Homework 11

ChBE 4411
November 22, 2021

Problem 1

1. See Fig. 1.

2. $$\mathbf{S}=\begin{bmatrix}1 & -0.8749\\ -0.8749 & 1\end{bmatrix},\ \mathbf{V}=\begin{bmatrix}-0.7071 & -0.7071\\ -0.7071 & 0.7071\end{bmatrix},\ \mathbf{D}=\begin{bmatrix}0.1251 & 0\\ 0 & 1.8749\end{bmatrix}.$$

   The larger eigenvalue is about 15 times of the smaller one, so we can choose $p=1$.

3. See Fig. 1. The data spreads widely along the direction of the eigenvector corresponding to the larger eigenvalue, and accumulates along the direction corresponding to the smaller eigenvalue.

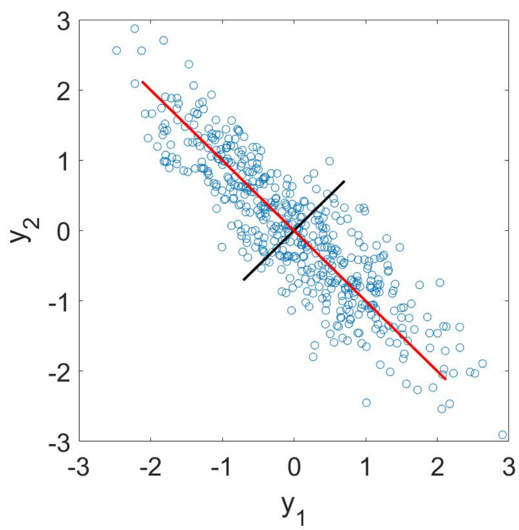


Figure 1: Scatter plot of $\mathbf{Y}$ and directions of two eigenvectors of $\mathbf{S}$.

Problem 2

1. $$\mathbf{V}=\begin{bmatrix}-0.0863 & 0.6987 & 0.2700 & -0.6138 & -0.2337\\ 0.6815 & 0.0815 & -0.0977 & 0.2158 & -0.6876\\ -0.4848 & -0.1028 & -0.7168 & -0.1924 & -0.4512\\ 0.5406 & -0.1205 & -0.4527 & -0.5675 & 0.4076\\ 0.0290 & 0.6929 & -0.4459 & 0.4664 & 0.3205\end{bmatrix},\mathbf{D}=\begin{bmatrix}2.0500 & 0 & 0 & 0 & 0\\ 0 & 1.4270 & 0 & 0 & 0\\ 0 & 0 & 0.9352 & 0 & 0\\ 0 & 0 & 0 & 0.5828 & 0\\ 0 & 0 & 0 & 0 & 0.0051\end{bmatrix},$$
   $$\tilde{\mathbf{V}}=\begin{bmatrix}-0.0863 & 0.6987 & 0.2700 & -0.6138\\ 0.6815 & 0.0815 & -0.0977 & 0.2158\\ -0.4848 & -0.1028 & -0.7168 & -0.1924\\ 0.5406 & -0.1205 & -0.4527 & -0.5675\\ 0.0290 & 0.6929 & -0.4459 & 0.4664\end{bmatrix},\qquad \tilde{\mathbf{D}}=\begin{bmatrix}2.0500 & 0 & 0 & 0\\ 0 & 1.4270 & 0 & 0\\ 0 & 0 & 0.9352 & 0\\ 0 & 0 & 0 & 0.5828\end{bmatrix}.$$

