NYU DATA SCIENCE

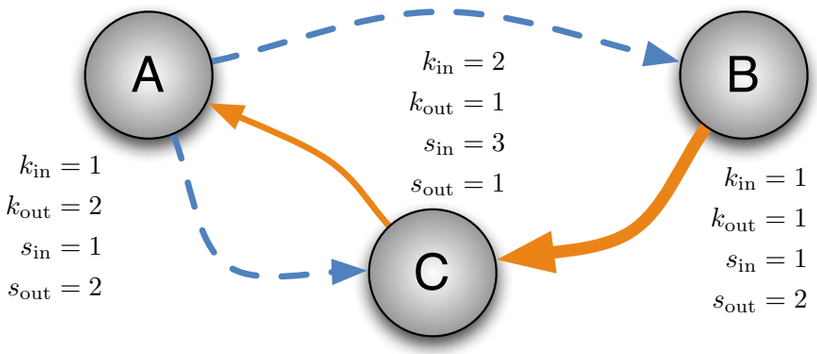
Network Science

Bruno Gonçalves

www.bgoncalves.com

https://github.com/bmtgoncalves/Binghamton

Networks


A
B
C
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$k_{\text{out}} = 2$
$s_{\text{in}} = 1$
$s_{\text{out}} = 2$
$k_{\text{in}} = 2$
$k_{\text{out}} = 1$
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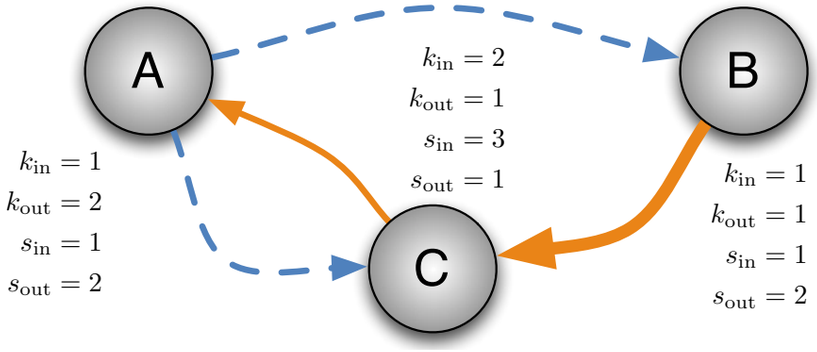
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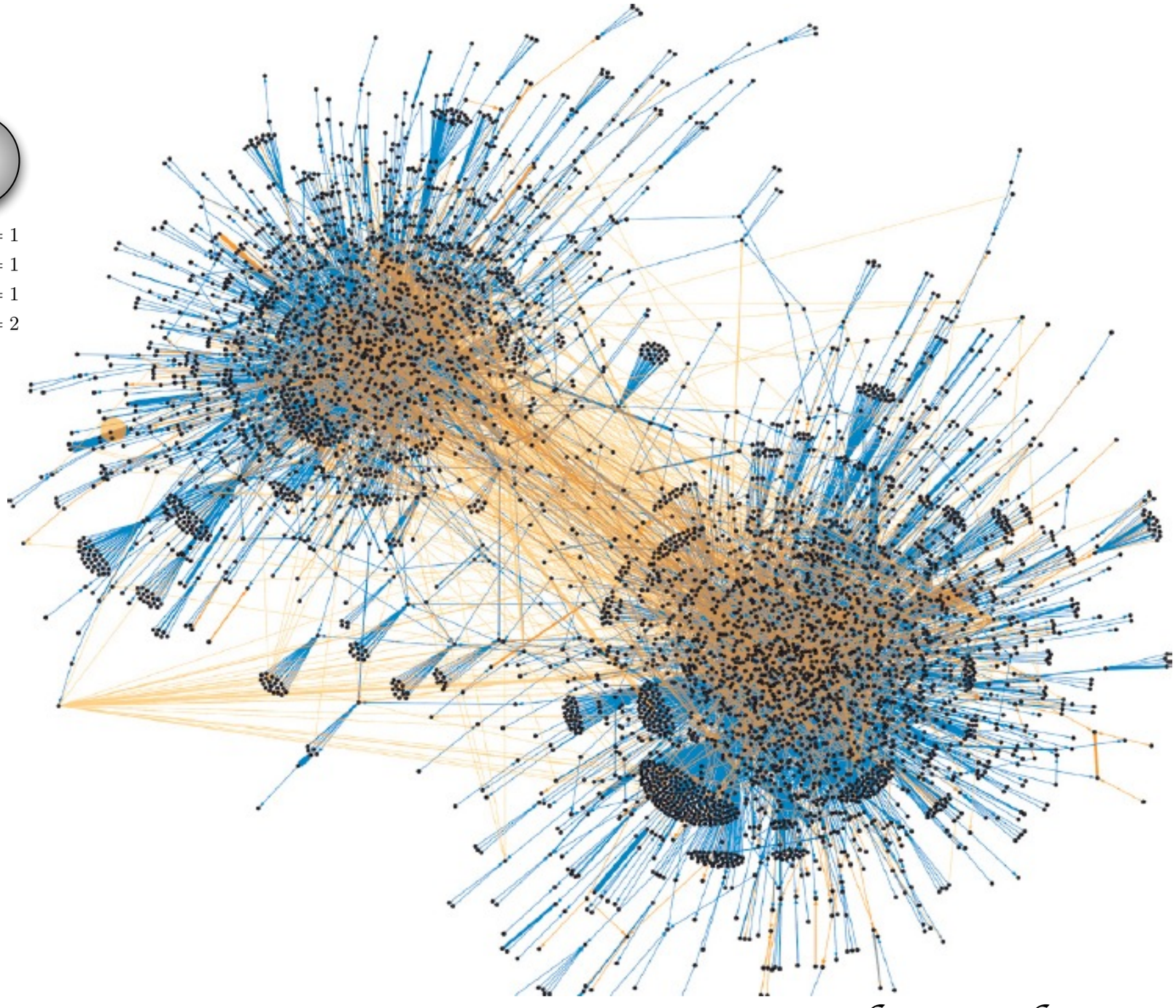
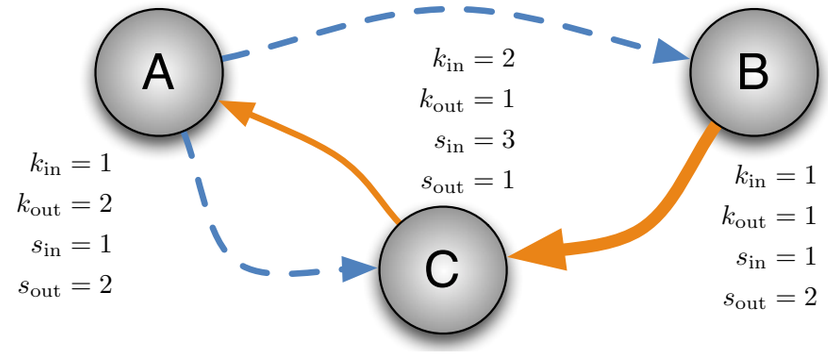
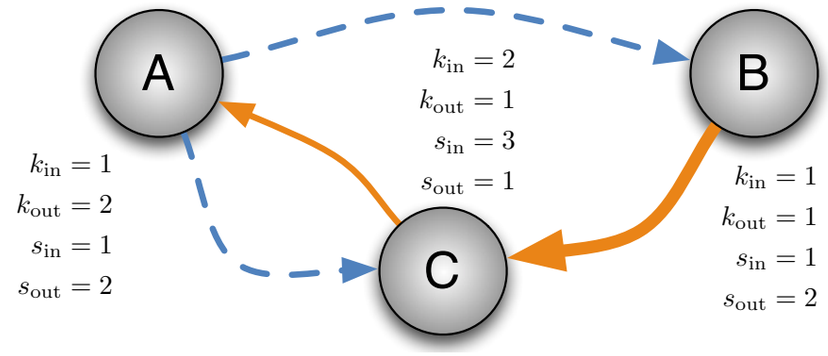


Networks - Basic Definitions



Networks - Basic Definitions

- Mathematical object, with set of nodes and edges



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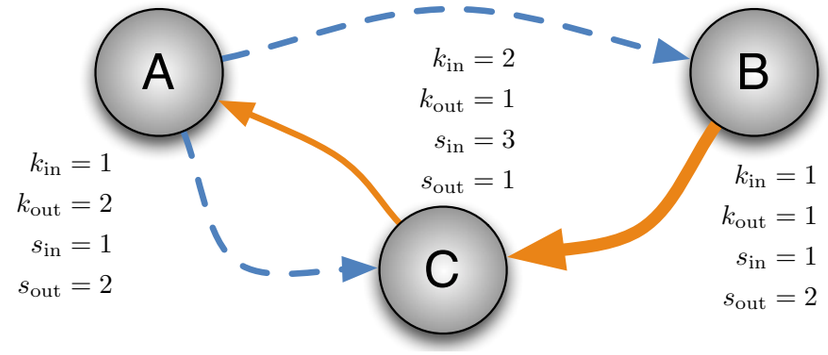

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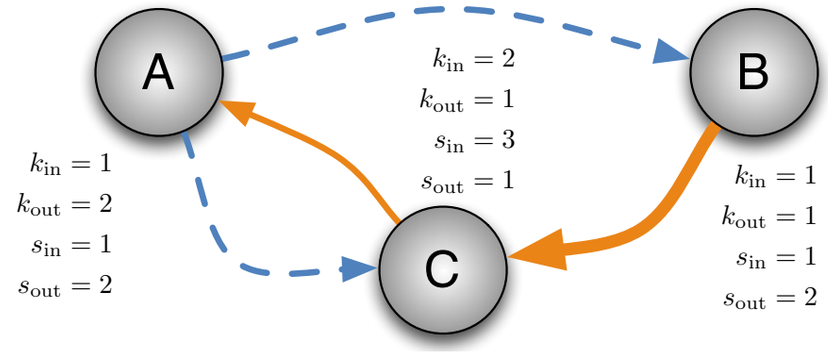

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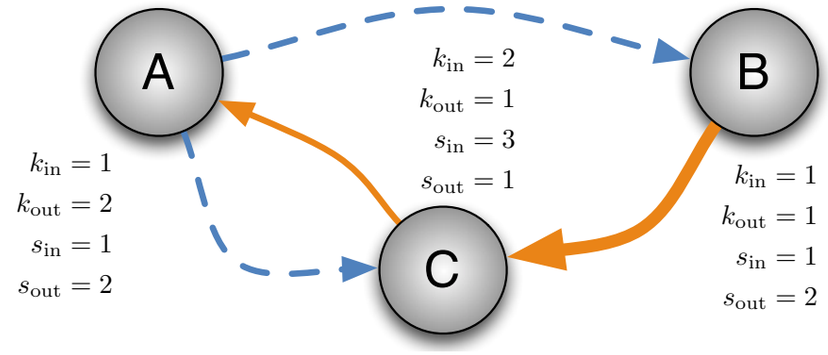

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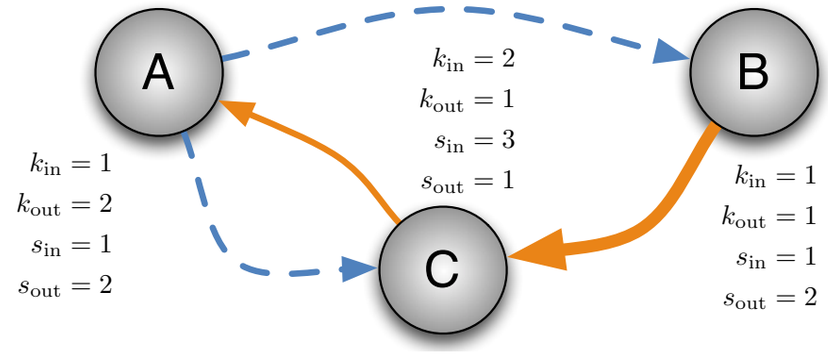

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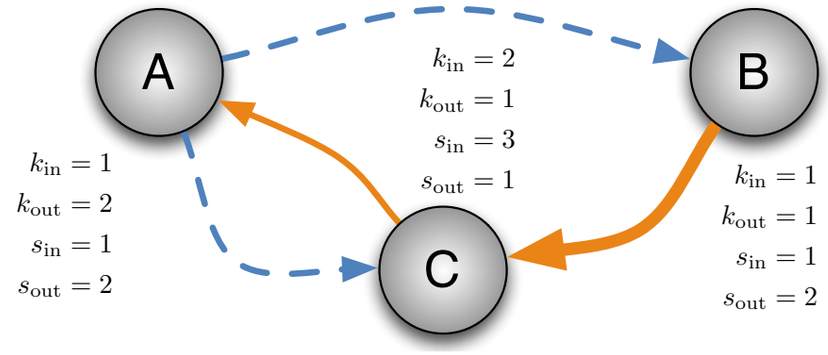

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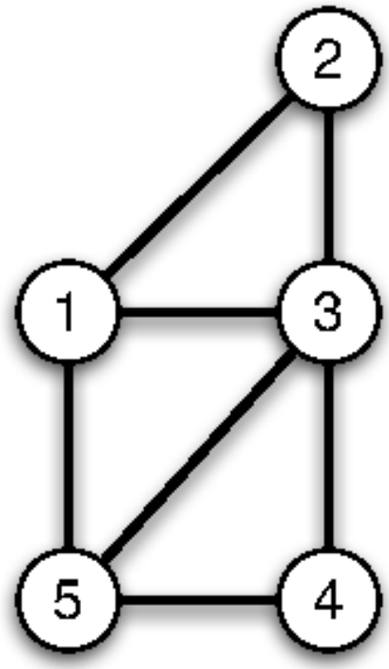
- Weighted Edge - Edge with a weight associated
- Direct Edge - "One way street"

Adjacency Matrix



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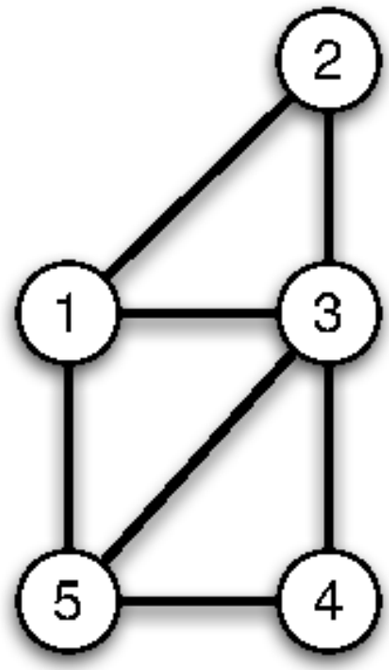

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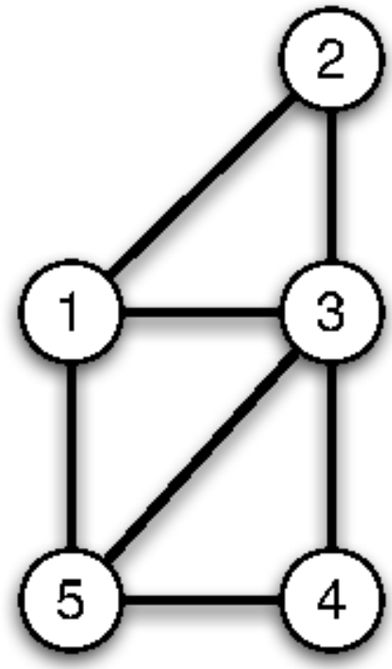

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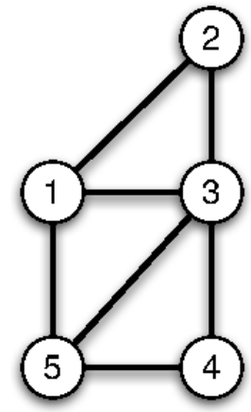

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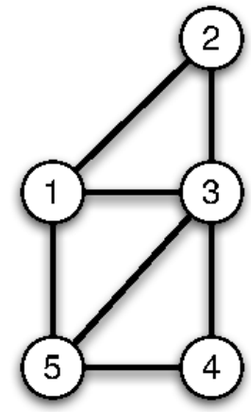

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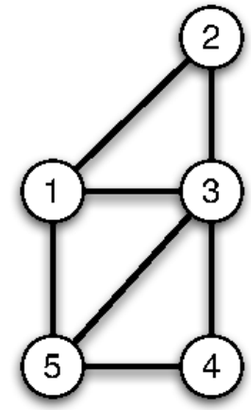

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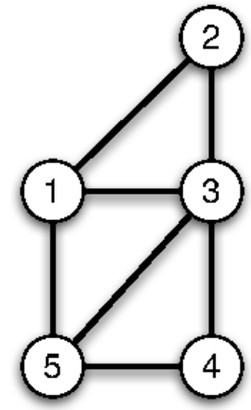

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- "The probability that a randomly chosen node has degree exactly k"

Paths and Cycles

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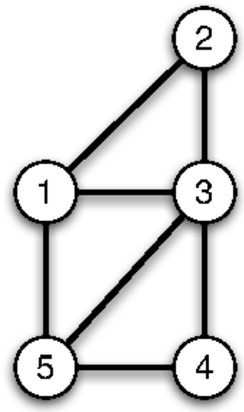

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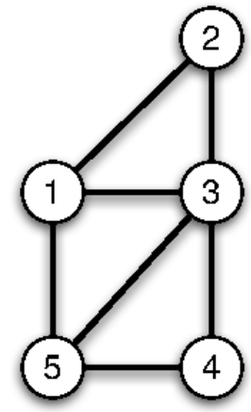

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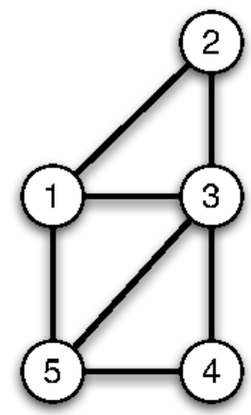

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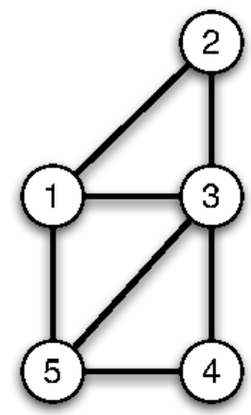

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$$A^2 = \begin{bmatrix} 3 & 1 & 2 & 2 & 1 \\ 1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 4 & 1 & 2 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 & 3 \end{bmatrix}$$

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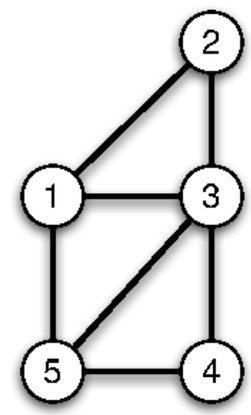

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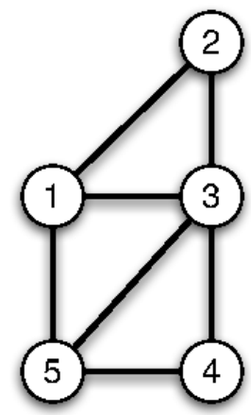

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- The diagonal elements of P indicate cycles, paths that start and end in the same node

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$$\langle C \rangle = \frac{1}{N}\sum_i C_i$$

