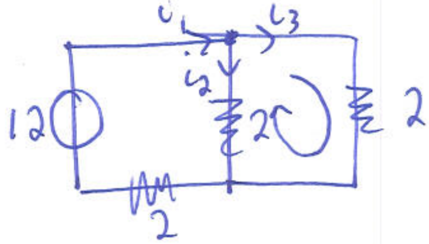
1a)



$i_1 = i_2 + i_3$

$12 = 2i_1 + 2i_2$

$0 = 2i_3 - 2i_2$

negative because you go backwods across the resistor (as you go around the loop you move opposite the current).

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 2 & 2 & 0 & 12 \\ 0 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3/-2 \end{array}$$

$$\left[\begin{array}{cccc} 1 & -1 & -1 & 0 \\ 0 & 4 & 2 & 12 \\ 0 & 1 & -1 & 0 \end{array}\right] \; R_2 - 4R_3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 6 & 12 \end{array}\right] \begin{array}{l} \\ \\ R_3/6 \end{array}$$

$$\left[\begin{array}{cccc} 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} R_1 + R_3 \\ R_2 + R_3 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc} 1 & -1 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} R_1 + R_2 \\ \\ \\ \end{array}$$

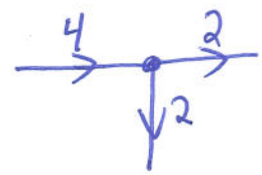
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$i_1 = 4$

$i_2 = 2$

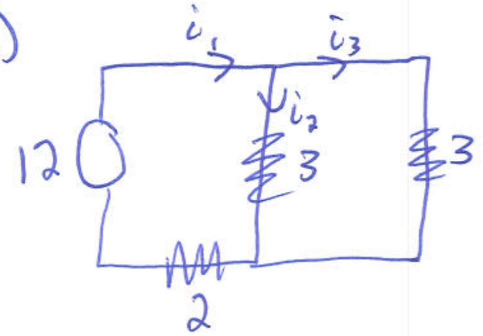
$i_3 = 2$

So the current in the left wire is 4 amps. When it hits the node



half goes each direction, which makes sense because the resistance is the same on both.

1b)



$$i_1 = i_2 + i_3$$
$$12 = 2i_1 + 3i_2$$
$$0 = 3i_3 - 3i_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 2 & 3 & 0 & 12 \\ 0 & -3 & 3 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3/-1 \end{array}$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 5 & 2 & 12 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{array}{l} \\ R_2 - 5R_3 \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 7 & 12 \end{bmatrix} \begin{array}{l} 7R_1 + R_3 \\ 7R_2 + R_3 \\ \\ \end{array}$$

$$\begin{bmatrix} 7 & -7 & 0 & 12 \\ 0 & 7 & 0 & 12 \\ 0 & 0 & 7 & 12 \end{bmatrix} \begin{array}{l} R_1 + R_2 \\ \\ \\ \end{array}$$

$$\begin{bmatrix} 7 & 0 & 0 & 24 \\ 0 & 7 & 0 & 12 \\ 0 & 0 & 7 & 12 \end{bmatrix} \begin{array}{l} /7 \\ /7 \\ /7 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 24/7 \\ 0 & 1 & 0 & 12/7 \\ 0 & 0 & 1 & 12/7 \end{array}\right]$$

$i_1 = 24/7$.

when i_1 hits the node, it splits evenly between i_2 & i_3, giving $i_2 = 12/7 = i_3$.

2a) $2x+3y=0$

$x-2y=1$

$$\left[\begin{array}{cc|c} 2 & 3 & 0 \\ 1 & -2 & 1 \end{array}\right]$$

$$x = \frac{\left|\begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix}\right|}{\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}} = \frac{0-3}{-4-3} = \frac{-3}{-7} = \frac{3}{7} = x$$

$$y = \frac{\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix}} = \frac{2-0}{-4-3} = \frac{2}{-7} = y$$

Remember, If you want to find the first variable, replace column 1 with the vector on the right, ~~[illegible]~~ find the determinant and divide by the det of the coefficient matrix.

3b) (0,1) (2,3) (-1,4)

$y = a_0 + a_1 x + a_2 x^2$ Plug in the 3pts to get 3 eqns.

