^{d}\right]$$

$$F = C\left[D^{2}, V^{2}, \rho,\ \ D^{-d} V^{-d} \rho^{-d} \mu^{d}\right]$$

$$\boxed{F = D^{2} V^{2} \rho\, \phi\left(\frac{\mu}{D V \rho}\right)}$$

Buckingham π method

Q depends on $\Delta P, \rho, D, d$, Mathematically

$Q = f(\Delta P, \rho, D, d)$

$f(Q, \Delta P, \rho, D, d) = 0$ ---------- (1)

No. of parameters (dependent & independent) 'n' = 5

Dimensions

independent variables

Q	ΔP	ρ	D	d
L^3T^{-1}	$ML^{-1}T^{-2}$	ML^{-3}	L	L

No of fundamental dimensions 'm' = 3

Numbers of π terms $= n - m$
$= 5 - 3$
$= 2$

Eqn (1) can be written as

$f(\pi_1, \pi_2) = 0$

To find π_1 & π_2

Each π term should contain $m+1$ term where m represents number of repeating variables.

ΔP, ρ, D are repeating variables

Q and d are non repeating variables

$$\pi_1 = \Delta P^{a_1}, \rho^{b_1}, D^{c_1}, Q$$

$$\pi_2 = \Delta P^{a_2}, \rho^{b_2}, D^{c_2}, d$$

$$\pi_1 = \Delta P^{a_1}, \rho^{b_1}, D^{c_1}, Q$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-2})^{a_1}, (ML^{-3})^{b_1}, L^{c_1}, L^3 T^{-1}$$

$$M \rightarrow 0 = a_1 + b_1$$

$$L \rightarrow 0 = -a_1 - 3b_1 + c_1 + 3$$

$$T \rightarrow 0 = -2a_1 - 1$$

$$a_1 = -\frac{1}{2}, \quad b_1 = +\frac{1}{2}, \quad c_1 = -2$$

$$\pi_1 = \Delta P^{a_1}, \rho^{b_1}, D^{c_1}, Q$$

$$\pi_1 = \Delta P^{-\frac{1}{2}}, \rho^{\frac{1}{2}}, D^{-2} Q$$

$$\boxed{\pi_1 = \frac{Q}{D^2}\sqrt{\frac{\rho}{\Delta p}}}$$

$$\pi_2 = \Delta p^{a_2},\ \rho^{b_2},\ D^{c_2},\ d$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-2})^{a_2},\ (ML^{-3})^{b_2},\ L^{c_2},\ L$$

$$M \rightarrow 0 = a_2 + b_2$$

$$L \rightarrow 0 = -a_2 - 3b_2 + c_2 + 1$$

$$T \rightarrow 0 = -2a_2$$

$$a_2 = 0,\ b_2 = 0,\ c_2 = -1$$

$$\pi_2 = \Delta p^{a_2},\ \rho^{b_2},\ D^{c_2},\ d$$

$$\pi_2 = \Delta p^{0},\ \rho^{0},\ D^{-1},\ d$$

$$\boxed{\pi_2 = \frac{d}{D}}$$

$$f(\pi_1, \pi_2) = 0$$

$$f\left(\frac{Q}{D^2}\sqrt{\frac{\rho}{\Delta p}}, \frac{d}{D}\right) = 0$$

$$\frac{Q}{D^2}\sqrt{\frac{\rho}{\Delta p}} = f\left(\frac{d}{D}\right)$$

$$\underline{\underline{Q = \sqrt{\frac{\Delta p}{\rho}}\; D^2\, \phi\left(\frac{d}{D}\right)}}$$

Buckingham's π method

Show that the power P developed in a water turbine can be expressed as $P = \rho N^3 D^5 \phi\left(\frac{D}{B}, \frac{\rho D^2 N}{\mu}, \frac{ND}{\sqrt{gH}}\right)$

$$P = f(\rho, N, D, B, \mu, g, H)$$

$$f_1(P, \rho, N, D, B, \mu, g, H) = 0$$

$P = ML^2T^{-3}$

$\rho = ML^{-3}$

$N = T^{-1}$

$D = L$

$B = L$

$\mu = ML^{-1}T^{-1}$

$g = LT^{-2}$

$H = L$

$n \rightarrow$ no. of variables (both independent and dependent)