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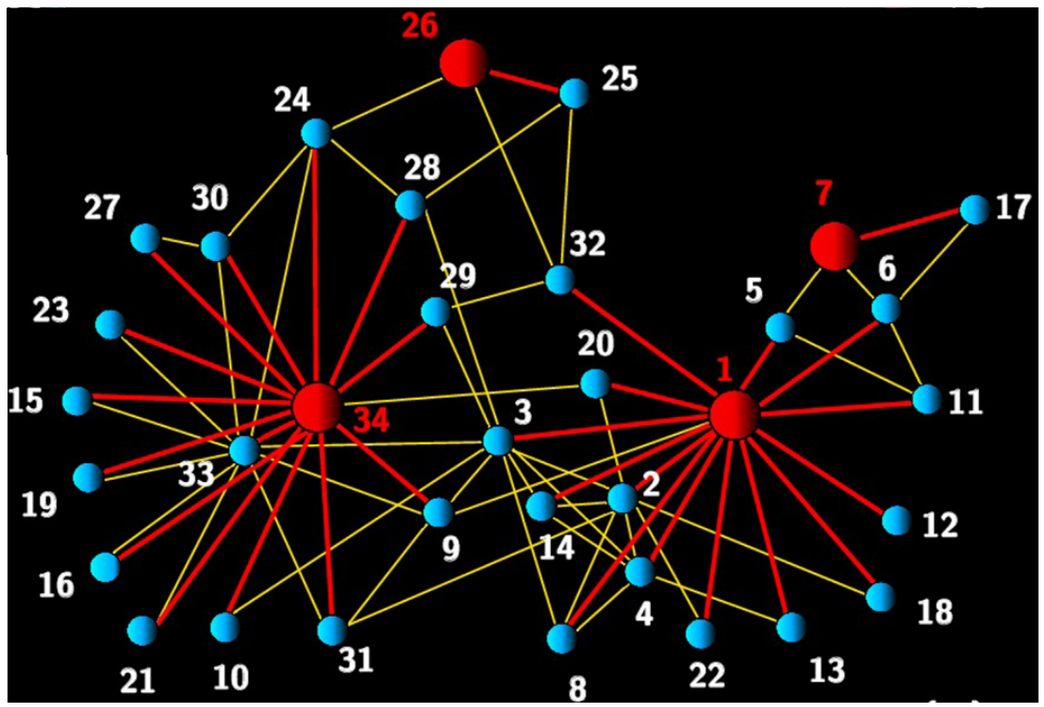
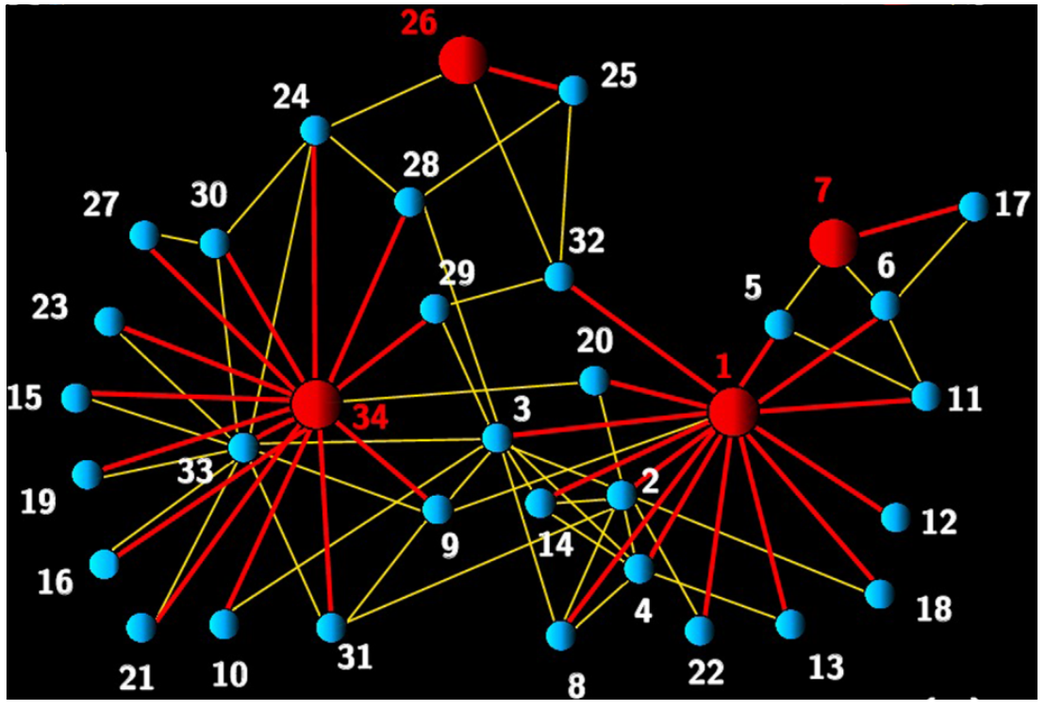


Clustering Coefficient

- Tendencially high in Social Networks

	WWW	Citations	Co-author	Ham Radio	Prison	High School Romance
Number of Nodes	325729	396	81217	44	67	572
Randomness: r	0.57	0.63	4.7	5.0	∞	∞
Avg. In-Degree: m	4.6	5.0	.84	3.5	2.7	.83
Avg. Clustering	.11	.07	.16	.47	.31	-

typically one or more orders of magnitude higher than expected in a random graph



Degree Correlations

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- Let us define a matrix $E_{kk'}$ that simply counts how many edges there are between nodes of degree k and nodes of degree k'. This matrix has several useful properties:

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- From which we can obtain the degree distribution (since $E_{kk'}$ contains all the connectivity information of the network).

$$P\left(k\right) = \frac{\langle k \rangle}{k} \sum_{k'} P\left(k,k'\right) \propto k^{-\gamma}$$

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- Using this conditional probability we can easily calculate $\langle k_{nn}\left(k\right)\rangle$ the average degree of the nearest neighbors of a node of degree k

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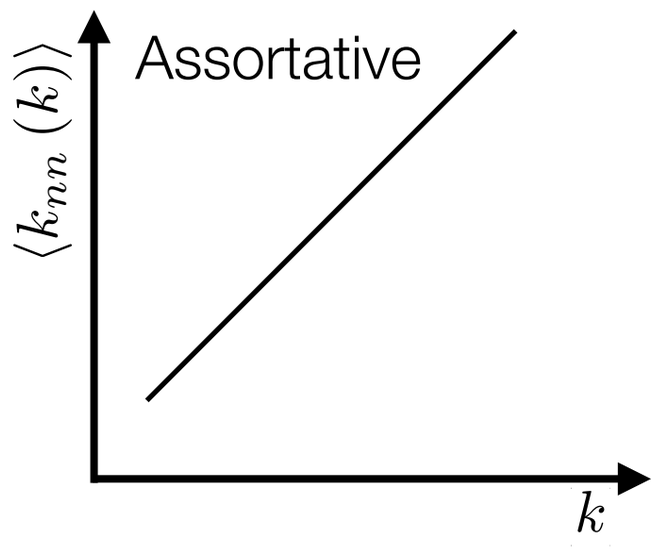

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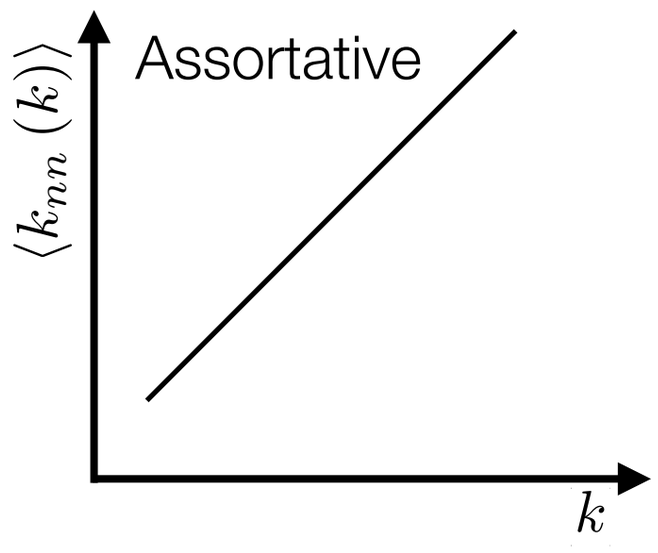

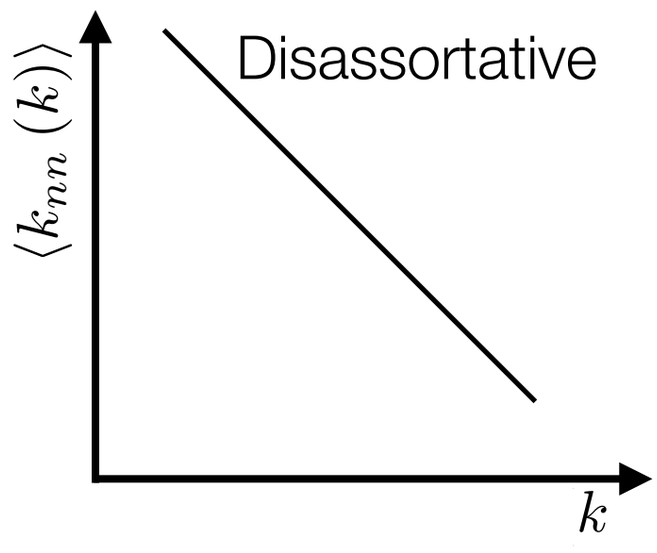

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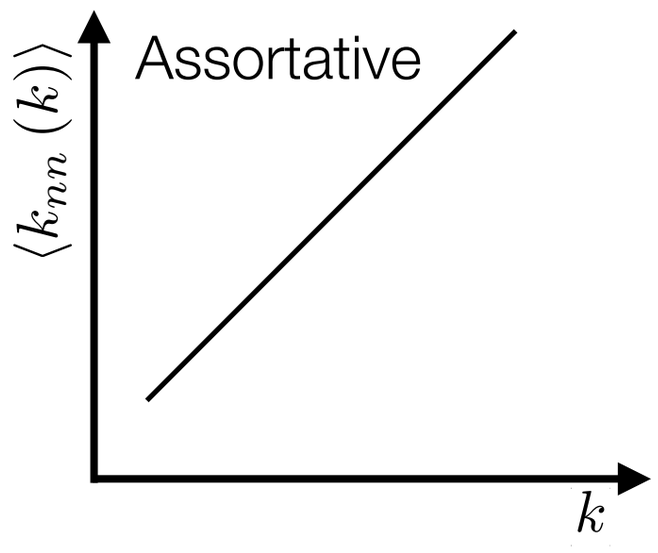

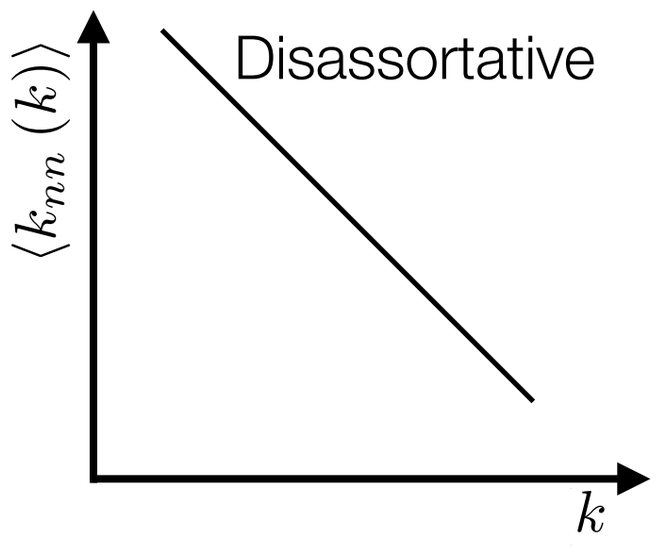

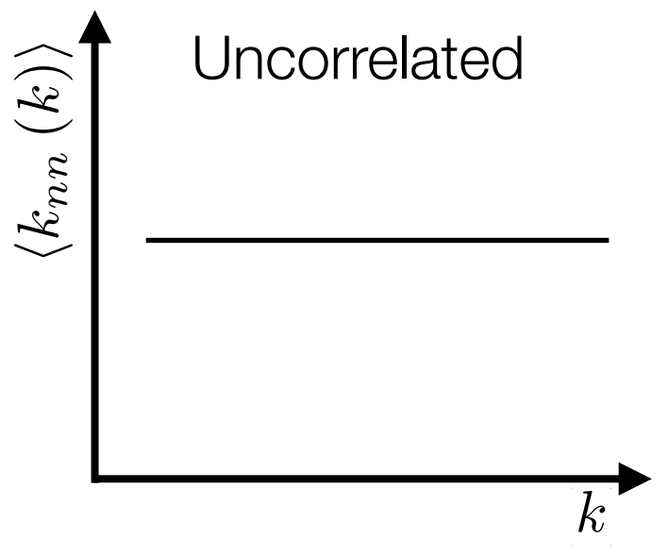

Degree Correlations

- The conditional probability that a node with degree k is connected to a node of degree k' is then:

$$P\left(k'|k\right) = \frac{\langle k \rangle}{k} \frac{P\left(k,k'\right)}{P\left(k\right)}$$

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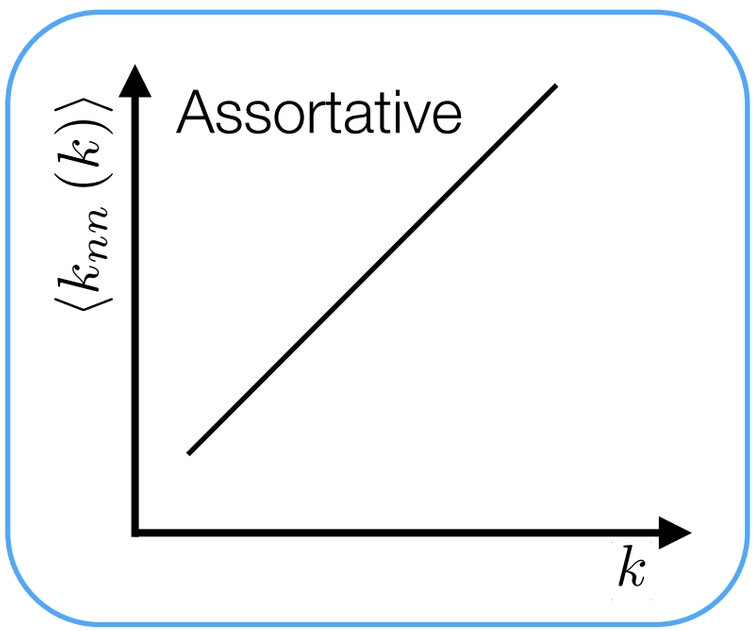

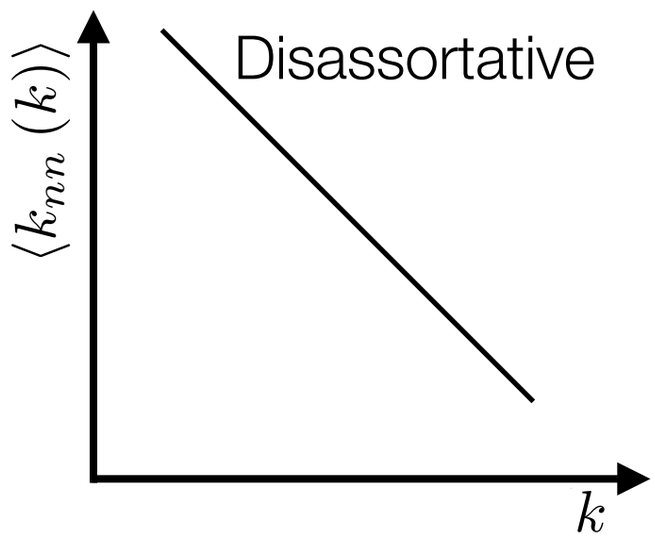

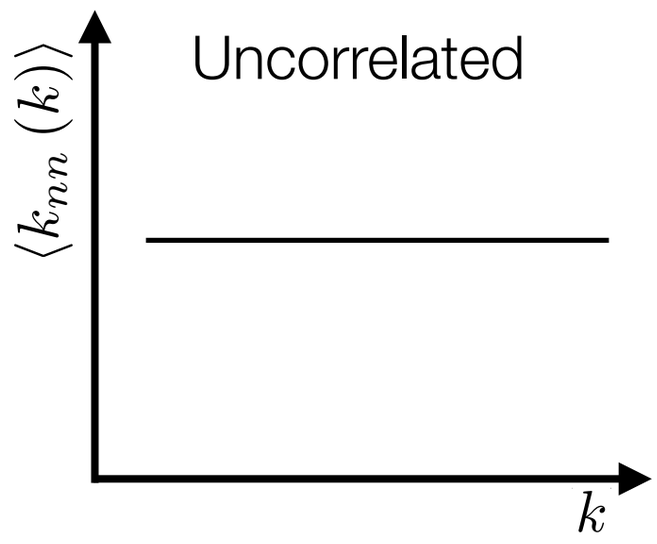

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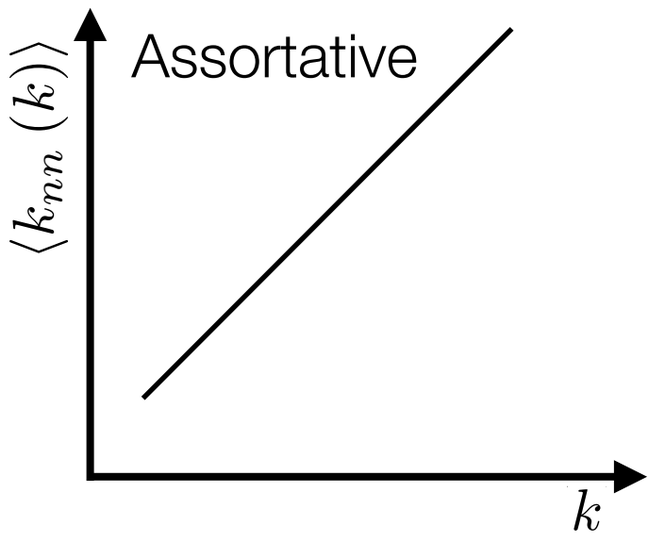

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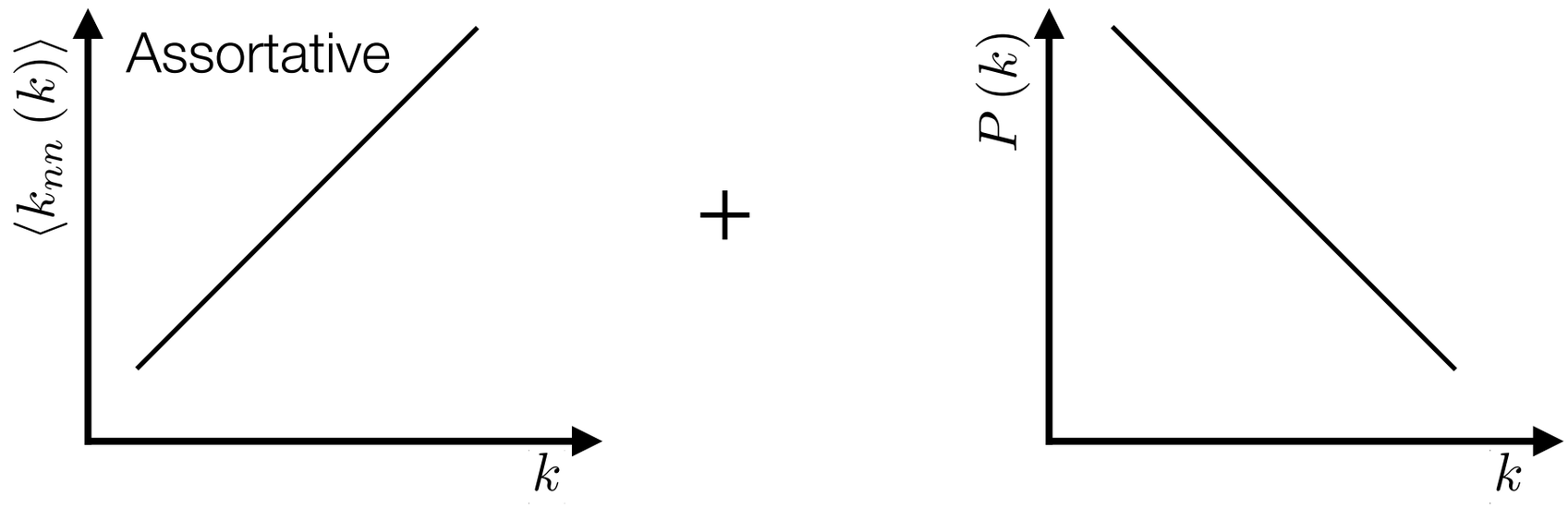

