Quadratic Equation

Given an equation $\mathbf{ax^2 + bx + c = 0.}$ The first thing to be done is to find $\mathbf{D = b^2 - 4ac.}$

If $\mathbf{D < 0}$ => no real roots

If $\mathbf{D = 0}$ => $x_1 = x_2 = -\frac{b}{2a}$

If $\mathbf{D > 0}$ => there are 2 distinct roots: $x_1 = \frac{-b + \sqrt{D}}{2a}$ $\qquad$ $x_2 = \frac{-b - \sqrt{D}}{2a}$

Here is a check:

$(x - x_1)(x-x_2) =$

$$\left(x - \left(\frac{-b + \sqrt{D}}{2a}\right)\right)\left(x - \left(\frac{-b - \sqrt{D}}{2a}\right)\right) = \left(\frac{(2ax + b) + \sqrt{D}}{2a}\right)\left(\frac{(2ax + b) - \sqrt{D}}{2a}\right) =$$

$$= \frac{(2ax + b)^2 - D}{4a^2} = \frac{\left(4a^2x^2 + 4abx + b^2 - \left(b^2 - 4ac\right)\right)}{4a^2} = x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

You'll get the original equation by multiplying by **a** the equation obtained above.

Vieta's Theorem

$$x_1x_2 = \left(\frac{-b + \sqrt{D}}{2a}\right)\left(\frac{-b - \sqrt{D}}{2a}\right) = \frac{\left(b^2 - D\right)}{4a^2} = \frac{4\,ac}{4a^2} = \frac{c}{a}$$

$$x_1 + x_2 = \left(\frac{-b + \sqrt{D}}{2a}\right) + \left(\frac{-b - \sqrt{D}}{2a}\right) = -\frac{2b}{2a} = -\frac{b}{a}$$