20) $y'' + 3y' + 2y = 4u(t-1) + 10\delta(t-2)$ $\quad y(0)=0 \quad y'(0)=0$

$$s^2 Y + 3sY + 2Y = \frac{4e^{-s}}{s} + 10e^{-2s}$$

$s^2+3s+2 = (s+2)(s+1)$

$$Y = \frac{4}{s(s+2)(s+1)}e^{-s} + \frac{10}{(s+2)(s+1)}e^{-2s}$$

$$\frac{4}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1}$$

$$4 = A(s+2)(s+1) + B(s)(s+1) + C(s)(s+2)$$

let $s=0$ $\quad 4 = A(2)(1) + 0 + 0 \quad A=2$

$s=-2 \quad 4 = 0 + B(-2)(-1) + 0 \quad B=2$

$s=-1 \quad 4 = 0 + 0 + C(-1)(+1) \quad C=4$

$$\frac{10}{(s+2)(s+1)} = \frac{D}{s+2} + \frac{E}{s+1}$$

$10 = D(s+1) + E(s+2)$

$s=-1 \quad 10 = 0 + E(1)$

$s=-2 \quad 10 = D(-1) + 0$

$E = 10$

$D = -10$

$$Y = \left(\frac{2}{s} + \frac{2}{s+2} + \frac{4}{s+1}\right)e^{-s} + \left(\frac{-10}{s+2} + \frac{10}{s+1}\right)e^{-2s}$$

$$y(t) = \left(2 + 2e^{-2(t-1)} + 4e^{-(t-1)}\right)u(t-1) + \left(-10e^{-2(t-2)} + 10e^{-(t-1)}\right)u(t-2)$$

That's it notice there are 2 Heavisides

22) $L = 0 \quad R = 2 \quad C = 1/5 \quad E = 12u(t-2) \quad I(0) = 0$

$E' = 12\delta(t-2)$

$\cancel{LI''} + RI' + \frac{1}{C} I = E'$

no inductor.

$$2I' + 5I = 12\delta(t-2).$$

$$2(sY - 0) + 5Y = 12e^{-2s}$$

$$Y = \frac{12e^{-2s}}{2s+5} = \frac{\cancel{12}^{\,6} e^{-2s}}{\cancel{2}(s + 5/2)}$$

$$y = 6e^{-5/2(t-2)} u(t-2)$$

Ignore the capacitor.

23] $L=1, R=2, C=0$ $E(t) = 12u(t-2)$ $I(0)=0.$

Use the equation $LI' + RI = E(t).$ (don't take derivatives)

$$1I' + 2I = 12u(t-2)$$

$$sY - 0 + 2Y = \frac{12e^{-2s}}{s}$$

$$Y = \frac{12e^{-2s}}{s(s+2)} = \left(\frac{A}{s} + \frac{B}{s+2}\right)e^{-2s}$$

$$= \left(\frac{6}{s} - \frac{6}{s+2}\right)e^{-2s}$$

$$\frac{12}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$12 = A(s+2) + Bs$$

Let $s=0$ $12 = 2A$ $A=6$

Let $s=-2$ $12 = 0 - 2B$ $B=-6$

invert

$$\boxed{I(t) = \left(6 - 6e^{-2(t-2)}\right)u(t-2)}$$

25] $L=1 \quad R=2 \quad C=\frac{1}{5} \quad E=12u(t-2) \quad I(0)=0$

$Q(0)=0.$

~~Ey''~~

$$1y' + 2y + 5\int y\,dt = 12u(t-2)$$

$$@0 \quad y'(0)+0+0=0 \quad \text{so} \quad \underline{y'(0)=0.}$$

now differentiate

$$y'' + 2y' + 5y = 12\,\delta(t-2)$$

$$s^2Y + 2sY + 5Y = 12e^{-2s}$$

Complete the square.

$\underline{s^2+2s+1}-1+5$

$$Y = \frac{12\,e^{-2s}}{s^2+2s+5} = \frac{12}{(s+1)^2+4}e^{-2s}$$

$$\frac{12}{(s+1)^2+4}\cdot\frac{2}{2}$$

$$y(t) = \frac{12}{2}\sin(2(t-2))e^{-1(t-2)}u(t-2)$$

27) This is the hardest one of all. $\boxed{y = I}$

★ Find $y'(0)$ first

$$y' + 2y + 5\int y\,dt = 4\cos 3t$$

$y(0)=0$
$\int y\,dt @0 = 0$

$$y'(0) + 0 + 0 = 4 \quad \text{so} \quad \underline{\underline{y'(0) = 4}}$$ ★. Important