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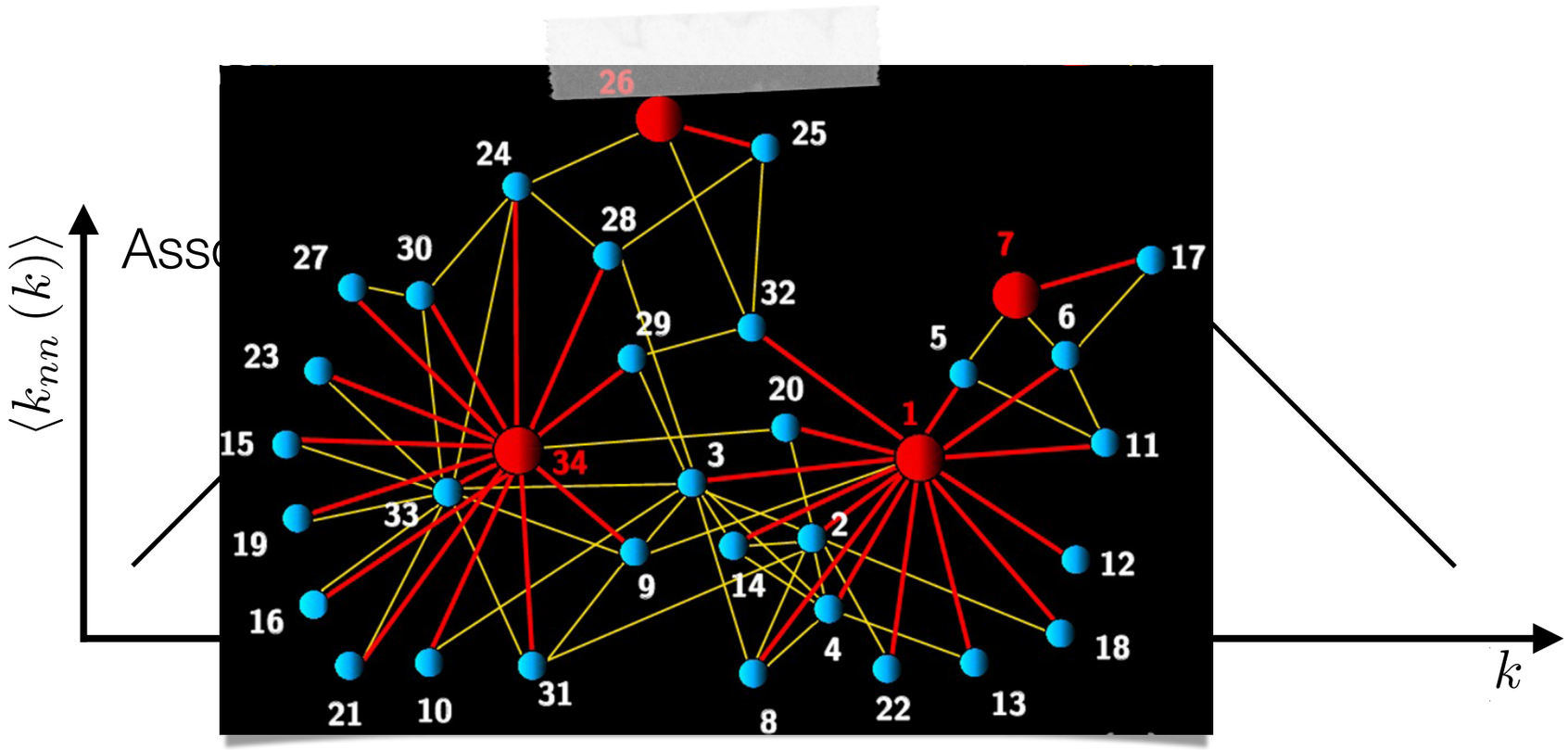

Your Friends Have More Friends Than You

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- And typically: $\mathcal{Y}_i\left(k\right) \propto k^{\alpha}$

How to get Social Network Data

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    - Applicable to historical records (Florentine weddings)

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- .connected_components() A list of nodes for each connected component

Social Network Models

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- So to build a network with a given average degree, we need:

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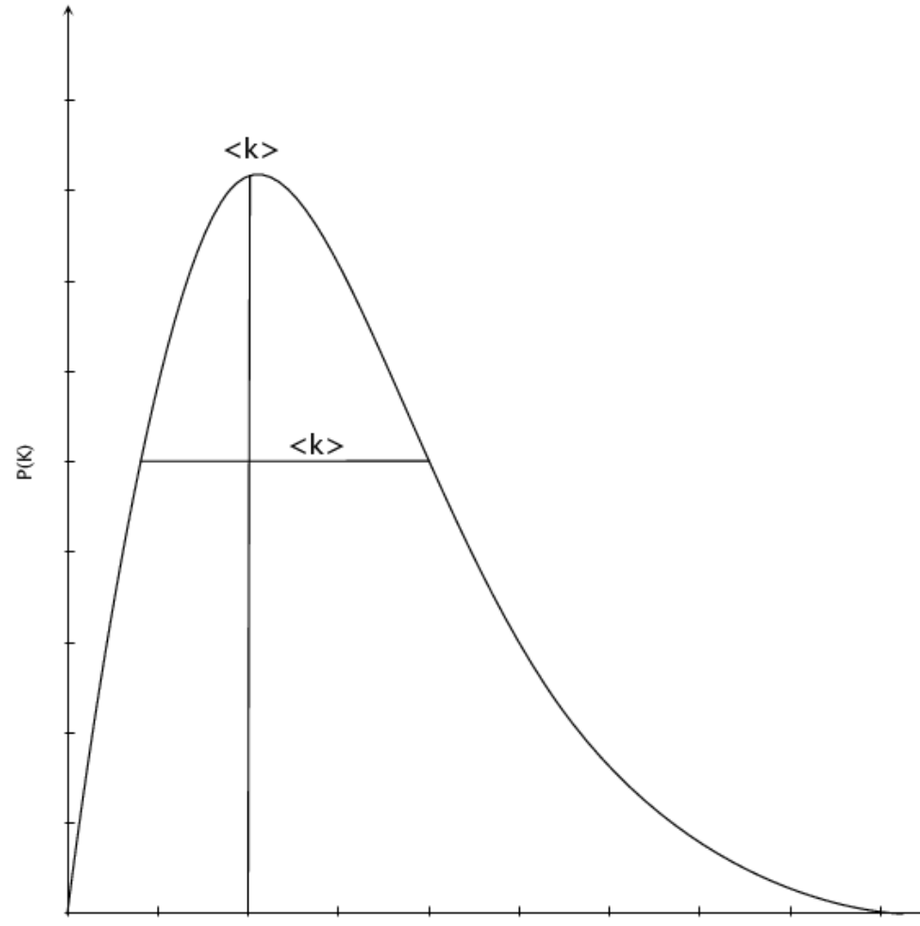

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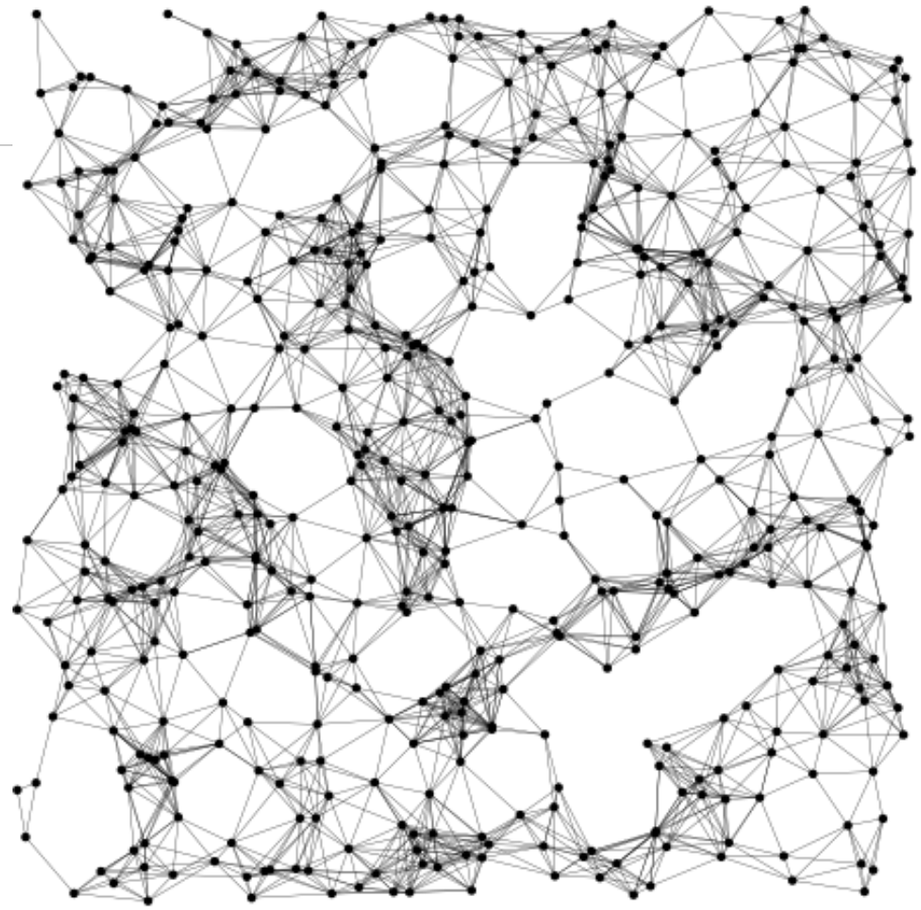

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https://networkx.github.io/documentation/networkx-1.10/examples/drawing/random_geometric_graph.html

```python
import networkx as nx
import matplotlib.pyplot as plt

G=nx.random_geometric_graph(200,0.125)
# position is stored as node attribute data for
random_geometric_graph
pos=nx.get_node_attributes(G,'pos')

# find node near center (0.5,0.5)
dmin=1
ncenter=0
for n in pos:
    x,y=pos[n]
    d=(x-0.5)**2+(y-0.5)**2
    if d<dmin:
        ncenter=n
        dmin=d

# color by path length from node near center
p=nx.single_source_shortest_path_length(G,ncenter)

plt.figure(figsize=(8,8))
nx.draw_networkx_edges(G,pos,nodelist=[ncenter],alpha=0.4)
nx.draw_networkx_nodes(G,pos,nodelist=list(p.keys()),
                       node_size=80,
                       node_color=list(p.values()),
                       cmap=plt.cm.Reds_r)

plt.xlim(-0.05,1.05)
plt.ylim(-0.05,1.05)
plt.axis('off')
plt.savefig('random_geometric_graph.png')
plt.show()
```

rgg.py

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# p
ran
pos

# f
dmi
nce
for

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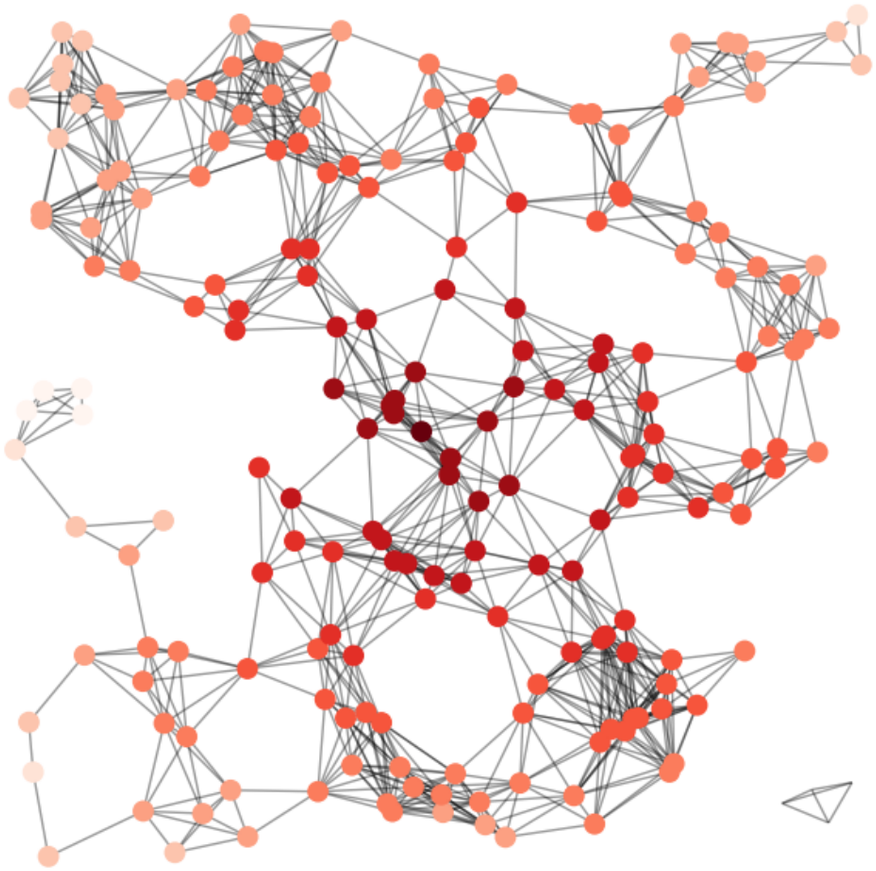

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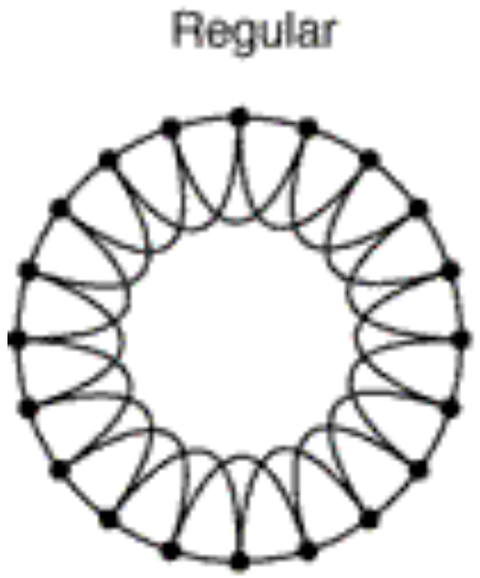
@bgoncalves

Small-World Effect: Watts-Strogatz Model



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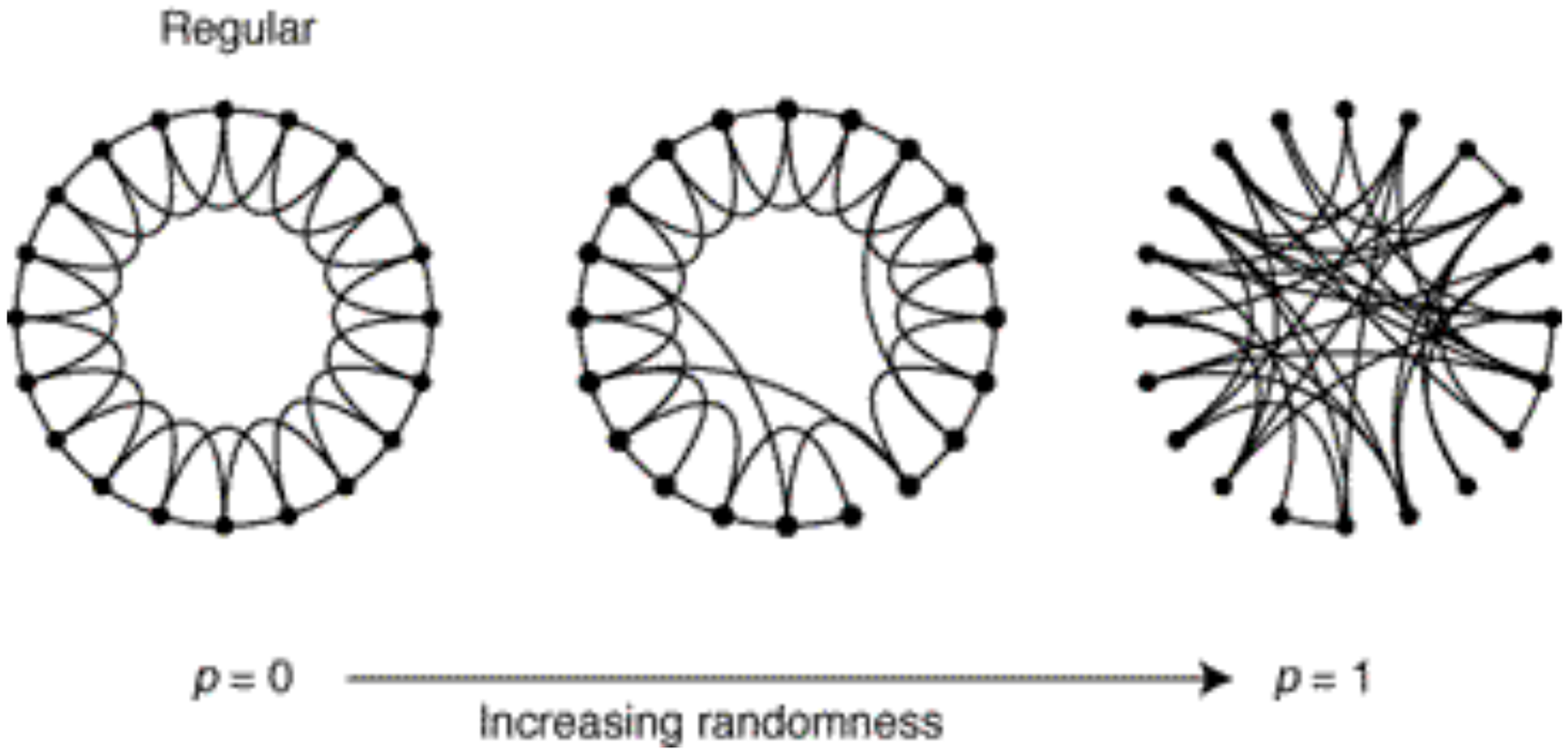

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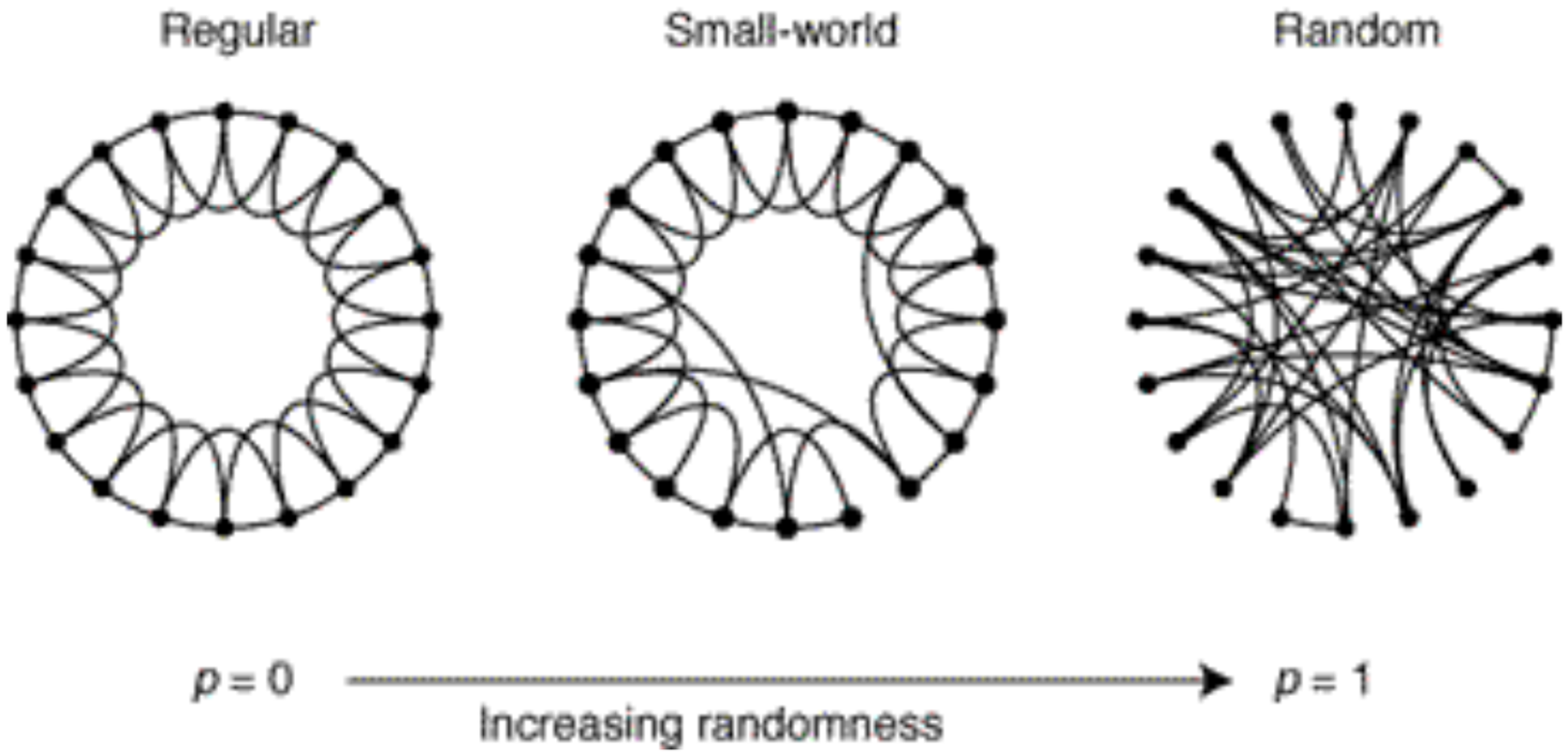