$m \rightarrow$ no. of fundamental dimensions

$n = 8$

$m = 3$

No. of π terms $= n - m$

$= 8 - 3$

$= 5$ terms

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0$$

Each term should have $m+1$ terms.

m is a repeating variable

D, N and ρ as repeating variables

$\pi_1 = D^{a_1}, N^{b_1}, \rho^{c_1}, P$

$\pi_2 = D^{a_2}, N^{b_2}, \rho^{c_2}, B$

$\pi_3 = D^{a_3}, N^{b_3}, \rho^{c_3}, \mu$

$\pi_4 = D^{a_4}, N^{b_4}, \rho^{c_4}, H$

$\pi_5 = D^{a_5}, N^{b_5}, \rho^{c_5}, g$

$\pi_1 = D^{a_1}, N^{b_1}, \rho^{c_1}, P$

$M^0 L^0 T^0 = L^{a_1}, (T^{-1})^{b_1}, (ML^{-3})^{c_1}, ML^2 T^{-3}$

$M \rightarrow 0 = c_1 + 1$

$L \rightarrow 0 = a_1 - 3c_1 + 2$

$T \rightarrow 0 = -b_1 - 3$

$c_1 = -1, \quad a_1 = -5, \quad b_1 = -3$

$\pi_1 = D^{a_1}, N^{b_1}, \rho^{c_1}, P$

$\pi_1 = D^{-5}, N^{-3}, \rho^{-1}, P$

$$\boxed{\pi_1 = \frac{P}{D^5 N^3 \rho}}$$

$\pi_2 = D^{a_2}, N^{b_2}, \rho^{c_2}, B$

$M^0 L^0 T^0 = L^{a_2}, (T^{-1})^{b_2} (ML^{-3})^{c_2} \quad L$

$M \rightarrow 0 = c_2$

$L \rightarrow 0 = a_2 - 3c_2 + 1$

$T \rightarrow 0 = -b_2$

$\pi_2 = D^{a_2}, N^{b_2}, \rho^{c_2}, B$

$M^0L^0T^0 = L^{a_2}, (T^{-1})^{b_2}, (ML^{-3})^{c_2}, L$

$M \rightarrow 0 = c_2$

$L \rightarrow 0 = a_2 - 3c_2 + 1$

$T \rightarrow 0 = -b_2$

$c_2 = 0$

$b_2 = 0$

$a_2 = 3c_2 - 1 = 3(0) - 1$

$a_2 = -1$

So $\pi_2 = D^{-1}, N^0, \rho^0, B$

$$\boxed{\pi_2 = \frac{B}{D}}$$

$\pi_3 = D^{a_3}, N^{b_3}, \rho^{c_3}, \mu$

$M^0L^0T^0 = L^{a_3}, (T^{-1})^{b_3}, (ML^{-3})^{c_3}, (ML^{-1}T^{-1})$

Solving

$c_3 = -1, \quad b_3 = -1, \quad a_3 = -2$

$\pi_3 = D^{-2}, N^{-1}, \rho^{-1}, \mu$

$$\boxed{\pi_3 = \frac{\mu}{D^2 N \rho}}$$

$\pi_4 = D^{a_4}, N^{b_4}, \rho^{c_4}, H$

$M^0 L^0 T^0 = L^{a_4}, (T^{-1})^{b_4}, (ML^{-3})^{c_4}, L$

$M \to 0 = c_4$

$L \to 0 = a_4 - 3c_4 + 1$

$T \to 0 = -b_4$

So $a_4 = 3c_4 - 1 = 0 - 1 = -1$

$a_4 = -1$

$c_4 = 0$

$b_4 = 0$

So $\pi_4 = D^{a_4}, N^{b_4}, \rho^{c_4}, H$

$\pi_4 = D^{-1}, N^0, \rho^0, H$

$$\boxed{\pi_4 = \frac{H}{D}}$$

$$\pi_5 = D^{a_5}, N^{b_5}, \rho^{c_5}, g$$

$$M^0L^0T^0 = L^{a_5}, (T^{-1})^{b_5}, (ML^{-3})^{c_5}, LT^{-2}$$

$$M \rightarrow 0 = c_5$$
$$L \rightarrow 0 = a_5 - 3c_5 + 1$$
$$T \rightarrow 0 = -b_5 - 2$$

