

DIJKSTRA'S ALGORITHM

Steps: Find shortest path from S to T

- 1) Given a connected, weighted graph G with n vertices. If G is not simple make it simple by removing self loops and parallel edge by keeping minimum weighted edge among parallel edges.
- 2) We start with origin (starting vertex) by assigning "0" as permanent label, remaining $(n-1)$ vertices are labelled as ∞ and treat as temporary labels.
- 3)
 - 3a) In each iteration another vertex gets a permanent label, according to the following rules.
 - Every vertex v_j that is not yet permanently labelled get a new temporary label by the formula

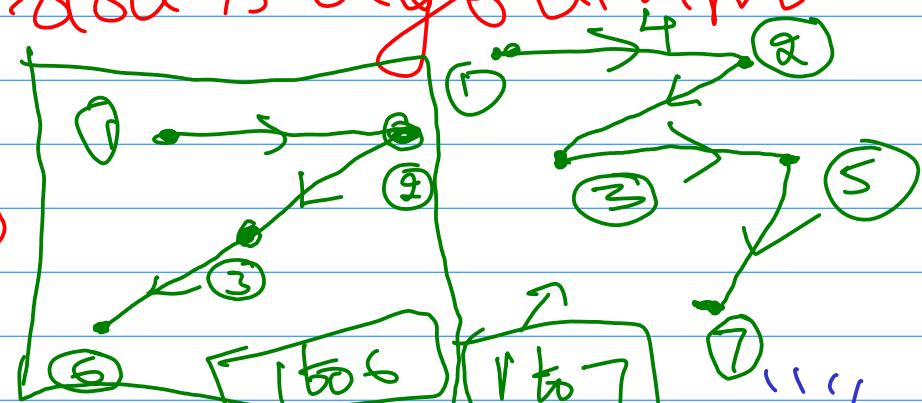
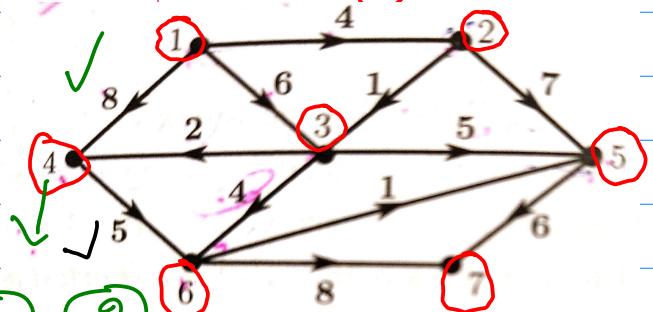
$$\text{Label}(v_j) = \min(\text{Permanent Label}, \text{Temporary label}(v_j) + \text{wt}(v_j \text{ to } x))$$

Here $\text{wt}(v_j \text{ to } x)$ means weight of the edge between v_j to x vertex
 - 3b) If the vertex v_i and v_j are not adjacent by any edge then take the weight as ∞ .
 - 3c) The smallest value among all the temporary label is found and this becomes the permanent label of the corresponding vertex.
If two or more smallest value, choose any one of them.
 - 3d) We repeat step 3 till destination vertex gets permanent label.
 - 3e) This algorithm gives shortest distance but not path. To find path we implement backward procedure from terminal vertex such where each vertex gets

permanent label.

11) Problem

Find shortest path from 'i' to all other
using Dijkstra's algorithm.



i) Start with vertex 1, assigning 0 as
Perman Label.

1	2	3	4	5	6	7
0	∞	∞	∞	∞	∞	∞

$$\text{Label}(2) = \min\{\infty, 0 + 4\} \\ = 4$$

edge(1 to 2)

$$\text{Label}(3) = \min\{\infty, 0 + 6\} = 6$$

$$\text{Label}(4) = \min\{\infty, 0 + 8\} = 8$$

$$\text{Label}(5) = \min\{\infty, 0 + \infty\} = \infty$$

$$\text{Label}(6) = \min\{\infty, 0 + \infty\} = \infty$$

1	2	3	4	5	6	7
0	4	6	8	∞	∞	∞

now 2 is having Permanent Label.

$$\text{Label}(3) = \min\{6, 4+1\} = 5$$

} edge from 2 to vertex

$$\text{Label}(4) = \min\{8, 4+\infty\} = 8$$

$$\text{Label}(5) = \min\{\infty, 4+7\} = 11$$

$$\text{For } 6, \min\{\infty, 4+\infty\} = \infty$$

$$\text{For } 7, \min\{\infty, 4+\infty\} = \infty$$

1	2	3	4	5	6	7
5	4	5	8	11	∞	∞

Now vertex 3 is having per. Label.