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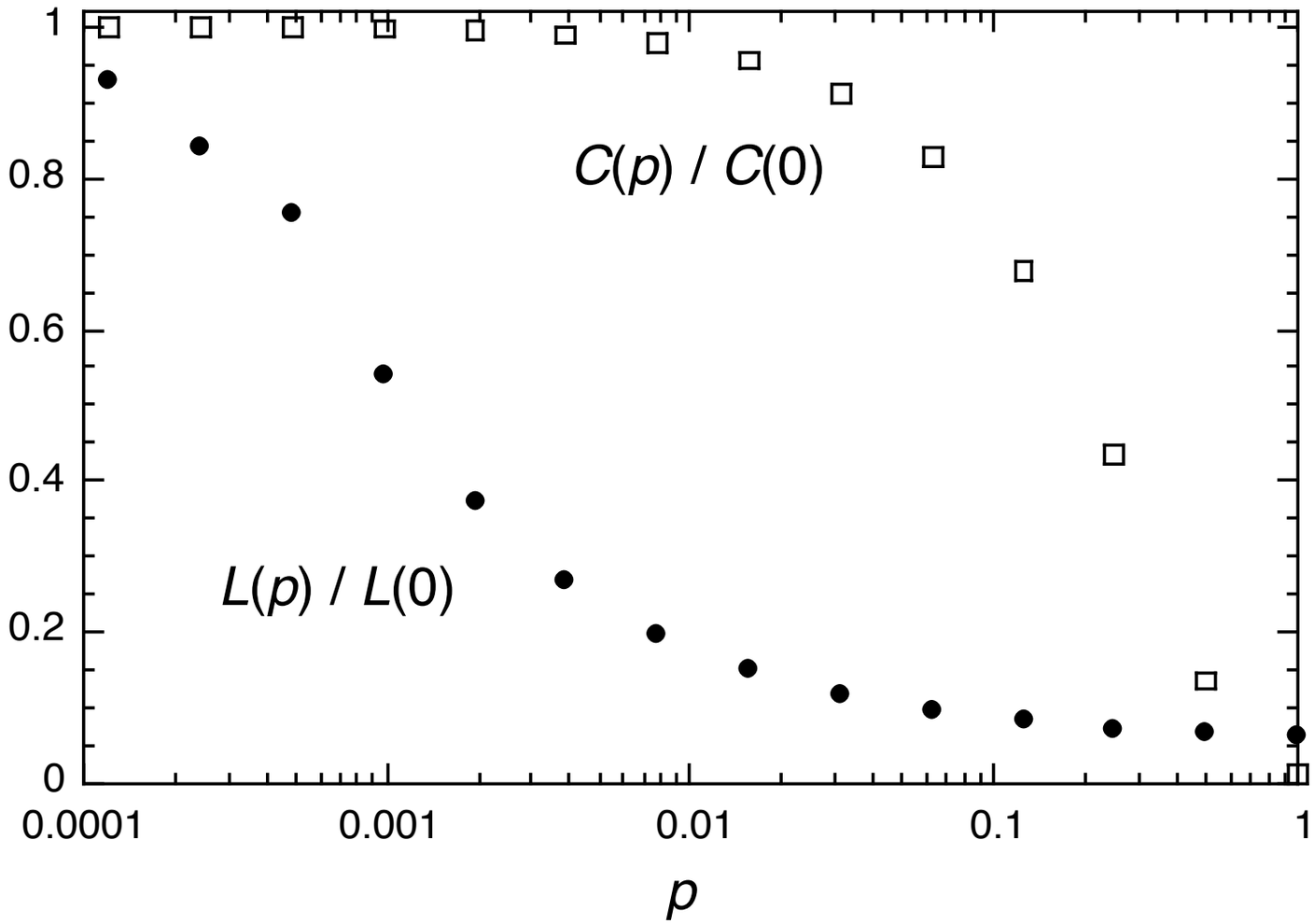

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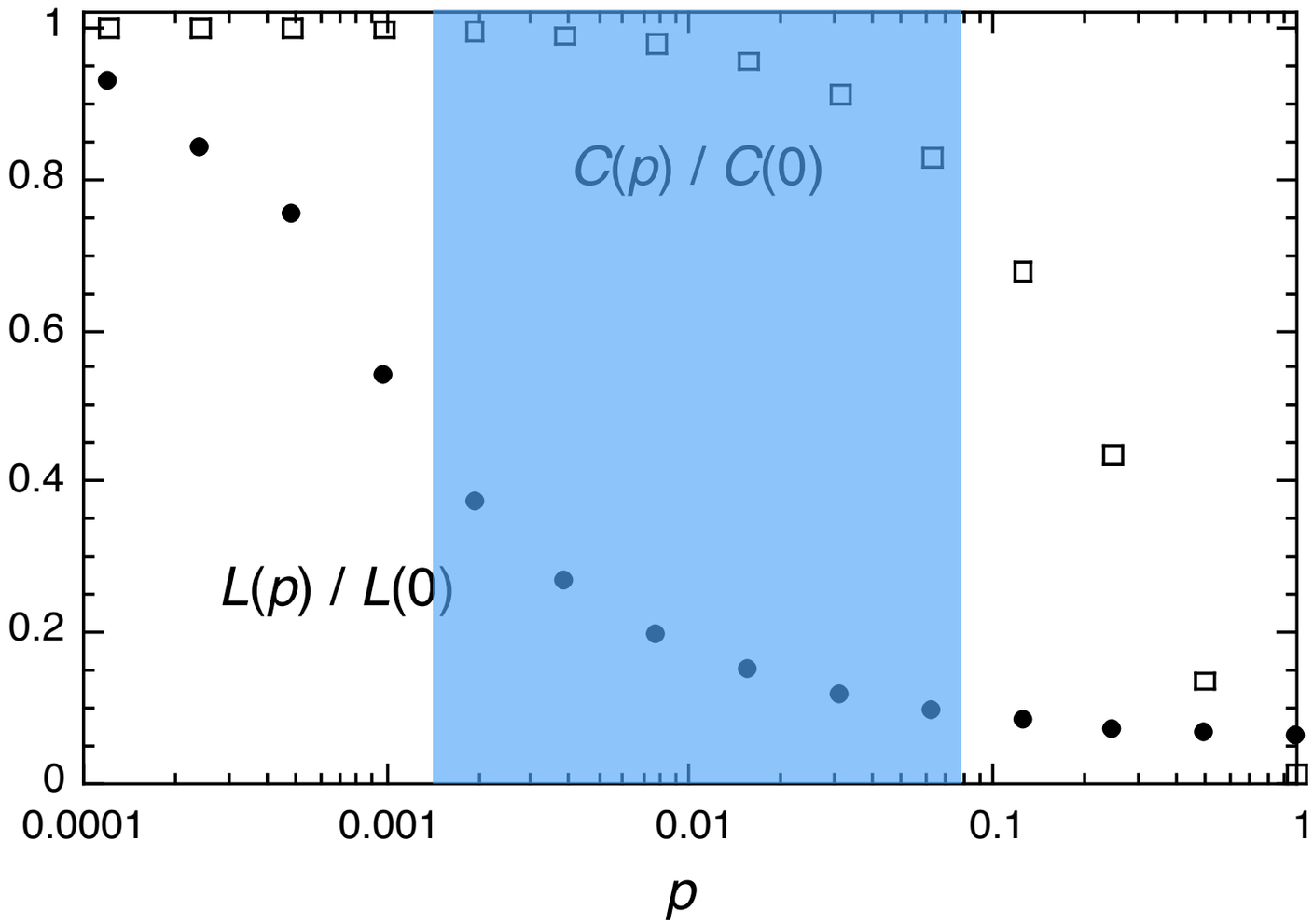

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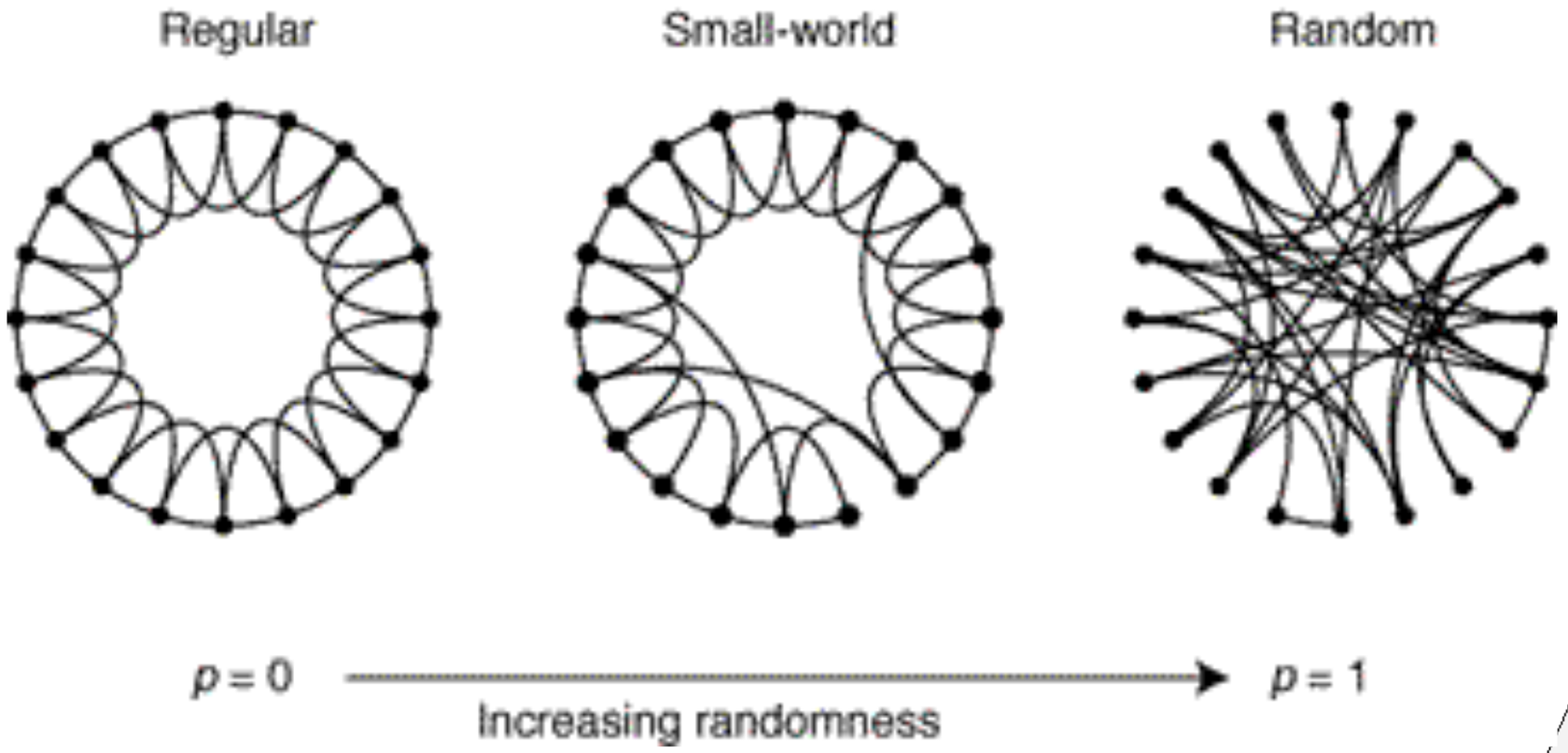

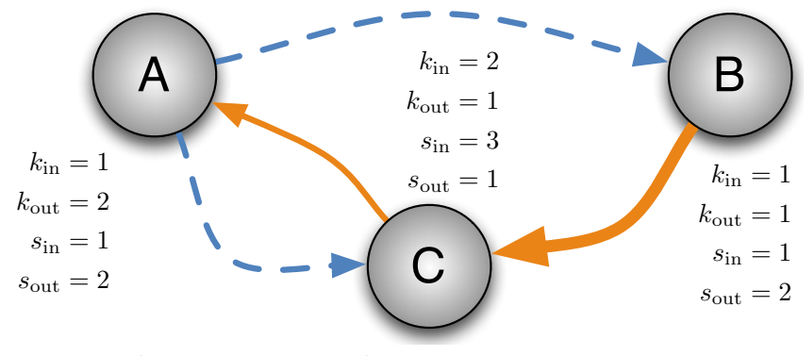

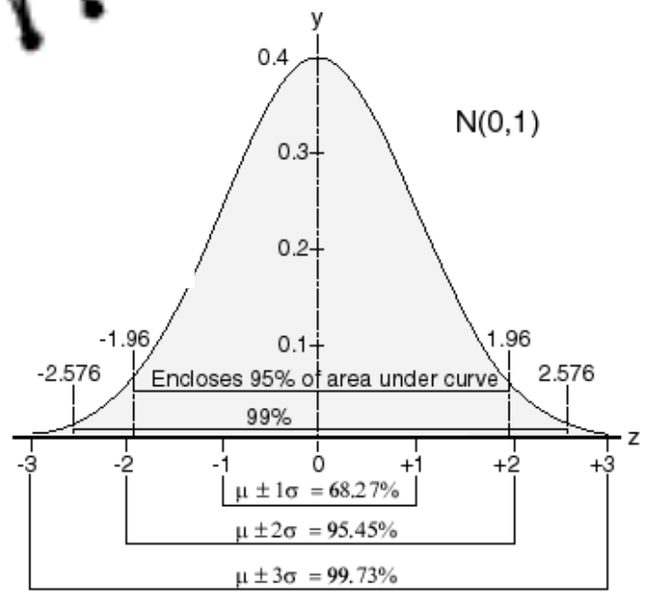

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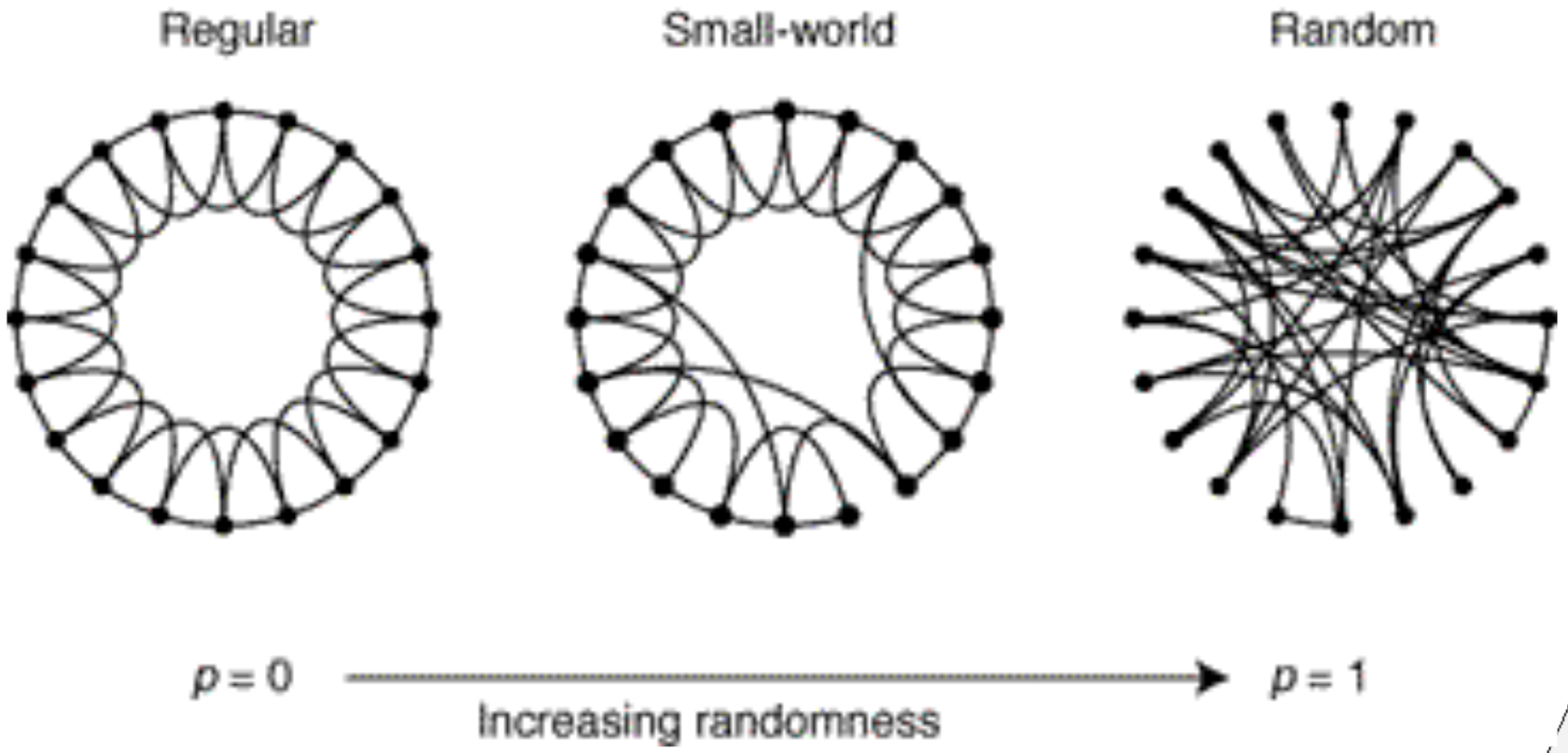
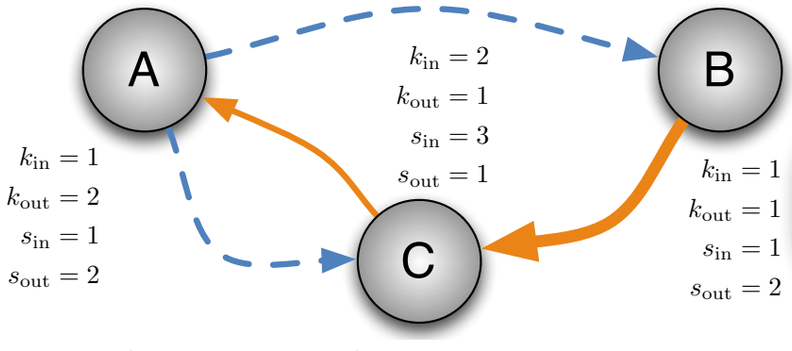
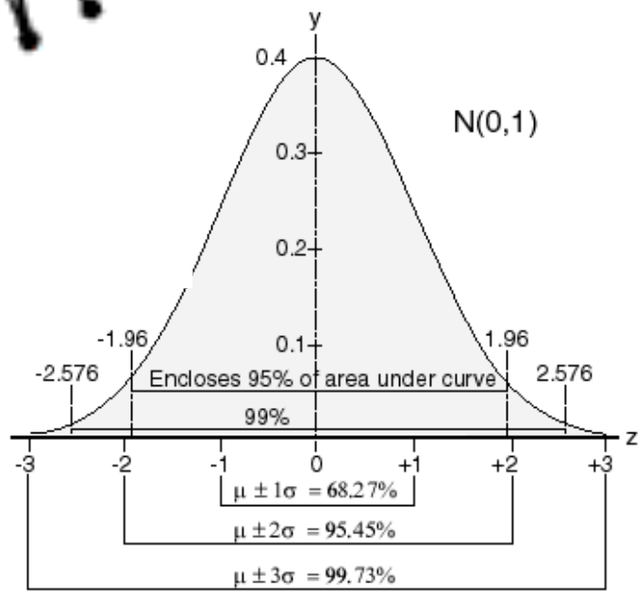




No Celebrities!



