11.9) $\begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$

$$\begin{vmatrix} 3-\lambda & -4 \\ 2 & -6-\lambda \end{vmatrix} = (3-\lambda)(-6-\lambda)+8$$
$$= -18+6\lambda-3\lambda+\lambda^2+8$$
$$= \lambda^2+3\lambda-10$$
$$= (\lambda+5)(\lambda-2)$$
$$\lambda = 2, -5$$

$\lambda = 2$

$$\left[\begin{array}{cc|c} 1 & -4 & 0 \\ 2 & -8 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -4 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

eigenvector

$x - 4y = 0 \quad x = 4y$
$y = y \quad y = y$

$\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\lambda = -5$

$$\left[\begin{array}{cc|c} 8 & -4 & 0 \\ 2 & -1 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & -1/2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x = \frac{1}{2}y$
$y = y$

$\begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$P = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$

check $P^{-1}AP = D$

$$\frac{1}{7}\begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$$
$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 8 & -5 \\ 2 & -10 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 & 0 \\ 0 & -35 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix} \checkmark$$

11.10) $B = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ $\quad \begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) - 8$

$= 3 - 3\lambda - \lambda + \lambda^2 - 8$

$= \lambda^2 - 4\lambda - 5 = (\lambda-5)(\lambda+1)$

$\lambda = 5, -1.$

A)

$\lambda = 5$

$\left[\begin{array}{cc|c} -4 & 4 & 0 \\ 2 & -2 & 0 \end{array}\right] \xrightarrow{rref} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad \begin{matrix} x = y \\ y = y \end{matrix}$ $\quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = -1$

$\left(\begin{array}{cc|c} 2 & 4 & 0 \\ 2 & 4 & 0 \end{array}\right) \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\quad x + 2y = 0 \quad \begin{matrix} x = -2y \\ y = y \end{matrix}$ $\quad \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

B) $P = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$ $\quad D = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$

Check $\quad P^{-1} B P = D \quad \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$

11.10 c) $\quad B = PDP^{-1}$

$B^5 = (PDP^{-1})^5 = (PD\underbrace{P^{-1})(P}_{I}D\underbrace{P^{-1})(P}_{I}D\underbrace{P^{-1})(P}_{I}D\underbrace{P^{-1})(P}_{I}DP^{-1})$

$= P D^5 P^{-1}$

$= \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5^5 & 0 \\ 0 & (-1)^5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}^{-1} =$ see book

11.11 $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ $\left|\begin{bmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix}\right| = (2-\lambda)(3-\lambda) - 2$

$= 6 - 5\lambda + \lambda^2 - 2 = \lambda^2 - 5\lambda + 4$

$(\lambda - 4)(\lambda - 1)$

$\lambda = 4, 1$

a) $\lambda = 4$ $\left[\begin{array}{cc|c} -2 & 2 & 0 \\ 1 & -1 & 0 \end{array}\right] \xrightarrow{\text{rref}} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = 1$ $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ \cancel{1} & \cancel{2} & \cancel{0} \end{array}\right]$ $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

b) $P = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$ check $P^{-1}AP = D$

$P^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

c) $A = PDP^{-1}$ $A^6 = PD^6P^{-1}$

$= \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4^6 & 0 \\ 0 & 1^6 \end{bmatrix} \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ = see book

$\sqrt{A} = P\sqrt{D}P^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ = see book

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11.12 $B = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$ $\begin{vmatrix} 1-\lambda & 3 \\ 5 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) - 15$

$= 3 - 4\lambda + \lambda^2 - 15 = \lambda^2 - 4\lambda - 12$

$= (\lambda - 6)(\lambda + 2)$

$\lambda = 6, -2$

The language just means find eigenvalues + eigenvectors.

$\lambda = 6$ $\left[\begin{array}{cc|c} -5 & 3 & 0 \\ 5 & -3 & 0 \end{array}\right] \xrightarrow{\text{rref}} \begin{bmatrix} 1 & -3/5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ so $x = \frac{3}{5}y$, $y = y$ $\begin{bmatrix} 3/5 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\lambda = -2$ $\begin{bmatrix} 3 & 3 & 0 \\ 5 & 5 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ so $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 3 & -1 \\ 5 & 1 \end{bmatrix}$ $S = \{(3,5), (-1,1)\}$ $D = \begin{bmatrix} 6 & 0 \\ 0 & -2 \end{bmatrix}$

check $P^{-1}BP = D$

11.13 $\begin{bmatrix} 5 & 6 \\ 3 & -2 \end{bmatrix}$ $\begin{vmatrix} 5-\lambda & 6 \\ 3 & -2-\lambda \end{vmatrix} = (5-\lambda)(-2-\lambda) - 18$

$= -10 - 5\lambda + 2\lambda + \lambda^2 - 18$

$= \lambda^2 - 3\lambda - 28 = (\lambda-7)(\lambda+4)$

$\lambda = 7, -4$

(a)

$\lambda = 7$ $\left[\begin{array}{cc|c} -2 & 6 & 0 \\ 3 & 9 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\lambda = -4$ $\left[\begin{array}{cc|c} 9 & 6 & 0 \\ 3 & 2 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2/3 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\begin{bmatrix} -2/3 \\ 1 \end{bmatrix} \approx \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\lambda = 7$ $\vec{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\lambda = -4$ $\vec{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(b) Notice the characteristic polynomial is

$\lambda^2 + 1$

which has <u>no</u> real zeros, so no real eigenvalues or eigenvectors

(c) $\lambda = 4$ is a double root. However There is only 1 lin indep eigenvector. Since there were not 2, this matrix is <u>not</u> diagonalizable

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14) $\lambda^2 + 1 = 0 \Rightarrow \lambda^2 = -1$ $\lambda = \pm\sqrt{-1} = \pm i$

there are 2 complex eigenvalues.

C^2 is the complex plane.

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all others. Their work is fine. Just realize they compute the characteristic polynomial differently. Just do it the way we have all semester. If it is order 3 or more, use software to factor.