$$\text{For } 4, \min\{8, 5+2\} = 7$$

$$\text{For } 5, \min\{11, 7+5\} = 10$$

$$\text{For } 6, \min\{\infty, 5+4\} = 9 \checkmark$$

$$\text{For } 7, \min\{\infty, 5+\infty\} = \infty$$

1	2	3	4	5	6	7
5	4	5	7	10	9	∞

Now 4 is having Per. Label and its value is "7".

$$\text{For } 5, \min\{10, 7+\infty\} = 10$$

$$\text{For } 6, \min\{9, 7+5\} = 9$$

$$\text{For } 7, \min\{\infty, 7+\infty\} = \infty$$

1	2	3	4	5	6	7
5	4	5	7	10	9	∞

Now 5 is having per. Label. and its value is "9".

For 5, $\min\{10, 9+1\} = 10$

For 7, $\min\{10, 9+8\} = 17$

1	2	3	4	5	6	7
10	4	5		10	9	17

Now 5 is having Per. Label.

For 7, $\min\{17, 10+6\} = 16$

1	2	3	4	5	6	7
10	4	5	13	7	10	9

shortest distance from 1

To 2 is 4

To 3 is 5

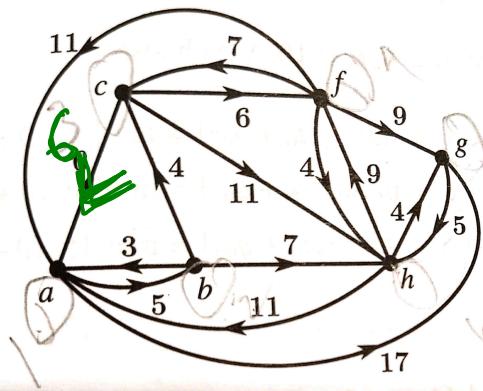
To 4 is 7

To 5 is 10

To 6 is 9

To 7 is 16

