With all these solutions, I will just show you the work I would use to get the solution. I will let you use Maple to invert matrices where appropriate. The point to providing these solutions is to show you an alternative way to do the problems than the book.

6.1 $S = \{(1,1),(2,3)\}$

Let $B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ then a) $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}[V]_S = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$ so $[V]_S = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}[V]_S = \begin{bmatrix} a \\ b \end{bmatrix}$ so $[V]_S = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}^{-1}\begin{bmatrix} a \\ b \end{bmatrix}$

6.2 $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ a) $I[V]_E = \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}$ so $[V]_E = \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}$

b) $I[V]_E = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ so $[V]_E = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

6.3 $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ a) $B[V]_S = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$ $[V]_S = B^{-1}\begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$

b) $[V]_S = B^{-1}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

6.4 $S = \{t^3-3t^2+3t-1,\ t^2-2t+1,\ t-1,\ 1\}$

or $\{(1,-3,3,-1),\ (0,1,-2,1),\ (0,0,1,-1),\ (0,0,0,1)\}$

$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix}$ a) rank $= 4$ so they are independent, hence a basis for $P_3(t)$

b) $B[V]_S = \begin{bmatrix} 3 \\ 4 \\ 2 \\ -5 \end{bmatrix}$ so $[V]_S = B^{-1}\begin{bmatrix} 3 \\ 4 \\ 2 \\ -5 \end{bmatrix}$

6.5 $E = \{t^3, t^2, t, 1\} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I \text{ so } I[V]_E = \begin{pmatrix} 3 \\ -4 \\ 2 \\ -5 \end{pmatrix} \text{ or } [V]_E = \begin{pmatrix} 3 \\ -4 \\ 2 \\ -5 \end{pmatrix}$$

6.6 $S = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ $\quad A = \begin{bmatrix} 2 & 3 \\ 4 & -7 \end{bmatrix}$ or $(2,3,4,-7)$

(Think of writing rows)

or $S = \{(1,1,1,1), (0,-1,1,0), (1,-1,0,0), (1,0,0,0)\}$

Let $B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ $\quad B[V]_S = \begin{bmatrix} 2 \\ 3 \\ 4 \\ -7 \end{bmatrix}$ so $[V]_S = B^{-1} \begin{bmatrix} 2 \\ 3 \\ 4 \\ -7 \end{bmatrix}$

6.7 Their solution is perfect as $B = I$

6.8 (use columns, not rows)

$[A]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 5 \end{bmatrix}$ $\quad [B]_E = \begin{bmatrix} 2 \\ 4 \\ 7 \\ 10 \\ 1 \\ 13 \end{bmatrix}$ $\quad [C]_E = \begin{bmatrix} 1 \\ 2 \\ 5 \\ 8 \\ 2 \\ 11 \end{bmatrix}$

Reduce $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 7 & 5 \\ 4 & 10 & 8 \\ 0 & 1 & 2 \\ 5 & 13 & 11 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Hence dependent

$$\boxed{-3[A] + 2B = C}$$

By using columns you can immediately discern how the vectors are related.

6.9 Reduce $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 7 & 5 \\ -2 & -2 & 2 \\ 4 & 5 & -2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ so dependent and

$-3u_1 + 2u_2 = u_3$

again: <u>Use Columns for vectors</u>

<u>not</u> rows

6.10 $\left(\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\\1\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right) \xrightarrow{\text{rref}} \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$ so $\underline{\underline{\text{yes}}}$

6.12) $E = \{(1,0),(0,1)\}$ $\quad S = \{(1,3),(1,4)\}$

$B = \begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\quad C = \begin{bmatrix}1&1\\3&4\end{bmatrix}$

$B[\vec{v}]_E = \vec{v}$ and $C[v]_S = \vec{v}$

so $B[v]_E = C[v]_S$ $\quad [v]_E = B^{-1}C[v]_S$ $\quad$ so $P = B^{-1}C = C$

a) from E to S. solve ↑ for $[v]_E$

so

b) from S to E

solve for $[v]_S = \underline{C^{-1}B}[v]_E$

$Q = C^{-1}B = C^{-1} = \begin{bmatrix}4&-1\\-3&1\end{bmatrix}$

c) $[v_s] = \begin{bmatrix}4&-1\\-3&1\end{bmatrix}\begin{bmatrix}5\\-3\end{bmatrix} = \begin{bmatrix}23\\-18\end{bmatrix}$

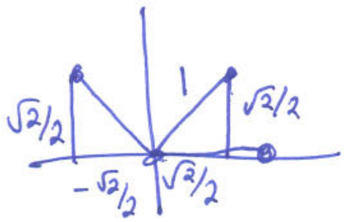
6.13 $B = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\quad C = \begin{pmatrix}1&1&0\\2&3&1\\0&2&3\end{pmatrix}$ $\quad \underline{B[v]_E = v \quad C[v]_S = v}$

Remember

a) $[v]_E = B^{-1}C[v]_s$ so $P = B^{-1}C = C$

b) $[v]_S = C^{-1}B[v]_E$ so $Q = C^{-1}B = C^{-1}$ (find C^{-1})

6.14



$E = \{(1,0),(0,1)\}$ $\quad S = \{(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}), (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})\}$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad C = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$

a) $P = B^{-1}C$

b) $Q = C^{-1}B$

6.15 $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$B[v_s] = v$

$[v]_s = B^{-1}[v]$

so $[v]_s = B^{-1}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

6.16 $B = \begin{bmatrix} 1 & 3 \\ -2 & -4 \end{bmatrix}$ $B[v]_s = v$ $\quad C = \begin{bmatrix} 1 & 3 \\ 3 & 8 \end{bmatrix}$ $C[v]_{s'} = \vec{v}$

a) $B[v]_s = \begin{bmatrix} a \\ b \end{bmatrix}$ so $[v]_s = B^{-1}\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -4 & -3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$

b) $[v]_s = B^{-1}C[v]_s$ so $P = B^{-1}C$

c) $[v]_{s'} = C^{-1}\begin{bmatrix} a \\ b \end{bmatrix} = -1\begin{bmatrix} 8 & -3 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$

d) $Q = C^{-1}B$ because $[v]_{s'} = C^{-1}B[v]_s$

e) $(P)^{-1} = (B^{-1}C)^{-1} = C^{-1}(B^{-1})^{-1} = C^{-1}B = Q$ ✓