Trigonometric Equations

1. Simplest trigonometric equations sre ones of the kind :
   $\sin(x) = a$, $\cos(x) = a \cdot (where\ |a| \le 1)$
   $\tan(x) = tg(x) = a$, $\cot(x) = ctg(x) = a \cdot (where\ -\infty < a < +\infty)$

   Solution formulas to them are below :
   $\sin(x) = sinx = a; x = (-1)^n \arcsin(a) + \pi n, n \in Z$
   $\cos(x) = cosx = a; x = \pm \arccos(a) + 2\pi n, n \in Z$
   $\tan(x) = tg(x) = tanx = tgx = a; x = \arctan(a) + \pi n = arctg(a) + \pi n, n \in Z$
   $\cot(x) = ctg(x) = cotx = ctgx = a; x = \text{arccot}(a) + \pi n = arcctg(a) + \pi n, n \in Z$

   In special cases, if $a = 0$, $a = 1$, $a = -1$, there are obtained the following formulas :

   $\sin(x) = 0 \quad x = \pi n, n \in Z$
   $\sin(x) = 1 \quad x = \frac{\pi}{2} + 2\pi n, n \in Z$
   $\sin(x) = -1 \quad x = -\frac{\pi}{2} + 2\pi n, n \in Z$

   $\cos(x) = 0 \quad x = \frac{\pi}{2} + \pi n, n \in Z$
   $\cos(x) = 1 \quad x = 2\pi n, n \in Z$
   $\cos(x) = -1 \quad x = \pi + 2\pi n, n \in Z$

   $\tan(x) = tg(x) = 0 \quad x = \pi n, n \in Z$
   $\cot(x) = ctg(x) = 0 \quad x = \frac{\pi}{2} + \pi n, n \in Z$

2. **Homogeneous equations** are the ones of the kind :
   $\text{asin}(\text{kx}) + \text{bcos}(\text{kx}) = 0$
   $\text{asin}^2(\text{kx}) + \text{bsin}(\text{kx})\cos(\text{kx}) + \text{ccos}^2(\text{kx}) = 0$
   $\text{asin}^3(\text{kx}) + \text{bsin}^2(\text{kx})\cos(\text{kx}) + \text{csin}(\text{kx})\cos^2(\text{kx}) + \text{dcos}^3(\text{kx}) = 0$

   Equation $\text{asin}^2(\text{kx}) + \text{bsin}(\text{kx})\cos(\text{kx}) + \text{ccos}^2(\text{kx}) = \text{d}$, if $\text{d} \ne 0$ is **NOT** homogeneous, can be reduced to type b by substitution of d by $\text{d}(\sin^2\text{x} + \cos^2\text{x})$.

   For solutions of equations of types a - c in case of $\text{a} \ne 0$,
   let's consider such values of x for which $\cos(\text{kx}) = 0$.
   Then from each equation follows that with the same values of x
   should be $\sin(\text{kx}) = 0$ which is impossible.
   Therefore, solutions of these equations are such values of x for which $\cos(\text{kx}) \ne 0$.
   Therefore, if $\text{a} \ne 0$, divide both sides of equation a by $\cos(\text{kx})$,
   both sides of equation b by $\cos^2\text{x}$,
   both sides of equation c by $\cos^3(\text{kx})$, there will be no loss of roots.

Z - set of all integer numbers.