1a 2π, π, $3\cdot 2\pi = 6\pi$, $\frac{2\pi}{k}$, $\frac{2\pi}{n\pi}\cdot 2 = \frac{4}{n}$

1b) π, $\pi/2$, 3π, π/k, $\frac{\pi}{n\pi/2} = \frac{2}{n}$ half of a.

2) $y = c$ $f(x) = c$ $f(x+p) = c$ so
$f(x) = f(x+p)$ for any p. but 0-periodic makes no sense

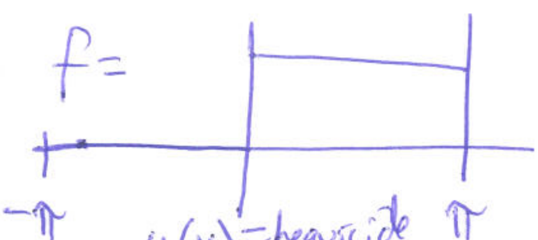
3 a) when the function is already a Fourier series, you don't have to do any this $\sin 2x$ is the answer

$a_0 = 0$ $a_n = 0$ $b_n = 0$ unless $n = 2$ in which case $b_2 = 1$

$$\frac{1}{\pi}\int_{-\pi}^{\pi} (\sin 2x)(\sin 2x)\,dx = \frac{1}{\pi}\cdot\pi = 1$$

b) $\cos 3x$ c) $\sin 2x + \cos 3x$ d) 4 e) $4 + 5\sin 2x - 7\cos 3x$

f) $f =$



$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_{0}^{\pi} 1\,dx = \frac{1}{2\pi}\cdot\pi = \frac{1}{2}$$

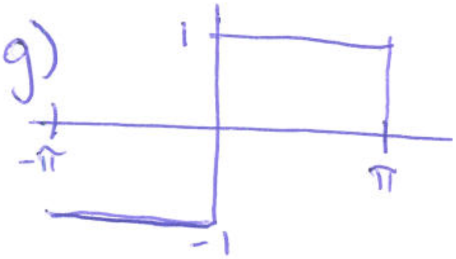
$$a_n = \frac{1}{\pi}\int_0^{\pi} 1\cos(nx)\,dx = \frac{1}{\pi n}\sin(nx)\Big|_0^{\pi} = \frac{1}{\pi n}(0-0) = 0.$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} 1\sin(nx)\,dx = \frac{1}{n\pi}\cdot -\cos(nx)\Big|_0^{\pi} = \frac{1}{n\pi}(-\cos n\pi + 1) = \left(\frac{1}{n\pi}\right)\underbrace{(1-(-1)^n)}_{2 \text{ or } 0}$$

$b_1 = \frac{1}{1\pi}(2)$ $b_2 = 0$ $b_3 = \frac{1}{3\pi}(2)$

$$f = \frac{1}{2} + \frac{2}{\pi}\sin x + \frac{2}{3\pi}\sin 3x + \frac{2}{5\pi}\sin 5x + \frac{2}{7\pi}\sin 7x + \dots$$

3g)



$f(x) = -1 + 2u(x-0)$ ← in mathematica.
Heaviside theta.

$\underline{a_0} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \underline{0}$ because area above cancels with area below

$$\underline{a_n} = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} -\cos nx\,dx + \int_{0}^{\pi} \cos nx\,dx\right)$$

$$= \frac{1}{\pi}\quad \underbrace{-\frac{\sin(nx)}{n}\Big|_{-\pi}^{0}}_{0-0} + \underbrace{\frac{\sin nx}{n}\Big|_{0}^{\pi}}_{0-0} = \underline{0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} -\sin(nx)\,dx + \int_{0}^{\pi} \sin(nx)\,dx\right)$$

$$= \frac{1}{\pi}\left(\frac{\cos nx}{n}\Big|_{-\pi}^{0} + \frac{-\cos nx}{n}\Big|_{0}^{\pi}\right)$$

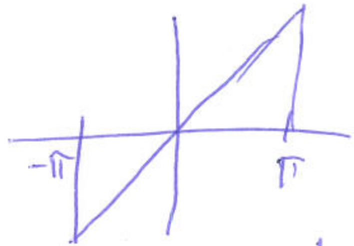
$$= \frac{1}{n\pi}\left(1 - \underbrace{\cos(-n\pi)}_{(-1)^n} + -\underbrace{\cos n\pi}_{(-1)^n} + 1\right)$$

$$= \frac{1}{n\pi}\left(2 - 2(-1)^n\right) = \begin{array}{l} 4 \text{ if } n \text{ is odd} \\ 0 \text{ if } n \text{ is even} \end{array}$$

