THE INFORMATIONAL BASIS OF SOCIAL CHOICE[1]

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ABSTRACT

Any procedure of social choice makes use of some types of information and ignores others. For example, the method of majority decision concentrates on people's votes, but pays no direct attention to, say, their social standings, or their prosperity or penury, or even the intensities of their preferences. The differences between distinct procedures lie, to a substantial extent, on the kind of information that each procedure uses and what it has to ignore. The informational bases of the different social choice procedures tell us a great deal about how they respectively work and what they can or cannot achieve.

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INTRODUCTION

Social choice theory addresses a wide range of decisional and judgmental problems, dealing with a variety of procedures—from voting to making normative social assessments. It encompasses theories of elections and balloting on one side, to welfare economics on the other, as well as theories of normative measurement, such as the evaluation of national incomes, measurement of inequality and poverty, and appraisal of social welfare. These distinct problems often demand quite dissimilar approaches, and there is little hope of getting some uniform approach that would work equally satisfactorily for all the different exercises.[2] Nevertheless, all the social choice problems have the shared feature of relating "social"—or group—assessment to the values, preferences, choices, or some other characteristics of the respective individuals who form the collectivity of that society or group. It is with the informational basis of that generic approach that this chapter is concerned.

Any procedure of social choice makes use of some types of information and ignores others. For example, the method of majority decision concentrates on people's votes and, by implication (given suitable assumptions), on their preferences. But it pays no direct attention to a variety of other kinds of information, such as their personal characters, their social standing, their prosperity or penury, or even the intensities of their preferences. But each of these ignored features can be very influential for other procedures of social choice. The differences between distinct procedures lie, to a substantial extent, on the kind of information that they respectively

use and what they ignore. The informational bases of the respective social choice procedures tell us a great deal about their nature and underlying motivations, and about the differences that distinguish them from each other.

Much the same thing can be said about *principles* of social judgment or of normative social decisions. Their respective informational bases qualify the nature of the judgments that can—or cannot—be made. The informational bases can be explicitly stated, or implicitly characterized, by the axiomatic requirements imposed on each respective approach. Indeed, many of the "conditions" of social decisions that have received intense attention in recent decades, especially since Arrow's (1950, 1951) pioneering departure in axiomatic social choice theory, can be helpfully analyzed in terms of their informational demands, including informational prohibitions.

We can, in fact, go a long way in characterizing any normative principle, or any well-defined procedure of social choice, by identifying the information that it uses and, no less importantly, the information it ignores, as is discussed in Sen (1977b, 1979). It is tempting to ask whether a procedure, or a principle, of social choice can be fully delineated by its informational basis *alone*? The answer is very definitely in the negative, for reasons that are not far to seek. No matter how tight the informational conditions are that help us to move closely towards a certain principle *p*, there are at least three other principles that are not excluded by the very same informational requirements. First, a principle, let us call it anti-*p*, can demand the exact opposite of what principle concludes (e.g., minimizing the utility sum-total does not demand any more information than the utilitarian

maximization of that sum). Second, to resolve that all alternatives be accepted as being equally good (what can be called *universal indifference*) does not demand any information at all (beyond the knowledge of that *blanket rule*). Third, resolving that no alternative can be ranked against any other (call it *universal unconnectedness*) also has little informational requirement.

However, the informational perspective can be used not only to make sure that there is enough information for the decisions involved, but also to demand that the processes or principles must actually respond to—or be sensitive to—particular types of information that characterize each specific principle or procedure. If we go beyond checking the sufficiency of available information to the necessity of taking discriminating note of particular information (a requirement of informational *responsiveness*), the last two of the three alternatives may turn out to be unacceptable. To illustrate with a simple example, an insistence on the necessity to be sensitive to the unanimous strict preferences of all individuals over a pair of social states $\{x, y\}$ may eliminate the possibility of being indifferent between them, or keeping them unranked, when everyone does in fact prefer x to y. But informational sensitivity alone, without a directional mandate, cannot yield the conclusion that x must, in this case, be socially preferred to y (or that x must be chosen over y, given that choice). Indeed, the exact opposite (that is, socially ranking y over x in response to every member of the society preferring x to y), while patently perverse, is also sensitive to the same information. Pareto and anti-Pareto do not, in fact, differ in informational

requirement or in informational responsiveness—only in the *direction* of the response.

Substantive social choice theory cannot, therefore, be understood merely in terms of informational bases. There is no way of getting rid of the different ways in which the same information can be used in different procedures or principles. And yet the differences between distinct social choice principles or processes often rest primarily on their contrasting informational bases.

