

Statement

↳ Given an integer array, `nums`, find and return all unique triplets $[nums[i], nums[j], nums[k]]$, such that $i \neq j$, $i \neq k$, and $j \neq k$ and $nums[i] + nums[j] + nums[k] == 0$.

Approach

↳ Preprocessing:

- Sort the array `nums` in ascending order
- Initialize an empty array `result` to store unique triplets
- Store the length of `nums` in `n`

↳ Iterate over array from index $i = 0$ to $n-2$

- Break the loop if $nums[i] > 0$. Since array is sorted, no triplet can sum to 0 after that.
- Continue only if $i == 0$ or $nums[i] != nums[i-1]$ to ensure the current number is either the first element or not a duplicate of the previous element.

- Initialize two pointers: $low = i+1$ and $high = n-1$

- Loop as long as $low < high$

↳ Calculate the $sum = nums[i] + nums[low] + nums[high]$

↳ If sum is less than 0, move `low` pointer forward

↳ If sum is greater than 0, move `high` pointer backward

↳ If sum is equal to 0, add $[nums[i], nums[low], nums[high]]$ to the `result`

↳ Skip duplicates when moving `low` and `high`

- Increment `low` while $nums[low] == nums[low-1]$

- Decrement `high` while $nums[high] == nums[high+1]$

↳ Return `result` after iterating through the whole array.

Visualization

↳ Find all triplets that sum to 0

i) Given array

-4	-1	2	2	-1	0
----	----	---	---	----	---

ii) Sort the array

-4	-1	-1	0	2	2
----	----	----	---	---	---

iii) Create empty array to store unique triplets

-4	-1	-1	0	2	2
----	----	----	---	---	---

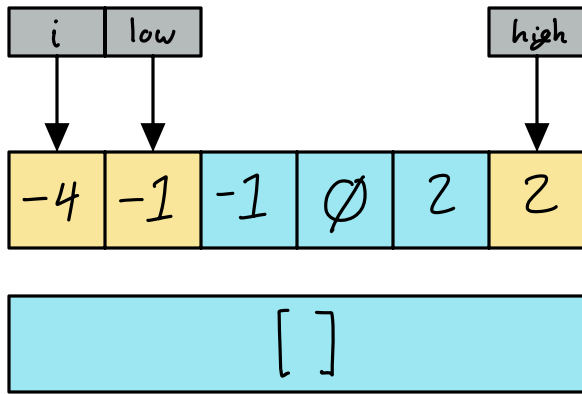
[]

iv) Start iterating through the *nums* array using *i* pointer. Then initialize *low* with the element next to *i* and *high* with the last element of *nums*.

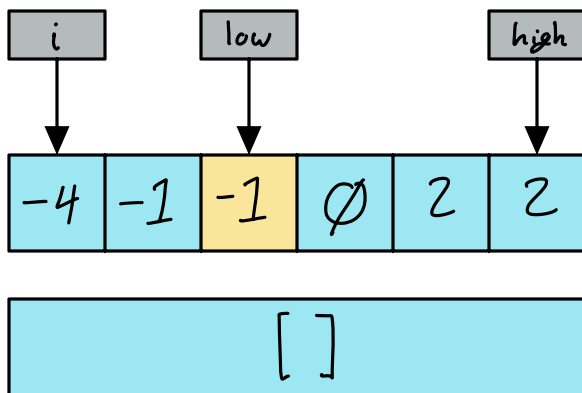
i	low				high
↓	↓				↓
-4	-1	-1	0	2	2

[]

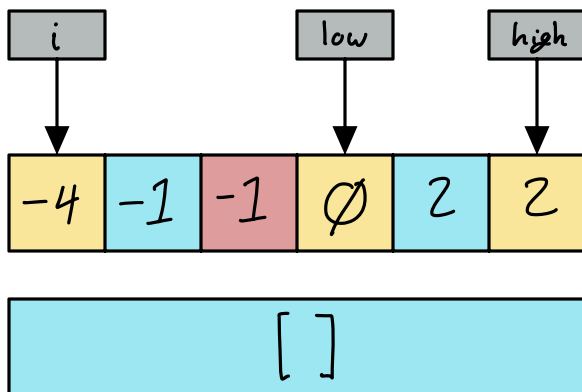
v) As $\text{nums}[i] + \text{nums}[\text{low}] + \text{nums}[\text{high}] = -4 + (-1) + 2 = -3$, which is less than \emptyset , we increment low .