Fat-tailed Degree Distribution

Network or subgraph	Number of vertices	Number of edges	γ	References
Collaboration network of movie actors	212,250	61,085,555	2.3	[55]
'——' (another fitting of the same data)			3.1	[102]
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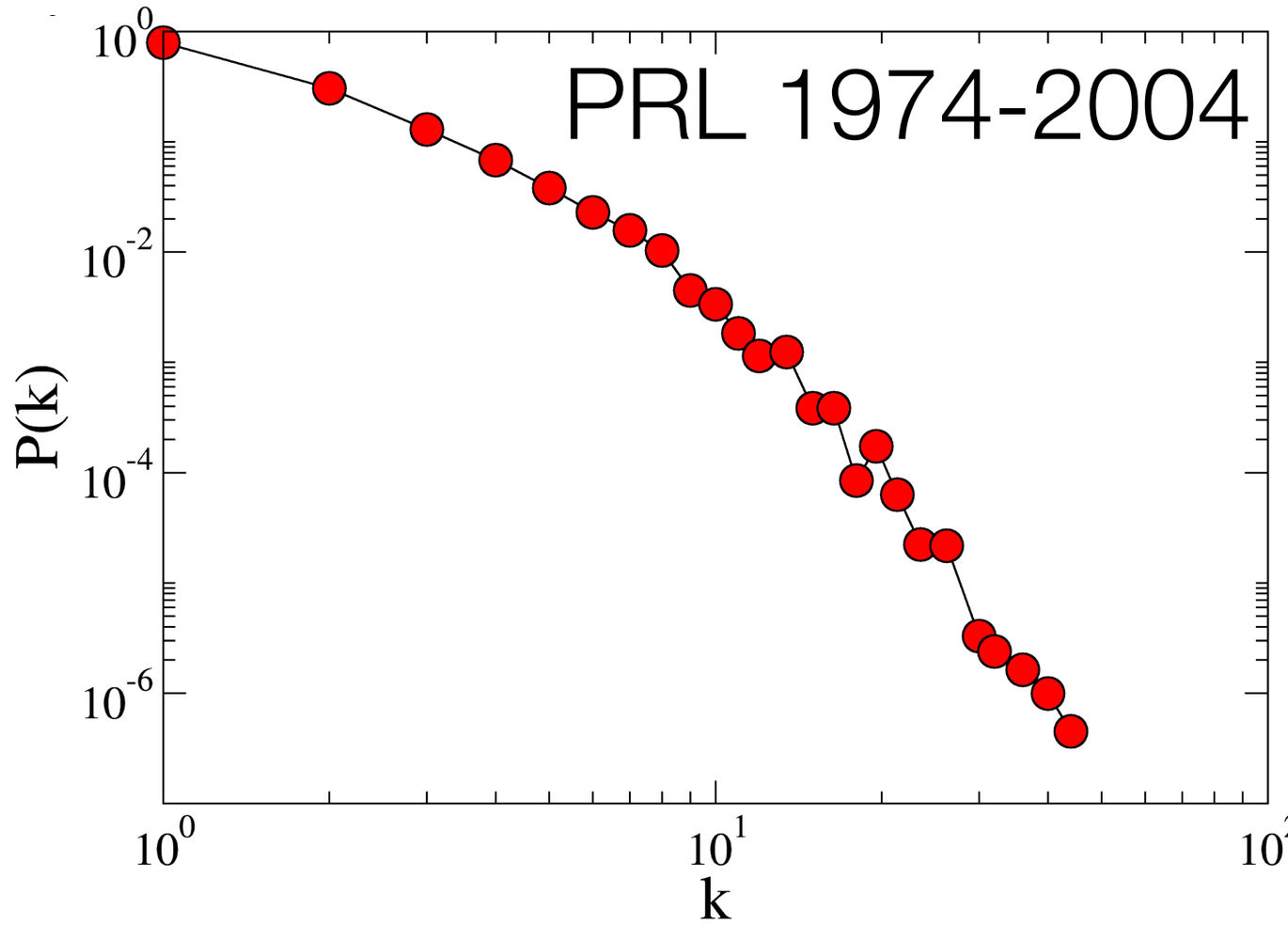

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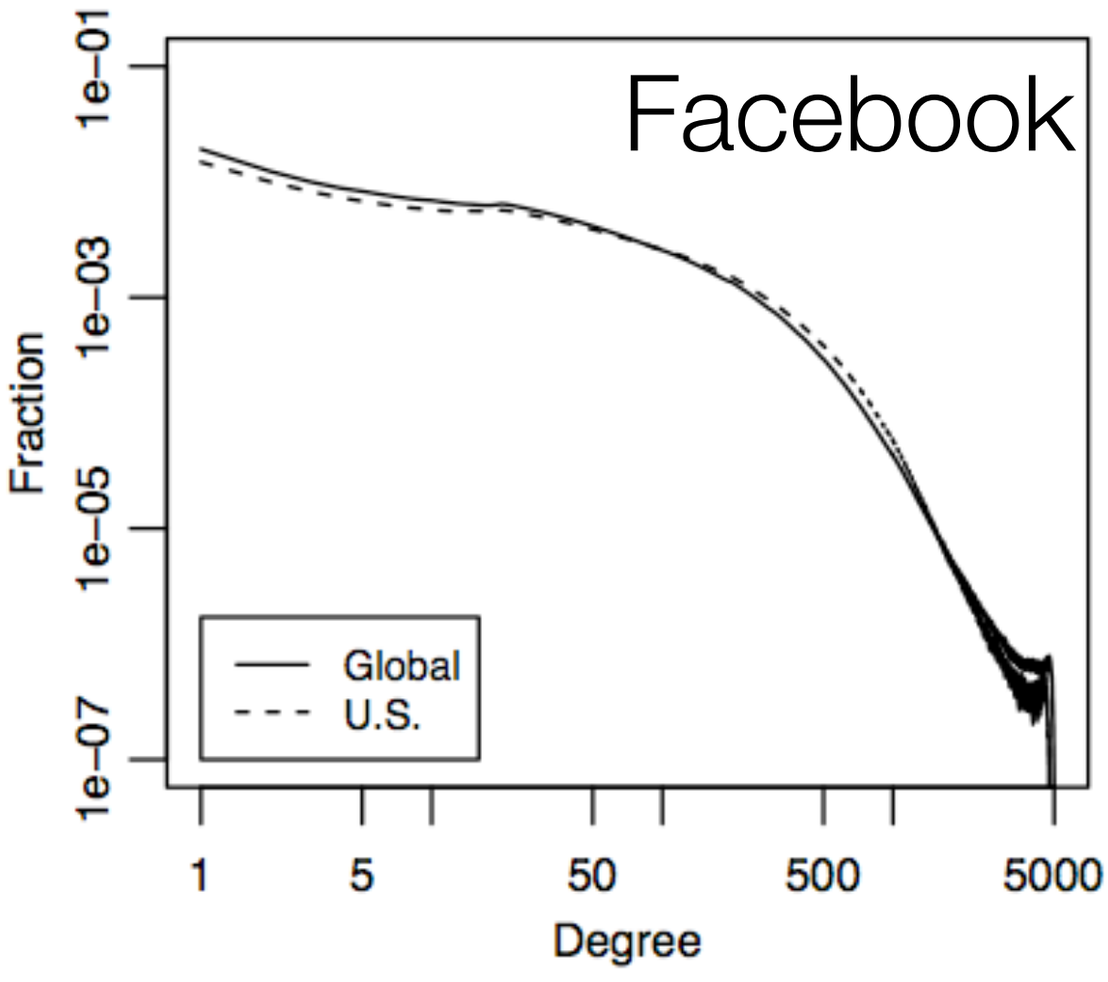

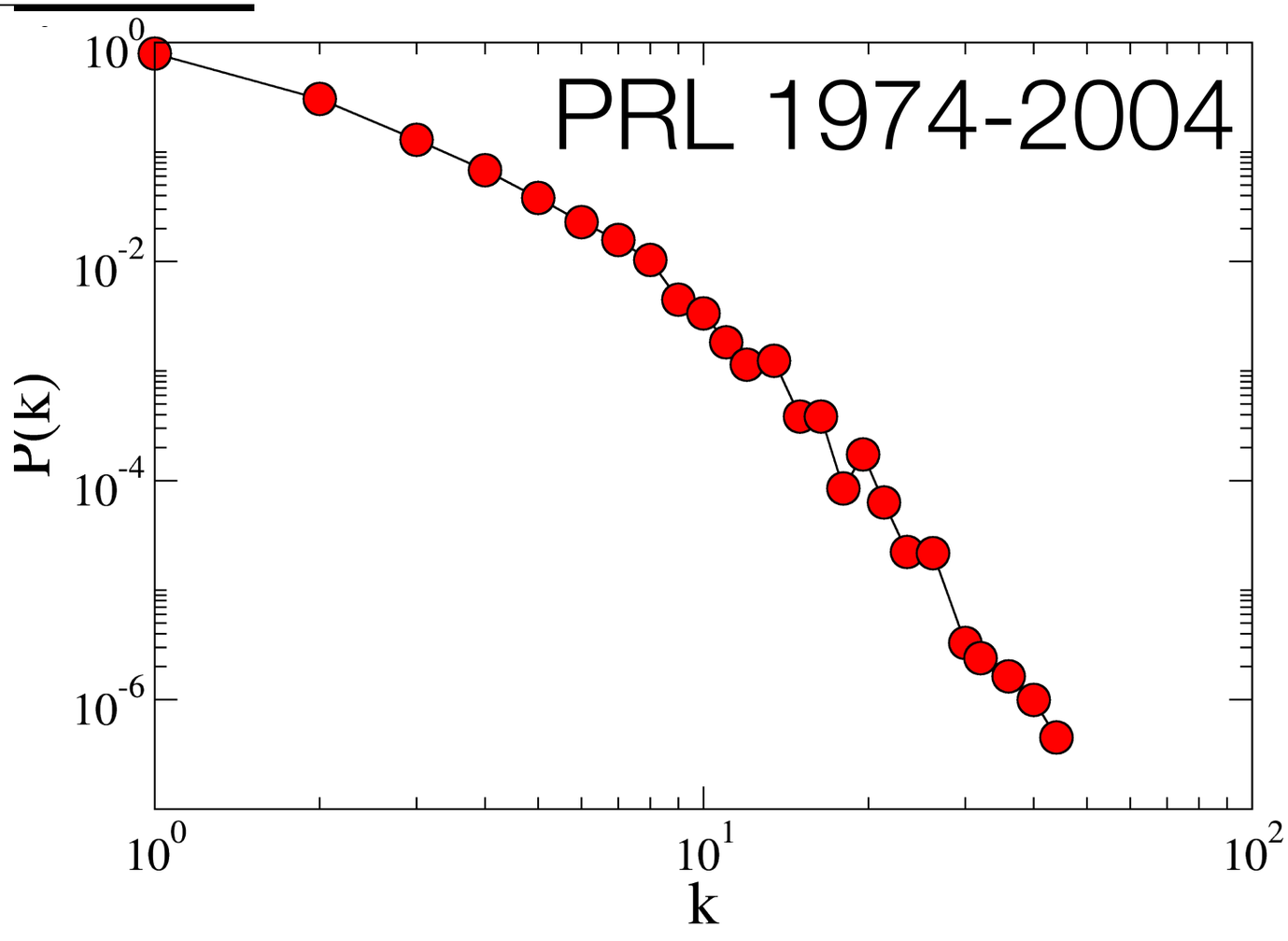

Preferential Attachment

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- In simplest case, power-law degree distribution with exponent 3
- "Younger" nodes connect to "older" nodes

NetworkX - Example

```
import networkx as NX
import numpy as np
from collections import Counter
import matplotlib.pyplot as plt

(…)

net = BarabasiAlbert()

degrees = net.degree()
Pk = np.array(list(Counter(degrees.values()).items()))

plt.loglog(Pk.T[0], Pk.T[1], 'b*')
plt.xlabel('k')
plt.ylabel('P[k]')
plt.savefig('Pk.png')
plt.close()

print("Number of nodes:", net.number_of_nodes())
print("Number of edges:", net.number_of_edges())
```


networkx_demo.py

NetworkX - Example

```
import net
import nur
from colle
import mat

(...)

net = Bara

degrees =
Pk = np.ar

plt.logloc
plt.xlabel
plt.ylabel
plt.savefi
plt.close

print("Nur
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```

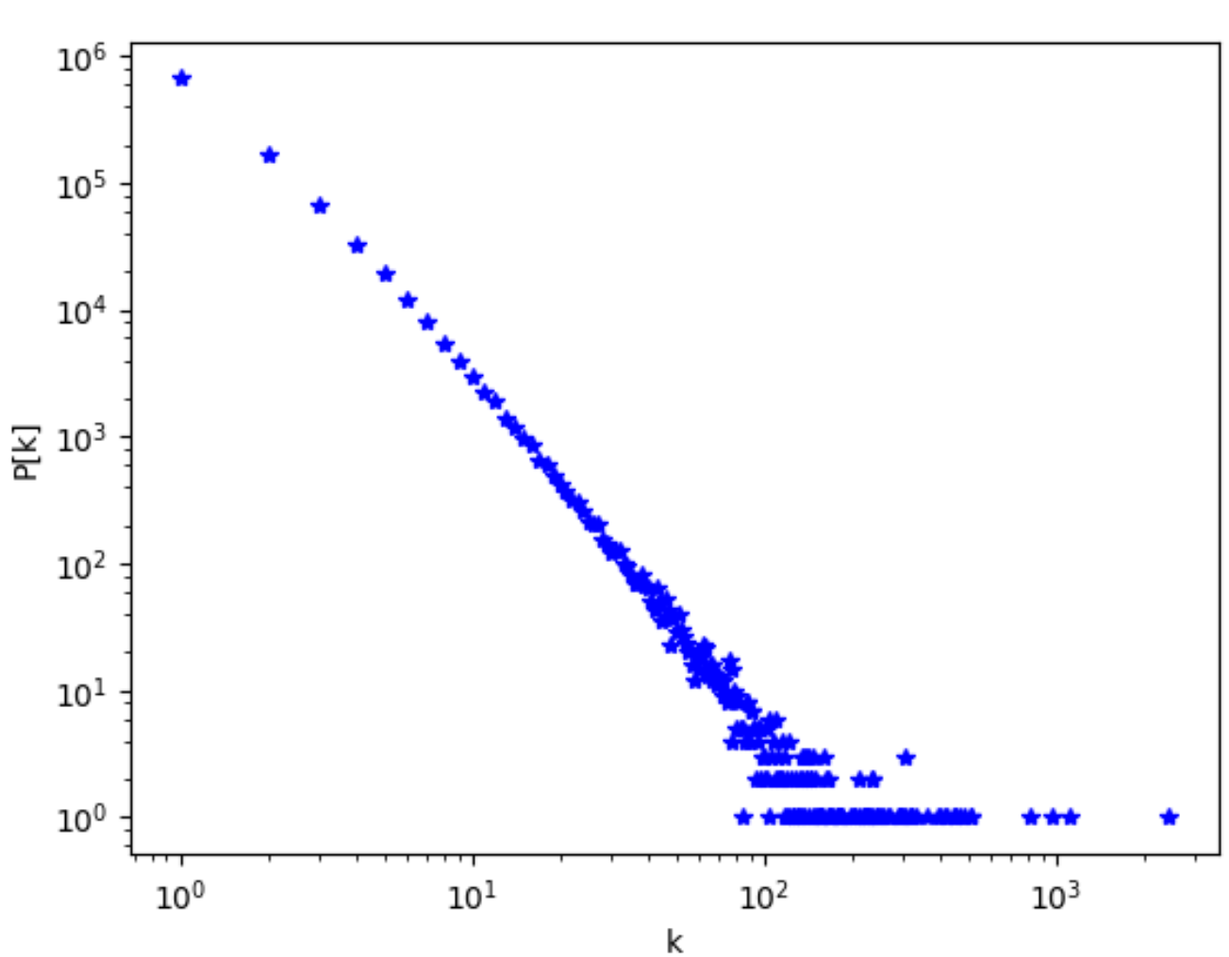

networkx_demo.py

MLE - Fitting a theoretical function to experimental data

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- In an experimental measurement, we expect (CLT) the experimental values to be normally distributed around the theoretical value with a certain variance. Mathematically, this means:

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Least Squares Fitting

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SIAM Rev. 51, 661 (2009)

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- We often find what look like power-law distributions in empirical data:

$$P(k) = \frac{\gamma - 1}{k_{min}} \left(\frac{k}{k_{min}} \right)^{-\gamma}$$

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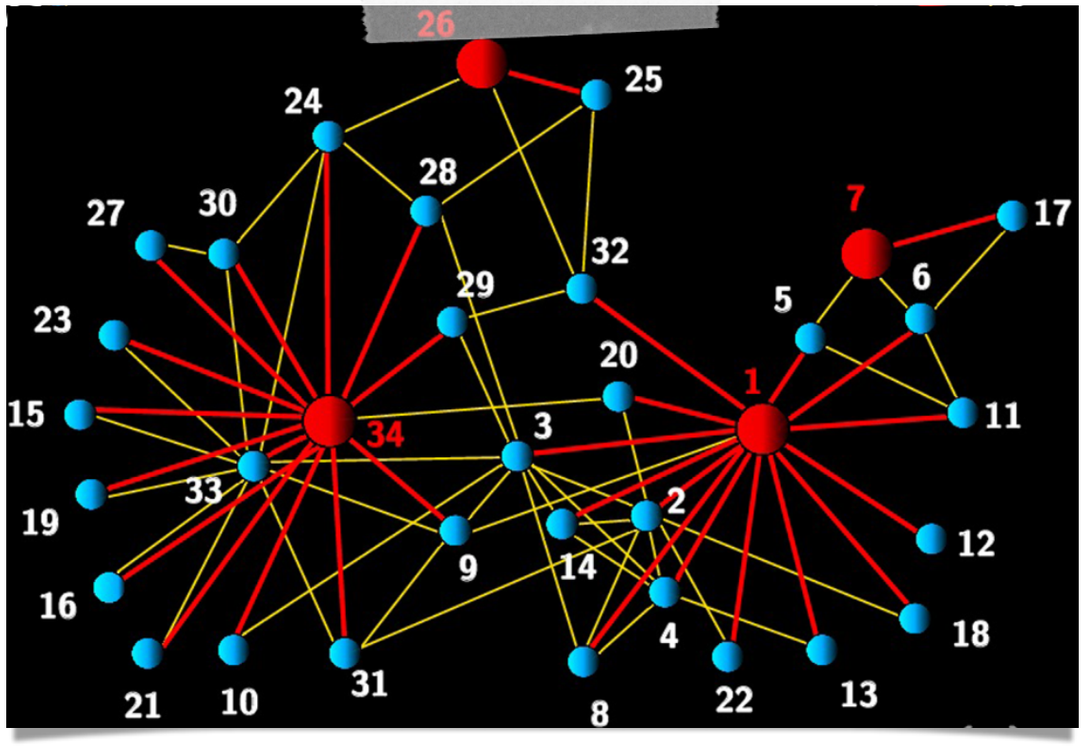
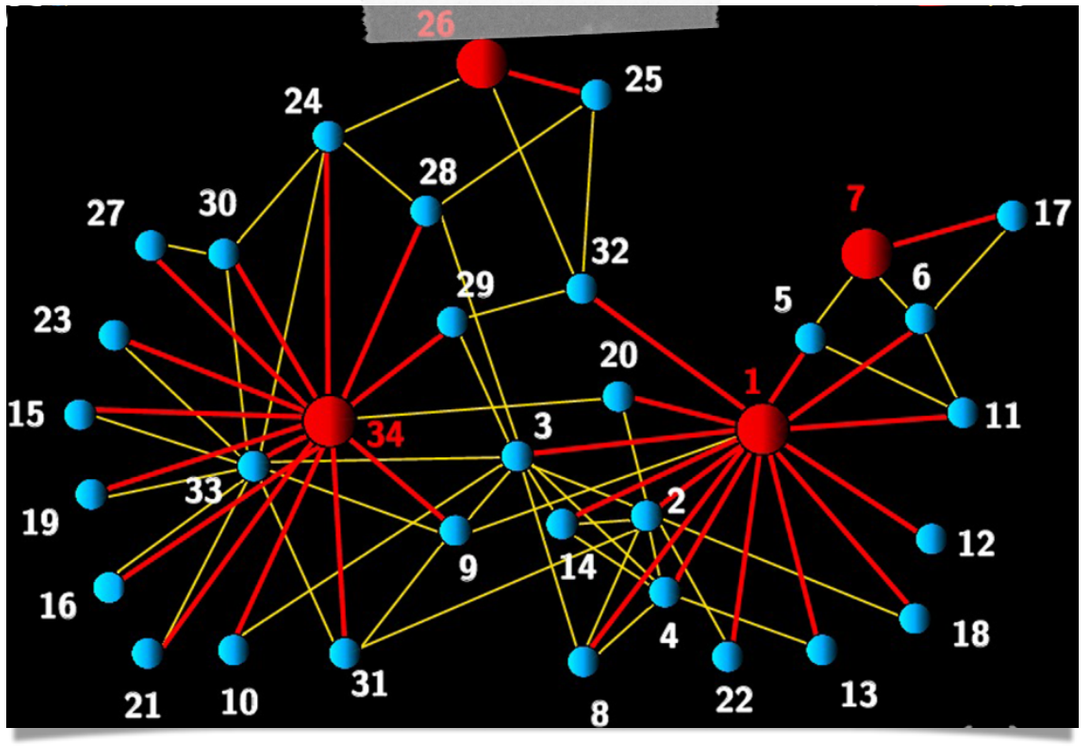
Communities



Communities

- Some network regions are denser than others



Communities

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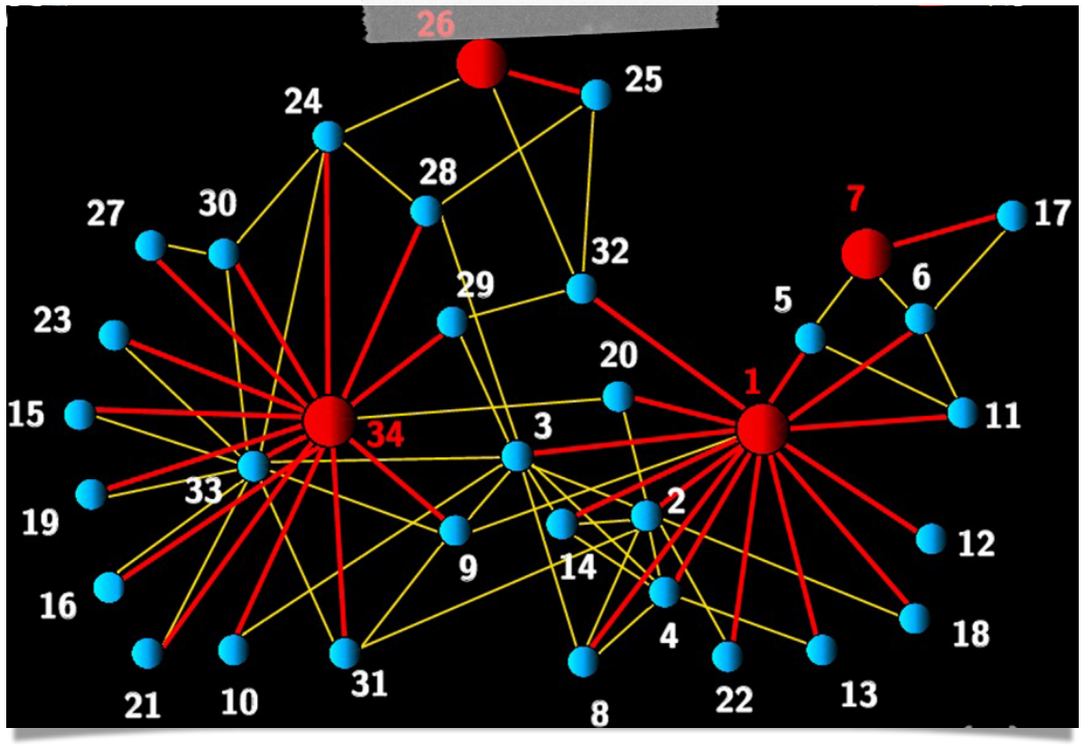

Communities

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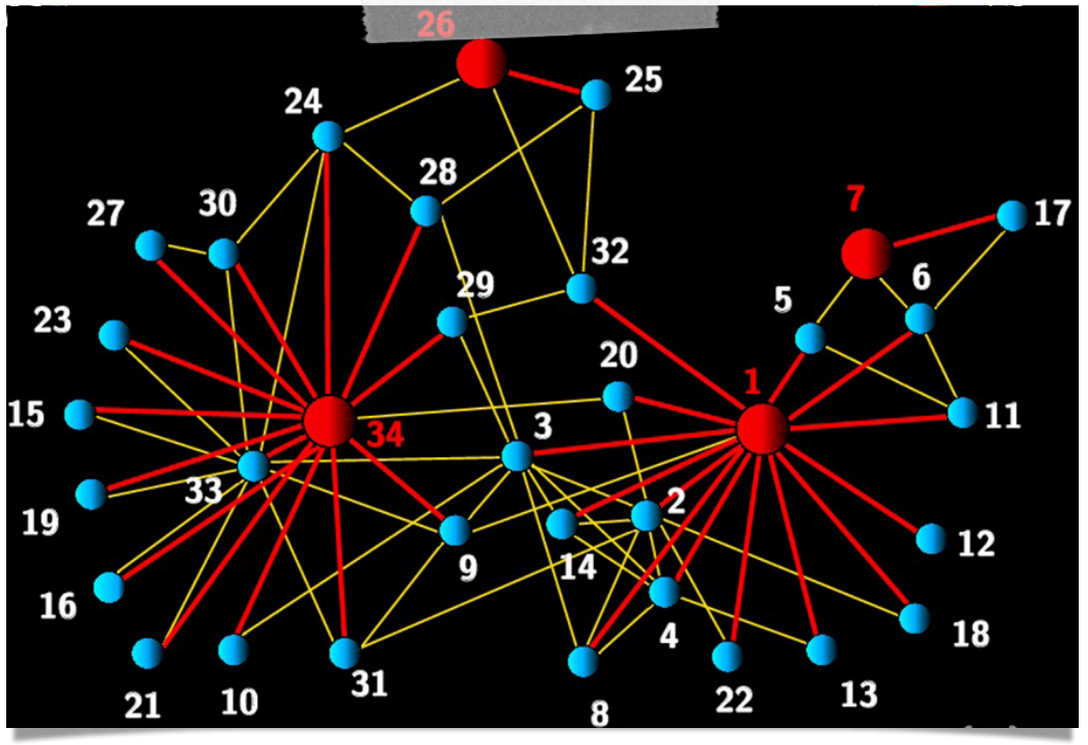
  - "Communities" should have more links within the community than without



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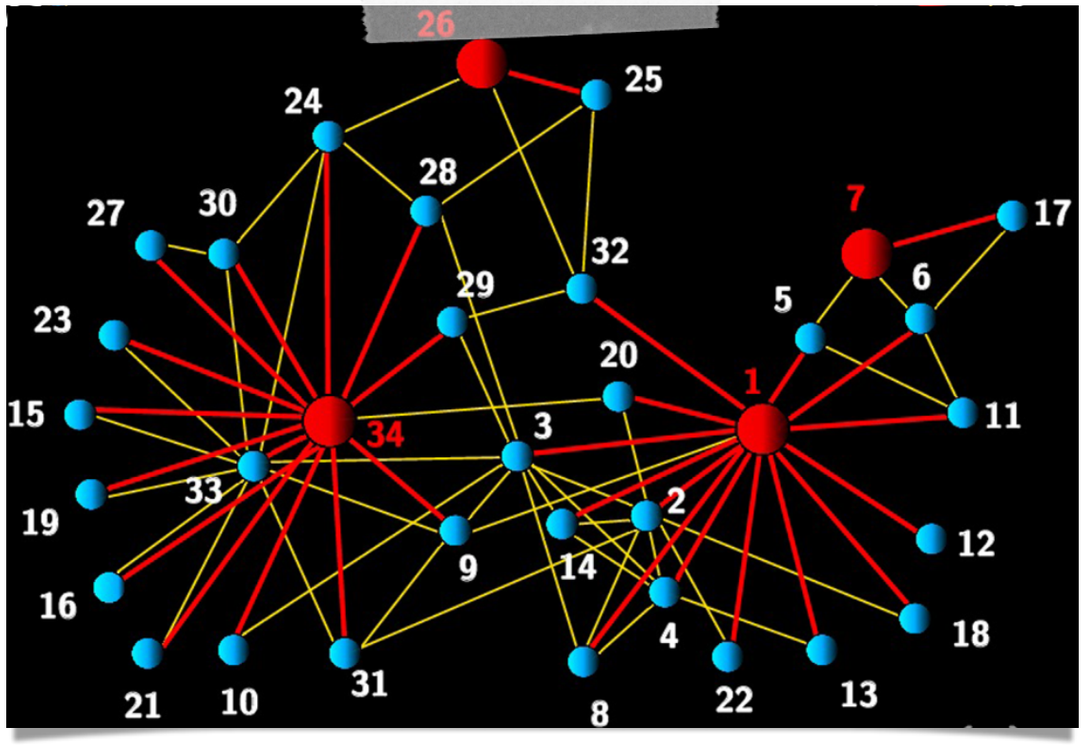
- Modularity:



$$Q = \sum_{i,j} \left[\frac{A_{ij}}{2m} - \frac{k_i k_j}{(2m)^2} \right]$$

Snowball Sampling

- Commonly used in Social Science and Computer Science

    1. Start with a single node (or small number of nodes)

    2. Get "friends" list

    3. For each friend get the "friend" list

    4. Repeat for a fixed number of layers or until enough users have been connected

- Generates a connected component from each seed

- Quickly generates a *lot* of data/API calls

Snowball Sampling