Now differentiate

$$y'' + 2y' + 5y = -12\sin 3t$$

$$(s^2Y - s0 - 4) + 2sY + 5Y = \frac{-12 \cdot 3}{s^2+9}$$

$$Y(s^2+2s+5) = \frac{-36}{s^2+9} + 4$$

Remember to complete the square.

$$s^2+2s+5 = (s+1)^2+4$$

$$Y = \frac{-36}{(s^2+2s+5)(s^2+9)} + \frac{4}{s^2+2s+5} \quad \text{OK.}$$

$$L^{-1}\left(\frac{4}{(s+1)^2+4}\cdot\frac{2}{2}\right) = \frac{4}{2}\sin(2t)$$

We need a partial fraction decomposition.

$$\frac{-36}{\text{stuff}} = \frac{As+B}{(s+1)^2+4} + \frac{Cs+D}{s^2+9}$$

$$-36 = (As+B)(s^2+9) + (Cs+D)(s^2+2s+5)$$

$$-36 = As^3 + 9As + Bs^2 + 9B + Cs^3 + 2s^2C + 5Cs + Ds^2 + 2Ds + 5D$$

use a calculator now.

s^3	s^2	s	1
A	B	9A	9B
C	2C	5C	
	D	2D	5D
0	0	0	-36

$A + C = 0$ $\boxed{A = -C}$
$B + 2C + D = 0$
$9A + 5C + 2D = 0$ $\quad -4C + 2D = 0$ so $\boxed{D = 2C}$
$9B + 5D = -36$

$B + 2C + 2D = 0$ or $B + 4C = 0$
$9B + 10C = -36$

$$B = \frac{\begin{vmatrix} 0 & 4 \\ -36 & 10 \end{vmatrix}}{\begin{vmatrix} 1 & 4 \\ 9 & 10 \end{vmatrix}} = \frac{36 \cdot 4}{-26}$$

$$C = \frac{\begin{vmatrix} 1 & 0 \\ 9 & -36 \end{vmatrix}}{-26} = \frac{-36}{-26}$$

Next page

$C = \frac{+36}{+26}$ $B = \frac{144}{-26}$ $A = -\frac{36}{26}$ $D = \frac{36}{13}$

$C = \frac{18}{13}$ $B = -\frac{72}{13}$ $A = \frac{-18}{13}$ $D = \frac{36}{13}$

$\cancel{y} =$

$4, 13 = 52$

$52 - 18 = 34$

$52 - 72 = -20$

So $Y = \dfrac{-18/13\, s + \frac{-\cancel{72}}{13}}{(s+1)^2+4} + \dfrac{18/13\, s + \frac{36}{13}}{s^2+9} + \dfrac{4}{(s+1)^2+4}$

$= \dfrac{(\overset{-18}{\cancel{34}}/13)s - \frac{\overset{-20}{\cancel{72}}}{13}}{(s+1)^2+4} + \dfrac{18/13\, s + \frac{36}{13}}{s^2+9}$

Now make s be an $s+1$.

$+18-20=-2$

$\frac{\overset{-18}{\cancel{34}}}{13}(s+1-1) - \frac{\overset{-20}{\cancel{72}}}{13} = \frac{\overset{-18}{\cancel{34}}}{13}(s+1) + \frac{\overset{18}{\cancel{34}}}{13} - \frac{\overset{20}{\cancel{72}}}{13} = \cancel{-106} = \frac{-2}{13}$

So $Y = \dfrac{\frac{\overset{-18}{\cancel{34}}}{13}(s+1) - \frac{\overset{2}{\cancel{106}}}{13}}{(s+1)^2+4} + \dfrac{\left(\frac{18}{13}\right)s + \frac{36}{13}}{s^2+9}$

$$\boxed{y(t) = \frac{\overset{18}{\cancel{34}}}{13}\cos(2t)e^{-t} + \left(\frac{\overset{-2}{\cancel{-106}}}{13}\right)\left(\frac{1}{2}\right)e^{-t}\sin 2t + \frac{18}{13}\cos(3t) + \frac{36}{13}\left(\frac{1}{3}\right)\sin(3t)}$$

$$y = \boxed{e^{-t}\left(\frac{18}{13}\cos 2t - \frac{1}{13}\sin 2t\right) + \frac{18}{13}\cos 3t + \frac{12}{13}\sin 3t}$$

This is the ugliest. If you can do this, you have it mastered.