So, $c_5 = 0$, $b_5 = -2$, $a_5 = -1$

$$\pi_5 = D^{-1}, N^{-2}, \rho^0, g$$

$$\boxed{\pi_5 = \frac{g}{DN^2}}$$

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0$$

$$f_1\left(\frac{P}{\rho N^3 D^5}, \frac{B}{D}, \frac{\mu}{\rho D^2 N}, \frac{H}{D}, \frac{g}{DN^2}\right) = 0$$

$$f_1\left(\frac{P}{\rho N^3 D^5}, \frac{B}{D}, \frac{\mu}{\rho D^2 N}, \frac{gH}{D^2N^2}\right) = 0$$

$$f_1\left(\frac{P}{\rho N^3 D^5}, \frac{B}{D}, \frac{\mu}{\rho D^2 N}, \frac{\sqrt{gH}}{DN}\right) = 0$$

$$f_1\left(\frac{P}{\rho N^3 D^5}, \frac{D}{B}, \frac{\rho D^2 N}{\mu}, \frac{DN}{\sqrt{gH}}\right) = 0$$

$$\frac{P}{\rho N^3 D^5} = \phi\left(\frac{D}{B}, \frac{\rho D^2 N}{\mu}, \frac{DN}{\sqrt{gH}}\right)$$

$$\boxed{P = \rho N^3 D^5 \phi\left(\frac{D}{B}, \frac{\rho D^2 N}{\mu}, \frac{DN}{\sqrt{gH}}\right)}$$

7.7 It is desired to determine the wave height when wind blows across a lake. The wave height, H, is assumed to be a function of the wind speed, V, the water density, ρ, the air density, ρ_a, the water depth, d, the distance from the shore, ℓ, and the acceleration of gravity, g, as shown in Fig. P7.7. Use d, V, and ρ as repeating variables to determine a suitable set of pi terms that could be used to describe this problem.

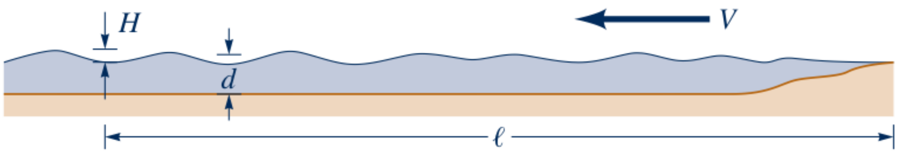


Figure P7.7

7.11 At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity, V, in the larger pipe, and the fluid density, ρ, and viscosity, μ. Use D_1, V, and μ as repeating variables to determine a suitable set of dimensionless parameters. ~~Why would it be incorrect to include the velocity in the smaller pipe as an additional variable~~?

ΔP is a function of

Diameter 1 D_1

Diameter 2 D_2

Velocity V

Density ρ

Viscosity μ

$$\Delta P = f(D_1, D_2, V, \rho, \mu)$$

$$f(\Delta P, D_1, D_2, V, \rho, \mu) = 0$$

Dimensions

ΔP	D_1	D_2	V	ρ	μ
$ML^{-1}T^{-2}$	L	L	LT^{-1}	ML^{-3}	$ML^{-1}T^{-1}$

So $n = 6$, $m = 3$

No. of π terms is $n - m = 6 - 3 = 3$

So, $f(\pi_1, \pi_2, \pi_3) = 0$

Here D_1, V, μ are repeating variables

So, $\pi_1 = D_1^{a_1}, V^{b_1}, \mu^{c_1}, \Delta P$

$\pi_2 = D_1^{a_2}, V^{b_2}, \mu^{c_2}, D_2$

$\pi_3 = D_1^{a_3}, V^{b_3}, \mu^{c_3}\ \rho$

$\pi_1 = D_1^{a_1}, V^{b_1}, \mu^{c_1}, \Delta P$

$M^0 L^0 T^0 = L^{a_1}, (L T^{-1})^{b_1}, (M L^{-1} T^{-1})^{c_1}, M L^{-1} T^{-2}$

