Syllabus

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Energy Equation, Specific Energy, Specific energy curve, momentum equation, specific force,
Energy and momentum correction factors

Critical flow : characteristics, critical flow in rectangular sections

Metering flumes : Venturi flume, Parshall Flume

Critical Flow

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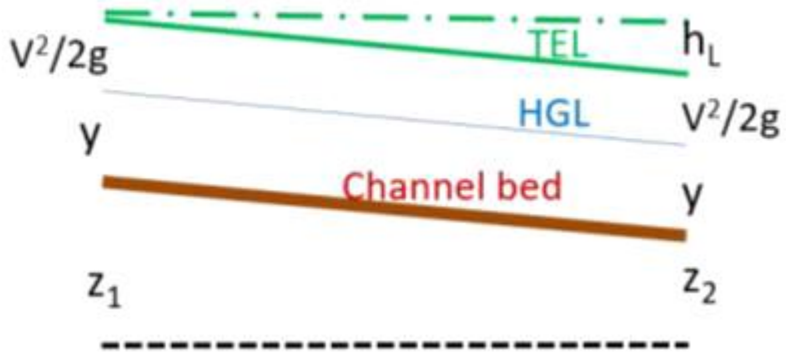
ENERGY IN OPEN CHANNEL FLOW



Total Energy
•Total energy per unit weight of water = Total head
$TE = P/\gamma + v^2/2g + z$
= Pressure head + Velocity head + Elevation head
•Because of friction, there will be head loss (h_L) from one section to another
•Plot of the TE against the channel position □Total Energy Line or Energy Grade Line
•Slope of this line □Energy gradient or Energy slope

Specific energy :
Energy per unit weight of water measured with respect to the channel bottom as the datum

$$E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2}$$

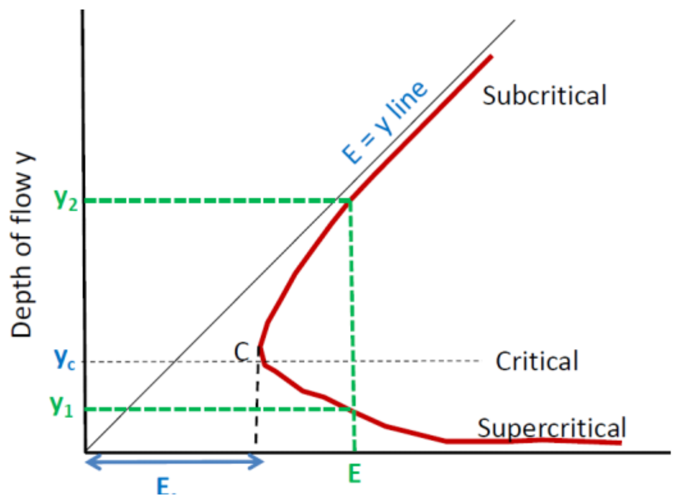
•Specific energy is a function of y
•For uniform flow, y remains constant along the channel
•Hence, for a given Q and A, specific energy is constant along the channel under uniform flow

For uniform flow condition
Total energy →Decreases along the direction of flow
Specific energy →Remains constant

Specific Energy Diagram

C: Corresponds to the minimum E for a given Q → Critical flow → Critical depth (y_c)

- If E is less than E_c → flow cannot occur
- For any other E→ two depth y_1 and y_2 → **Alternate depths**



$y_1 < y_c$→super critical flow
$y_2 > y_c$→subcritical flow

Critical Flow Continued...

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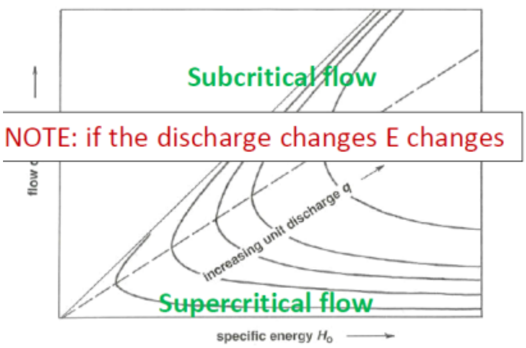
Critical Flow

1. For a given Q and A, Specific energy is the minimum
2. Depth of flow yc: Critical depth
3. Velocity vc: Critical velocity
4. Froude's number =1

