

## Chapter-2

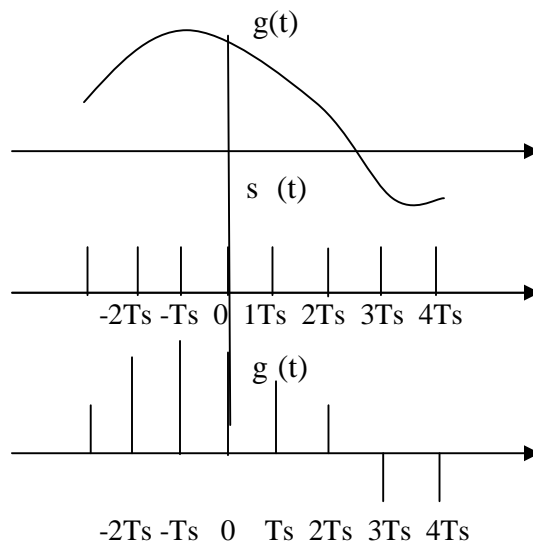
# **SAMPLING PROCESS**

**SAMPLING:** A message signal may originate from a digital or analog source. If the message signal is analog in nature, then it has to be converted into digital form before it can be transmitted by digital means. The process by which the continuous-time signal is converted into a discrete-time signal is called Sampling.

Sampling operation is performed in accordance with the sampling theorem.

### **SAMPLING THEOREM FOR LOW-PASS SIGNALS:-**

Statement:- “If a band –limited signal  $g(t)$  contains no frequency components for  $f > W$ , then it is completely described by instantaneous values  $g(kT_s)$  uniformly spaced in time with period  $T_s = 1/2W$ . If the sampling rate,  $f_s$  is equal to the Nyquist rate or greater ( $f_s \geq 2W$ ), the signal  $g(t)$  can be exactly reconstructed.



**Fig 2.1: Sampling process**

Proof:- Consider the signal  $g(t)$  is sampled by using a train of impulses  $s(t)$ .

Let  $g(t)$  denote the ideally sampled signal, can be represented as

$$g(t) = g(t) \cdot s(t) \quad \text{-----} \quad 2.1$$

where  $s(t)$  – impulse train defined by

$$s(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT_s) \quad \text{-----} \quad 2.2$$

Therefore 
$$g(t) = g(t) \cdot \sum_{k=-\infty}^{+\infty} \delta(t - kT_s)$$

$$= \sum_{k=-\infty}^{+\infty} g(kT_s) \delta(t - kT_s) \quad \text{-----} \quad 2.3$$

The Fourier transform of an impulse train is given by

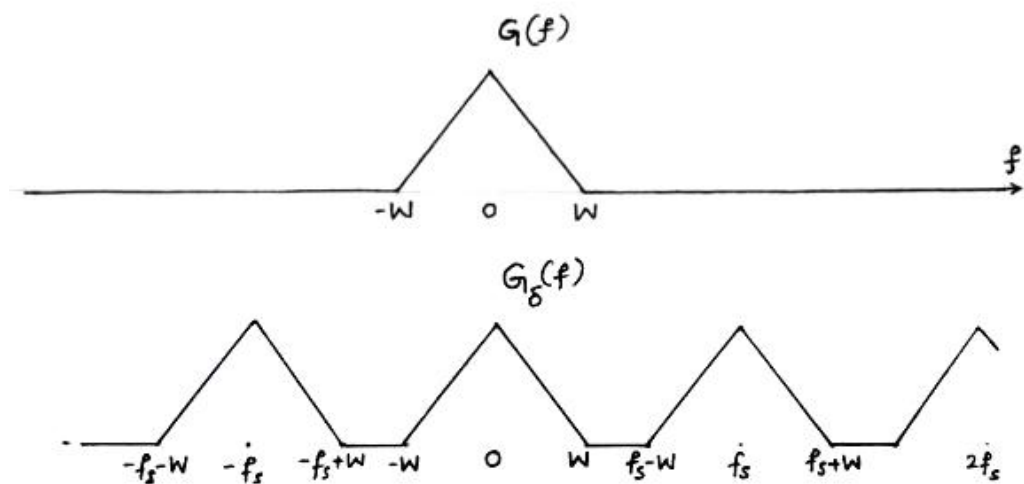
$$S(f) = F[s(t)] = f_s \sum_{n=-\infty}^{+\infty} \delta(f - nf_s) \quad \text{-----} \quad 2.4$$

Applying F.T to equation 2.1 and using convolution in frequency domain property,

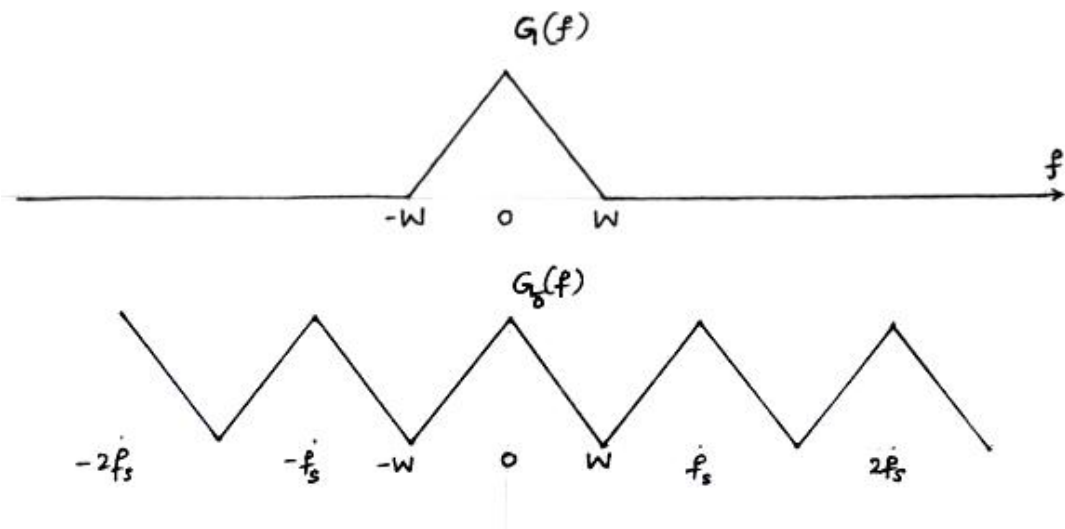
$$G(f) = G(f) * S(f)$$

Using equation 2.4, 
$$G(f) = G(f) * f_s \sum_{n=-\infty}^{+\infty} \delta(f - nf_s)$$

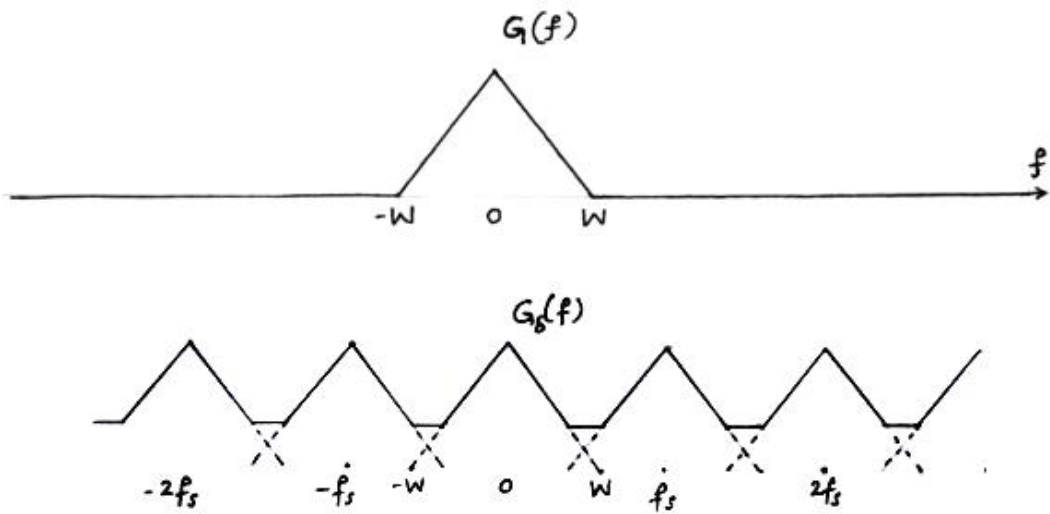
$$G(f) = f_s \sum_{n=-\infty}^{+\infty} G(f - nf_s) \quad \text{-----} \quad 2.5$$



**Fig. 2.2 Over Sampling ( $f_s > 2W$ )**



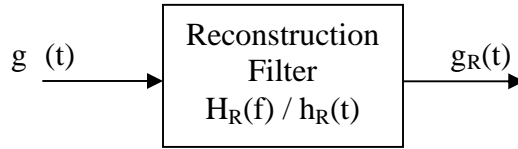
**Fig. 2.3 Nyquist Rate Sampling ( $f_s = 2W$ )**



**Fig. 2.4 Under Sampling ( $f_s < 2W$ )**

### Reconstruction of $g(t)$ from $g_s(t)$ :

By passing the ideally sampled signal  $g_s(t)$  through an low pass filter ( called Reconstruction filter ) having the transfer function  $H_R(f)$  with bandwidth,  $B$  satisfying the condition  $W \leq B \leq (f_s - W)$  , we can reconstruct the signal  $g(t)$ . For an ideal reconstruction filter the bandwidth  $B$  is equal to  $W$ .



