# Similar Matrices Problems

(I) For each of the following linear transformations, find a basis $S$ for the domain and a basis $S'$ for the range so that the matrix representation relative to $S$ and $S'$ consists of all zeros and 1's, where the 1's occur on the diagonal of a sub matrix located in the upper left corner of the matrix. In other words, the matrix representation consists of adding rows and/or columns of zeros to the identity matrix (where the number of 1's equals the rank of the matrix). [Hint: extend a basis for the kernel to a basis for the domain, and extend the images of these basis vectors to a basis for the range. See Examples 9 and 10 in the handout.]

(1) $T(x, y, z) = (x + 2y - z, y + z)$

(2) $T(x, y, z) = (x + 2y - z, y + z, x + 3y)$

(3) $T(x, y, z) = (x + 2y - z, 2x + 4y - 2z)$

(4) $T(x, y) = (x + 2y, y, 3x - y)$

(5) $T(x, y) = (x + 2y, 2x + 4y, 3x + 6y)$

(II) Prove the following (which requires checking 3 things). Again, the proofs are on the first 2 pages of the handout:

(6) The kernel of a linear transformation is a subspace of the domain.

(7) The image of a linear transformation is a subspace of the range.

(8) The eigenspace corresponding to an eigenvalue of a linear transformation is a subspace of both the domain and range.