With all these solutions, I will just show you the work I would use to get the solution. I will let you use Maple to invert matrices where appropriate. The point to providing these solutions is to show you an alternative way to do the problems than the book.

6.1 $S = \{(1,1),(2,3)\}$

Let $B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ then a) $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}[V]_S = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$ so $[V]_S = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}[V]_S = \begin{bmatrix} a \\ b \end{bmatrix}$ so $[V]_S = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}^{-1}\begin{bmatrix} a \\ b \end{bmatrix}$

6.2 $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ a) $I[V]_E = \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}$ so $[V]_E = \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}$

b) $I[V]_E = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ so $[V]_E = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

6.3 $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ a) $B[V]_S = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$ $[V]_S = B^{-1}\begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$

b) $[V]_S = B^{-1}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

6.4 $S = \{t^3-3t^2+3t-1,\ t^2-2t+1,\ t-1,\ 1\}$

or $\{(1,-3,3,-1),\ (0,1,-2,1),\ (0,0,1,-1),\ (0,0,0,1)\}$

$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix}$ a) rank = 4 so they are independent, hence a basis for $P_3(t)$

b) $B[V]_S = \begin{bmatrix} 3 \\ 4 \\ 2 \\ -5 \end{bmatrix}$ so $[V]_S = B^{-1}\begin{bmatrix} 3 \\ 4 \\ 2 \\ -5 \end{bmatrix}$

6.5 $E = \{t^2, t^3, t, 1\} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$

$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$ so $I[V]_E = \begin{pmatrix} 3 \\ -4 \\ 2 \\ -5 \end{pmatrix}$ or $[V]_E = \begin{pmatrix} 3 \\ -4 \\ 2 \\ -5 \end{pmatrix}$

6.6 $S = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ $A = \begin{bmatrix} 2 & 3 \\ 4 & -7 \end{bmatrix}$ or $(2,3,4,-7)$

(Think of writing rows)

or $S = \{(1,1,1,1), (0,-1,1,0), (1,-1,0,0), (1,0,0,0)\}$

Let $B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ $B[V]_S = \begin{bmatrix} 2 \\ 3 \\ 4 \\ -7 \end{bmatrix}$ so $[V]_S = B^{-1}\begin{bmatrix} 2 \\ 3 \\ 4 \\ -7 \end{bmatrix}$

6.7 Their solution is perfect as $B = I$

6.8 (use columns, not rows)

$[A]_E = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 5 \end{bmatrix}$ $[B]_E = \begin{bmatrix} 2 \\ 4 \\ 7 \\ 10 \\ 1 \\ 13 \end{bmatrix}$ $[C]_E = \begin{bmatrix} 1 \\ 2 \\ 5 \\ 8 \\ 2 \\ 11 \end{bmatrix}$

Reduce $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 7 & 5 \\ 4 & 10 & 8 \\ 0 & 1 & 2 \\ 5 & 13 & 11 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Hence dependent

$\boxed{-3[A] + 2B = C}$

By using columns you can immediatly discern how the vectors are related.

6.9 Reduce $\begin{bmatrix} 1 & 3 & 1 \\ 3 & 7 & 5 \\ -2 & -2 & 2 \\ 4 & 5 & -2 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ so dependent and $-3u_1 + 2u_2 = u_3$

Again: <u>Use Columns for vectors</u>

<u>not</u> rows

6.10 $\left(\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\\1\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right) \xrightarrow{\text{rref}} \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$ so <u>yes</u>

6.12) $E = \{(1,0),(0,1)\}$ $\quad S = \{(1,3),(1,4)\}$

$B = \begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\quad C = \begin{bmatrix}1&1\\3&4\end{bmatrix}$

$B[\vec{V}]_E = \vec{V}$ and $C[V]_S = \vec{V}$

so $B[V]_E = C[v]_S$ $\quad [V]_E = B^{-1}C[V]_S$ $\quad$ so $P = B^{-1}C = C$

a) from E to S. solve ↑ for $[v]_E$

so

b) from S to E

solve for $[v]_S = \underline{C^{-1}B}[v]_E$

$Q = C^{-1}B = C^{-1} = \begin{bmatrix}4&-1\\-3&1\end{bmatrix}$

c) $[v_s] = \begin{bmatrix}4&-1\\-3&1\end{bmatrix}\begin{bmatrix}5\\-3\end{bmatrix} = \begin{bmatrix}23\\-18\end{bmatrix}$