The output of LPF is,  $g_R(t) = g_s(t) * h_R(t)$

where  $h_R(t)$  is the impulse response of the filter.

In frequency domain,  $G_R(f) = G_s(f) \cdot H_R(f)$ .

$$\text{For the ideal LPF } H_R(f) = \begin{cases} K & -W \leq f \leq +W \\ 0 & \text{otherwise} \end{cases}$$

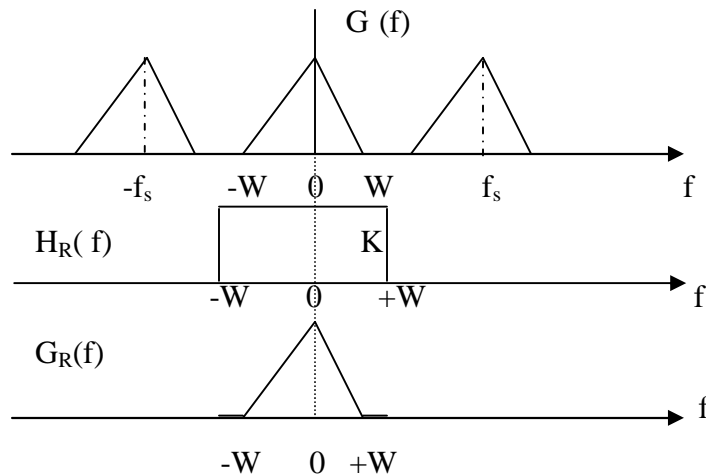
then impulse response is  $h_R(t) = 2WT_s \cdot \text{Sinc}(2Wt)$

Correspondingly the reconstructed signal is

$$g_R(t) = [2WT_s \text{Sinc}(2Wt)] * [g_s(t)]$$

$$g_R(t) = 2WT_s \sum_{K=-\infty}^{+\infty} g_s(kT_s) \cdot \text{Sinc}(2Wt) * u(t - kT_s)$$

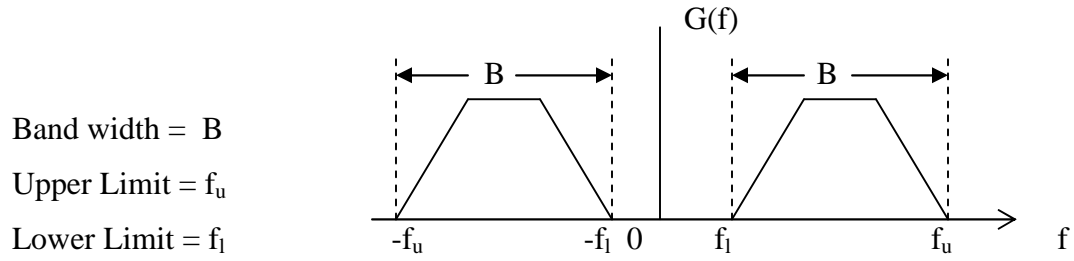
$$g_R(t) = 2WT_s \sum_{K=-\infty}^{+\infty} g_s(kT_s) \cdot \text{Sinc}[2W(t - kT_s)]$$



**Fig: 2.5 Spectrum of sampled signal and reconstructed signal**

### Sampling of Band Pass Signals:

Consider a band-pass signal  $g(t)$  with the spectrum shown in figure 2.6:



**Fig 2.6: Spectrum of a Band-pass Signal**

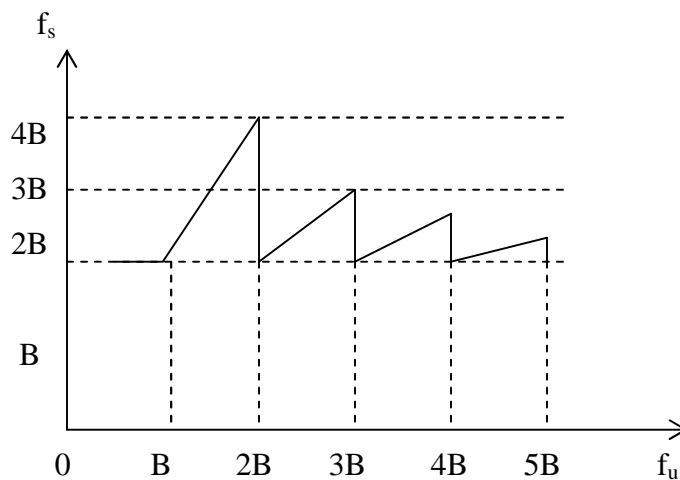
The signal  $g(t)$  can be represented by instantaneous values,  $g(kTs)$  if the sampling rate  $f_s$  is  $(2f_u/m)$  where  $m$  is an integer defined as

$$((f_u / B) - 1) < m \quad (f_u / B)$$

If the sample values are represented by impulses, then  $g(t)$  can be exactly reproduced from it's samples by an ideal Band-Pass filter with the response,  $H(f)$  defined as

$$H(f) = \begin{cases} 1 & f_l < |f| < f_u \\ 0 & \text{elsewhere} \end{cases}$$

If the sampling rate,  $f_s = 2f_u$ , exact reconstruction is possible in which case the signal  $g(t)$  may be considered as a low pass signal itself.



**Fig 2.7: Relation between Sampling rate, Upper cutoff frequency and Bandwidth.**

Example-2.1 :

Consider a signal  $g(t)$  having the Upper Cutoff frequency,  $f_u = 100\text{KHz}$  and the Lower Cutoff frequency  $f_l = 80\text{KHz}$ .

The ratio of upper cutoff frequency to bandwidth of the signal  $g(t)$  is

$$f_u / B = 100\text{K} / 20\text{K} = 5.$$

Therefore we can choose  $m = 5$ .

Then the sampling rate is  $f_s = 2f_u / m = 200\text{K} / 5 = 40\text{KHz}$

Example-2.2 :

Consider a signal  $g(t)$  having the Upper Cutoff frequency,  $f_u = 120\text{KHz}$  and the Lower Cutoff frequency  $f_l = 70\text{KHz}$ .

The ratio of upper cutoff frequency to bandwidth of the signal  $g(t)$  is

$$f_u / B = 120\text{K} / 50\text{K} = 2.4$$

Therefore we can choose  $m = 2$ . ie..  $m$  is an integer less than  $(f_u / B)$ .

Then the sampling rate is  $f_s = 2f_u / m = 240\text{K} / 2 = 120\text{KHz}$

### **Quadrature Sampling of Band – Pass Signals:**

This scheme represents a natural extension of the sampling of low – pass signals.

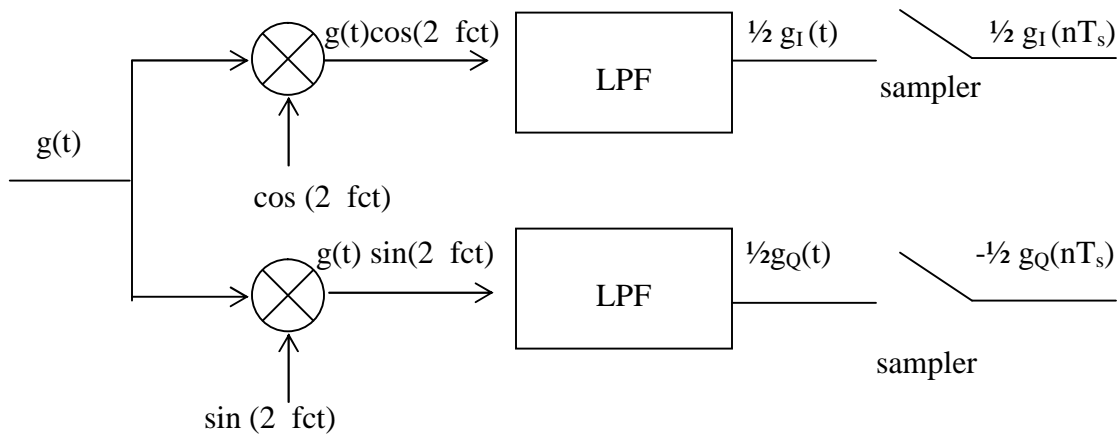
In this scheme, the band pass signal is split into two components, one is in-phase component and other is quadrature component. These two components will be low-pass signals and are sampled separately. This form of sampling is called quadrature sampling.

