INTRODUCTION[1]

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Since its publication more than six decades ago, Kenneth J. Arrow's (1950, 1951) impossibility theorem has profoundly influenced the thinking of all who are interested in issues relating to social choice and welfare, and the contributions of Eric Maskin and Amartya Sen to the vast literature, which followed Arrow's theorem, have been of fundamental importance. It is a great pleasure for me to write an introduction to this volume based on Eric Maskin's and Amartya Sen's lectures at Columbia University on Arrow's impossibility theorem, especially since, as a graduate student, I was first introduced to Arrow's impossibility theorem (and much else in welfare economics) by Amartya Sen, and I have known Eric Maskin for a long time.

WELFARE ECONOMICS AND THE THEORY OF SOCIAL CHOICE AT THE TIME OF PUBLICATION OF ARROW'S IMPOSSIBILITY THEOREM

Arrow's (1950, 1951) impossibility theorem is one of those rare intellectual contributions that virtually transform entire disciplines or subdisciplines. Welfare economics and the theory of social choice were the main areas impacted in this fashion by Arrow's theorem, though the theorem has also had far-reaching influence on political philosophy and political theory. It may be helpful to take a quick look at the state of welfare economics at the time when Arrow (1950, 1951) published his celebrated theorem. The important contributions of Bergson (1938) and Samuelson (1947), with their precise notion of a social welfare function, had already made it amply clear that economists needed to use value judgments if they wanted to engage in policy prescription or evaluation of social states, but the set of value judgments that welfare economists focused on in the 1930s and 1940s was remarkably small. The earlier utilitarian tradition was out of favor thanks to the rise of ordinal analysis in the theory of consumers' behavior and widespread skepticism about the possibility of interpersonal comparisons of utility. Much of welfare economics centered on the Pareto Principle, which says that if everybody in the society strictly prefers a social state x to another social state y, then x is strictly better than y for the society.[2] A major problem with the Pareto Principle, of course, is that, whenever any two individuals differ in their preference-based rankings over two social states, the Pareto Principle fails to compare those two social states. In an attempt to fill at least some of the

numerous gaps in comparisons under the Pareto Principle, Kaldor (1939) and Hicks (1939) introduced the "compensation criterion," but it was soon clear that not only did the compensation criterion have rather limited ethical appeal but it could also sometimes yield contradictory results, such as that social state x was better for the society than social state y, and, simultaneously, social state y was also socially better than social state x. A few years before the publication of Arrow's impossibility theorem, there were several contributions[3] to the literature on social decisions through voting,[4] the papers of Bowen (1943) and Black (1948a) being particularly important. Though these contributions appeared in some of the most respected economics journals, it will perhaps be fair to say that, by and large, welfare economists did not take much interest in them until Arrow's (1950, 1951) theorem stimulated their interest in voting procedures.

ARROW'S THEOREM AND SOME INTERPRETATIONS OF THE PROBLEM OF PREFERENCE AGGREGATION

Let X be the set of all possible social states or states of affairs in the society. At any given time, the set A of actually available social states is a nonempty subset (likely a proper subset) of X. It is assumed that each individual has a preference ordering over X. Arrow's (1951) impossibility theorem deals with the problem of specifying a social ranking or ordering R of the social states in X. The social ordering R is intended to serve as the basis of social choice in the following sense: given any set A of available social states, the society chooses

from A a social state that R ranks highest among the social states belonging to A.

Arrow (1951) starts with the intuition that R must be based on the profile of individual preference orderings over X (with exactly one preference ordering for each individual in the society). As Sen (2014) notes, this is clearly an intuition derived from the democratic tradition of social choice, and it underlies Arrow's (1951) formal concept of a social welfare function. Arrow's social welfare function is a function the domain of which is a class of profiles of individual orderings and the range of which is a class of social orderings. Thus, a social welfare function specifies exactly one social ordering over X for every profile of individual orderings in its domain. The concept of a social welfare function provides a formal framework for analyzing the ethical problem of how the social ordering of social states in X is to be arrived at, given the individual orderings over X. The framework would not be of much interest unless one is prepared to introduce specific restrictions or properties that one may like to impose on the social welfare function. It is, however, important to note that, intuitively, the definition of a social welfare function itself implies that cardinal information about individual preferences and interpersonal comparisons with respect to preference satisfaction cannot play any role in the determination of the social ordering. By definition, a social welfare specifies exactly one social ordering for each profile of individual preference orderings in its domain. Therefore, intuitively, if the individuals' preference orderings of social states remain the same, then the social ordering of social states must remain the same, irrespective of any changes in the cardinal