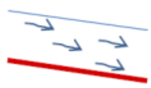
- All flows falls at some point on one of the curves
- Sometimes, the flow is forced from one state to the other through critical state
- Transition behaviour is different depending on the state of flow (subcritical or supercritical)



NOTE: if the discharge changes E changes

- Super critical flow (rapid flow)
  - $Y < y_c$
  - As y increases, E decreases
  - Froude's number Fr >1

- Subcritical flow (tranquil flow)
  - $y > y_c$
  - As y increases, E increases
  - Froude's number Fr < 1



- Velocity of flow = Celerity $(v = \sqrt{gD})$
  - Celerity = Speed at which the disturbance of the water surface moves through shallow water
  - Celerity in open channel flow $C = \sqrt{gD}$
  - Where D = hydraulic mean depth = A/T
- $\frac{v}{\sqrt{gD}} = 1 = Froude's\ number\ Fr$
- Critical depth is independent of the channel slope S_o

Critical flow → For a given Q, E is the minimum

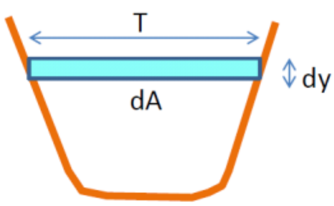
$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = \frac{d}{dy}\left(y + \frac{Q^2}{2gA^2}\right)$$

$$= 1 + \frac{Q^2}{2g}\frac{d}{dy}\left(\frac{1}{A^2}\right)$$

$$= 1 + \frac{Q^2}{2g}\frac{-2}{A^3}\frac{dA}{dy} = 0$$

$$1 = \frac{Q^2}{gA^3}\frac{dA}{dy}$$



$$dA = dy\,T$$

$$\frac{dA}{dy} = T$$

$$1 = \frac{Q^2}{gA^3}\frac{dA}{dy}$$

$$1 = \frac{Q^2}{gA^3}T$$

$$\boxed{\frac{Q^2}{g} = \frac{A^3}{T}}$$

Condition for the Maximum discharge

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Condition for maximum discharge through a channel

- For a given channel section, for a given E
- Maximum discharge when $\frac{dQ}{dy} = 0$

$$E = y + \frac{Q^2}{2gA^2}$$

$$Q = \sqrt{(E-y)2gA^2} \;........(1)$$

$$\frac{dQ}{dy} = 0 \rightarrow \sqrt{2g}\left[\sqrt{E-y}\frac{dA}{dy} - \frac{A\,1/2}{\sqrt{E-y}}\right] = 0$$

$$\frac{dA}{dy} = T$$

$$T = \frac{A}{2(E-y)} \Rightarrow E = y + \frac{A}{2T}$$

Specific energy for maximum discharge is

$$E = y + \frac{A}{2T}$$

Substituting E in (1), Q_{max} is given by

$$\frac{Q^2}{g} = \frac{A^3}{T} \Rightarrow \text{Critical flow}$$

Q is max. when flow is critical

Critical flow ....

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

Q/ A = v

A /T = D

$$v^2 = \frac{Ag}{T} = gD$$

$$\Rightarrow v = \sqrt{gD}$$

$$\frac{v}{\sqrt{gD}} = 1 = Fr$$

For critical flow

$v = v_c$

$$\frac{v_c}{\sqrt{gD}} = 1 = Fr$$

Froude's number = 1

Super critical flow

$v > v_c$

$$\frac{v}{\sqrt{gD}} > 1$$

Froude's number > 1

Subcritical flow

$v < v_c$

$$\frac{v}{\sqrt{gD}} < 1$$

Froude's number < 1

- Specific energy is minimum for a given Q
- $\frac{Q^2}{g} = \frac{A^3}{T}$
- Froude's number F_r =1
- For a given E, discharge is maximum

Critical slope

For a given discharge, it is possible to adjust the bed slope such that uniform flow condition is maintained at yc. This value of bed slope is known as critical slope.