Let  $g(t)$  be a band pass signal, of bandwidth ' $2W$ ' centered around the frequency,  $f_c$ , ( $f_c > W$ ). The in-phase component,  $g_I(t)$  is obtained by multiplying  $g(t)$  with  $\cos(2 f_c t)$  and then filtering out the high frequency components. Parallely a quadrature phase component is obtained by multiplying  $g(t)$  with  $\sin(2 f_c t)$  and then filtering out the high frequency components..

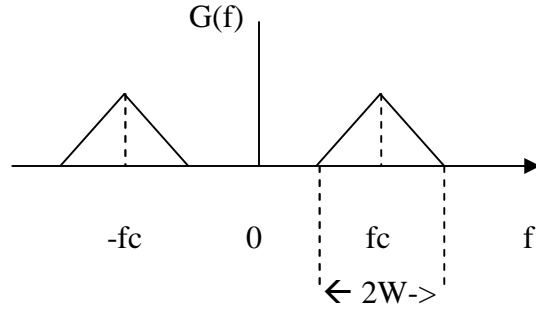
The band pass signal  $g(t)$  can be expressed as,

$$g(t) = g_I(t) \cdot \cos(2 f_c t) - g_Q(t) \sin(2 f_c t)$$

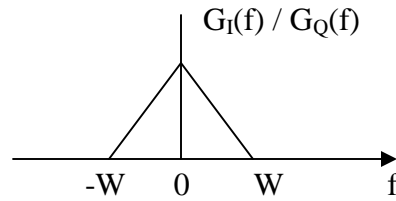
The in-phase,  $g_I(t)$  and quadrature phase  $g_Q(t)$  signals are low-pass signals, having band limited to  $(-W < f < W)$ . Accordingly each component may be sampled at the rate of  $2W$  samples per second.



**Fig 2.8: Generation of in-phase and quadrature phase samples**



a) Spectrum of a Band pass signal.



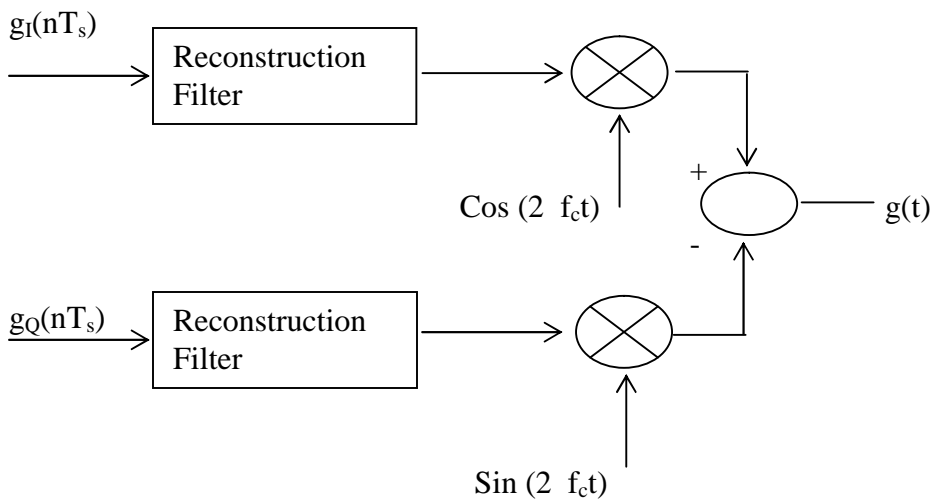
b) Spectrum of  $g_I(t)$  and  $g_Q(t)$

**Fig 2.9 a) Spectrum of Band-pass signal  $g(t)$**

**b) Spectrum of in-phase and quadrature phase signals**

#### RECONSTRUCTION:

From the sampled signals  $g_I(nT_s)$  and  $g_Q(nT_s)$ , the signals  $g_I(t)$  and  $g_Q(t)$  are obtained. To reconstruct the original band pass signal, multiply the signals  $g_I(t)$  and  $g_Q(t)$  by  $\cos(2 f_c t)$  and  $\sin(2 f_c t)$  respectively and then add the results.



**Fig 2.10: Reconstruction of Band-pass signal  $g(t)$**



### Natural Sampling:

In this method of sampling, an electronic switch is used to periodically shift between the two contacts at a rate of  $f_s = (1/T_s)$  Hz, staying on the input contact for  $C$  seconds and on the grounded contact for the remainder of each sampling period.

The output  $x_s(t)$  of the sampler consists of segments of  $x(t)$  and hence  $x_s(t)$  can be considered as the product of  $x(t)$  and sampling function  $s(t)$ .

$$x_s(t) = x(t) \cdot s(t)$$

The sampling function  $s(t)$  is periodic with period  $T_s$ , can be defined as,

$$S(t) = \begin{cases} 1 & -\tau/2 < t < \tau/2 \\ 0 & \tau/2 < t < T_s/2 \end{cases} \quad \text{----- (1)}$$

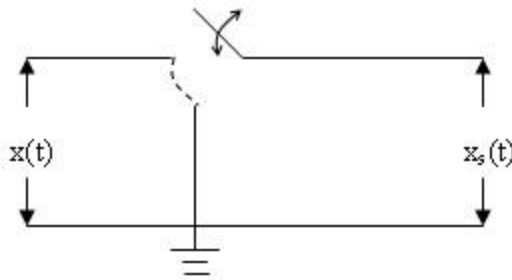


Fig: 2.11 Natural Sampling – Simple Circuit.

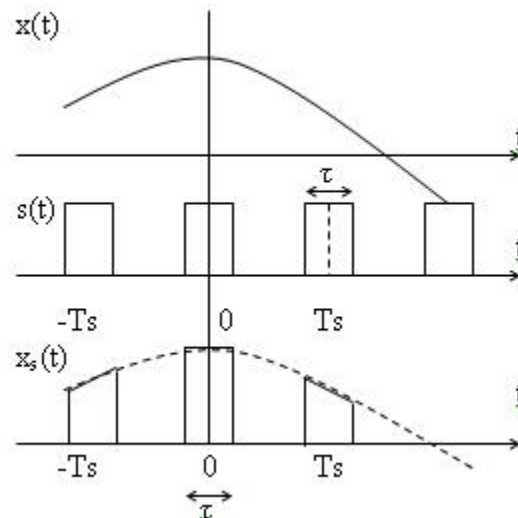


Fig: 2.12 Natural Sampling – Waveforms.

