Properties of Logarithms

1. If $x > 0 \Rightarrow x = a^{\log_a x}$ $(\forall\ a>0,\ a\neq 1)$
2. $\log_a a = 1$ $(\forall\ a>0,\ a\neq 1)$
3. $\log_a 1 = 0$ $(\forall\ a>0,\ a\neq 1)$
4. If $x_1 > 0$ and $x_2 > 0$:

   $\log_a(x_1 x_2) = \log_a x_1 + \log_a x_2$

   $\log_a\left(\dfrac{x_1}{x_2}\right) = \log_a x_1 - \log_a x_2$

   $(\forall a>0 \text{ and } a\neq 1)$
5. If $x > 0 \Rightarrow \log_a x^p = p\log_a x$ $(\forall\ a>0,\ a\neq 1)$
6. If $x > 0 \Rightarrow \log_a x = \dfrac{\log_b x}{\log_b a}$ $(\forall\ b>0,\ b\neq 1)$ **Change of Base**
7. $\log_a b = \dfrac{1}{\log_b a} \iff \log_a b \cdot \log_b a = 1$ $(\forall\ a>0,\ a\neq 1,\ b>0,\ b\neq 1)$
8. $\log_{a^m} b = \dfrac{1}{m}\log_a b$ $(a>0,\ a\neq 1,\ m\neq 0,\ b>0)$
9. $\log_a b = m\log_{a^m} b = \log_{a^m} b^m$ $(a>0,\ a\neq 1,\ m\in\mathbb{R} \text{ and } m\neq 0)$

Exponential Function Properties

$y = a^x$, $a>0$, $a\neq 1$

1. Domain: $\mathbb{R}$ (set of all real numbers)
2. Range: $\mathbb{R}^+$ (set of all positive numbers: $a^x > 0,\ \forall x$)
3. If $a>1$ $y\uparrow$, i.e. if $x_1 < x_2 \Rightarrow a^{x_1} < a^{x_2}$

   If $0<a<1$ $y\downarrow$, i.e. if $x_1 < x_2 \Rightarrow a^{x_1} > a^{x_2}$
4. If $a^{x_1} = a^{x_2} \Rightarrow x_1 = x_2$

Logarithmic Function Properties

$y = \log_a x$, $a>0$, $a\neq 1$

1. Domain: $\mathbb{R}^+$
2. Range: $\mathbb{R}$
3. If $a>1$ $y\uparrow$, i.e. if $x_2 > x_1 > 0 \Rightarrow \log_a x_2 > \log_a x_1$
4. If $0<a<1$ $y\downarrow$, i.e. if $x_2 > x_1 > 0 \Rightarrow \log_a x_2 < \log_a x_1$