(0,1) $1 = a_0 + a_1(0) + a_2(0)^2$

(2,3) $3 = a_0 + a_1(2) + a_2(2)^2$

(-1,4) $4 = a_0 + a_1(-1) + a_2(-1)^2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & 2 & 4 & 3 \\ 1 & -1 & 1 & 4 \end{array}\right]$$

I'll use cramers rule to solve.
You could also row reduce to get the answer.

$$a_0 = \frac{\begin{vmatrix} 1 & 0 & 0 \\ 3 & 2 & 4 \\ 4 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 1 \end{vmatrix}} = \frac{1\begin{vmatrix} 2 & 4 \\ -1 & 1 \end{vmatrix}}{1\begin{vmatrix} 2 & 4 \\ -1 & 1 \end{vmatrix}} = \frac{1(2+4)}{1(2+4)} = \frac{6}{6} = 1$$

$$a_1 = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 3 & 4 \\ 1 & 4 & 1 \end{vmatrix}}{6} = \frac{1\begin{vmatrix} 3 & 4 \\ 4 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix}}{6} = \frac{\overset{-10}{1\overset{-13}{(3-16)} - 1\overset{+3}{\overset{-3}{(1-4)}}}}{6} = -10/6 = -5/3$$

$$a_2 = \frac{\begin{vmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & 4 \end{vmatrix}}{6} = \frac{1\begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}}{6} = \frac{\overset{11}{(8+3)} + \overset{-3}{(-1-2)}}{6} = \frac{8}{6} = \frac{4}{3}$$

$$\boxed{\text{so } y = 1 - \tfrac{5}{3}x + \tfrac{4}{3}x^2}$$

3f) $(0,1), (1,3), (-1,4), (2,4)$

Here the polynomial is a cubic.

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$1 = a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3$$

$$3 = a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3$$

$$4 = a_0 + a_1(-1) + a_2(-1)^2 + a_3(-1)^3$$

$$4 = a_0 + a_1(2) + a_2(2)^2 + a_3(2)^3$$

Because its large, I'll row reduce instead of Cramers rule.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & -1 & 1 & -1 & 4 \\ 1 & 2 & 4 & 8 & 4 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - R_1 \\ R_4 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & -1 & 1 & -1 & 3 \\ 0 & 2 & 4 & 8 & 3 \end{bmatrix} \begin{array}{l} \\ \\ R_3 + R_2 \\ R_4 - 2R_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & 0 & 5 \\ 0 & 0 & 2 & 6 & -1 \end{bmatrix} \begin{array}{l} \\ \\ \\ R_4 - R_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 2 & 0 & 5 \\ 0 & 0 & 0 & \underset{1}{\cancel{6}} & \underset{-1}{\cancel{-6}} \end{bmatrix} \begin{array}{l} R_2 - R_4 \\ \\ \\ R_4/6 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 2 & 0 & 5 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{array}{l} \\ 2R_2 - R_3 \\ \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 5 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{array}{l} \\ /2 \\ /2 \\ \\ \end{array}$$

$a_0 = 1$
$a_1 = 1/2$
$a_2 = 5/2$
$a_3 = -1$

So $\boxed{y = 1 + \tfrac{1}{2}x + \tfrac{5}{2}x^2 - x^3}$

4c) $(1,2)\ (3,0)\ (5,1)$

$$y = a_0 + a_1 x$$

$$\begin{aligned} 2 &= a_0 + a_1(1) \\ 0 &= a_0 + a_1(3) \\ 1 &= a_0 + a_1(5) \end{aligned} \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & 3 & 0 \\ 1 & 5 & 1 \end{array}\right]$$

Now multiply on left by $A^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & 3 & 0 \\ 1 & 5 & 1 \end{array}\right] = \left[\begin{array}{cc|c} 3 & 9 & 3 \\ 9 & 35 & 7 \end{array}\right]$$

solve. I'll use cramer's rule.

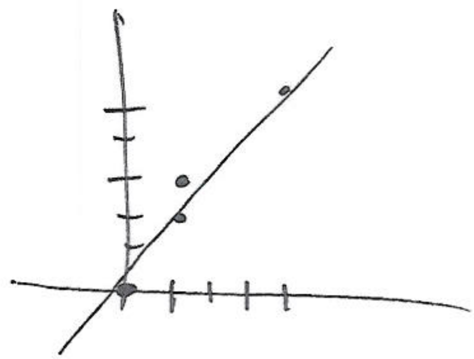
$$a_0 = \frac{\begin{vmatrix} 3 & 9 \\ 7 & 35 \end{vmatrix}}{\begin{vmatrix} 3 & 9 \\ 9 & 35 \end{vmatrix}} = \frac{105 - 63}{105 - 81} = \frac{42}{24} = 7/4$$

$$a_1 = \frac{\begin{vmatrix} 3 & 3 \\ 9 & 7 \end{vmatrix}}{24} = \frac{21 - 27}{24} = \frac{-6}{24} = -1/4$$