information about the individuals' preferences. Arrow (1951) proposed several properties for the social welfare function. While there are alternative versions of these properties, I shall follow Sen's (2014) elegant adaptation of Arrow's properties.[5] One of the conditions that Arrow proposed was that the domain of the social welfare function should be the set of all logically possible profiles of individual orderings. Given the definition of a social welfare function, this condition amounts to the requirement that, for every logically possible profile of individual orderings, the social welfare function should specify a unique social ordering—a requirement that Sen (2014) has called the axiom of "unrestricted domain." Arrow required the social welfare function to satisfy the familiar Pareto Principle widely accepted in welfare economics. Arrow also imposed two additional properties, namely, "independence of irrelevant alternatives" and "nondictatorship," on the social welfare function (see Sen [2013] for statements of these conditions). What Arrow's impossibility theorem says is that if X contains at least three distinct social states and the society consists of a finite number of individuals, then no social welfare function can simultaneously satisfy the conditions of unrestricted domain, the Pareto Principle, independence of irrelevant alternatives, and nondictatorship. Since the qualifications that there are at least three distinct possible social states and a finite number of individuals in the society are not particularly restrictive, and since each of the four conditions seems to have some prima facie plausibility, the theorem has the flavor of a paradox. The usefulness and importance of a paradoxical result, such as Arrow's, which demonstrates that certain plausible

assumptions or axioms lead to a logical contradiction, often lies in the intellectual challenge that it poses to come up with a resolution of the paradox and the resulting scrutiny of the axioms to find reasons why one should discard or modify some of them. In the case of Arrow's impossibility theorem, the challenge was tough and led to a vast and rich literature that continues to expand in various directions.

An important issue relating to Arrow's impossibility theorem involves the intuitive interpretation of the social welfare function, which, in turn, is closely linked to the interpretation of the social ordering R and the individual preferences that figure in the definition of a social welfare function.

One can think of at least two distinct interpretations of the social ordering. First, one can interpret R as reflecting the results of comparisons of social states under a procedure or rule adopted by the society to rank social states for the purpose of taking decisions; when R is interpreted in this fashion, the social welfare function is simply a decision procedure that the society uses to rank different social states. Alternatively, R can be interpreted as reflecting an individual's social welfare judgments, i.e., an individual's ethical judgments about the relative goodness or badness of social states; the individual may be an individual belonging to the society or a central planner or someone else from outside the society. The two interpretations are very different: an individual may agree to the use of a decision rule in the society that ranks two social states x and y differently from the ranking in terms of his own social welfare judgment. The distinction between the two different interpretations of the social ordering was at the center of some of the earliest assessments

(see, for example, Little, 1952; and Bergson, 1954) of Arrow's theorem,[6] and it has been discussed at length by Sen (1977, 2011). Arrow (2nd edition, 1963, p. 106) accepted the validity of the distinction between the two interpretations[7] and opted for the former interpretation since he felt that the final social choice would ultimately be determined by the results of the decision procedure adopted by the society to aggregate the preference orderings of individuals.[8,9]

Like the social orderings, individual preferences also lend themselves to a variety of interpretations. Sen (2013) distinguishes three different ways of interpreting an individual's preferences: preferences as expressed through votes, preferences as reflecting the interests of the person, and preferences reflecting ethical judgments. For Arrow, an individual's preferences reflected "whatever standards he deems relevant" (see Arrow, 1951, p. 7). In many ways, Arrow's (1951) interpretation of individual preferences corresponds closely to votes. The vote cast by a voter is based on whatever the voter considers relevant: it may represent neither her interest/personal well-being exclusively nor her moral judgments exclusively.