Using Fourier series, we can rewrite the signal  $S(t)$  as

$$S(t) = C_0 + \sum_{n=1}^{\infty} 2C_n \cos(n\omega_s t)$$

where the Fourier coefficients,  $C_0 = \frac{1}{T_s}$  &  $C_n = \frac{1}{T_s} \text{Sinc}(n f_s T_s)$

$$\text{Therefore: } x_s(t) = x(t) \left[ C_0 + \sum_{n=1}^{\infty} 2C_n \cos(n\omega_s t) \right]$$

$$x_s(t) = C_0 x(t) + 2C_1 x(t) \cos(\omega_s t) + 2C_2 x(t) \cos(2\omega_s t) + \dots$$

Applying Fourier transform for the above equation

$$\left\{ \begin{array}{l} \text{Using } x(t) \xleftrightarrow{\text{FT}} X(f) \\ x(t) \cos(2\pi f_0 t) \xleftrightarrow{\text{FT}} \frac{1}{2} [X(f-f_0) + X(f+f_0)] \end{array} \right.$$

$$X_s(f) = C_0 X(f) + C_1 [X(f-f_0) + X(f+f_0)] + C_2 [X(f-f_0) + X(f+f_0)] + \dots$$

$$X_s(f) = C_0 X(f) + \sum_{n=1}^{\infty} C_n X(f - n f_s)$$

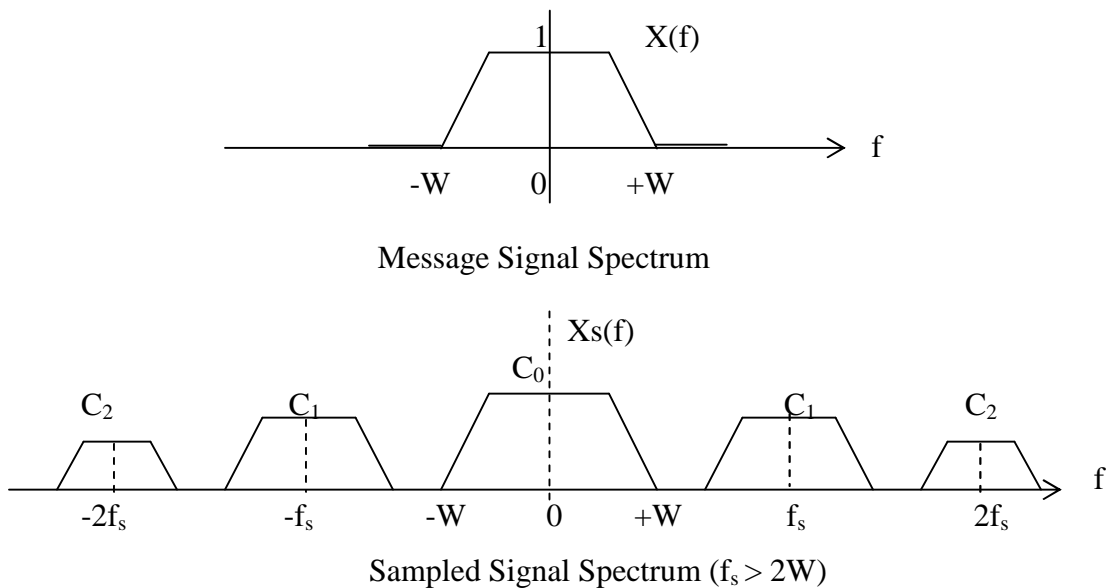
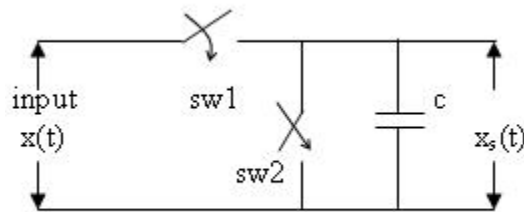


Fig.2.13 Natural Sampling Spectrum

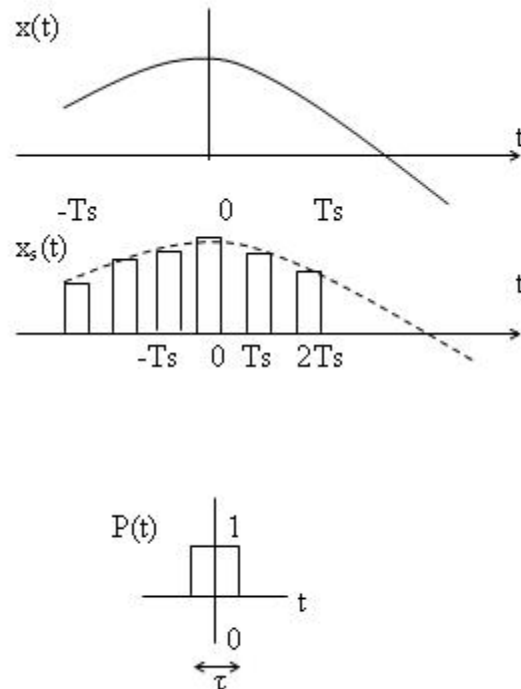
The signal  $x_s(t)$  has the spectrum which consists of message spectrum and repetition of message spectrum periodically in the frequency domain with a period of  $f_s$ . But the message term is scaled by 'Co'. Since the spectrum is not distorted it is possible to reconstruct  $x(t)$  from the sampled waveform  $x_s(t)$ .

### **Flat Top Sampling:**

In this method, the sampled waveform produced by practical sampling devices, the pulse  $p(t)$  is a flat – topped pulse of duration,  $\tau$  .



*Fig. 2.14: Flat Top Sampling Circuit*



*Fig. 2.15: Waveforms*

Mathematically we can consider the flat – top sampled signal as equivalent to the convolved sequence of the pulse signal  $p(t)$  and the ideally sampled signal,  $x_s(t)$ .

$$x_s(t) = p(t) * x(t)$$

$$x_s(t) = p(t) * \left[ \sum_{k=-\infty}^{+\infty} x(kTs) \delta(t - kTs) \right]$$

Applying F.T,

$$X_s(f) = P(f) \cdot X(f)$$

$$= P(f) \cdot \sum_{n=-\infty}^{+\infty} X(f - nfs)$$

where  $P(f) = \text{FT}[p(t)]$  and  $X(f) = \text{FT}[x(t)]$

#### Aperture Effect:

The sampled signal in the flat top sampling has the attenuated high frequency components. This effect is called the Aperture Effect.

The aperture effect can be compensated by:

1. Selecting the pulse width  $\dagger$  as very small.
2. by using an equalizer circuit.

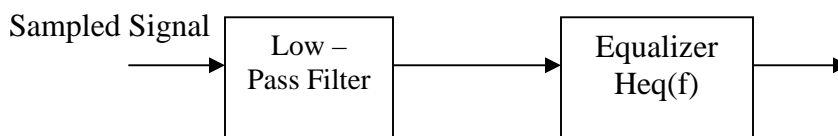


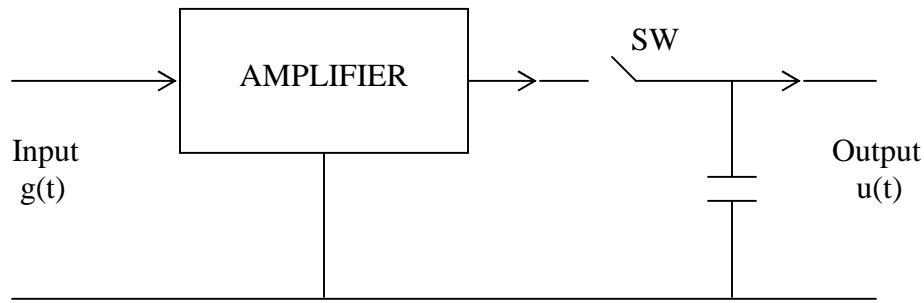
Fig:2.16

Equalizer decreases the effect of the in-band loss of the interpolation filter (lpf). As the frequency increases, the gain of the equalizer increases. Ideally the amplitude response of the equalizer is

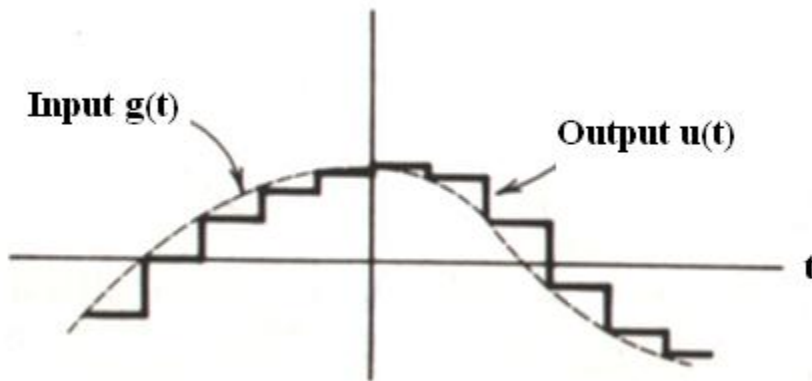
$$|H_{eq}(f)| = 1 / |P(f)| = \frac{1}{\dagger \cdot \text{Sinc}(f\dagger)} = \frac{f\dagger}{\text{Sinc}(f\dagger)}$$

## Sample and Hold Circuit for Signal Recovery.

In both the natural sampling and flat-top sampling methods, the spectrum of the signals are scaled by the ratio  $\tau/T_s$ , where  $\tau$  is the pulse duration and  $T_s$  is the sampling period. Since this ratio is very small, the signal power at the output of the reconstruction filter is correspondingly small. To overcome this problem a sample-and-hold circuit is used.



a) Sample and Hold Circuit



b) Idealized output waveform of the circuit

**Fig: 2.17 Sample Hold Circuit with Waveforms.**

The Sample-and-Hold circuit consists of an amplifier of unity gain and low output impedance, a switch and a capacitor; it is assumed that the load impedance is large. The switch is timed to close only for the small duration of each sampling pulse, during which time the capacitor charges up to a voltage level equal to that of the input sample. When the switch is open, the capacitor retains the voltage level until the next closure of the switch. Thus the sample-and-hold circuit produces an output waveform that represents a staircase interpolation of the original analog signal.

