9.1

a) $A = \begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix}$

b) let $P = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ so that $P[v]_s = v$

Then $A\vec{v} = T(\vec{v})$

becomes $A(P[v]_s) = P[T(v)]_s$

or $P^{-1}AP[v]_s = [T_v]_s$

So $B = P^{-1}AP$

c) we used coordinates to create it No need to verify

9.2 always start by getting the standard representation

$[G]_E = A = \begin{bmatrix} 2 & -7 \\ 4 & 3 \end{bmatrix}$

a) let $P = \left[\begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix}\right]$ so that $P[v]_s = v$

Then $Av = T(\vec{v})$ means

$A[P[v]_s] = P[T(v)]_s$ or

Here is the verification

$\underline{P^{-1}AP}[v_s] = [G]_s$ perform multiplication.

$\frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -7 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = [G]_s$

b) or do what they did in the book

9.3

let $P = \begin{bmatrix} 1 & 3 \\ -2 & -7 \end{bmatrix}$ Then $B = P^{-1}AP$. <u>Done</u> (see method 2)

9.4 see book. theirs is fine

9.5 $[G]_E = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -4 & 0 \\ 3 & 0 & 0 \end{bmatrix} = A$

a) let $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ then $[G]_S = P^{-1}AP$ <u>done</u>.

always start by getting the standard representation A, and then use a similarity transformation

b) let $v = (a,b,c)$

$[G][v] = (P^{-1}AP)\begin{bmatrix} a \\ b \\ c \end{bmatrix}_S =$ ~~[illegible]~~

you have to first write any vector (a,b,c) in terms of its coordinates relative to S.

~~$[G(v)] =$~~

Hence $P\begin{bmatrix} a \\ b \\ c \end{bmatrix}_S = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

so $P^{-1}AP\begin{bmatrix} a \\ b \\ c \end{bmatrix}_S = P^{-1}A\begin{pmatrix} a \\ b \\ c \end{pmatrix} = P^{-1}G(a,b,c)$

we know $G(a,b,c) = A\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ so

we also know $P[G(a,b,c)]_S = G(a,b,c)$

so $P^{-1}G(a,b,c) = P^{-1}(P[G(a,b,c)]_S) = \overset{I}{\cancel{P^{-1}P}}[G(a,b,c)]_S$

$= G(a,b,c)$.

On this one, please try to follow the book as well.

9.6) $A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -1 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ so $P[u]_s = u$

$B = P^{-1}AP$: why? $Au = Au$

so $A\,P[u]_s = P[Au]_s$

Hence $\underbrace{P^{-1}AP}_{B}[u]_s = [Au]_s$

Use maple to invert. (or this one isn't too bad by hand)

9.7) a) $L(1,0) = (2,4)$ $L(0,1) = (5,8)$ $A = \begin{bmatrix} 2 & 5 \\ 4 & 8 \end{bmatrix}$

b) $L(1,0) = (0,1)$ $L(0,1) = (-1,0)$ $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

c)  $L(1,0) = (0,-1)$ $L(0,1) = (-1,0)$ $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

9.8 $S = \{(1,1), (1,2)\}$ $L(1,1) = (4,7)$ $L(1,2) = (1,6)$ Standard satisfies $A\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix}$

so $A = \begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$ (This is the standard matrix).

To get the matrix relative to S, we let $P = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$,

Then $B = P^{-1}AP = P^{-1}\begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix}\overbrace{\cancel{\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}}}^{I}$

$= P^{-1}\cancel{A}\begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & 5 \end{bmatrix}$

The books work is equivalent, it just doesn't focus on matrices.

9.9) Start by deciding on a "standard" representation.

let $(1,0,0) = e^{3t}$, $(0,1,0) = te^{3t}$, $(0,0,1) = t^2e^{3t}$.

Then $D(e^{3t}) = 3e^{3t} = 3(1,0,0) = (3,0,0)$

so $D(1,0,0) = (3,0,0)$

also $D(te^{3t}) = t \cdot 3e^{3t} + 1 \cdot e^{3t} = (1,3,0)$

and $D(t^2e^{3t}) = t^2 3e^{3t} + 2te^{3t} = (0,2,3)$

Hence $[D] = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

9.10 $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = [1,2,3,4]_E$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = (1,0,0,0)$
$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = (0,1,0,0)$
$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = (0,0,1,0)$
$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = (0,0,0,1)$

Let $T(A) = MA$

$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = \underline{(1,0,3,0)}$

$T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix} = \underline{(0,1,0,3)}$

$T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix} = \underline{(2,0,4,0)}$

$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix} = \underline{(0,2,0,4)}$