```python
import networkx as NX

def snowball(net, seed, max_depth = 3, maxnodes=1000):
    seen = set()
    queue = set()

    queue.add(seed)
    queue2 = set()

    for _ in range(max_depth+1):
        while queue:
            user_id = queue.pop()
            seen.add(user_id)

            NN = net.neighbors(user_id)

            for node in NN:
                if node not in seen:
                    queue2.add(node)

        queue.update(queue2)
        queue2 = set()

    return seen

net = NX.connected_watts_strogatz_graph(10000, 4, 0.01)
neve = snowball(net, 0)

print(neve)
```

snowball.py

Random Walk Sampling

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- Similar to Snowball Sampling

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- Follow just one link instead of all links

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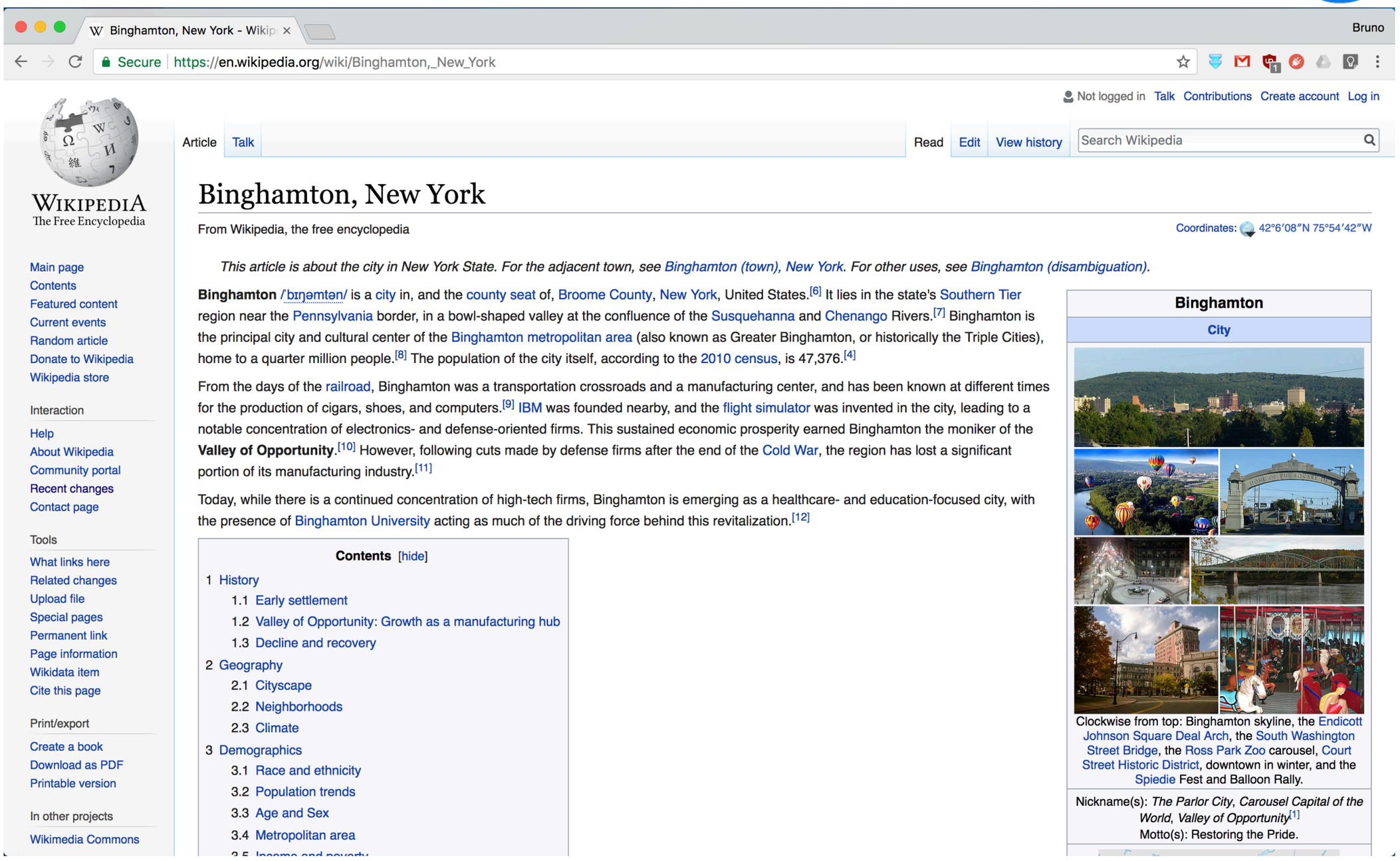
- Asymptotic sampling probability of vertices in a Random Walk is proportional to vertex degree
- Asymptotic regime reached quickly even for relatively small sample sizes

PageRank


Google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Contributions
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The Free Encyclopedia
Main page
Contents
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Current events
Random article
Donate to Wikipedia
Wikipedia store
Interaction
Help
About Wikipedia
Community portal
Recent changes
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Related changes
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Binghamton, New York
From Wikipedia, the free encyclopedia
Coordinates: 42°6′08″N 75°54′42″W
This article is about the city in New York State. For the adjacent town, see Binghamton (town), New York. For other uses, see Binghamton (disambiguation).
Binghamton /ˈbɪŋəmtən/ is a city in, and the county seat of, Broome County, New York, United States.[6] It lies in the state's Southern Tier region near the Pennsylvania border, in a bowl-shaped valley at the confluence of the Susquehanna and Chenango Rivers.[7] Binghamton is the principal city and cultural center of the Binghamton metropolitan area (also known as Greater Binghamton, or historically the Triple Cities), home to a quarter million people.[8] The population of the city itself, according to the 2010 census, is 47,376.[4]
From the days of the railroad, Binghamton was a transportation crossroads and a manufacturing center, and has been known at different times for the production of cigars, shoes, and computers.[9] IBM was founded nearby, and the flight simulator was invented in the city, leading to a notable concentration of electronics- and defense-oriented firms. This sustained economic prosperity earned Binghamton the moniker of the Valley of Opportunity.[10] However, following cuts made by defense firms after the end of the Cold War, the region has lost a significant portion of its manufacturing industry.[11]
Today, while there is a continued concentration of high-tech firms, Binghamton is emerging as a healthcare- and education-focused city, with the presence of Binghamton University acting as much of the driving force behind this revitalization.[12]
Contents [hide]
1 History
1.1 Early settlement
1.2 Valley of Opportunity: Growth as a manufacturing hub
1.3 Decline and recovery
2 Geography
2.1 Cityscape
2.2 Neighborhoods
2.3 Climate
3 Demographics
3.1 Race and ethnicity
3.2 Population trends
3.3 Age and Sex
3.4 Metropolitan area
Binghamton
City
Clockwise from top: Binghamton skyline, the Endicott Johnson Square Deal Arch, the South Washington Street Bridge, the Ross Park Zoo carousel, Court Street Historic District, downtown in winter, and the Spiedie Fest and Balloon Rally.
Nickname(s): The Parlor City, Carousel Capital of the World, Valley of Opportunity[1]
Motto(s): Restoring the Pride.

PageRank

"A page is important if it is pointed to by other important pages"

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$$\pi_i = \sum_j a_{ji} \frac{\pi_j}{k_j}$$

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Stationary state of a discrete time random walk

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Problems:

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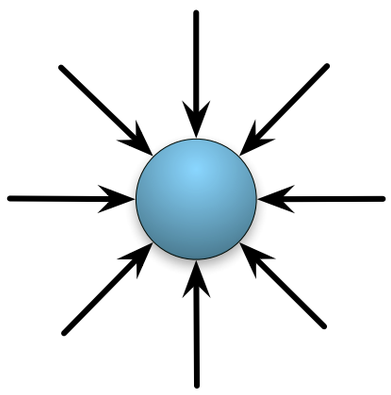
"Dangling nodes"



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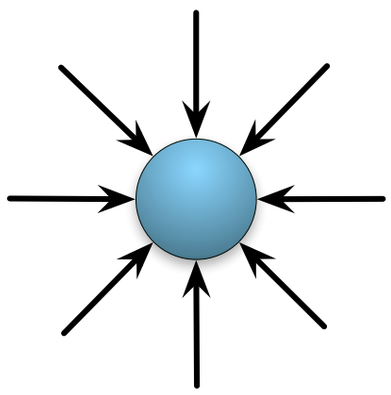

Solution:

Connect dangling nodes to every other node

PageRank

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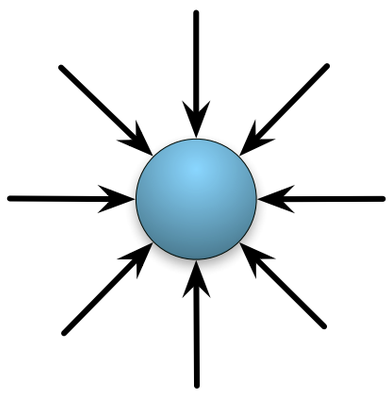
Problems:

"Dangling nodes"



Slow convergence

Solution:

Connect dangling nodes to every other node

PageRank

"A page is important if it is pointed to by other important pages"

$$\pi_i = \sum_j a_{ji} \frac{\pi_j}{k_j}$$

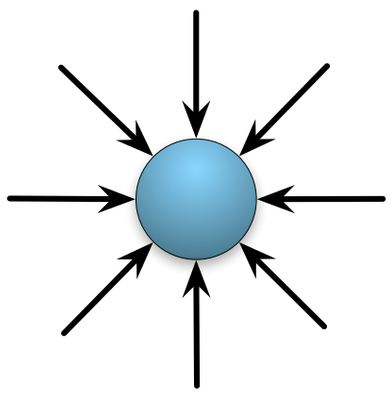
Stationary state of a discrete time random walk

$$\vec{\pi}\left(t+1\right) = \mathcal{T}\vec{\pi}\left(t\right)$$

$$\mathcal{T} = K_o^{-1} A^T$$

Problems:

"Dangling nodes"



Slow convergence

Solution:

Connect dangling nodes to every other node

Add damping factor

Markov Chain

Markov Chain

- Memoryless - "Future depends only on present state and not on past history"

$$\vec{\pi}\left(t+1\right) = \left[\left(1-\alpha\right)\mathcal{S} + \alpha\mathcal{E}\right]\vec{\pi}\left(t\right)$$

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Fully connected matrix

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- Has a stationary state if matrix is:

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  - Aperiodic - Return to node i does not occur periodically

Fully connected matrix

Damping Factor

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- Has a stationary state if matrix is:

$$\vec{\pi} = \mathcal{G}\vec{\pi}$$

Fully connected matrix

Damping Factor

  - Stochastic - Column vectors normalized to 1.
  - Irreducible - All nodes are accessible (graph is connected)
  - Aperiodic - Return to node i does not occur periodically

Power Method

$$\vec{\pi} = \mathcal{G}\vec{\pi}$$

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Eigenvector corresponding to Eigenvalue 1.

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Iterate...

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Iterate...

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Stop when:

Power Method

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Eigenvector corresponding to Eigenvalue 1.

Iterate...

$$\vec{\pi}^{(n+1)} = \mathcal{G}\vec{\pi}^{(n)}$$

Stop when:

$$\vec{\pi}^{(n+1)} \approx \vec{\pi}^{(n)}$$

Power Method

$$\vec{\pi} = \mathcal{G}\vec{\pi}$$

Eigenvector corresponding to Eigenvalue 1.

Iterate...

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Stop when:

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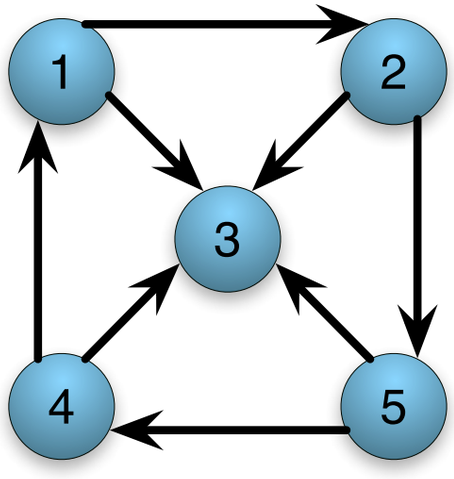
Example:

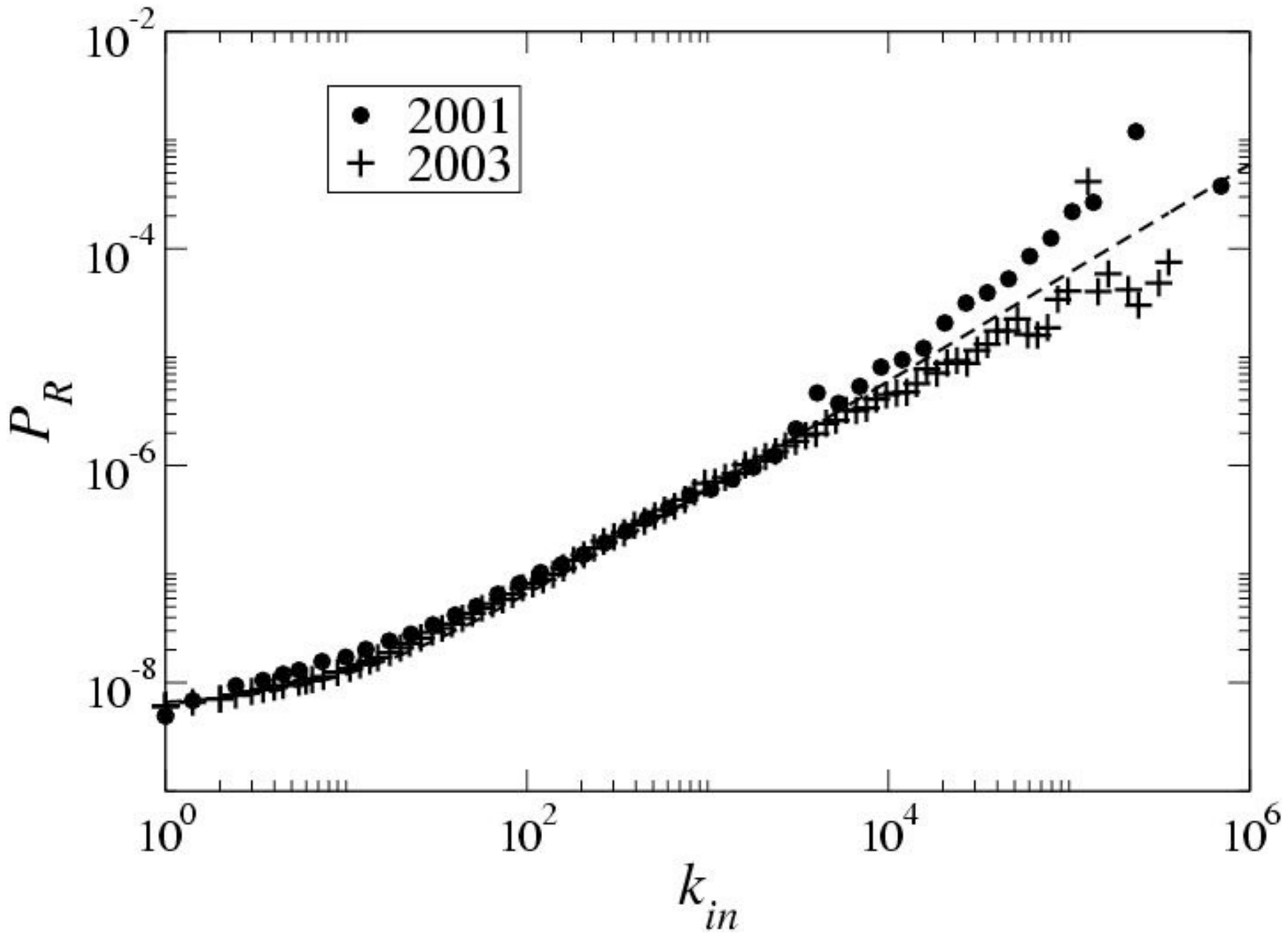


PageRank