The output of a Sample-and-Hold circuit is defined as

$$u(t) = \sum_{n=-\infty}^{+\infty} g(nTs) h(t - nTs)$$

where  $h(t)$  is the impulse response representing the action of the Sample-and-Hold circuit; that is

$$h(t) = \begin{cases} 1 & \text{for } 0 < t < Ts \\ 0 & \text{for } t < 0 \text{ and } t > Ts \end{cases}$$

Correspondingly, the spectrum for the output of the Sample-and-Hold circuit is given by,

$$U(f) = f_s \sum_{n=-\infty}^{+\infty} H(f) G(f - nf_s)$$

where  $G(f)$  is the FT of  $g(t)$  and  
 $H(f) = Ts \text{ Sinc}(fTs) \exp(-j\pi fTs)$

To recover the original signal  $g(t)$  without distortion, the output of the Sample-and-Hold circuit is passed through a low-pass filter and an equalizer.

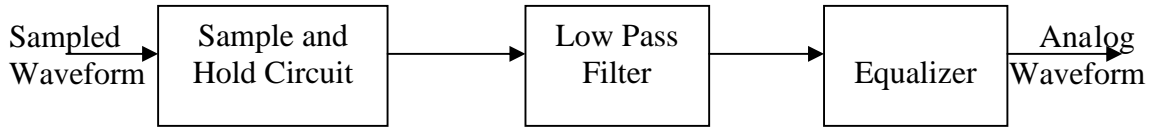


Fig. 2.18: Components of a scheme for signal reconstruction

## Signal Distortion in Sampling.

In deriving the sampling theorem for a signal  $g(t)$  it is assumed that the signal  $g(t)$  is strictly band-limited with no frequency components above 'W' Hz. However, a signal cannot be finite in both time and frequency. Therefore the signal  $g(t)$  must have infinite duration for its spectrum to be strictly band-limited.

In practice, we have to work with a finite segment of the signal in which case the spectrum cannot be strictly band-limited. Consequently when a signal of finite duration is sampled an error in the reconstruction occurs as a result of the sampling process.

Consider a signal  $g(t)$  whose spectrum  $G(f)$  decreases with the increasing frequency without limit as shown in the figure 2.19. The spectrum,  $G_\delta(f)$  of the ideally sampled signal,  $g_\delta(t)$  is the sum of  $G(f)$  and infinite number of frequency shifted replicas of  $G(f)$ . The replicas of  $G(f)$  are shifted in frequency by multiples of sampling frequency,  $f_s$ . Two replicas of  $G(f)$  are shown in the figure 2.19.

The use of a low-pass reconstruction filter with its pass band extending from  $(-f_s/2$  to  $+f_s/2)$  no longer yields an undistorted version of the original signal  $g(t)$ . The portions of the frequency shifted replicas are folded over inside the desired spectrum. Specifically, high frequencies in  $G(f)$  are reflected into low frequencies in  $G_\delta(f)$ . The phenomenon of overlapping in the spectrum is called as Aliasing or Foldover Effect. Due to this phenomenon the information is invariably lost.

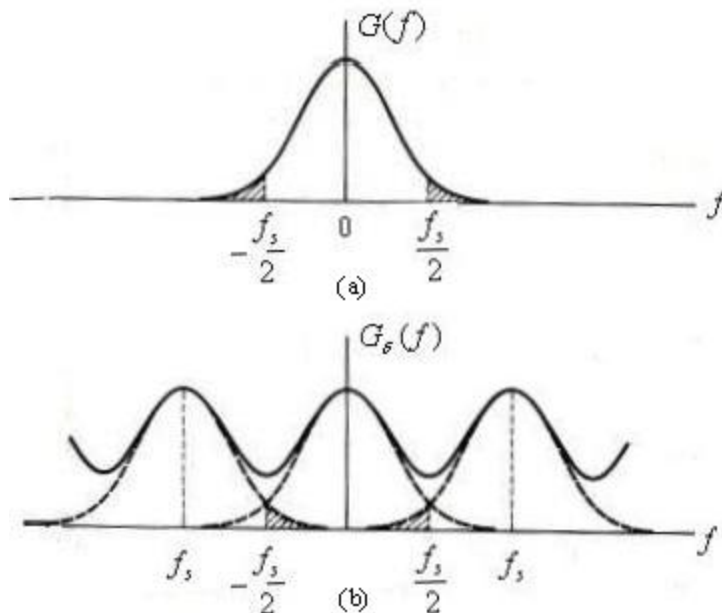


Fig. 2.19 :      a) Spectrum of finite energy signal  $g(t)$   
                         b) Spectrum of the ideally sampled signal.

### Bound On Aliasing Error:

Let  $g(t)$  be the message signal,  $g(n/f_s)$  denote the sequence obtained by sampling the signal  $g(t)$  and  $g_i(t)$  denote the signal reconstructed from this sequence by interpolation; that is

$$g_i(t) = \sum_n g\left(\frac{n}{f_s}\right) \text{Sinc}(f_s t - n)$$

Aliasing Error is given by,  $\varepsilon = |g(t) - g_i(t)|$

Signal  $g(t)$  is given by

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi f t) df$$

Or equivalently

$$g(t) = \sum_{m=-\infty}^{+\infty} \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f) \exp(j2\pi f t) df$$

Using Poisson's formula and Fourier Series expansions we can obtain the aliasing error as

$$\varepsilon = \left| \sum_{m=-\infty}^{+\infty} [1 - \exp(-j2\pi m f_s t)] \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f) \exp(j2\pi f t) df \right|$$

Correspondingly the following observations can be done :

1. The term corresponding to  $m=0$  vanishes.
2. The absolute value of the sum of a set of terms is less than or equal to the sum of the absolute values of the individual terms.
3. The absolute value of the term  $1 - \exp(-j2\pi m f_s t)$  is less than or equal to 2.
4. The absolute value of the integral in the above equation is bounded as

$$\left| \int_{(m-1/2)f_s}^{(m+1/2)f_s} G(f) \exp(j2\pi f t) df \right| < \int_{(m-1/2)f_s}^{(m+1/2)f_s} |G(f)| df$$

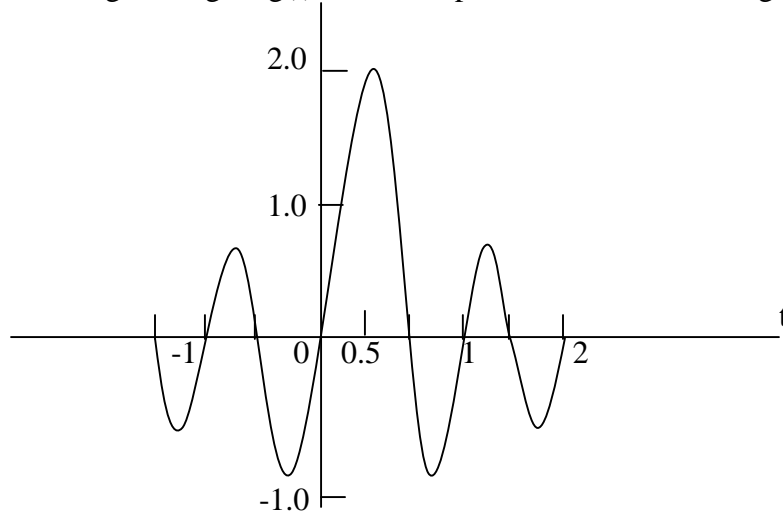
Hence the aliasing error is bounded as

$$\varepsilon \leq 2 \int_{|f| > f_s/2} |G(f)| df$$

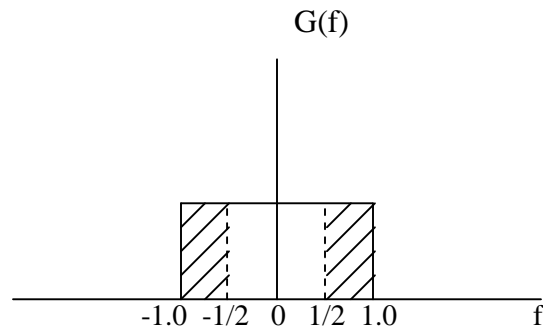


Example: Consider a time shifted sinc pulse,  $g(t) = 2 \text{ sinc}(2t - 1)$ . If  $g(t)$  is sampled at rate of 1 sample per second that is at  $t = 0, \pm 1, \pm 2, \pm 3$  and so on, evaluate the aliasing error.

