23: 8 $L(u(x-c)) = \int_0^{\infty} e^{-sx} u(x-c)\,dx$

$= \int_c^{\infty} e^{-sx}\,dx$ (since $u(x-c)=0$ if $x<c$)

$= \left.\dfrac{e^{-sx}}{-s}\right|_c^{\infty} = (0) - \left(\dfrac{e^{-sc}}{-s}\right) = \dfrac{e^{-sc}}{s}$

23.14 $g = \begin{cases} 0 & x<4 \\ x^2 & x \geq 4 \end{cases}$ Turn on x^2 @ $x=4$

so $g(x) = x^2 u(x-4)$ need an $x-4$

rewrite as $g(x) = (\underline{x-4}+4)^2 u(\underline{x-4})$ replace $x-4$ with t.

$L((t+4)^2)$

$= L(t^2+8t+16) = \dfrac{2}{s^3} + \dfrac{8}{s^2} + \dfrac{16}{s}$

So $\boxed{L(x^2 u(x-4)) = \left(\dfrac{2}{s^3} + \dfrac{8}{s^2} + \dfrac{16}{s}\right)e^{-4s}}$

23.6 (not 24)

$L^{-1}\left(\dfrac{1}{s(s^2+4)}\right) = L^{-1}\left(\dfrac{1}{s}\right) * L^{-1}\left(\dfrac{1}{s^2+4}\right) = 1 * \dfrac{1}{2}\sin(2x)$

$= \dfrac{1}{2}\sin(2x) * 1$

$\int_0^x \dfrac{1}{2}\sin(2p)\cdot 1\,dp = -\dfrac{1}{4}\cancel{\sin}\cos(2p)\Big|_0^x$

$\boxed{= -\dfrac{1}{4}\cos(2x) + \dfrac{1}{4}}$

23.20

$$\cancel{x * x = \int_0^x (p-x)\, p\, dp = \int_0^x t}$$

$$\underset{f(x)}{x} * \underset{g(x)}{x} = \int_0^x f(p)\, g(x-p)\, dp = \int_0^x p\,(x-p)\, dp$$

$$= \int_0^x px - p^2\, dp$$

$$= \frac{p^2}{2}x - \frac{p^3}{3}\Big|_0^x = \frac{x^3}{2} - \frac{x^3}{3} = \frac{x^3}{6}$$

23.21

$$2 * x = x * 2$$

$$\int_0^x 2(x-p)\, dp \qquad \int_0^x p \cdot 2\, dp = p^2\Big|_0^x = x^2$$

easier

$$\int_0^x 2x - 2p\, dp = 2xp - \frac{\cancel{2}p^2}{\cancel{2}}\Big|_0^x = 2x^2 - \frac{\cancel{2}x^2}{\cancel{2}} = x^2$$

23.22

$$4x * e^{2x} = e^{2x} * 4x$$

$$\int_0^x 4p e^{2(x-p)}\, dp \qquad \int_0^x e^{2p}\, 4(x-p)\, dp$$

I prefer this one

	D	I
+	$4(x-p)$	e^{2p}
−	-4	$e^{2p}/2$
+	0	$e^{2p}/4$

$$\overset{2}{\cancel{4}}(x-p)\frac{e^{2p}}{\cancel{2}} + \frac{4e^{2p}}{4}\Big|_0^x$$

$$(0 + e^{2x}) - (2x + 1)$$

$$\boxed{e^{2x} - 2x - 1}$$

23.23

$$e^{4x} * e^{-2x} = e^{-2x} * e^{4x} = \int_0^x e^{-2p} e^{4(x-p)}\, dp = \int_0^x e^{-2p+4x-4p}\, dp$$

$$= \int_0^x e^{4x} e^{-6p}\, dp = e^{4x}\frac{e^{-6p}}{-6}\Big|_0^x = \frac{e^{4x}e^{-6x}}{-6} - \frac{e^{4x}}{-6} = -\frac{1}{6}e^{-2x} + \frac{e^{4x}}{6}$$

23.28

$$\frac{1}{(s-1)(s-2)} = \frac{1}{(s-1)} \cdot \frac{1}{(s-2)}$$

L^{-1} ↓ ↓ ↓

$$e^{x} * e^{2x} = e^{2x} * e^{x} = \int_0^x e^{2p} e^{x-p}\,dp$$

$$= \int_0^x e^{x} e^{p}\,dp = e^{x} \frac{e^{p}}{1}\Big|_0^x = \boxed{e^{2x} - e^{x}}$$

23.29

$$\frac{1}{(s)(s)} = \left(\frac{1}{s}\right)\left(\frac{1}{s}\right)$$

L^{-1} ↓ ↓

$$1 * 1 = \int_0^x 1 \cdot 1\,dp = p\Big|_0^x = X$$

23.30 $$\frac{2}{s(s+1)} = \frac{2}{s} \quad \frac{1}{s+1}$$

↓ ↓

$$2 * e^{-x} = e^{-x} * 2 = \int_0^x e^{-p} \cdot 2\,dp = \frac{2e^{-p}}{-1}\Big|_0^x$$

$$= \frac{2e^{-x}}{-1} - \frac{2 \cdot 1}{-1} = \boxed{2 - 2e^{-x}}$$

23.31

$$\frac{1}{}$$

23.33

$$L^{-1}\left(\frac{1}{s(s^2+4)}\right) = L^{-1}\left(\frac{s}{s^2(s^2+4)}\right) = L^{-1}\left(\frac{1}{s^2}\right) * L^{-1}\left(\frac{s}{s^2+4}\right)$$

$$= x * \cos(2x)$$

$$= \cos(2x) * x$$

$$= \int_0^x \cos(2p)(x-p)\,dp$$ Integration by parts

	D	I
+	$x-p$	$\cos(2p)$
−	-1	$\sin(2p)/2$
−	0	$-\cos(2p)/4$

$$= (x-p)\frac{\sin 2p}{2} - \frac{\cos 2p}{4}\Big|_0^x$$

$$= \left(0 - \frac{\cos 2x}{4}\right) - \left(x \cdot 0 - \frac{1}{4}\right)$$

$$= -\frac{\cos 2x}{4} + \frac{1}{4}$$ same as #6.