Indeed, many of the decisive steps in the history of social choice theory have turned on invoking some types of information and disavowing others. This chapter is concerned with tracing some of these historical steps, beginning with the early origins of the subject in the hands of eighteenth-century French mathematicians (such as Condorcet and Borda), then proceeding to the founding of modern social choice theory in the middle of the twentieth century (with the pioneering work of Kenneth Arrow in 1950 and 1951), and finally looking briefly at some of the more recent trends in the subject, particularly in normative social choice theory.[3]

DEMOCRATIC PRIORITIES AND INFORMATIONAL INCLUSIVENESS

The informational foundation of modern social choice theory relates closely to the basic democratic conviction that social judgments and public decisions must depend, in some transparent way, on individual preferences, broadly understood. The emergence of this democratic instinct relates closely to

the ideas and events that surrounded the European Enlightenment. While it drew on various antecedent sources and inspirations, the democratic perspective received adequate delineation and wide public acknowledgement only during the Enlightenment, particularly in the late-eighteenth century, which also saw the French Revolution and American independence. The works of Borda (1781) and Condorcet (1785) on the properties of voting systems and of Bentham (1789) on the demands of utilitarian social aggregation were clearly influenced by this general intellectual climate.

Individual preferences can, of course, be variously interpreted in different democratic exercises, and this is well illustrated by the contrast between focusing on votes—and through them on people's voices (explored in the classic works of Borda or Condorcet)—on the one hand, and concentrating on the interests or judgments (explored in the pioneering writings of Adam Smith, Jeremy Bentham, or John Stuart Mill), on the other. These contrasts—between alternative interpretations of preferences—can be very important for some purposes, but in the present context, that of noting the democratic foundations of social choice theory, their differences are less important than what they have in common. For the moment I shall use the generic term *preference* to cover all these different interpretations of individual concerns that could be invoked, in one way or another, to serve as the informational bases of public decisions or social judgments.

In this elementary democratic foundation, there is also a strong understanding that at least in principle the preferences of everyone could count, without any a priori exclusions

(even if it were to turn out, on the basis of further analyses, that some preferences would be more effective than others, sometimes radically so). No member of the collectivity could be eliminated as being foundationally irrelevant. That informational underpinning, which was established in the first generation of formal social choice theory that emerged in the eighteenth century, has been powerfully reinforced in modern social choice theory pioneered by Arrow (1951). While this rudimentary democratic feature may seem today to be rather straightforward, perhaps even mundane, it was a radical enough step at the time of its first consolidation. It firmly established a democratic inclusiveness in the informational basis of social choice.

The force of this departure can be brought out by examining the contrast between these new presumptions and the earlier interpretations of social choice in pre-Enlightenment politics or political economy. For example, Aristotle in ancient Greece and Kautilya in ancient India, both of whom lived in the fourth century before Christ, had explored various procedures for making appropriate social decisions, in their books respectively called *Politics* and *Economics*.[4] Aristotle had no great difficulty in excluding women and slaves from the process of social decision making, and Kautilya had little problem in confining critical decisions in the hands of those blessed by high social status (related to caste or political authority). Enormous informational exclusions were, in effect, authorized *before* the substantive investigation of social procedures properly began. The world of late-eighteenth century Europe broadened the needed informational foundation by overturning that exclusionary authorization.

The pioneers of modern social choice theory were guided by their firm conviction that every member of a collectivity must, in principle, count in the decisions of that collectivity.[5]

This is meant to apply to choices for the society as a whole, as well as to those for a committee, or an academy, or a set of juries, or some other organizational panel or board. There is an insistence on no antecedent exclusion of the concerns of any individual member of the respective collectivity. The different decisional structures that were explored, such as majority rule, or utilitarian aggregation, or rank-order voting (the so-called "Borda rule") differed in the significance that can be attached to particular aspects of individual preferences: for example, whether to take note only of the ranking of each pair considered separately (ignoring other "irrelevant" alternatives), or to attach significance to the rank of a particular alternative in an overall ranking (as in Borda's well-known formula).[6] But in not excluding anyone's preference ordering from counting, they shared a basic belief in democratic inclusiveness in an elementary but powerful form.

Since contemporary social choice theory, pioneered by Arrow, emphatically shares this foundational democratic value, the discipline has continued to be loyal to this basic informational presumption. For example, when an axiomatic structure yields the existence of a dictator (Arrow 1951), as a joint implication of chosen axioms that seemed plausible enough (seen on their own), this is immediately understood as something of a major embarrassment for that set of axioms, rather than being taken to be just fine on the ground that it is a logical corollary of axioms that have

been already accepted and endorsed. We cannot begin to understand the intellectual challenge involved in Arrow's impossibility theorem without coming to grips with the focus on inclusiveness that goes with a democratic commitment, which is deeply offended by a dictatorial procedure, even when it is entailed by axiomatic requirements that seem eminently acceptable.

