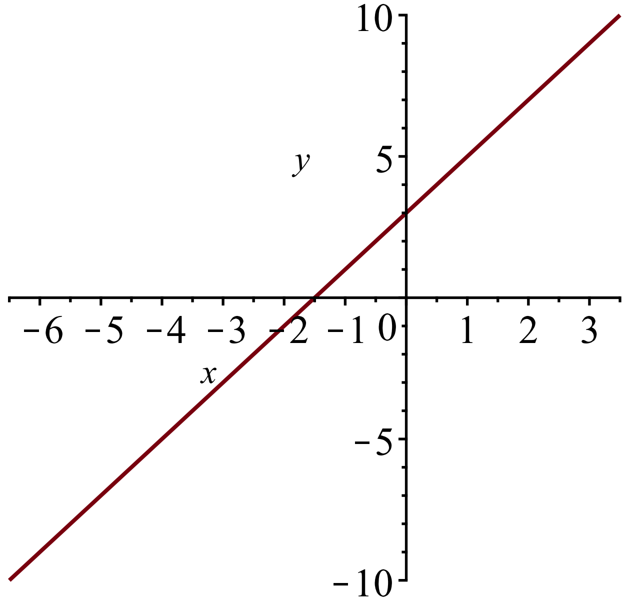
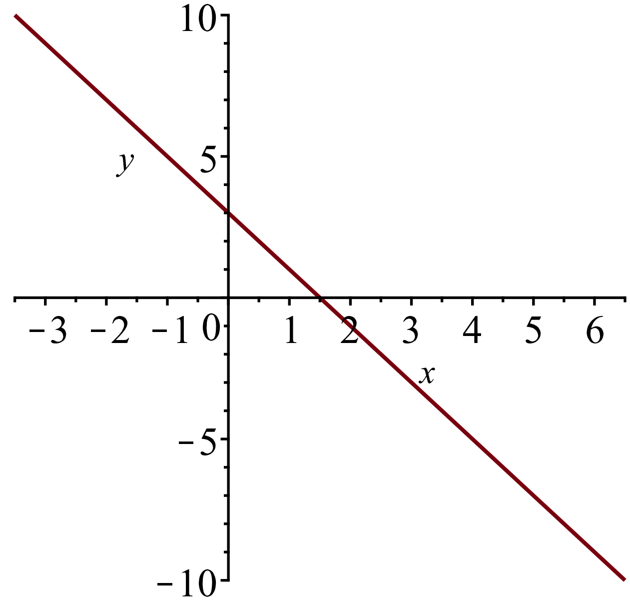
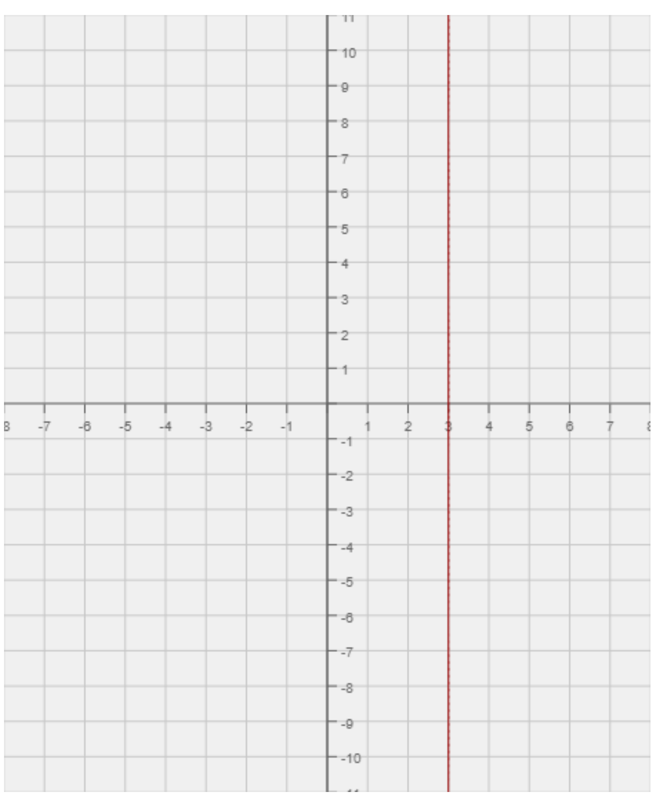
Straight Line Equation