Critical Flow in Rectangular Channel

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For critical flow $\frac{Q^2}{g} = \frac{A^3}{T}$

For rectangular channels

T= B

Q /B = q (Discharge per unit width)

$$\frac{q^2B^2}{g} = \frac{B^3{y_c}^3}{B}$$

$$\boxed{y_c = \sqrt[3]{\frac{q^2}{g}}}$$

Specific energy $E = y + \frac{Q^2}{2gA^2}$

$$E_c = yc + \frac{q^2B^2}{2g(B^2{y_c}^2)}$$

$$E_c = yc + \frac{q^2}{2gyc^2}$$

$$q = \sqrt{2g(E_c - yc){y_c}^2}$$

$$q = \sqrt{2g(E - y)y^2}$$

Critical flow :
For a given E, Q is max→ q is max

$$\frac{dq}{dy} = 0 \rightarrow \frac{d}{dy}\sqrt{2g(E - y)y^2} = 0$$

$$\boxed{E = \frac{3}{2}y}$$

For critical flow in rectangular channel

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

$$E = \frac{3}{2}yc$$

Problem : Discharge of water through a rectangular channel of width 8 m is 15 m3/s., when depth of flow of water is 1.2 m.
Calculate
i) specific energy
ii) critical depth and critical velocity
Iii) Minimum specific energy for the given discharge

Momentum Correction Factor

17 July 2023 10:06

The momentum correction factor is the ratio of momentum rate based on the actual velocity to the momentum rate based on average velocity. It shows how momentum rate based on actual velocity and the momentum rate based on average velocity are related.
It is a unitless quantity and is denoted by the symbol ‘β’.

convenient to express the momentum flux flowing through any cross-section in terms of the mean velocity of flow. But the actual momentum flux is always greater than that computed by using the mean velocity of flow. Hence in order to account for this difference in the values of the momentum flux due to the non-uniform velocity distribution at any cross-section a factor called momentum correction factor represented by β (Greek ‘beta’) is introduced, so that the momentum flux computed by using the mean velocity V may be expressed as ($\beta \rho A V^2$) and it is then equal to the actual total momentum flux flowing through the entire cross-section. Thus equating the two, the value of the momentum correction factor may be obtained as

$$\beta \rho A V^2 = \rho \int_A v^2 dA$$

$$\beta = \frac{1}{AV^2} \int_A v^2 dA$$

Mathematically the square of the average is less than the average of the squares. the numerical value of β will always be greater than 1. The actual value of β depends on the velocity distribution at the flow section. If the velocity is uniform over the entire cross-section, β will be equal to 1. For turbulent flow in pipes the value of β lies between 1.02 and 1.05. However, for laminar flow in pipes the value of β is 1.33.

Energy Correction Factor

17 July 2023 10:34

in the case of flow of real fluids the velocity distribution across any cross-sectional area of the flow passage is not uniform. Therefore along any cross-section the velocities of flow will be different at different points and the total kinetic energy possessed by flowing fluid at any section will be obtained by integrating the kinetic energies possessed by different fluid particles
kinetic energy of this mass of fluid will be

$$(\rho v dA)\,\frac{v^2}{2}$$

The total kinetic energy possessed by the flowing fluid across the entire cross-section A is

$$\int_A \rho \frac{v^3}{2} dA = \frac{w}{2g}\int_A v^3 dA$$

convenient to express the kinetic energy of the flowing fluid in terms of the mean velocity of flow. But the actual kinetic energy possessed by the flowing fluid is greater than that computed by using the mean velocity. Hence a factor called kinetic energy correction factor represented by α (Greek 'alpha') is introduced, so that the kinetic energy computed by using the mean velocity V may be expressed as

$$\left(\alpha \frac{w}{2g} A V^3\right)$$

and it is equal to the actual total kinetic energy possessed by the flowing fluid. Thus equating the two, the value of the kinetic energy correction factor α may be obtained as

$$\alpha \frac{w}{2g} A V^3 = \frac{w}{2g}\int_A v^3 dA$$

$$\alpha = \frac{1}{AV^3}\int_A v^3 dA$$

the numerical value of α will always be greater than 1. The actual value of α depends on the velocity distribution at the flow section. The value of α for turbulent flow in pipes lies between 1.03 to 1.06, which is very close to 1, because in turbulent flow the velocity distribution is very close to uniform velocity distribution. However, for laminar flow in pipes the value of α is 2.

$$\frac{p_1}{w} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

in which α1 and α2 are the energy correction factors at sections 1 and 2 respectively. However, in most of the problems of turbulent flow the value of α is nearly equal to 1, and therefore it may be assumed as
one without any appreciable error being introduced.