$b_1 = \frac{4}{\pi}$ $b_2 = 0$ $b_3 = \frac{4}{3\pi}$ $b_4 = 0$ $b_5 = \frac{4}{5\pi}$

$$f(x) = 0 + \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin 3x + \frac{4}{5\pi}\sin 5x + \frac{4}{7\pi}\sin 7x + \cdots$$

3i) $f(x) = x$



$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = 0$ by area argument

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx\,dx$

D	I
$+\ x$	$\cos nx$
$-\ 1$	$(\sin nx)/n$
$+\ 0$	$-\cos nx/n^2$

$= \frac{1}{\pi}\left(\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right)\Big|_{-\pi}^{\pi} = \frac{1}{\pi}\left(\left(\frac{\pi\overset{0}{\sin n\pi}}{n} + \frac{\overset{(-1)^n}{\cos n\pi}}{n}\right) - \left(\frac{\overset{0}{\sin \pi}}{} + \frac{\overset{(-1)^n}{\cos -n\pi}}{n}\right)\right)$

$= \frac{1}{\pi}\left(\frac{(-1)^n}{n} - \frac{(-1)^n}{n}\right) = \boxed{0}$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx\,dx$

D	I
$+\ x$	$\sin nx$
$-\ 1$	$-\cos nx/n$
$+\ 0$	$-\sin nx/n^2$

$-\frac{x\cos nx}{n} + \cancel{\frac{\sin nx}{n^2}}\Big|_{-\pi}^{\pi}$

plugging π into $\sin nx$ gives 0

so $\frac{1}{\pi}\,\frac{-x\cos nx}{n}\Big|_{-\pi}^{\pi} = \frac{1}{\pi}\left(\frac{-\pi\cos(n\pi)}{n} - \left(\frac{+\pi\cos(n(-\pi))}{n}\right)\right)$

$= \frac{1}{\pi}\,\frac{\pi}{n}\left(\overset{-(-1)^n}{-\cos n\pi} - \overset{(-1)^n}{\cos(-n\pi)}\right) = \frac{1}{n}(-2)(-1)^n$

$b_n = \frac{-2}{n}(-1)^n \quad b_1 = +\frac{2}{1} \quad b_2 = -\frac{2}{2} \quad b_3 = +\frac{2}{3} \quad b_4 = -\frac{2}{4}$

$f(x) = 0 + \ +2\sin x - \frac{2}{2}\sin 2x + \frac{2}{3}\sin 3x - \frac{2}{4}\sin 4x + \frac{2}{5}\sin 5x + \cdots$

Compare to 4c

4c $f(x) = x$ for $-1 < x < 1$ period $= 2$ $L = 1$

half the period.

$$a_0 = \frac{1}{L=1}\int_{-L=1}^{L=1} f(x)\,dx = 0 \text{ by area argument}$$

$+ x$	$\sin n\pi x$
-1	$-\cos(n\pi x)/n\pi$
$+0$	$-\sin(n\pi x)/(n\pi)^2$

$$a_n = \frac{1}{L}\int_{-L}^{L} x \sin\left(\frac{n\pi x}{L=1}\right)dx = \frac{1}{1}\int_{-1}^{1} x \sin(n\pi x)\,dx$$

$$= \frac{-x\cos(n\pi x)}{n\pi} + \cancel{\frac{\sin n\pi x}{(n\pi)^2}}\Big|_{-1}^{1}$$

0

$\sin n\pi = 0$
$\sin n\pi = 0$
so

$$= \left(\frac{-1\cos n\pi}{n\pi}\right)_{x=1} - \left(\frac{-(-1)\cos(-n\pi)}{n\pi}\right)_{x=-1} = \frac{-1}{n\pi}\left((-1)^n + (-1)^n\right)$$

$$= \frac{-1}{n\pi}\,2(-1)^n$$

$$b_1 = \frac{2}{\pi} \quad b_2 = \frac{-2}{2\pi} \quad b_3 = \frac{2}{3\pi} \quad b_4 = \frac{-2}{4\pi} \quad b_5 = \frac{+2}{5\pi}$$

$$f(x) = 0 + \frac{2}{\pi}\sin(1\pi x) - \frac{2}{2\pi}\sin(2\pi x) + \frac{2}{3\pi}\sin(3\pi x) - \frac{2}{4\pi}\sin(4\pi x) + \cdots$$

compare to 3i