So the line is $\boxed{y = 7/4 - 1/4 x}$

4d) $(0,0), (1,3), (1,2), (4,5)$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 4 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 5 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 6 \\ 6 & 18 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 10 \\ 25 \end{bmatrix}$$

Use cramer's rule

$$a_0 = \frac{\begin{vmatrix} 10 & 6 \\ 25 & 18 \end{vmatrix}}{\begin{vmatrix} 4 & 6 \\ 6 & 18 \end{vmatrix}} = \frac{180-150}{72-36} = \frac{30}{36}$$

$$a_1 = \begin{vmatrix} 4 & 10 \\ 6 & 25 \end{vmatrix} \Big/ 36 = \frac{100-60}{36} = \frac{40}{36}$$

Using inverse

$$\frac{1}{72-36} \begin{bmatrix} 18 & -6 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} 10 \\ 25 \end{bmatrix} = \frac{1}{36} \begin{bmatrix} 180 - 150 \\ -60 + 100 \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \end{bmatrix} \Big/ 36$$

$$\begin{bmatrix} 30/36 \\ 40/36 \end{bmatrix} = \begin{bmatrix} 5/6 \\ 10/9 \end{bmatrix} \qquad \boxed{y = 5/6 + \frac{10}{9} x}$$

There's more than one way to do the problem.

The inverse of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \dfrac{\begin{vmatrix} d & -b \\ -c & a \end{vmatrix}}{\det A}$

5b)

$$\frac{2x+3}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} \qquad \text{mult by } (x-3)(x+2)$$

$$2x+3 = A(x+2) + B(x-3)$$

$$= Ax + 2A + Bx - 3B$$

$$2x+3 = x(A+B) + (2A-3B)$$

Slopes must match

Intercepts must match.

So

$$2 = A + B$$
$$3 = 2A - 3B$$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & -3 & 3 \end{array}\right]$$

$$A = \frac{\begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix}}{-5} = \frac{-6-3}{-5} = +9/5$$

$$B = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}}{-5} = \frac{3-4}{-5} = 1/5$$

Now integrate

$$\int \frac{2x+3}{(x-3)(x+2)}\,dx = \int \frac{9/5}{x-3} + \frac{1/5}{x+2}\,dx = \boxed{\frac{9}{5}\ln|x-3| + \frac{1}{5}\ln|x+2| + C}$$

6a)

Make sure your order on Rows & colums remains the same.

$$\begin{array}{cc} & \begin{array}{ccc} R & C & I \end{array} \\ \begin{array}{c} \text{to } R \\ \text{to } C \\ \text{to } I \end{array} & \begin{bmatrix} .85 & .10 & .15 \\ .10 & .70 & .25 \\ .05 & .20 & .60 \end{bmatrix} = A. \end{array}$$

transition matrix.

Current state is $\vec{X}_0 = \begin{bmatrix} 40 \\ 30 \\ 30 \end{bmatrix}$.

after 5 years $\quad \vec{X}_1 = A\vec{X} = \begin{bmatrix} .85 & .10 & .15 \\ .10 & .70 & .25 \\ .05 & .20 & .60 \end{bmatrix} \begin{bmatrix} 40 \\ 30 \\ 30 \end{bmatrix} = \begin{pmatrix} 41.5 \\ 32.5 \\ 26 \end{pmatrix}$

after 10 years $\quad \vec{X}_2 = A^2 x = A\vec{X}_1 = \left[A \right] \begin{pmatrix} 41.5 \\ 32.5 \\ 26 \end{pmatrix} = \begin{pmatrix} 42.425 \\ 33.4 \\ 24.175 \end{pmatrix}$

Please use a calculator here

Try to get the eigenvector without a calc.