More than anyone else, Sen has made us aware of the importance of these distinctions between the different senses in which one can talk about the social ordering and individual preferences (see Sen, 1977, 2011, 2013). The pairing of alternative interpretations of a social ordering with alternative notions of individual preferences yields various preference aggregation problems,[10] which, despite the similarity of their formal structures, are often intuitively very different. Obviously, the formal validity of Arrow's theorem does not depend on any specific interpretation of his

overall formal framework. But the appeal of the framework and the appeal of specific axioms may depend significantly on the interpretation under consideration. Thus, when one interprets individual preference as the individuals' ethical assessments of social states and considers how to aggregate them to reach an ordering that will serve as the basis of the society's decisions, the fact that Arrow's definition of a social welfare function rules out cardinal aspects of individual preferences and interpersonal comparison of preference satisfaction does not seem to be a particularly restrictive feature. It is not clear that in aggregating individual moral judgments to reach social decisions one needs to compare either the intensities with which individuals consider one social state to be ethically superior to another or the individuals' levels of "moral satisfaction." Things are, however, very different when individuals' preferences are interpreted in terms of the well-being of the respective individuals and an ethical evaluator is concerned with the problem of aggregating such preferences to reach an evaluation of social states in terms of social welfare; we feel that interpersonal comparison of well-being is essential for such an exercise.

Sen (2014) has emphasized how Arrow draws on the democratic tradition in voting theory. It may be useful to distinguish here between two aspects of democracy. One is the phase of democratic deliberations, where individual judgments about alternative social states are discussed and debated; interpersonal comparisons of well-being typically play an important role in the formation of these individual judgments. As a result of deliberations and debates (or what Arrow [2014] calls "conversation and dialogue" in

his commentary) in this phase, a person's initial judgments about the social states may or may not change. But a time comes when final decisions need to be taken in the society. At that stage, votes are taken to aggregate the judgments as they stand at the end of the first stage, i.e., the stage of deliberation over individual judgments. Arrow's own interpretation of his theorem was in terms of this latter phase of decision making in a democracy; it is not therefore surprising that he excluded from his formal framework the notion of cardinal preference and interpersonal comparisons of preference satisfaction.

SOME RESPONSES TO ARROW'S IMPOSSIBILITY THEOREM

One can think of two distinct types of responses to Arrow's impossibility theorem. First, one can scrutinize Arrow's overall framework and his conditions to see whether there may be a case, at least under some interpretation of the problem of preference aggregation, for modifying them, thus opening up possible routes of escape from the impossibility result. Second, one can agree that the overall framework as well as the conditions figuring in the theorem are all reasonable, at least under some interpretations of the problem of social choice, but argue that, since all preference aggregation rules are flawed insofar as none of them can possibly satisfy all those reasonable conditions, it is important to explore further the properties of these admittedly flawed rules for preference aggregation and to see whether some of them might perform better than others.

Let me briefly review some examples of the first type of response to Arrow's theorem. We have already considered how, in the context of some interpretations of the preference aggregation problem, it may be desirable to admit cardinal individual preferences and interpersonal comparisons of preference satisfaction, which are ruled out by Arrow's definition of a social welfare function. In such cases, it will also be additionally necessary to relax Arrow's condition of independence of irrelevant alternatives. Even if the definition of a social welfare function is modified to admit cardinal preferences of individuals and interpersonal comparisons of preference satisfaction, Arrow's independence of irrelevant alternatives will still rule out all such interpersonal comparisons by making the social ranking of any two social states dependent exclusively on the individuals' *rankings* of those two social states.