2) Find shortest path from c to all other

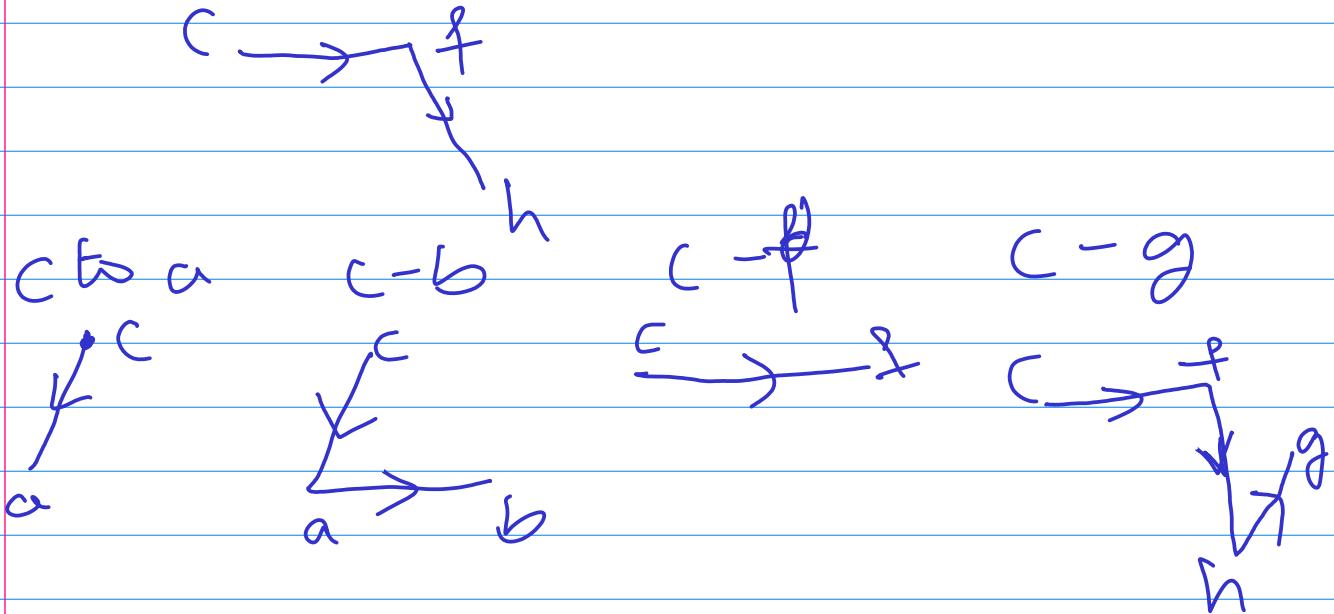


a	b	c	d	e	f	g	h
∞							
6	∞	0	6	∞	11		
6	∞	0	6	15	10		
6	11	0	6	15	10		

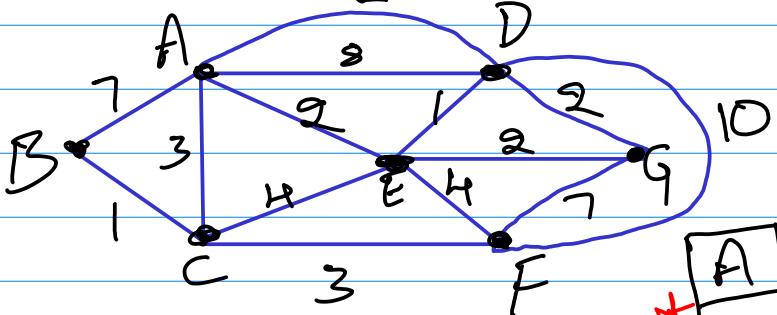
$\min\{\text{Per. Label}, \text{Per. L} + \text{edge}\}$

G	H	I	J	K	L	M	N	O	P	Q	R	S
G	H	I	J	K	L	M	N	O	P	Q	R	S

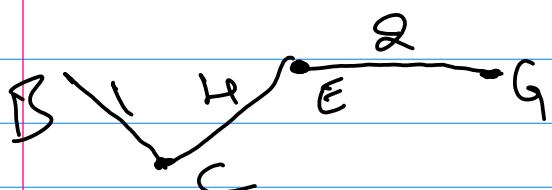
shortest path from c to h



3) Find shortest path from B to G



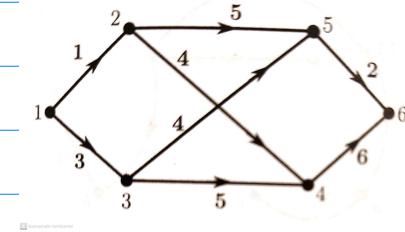
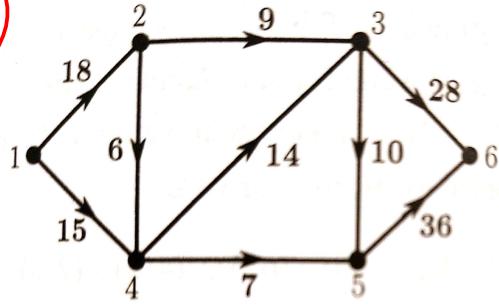
using T.L, P.L + wt(e)
2, 1+4



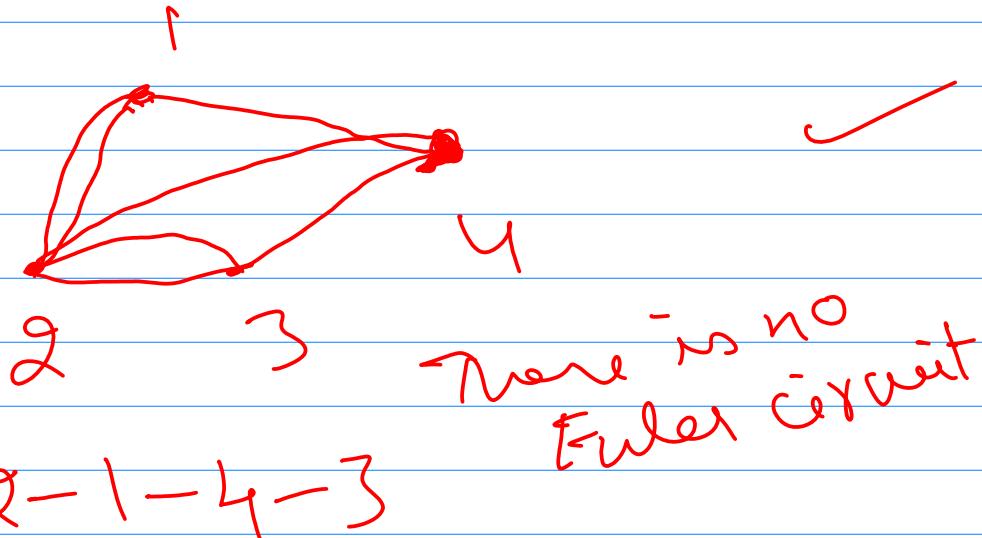
B - C - E - G

This is required shortest path and its length "7"

A	B	C	D	E	F	G
∞	0	2	∞	∞	∞	∞
7	0	1	∞	∞	∞	∞
4	0	1	∞	5	4	∞
4	0	1	14	5	4	11
4	0	1	6	5	4	11
4	0	1	6	5	4	7



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Example 6)
Königsberg bridge



1-2-3-2-1-4-3