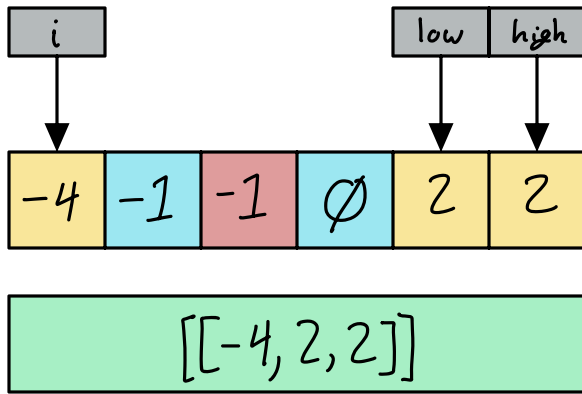
vi) As $\text{nums}[\text{low}]$ is equal to $\text{nums}[\text{low}-1]$, we skip this to avoid duplication and increment low again.



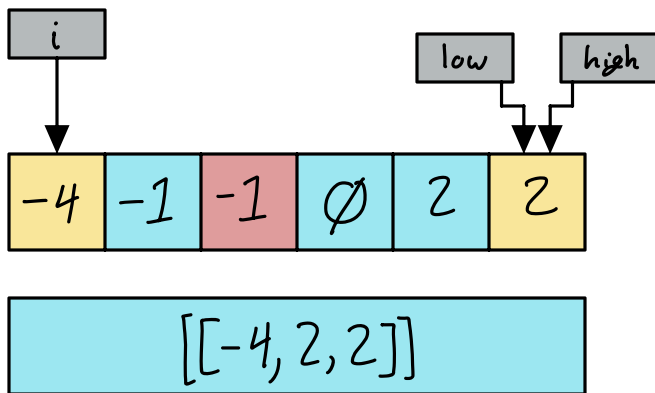
vii) As $\text{nums}[i] + \text{nums}[\text{low}] + \text{nums}[\text{high}] = -4 + \emptyset + 2 = -2$, which is less than \emptyset , increment low .



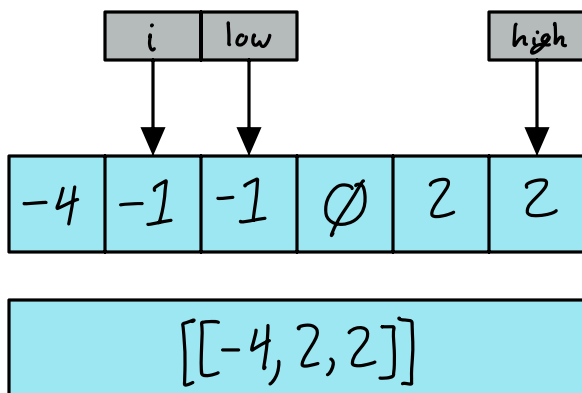
viii) As $nums[i] + nums[low] + nums[high] = -4 + 2 + 2 = 0$, which is equal to 0, add the triple to result and increment low .



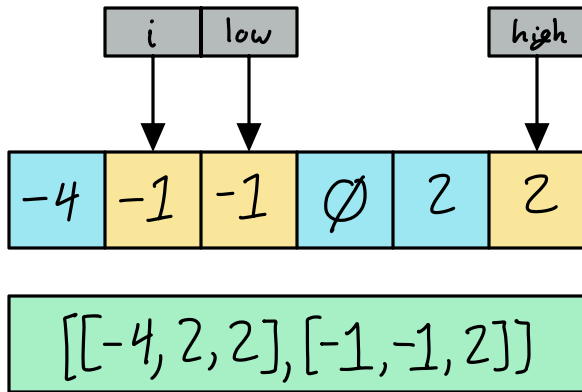
ix) Stop the two pointers iteration because low is not less than $high$ anymore.