$M \rightarrow 0 = c_1 + 1$

$L \rightarrow 0 = a_1 + b_1 - c_1 - 1$

$T \rightarrow 0 = -b_1 - c_1 - 2$

Here $c_1 = -1$

$b_1 = -c_1 - 2 = -(-1) - 2 = -1$

$b_1 = -1$

$a_1 = -b_1 + c_1 + 1$

$a_1 = -(-1) + (-1) + 1$

$a_1 = +1$

$$\pi_1 = D_1^1,\ V^{-1},\ \mu^{-1},\ \Delta P$$

$$\boxed{\pi_1 = \frac{\Delta P \cdot D_1}{V\mu}}$$

$$\pi_2 = D_1^{a_2},\ V^{b_2},\ \mu^{c_2},\ D_2$$

$$M^0 L^0 T^0 = L^{a_2},\ (LT^{-1})^{b_2},\ (ML^{-1}T^{-1})^{c_2},\ L$$

$M \rightarrow 0 = c_2$

$L \rightarrow 0 = a_2 + b_2 - c_2 + 1$

$T \rightarrow 0 = -b_2 - c_2$

So $c_2 = 0$

$b_2 = -c_2 = 0$

$b_2 = 0$

$a_2 = -b_2 + c_2 - 1$

$= 0 + 0 - 1$

$a_2 = -1$

So $\pi_2 = D_1^{a_2},\ V^{b_2},\ \mu^{c_2},\ D_2$

$\pi_2 = D_1^{-1},\ V^0,\ \mu^0,\ D_2$

$$\boxed{\pi_2 = \frac{D_2}{D_1}}$$

$$\pi_3 = D_1^{a_3}, V^{b_3}, \mu^{c_3}, \rho$$

$$M^0 L^0 T^0 = L^{a_3}, (LT^{-1})^{b_3}, (ML^{-1}T^{-1})^{c_3}, ML^{-3}$$

$$M \to 0 = c_3 + 1$$

$$L \to 0 = a_3 + b_3 - c_3 - 3$$

$$T \to 0 = -b_3 - c_3$$

So $c_3 = -1$

$$b_3 = -c_3 = -(-1) = 1$$

$$b_3 = 1$$

$$a_3 = -b_3 + c_3 + 3$$

$$= -(1) + (-1) + 3$$

$$= -1 - 1 + 3$$

$$= +1$$

$$a_3 = 1$$

$$\pi_3 = D_1^{a_3}, V^{b_3}, \mu^{c_3}, \rho$$

$$\pi_3 = D_1^1, V^1, \mu^{-1}, \rho$$

$$\boxed{\pi_3 = \frac{D_1 V \rho}{\mu}}$$

But $f(\pi_1, \pi_2, \pi_3) = 0$

$$f\left(\frac{\Delta P, D_1}{V \mu}, \frac{D_2}{D_1}, \frac{D_1 V \rho}{\mu}\right) = 0$$

$$\frac{\Delta P D_1}{V \mu} = f\left(\frac{D_2}{D_1}, \frac{D_1 V \rho}{\mu}\right)$$

$$\boxed{\Delta P = \frac{V \mu}{D_1} \phi\left(\frac{D_2}{D_1}, \frac{D_1 V \rho}{\mu}\right)}$$

7.13 The drag, $\mathfrak{D}$, on a washer-shaped plate placed normal to a stream of fluid can be expressed as

$$\mathfrak{D} = f(d_1, d_2, V, \mu, \rho)$$

where d_1 is the outer diameter, d_2 the inner diameter, V the fluid velocity, μ the fluid viscosity, and ρ the fluid density. Some experiments are to be performed in a wind tunnel to determine the drag. What dimensionless parameters would you use to organize these data?

7.26 A viscous fluid is poured onto a horizontal plate as shown in Fig. P7.26. Assume that the time, t, required for the fluid to flow a certain distance, d, along the plate is a function of the volume of fluid poured, $\forall$, acceleration of gravity, g, fluid density, ρ, and fluid viscosity, μ. Determine an appropriate set of pi terms to describe this process. Form the pi terms by inspection.