There are, however, other interpretations of the problem of aggregating individual preferences, where the absence of interpersonal comparison of preference satisfaction does not seem unreasonable. This seems to be the case where individual preferences are interpreted as judgments or "votes" about social states, and the social ranking is simply the result of the decision procedure adopted by the society. Let us consider Arrow's conditions in such contexts. Since the Pareto Principle and nondictatorship seem to be fairly reasonable restrictions, attempts to find a way out of Arrow's impossibility result have often concentrated on the reasonableness of other conditions. The main justification for independence of irrelevant alternatives seems to be based on convenience: if the condition is not satisfied, then, to compare any two

social states, one would need information about individual preferences with respect to other ("irrelevant") social states, and this may be considered too stringent a demand for information about individual preferences.

Unrestricted domain and the definition of a social welfare function, together, imply that there must be a *social ordering* for *every logically possible* profile of individual orderings. The motivation that Arrow provided for requiring that there must be a social ordering (i.e., a binary social weak preference relation satisfying reflexivity, connectedness, and transitivity) was that, if the social weak preference relation is reflexive, connected, and transitive, then it would define a best alternative for every finite set of feasible alternatives.[11] Many rules, such as the simple majority rule, for aggregating individual preference orderings, give rise to cyclical strict social preferences, and, therefore, fail to define a best social state for some finite set of feasible social states. If the purpose of comparing social states is to make choices from different sets of feasible social states, the purpose is defeated when the social weak preference relation does not define a best social option for some sets of options. In the literature that followed Arrow (1950, 1951), it was soon noted that, while a social ordering did indeed define a best option for every finite set of feasible options, transitivity was not a necessary condition for the social weak preference relation to generate a best option for every finite set of feasible options. A number of contributions explored whether one could find a way out of Arrow's impossibility theorem by relaxing the requirement of transitivity. But it turned out that even considerably weaker restrictions than transitivity of the social

weak preference relation were incompatible with other appealing properties of a preference aggregation rule. Two of the most celebrated impossibility results that used such weaker rationality conditions for the social weak preference relation were those of Gibbard (1969), who relaxed the transitivity of the social weak preference relation to transitivity of the social strict preference relation, and Sen (1970 a, b) who assumed only acyclicity of the social strict preference relation—a property that is even weaker than the transitivity of the social strict preference relation. Is there any compelling reason why one should assume that, given the individual preference orderings, social choices for every possible set of feasible social states must be based on a fixed social weak preference relation defined over the universal set of social states? The requirement that, given the individual orderings, social choices must be based on a fixed binary social weak preference relation defined over X imposes certain "consistency properties" on the society's choices. Using the notion of social choice rather than social weak preference relation as the primitive concept, Sen (1993) has shown that a counterpart of Arrow's theorem can be proved without imposing consistency properties on social choices. It can also be argued that the requirement of consistency for social choices conflicts with some of our deepest intuitions. Thus, if one agrees with John Stuart Mill (1859) that an individual should be left free to take decisions in matters involving her own "personal" life, then it is easy to demonstrate that the social states that emerge from such decision making by individuals in their respective personal spheres will sometimes violate even the weakest consistency properties for social choice

discussed in the literature. Sugden (1985) gives a highly interesting example involving the choice of marriage partners, where the "social decisions" resulting from a man's freedom to propose or not to propose to a woman and a woman's freedom to accept or not to accept a man's proposal for marriage can violate even the weakest properties of choice consistency.

There is another aspect of Arrow's condition of unrestricted domain, which has been investigated at some length in the literature and to which Maskin (2014) draws our attention.[12] The condition of unrestricted domain demands that for every possible profile of individual orderings, the social weak preference relation should be an ordering. But what if some profiles of individual orderings are quite improbable? We know that the simple majority rule satisfies the Pareto Principle, independence of irrelevant alternatives, and nondictatorship. Then, it may be argued that perhaps we should not worry too much if, only for some "improbable" profiles of individual orderings, the simple majority rule yields cyclical social strict preferences, and, therefore, fails to specify a majority winner for some set of feasible alternatives. As Maskin (2014) writes, "That's the sense in which the impossibility theorem is too gloomy: if rankings are restricted in an arguably plausible way, then the five axioms are no longer collectively inconsistent." We do know of restrictions on profiles of individual orderings, which are plausible in certain contexts and which will rule out voting cycles under the majority rule. Thus, suppose the social options are political parties, all voters have a common ranking of the parties in terms of the criterion of how rightist or leftist the parties are, and the degrees of "rightism" or "leftism" of the parties