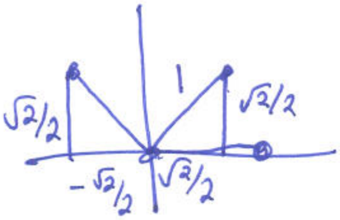
6.13 $B = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\quad C = \begin{pmatrix}1&1&0\\2&3&1\\0&2&3\end{pmatrix}$ $\quad \underline{B[V]_E = V \quad C[V]_S = V}$

Remember

a) $[v]_E = B^{-1}C[v]_S$ so $P = B^{-1}C = C$

b) $[v]_S = C^{-1}B[v]_E$ so $Q = C^{-1}B = C^{-1}$ (find C^{-1})

6.14



$E = \{(1,0),(0,1)\}$ $\quad S = \{(\frac{\sqrt2}{2}, \frac{\sqrt2}{2}), (-\frac{\sqrt2}{2}, \frac{\sqrt2}{2})\}$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad C = \begin{pmatrix} \sqrt2/2 & -\sqrt2/2 \\ \sqrt2/2 & \sqrt2/2 \end{pmatrix}$

a) $P = B^{-1}C$

b) $Q = C^{-1}B$

6.15 $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

$B[v_S] = v$

$[v]_S = B^{-1}(v)$

so $[v]_S = B^{-1}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

6.16 $B = \begin{pmatrix} 1 & 3 \\ -2 & -4 \end{pmatrix}$ $\quad B[v]_S = v$ $\quad C = \begin{bmatrix} 1 & 3 \\ 3 & 8 \end{bmatrix}$ $\quad C[v]_{S'} = \vec{v}$

a) $B[v]_S = \begin{bmatrix} a \\ b \end{bmatrix}$ so $[v]_S = B^{-1}\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -4 & -3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$

b) $[v]_S = B^{-1}C[v]_S$ so $P = B^{-1}C$

c) $[v]_{S'} = C^{-1}\begin{bmatrix} a \\ b \end{bmatrix} = -1\begin{bmatrix} 8 & -3 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$

d) $Q = C^{-1}B$ because $[v]_{S'} = C^{-1}B[v]_S$

e) $(P)^{-1} = (B^{-1}C)^{-1} = C^{-1}(B^{-1})^{-1} = C^{-1}B = Q$ ✓

9.1

a) $A = \begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix}$

b) let $P = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ so that $P[v]_s = v$

Then $A\vec{v} = T(\vec{v})$

becomes $A(P[v]_s) = P[T(v)]_s$

or $P^{-1}AP[v]_s = [Tv]_s$

So $B = P^{-1}AP$

No need to verify

c) we used coordinates to create it.

9.2 always start by getting the standard representation

$[G]_E = A = \begin{bmatrix} 2 & -7 \\ 4 & 3 \end{bmatrix}$

a) let $P = \left[\begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix}\right]$ so that $P[v]_s = v$

Then $Av = T(\vec{v})$ means

$A[P[v]_s] = P[T(v)]_s$ or

Here is the verification

$\underline{\underline{P^{-1}AP}}[v_s] = [G]_s$ perform multiplication.

$$\frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -7 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = [G]_s$$

b) or do what they did in the book

9.3

let $P = \begin{bmatrix} 1 & 3 \\ -2 & -7 \end{bmatrix}$ Then $B = P^{-1}AP$. Done (see method 2)

9.4 see book. Theirs is fine

9.5 $[G]_E = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -4 & 0 \\ 3 & 0 & 0 \end{bmatrix} = A$

a) let $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ then $[G]_S = P^{-1}AP$ done.

always start by getting the standard representation A, and then use a similarity transformation

b) let $v = (a,b,c)$

$[G][v] = (P^{-1}AP)\begin{bmatrix} a \\ b \\ c \end{bmatrix}_S =$ ~~[scribbled]~~

you have to first write any vector (a,b,c) in terms of its coordinates relative to S.

~~$[G(v)]_S =$~~

Hence $P\begin{bmatrix} a \\ b \\ c \end{bmatrix}_S = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

so $P^{-1}AP\begin{bmatrix} a \\ b \\ c \end{bmatrix}_S = P^{-1}A\begin{pmatrix} a \\ b \\ c \end{pmatrix} = P^{-1}G(a,b,c)$

we know $G(a,b,c) = A\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ so

we also know $P\,[G(a,b,c)]_S = G(a,b,c)$

so $P^{-1}G(a,b,c) = P^{-1}(P[G(a,b,c)]_S) = \underbrace{\cancel{P^{-1}P}}_{I}[G(a,b,c)]_S$

$= G(a,b,c)$.