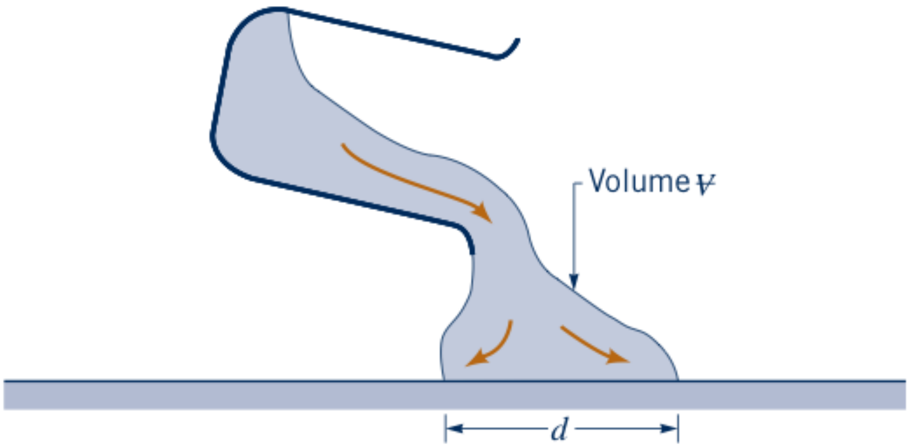


Figure P7.26

Distance d depends on

$\forall \rightarrow$ volume

$g \rightarrow$ accln due to gravity

$\rho \rightarrow$ density

$\mu \rightarrow$ viscosity

$d = f(\forall, g, \rho, \mu)$

$f(d, \forall, g, \rho, \mu) = 0$

Dimensions

d	$\forall$	g	ρ	μ
L	L^3	LT^{-2}	ML^{-3}	$ML^{-1}T^{-1}$

$n = 5, \quad m = 3$

No. of π terms $= n - m$

$= 5 - 3 = 2$

So $f(\pi_1, \pi_2) = 0$

Assuming g, ρ, μ are repeating variables.

$\pi_1 = g^{a_1}, \rho^{b_1}, \mu^{c_1}, d$

$\pi_2 = g^{a_2}, \rho^{b_2}, \mu^{c_2}, \forall$

So $\pi_1 = g^{a_1}, \rho^{b_1}, \mu^{c_1}, d$

$$M^0L^0T^0 = (LT^{-2})^{a_1}, (ML^{-3})^{b_1}, (ML^{-1}T^{-1})^{c_1}, L$$

$M \to 0 = b_1 + c_1$

$L \to 0 = a_1 - 3b_1 - c_1 + 1$

$T \to 0 = -2a_1 - c_1$

So $c_1 = -b_1$

$2a_1 = -c_1$

$= -(-b_1)$

$a_1 = \frac{b_1}{2}$

So $a_1 - 3b_1 - c_1 + 1 = 0$

$+\frac{b_1}{2} - 3b_1 - (-b) + 1 = 0$

$\frac{b_1}{2} - 2b_1 + 1 = 0$

$\frac{b_1 - 4b_1}{2} = -1$

$$-3b_1 = -2$$

$$\boxed{b_1 = \frac{2}{3}}$$

$$a_1 = \frac{b_1}{2} = \frac{\frac{2}{3}}{2} = \frac{1}{3}$$

$$\boxed{a_1 = \frac{1}{3}}$$

$$c_1 = -b_1$$

$$\boxed{c_1 = -\frac{2}{3}}$$

So $\pi_1 = g^{a_1}, \rho^{b_1}, \mu^{c_1}, d$

$$\pi_1 = g^{1/3}, \rho^{2/3}, \mu^{-2/3}, d$$

$$\boxed{\pi_1 = \frac{d\, g^{1/3}\, \rho^{2/3}}{\mu^{2/3}}}$$

Now $\pi_2 = g^{a_2}, \rho^{b_2}, \mu^{c_2}, \forall$

$$M^0 L^0 T^0 = (L T^{-2})^{a_2} \; (M L^{-3})^{b_2}, (M L^{-1} T^{-1})^{c_2}, L^3$$