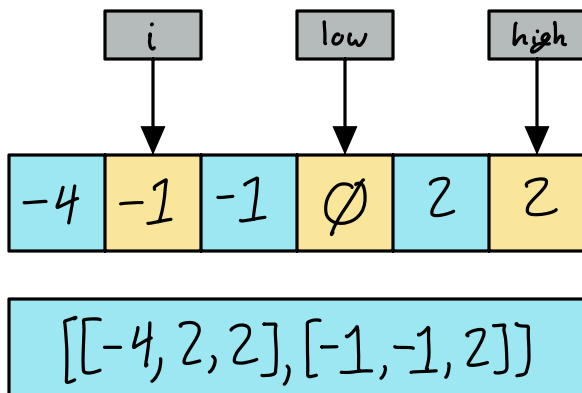
x) Move i to the next index. Initialize low with the element next to the i and $high$ with the last element of $nums$.



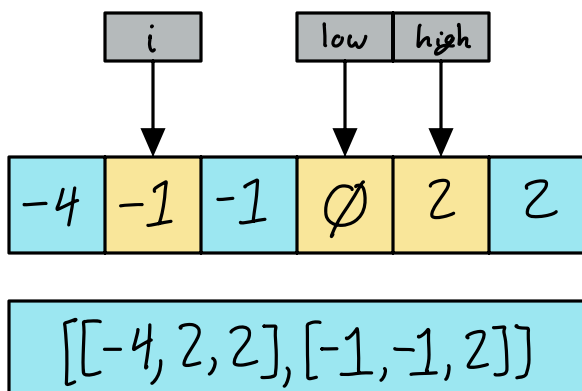
xi) As $\text{nums}[i] + \text{nums}[\text{low}] + \text{nums}[\text{high}] = -1 + (-1) + 2 = \emptyset$, add it to **result** and increment **low**.



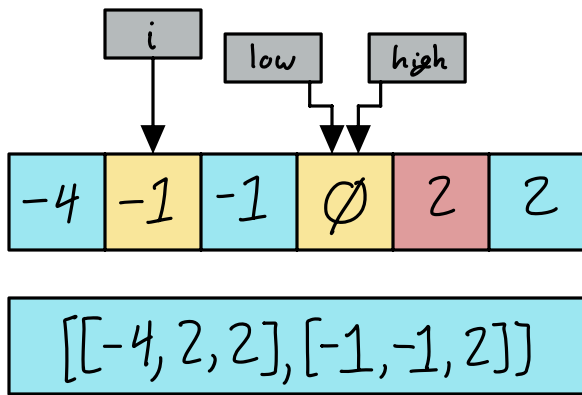
xii) As $\text{nums}[i] + \text{nums}[\text{low}] + \text{nums}[\text{high}] = -1 + \emptyset + 2 = 1$, which is greater than \emptyset , decrement **high**.



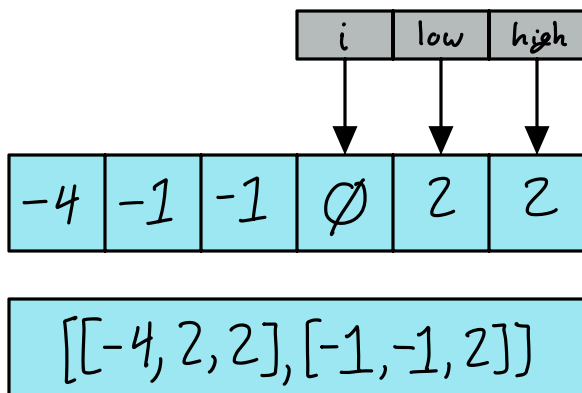
xiii) As $\text{nums}[\text{high}]$ equals $\text{nums}[\text{high}+1]$, decrement **high** to skip this value in order to avoid duplication.