On this one, please try to follow the book as well.

9.6) $A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -1 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ so $P[u]_S = u$

$B = P^{-1}AP$: why? $\quad Au = Au$

So $A P[u]_S = P[Au]_S$

Hence $\underbrace{P^{-1}AP}_{B}[u]_S = [Au]_S$

Use maple to invert. (or this one isn't too bad by hand)

9.7) a) $L(1,0) = (2,4)$ $\quad A = \begin{bmatrix} 2 & 5 \\ 4 & 8 \end{bmatrix}$

$L(0,1) = (5,8)$

b) $L(1,0) = (0,1)$ $\quad A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$L(0,1) = (-1,0)$

c)  $L(1,0) = (0,-1)$ $\quad A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$L(0,1) = (-1,0)$

9.8 $S = \{(1,1), (1,2)\}$ $\quad L(1,1) = (4,7)$ $\quad$ Standard satisfies

$L(1,2) = (1,6)$ $\quad A\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix}$

so $A = \begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$ (This is the standard matrix).

To get the matrix relative to S, we let $P = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$,

Then $B = P^{-1}AP = P^{-1}\begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix}\overbrace{\cancel{\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}}}^{I}$

$= P^{-1}\begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & 5 \end{bmatrix}$

The books work is equivalent, it just doesn't focus on matrices.

9.9) Start by deciding on a "standard" representation.

let $(1,0,0) = e^{3t}$, $(0,1,0) = te^{3t}$, $(0,0,1) = t^2e^{3t}$.

Then $D(e^{3t}) = 3e^{3t} = 3(1,0,0) = (3,0,0)$

so $D(1,0,0) = (3,0,0)$

also $D(te^{3t}) = t\cdot 3e^{3t} + 1\cdot e^{3t} = (1,3,0)$

and $D(t^2e^{3t}) = t^2 3e^{3t} + 2te^{3t} = (0,2,3)$

Hence $[D] = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

9.10 $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = [1,2,3,4]_E$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = (1,0,0,0)$
$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = (0,1,0,0)$
$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = (0,0,1,0)$.
$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = (0,0,0,1)$.

Let $T(A) = MA$

$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = (1,0,3,0)$

$T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix} = (0,1,0,3)$

$T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix} = (2,0,4,0)$

$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix} = (0,2,0,4)$

Place in columns

So $[T] = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 3 & 0 & 4 & 0 \\ 0 & 3 & 0 & 4 \end{bmatrix}$

9.11) standard $A = \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix}$ E let $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $S = \{(1,4),(2,7)\}$

$B[u]_E = u$ $C = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$ $C[u]_S = u$

a) from E to S. Solve $B[u]_E = C[u]_S$ for $[u]_E = B^{-1}C[u]_S = C[u]_S$

$$P = C = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$$

9.11 continued.

from S to E requires we solve
$$B[u]_E = C[u]_S \quad \text{for } [u]_S = C^{-1}B[u]_E$$
$$\text{so } Q = C^{-1}B = C^{-1} = \frac{1}{-1}\begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix}$$
$$\text{Notice } (P)^{-1} = (B^{-1}C)^{-1} = C^{-1}B = Q.$$

b) $A = \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix}$ ~~$A[u]_E = F(u)$~~

$$B = \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -2 & 1 \end{bmatrix}$$

c) trace $A = 6$ = trace B

det $A = 5 + 2 = 7$ det $B = 5 + 2 = 7$

Trace and det are always the same for similar matrices.