Problem on correction factor

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In a circular pipe the velocity profile is given as where u is the velocity at any radius r, Um is the velocity at the pipe axis, and R is the radius of the pipe.

$$u = U_m\left[1-\left(\frac{r}{R}\right)^2\right]$$

Find: • (i) Average velocity, • (ii) Energy correction factor, and • (iii) Momentum correction factor.

Consider an elementary area dA in the form of a ring at a radius r and of thickness dr, then dA = 2π r.dr
Flow rate through the ring = dQ = Elemental area × local velocity = 2πr.dr.u

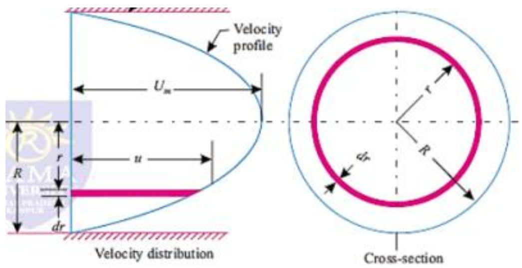
$$\text{Total flow } Q = \int_0^R 2\pi r.u.dr$$

$$= \int_0^R 2\pi\, U_m\left(1-\frac{r^2}{R^2}\right) r\, dr$$

$$= 2\pi U_m \int_0^R \left(r-\frac{r^3}{R^2}\right) dr = 2\pi U_m\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R$$

$$= 2\pi U_m\left(\frac{R^2}{2}-\frac{R^2}{4}\right) = 2\pi U_m\left(\frac{R^2}{4}\right)$$

$$Q = 2\pi U_m\left(\frac{R^2}{4}\right)$$



Average velocity, $\bar{u}$:

If $\bar{u}$ is the average flow velocity, then:

$$Q = A\bar{u} = \pi R^2 \bar{u}$$

From (*i*) and (*ii*), we get:

$$\pi R^2 \bar{u} = 2\pi U_m\left(\frac{R^2}{4}\right)$$

$$\bar{u} = \frac{2\pi U_m\left(\frac{R^2}{4}\right)}{\pi R^2} = \frac{U_m}{2}$$

(iii) Kinetic energy correction factor, α

$$\alpha = \frac{1}{A\bar{u}^3}\int_0^R u^3\, dA$$

$$= \frac{1}{A\bar{u}^3}\int_0^R U_m^3\left[1-\left(\frac{r}{R}\right)^2\right]^3 2\pi\, r dr$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\int_0^R \left(1-\frac{3r^2}{R^2}+\frac{3r^4}{R^4}-\frac{r^6}{R^6}\right) r\, dr$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\int_0^R \left(r-\frac{3r^3}{R^2}+\frac{3r^5}{R^4}-\frac{r^7}{R^6}\right) dr$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\left[\frac{r^2}{2}-\frac{3r^4}{4R^2}+\frac{3r^6}{6R^4}-\frac{r^8}{8R^6}\right]_0^R$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\left[\frac{R^2}{2}-\frac{3}{4}R^2+\frac{R^2}{2}-\frac{R^2}{8}\right]$$

$$= \frac{2\pi U_m^3}{A\bar{u}^3}\left(\frac{R^2}{8}\right)$$

$A = \pi R^2$ and $\bar{u} = \frac{U_m}{2}$, we get:

$$\alpha = \frac{2\pi U_m^3}{\pi R^2 \times \left(\frac{U_m}{2}\right)^3} \times \left(\frac{R^2}{8}\right) = 2 \text{ (Ans.)}$$

(iv) Momentum correction factor, β:

$$\beta = \frac{1}{A\bar{u}^2}\int u^2\, dA$$

$$= \frac{1}{A\bar{u}^2}\int_0^R U_m^2\left[1-\left(\frac{r}{R}\right)^2\right]^2 2\pi r\, dr$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\int_0^R \left(1-2\times\frac{r^2}{R^2}+\frac{r^4}{R^4}\right) r\, dr$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\int_0^R \left(r-2\times\frac{r^3}{R^2}+\frac{r^5}{R^4}\right) dr$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\left[\frac{r^2}{2}-2\times\frac{r^4}{4R^2}+\frac{1}{6}\times\frac{r^6}{R^4}\right]_0^R$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\left[\frac{R^2}{2}-\frac{R^2}{2}+\frac{R^2}{6}\right]$$

$$= \frac{2\pi U_m^2}{A\bar{u}^2}\left(\frac{R^2}{6}\right)$$

tuting the values $A = \pi R^2$ and $\bar{u} = \frac{U_m}{2}$, we get:

$$\beta = \frac{2\pi U_m^2}{\pi R^2 \times \left(\frac{U_m}{2}\right)^2}\left(\frac{R^2}{6}\right) = 1.33$$