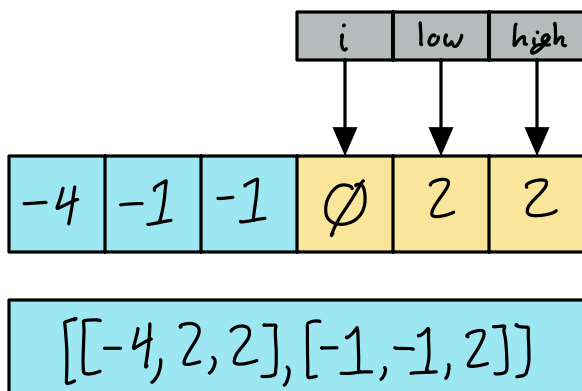
xiv) Stop iterating the two pointers because $low \geq high$.



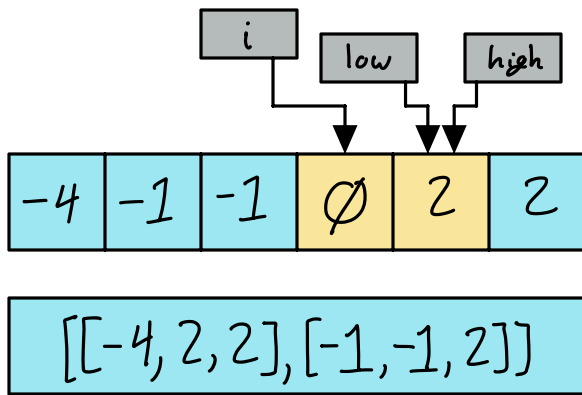
xv) Next, move `i` forward. As `nums[2]` equals `nums[1]`, skip `nums[2]` and move to `nums[3]` to avoid duplicates. Then, initialize `low` with the element next to `i` and `high` with the last element of `nums`.



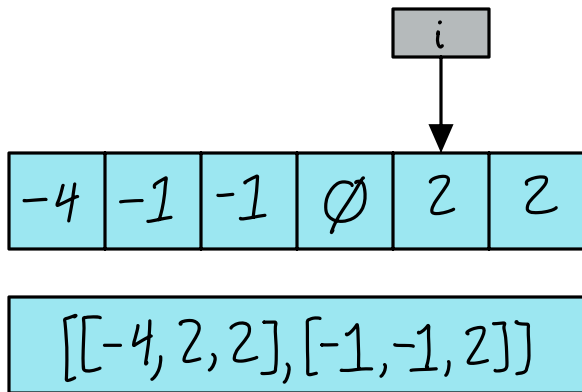
xvi) As $nums[i] + nums[low] + nums[high] = \emptyset + 2 + 2 = 4$, is greater than \emptyset , decrement `high`.



xvii) Stop iterating the two pointers because $low \geq high$. Then, increment i .



xviii) At this point i has reached the second-to-last element of the array. As a valid triplet requires three distinct elements, no further triplets can be formed with only two remaining. Therefore, stop iterating through the array and return the **result** array, which contains all the unique triplets that sum to zero.



Code

```
vector<vector<int>>> threeSum(vector<int>& nums) {
    sort(nums.begin(), nums.end());

    vector<vector<int>>> result;

    int n = nums.size();

    for (int i = 0; i < n-2; i++) {
        if (nums[i] > 0) {
            break;
        }

        if (i == 0 || nums[i] != nums[i-1]) {
            int low = i+1, high = n-1;

            if (sum < 0) {
                low++;
            } else if (sum > 0) {
                high--;
            } else {
                result.push_back({nums[i], nums[low], nums[high]});

                low++;
                high--;

                while (low < high && nums[low] == nums[low-1]) {
                    low++;
                }
                while (low < high && nums[high] == nums[high+1]) {
                    high--;
                }
            }
        }
    }

    return result;
}
```


Time Complexity

↳ Let n represent the length of the `nums` array.

- Sorting takes $O(n \log n)$

- The nested iteration takes $O(n^2)$, where each `nums[i]` is paired with a two pointer traversal over the remaining elements of the array.

↳ Therefore, the overall time complexity is $O(n^2)$.

Space Complexity

↳ Apart from the space used by the built-in sorting algorithm, the algorithm's space complexity is constant $O(1)$.