Solution: The given signal  $g(t)$  and it's spectrum are shown in fig. 2.20.



a) Sinc Pulse



(b) Amplitude Spectrum,  $G(f)$

**Fig. 2.20**

The sampled signal  $g(nTs) = 0$  for  $n = 0, \pm 1, \pm 2, \pm 3 \dots$  and reconstructed signal

$$g_i(t) = 0 \text{ for all } t.$$

From the figure, the sinc pulse attains its maximum value of 2 at time  $t$  equal to  $1/2$ . The aliasing error cannot exceed  $\max|g(t)| = 2$ .

From the spectrum, the aliasing error is equal to unity.

## Time Division Multiplexing (TDM)

TDM is a technique used for transmitting several message signals over a single communication channel by dividing the time frame into slots, one slot for each message signal.

The concept of TDM is indicated in the figures 2.21 and 2.22. Each message signal is first restricted in bandwidth by a low pass pre-alias filter to remove the frequencies that are not essential which helps in reducing the aliasing problem. The outputs of these filters are then applied to a commutator. The functions of the commutator are:

- (i) allows narrow samples of each of the  $N$  input messages at a rate of  $f_s$  and
- (ii) sequentially interleaves these  $N$  samples inside a sampling interval  $T_s$ .

The multiplexed signal is then applied to a pulse amplitude modulator, which transforms the multiplexed signal into a form suitable for transmission over the communication channel.

The time division scheme squeezes  $N$  samples derived from different  $N$  independent message signals into a time slot equal to one sampling interval. Thus the use of TDM introduces a bandwidth expansion factor  $N$ .

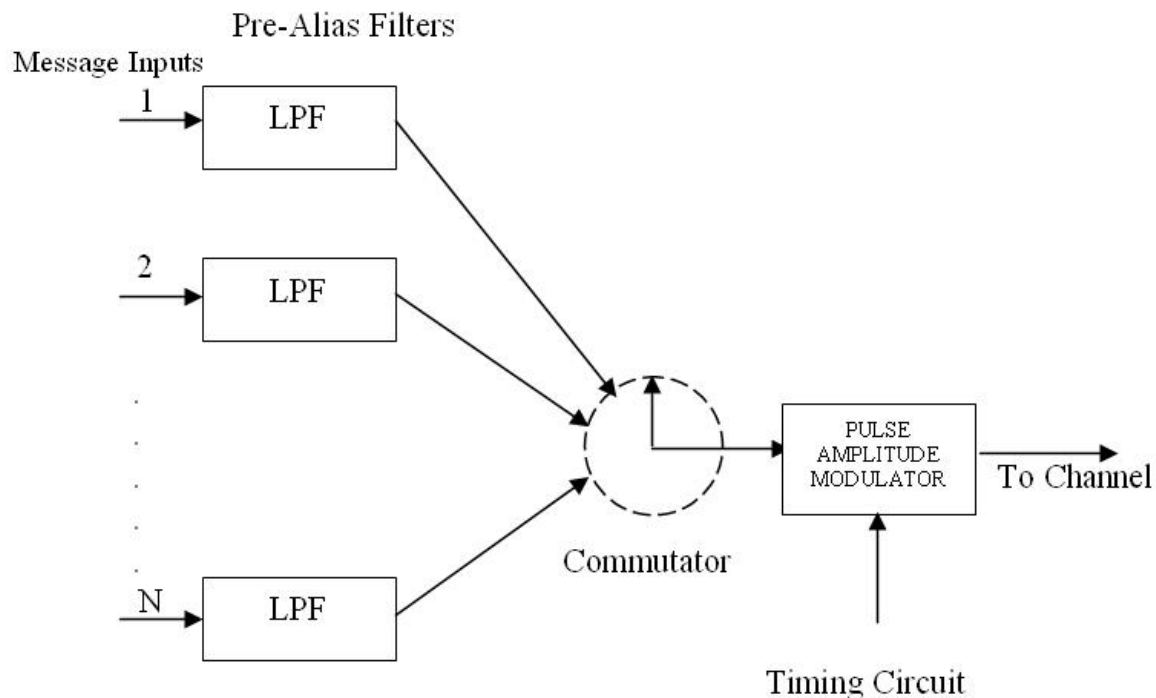


Fig-2.21 : TDM-PAM: Transmitter

**TDM-PAM Receiver:** At the receiver end of the system, the received signal is applied to a pulse amplitude demodulator, which performs the reverse operation of the pulse amplitude modulator. The decommutator distributes the appropriate pulses to the respective reconstruction filters. The decommutator operates in synchronism with the commutator in the transmitter.

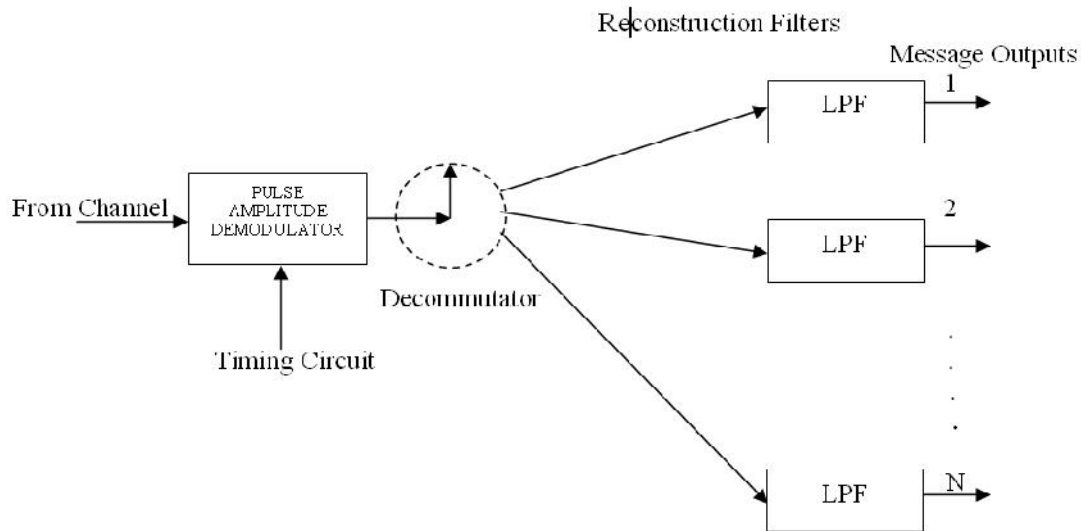


Fig-2.22 : TDM-PAM: Receiver

The multiplexed signal, considering four message signals is shown in the figure 2.23 and the corresponding commutator and decommutator arrangements are shown in the figures 2.24 and 2.25.

