Drone SNR calculation

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Assume drone flies at heigh $h \approx 150$m, emits power P_e into bandwidth $\Delta\nu$ over solid angle $\Omega_e \approx 2\pi$, which is received by a dish of radius $r_d = 2$ and is beam suppressed by $A = 10^{-3} - 1$ (assuming we want to measure beam to -30dB). Received signal power is given by

$$P_S = P_e A \frac{\pi r_d^2}{h^2 \Omega_e} \approx 10^{-4} P_e A \tag{1}$$

The noise power is given by sum over system $T_{\rm sys} \approx 50$K and sky $T_{\rm sky} \approx 10$K temperatures, giving total temperature $T_T \approx 60$K

$$P_N = 2 k_B T_T \Delta\nu \tag{2}$$

We want signal to be always dominated by drone, rather than the changing sky background, etc, which implies $P_S \gg P_N$, or

$$\Delta\nu \ll A \left(\frac{P_e}{1\text{mW}} \right) 6 \times 10^{13}\text{Hz} \tag{3}$$

So, we are always very safely into this regime up to $A = 10^{-4}$

Now say we want to map a square of $30^\circ \times 30^\circ$ with resolution 0.1°. This gives $N_{\rm pix} = 10^5$ pixels. Flying for $T = 10$ minutes, this gives integration time

$$t_i = \frac{T}{N_{\rm pix}} \approx 0.4\text{s} \tag{4}$$

per resolution element.

Given that we have established that noise never matters, we have SNR that is purely "mode counting"

$$\text{SNR} = \frac{P_S}{(P_S + P_N)/\sqrt{t_i \Delta\nu}} \approx \sqrt{t_i \Delta\nu} \tag{5}$$

This is somewhat counter-intuitive, because I would have thought that for a delta function $\Delta\nu$ I should have meausured the amplitude perfectly. So I think this indicates breakdown of the "Gaussian field" approximation. So, let's do two calculations:

Broadband signal

With $\Delta\nu = 500$MHz, $P_N \approx 10^{-12}$W and $P_S = A \times 10^{-7}$W for mW source, giving

$$\text{SNR} = 10^4 \frac{A}{A + 10^{-5}} \tag{6}$$

So essentially a very good SNR down to -50dB. Emitting 10μW would give us signal to -30dB.

Narrowband signal

Say you have a narrowband signal, a pure tone with infinite precision that goes into a single FFT bin. The signal value in volts in that FFT bin after integration t_i is going to be $V = \sqrt{2P_sAR}$, where $R = 50\Omega$ is the line impendance.

At the same time the thermal noise injects $2k_BT_T/t_i$ power into it (since the width of the FFT bin is $\sim 1/t_i$), giving rms noise contribution of $\sigma(V_n) \sim \sqrt{2Rk_BT_t/t_i}$

The ratio gives SNR

$$\text{SNR} = \sqrt{\frac{P_sA}{k_BT_T}t_i} = 4 \times 10^8\sqrt{A}\sqrt{\frac{P_s}{1\text{mW}}} \tag{7}$$

If this calculation is right, which I'm not sure it is, then the scaling is very favorable and and we emit very very small powers and still get away with it, even for much decreased t_i and deep into small values of A.

Note that scaling with A is more favorable. This squares with my intuition that if measuring amplitude of a known signal is much easier than measuring "power".