```python
def Google_Matrix(A, m):
    N = A.shape[0]
    v = np.ones(N)

    KT = np.dot(A, v)
    A = A.T

    for i in range(N):
        if KT[i] != 0:
            A.T[i] = A.T[i]/KT[i]
        else:
            A.T[i] = np.ones(N)/float(N)

    E = np.ones((N,N))/N
    G = (1-m)*A+m*E

    return G

def Power_Method(G, iter):
    N = G.shape[0]
    x0 = np.ones(N)/N

    print(0, x0, np.sum(x0))

    for i in range(iter):
        x1 = np.copy(x0)
        x0 = np.dot(G,x0)
        print(i+1, x0, np.sum(x0))
        if np.sum(np.abs(x1-x0)) < 1e-10:
            break
```

pagerank.py

Web Graph



Network Backbone

PNAS 106, 6487 (2009)

- Algorithm to identify the most relevant weights in a network.

- Applies a "disparity filter" to identify which edges should be preserved

- For each edge, calculate:

$$\alpha_{ij} = 1 - (k-1)\int_0^{p_{ij}} (1-x)^{k-2}\,\mathrm{d}x$$

- Keep edge if and only if:

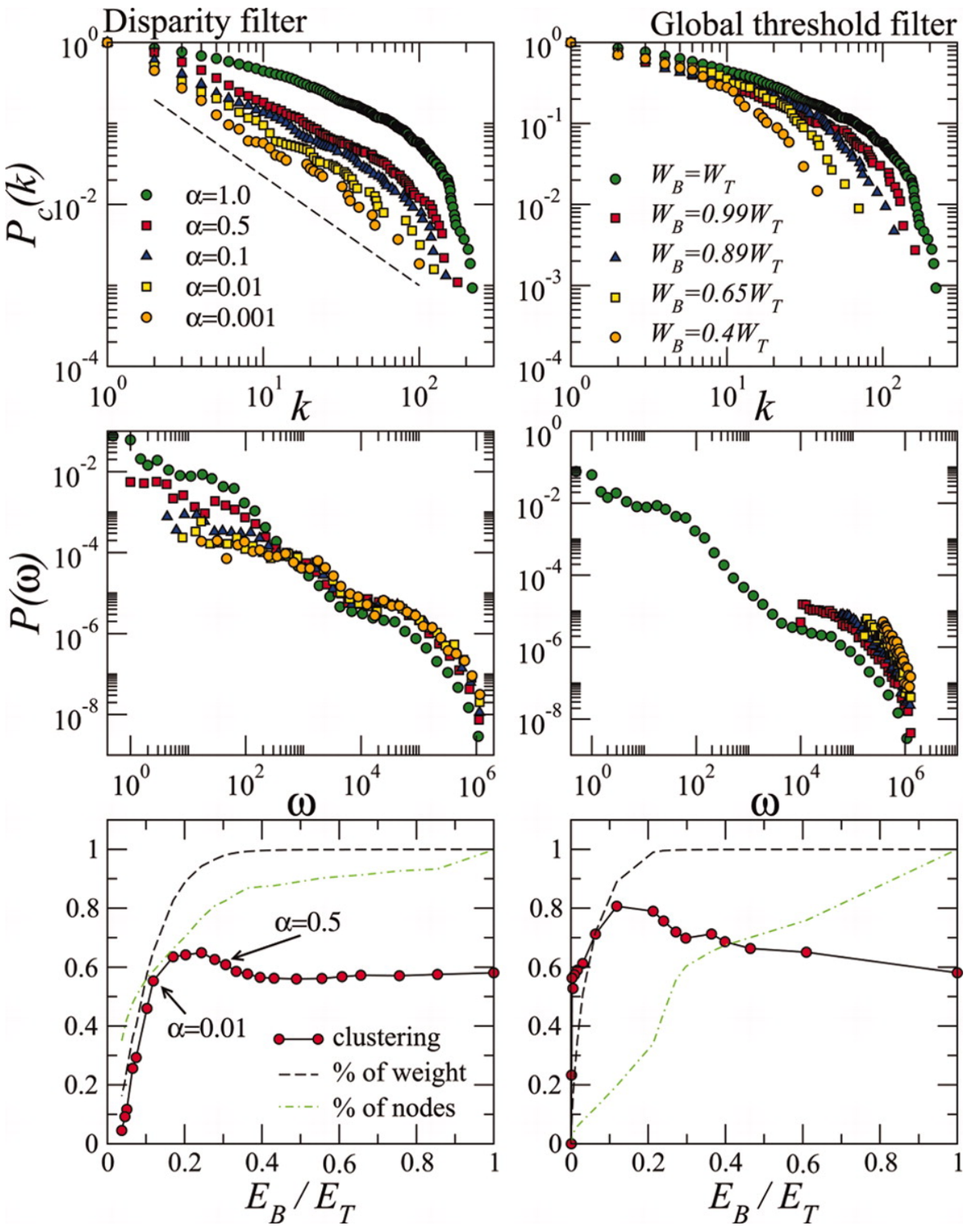
$$\alpha_{ij} < \alpha$$

- For some specified significance value, α

- Similar to p value in statistics.

Network Backbone

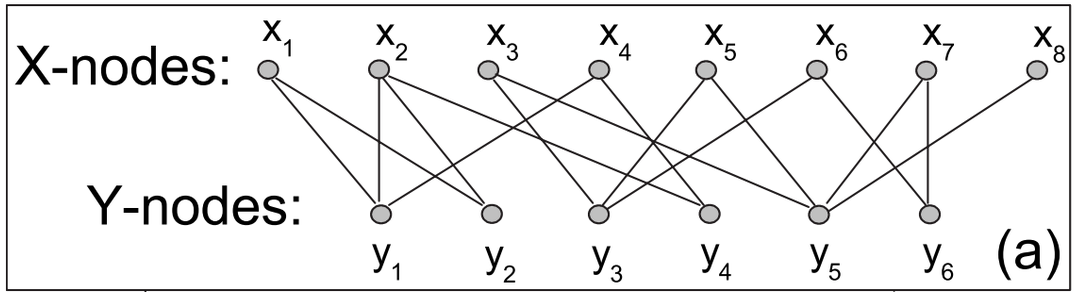
PNAS 106, 6487 (2009)



Bipartite Networks

Bipartite Networks

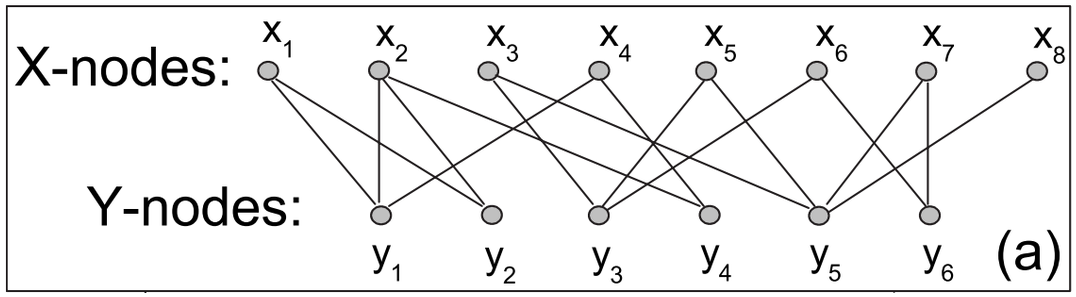
- Two kinds of nodes:



Bipartite Networks

- Two kinds of nodes:

- Edges only allowed between nodes of different kinds



Bipartite Networks

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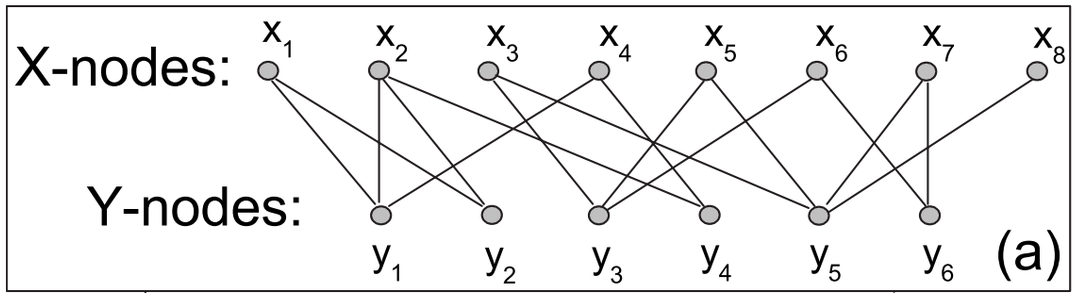
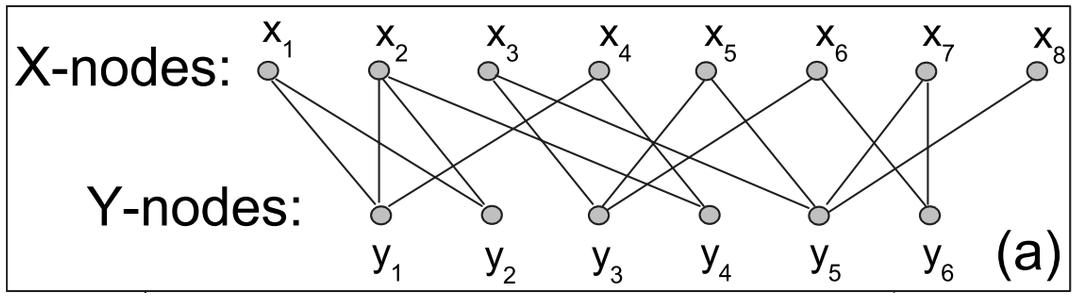


- Edges only allowed between nodes of different kinds

- Typically represent affiliation or collaboration:

Bipartite Networks

- Two kinds of nodes:



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  - actors in movies

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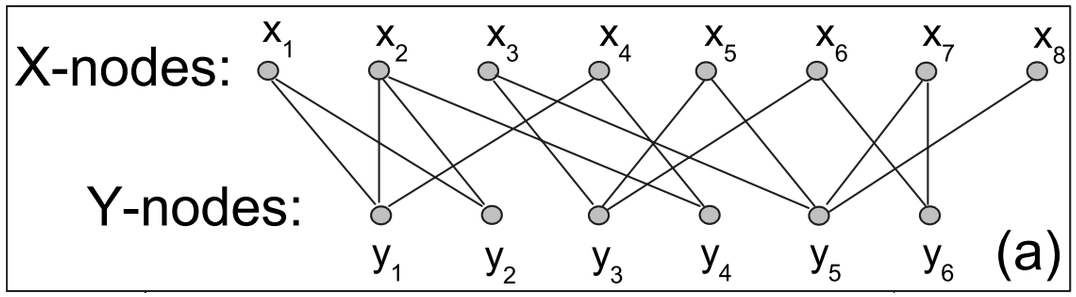

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Bipartite Networks

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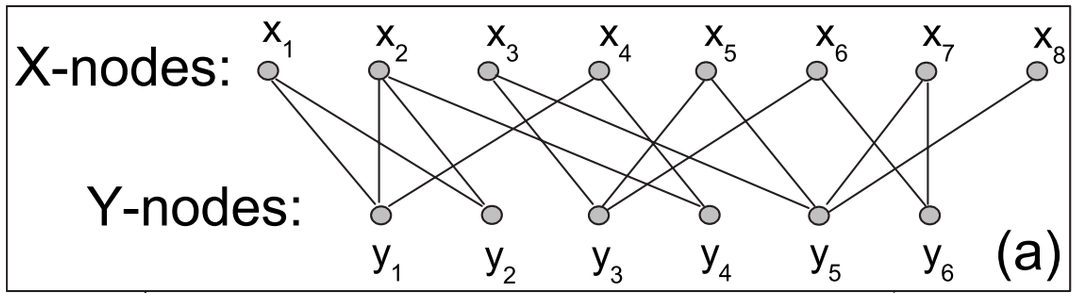

- Edges only allowed between nodes of different kinds

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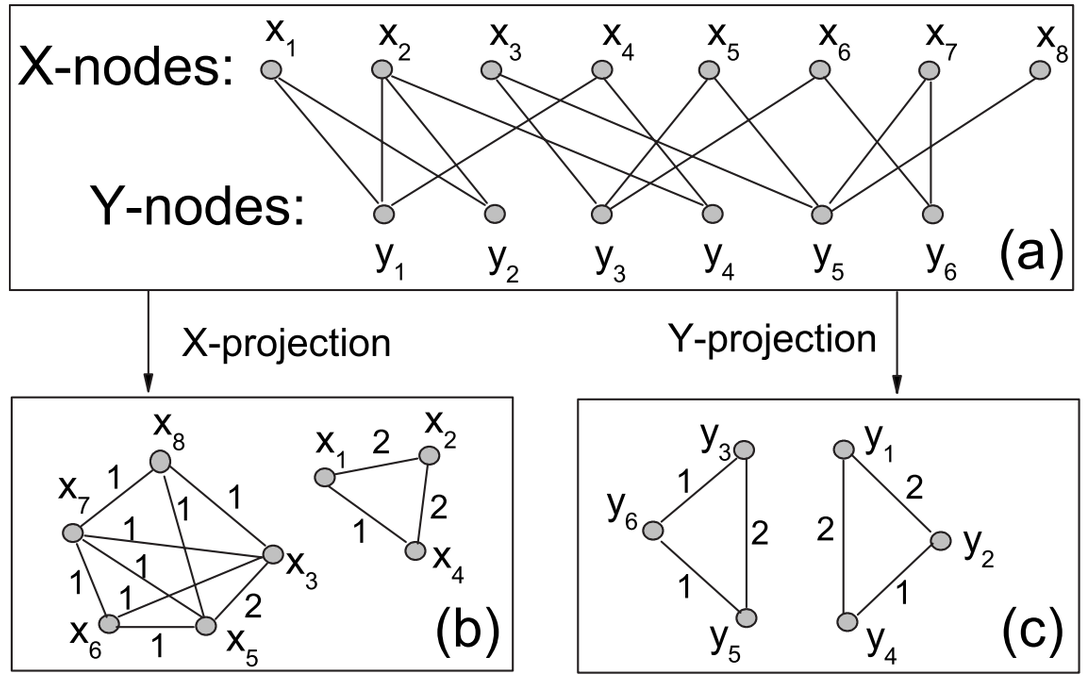
  - actors in movies
  - authors in papers
  - etc

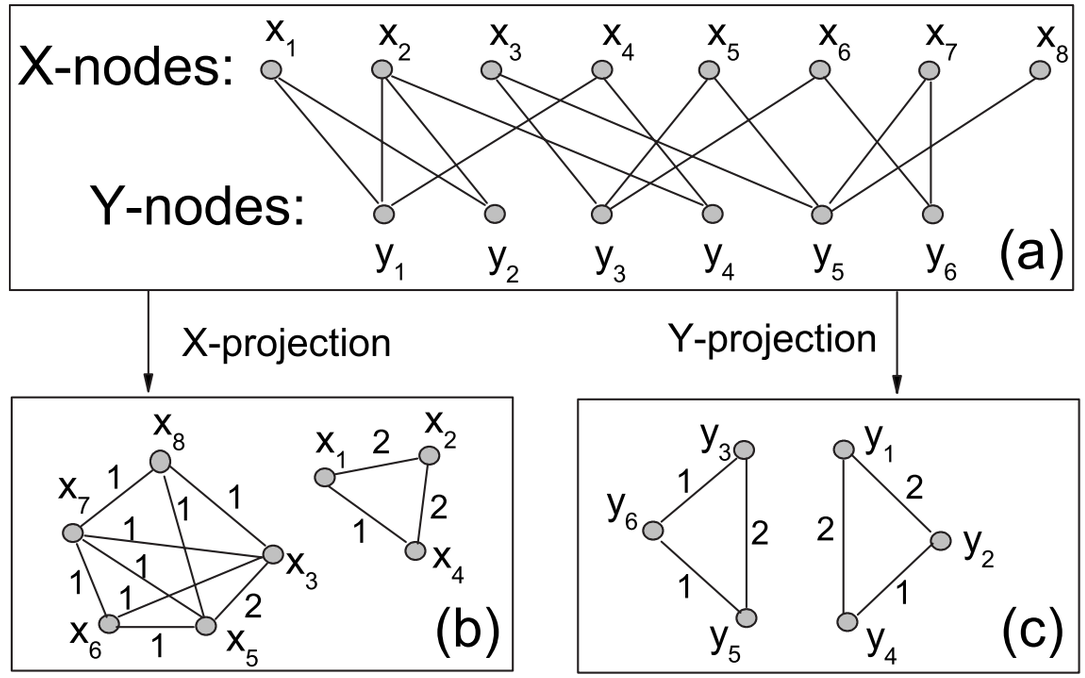
Bipartite Networks

- Two kinds of nodes:
- Edges only allowed between nodes of different kinds
- Typically represent affiliation or collaboration:
  - actors in movies
  - authors in papers
  - etc
- Usually the first step is to project them to a single node type



Bipartite Networks - Adjacency Matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$



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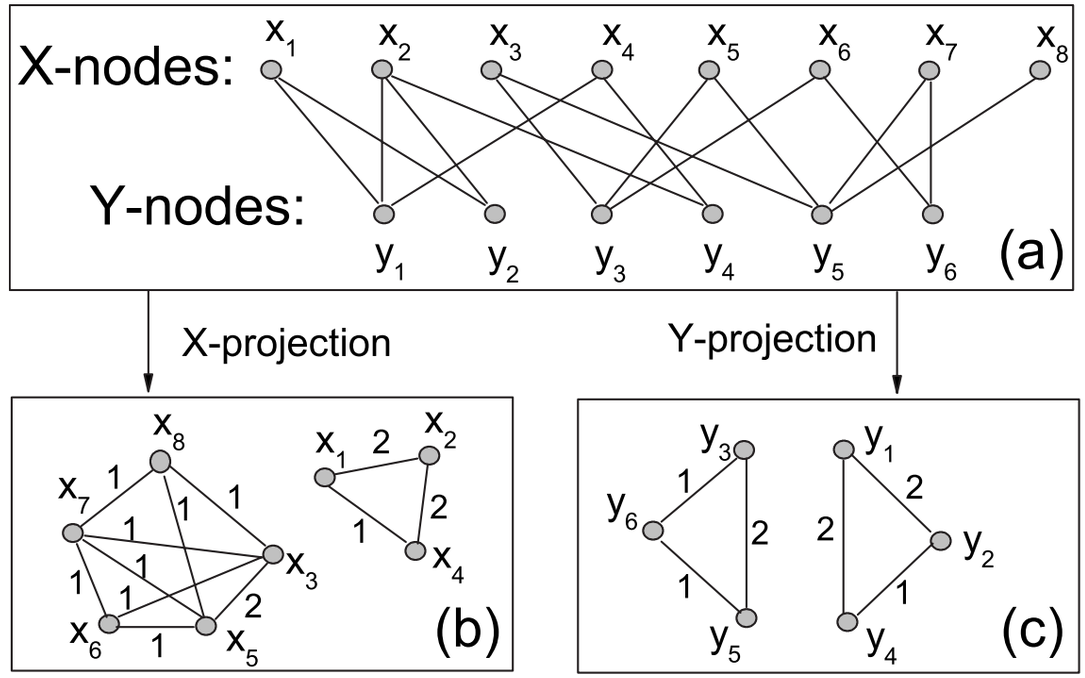
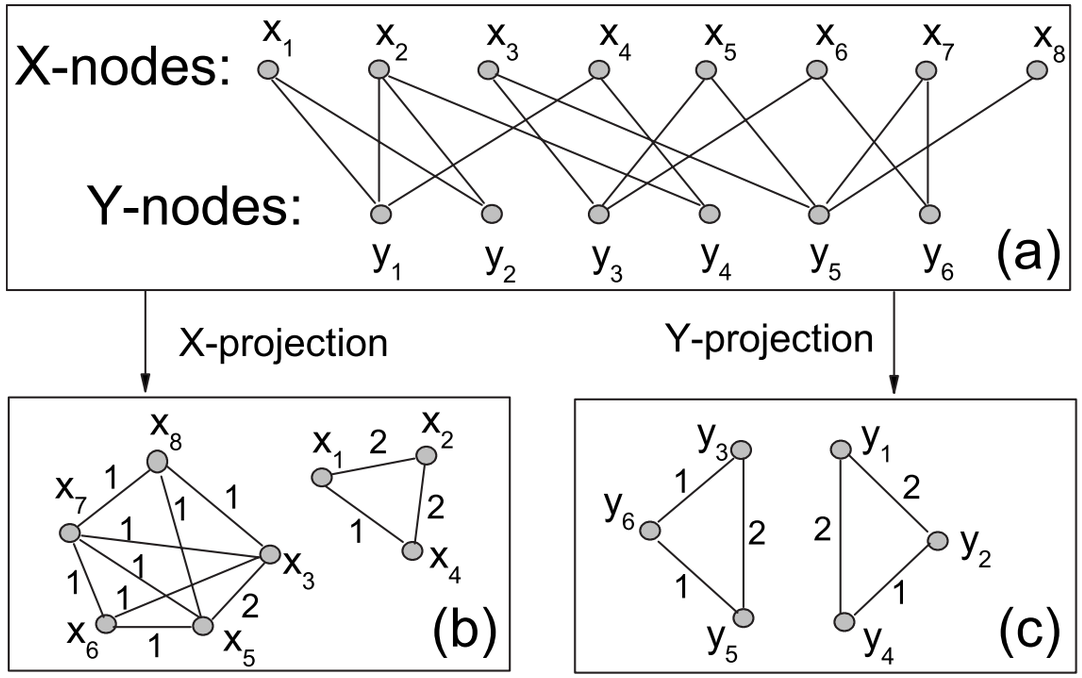
$$A = \begin{pmatrix} & & & y & & \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$



Bipartite Networks - Adjacency Matrix

$$A = \begin{matrix} \\ x \end{matrix} \overset{\overset{y}{\begin{matrix} \end{matrix}}}{\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}}$$



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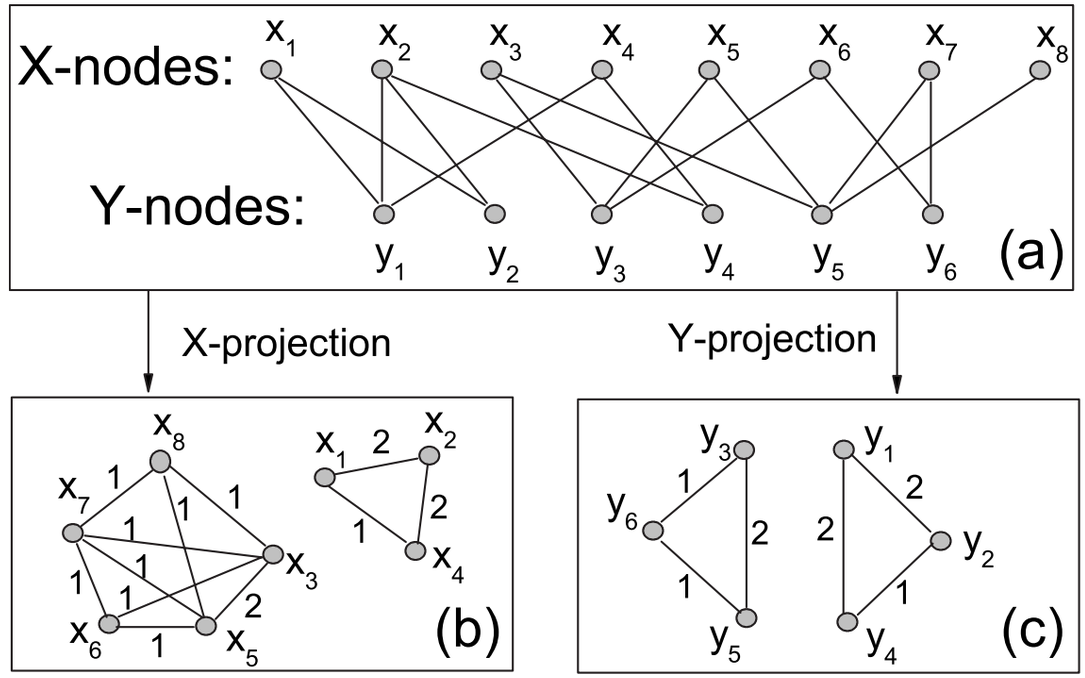
$$\begin{matrix} \\ y \end{matrix}$$



$$X = AA^T$$

Bipartite Networks - Adjacency Matrix