Application of Specific Energy

17 July 2023 11:02

In the case of long channels often it becomes necessary to provide transitions. A transition is the portion of a channel with varying cross-section, which connects one uniform channel to another, which may or may not have the same cross-sectional form. The variation of a channel section may be caused either by reducing or increasing the width or by raising or lowering the bottom of the channel. Various channel transitions may be broadly classified as sudden transitions and gradual transitions. Sudden transitions are those in which the change of cross-sectional dimensions occur in a relatively short length. On the other hand in the case of a gradual transition the change of cross-sectional area takes place gradually in a relatively long length of the channel. Some of the functions which channel transitions are made to serve are metering of flow, dissipation of energy, reduction or increase of velocities, change in channel section or alignment with a minimum of energy dissipation etc.

(a) Transition with Reduction in Width of a Rectangular Channel

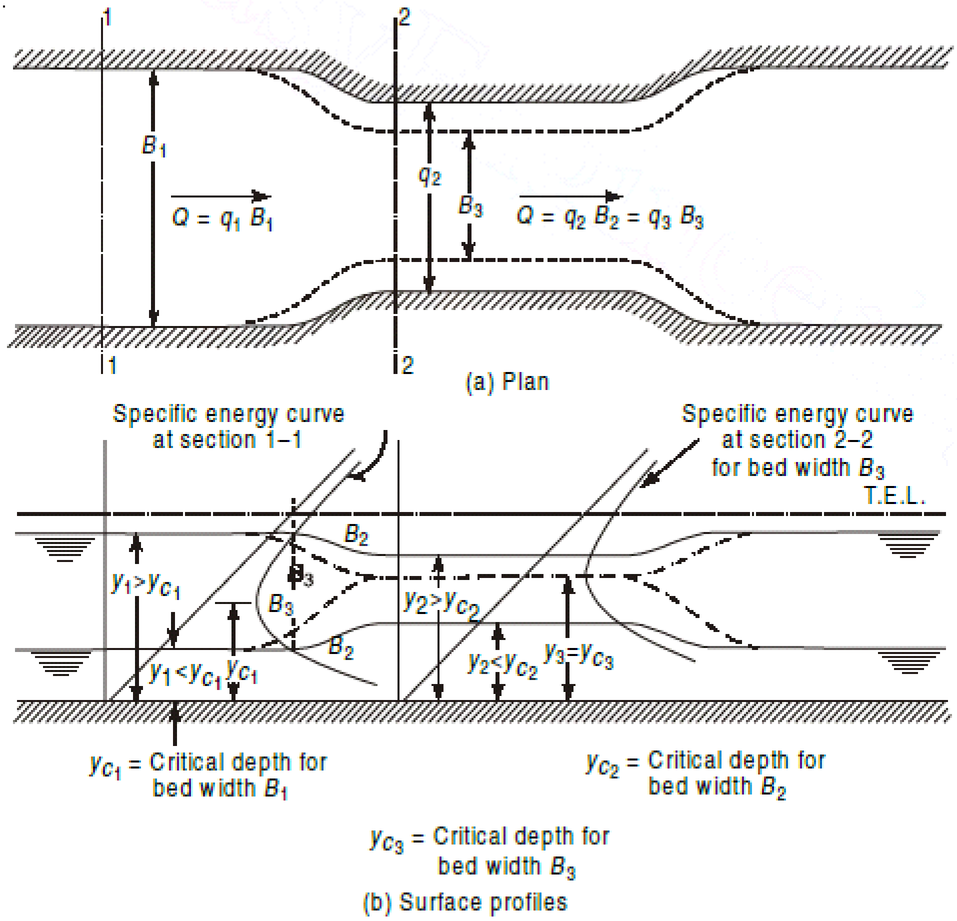


Figure 15.13 *Flow in rectangular channel with reduction in width*

For a given discharge flowing in a rectangular channel, there is a certain limiting value of the contracted width (say B3) which when provided for the throat, then there will be critical depth of flow yc (corresponding to the given discharge and the contracted section of the channel) developed at this section. If for the same total discharge flowing through the channel, the contraction at the throat is less than the above indicated value, then for subcritical flow entering the throat the depth of flow in the throat is greater than critical depth, and for a supercritical flow entering the throat, the depth of flow in the throat is less than critical depth. On the contrary for the same total discharge if the contraction at the throat is more than the above noted limiting value, then for a subcritical flow entering the throat there is heading up of water on the upstream side, and for a supercritical flow entering the throat the water surface on the upstream side is lowered; and in both these cases the depth of flow at the contracted section will be equal to the critical depth corresponding to the given discharge in the contracted section of the channel

