$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$

cos(3α) $= \cos(2\alpha+\alpha) = \cos2\alpha\cos\alpha - \sin2\alpha\sin\alpha = (2\cos^2\alpha - 1)\cos\alpha - 2\sin^2\alpha\cos\alpha =$
$= (2\cos^2\alpha - 1)\cos\alpha - 2\cos\alpha(1-\cos^2\alpha) = 2\cos^3\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha =$
$= \mathbf{4\cos^3\alpha - 3\cos\alpha = \cos(3\alpha)}$

sin(3α) $= \sin(2\alpha+\alpha) = \sin2\alpha\cos\alpha + \cos2\alpha\sin\alpha = 2\sin\alpha\cos^2\alpha + \cos2\alpha\sin\alpha =$
$= \sin\alpha(2\cos^2\alpha + \cos2\alpha) = \sin\alpha(2(1-\sin^2\alpha) + (1 - 2\sin^2\alpha)) =$
$= \sin\alpha(2 - 2\sin^2\alpha + 1 - 2\sin^2\alpha) = \sin\alpha(3 - 4\sin\alpha) = \mathbf{3\sin\alpha - 4\sin^3\alpha} =$
$= \mathbf{\sin(3\alpha)}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Addition Theorems

Now, let $x = \frac{\alpha+\beta}{2}$ and $y = \frac{\alpha-\beta}{2}$ => $\alpha = \frac{x+y}{2}$ and $\beta = \frac{x-y}{2}$

$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha - \beta)$ => $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$2\sin\beta\cos\alpha = \sin(\alpha+b) - \sin(\alpha-\beta)$ => $\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$

$-2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$ => $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$ => $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$