The same applies, in one way or another, to the various subsequent results that followed Arrow's impossibility theorem. Arrow's specific impossibility result—with dictatorship's being implied by his other conditions—is not extendable to the case in which the transitivity of social preference is weakened, even rather slightly, to just "quasitransitivity" (or the transitivity of strict preference only), unless the other conditions are redefined.[7] But as the requirement of "collective rationality" in the form of social transitivity or binariness is gradually relaxed, new results emerge that show that there must now be an oligarchy, or the existence of someone with veto power, or some other violation of what democracy demands.[8] Again, the tension that is generated by these results relates to the violation of informational inclusion that a democratic commitment entails.

It is important to recognize how radically the nature of the social choice search for a minimally acceptable social decision procedure has been shaped by the informational implications of a basic democratic conviction that was getting firmly established in the second half of the eighteenth century, just as social choice theory, in its early form, was being founded. Given the centrality of Arrow's result, which has profoundly shaped the direction that the development

of social choice theory took over the second half of the twentieth century, it is perhaps useful to examine sequentially how the tension with democratic inclusiveness emerges from what looks merely like minimal demands of systematic and sensitive social choice. After some clarificatory discussion of the informational aspects of Arrow's social choice framework in the next section, a simple way of understanding and establishing Arrow's theorem (seen specifically in an informational perspective) is presented in section 4.

INFORMATIONAL EXCLUSIONS AND SOCIAL CHOICE FRAMEWORK

In the general Arrovian framework, the social ranking R of the alternative social states is taken to be a function of the n-tuple of individual rankings $\{R_i\}$ of those states:

$$R = f(\{R_i\}) \qquad (1)$$

The functional relation f, which we can call a "collective choice rule," is an Arrovian "social welfare function" when there is the further requirement that R as well as each R_i be a complete ordering of feasible social states.[9] In the discussion that follows, the immediate reference will be specifically to social welfare functions (SWF), but much of the discussion applies to collective choice rules in general.

Since conflict with informational inclusiveness is such a central feature of the Arrow impossibility theorem, it is important to be sure that the impossibility result is not

being achieved simply by beginning with patently informational restrictions (in the formulation of social welfare functions). We have to ask: How inclusive is this general Arrovian formulation regarding the information that can be accommodated in the process of social choice? In answering this question, it is convenient to distinguish between "utility information" in the general sense (including information about preference rankings) and "nonutility information" regarding other features of states of affairs. It is easily checked that while the utility information that is allowed to be accommodated in an SWF is rather restricted, there is nothing in the form of an SWF itself that limits the admissibility of nonutility information.

Consider first the nature of allowable utility information. In (1), the form of $f(\{R_i\})$ does not allow the use of interpersonal comparison of utilities.[10] This is certainly a start-off restriction. In his initial formulation of the problem of social choice, Kenneth Arrow was moved by the view, common in positivist philosophy that was then influential in welfare economics, that "interpersonal comparison of utilities has no meaning" (Arrow 1951, p. 9). The utility information that is usable in this structure of social choice consists of n-tuples of individual preferences (or utility orderings) of the respective individuals—considered separately. This is a momentous informational exclusion, the removal of which can open up many constructive possibilities (as was discussed in Sen 1970a).

However, as far as nonutility information is concerned, the format of social welfare functions is remarkably permissive. Unless eliminated by specific axioms to be imposed on

social welfare functions (on which more will be discussed presently), the framework can accommodate sensitivity to any part of the informational content of social states. There is a real comprehensiveness here, which is worth emphasizing, since it can be easily missed because of the apparent insistence, in the formulation of (1), that the n-tuple of individual preferences $\{R_i\}$ be the sole input into the choice process. The implications of this formulation require some elucidation, particularly since they are, in fact, critically important for later social choice theory, involving the use of nonutility information related to liberties, rights, and nonwelfarist interpretations of justice, equity, and poverty.

The informational content of social states is not arbitrarily restricted in any way, and the social welfare function can take note of any information that can be accommodated within the specification of social states. There is need for some clarification here. Given the form of (1), with $R = f(\{R_i\})$, it may appear that no feature of social states can be influential in the choice over these states unless the individual preferences $\{R_i\}$ respond to that feature. In this interpretation, if a specific feature of a state of affairs (say, the level of income inequality, or the violation of some liberties, or the infringement of civil rights) is going to be directly influential in social choice, it must be *through* the impact of that feature on individual preferences over states of affairs. Indeed, in this interpretation, no feature of the states can have an influence on social choice through any channel *other than* individual preferences.