are the only relevant concerns of the voters. Then we know that the individual preference orderings will satisfy Black's (1958, pp. 7–10) property of "single-peakedness," which rules out cyclicity of the social strict preference relation under the simple majority rule. But once we go beyond cases where the voters' preferences are based on the consideration of a single dimension of the options, it is difficult to think of plausible restrictions on preference profiles that will rule out such cycles under the simple majority rule. In fact, an elegant and deep result of Kramer (1973) demonstrates that, in the general case, where the options have multiple dimensions considered relevant by the voters, most restrictions on profiles of individual preference orderings, which have been discussed in the literature, are unlikely to be satisfied.

It is important to note one implication of Arrow's axioms, to which Sen (1977, 2011, 2014) has often drawn attention and which follows not from any single axiom of Arrow but from the conjunction of unrestricted domain, the Pareto Principle, and independence of irrelevant alternatives. These three axioms, together, rule out the possibility of taking into account, in the determination of the social ranking over social states, any information (e.g., information about the social states under consideration) other than information about individual preferences. Sen (2014) persuasively argues that this, by itself, constitutes a restrictive feature of a social welfare function since it conflicts with Mill's (1859) notion that a person should have the right to make his own choices with respect to matters relating to his "personal life" irrespective of how others may feel about such

choice by her. The point remains valid even if we modify the definition of a social welfare function to introduce cardinal individual preferences and interpersonal comparisons of preference satisfaction. Following Sen's (1970a, 1970b) fundamental work on individual rights, a large number of writers in welfare economics and the theory of social choice have explored issues relating to individual rights, freedom, and responsibility. These contributions have extended the informational basis of social choice much beyond what is permissible under Arrow's axioms. Their intellectual origin, however, seems to lie in the intense scrutiny of the problem of social choice and social welfare evaluation that followed Arrow's impossibility theorem.

Let me now briefly comment on another type of response to Arrow's impossibility theorem. One can argue that, though every voting procedure must violate some conditions of Arrow, it is still important to know the structural properties of different voting procedures and to study whether, in some ways, some voting procedures perform better than others. While the study of individual voting procedures and their possible drawbacks goes back to Borda (1781) and Condorcet (1785), Arrow's theorem provided the intellectual stimulus for a vast number of studies of specific voting procedures and the "paradoxical" results that may arise under them.[13] These studies are of much interest. One particular line of investigation, which started with Condorcet (1985) himself, has been to calculate, on the basis of alternative assumptions, the probabilities of certain paradoxes arising under particular voting procedures (see Gehrlein and Lepelley, 2012) for an account

of several results in this strand of the literature). An important study of Dasgupta and Maskin (2008), which is lucidly summarized in Maskin (2014), is concerned with the issue of how well the simple majority voting procedure performs vis-à-vis other voting procedures. The analytical framework of Dasgupta and Maskin (2008) is somewhat different from that of Arrow (1951) insofar as their analysis is presented, not in terms of social welfare functions, but in terms of what they call voting rules. A voting rule, as Dasgupta and Maskin define it, is a function, which, for every subset A, of the universal set X (assumed to be finite) of social states, and, for every profile of individual orderings over X, specifies exactly one (possibly empty) subset of A, the specified subset of A being interpreted as the set of social states that the society chooses from A, given the profile of individual orderings. It is assumed that individual preference orderings are all drawn from some nonempty subset $\mathfrak{R}$, of the set of all possible strict orderings over X. For every such $\mathfrak{R}$, five plausible properties of voting rules are defined with reference to $\mathfrak{R}$; and, for every such $\mathfrak{R}$, a voting rule is said to work well on $\mathfrak{R}$ if it satisfies all the five properties with reference to $\mathfrak{R}$. Essentially, Dasgupta and Maskin (2008) demonstrate the following:

1. If $\mathfrak{R}$ is the set of all logically possible strict preference orderings over X, then no voting rule works well on $\mathfrak{R}$ (like Arrow's impossibility theorem, this is, of course, an impossibility result, but the result is proved for Dasgupta and Maskin's voting rules rather than for social welfare functions and the axioms are intuitively somewhat different).

