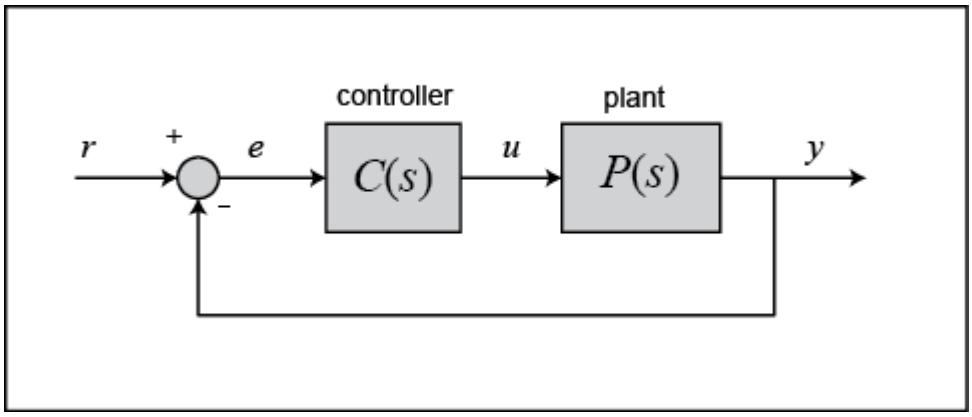
Phase-lead compensator to explain phase-leading behavior in *Drosophila* head movements

```
clear; close all; clc
syms s I c k K_p K_d K_i a_2 tau K
assume([I c k s]>=0)
assume([I c k],'real')
freqRange = logspace(-1,2,10000);
```

Contol Framework



Plant

Natural Frequency (rad/s)

```
wn = 50;
```

Damping Ratio

```
z = 1.2;
```

Head inertia about rotation point (kg*m^2)

```
I = 1;
```

Head stiffness (N*m/rad)

```
k = wn^(2)*I;
```

Head critical damping (N*m*s/rad)

```
cc = 2*sqrt(k*I);
```

Head damping (N*m*s/rad)

```
c = z*cc;
P(s) = 1/(I*s^(2) + c*s + k)
```

```
P(s) =
```

$$\frac{1}{s^2 + 120\,s + 2500}$$

Nominal Controller

```
C1(s) = sym(1)
```

```
C1(s) = 1
```

Closed-Loop Model

$$G(s) = \frac{Y(s)}{R(s)} = \frac{P(s)C(s)}{1 + P(s)C(s)}$$

```
G(s) = 1000*collect(simplify(P(s)*C1(s)/(1 + P(s)*C1(s))),s)
```

```
G(s) =
```

$$\frac{1000}{s^2 + 120\,s + 2501}$$

```
[MAG{1},PHS{1},FREQ{1}] = bode(sym2tf(G(s)),freqRange);
```

Phase-Lead Compensator Controller Design

The controller is of the form

$$C(s) = \frac{a_2 \tau s + 1}{\tau s + 1}$$

```
C2(s) = (a_2*tau*s + 1)/(tau*s + 1)
```

```
C2(s) =
```

$$\frac{a_2\,s\,\tau + 1}{s\,\tau + 1}$$

The phase addition and frequency peak are defined as

$$\phi_{\max} = \sin^{-1}\left(\frac{a_2 - 1}{a_2 + 1}\right) \quad \& \quad \omega_{\max} = \frac{1}{\tau\sqrt{a_2}}$$

```
wm = 0.1;
phi = deg2rad(20);
a2_num = double(solve(phi == asin((a_2-1)/(a_2+1))))
```

```
a2_num = 2.0396
```

```
tau_num = 1/(wm*sqrt(a2_num))
```

```
tau_num = 7.0021
```

The compensator controller becomes

```
C2(s) = simplify(expand(subs(C2(s),[a_2 tau],[a2_num tau_num])));
```

And the combined controller

```
C(s) = C1(s)*C2(s);
```

The new closed-loop model

```
G(s) = 1000*simplify(expand(P(s)*C(s)/(1 + P(s)*C(s))));

[MAG{2},PHS{2},FREQ{2}] = bode(sym2tf(C(s)),freqRange);
[MAG{3},PHS{3},FREQ{3}] = bode(sym2tf(G(s)),freqRange);

figure (1) ; clf
for kk = 1:length(MAG)
    ax(1) = subplot(2,1,1);
    h(kk) = semilogx(FREQ{kk},squeeze(MAG{kk}),'LineWidth',2);
    hold on
    ylabel('Magnitude')

    ax(2) = subplot(2,1,2);
    h(kk) = semilogx(FREQ{kk},squeeze(PHS{kk}),'LineWidth',2);
    hold on
    ylabel('PHS')
    xlabel('Frequency (Hz)')
end
set(ax,'XLim',10.^[-1 2],'XGrid','on')
leg = legend([h(2) h(1) h(3)],'Compensator','Closed-loop Model: no compensator','Closed-loop Mo
set(leg,'Location','southwest')
```