This interpretation, however, is not correct. Even though a social welfare function insists on a tight functional

relation f between R and $\{R_i\}$, there is nothing in the mathematics of this requirement that would prevent the nature of the functional relation f to be itself responsive to any information that is included in the content of the respective social states that R and R_i order. The individual preference orderings are rankings of substantive social states, and the information about the social states can be taken into account in deciding on the mapping between the set of n-tuples of individual preferences and the set of social rankings to be determined by f.

However, this permissive format can be made informationally more restrictive through the effects of axioms that may be imposed on a social welfare function. Indeed, through this route it is possible to end up eliminating the direct usability of *all* nonutility information, so that the characteristics of social states are made totally inconsequential: social choice over them will then be determined only by their placing in the individual preference rankings. This condition is sometimes called "neutrality." This is perhaps an oddly reverent name for what is after all only an informational restriction, but the requirement can be seen as "neutralizing"—indeed eliminating—the influence of all nonutility (or nonpreference) information regarding social states. In effect, it yields an insistence that social decisions be taken only on the basis of individual preferences over the states, without paying any attention to the nature of these states (and the nonutility or nonpreference information about these states).

In the literature of moral philosophy, this "neutrality" condition and similar requirements are sometimes called

"welfarism" (see Sen and Williams 1982), and that term has been in increasing use in social choice theory as well. Welfarism, narrowly defined, is the demand that social welfare (or whatever is taken as the social maximand) depends only on individual utilities: Other features of states of affairs have no direct influence on social welfare (or the social maximand). In somewhat broader formulations, welfarism, corresponding to "neutrality," can be seen as a more permissive insistence that the social maximand depends only on individual utilities, or individual welfares, or individual evaluations of the worth of states of affairs (more on this presently).

It is possible to combine welfarism with very rich utility information (such as interpersonal comparability and cardinality), and indeed such enrichment of information would be particularly important for normative social judgments, including welfare economic assessments.[11] But when it is applied to social welfare functions that use individual utility information only in the form—as in (1)—of n-tuples of individual preferences (corresponding to noncomparable ordinal individual utilities), we get a combination that attempts to make do with very little information indeed. It must, however, be noted that Arrow does not invoke neutrality or welfarism in any form as a prior requirement. In a limited form that restriction emerges as an implication of other conditions, and a substantial part of the unexpected nature of Arrow's impossibility result (or the "General Possibility Theorem" as Arrow called it) relates, in fact, to this analytical demonstration.[12]

AXIOMATIC EXCLUSIONS AND ARROW'S IMPOSSIBILITY THEOREM

ARROW'S THEOREM

It is useful, in this perspective, to go through a simple proof of Arrow's impossibility theorem not merely because the result is so central to social choice theory, but also because of the light it throws on the way apparently mild axioms can, acting in combination, end up as very severe informational constraints. Arrow considered a set of very plausible-looking conditions relating social choice to the n-tuple of individual preferences, and showed that it is impossible to satisfy those conditions simultaneously.

The axioms used by Arrow (in the later, and neater, version in Arrow 1963) include *unrestricted domain, weak Pareto Principle, nondictatorship,* and *the independence of irrelevant alternatives* (in addition to the structural conditions requiring that the set of individuals is finite and that the set of social states includes at least three distinct states). We define xR_iy as the statement that person i weakly prefers x to y (that is, either strictly prefers x to y, or is indifferent between them), and xP_iy as person i strictly prefers x to y. The weak and strict social preferences are denoted R and P, respectively.

Unrestricted domain (U) demands that the domain of the social welfare function, that is f in (1), includes all possible n-tuples of individual preferences $\{R_i\}$. The *weak Pareto Principle* (P) says that if all persons prefer any x to any y, then x is socially preferred to y. *Nondictatorship* (D) excludes the possibility that any individual j could be so powerful that

whenever, over the domain of f, he or she prefers any x to any y, society too strictly prefers x to y. And *independence of irrelevant alternatives* (I) can be seen as demanding that the social ranking of any pair $\{x, y\}$ must depend only on individual preferences over $\{x, y\}$. The Arrow impossibility theorem states that there does not exist any social welfare function f that can simultaneously fulfill U, P, D, and I.