$M \rightarrow 0 = b_2 + c_2$

$L \rightarrow 0 = a_2 - 3b_2 - c_2 + 3$

$T \rightarrow 0 = -2a_2 - c_2$

Here $\boxed{c_2 = -b_2}$

$0 = -2a_2 - c_2$

$= -2a_2 - (-b_2)$

$2a_2 = b_2$

$\boxed{a_2 = \frac{b_2}{2}}$

$0 = a_2 - 3b_2 - c_2 + 3$

$= \frac{b_2}{2} - 3b_2 - (-b_2) + 3$

$= \frac{b_2}{2} - 3b_2 + b_2 + 3$

$\Rightarrow \frac{b_2}{2} - 2b_2 = -3$

$\Rightarrow \frac{b_2 - 4b_2}{2} = -3$

$\Rightarrow -\frac{3b_2}{2} = -3$

$$3b_2 = 6$$

$$b_2 = \frac{6}{3} = 2$$

$$\boxed{b_2 = 2}$$

$$a_2 = \frac{b_2}{2} \Rightarrow a_2 = \frac{2}{2} = 1$$

$$\boxed{a_2 = 1}$$

$$c_2 = -b_2$$

$$\boxed{c_2 = -2}$$

So $\pi_2 = g^{a_2}, \rho^{b_2}, \mu^{c_3}, \forall$

$$\pi_2 = g^1, \rho^2, \mu^{-2}, \forall$$

$$\boxed{\pi_2 = \frac{\forall g \rho^2}{\mu^2}}$$

So $f(\pi_1, \pi_2) = 0$

$$f\left(\frac{d\, g^{1/3}\, \rho^{2/3}}{\mu^{2/3}}, \frac{\forall g \rho^2}{\mu^2}\right) = 0$$

$$\frac{d\, g^{1/3}\, \rho^{2/3}}{\mu^{2/3}} = f\left(\frac{\forall g \rho^2}{\mu^2}\right)$$

$$d = \frac{\mu^{2/3}}{g^{1/3}\rho^{2/3}}\, \phi\left[\frac{\forall g \rho^2}{\mu^2}\right]$$

Dimensionless numbers

1. Reynold's number
2. Froude's number
3. Euler's number
4. Weber's number
5. Mach's number
6. Knudsen number

Forces acting on the fluid

1. Inertia Force (F_i) = mass x acceleration of the fluid
2. Viscous force (F_v) = τ x surface area
3. Gravity force (F_g) = mass x g
4. Pressure force (F_p) = $p \times a$
5. Surface tension force (F_s) = $\sigma \times L$
6. Elastic force (F_e) = $K \times a$

---

1. Reynold's number (R_N)

$$R_N = \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$R_N = \frac{m \times accl^n}{\tau \times a} = \frac{\frac{m}{t} \times \text{Velocity}}{\tau \times a}$$

$$R_N = \frac{\rho a v \times v}{\mu \frac{du}{dy} \times a} = \frac{\rho \cancel{a} v^{\cancel{2}}}{\mu \frac{\cancel{v}}{L} \times \cancel{a}} = \frac{\rho v L}{\mu}$$ or D characteristic dimensions

$$R_N = \frac{\rho v L}{\mu} \text{ or } \frac{\rho v D}{\mu} \text{ or } \frac{vD}{\frac{\mu}{\rho}} = \frac{vD}{\nu}$$

$$R_N = \frac{\rho v D}{\mu} = \frac{vD}{\nu}$$

ν is the kinematic viscosity

2. Froude's Number (F_N)

$$F_N = \sqrt{\frac{\text{Inertia force}}{\text{Gravity force}}}$$

$$F_N = \sqrt{\frac{\rho a v^2}{m \times g}} = \sqrt{\frac{\rho a v^2}{\rho a L g}} = \sqrt{\frac{v^2}{Lg}}$$

$$F_N = \frac{v}{\sqrt{Lg}}$$

3. Euler's Number (E_N)

$$E_N = \sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}}$$

$$E_N = \sqrt{\frac{\rho a v^2}{p \times a}} = \sqrt{\frac{\rho v^2}{p}} = \frac{v}{\sqrt{p/\rho}}$$

4. Weber's Number (W_N)

$$W_N = \sqrt{\frac{\text{Inertia force}}{\text{Surface tension force}}}$$

$$W_N = \sqrt{\frac{\rho a v^2}{\sigma \times L}} = \sqrt{\frac{\rho\, L \times L\, v^2}{\sigma \times L}} = \sqrt{\frac{\rho L v^2}{\sigma}}$$

$$W_N = \frac{v}{\sqrt{\frac{\sigma}{\rho L}}}$$

5. Mach number (M)

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}}$$

$$M = \sqrt{\frac{\rho a v^2}{K \times a}} = \sqrt{\frac{\rho v^2}{K}} = \frac{v}{\sqrt{K/\rho}}$$

$$M = \frac{V}{C} \quad \text{where } C = \sqrt{K/\rho} = \text{velocity of sound in the fluid}$$

6. Knudsen number (K_N)

$$K_N = \frac{\text{molecular free path length}}{\text{Representative physical length scale}}$$

$$K_N = \frac{\lambda}{L}$$

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