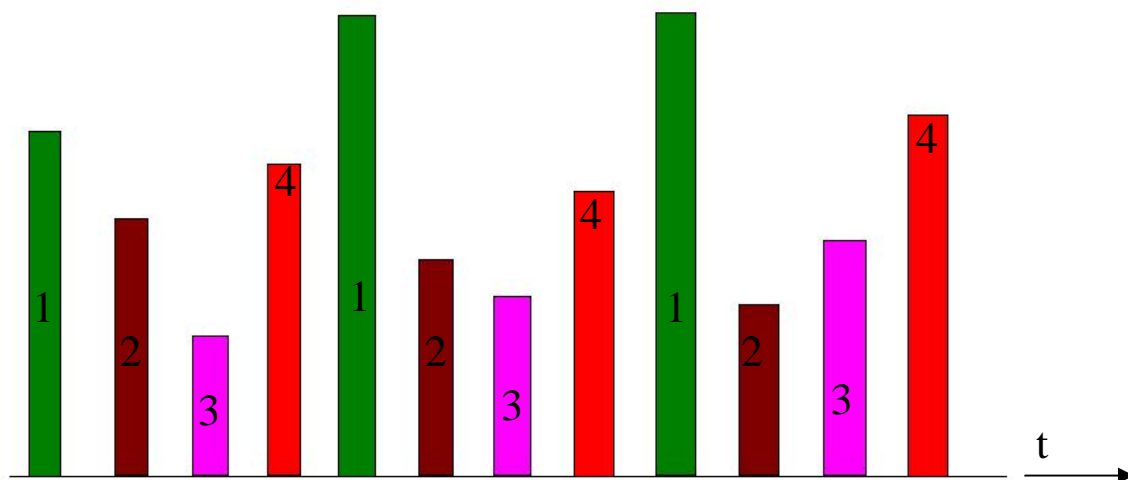
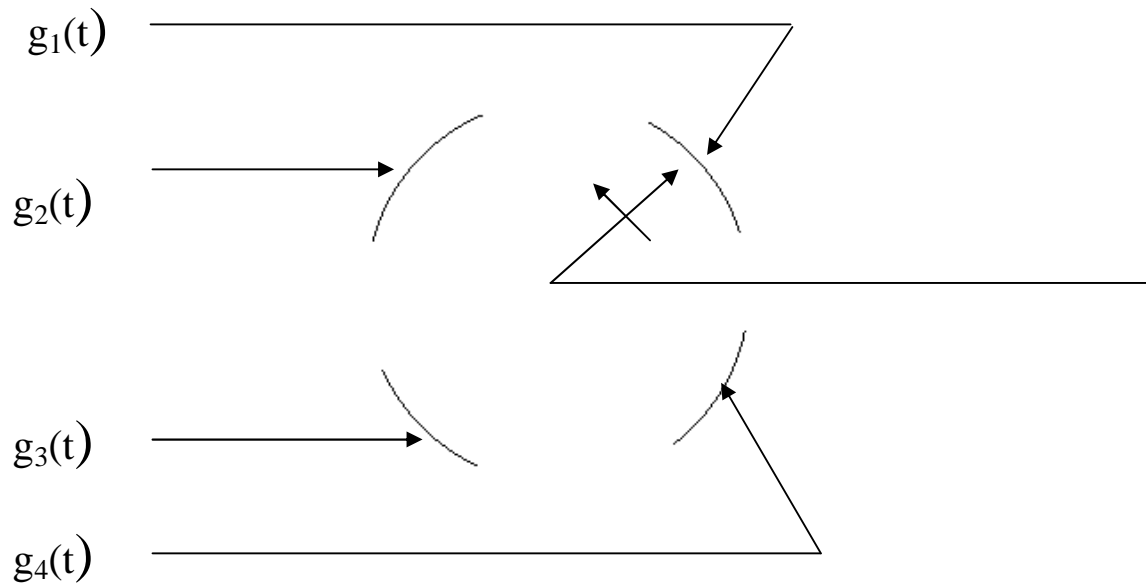


Fig: 2.23 Multiplexing of FOUR signals.

Commutator Arrangement ( for four signals )



*Fig: 2.24 Commutator Arrangement Multiplexing of FOUR signals.*

Decommutator Arrangement ( for four signals )

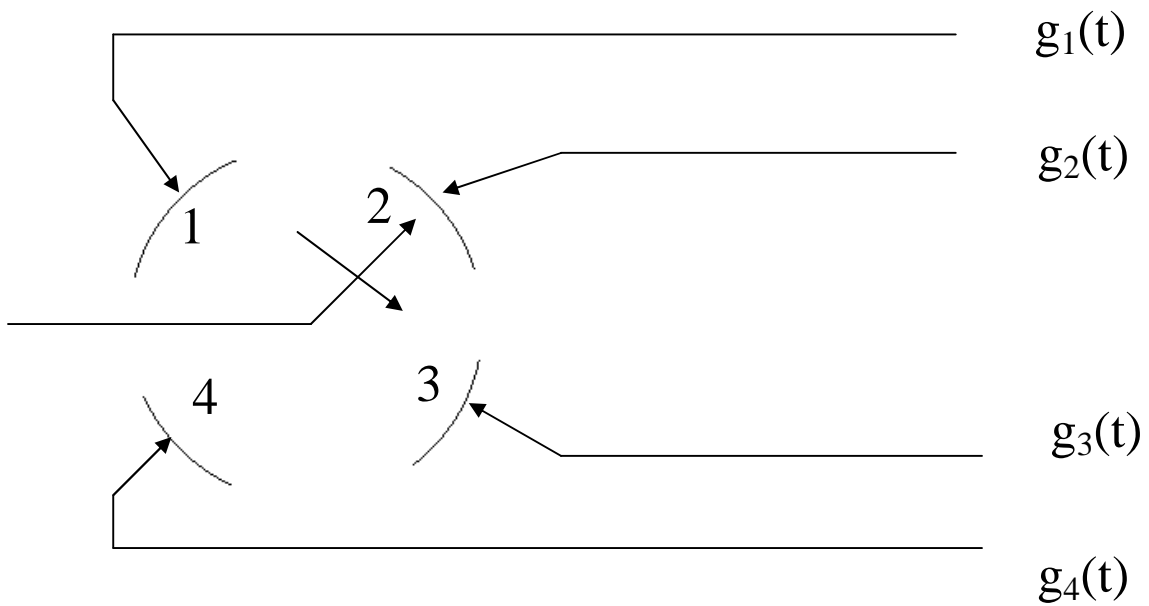


Fig: 2.25 DeCommutator Arrangement Multiplexing of FOUR signals.

## **TDM – PAM:**

There are two types of TDM:

1. Synchronous TDM
2. Asynchronous TDM

### **Synchronous TDM**

1. Same Sampling rate for all signals.
2. Minimum Sampling rate = twice the maximum frequency of all the signals.
3. Total number of samples transmitted per second is equal to N times the sampling rate,  $F_s$  plus sync pulses.
4. Transmission Bandwidth =  $N \cdot F_s/2$

### **Asynchronous TDM:**

1. Different Sampling rate for different. signals.
2. Sampling rate of a signal = twice the maximum frequency of that signal.
3. Total number of samples transmitted per second is equal to Sum of samples of all the signals plus sync pulses
4. Transmission Bandwidth = Half the total number of samples transmitted.
5. Bandwidth is less for Asynchronous TDM.
6. Design of Commutator / Decommutator is difficult.

## **PROBLEM-1:**

Two low-pass signals of equal bandwidth are sampled and time division multiplexed using PAM. The TDM signal is passed through a Low-pass filter & then transmitted over a channel with a bandwidth of 10KHz.

- a) What is maximum Sampling rate for each Channel?
- b) What is the maximum frequency content allowable for each signal?

Solution:

Channel Bandwidth = 10 KHz.

Number of samples that can be transmitted through the channel = 20K

Maximum Sampling rate for each channel = 10K Samples/sec.

Maximum Frequency for each Signal = 5KHz

### PROBLEM-2:

Two signals  $g_1(t)$  and  $g_2(t)$  are to be transmitted over a common channel by means of TDM. The highest frequency of  $g_1(t)$  is 1KHz and that of  $g_2(t)$  is 1.3KHz. What is the permissible sampling rate?

Solution: Choosing the highest frequency of the signal as 1.3KHz, the permissible sampling rate is

2.6K samples/sec and above. { Synchronous TDM }

### PROBLEM-3:

24 voice signals are sampled uniformly and then time division multiplexed. The sampling operation uses the flat-top samples with 1microsec duration. The multiplexing operation includes provision for Synchronization by adding an extra pulse of sufficient amplitude and also 1micro second. Assuming a sampling rate of 8KHz, calculate the spacing between successive pulses of the multiplexed signal.

Solution:

In One frame, Total number of pulses = 25.

Time duration for one time frame =  $T_s = 125\mu$  seconds.