We can define a set G of individuals as being "decisive" over the ordered pair $\{x, y\}$, denoted $DG(x, y)$, if and only if whenever everyone in G prefers x to y, we must have xPy no matter what others prefer. Further, if G is decisive over every ordered pair, then G is simply called "decisive," denoted DG. It is readily seen that nondictatorship is the requirement that no individual is decisive, whereas the weak Pareto Principle is the requirement that the set of all individuals is decisive. The proof used here goes via two lemmas, which establish the implied informational exclusions, to obtain dictatorship from the weak Pareto Principle.[13]

PROOF OF ARROW'S THEOREM

Lemma L.1 *If $DG(x, y)$ for any ordered pair $\{x, y\}$, then DG.*

To establish this, we have to show that $DG(x, y) \rightarrow DG(a, b)$, for all a and b. The demonstration proceeds by repetitions of essentially the same strategy in different possible cases depending on whether or not x or y is identical with either b or a. Consider the case in which the four states x, y, a, b are all distinct. Assume the following pattern of individual preferences: for all persons j in G: $aP_j x$, $xP_j y$, and $yP_j b$, and for all persons i not in G: $aP_i x$ and $yP_i b$

(with nothing being presumed about the ranking of the other pairs). By $DG(x, y)$, we have xPy, and by the weak Pareto Principle, we obtain aPx and yPb. Hence, through the transitivity of strict preference, we get: aPb. By the independence of irrelevant alternatives, aPb must depend on individual preferences only over $\{a, b\}$, and since only the preferences of people in G have been specified, clearly $DG(a, b)$.

Lemma L.2 *For any G, if DG, and if G has more than one person in it and can be, thus, partitioned into two nonempty parts* G_1 *and* G_2*, then either* DG_1 *or* DG_2.

Assume that for all i in G_1: xP_iy, and xP_iz, with any possible ranking of y, z, and that for all j in G_2: xP_jy, and zP_jy, with any possible ranking of x, z. Nothing is required from the preferences of those not in G. Clearly, xPy by the decisiveness of G. If, now, xPz, then group G_1 would be decisive over this pair, since they alone definitely prefer x and z (the others can rank this pair in any way). If G_1 is not to be decisive (and thus by Lemma L.1 not to be decisive over any pair), we must have zRx for *some* set of individual preferences over x, z of nonmembers of G_1. Take that case. So we have zRx and also xPy. We thus have, by transitivity of preferences, zPy. Since only G_2 members definitely prefer z to y, this entails that G_2 is decisive over this pair $\{z, y\}$. But, then, by L.1, G_2 is generally decisive. So either G_1 or G_2 must be decisive.

Completing the proof of Arrow's theorem can now proceed very rapidly. By the weak Pareto Principle, the group of all individuals is decisive. It is, by assumption, finite. By successive twofold partitionings, and each time picking the decisive part (which exists, guaranteed by L.2), we arrive at a decisive individual, who must, thus, be a dictator.

INTERPRETATION OF THE PROOF

The proof just presented works through a sequential compounding of informational exclusions, beginning in a small way and ending with such a massive prohibition that the weak Pareto Principle cannot be effectively distinguished from the existence of a dictator. In socially ranking x and y, the condition of independence of irrelevant alternatives excludes the use of information—both preferences related and any other—except what relates directly to the "relevant" alternatives, that is, x and y only. Starting with that small beginning and afforced by the unrestricted domain and the weak Pareto Principle, we get to the result in Lemma L.1 that any group that is socially decisive over any pair of social states must be socially decisive over every pair of social states—no matter what these states are. So the specific information about the respective states, which we may have plentifully, will not be allowed to make any difference as far as decisiveness (based on individual preferences) is concerned.

Armed with this informational exclusion established in Lemma L.1, the proof proceeds in Lemma L.2 to economize also on preference information itself. If the information about the unanimous preference of a set G of individuals is adequate to rank social states (no matter what others want), then the information about the unanimous preference of some proper subset G^* of individuals, excluding some others in G, will be adequate as well. This opens the door, through sequential use, to go from the decisiveness of unanimous strict preference of all (thanks to the weak Pareto Principle) to the decisiveness of strict preference of some one individual

(that is, a dictator). The informational inclusiveness of a foundational democratic commitment is, thus, caught in a fierce internal contradiction: to empower *all* without discrimination (as incorporated in the weak Pareto Principle) is to empower *one* irrespective of what others want (that is, a dictatorship).

ENRICHING INFORMATION FOR THE POSSIBILITY OF SOCIAL CHOICE

As was mentioned earlier, the *entailed* exclusion of nonutility information through Arrow's axioms adds to the *antecedent* exclusions directly incorporated in the formulation of a social welfare function through the nonadmissibility of interpersonal comparability and cardinality. Informational enrichment can be sought *either* through the route of enriching utility information *or* through that of admitting nonutility information.