(b) Transition with Raised Bottom in a Rectangular Channel

In this case since the width of the channel is not changing, the
discharge per unit width will be same at different sections for a total discharge Q flowing through the channel. Since specific energy is measured with respect to channel bottom as the datum, a rise in the bottom of the channel causes a decrease in specific energy.

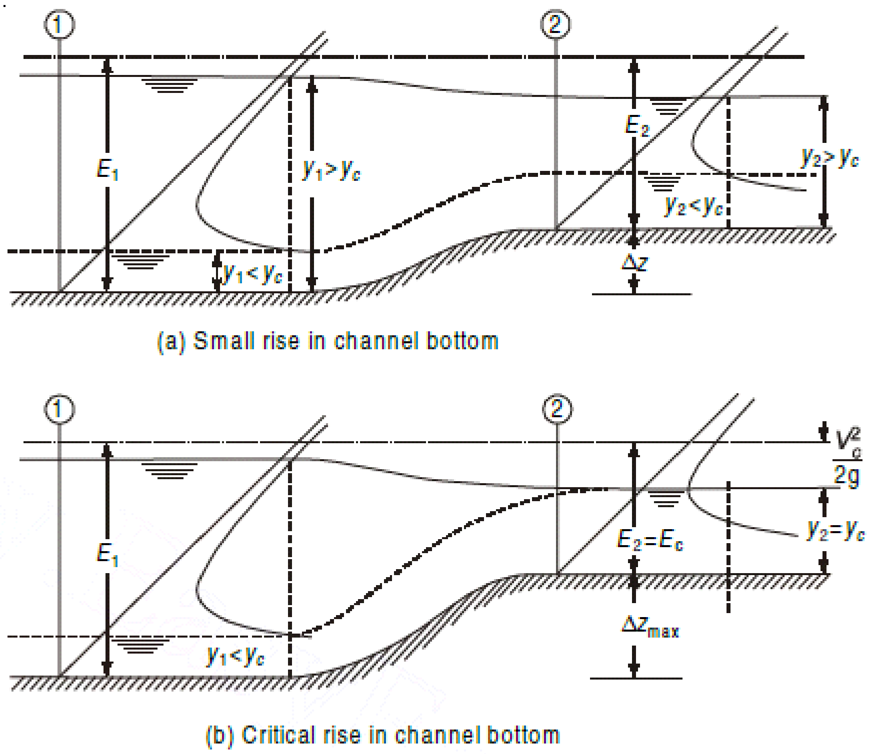


Figure 15.14 *Flow in a rectangular channel with a hump in the bottom*

Now if the flow approaching the hump is in subcritical state, the free surface drops down at the hump. This drop in the free surface at the hump can be explained with the help of specific energy diagram shown in Fig. 15.14. It can be seen from this diagram that as long as the flow is in subcritical state a decrease in specific energy is associated with a decrease in the depth of flow and increase in the velocity. Thus as shown in Fig. 15.14 (a) over the hump the depth of flow decreases and the velocity head increase. However, if the flow approaching the hump is in supercritical state, the depth of flow at the hump is increased. This increase in the depth of flow at the hump can also be explained with the help of specific energy diagram, from which it is indicated that as long as the flow is in supercritical state a decrease in specific energy is associated with an increase in the depth of flow and decrease in the velocity. It may be observed from the specific energy diagram that there is a limit upto which the specific energy for a given discharge can be reduced by increasing the height of the hump to a certain value ΔZ. Thus for given condition of flow at section 1 there is limiting or maximum value of ΔZ, as shown in Fig. 15.14 (b), at which the specific energy at section 2 is equal to the minimum specific energy for the given discharge. In other words, the height of hump can be increased to a certain maximum value without altering the condition of the approaching flow upstream of the hump.