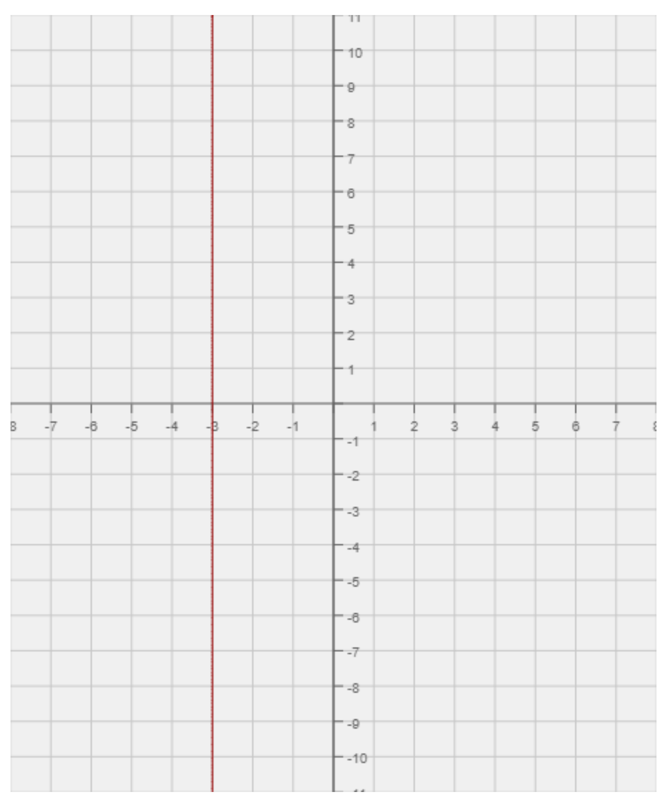
y=$k \cdot x + b$ (k>0, d is a segment on y-axis)



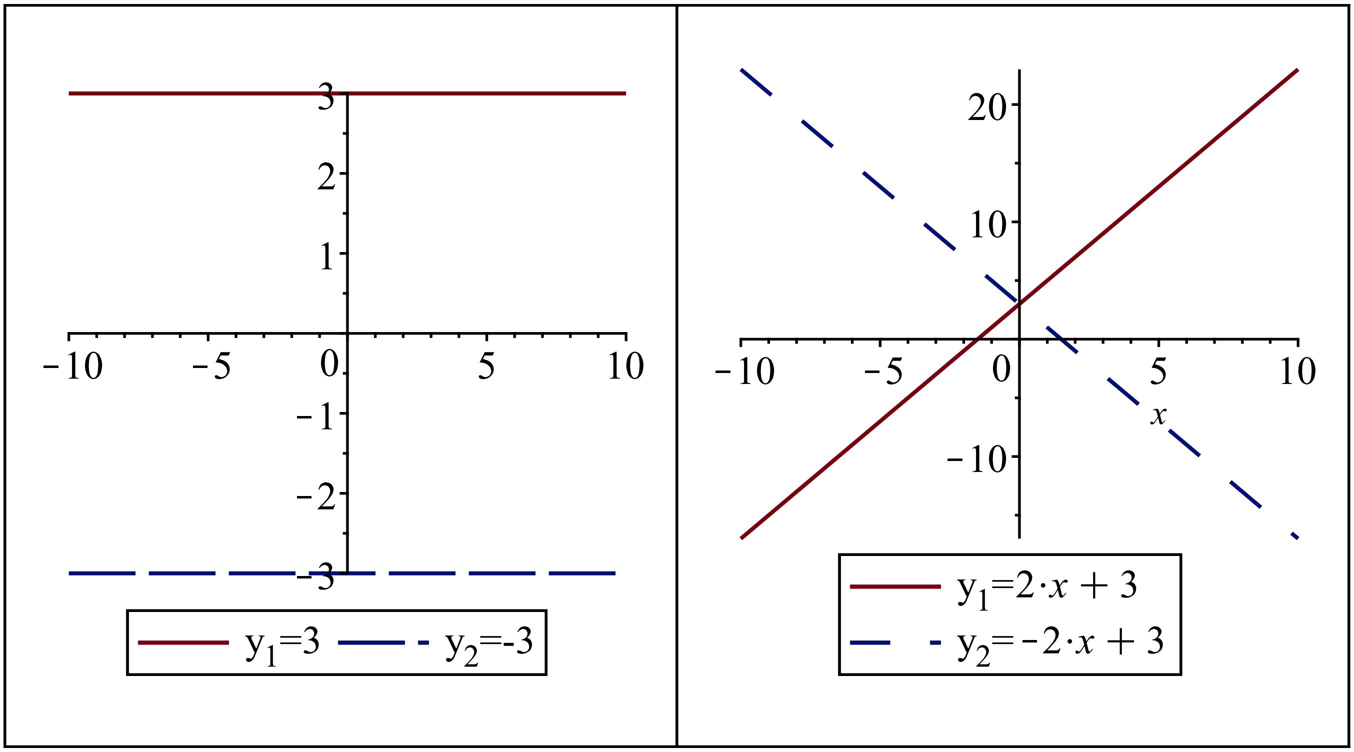
y=$m \cdot x + d$ (m<0, d is a segment on y-axis)