The former route has been particularly explored in social choice theory since the 1970s.[14] The formal structure is that of a "social welfare functional," which functionally relates social ranking R to n-tuples of individual utility functions $\{Ui\}$. The extent of measurability and cardinality of utilities is specified by "invariance conditions" imposed on social welfare functionals.[15] It can be shown that all the Arrow axioms, if translated into a broadened framework of "social welfare functionals" that allows richer utility information, can be simultaneously satisfied, even with just ordinal comparability of individual utilities, even without any cardinality.

Furthermore, many other constructive possibilities are opened up once cardinal comparability is also allowed.[16] These extensions are hugely important for welfare economics and for normative social judgments in general.

This route (that is, enriching utility information while still keeping out nonutility information) does not immediately raise issues of inconsistency. But while the Arrow impossibility may be circumvented this way, this path does not still accommodate the use of nonutility information needed for specification of rights or liberties or nonwelfarist assessment of inequality or fairness. This is unfortunate since these norms have considerable appeal, and many "non-neutral" concerns have figured in the informal literature on social decisions and choices for a very long time. Even as formal social choice theory was getting founded through the pioneering works of Borda (1781), Condorcet (1785), and other mathematical analysts of voting and electoral processes, and also through the parallel line of investigation pursued by utilitarians such as Bentham (1789), other innovative departures were being made in the understanding of justice in a way that could not be fully translated into axioms defined in the neutral space of preferences. For example, the relevance and reach of the idea of rights were extensively explored by such pioneering authors as Mary Wollstonecraft (1790, 1792) and Thomas Paine (1791).[17] These concentrations were well reflected in the practical politics related to the French Revolution as well as American Independence, both of which made extensive use of the idea of fundamental rights.

Some of these rights involve conditions that relate to individual preferences of the people involved (for example, what

one prefers in one's "personal domain" of liberty), but even here nonutility characteristics of the states of affairs have to be taken into account to give more effectiveness to each person over his or her own personal domain. These concerns were not explicitly accommodated in formal social choice theory in its classic formulations, but they have figured prominently in more recent developments in the social choice literature.[18]

ON COMBINING UTILITY AND NONUTILITY INFORMATION

There are, however, important problems in combining utility and nonutility information, since their disparate roles can yield possible inconsistencies. Indeed, this is one way of interpreting the so-called liberal paradox, which involves a consistency problem in simultaneously accommodating, along with unrestricted domain, a minimal condition of liberty (involving the use of some nonutility information regarding personal features in social states) and the weak Pareto Principle (involving very modest use of utility information).[19] Indeed, the "impossibility of the Paretian liberal" brings out this tension in a very simple case, and the conflicts can be much more complex when it is attempted to use richer utility information *along with* substantive use of nonutility descriptions of states of affairs.

These "hybrid" frameworks have not been extensively investigated yet, and there has in fact been some reluctance to leave the simplicity of welfarism even when trying to

accommodate principles or procedures of social choice that are quintessentially nonwelfarist. Consider, for example, John Rawls's (1971) well-known theory of justice, which involves "the priority of liberty" as the first principle (a substantively nonwelfarist requirement), and also the Difference Principle, which uses lexicographic maximin in the space of primary goods, not utilities. While Rawls has been much invoked in social choice theory, nevertheless the axiomatizations of Rawls in welfare economics (and in social choice theory related to welfare economics) have tended to ignore his first principle altogether (except indirectly in the context of the so-called liberal paradox) and have also redefined the Difference Principle in terms of utilities, in contrast with Rawls's own focus on primary goods.[20]

These recharacterizations of nonwelfarist principles (like Rawls's) in "welfarist" terms, while strictly speaking inaccurate, do have significant usefulness, for several distinct reasons. First, welfarism does appeal to the intuition of many social analysts.[21] In fact, many seem to find the welfarist version of Rawls's lexicographic maximin more acceptable than Rawls's own insistence on operating in the space of primary goods.[22]

Second, the utility-based formulation is open to alternative interpretations and can be relatively easily integrated with decision-theoretic normative reasoning. Indeed, as d'Aspremont and Gevers (2002) point out in their masterly critical survey of the literature on "social welfare functionals" in the first volume of the *Handbook of Social Choice and Welfare* this literature "can be reinterpreted as an application of multi-objective decision theory to the ethical observer's problem" (p. 464).