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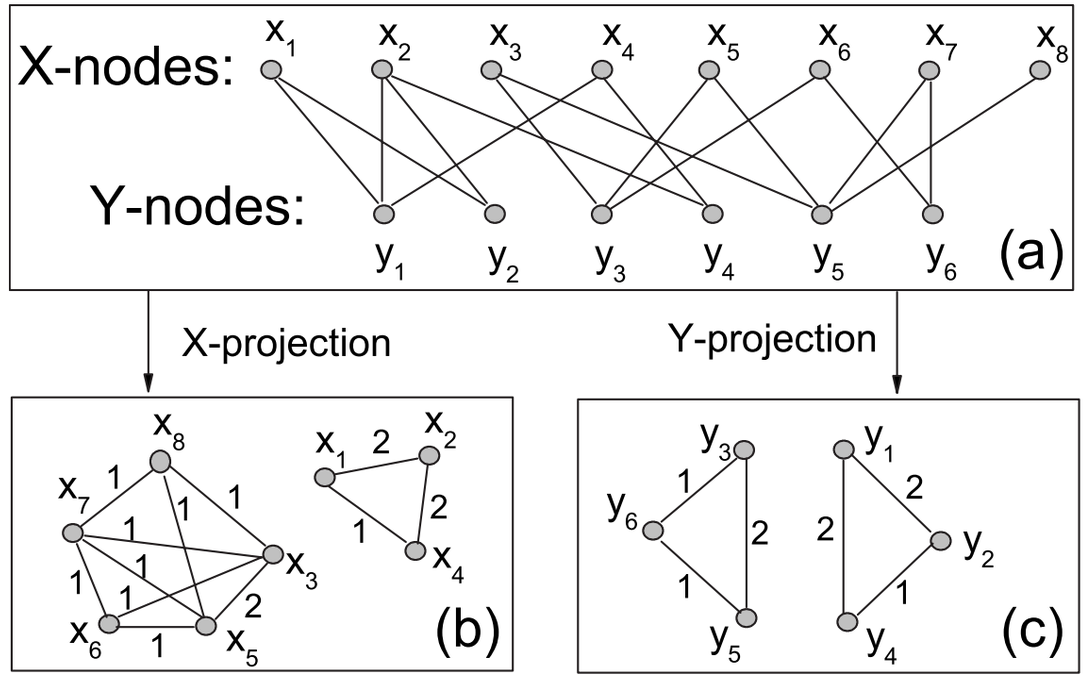

$$X = AA^T$$

$$Y = A^T A$$

Bipartite Networks - Similarities

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Bipartite Networks - Similarities

We can treat the adjacency matrix as a feature matrix!

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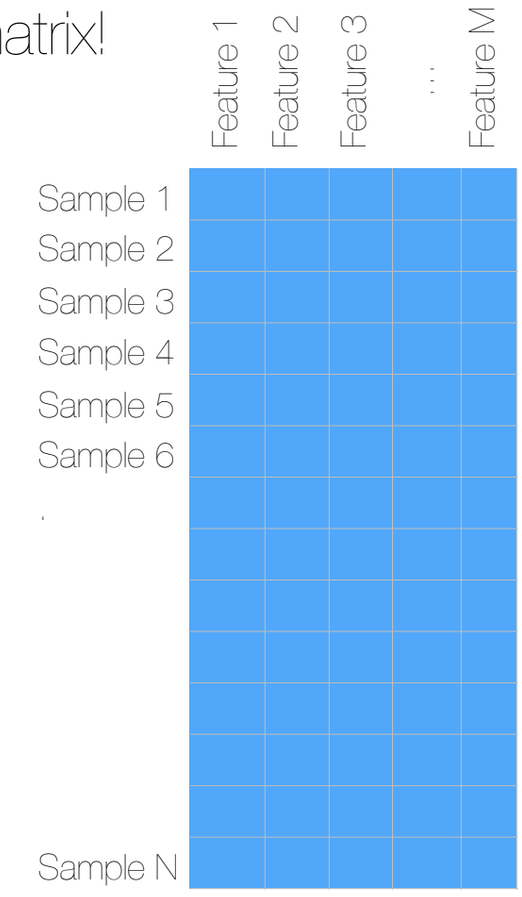

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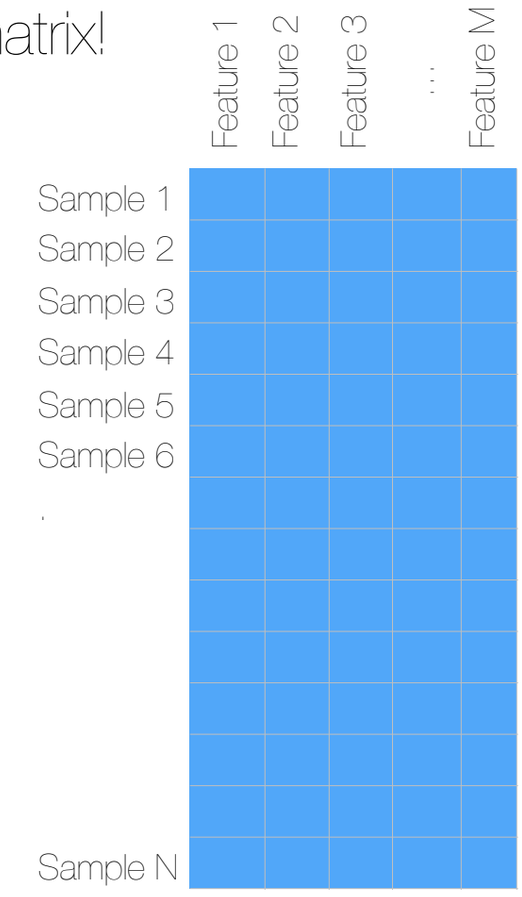
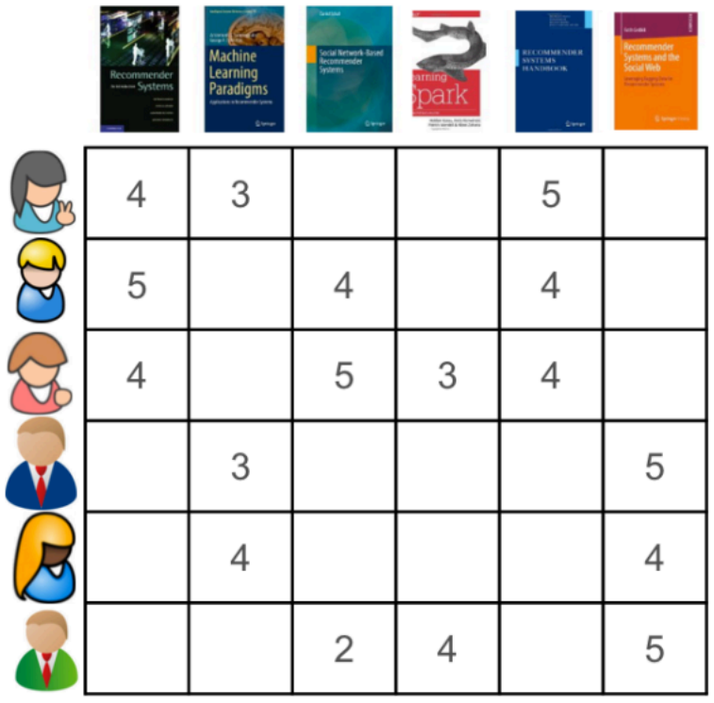


And generate a new matrix based on the similarities between items

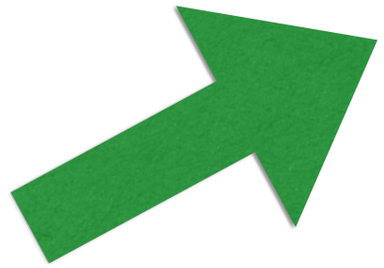
Bipartite Networks - Similarities

	4	3			5	
	5		4		4	
	4		5	3	4	
		3				5
		4				4
			2	4		5

Bipartite Networks - Similarities

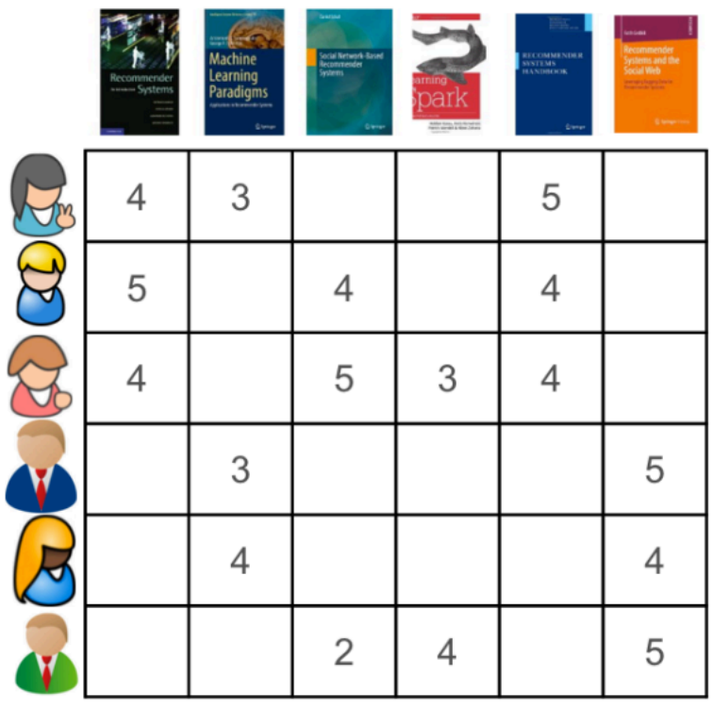


4	3			5	
5		4		4	
4		5	3	4	
	3				5
	4				4
		2	4		5



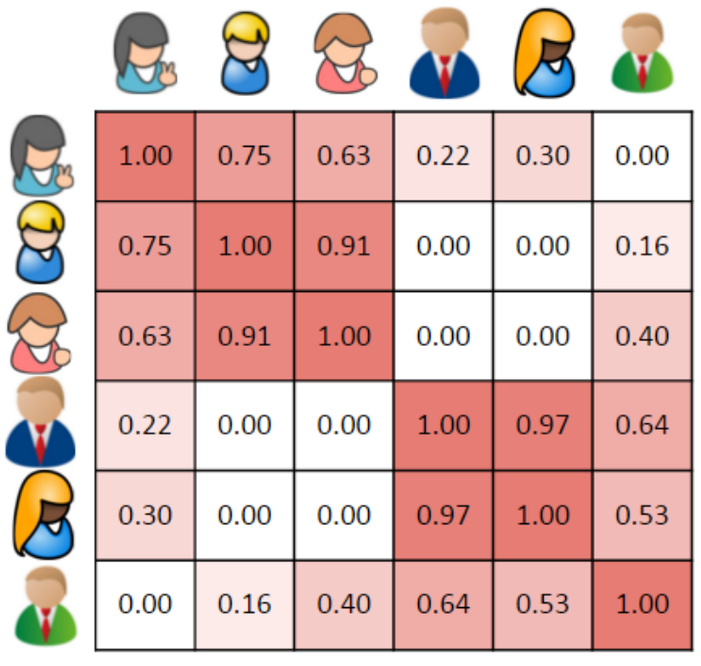
1.00	0.00	0.58	0.00	0.67	0.58
0.00	1.00	0.00	0.41	0.00	0.00
0.58	0.00	1.00	0.00	0.58	0.75
0.00	0.41	0.00	1.00	0.00	0.00
0.67	0.00	0.58	0.00	1.00	0.58
0.58	0.00	0.75	0.00	0.58	1.00

Bipartite Networks - Similarities



4	3			5	
5		4		4	
4		5	3	4	
	3				5
	4				4
		2	4		5

1.00	0.00	0.58	0.00	0.67	0.58
0.00	1.00	0.00	0.41	0.00	0.00
0.58	0.00	1.00	0.00	0.58	0.75
0.00	0.41	0.00	1.00	0.00	0.00
0.67	0.00	0.58	0.00	1.00	0.58
0.58	0.00	0.75	0.00	0.58	1.00



1.00	0.75	0.63	0.22	0.30	0.00
0.75	1.00	0.91	0.00	0.00	0.16
0.63	0.91	1.00	0.00	0.00	0.40
0.22	0.00	0.00	1.00	0.97	0.64
0.30	0.00	0.00	0.97	1.00	0.53
0.00	0.16	0.40	0.64	0.53	1.00

Bipartite Networks - Similarities

Sparse

4	3			5	
5		4		4	
4		5	3	4	
	3				5
	4				4
		2	4		5

Dense

1.00	0.00	0.58	0.00	0.67	0.58
0.00	1.00	0.00	0.41	0.00	0.00
0.58	0.00	1.00	0.00	0.58	0.75
0.00	0.41	0.00	1.00	0.00	0.00
0.67	0.00	0.58	0.00	1.00	0.58
0.58	0.00	0.75	0.00	0.58	1.00

1.00	0.75	0.63	0.22	0.30	0.00
0.75	1.00	0.91	0.00	0.00	0.16
0.63	0.91	1.00	0.00	0.00	0.40
0.22	0.00	0.00	1.00	0.97	0.64
0.30	0.00	0.00	0.97	1.00	0.53
0.00	0.16	0.40	0.64	0.53	1.00

RECOMMENDER SYSTEM ALGORITHMS

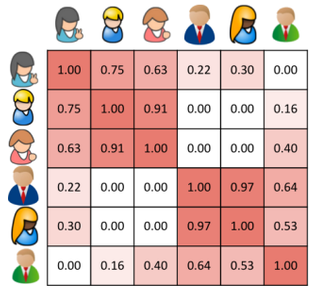
INPUT

4	3			5	
5		4		4	
4		5	3	4	
	3				5
	4				4
		2	4		5

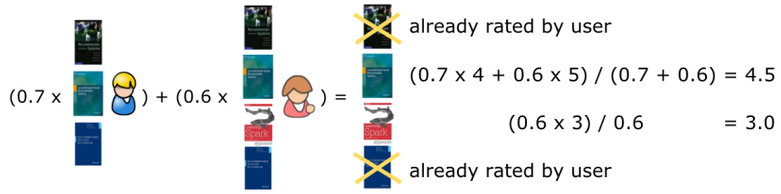
Introduction to Recommender Systems
Machine learning Paradigms
Social Network-based Recommender Systems
Learning Spark
Recommender Systems Handbook
Recommender Systems and the Social Web

FOUR RECOMMENDER ALGORITHMS ARE FED THE SAME INPUT AND PRODUCE DIFFERENT OUTPUTS.
THEIR DEFINITIONS OF USER AND/OR ITEM SIMILARITY ACCOUNT FOR MOST OF THE DIFFERENCES.

USER-BASED COLLABORATIVE FILTERING



1.00	0.75	0.63	0.22	0.30	0.00
0.75	1.00	0.91	0.00	0.00	0.16
0.63	0.91	1.00	0.00	0.00	0.40
0.22	0.00	0.00	1.00	0.97	0.64
0.30	0.00	0.00	0.97	1.00	0.53
0.00	0.16	0.40	0.64	0.53	1.00



ITEM-BASED COLLABORATIVE FILTERING

1.00	0.27	0.79	0.32	0.98	0.00
0.27	1.00	0.00	0.00	0.34	0.65
0.79	0.00	1.00	0.69	0.71	0.18
0.32	0.00	0.69	1.00	0.32	0.49
0.98	0.34	0.71	0.32	1.00	0.00
0.00	0.65	0.18	0.49	0.00	1.00

CONTENT-BASED FILTERING

1.00	0.00	0.58	0.00	0.67	0.58
0.00	1.00	0.00	0.41	0.00	0.00
0.58	0.00	1.00	0.00	0.58	0.75
0.00	0.41	0.00	1.00	0.00	0.00
0.67	0.00	0.58	0.00	1.00	0.58
0.58	0.00	0.75	0.00	0.58	1.00

HYBRID

OUTPUTS