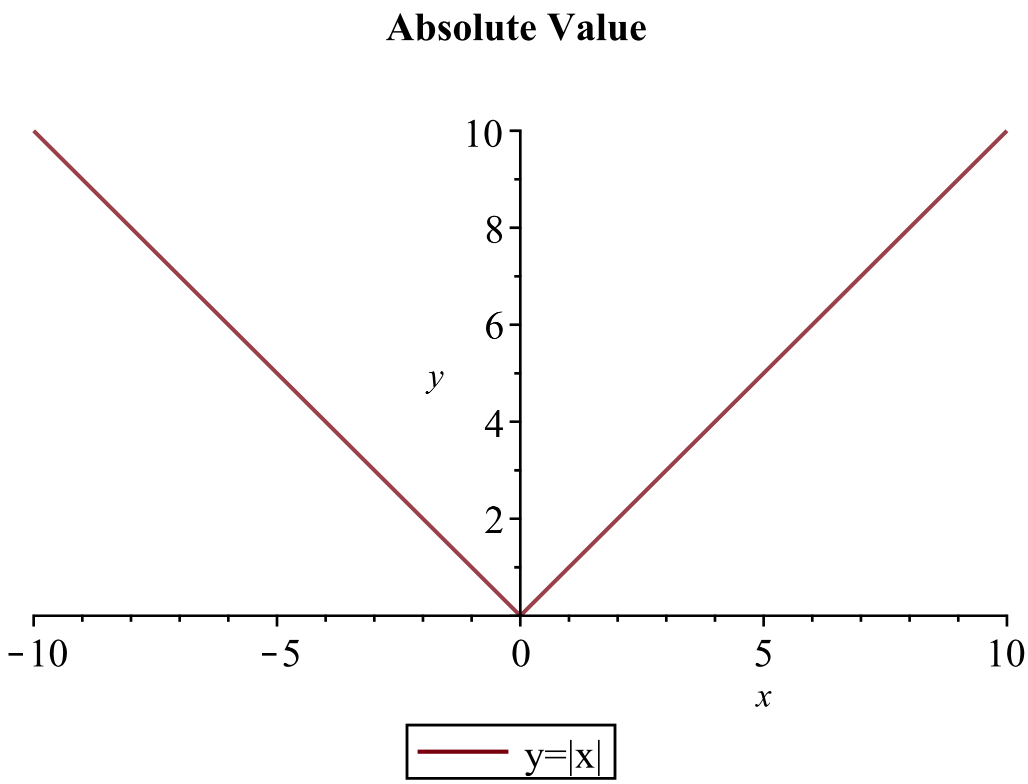
x=3



x=-3



Only 2 points are required to draw a straight line thought them.
A few more special cases are shown below.