Metering Flume

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that critical depth of flow may be obtained at certain sections in an
open channel where the channel bottom is raised by the construction of a low hump or the channel is constricted by reducing its width. Since at critical state of flow the relationship between the depth of flow and discharge is definite and is independent of the channel roughness and other uncontrollable factors, it provides a theoretical basis for the measurement of discharge in open channels. As such various devices which have been developed for flow measurement are based on the principle of critical flow. Some of the devices commonly used for the measurement of flow in open channels

Venturi Flume

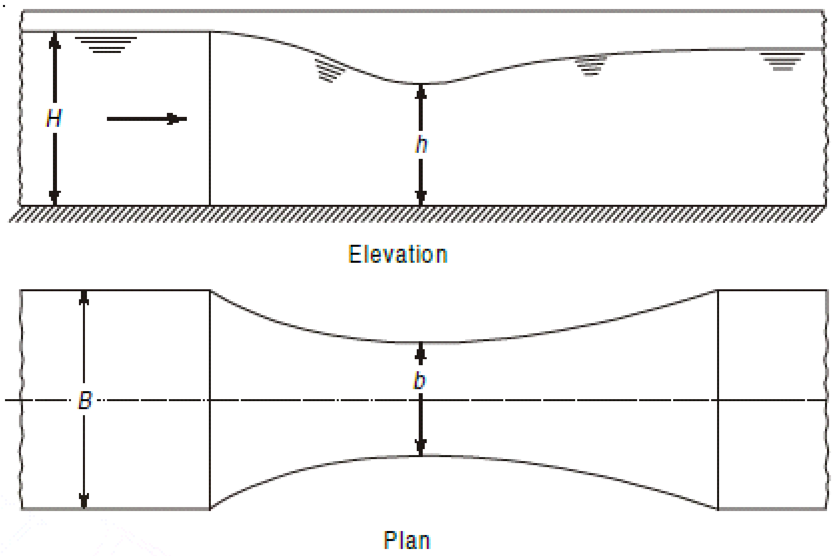
17 July 2023 10:17

a venturi flume is a structure in a channel which has a contracted section called throat, downstream of which follows a flared transition section designed to restore the stream to its original width. It is an open channel counterpart of a venturi meter, which is used for measuring discharge in open channels. At the throat section there will be a drop in the water surface and this drop in water surface may be related to the discharge. The velocity of flow at the throat is always less than critical velocity and hence the discharge passing through it will be a function of the difference between the depths of flow upstream of the entrance section and at the throat. Further since the velocity of flow at the throat is less than critical velocity, standing wave or hydraulic jump will not be formed at any section in the venturi flume.
The discharge Q flowing through the channel can be calculated by measuring the depths of flow at the entrance and the throat of the flume and applying the following formula which may be readily derived

$$Q = k\frac{Aa\sqrt{2g}}{\sqrt{A^2 - a^2}}\left(\sqrt{H-h}\right)$$

$$Q = k\frac{BH \times bh \times \sqrt{2g}}{\sqrt{(BH)^2 - (bh)^2}}\left(\sqrt{H-h}\right)$$

(RECTANGULAR CHANNEL)



