Epidemiology modeling - systems of ODE (ordinary differential equations) - SEIRQDH model

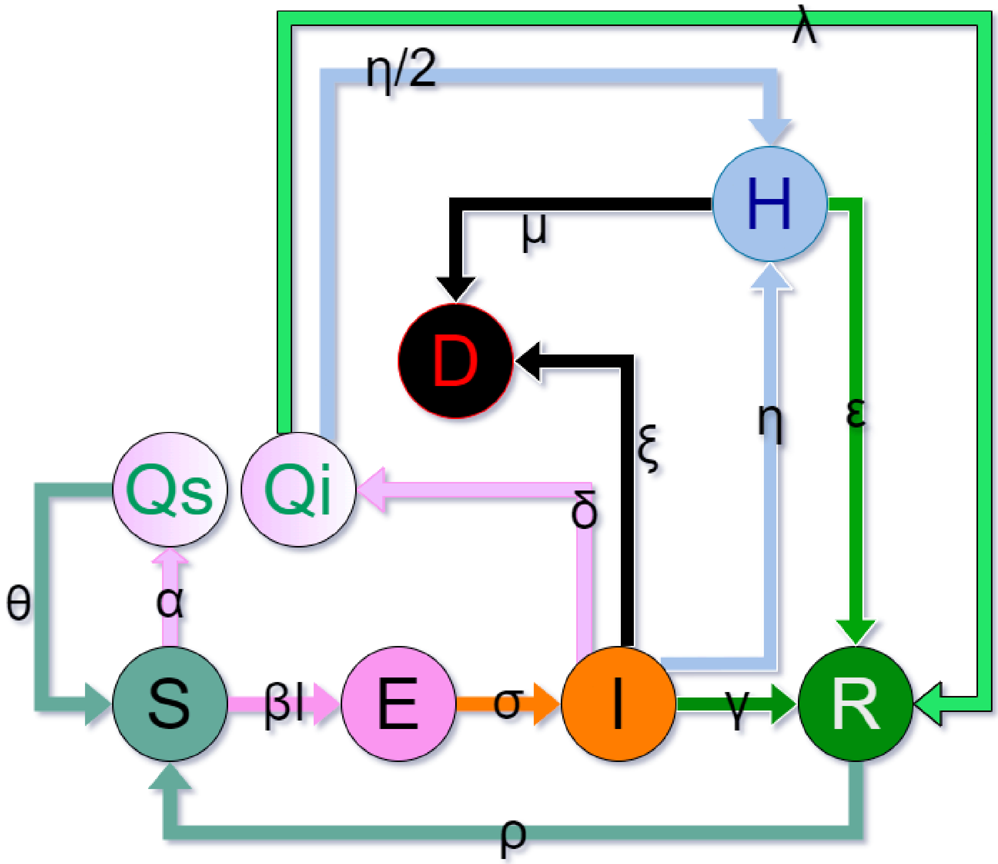
Bartosz Skowronek

Introduction

The aspect I elected to focus my modeling on is social distancing and severity, via introducing compartments: Qi and Qs, (quarantined infectious/susceptible), H(hospitalized) and D(deceased) to a standard SEIR model. The D (deceased) serves mostly an illustrative purpose, because of course a person categorized by that status can no longer change to any other, and the H(hospitalized) compartment allows for more accurate illustration of hospital capacity scenarios. Manipulating the parameters, presented in the next section, enables us to attempt simulating how social distancing can prevent infection spread, or how the severity/mortality of a disease demands a longer period of isolation from a larger proportion of the population to survive. A SEIR+Q+D+H model seems to be a viable representation of the early stages of a pandemic, when the community affected by it attempts various measures to protect themselves, but a vaccine is not yet available.

Model overview

The Qi/Qs compartments take in individuals from compartments S and I that are perceived to be (potentially) infectious and reduce their ability to spread infections. The severity of the modeled disease, described by parameter **η**(eta), influences how often infected patients require to be hospitalized (in other words, how intense is the flow from compartment I (infectious) to H (hospitalized) . In this model, the term “quarantine” describes a state of social distancing that reduces social contact, due to voluntary isolation of a susceptible person (compartment Qs) or medication at home, in quarantine, of an infected person (Qi). The infected in Qi retain a possibility of hospitalization due to severity, but due to the assumption that staying at home and treating the infection in isolation reduces strain on the immune system of an individual, I reduced severity in this case to ½ of the parameter, even if perhaps this may be either an incorrect assumption or proportion. Below is a graph that illustrates the flow between compartments in the presented model, and next are descriptions of compartments, parameters and the equations that define them. Various studies on the COVID-19 pandemic can provide reference data for this model, for example seasonal mortality rate distributions.



Compartments:

S: Susceptible (normal engagement in social activities)

E: Exposed (susceptible individuals that had contact with virus and are developing an infection)

I: Infectious (no special social distancing measures implemented)

R: Recovered (infection healed, immunity developed)

H: Hospitalized (due to severity of infection; quarantined at hospital)

Qi: Quarantined (infection treatment at home; social distancing)

Qs: Quarantined (social distancing of a susceptible individual)

D: Deceased

Parameters (all parameters are numbers within range $[0, 1]$**):**

β(beta): Transmission rate $\frac{(average\ number\ of\ contacts\ per\ individual) \bullet (probability\ of\ infection\ on\ contact)}{total\ population}$

α(alpha): % of susceptible population that goes into quarantine (social distancing)

θ(theta): 1/(average time of social distancing of a quarantined susceptible individual)

σ(sigma): Incubation rate (1/ time for exposed individual to become infectious)

γ(gamma): Recovery rate (1/time of recovery from infection) without any distancing measures

δ(delta): % of infected population that goes into quarantine (medication at home)

η(eta): % of infected individuals that require hospitalization (disease severity)

μ(mu): Mortality rate for hospitalized patients

ξ (xi): Mortality rate for infected patients (death before hospitalization)

ρ(rho): Rate at which recovered individuals lose immunity (1/days to become susceptible)

ε(epsilon): Recovery rate for hospitalized patients (1/time of recovery)

λ(lambda): Recovery rate for infected individual in quarantine

Model Equations:

$$\frac{dS}{dt} = -\beta SI + \rho R - \alpha S + \theta Q_s$$

$$\frac{dE}{dt} = \beta SI - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I - \eta I - \xi I - \delta I$$

$$\frac{dR}{dt} = \gamma I + \epsilon H + \lambda Q_i - \rho R$$

$$\frac{dQ_i}{dt} = \delta I - \lambda Q_i - \frac{\eta Q_i}{2}$$

$$\frac{dQ_s}{dt} = \alpha S - \theta Q_s$$

$$\frac{dH}{dt} = \eta I + \frac{\eta Q_i}{2} - \mu H - \epsilon H$$

$$\frac{dD}{dt} = \xi I + \mu H$$

Key Model Features:

Social distancing is represented by variations in the β parameter. Lower β values indicate stronger social distancing measures.

Disease severity is captured by the η parameter, representing the rate at which infectious individuals require hospitalization.

The quarantine compartment (Q) reduces the number of infectious individuals actively spreading the disease.

The hospitalized compartment (H) allows for tracking of severe cases and healthcare system load.

The deceased compartment (D) provides a way to track mortality over time.