$$y=|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

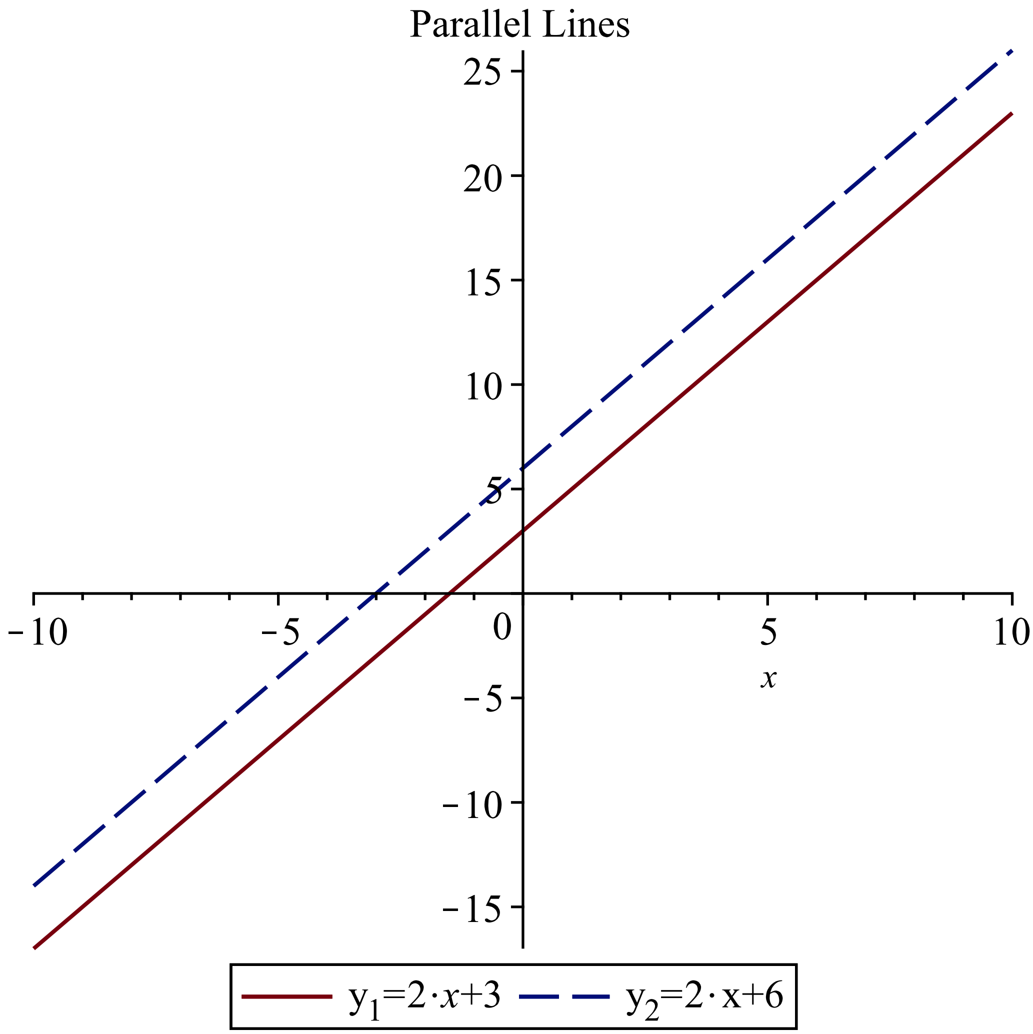
$y_1 = x = k_1 \cdot x + b_1 = 1 \cdot x + 0 \Rightarrow k_1 = 1, b_1 = 0$
$y_2 = -x = k_2 \cdot x + b_2 = -1 \cdot x + 0 \Rightarrow k_2 = -1, b_2 = 0$

Since $y_1 \cdot y_2 = -1 \Rightarrow y_1 \perp y_2$

More analysis of lines' equations is shown below:

$y = k \cdot x + b$

1. If x=0 => y=b
2. If b=0 => $y = k \cdot x$, this line goes through the origin (0,0).
3. Given 2 lines's equations: $y_1 = k_1 \cdot x + b_1$ and $y_2 = k_2 \cdot x + b_2$. When are they parallel?



They are parallel iff $k_1=k_2$ and $b_1 \neq b_2$.

If $k_1=k_2$ and $b_1=b_2$, both equations represent the same line.

List of Typical Problems

1. Line equation via 2 points $A(x_1,y_1)$ and $B(x_2, y_2)$

$$tg\alpha = k = \frac{y_2 - y_1}{x_2 - x_1} = tan\alpha$$

To find b: using $A(x_1, y_1)$: $y_1 = k\cdot x_1+b \Rightarrow b = y_1-k\cdot x_1$

$y = k\cdot x + b$

2. Given a line equation and a point on this line. Find an equation of a line ⊥ to this line and intersecing it at a given point $A(x_0, y_0)$.

$y = k\cdot x_1 + b_1$ - given line.

$A(x_0, y_0)$ is a given intersection point of both lines.

The equation of a ⊥ line is $y = k_2\cdot x + b_2$ where $k_1\cdot k_2 = -1 \Rightarrow k_2 = -\frac{1}{k_1}$

$b_2 = y - k_2\cdot x \Rightarrow$ For point $A(x_0, y_0)$: $b_2 = y_0 - k_2\cdot x_0$

3. Solving system of 2 linear equations graphically

$$\begin{cases} a_1\cdot x + b_1\cdot y = c_1 \\ a_2\cdot x + b_2 . y = c_2 \end{cases} \quad <\equiv> \quad \begin{cases} y = \dfrac{c_1 - a_1\cdot x}{b_1} \\ y = \dfrac{c_2 - a_2\cdot x}{b_2} \end{cases}$$