9.12 The books solution is great.

9.16 The standard representation is $A = \begin{bmatrix} 3 & 2 & -4 \\ 1 & -5 & 3 \end{bmatrix}$

a) Let $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $C^{-1} = \frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

so $B[u]_S = u$ and $C[v]_{S'} = v$

we have $Au = F(u)$ so $AB[u]_S = C[F(u)]_{S'}$

$$\begin{bmatrix} -7 & -33 & -13 \\ 4 & 19 & 8 \end{bmatrix} \quad \text{or} \quad C^{-1}AB = [F]_{S,S'} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 & -4 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

9.16 b) let $v = (x, y, z)$

$B[v]_S = v$ or $[v]_S = B^{-1}v$

I need to compute

- $[F]_{S,S'}[v]_S = \begin{bmatrix} -7 & -33 & -13 \\ 4 & 19 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13x - 20y + 26z \\ 8x + 11y - 15z \end{bmatrix}$

- $[F(v)]_{S'} = C^{-1}F(x,y) = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 & -4 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$= \begin{pmatrix} -13x - 20y + 26z \\ 8x + 11y - 15z \end{pmatrix}$

(recall $C[u]_{S'} = u$
so $[u]_{S'} = C^{-1}[u]$)

Notationally we just have

$$[F]_{S,S'}[v]_S = (C^{-1}AB)B^{-1}v = C^{-1}Av$$

and $[F(v)]_{S'} = C^{-1}(Av)$

They are the same

9.17 (1) $\begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 5 & -6 \end{bmatrix}$ (2) $\begin{bmatrix} 3 & -4 & 2 & -5 \\ 5 & 7 & -1 & -2 \end{bmatrix}$ (3) $\begin{bmatrix} 2 & 3 & -8 \\ 1 & 1 & 1 \\ 4 & 0 & -5 \\ 0 & 6 & 0 \end{bmatrix}$

9.18 a) $A = \begin{bmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}$

b) $S = \{(1,1,1), (1,1,0), (1,0,0)\}$ $\quad S' = \{(1,3), (2,5)\}$

$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\quad C = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

$$[F]_{S,S'} = C^{-1}AB = \frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & -41 & -8 \\ 8 & 24 & 5 \end{bmatrix}$$

Check the multiplication

9.19 $E = \{(1,0), (0,1)\}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad S = \{(1,3), (2,5)\}$ $C = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ $A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$

a) $$[T]_{E,S} = C^{-1}AB = \frac{1}{-1}\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 23 \\ 5 & -13 \end{bmatrix} \longleftarrow -1\begin{bmatrix} 10-2 & -15-8 \\ -6+1 & 9+4 \end{bmatrix}$$

b) $$[T]_{S,E} = B^{-1}AC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & -11 \\ 13 & 22 \end{bmatrix}$$

9.20 $\{\sin\theta, \cos\theta\}$ let $\sin\theta = (1,0)$ $\quad \cos\theta = (0,1)$

$D(\sin\theta) = \cos\theta$ so $D(1,0) = (0,1)$

$D(\cos\theta) = -\sin\theta$ so $D(0,1) = (-1,0)$

a) $A = [D]_S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\quad$ or $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ see book

b) $(D^2+I)\sin\theta = \underbrace{(\sin\theta)''}_{-\sin\theta} + \sin\theta = 0$ $\quad (D^2+I)\cos\theta = \underbrace{(\cos\theta)''}_{-\cos\theta} + \cos\theta = 0.$

11.9) $\begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$

$$\begin{vmatrix} 3-\lambda & -4 \\ 2 & -6-\lambda \end{vmatrix} = (3-\lambda)(-6-\lambda)+8$$
$$= -18+6\lambda-3\lambda+\lambda^2+8$$
$$= \lambda^2+3\lambda-10$$
$$= (\lambda+5)(\lambda-2)$$
$$\lambda = 2, -5$$

$\lambda = 2$

$$\left[\begin{array}{cc|c} 1 & -4 & 0 \\ 2 & -8 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -4 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x - 4y = 0 \quad x = 4y$

$y = y \quad y = y$

eigenvector

$$\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$\lambda = -5$

$$\left(\begin{array}{cc|c} 8 & -4 & 0 \\ 2 & -1 & 0 \end{array}\right) \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x = \frac{1}{2}y \quad \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$y = y$

$$P = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$$

check $P^{-1}AP = D$

$$\frac{1}{7}\begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$$
$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 8 & -5 \\ 2 & -10 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 & 0 \\ 0 & -35 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix} \checkmark$$

11.10) $B = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ $\qquad \begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) - 8$

$$= 3 - 3\lambda - \lambda + \lambda^2 - 8$$

$$= \lambda^2 - 4\lambda - 5 = (\lambda - 5)(\lambda + 1)$$

$$\lambda = 5, -1.$$

A)