Third, the restricting of the entire informational basis of all the normative principles to one basic class of evaluative data (such as individual utilities or individual overall evaluations) can be a simple way of keeping the possibility of inconsistency (under discussion here) at bay, and this can be seen to be a considerable merit in itself. Indeed, even within such restrictions, a variety of different concerns can be accommodated, without internal tension. As d'Aspremont and Gevers (2002) point out, even the alternative approach of capabilities (on which see chapter 16 by Kaushik Basu and Luis Felipe Lopez-Calva in *Handbook of Social Choice and Welfare*, vol. 1) can do this with its exploration of the possibility of accommodating "doings and beings" *within* the approach of "social welfare functionals." The problems that may have to be faced would arise not from the *objects* that influence individual utility or individual welfare or individual evaluation, but from any proposed use of other information to determine the relevance of—and the weights to be placed on—utility or welfare, or on evaluative conclusions of individuals.

Possible tensions arise when *other* data from the states of affairs are invoked in making social judgments or social choices: for example, in giving priority to a person's evaluation over her own "personal domain" (as in Rawlsian "priority of liberty" or in various conditions of "minimal liberty" used in social choice theory), or in attaching special importance to the centrality of certain capabilities (as in the philosophical or developmental approaches that give a special role to the fulfilment of certain basic capabilities).[23] The critical issue, as was discussed earlier, is so-called neutrality—a condition that can be directly imposed or

indirectly precipitated through the use of other axioms, which restricts the class of permissible social welfare functions and social welfare functionals to reliance on utility information.[24] This requirement runs counter to the invocation of various foundational norms, reflected in, say, the Rawlsian principles of justice, or the Aristotelian focus on capabilities, or the Wollstonecraft–Paine concentration on the "vindication of rights."

The possibility of combining these different classes of foundational information reflected in different types of principles of social justice and equity—involving both utility and nonutility information—has not yet been investigated adequately. While celebrating what has already been achieved, it is important to identify this as an area in which more investigation will be needed in the future.[25] That would be of particular importance in making further use of social choice theory in analyzing and exploring theories of justice.

NOTES

1. For helpful discussions I am grateful to Kenneth Arrow and Kotaro Suzumura.
2. I have discussed elsewhere, in Sen (1977a), why the distinctions involved are very important to acknowledge and accommodate. See also Suzumura (1982, 1983).
3. The balance of attention in this chapter is somewhat tilted in the direction of relatively broader results rather than more particularized findings. I have had the occasion to discuss and illustrate in another essay on social choice theory (Sen, 1986) how the classical debates on the principles and procedures of social choice have continuing relevance to the formal analyses and

technical results in the more recent works that often deal with very specialized issues. The "older" contentious issues, related to informational bases, have similar resilience and durability.

4. For English translations of Aristotle's *Politics* and of Kautilya's *Arthashastra*, see, respectively, Barker (1958) and Shama Sastry (1967). *Arthashastra*, the Sanskrit title of Kautilya's book, which literally means "treatise on material wealth," is perhaps best translated as "economics," even though much of the book is devoted to studying systematic statecraft.
5. On the intellectual debates that engaged Enlightenment authors, including Condorcet, see Rothschild (2001).
6. The so-called "Borda rule" belongs to a general class of "positional rules," the properties of which have been extensively investigated by Gardenfors (1973), Fine and Fine (1974), and others; see Pattanaik's (2002) superb critical survey of this literature, in Vol. 1 of *Handbook of Social Choice and Welfare*, vol. 1. Condorcet's voting principles have been well discussed by Arrow (1963), Fishburn (1973), Suzumura (1983), and Young (1988), among others.
7. This nonextendability is shown in Sen (1969, 1970a). However, it can also be shown that in a choice-functional framework, Arrow's conditions can be suitably recharacterized, without changing their motivational justifications, to precipitate the dictatorship result without any condition of internal consistency of social choice whatsoever (as shown in Sen 1993). Thus reinterpreted, Arrow's conditions of independence, unrestricted domain, and the weak Pareto Principle, together, continue to contradict democratic inclusiveness, even without any demand for *internal* consistency of social choice.
8. Different types of results in this general line of investigation have been presented—or scrutinized—in Gibbard (1969, 1973), Sen (1970a, 1977a, 1993), Mas-Colell and Sonnenschein (1972), Fishburn (1973, 1974), Brown (1974, 1975), Binmore (1975, 1994), Campbell (1976), Deb (1976, 1977), Suzumura (1976a, b, 1983),

Blau and Deb (1977), Kelly (1978), Blair and Pollak (1979, 1982), Grether and Plott (1982), Chichilnisky (1982), Chichilnisky and Heal (1983), Moulin (1983), Pattanaik and Salles (1983), Peleg (1984), Hammond (1985), Kelsey (1985), and Campbell and Kelly (1997).