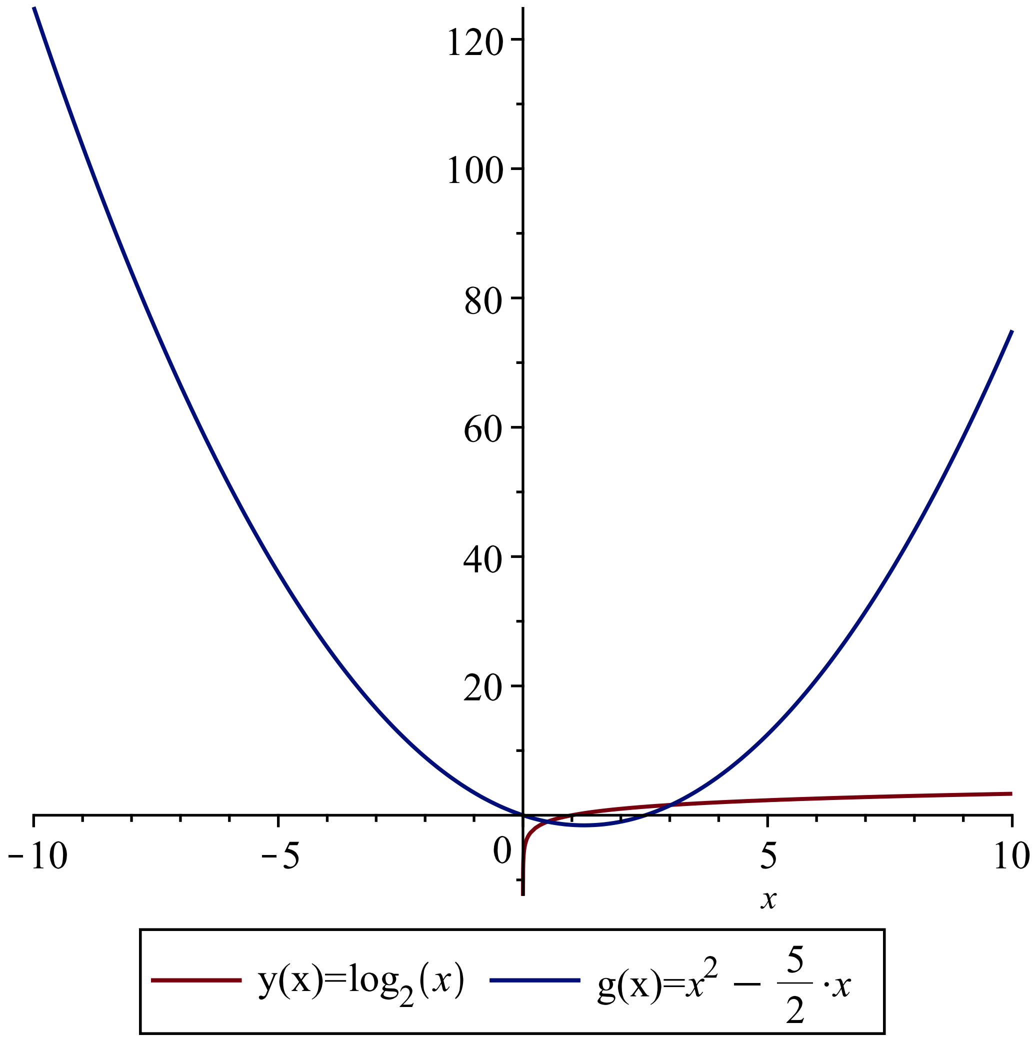
The intersection of these lines at point $A(x_0=x, y_0=y)$ is solution of this system.

4. Solving equation where both sides are different functions: i.e. linear, logarithmic, polynomial, exponential, etc.

$\log_2(x) = x^2 - \frac{5}{2}\cdot x$ Answer: $\frac{1}{2}$

Consider the equation above as $y(x) = \log_2(x) = g(x) = x^2 - \frac{5}{2}\cdot x$.

Graph $y(x) = \log_2(x)$ and $g(x) = x^2 - \frac{5}{2}\cdot x$



The intersection of y(x) and g(x) is at some point A(x_0, y_0) where x=x_0 is a solution of this equation.

5. Do lines $y_1 = k_1.x_1 + b_1$ and $y_2 = k_2 \cdot x_2 + b_2$ intersect?
   Look if $k_1 \cdot k_2 = -1$ ($\perp$ lines) or $k_1=k_2$ ($\|$ lines).

   Otherwise, solve equation $\mathbf{y_1} = k \cdot x_1 + b_1 = \mathbf{y_2} = k_2 \cdot x_2 + b_2$.
   x_0 is the intersection point, $\mathbf{y_1} = k_1 \cdot x_0 + b_1 = k_2 \cdot x_0 + b_2 = \boldsymbol{y_2}$.

Relations with Inequalities