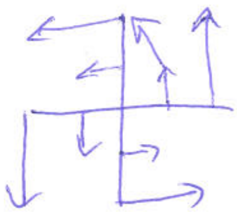
General functions.

$f = x^2+y^2+z^2$

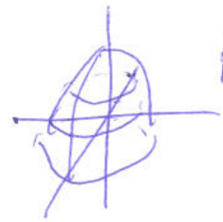
4D 3D level surfaces

vector fields

$F = \langle -y, x\rangle$



$r(x,y) = \langle x, y, 9-x^2-y^2\rangle$



parametric surfaces.

General Derivatives.

Df = a matrix where each column is a partial derivative

$f = 9-x^2-y^2 \qquad Df(x,y) = \begin{bmatrix} -2x & -2y \end{bmatrix}$

$r(t) = \langle \cos t, \sin t, t\rangle \qquad Dr(t) = \begin{bmatrix} -\sin t \\ \cos t \\ 1 \end{bmatrix}$.

$g(r,\theta) = \langle r\cos\theta, r\sin\theta\rangle \qquad Dg(r,\theta) = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$

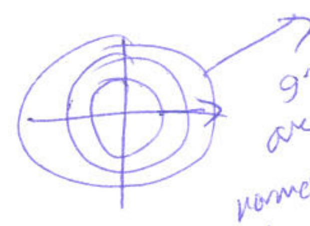
Chain rule.

$$D(f\circ g)(r,\theta) = Df\cdot Dg = \begin{bmatrix} -2x & -2y \end{bmatrix}\begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} -2(r\cos\theta)\cos\theta - 2y\sin\theta & +2x\,r\sin\theta - 2y\,r\cos\theta \end{bmatrix}$$

Gradients, potentials, exact diff forms

$f = 9-x^2-y^2$ has gradient $\nabla f = \langle -2x, -2y\rangle$, a vector field.



gradients are normal to level curves, and point in direction of greatest increase.

$F = \langle x+y, x+y^2\rangle$ has a potential f

since $f = \frac{x^2}{2} + xy + \frac{y^3}{3}$ satisfies $\nabla f = \vec{F}$.

$df = (x+y)dx + (x+y^2)dy$ is called the differential of f.

$\int_A^B df = f(B) - f(A) \leftarrow$ called the fundamental thm of line integrals

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Graphs

Graphs	derivatives.
$y = x^2$	$2x$
x^3	$3x^2$
x^4	$4x^3$
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\ln x$	$\frac{1}{x}$
e^x	e^x

$(\csc x)' = -\csc x \cot x$

$(\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \tan x$

$(\sin^{-1} x)' =$

$(\cos^{-1}(x))' =$

use implicit diff.

product rule

$(fg)' = fg' + f'g$

Quotient $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$

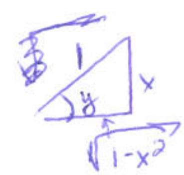
Chain rule $\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$

$y = \sin^{-1} x$

$x = \sin y$

$1 = \cos y \, y'$

$y' = 1/\cos y = \sec y = \frac{1}{\sqrt{1-x^2}}$



Integrals

u sub.

$$\int \frac{y}{4+y^2}\,dy \qquad u = 4+y^2 \qquad du = 2y\,dy$$

so $dy = \frac{du}{2y}$ $\quad \int \frac{y}{4+y^2}\,dy = \int \frac{\cancel{y}}{u}\frac{du}{2y}$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|4+y^2| + C$$

Int by parts. $\int u\,dv = uv - \int v\,du$.

$$\int x e^x\,dx = xe^x - e^x + C$$

x	e^x
1 $+$	e^x
0 $-$	e^x

$$\int \ln x\,dx = x\ln x - \int \frac{1}{x} x\,dx$$

$$= x\ln x - x + C.$$

$\ln x$ $+$	1
$\frac{1}{x}$ $-$	x

General functions:

$y = x^2$

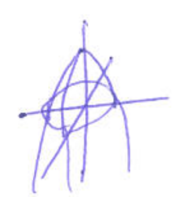
$f: \mathbb{R}^1 \to \mathbb{R}^1$

$r(t) = \langle 3\cos t, 2\sin t\rangle$

$r(t) = \langle \cos t, \sin t, t\rangle$

space curves

$f(x,y) = 9 - x^2 - y^2$



or

level curves