$$\left[\begin{array}{ccc|c} .85-1 & .10 & .15 & 0 \\ .10 & .70-1 & .25 & 0 \\ .05 & .20 & .60-1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} -.15 & .10 & .15 & 0 \\ .10 & -.3 & .25 & 0 \\ .05 & .2 & -.4 & 0 \end{array}\right] \begin{array}{l} \times 100 \\ \times 100 \\ \times 100 \end{array}$$

$$\left[\begin{array}{cccc} -15 & 10 & 15 & 0 \\ 10 & -30 & 25 & 0 \\ 5 & 20 & -40 & 0 \end{array}\right] \begin{array}{l} /5 \\ /5 \\ /5 \end{array} \rightarrow \left[\begin{array}{ccc|c} -3 & 2 & 3 & 0 \\ 2 & -6 & 5 & 0 \\ 1 & 4 & -8 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 4 & -8 & 0 \\ 2 & -6 & 5 & 0 \\ -3 & 2 & 3 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 + 3R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{cccc} 1 & 4 & -8 & 0 \\ 0 & -14 & 21 & 0 \\ \cancel{0} & \cancel{14} & \cancel{-21} & \cancel{0} \end{array}\right] \begin{array}{l} \\ R_2/-7 \\ R_3 + R_2 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 4 & -8 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_1 - 2R_2 \quad \begin{array}{cccc} 1 & 0 & -2 & 0 \\ 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

$x = 2z$
$y = 3z/2$
$z = z$

$\begin{bmatrix} 2 \\ 3/2 \\ 1 \end{bmatrix}$

or times by 2 to get $\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$

$\begin{pmatrix} 4/9 \\ 3/9 \\ 2/9 \end{pmatrix} \begin{array}{l} - R \\ - C \\ - I \end{array}$

6 b)

$$\begin{array}{c c} & \begin{array}{cc} R & IF \end{array} \\ \begin{array}{c} \text{to } R \\ \text{to } IF \end{array} & \begin{bmatrix} .60 & .20 \\ .40 & .80 \end{bmatrix} \end{array} = A \qquad \overset{\text{currently}}{\downarrow} \; \vec{x}_0 = \begin{bmatrix} 60 \\ 140 \end{bmatrix}$$

next week $\vec{x}_1 = A\vec{x}_0 = \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix}\begin{bmatrix} 60 \\ 140 \end{bmatrix} = \begin{bmatrix} 64 \\ 136 \end{bmatrix}$

use calculator

2 weeks $\vec{x}_2 = A\vec{x}_1 = \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix}\begin{bmatrix} 64 \\ 136 \end{bmatrix} = \begin{bmatrix} 66.24 \\ 133.76 \end{bmatrix}$ so about $\begin{pmatrix} 66 \\ 134 \end{pmatrix}$

Steady state (do by hand).

$$\left[\begin{array}{cc|c} .6-1 & .2 & 0 \\ .4 & .8-1 & 0 \end{array}\right] = \left[\begin{array}{cc|c} -.4 & .2 & 0 \\ .4 & -.2 & 0 \end{array}\right] \begin{array}{l} R_1 \cdot 5 \\ R_2 + R_1 \end{array} \Rightarrow \left[\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$-2x + y = 0$

$x = 1$
$y = 2$ solves

so eigenvector are multiples of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$

33%
66%

33% of 200 cars is 66.7 cars, so 66 to 67 will be in Rex, and 134 to 133 in IF.

7d) $f = x^2 - 4x + y^2 + 2y + 1$

$Df = [\underbrace{2x-4}_{=0} \; \vdots \; \underbrace{2y+2}_{=0}] = [0 \;\; 0]$

$2x = 4 \qquad 2y = -2$

$\boxed{x = 2 \qquad y = -1}$ ← critical pt, where the max, min, or saddle occurs.

$D^2f = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

find eigenvalues. $\left| \begin{bmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} \right| = (2-\lambda)(2-\lambda) = 0$

$\lambda = \underline{2, 2}$

both positive

∪ ∪ so always

<u>concave up.</u>

$$\boxed{\text{min @ } (2,-1)}$$

7g $f = x^3 - 3x + y^2 - 2y$

$$Df = \left[3x^2 - 3 \mid 2y - 2\right] = [0 \quad 0]$$

$3x^2 = 3$
$x^2 = 1$
$x = \pm 1$

$2y = 2$
$y = 1$

so $(1,1)$ ad $(-1,1)$ are critical pts

$$D^2 f = \begin{bmatrix} 6x & 0 \\ 0 & 2 \end{bmatrix}$$

$D^2 f(1,1) = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$ eigenvalues are $\underset{\cup}{6}, \underset{\cup}{2}$. both concave up

so $\boxed{\text{min @ } (1,1)}$

$D^2 f(-1,1) = \begin{bmatrix} -6 & 0 \\ 0 & 2 \end{bmatrix}$ $\lambda = \overset{\cup}{\underset{\text{up}}{-6}}, \overset{\wedge}{\underset{\text{down}}{2}}$ $\boxed{\text{saddle @ } (-1,1)}$