2. If $\mathfrak{R}$ is such that some voting rule works well on $\mathfrak{R}$, then the simple majority voting rule also works well on $\mathfrak{R}$.
3. Let V be any given voting rule of the Dasgupta-Maskin type. If certain conditions are fulfilled, then there exists a subset $\mathfrak{R}'$ of the set of all logically possible strict orderings such that the simple majority voting rule works well on $\mathfrak{R}'$ but V does not work well on $\mathfrak{R}'$.

In this specific sense, the performance of the simple majority voting rule, judged in terms of the five properties specified by Dasgupta and Maskin, dominates the performance of every other voting rule. This is so despite the fact that, when all logically possible strict individual orderings are admissible, the simple majority voting rule sometimes fails to yield a social decision for some sets of feasible social states and some profiles of individual orderings.

CONCLUDING REMARKS

Even a cursory study of the literature on the subject of social choice and social welfare evaluation reveals how deeply and extensively Arrow's impossibility theorem has influenced our thinking about every dimension of the subject. The lectures of Amartya Sen and Eric Maskin highlight, in an exceptionally lucid fashion, some major aspects of the theorem and the subsequent developments to which they themselves have contributed immensely.

NOTES

1. I am grateful to John Weymark for several helpful comments.
2. Welfare economists often use a version of the Pareto Principle, which is slightly stronger than the version given here.
3. See, for instance, Bowen (1943) and Black (1948a, 1948b, 1948c). For an integrated presentation of the material in Black's papers, as well as a history of the theory of committees and elections, see Black (1958).
4. The literature originated in the works of Borda (1781) and Condorcet (1785).
5. Maskin's (2014) versions of Arrow's restrictions are formulated for use in a framework specified in terms of social choice rather than social preference.
6. Little (1952) and Bergson (1954) felt that welfare economics should be concerned only with social welfare judgments and not with procedures or rules that the society may adopt to arrive at decisions. It is not, however, clear why welfare economics should exclude studies of the latter.
7. Arrow (2nd ed., 1963, p. 106) observes, "A welfare judgment requires that some one person is judge; a rule for arriving at social decisions may be agreed upon for reasons of convenience and necessity without its outcome being treated as evaluation by anyone in particular."
8. Arrow (2nd ed., 1963, p. 106) wrote: "'Social welfare' is related to social policy in any sensible interpretation; the welfare judgments formed by any single individual are unconnected with action and therefore sterile.'"
9. Arrow felt that his formal theorem was applicable under the second interpretation as well: "the body of welfare judgments made by a single individual are determined, in effect, by the social decision process which the individual would have society adopt if he could" (Arrow, 1963, 2nd ed., p. 106).
10. Not every such pairing may yield an intuitively interesting prob-

lem of preference aggregation. An example is the problem of an ethical evaluator of social states aggregating the individuals' ethical orderings of those social states to reach her own social welfare evaluations. While it seems highly plausible that, in reaching her own ethical evaluation of social states in terms of social welfare, the evaluator should take into account the personal well-being of all individuals in the society, it is not clear why she should take into account the moral sentiments of all those individuals.

11. Arrow (1951) also had a second justification for the requirement that the social weak preference relation should be an ordering. He argued that, if the social weak preference relation was an ordering, then the society could make its choice from a feasible set of options by using a sequence of pairwise comparisons without the final choices being dependent on the specific "path" or sequence of pairwise comparisons.
12. The central point of Maskin (2014), however, lies elsewhere, and I shall come to it a little later.
13. For a list of alternative voting rules and various voting paradoxes, see Felsenthal (2012).

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