Place in columns

So $[T] = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 3 & 0 & 4 & 0 \\ 0 & 3 & 0 & 4 \end{bmatrix}$

9.11 standard $A = \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix}$ E let $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $S = \{(1,4),(2,7)\}$

$B[u]_E = u$ $C = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$ $C[u]_S = u$

a) from E to S. Solve $B[u]_E = C[u]_S$ for $[u]_E = B^{-1}C[u]_S = C[u]_S$

$$P = C = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$$

9,11 continued.

from S to E requires we solve

$$B[u]_E = C[u]_S \quad \text{for} \quad [u]_S = C^{-1}B[u]_E$$

$$\text{so} \quad Q = C^{-1}B = C^{-1} = \frac{1}{-1}\begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix}$$

Notice $(P)^{-1} = (B^{-1}C)^{-1} = C^{-1}B = Q.$

b) $A = \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix}$ $\cancel{A[u]_E = T(u)}$

$$B = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -2 & 1 \end{bmatrix}$$

c) $\text{trace } A = 6 = \text{trace } B$

$\det A = 5 + 2 = 7$ $\quad \det B = 5 + 2 = 7$

Trace and det are always the same for similar matrices.

9.12 The books solution is great.

9.16 The standard representation is $A = \begin{bmatrix} 3 & 2 & -4 \\ 1 & -5 & 3 \end{bmatrix}$

a) Let $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $C^{-1} = \frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

so $B[u]_S = u$ and $C[v]_{S'} = v$

we have $Au = F(u)$ so $AB[u]_S = C[F(u)]_{S'}$

$\begin{bmatrix} -7 & -33 & -13 \\ 4 & 19 & 8 \end{bmatrix} \leftarrow$ or $C^{-1}AB = [F]_{S,S'} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 & -4 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

9.16 b) let $v = (x, y, z)$

$B[v]_S = v$ or $[v]_S = B^{-1}v$

I need to compute

- $[F]_{S,S'}[v]_S = \begin{bmatrix} -7 & -33 & -13 \\ 4 & 19 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13x - 20y + 26z \\ 8x + 11y - 15z \end{bmatrix}$

- $[F(v)]_{S'} = C^{-1}F(x,y) = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 & -4 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$= \begin{pmatrix} -13x - 20y + 26z \\ 8x + 11y - 15z \end{pmatrix}$

(recall $C[u]_{S'} = u$ so $[u]_{S'} = C^{-1}[u]$)

Notationally we just have

$$[F]_{S,S'}[v]_S = (C^{-1}AB)B^{-1}v = C^{-1}Av$$

and

$$[F(v)]_{S'} = C^{-1}(Av)$$

They are the same

9.17 (1) $\begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 5 & -6 \end{bmatrix}$ (2) $\begin{bmatrix} 3 & -4 & 2 & -5 \\ 5 & 7 & -1 & -2 \end{bmatrix}$ (3) $\begin{bmatrix} 2 & 3 & -8 \\ 1 & 1 & 1 \\ 4 & 0 & -5 \\ 0 & 6 & 0 \end{bmatrix}$

9.18 a) $A = \begin{bmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}$

b) $S = \{(1,1,1), (1,1,0), (1,0,0)\}$ $\quad S' = \{(1,3), (2,5)\}$

$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\quad C = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

$$[F]_{S,S'} = C^{-1}AB = \frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -41 & -8 \\ 8 & 24 & 5 \end{bmatrix}$$

Check the multiplication

9.19 $E = \{(1,0), (0,1)\}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad S = \{(1,3), (2,5)\}$ $C = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ $A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$

a) $$[T]_{E,S} = C^{-1}AB = \frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 23 \\ 5 & -13 \end{bmatrix} \leftarrow -1\begin{bmatrix} 10-2 & -15-8 \\ -6+1 & 9+4 \end{bmatrix}$$

b) $$[T]_{S,E} = B^{-1}AC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -11 \\ 13 & 22 \end{bmatrix}$$

9.20 $\{\sin\theta, \cos\theta\}$ let $\sin\theta = (1,0)$ $\quad \cos\theta = (0,1)$

$D(\sin\theta) = \cos\theta$ so $D(1,0) = (0,1)$

$D(\cos\theta) = -\sin\theta$ so $D(0,1) = (-1,0)$

a) $A = [D]_S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\quad$ or $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ see book

b) $(D^2 + I)\sin\theta = \underbrace{(\sin\theta)''}_{-\sin\theta} + \sin\theta = 0$ $\quad (D^2 + I)\cos\theta = \underbrace{(\cos\theta)''}_{-\cos\theta} + \cos\theta = 0.$