Drone SNR calculation

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Assume drone flies at heigh $h \approx 150$m, emits power P_e into bandwidth $\Delta\nu$ over solid angle $\Omega_e \approx 2\pi$, which is received by a dish of radius $r_d = 2$ and is beam suppressed by $A = 10^{-3} - 1$ (assuming we want to measure beam to -30dB). Received signal power is given by

$$P_S = P_e A \frac{\pi r_d^2}{h^2 \Omega_e} \approx 10^{-4} P_e A \tag{1}$$

The noise power is given by sum over system $T_{\rm sys} \approx 50$K and sky $T_{\rm sky} \approx 10$K temperatures

$$P_N = 2k_b(T_{\rm sky} + T_{\rm sys})\Delta\nu \tag{2}$$

We want signal to be always dominated by drone, rather than the changing sky background, etc, which implies $P_S \gg P_N$, or

$$\Delta\nu \ll 6 \times 10^{13} \mathrm{Hz} A \left(\frac{P_e}{1\mathrm{mW}}\right) \tag{3}$$

So, we are always very safely into this regime up to $A = 10^{-4}$

Now say we want to map a square of $30^\circ \times 30^\circ$ with resolution 0.1°. This gives $N_{\rm pix} = 10^5$ pixels. Flying for $T = 10$ minutes, this gives integration time

$$t_i = \frac{T}{N_{\rm pix}} \approx 0.4\mathrm{s} \tag{4}$$

per resolution element.

Given that we have established that noise never matters, we have SNR that is purely “mode counting”

$$\mathrm{SNR} = \frac{P_S}{(P_S + P_N)/\sqrt{t_i \Delta\nu}} \approx \sqrt{t_i \Delta\nu} \tag{5}$$

This is somewhat counter-intuitive, because I would have thought that for a delta function $\Delta\nu$ I should have meausured the amplitude perfectly. So I think this indicates breakdown of the “Gaussian field” approximation. So, let’s do two calculations:

Broadband signal

With $\Delta\nu = 500\text{MHz}$, $P_N \approx 10^{-12}\text{W}$ and $P_S = 10^{-7}A\text{W}$ for mW source, giving

$$\text{SNR} = 10^4 \frac{A}{A + 10^{-5}} \tag{6}$$

So essentially a very good SNR down to -50dB. Emitting 10μW would give us signal to -30dB.

Narrowband signal

TODO