Time duration utilized by pulses =  $25\mu$  sec

Time spacing between successive pulses =  $(125 - 25)/25 = 4\mu$  sec

### PROBLEM-4:

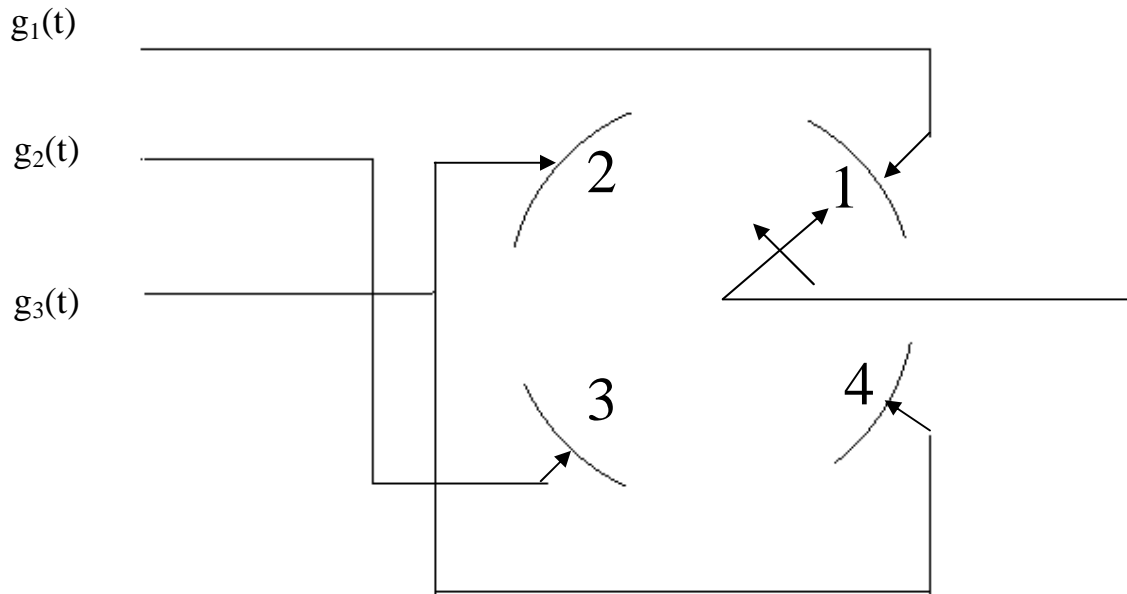
Three independent message signals of bandwidths 1KHz, 1KHz and 2KHz respectively are to be transmitted using TDM scheme. Determine

- Commutator segment arrangement
- Speed of the commutator if all the signals are sampled at its Nyquist rate.
- Minimum Transmission bandwidth

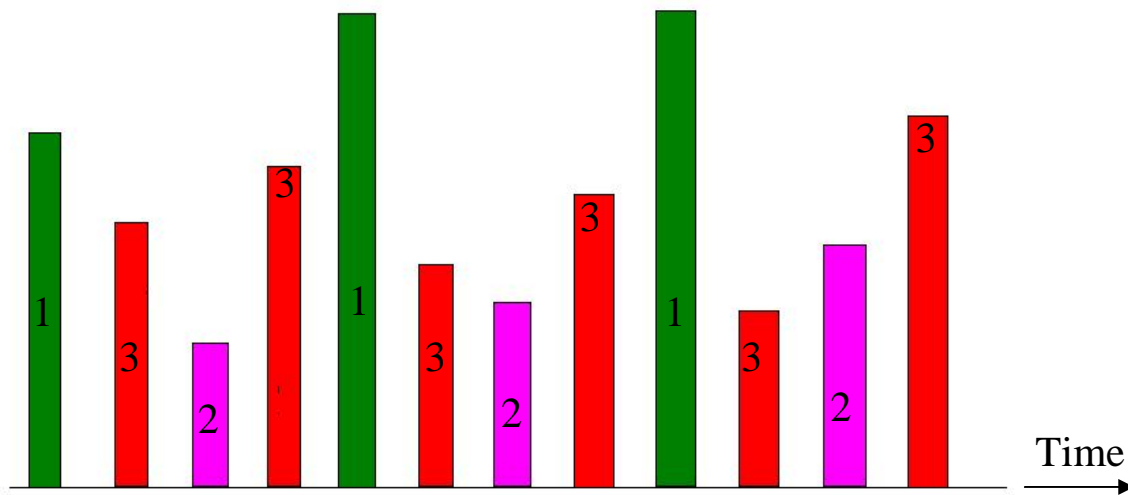
Solution:

- Commutator Segment arrangement - refer figure-2.26
- Total number of Samples to be transmitted per second = 8K samples/sec.  
Number of Commutator segments = 4  
Speed of Commutator = 2000 rotations/sec.
- Transmission Bandwidth = 4000 Hz.

The corresponding TDM signal is shown in the figure below:



*Fig 2.26 – commutator arrangement for problem-4.*



*Fig 2.27 – TDM Signal for problem-4.*

### ASSIGNMENT PROBLEM:

Eight independent message signals are sampled and time multiplexed using PAM. Six of the message signals are having a bandwidth of 4KHz and other two have bandwidth of 12KHz. Compare the transmission bandwidth requirements of Synchronous TDM and Asynchronous TDM.

### PROBLEM-5:

Four independent message signals of bandwidths  $W$ ,  $2W$ ,  $2W$  and  $4W$  hertz are to be transmitted on a TDM basis using a common channel.

Design a suitable ASYNCHRONOUS – TDM system.

Solution:

In asynchronous TDM all the signals will be sampled at their respective Nyquist rate. This is possible only if there is a common factor for all the sampling frequencies. In this case the common factor value is  $2W$ . Hence the number of segments required for each signal and the respective segments are all shown in the table below.

| Signal | Bandwidth of the Signal | Sampling Frequency | Number of Segments | Segments    |
|--------|-------------------------|--------------------|--------------------|-------------|
| g1     | $W$                     | $2W$               | 1                  | 5           |
| g2     | $2W$                    | $4W$               | 2                  | 2, 8        |
| g3     | $2W$                    | $4W$               | 2                  | 3, 9        |
| g4     | $4W$                    | $8W$               | 4                  | 1, 4, 7, 10 |

### PROBLEM-6:

Q. Twenty four analog signals, each having a bandwidth of 15KHz, are to be time-division multiplexed and transmitted via PAM/AM. A guard band of 10 KHz is required for signal reconstruction from the PAM samples of each signal.

- Find the sampling rate for each channel
- Calculate the transmission bandwidth

Solution:

- Bandwidth of the signal  $W = 15$  KHz.

Guard band =  $f_s - 2W$  ( refer the figure2.28 ) = 10KHz

Sampling frequency,  $f_s = 2W + \text{Guard band} = 40$  KHz

- No. of samples transmitted per second =  $N.f_s = 24 \times 40 \text{ K} = 960\text{K}$

Transmission Bandwidth = 480 KHz.



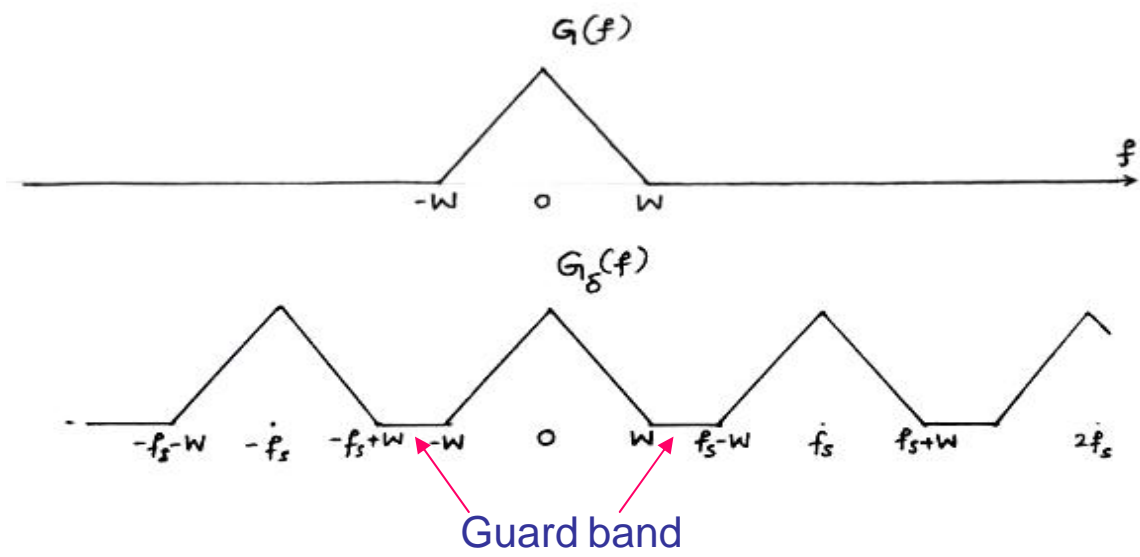


Fig: 2.28 Spectrum of the sampled signal with guard band indicated.

---

-- END --

---