7.1 $A = \begin{bmatrix} 3 & 5 \\ -2 & -4 \end{bmatrix}$

$$\det \begin{bmatrix} 3-\lambda & 5 \\ -2 & -4-\lambda \end{bmatrix} = (3-\lambda)(-4-\lambda) + 10 = 0$$

$$-12 + 4\lambda - 3\lambda + \lambda^2 + 10 = 0$$

$$\lambda^2 + \lambda - 2 = 0$$

$$(\lambda+2)(\lambda-1) = 0$$

$\underline{\lambda = -2,\ \lambda = 1}$ Eigen values

To get eigen vectors

$\lambda = -2$

$$\left[\begin{array}{cc|c} 3-(-2) & 5 & 0 \\ -2 & -4-(-2) & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 5 & 5 & 0 \\ -2 & -2 & 0 \end{array}\right] 5R_2 + 2R_1$$

$$\left[\begin{array}{cc|c} 5 & 5 & 0 \\ 0 & 0 & 0 \end{array}\right] /5$$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x + y = 0$

$y = y$

$\begin{bmatrix} x \\ y \end{bmatrix} = y\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

eigen-vectors for $\lambda = -2$

$\lambda = 1$

$$\left[\begin{array}{cc|c} 3-(1) & 5 & 0 \\ -2 & -4-1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 2 & 5 & 0 \\ -2 & -5 & 0 \end{array}\right] R_1 + R_2$$

$$\left[\begin{array}{cc|c} 2 & 5 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$2x + 5y = 0 \quad x = -\frac{5}{2}y$

$y = y \quad y = y$

$\begin{bmatrix} x \\ y \end{bmatrix} = y\begin{bmatrix} -5/2 \\ 1 \end{bmatrix}$

or $\boxed{\frac{y}{2}\begin{bmatrix} -5 \\ 2 \end{bmatrix}}$ eigen vectors for $\lambda = 1$

7.5

In problem 1 we got

for $\lambda = -2$, $y \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ as eigen values. for any y. Pick one of these. (Let $y = 1$) and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is an eigen value.

for $\lambda = 1$, we found $y \begin{bmatrix} -5/2 \\ 1 \end{bmatrix}$ as eigen values

If we let $y = 2$ (to get rid of fractions)

then $\begin{bmatrix} -5 \\ 1 \end{bmatrix}$ is an eigen value.

7.6.

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Notice all the zeros.
Use the first column to find determinant

$$\det \begin{bmatrix} 2-\lambda & 1 & 0 & 0 & 0 \\ 0 & 2-\lambda & 0 & 0 & 0 \\ 0 & 0 & 2-\lambda & 0 & 0 \\ 0 & 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 0 & 0 & 2-\lambda \end{bmatrix}$$

$$= (2-\lambda)\begin{vmatrix} 2-\lambda & 0 & 0 & 0 \\ 0 & 2-\lambda & 0 & 0 \\ 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 0 & 2-\lambda \end{vmatrix} =$$

$$= (2-\lambda)(2-\lambda)(2-\lambda)(2-\lambda)(2-\lambda) = 0$$

So $\lambda = 2, 2, 2, 2, 2$ are the eigen values.
They are all 2.

$$\left[\begin{array}{ccccc|c} 2-2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2-2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2-2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2-2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2-2 & 0 \end{array}\right] \to \left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccccc|c} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_1, x_3, x_4 are free. $x_2 = 0$ $x_5 = 0$.

so
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \\ x_3 \\ x_4 \\ 0 \end{bmatrix} = x_1\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

eigenvalues $x_1\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$