2. See Fig. 2. The false alarm rate is 4.8%.

3. The fault is detected here. See Fig. 3a

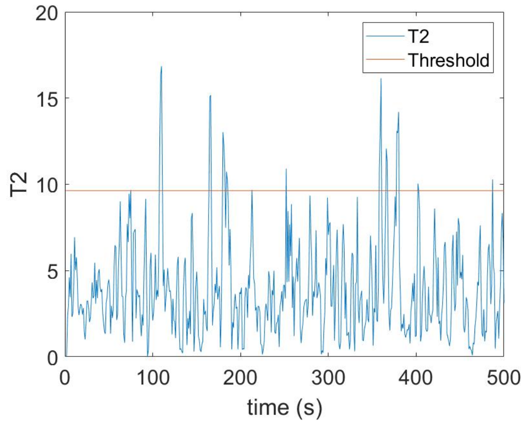
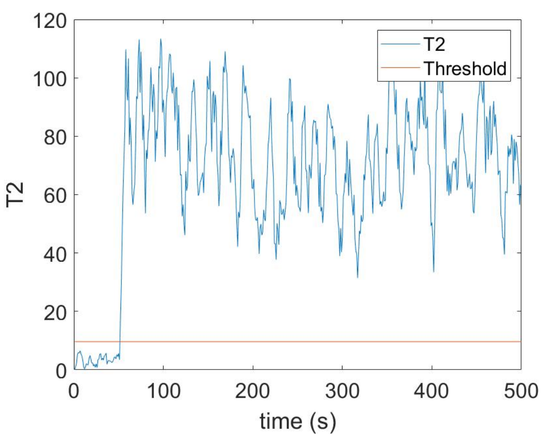


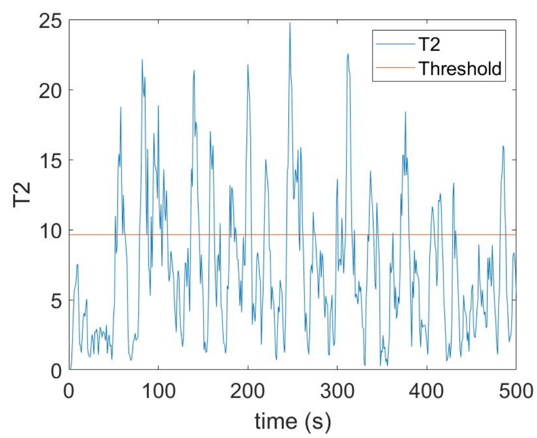
Figure 2: Plot of T^2 versus time for the fault-free model.

4. The fault is not as evident as in the previous one. The T^2 plot looks like one in a fault-free model with a high false alarm rate. See Fig. 3b.

5. Another two faulty models are considered here. In the first one, the measurement noises change such that the mean value shift by 10. This fault can be detected by PCA. See Fig. 4a. In the second one, the matrix A_{22} takes a zero matrix, and the fault cannot be detected. See Fig. 4b.

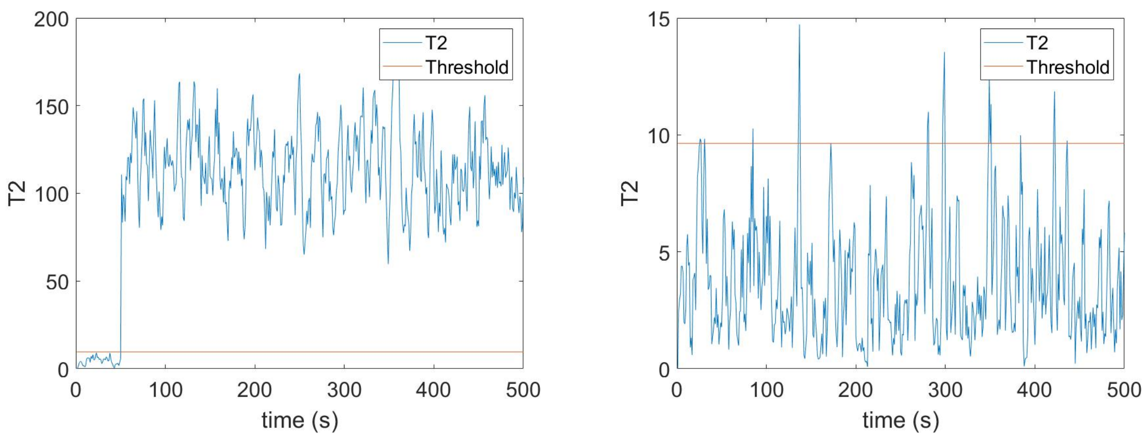


(a) The first faulty model.



(b) The second faulty model.

Figure 3: The T^2 plot in two faulty models.



(a) A faulty model that can be detected. (b) A faulty model that cannot be detected.

Figure 4: The T^2 plot in two new faulty models.