$\lambda = 5$

$$\left[\begin{array}{cc|c} -4 & 4 & 0 \\ 2 & -2 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x = y \\ y = y \end{array} \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda = -1$

$$\left[\begin{array}{cc|c} 2 & 4 & 0 \\ 2 & 4 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad x + 2y = 0 \quad \begin{array}{l} x = -2y \\ y = y \end{array} \quad \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

B) $P = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$

Check $\quad P^{-1} B P = D \quad \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$

11.10 c) $\qquad B = PDP^{-1}$

$$B^5 = (PDP^{-1})^5 = (PD\underbrace{P^{-1})(P}_{I}D\underbrace{P^{-1})(P}_{I}D\underbrace{P^{-1})(P}_{I}D\underbrace{P^{-1})(P}_{I}DP^{-1})$$

$$= P D^5 P^{-1}$$

$$= \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5^5 & 0 \\ 0 & (-1)^5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}^{-1} = \text{see book}$$

11.11 $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ $\left|\begin{bmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix}\right| = (2-\lambda)(3-\lambda) - 2$

$= 6 - 5\lambda + \lambda^2 - 2 = \lambda^2 - 5\lambda + 4$

$(\lambda-4)(\lambda-1)$

$\lambda = 4, 1$

a) $\lambda = 4$ $\left[\begin{array}{cc|c} -2 & 2 & 0 \\ 1 & -1 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = 1$ $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ \cancel{1} & \cancel{2} & \cancel{0} \end{array}\right]$ $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

b) $P = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$ check $P^{-1}AP = D$

$P^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

c) $A = PDP^{-1}$ $A^6 = PD^6P^{-1}$

$= \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4^6 & 0 \\ 0 & 1^6 \end{bmatrix} \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ = see book

$\sqrt{A} = P\sqrt{D}P^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ = see book

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11.12 $B = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$ $\begin{vmatrix} 1-\lambda & 3 \\ 5 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) - 15$

$= 3 - 4\lambda + \lambda^2 - 15 = \lambda^2 - 4\lambda - 12$

$= (\lambda-6)(\lambda+2)$

$\lambda = 6, -2$

The language just means find eigenvalues + eigenvectors.

$\lambda = 6$ $\left[\begin{array}{cc|c} -5 & 3 & 0 \\ 5 & -3 & 0 \end{array}\right] \xrightarrow{\text{rref}} \begin{bmatrix} 1 & -3/5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ so $x = \frac{3}{5}y$, $y = y$ $\begin{bmatrix} 3/5 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\lambda = -2$ $\begin{bmatrix} 3 & 3 & 0 \\ 5 & 5 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ so $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 3 & -1 \\ 5 & 1 \end{bmatrix}$ $S = \{(3,5), (-1,1)\}$ $D = \begin{bmatrix} 6 & 0 \\ 0 & -2 \end{bmatrix}$

check $P^{-1}BP = D$

11.13 $\begin{bmatrix} 5 & 6 \\ 3 & -2 \end{bmatrix}$ $\begin{vmatrix} 5-\lambda & 6 \\ 3 & -2-\lambda \end{vmatrix} = (5-\lambda)(-2-\lambda) - 18$

$= -10 - 5\lambda + 2\lambda + \lambda^2 - 18$

$= \lambda^2 - 3\lambda - 28 = (\lambda-7)(\lambda+4)$

$\lambda = 7, -4$

(a)

$\lambda = 7$ $\left[\begin{array}{cc|c} -2 & 6 & 0 \\ 3 & 9 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc} 1 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\lambda = -4$ $\left[\begin{array}{cc|c} 9 & 6 & 0 \\ 3 & 2 & 0 \end{array}\right] \to \left(\begin{array}{cc|c} 1 & 2/3 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\begin{bmatrix} -2/3 \\ 1 \end{bmatrix} \approx \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\lambda = 7$ $\vec{x} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\lambda = -4$ $\vec{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(b) Notice the characteristic polynomial is

$\lambda^2 + 1$

which has <u>no</u> real zeros, so no real eigenvalues or eigen vectors

(c) $\lambda = 4$ is a double root. However There is only 1 lin indep eigen vector. Since there were not 2, this matrix is <u>not</u> diagonalizable

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14) $\lambda^2 + 1 = 0 \Rightarrow \lambda^2 = -1$ $\lambda = \pm\sqrt{-1} = \pm i$

there are 2 complex eigenvalues.

C^2 is the complex plane.

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all others. Their work is fine. Just realize they compute the characteristic polynomial differently. Just do it the way we have all semester. If it is order 3 or more, use soft ware to factor.