Parshall Flume

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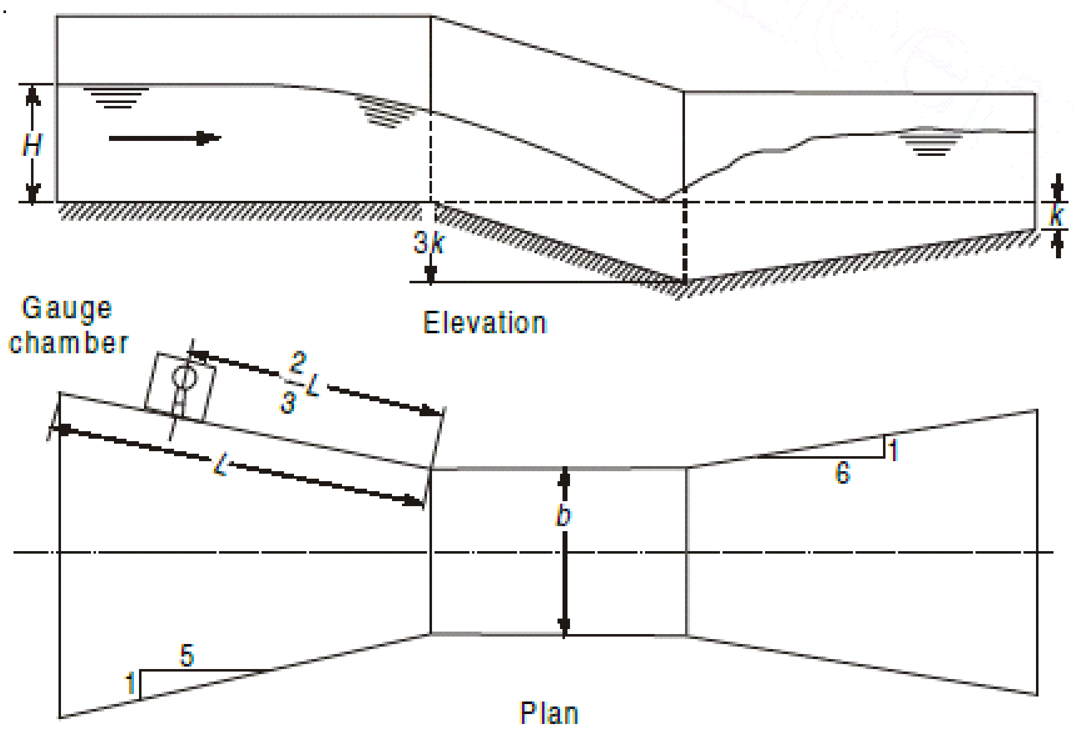
The Parshall flume is one of the most widely used standing wave flume in U.S.A., which was developed by R.L. Parshall in 1920. The converging section of the flume has a level floor, the throat section has a downward sloping floor, while the floor in the diverging section slopes upwards.



The flume has no raised floor as in the case of an ordinary standing wave flume, but the upward sloping floor of the diverging section facilitates the formation of a standing wave in this portion of the flume. As such for the flumes of intermediate size having downstream depth of flow or submergence less than 0.7 times the upstream depth of flow, free flow will occur and the discharge flowing through the flume is a function of only the upstream depth. The depth–discharge relationship for such flumes may be expressed as

$$Q = CbH^n$$

Q is the free discharge, b is the width of the throat, H is the upstream depth of flow measured at gauge chamber and C and n are the constants of the flume which are obtained by calibrations. When the submergence exceeds the above mentioned limit the discharge gets reduced and now if the equation for the free flow condition is used to compute the discharge, the obtained value of Q will have to be corrected by applying a suitable correction factor. Standard design for wood and concrete flumes of various capacities have been worked out for field use and the same are readily available.

Tutorial 2

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Specific energy and critical flow

1. A trapezoidal channel has a bottom width of 6 m and side slopes of 2 horizontal: 1 vertical. If the depth of flow is 1.2 m at a discharge of 10 m3/s, compute the specific energy and the critical depth.
2. A trapezoidal channel with a bed width of 4.0 m and side slopes of 1.5 H: 1V carries a certain discharge. (i) Based on observations, if the critical depth of flow is estimated as 1.70 m, calculate the discharge in the channel. (ii) Identify the state of flow when the flow depth is 2.50 m. (Ans: Q= 38.579 m3/sec, Fr= 0.490, Super critical)
3. The discharge of water through a rectangular channel of width 8 m is 15 m3/s, when the depth of flow of water is 1.2 m. Calculate (i) specific energy of the flowing water (ii) critical depth and critical velocity (iii) value of minimum specific energy
4. A 2.5 m wide rectangular channel has a specific energy of 1.50 m when carrying a discharge of 6.48 m3/s. calculate the alternate depths and the corresponding Froude's number (Ans: 1.296 m, 0.625 m, Fr = 0.561 , 1.675)
5. Calculate the bottom width of a channel required to carry a discharge of 15 m3/sec as a critical flow at a depth of 1.2 m, if the channel section is (a) rectangular (b) trapezoidal with side slope 1.5H: 1V (Ans: 3.643m , 2.535 m)