9. Different types of collective choice rules can be distinguished and individually investigated, as discussed in Sen (1970a). See also Fishburn (1973), Kelly (1978), and Suzumura (1983), among other treatises, for different classificatory systems.
10. Nor are cardinal utilities (whether or not comparable) admissible in this framework. This is not, however, a critical constraint for Arrow's impossibility theorem, since that result can be extended to the case of noncomparable cardinal utilities (see Theorem 8.2 in Sen 1970a), even though it does severely restrict the class of permissible social choice procedures (on which see Gevers 1979 and Roberts 1980a).
11. This general issue is discussed more fully in Sen (1970a, 1977a, 1999).
12. Since it is often said that Arrow's impossibility theorem is a generalization of the old "paradox of voting," it is worth noting that this is only partly true. It is important, in particular, to recognize that while voting rules must satisfy neutrality, neither the form of Arrow's social welfare function, nor any of the individual axioms imposed by Arrow on that function, make any demand of neutrality. We are moved in the direction of neutrality by the combination of the axioms, and indeed the main work in proving Arrow's impossibility theorem consists, it can be argued, in deriving a property of neutrality from the *combination* of different axioms, through the use of axiomatic reasoning. In what follows, see Lemma L.1.
13. The brief proof used here corresponds to the one outlined in Sen (1995, fns. 9 and 10, p. 4). Note that this proof drops the need to introduce the intermediate concept of "almost decisiveness" (used in Arrow 1951, 1963, and Sen 1970a), since it is redundant.

14. Vickrey (1945) and Harsanyi (1955) had earlier identified representation results that make use of information based on expected utility, interpreted as interpersonally comparable cardinal utilities, to obtain the value of social welfare in a summational form.
15. The operation and use of invariance conditions is investigated in Sen (1970a, b), d'Aspremont and Gevers (1977), Gevers (1979), Maskin (1979), and Roberts (1980a), among other contributions.
16. See, among other contributions, Sen (1970a, 1977b), Hammond (1976, 1985), d'Aspremont and Gevers (1977, 2002), Arrow (1977), Maskin (1978, 1979), Gevers (1979), Roberts (1980a, b), Suzumura (1983, 1996), Blackorby, Donaldson, and Weymark (1984), d'Aspremont (1985), Blackorby, Bossert, and Donaldson (2002), and d'Aspremont and Mongin (2008).
17. The far-reaching relevance of these perspectives is discussed in Sen (2009).
18. See Arrow, Sen, and Suzumura (1996/1997) and other chapters of *Handbook of Social Choice Theory*, vol. 2, especially part 7, including the chapters by William Thomson, Marc Fleurbaey and Francois Maniquet, and Kotaro Suzumura.
19. The literature on the liberal paradox is by now quite vast. The special number of *Analyse & Kritik*, September 1996, includes a fine collection of papers on this subject, as well as extensive bibliographies of publications in this field
20. See Phelps (1973, 1977), Hammond (1976), Maskin (1978, 1979), Meade (1976), Strasnick (1976), Arrow (1977), d'Aspremont and Gevers (1977), Sen (1977b), Gevers (1979), Roberts (1980a, 1980b), Atkinson (1983), Suzumura (1983, 1996), Blackorby, Donaldson, and Weymark (1984), d'Aspremont (1985). However, social choice theoretic reasoning has been used to question or defend the possibility of having a consistent index of bundles of diverse primary goods, on which see Plott (1978), Gibbard (1979), Blair (1988), and Sen (1991).

21. On this see Hammond (1976, 1982), d'Aspremont and Gevers (1977, 2002), Gevers (1979), Suzumura (1983, 1996), Broome (1991, 2004), Pattanaik and Suzumura (1994), d'Aspremont and Mongin (2008), Blackorby, Bossert, and Donaldson (2002), among other contributions.
22. I should explain that this is definitely not the judgment of this author, but he is able to distinguish between his own assessment and that of many analysts whose judgments he respects.
23. See Sen (1982), Atkinson (1983), Suzumura (1983), Nussbaum and Sen (1993), Dutta (2002), among other contributions.
24. In the format of social welfare functionals, strong neutrality follows from unrestricted domain, Pareto indifference, and independence in a binary form, as was established by d'Aspremont and Gevers (1977); see Theorem 3.7 in d'Aspremont and Gevers (2002, pp. 493–494). The conditions that yield something quite close to neutrality for social welfare functions, as shown in Lemma L.1 above, draw on similar informational demands imposed on that framework.
25. This has been discussed in the exploration of social choice–based ideas of justice in Sen (2009).

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