COMPUTER VISION
LECTURE 8 27.9.2019

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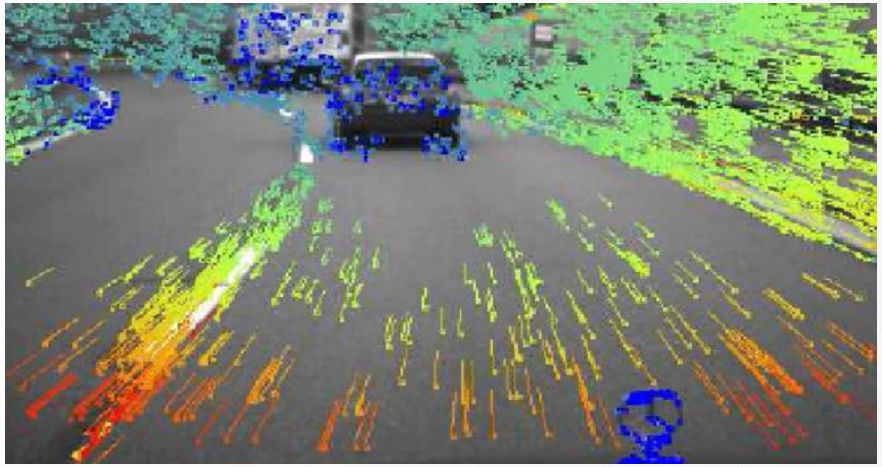
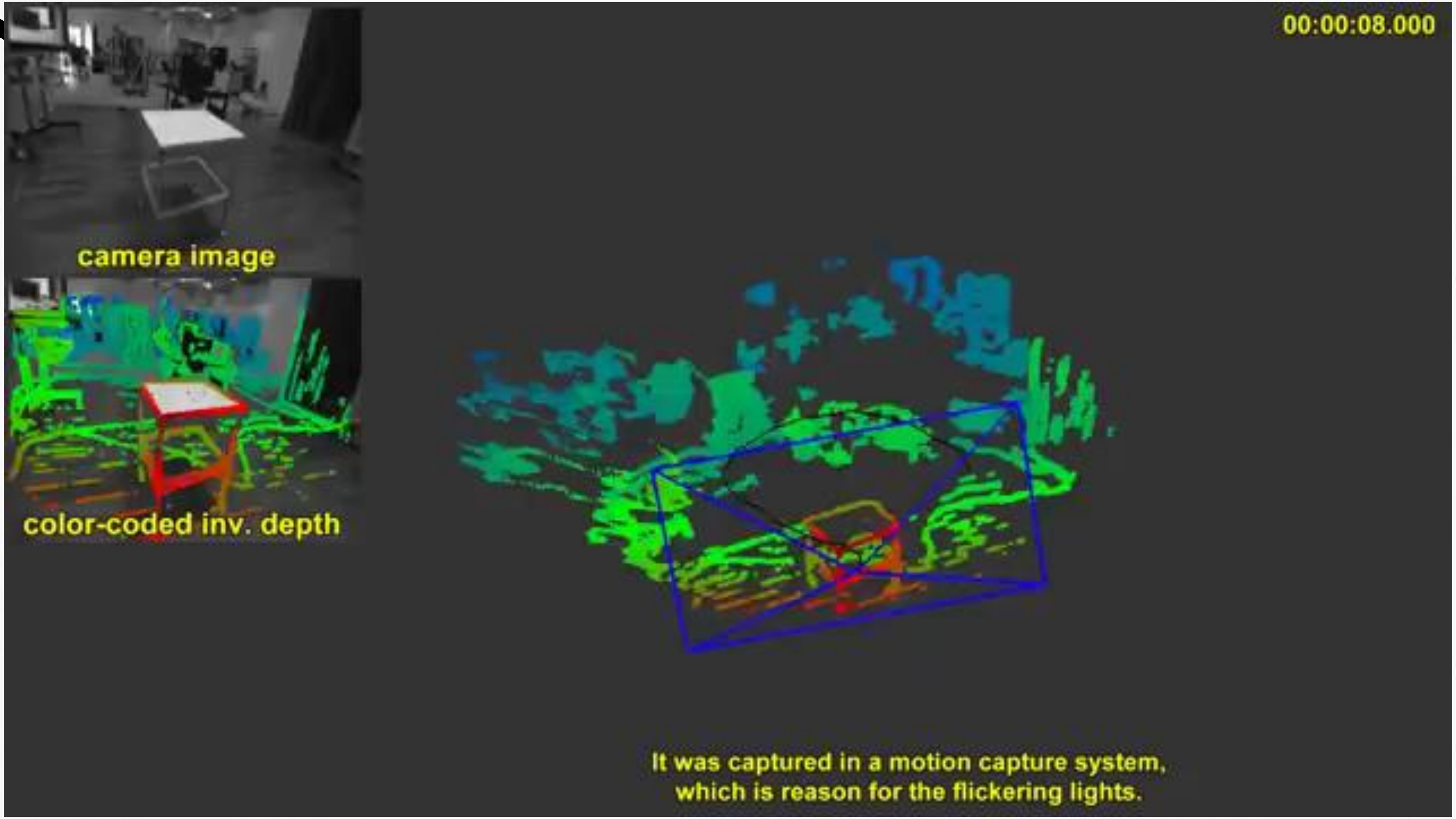
Department of Computer Science

TODAY’S LECTURE: OPTICAL FLOW



- Szeliski 8.4
- Feature tracking (this we have looked at before)
  - Extracting and matching features (corners, SIFT descriptors, …)
  - Tracking them over two or more frames
  - Applications: object tracking, structure from motion (SFM)
  - Performs well when motion is relatively small
- Optical flow recovers image motion at each pixel from spatiotemporal image brightness variations

optical flow used for motion estimation in visual odometry



Slide credits: Kris Kitani

OPTICAL FLOW

Problem Definition

Given two consecutive image frames,
estimate the motion of each pixel

Assumptions

Brightness constancy

Small motion

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$$\begin{bmatrix} I_x(\boldsymbol{p}_1) & I_y(\boldsymbol{p}_1) \\ I_x(\boldsymbol{p}_2) & I_y(\boldsymbol{p}_2) \\ \vdots & \vdots \\ I_x(\boldsymbol{p}_{25}) & I_y(\boldsymbol{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\boldsymbol{p}_1) \\ I_t(\boldsymbol{p}_2) \\ \vdots \\ I_t(\boldsymbol{p}_{25}) \end{bmatrix}$$

Constant Flow
(Lucas- Kanade)

$$\min_{\boldsymbol{u},\boldsymbol{v}} \sum_{ij} \left\{ E_d(i,j) + \lambda E_s(i,j) \right\}$$

Horn-Schunck

Optical Flow

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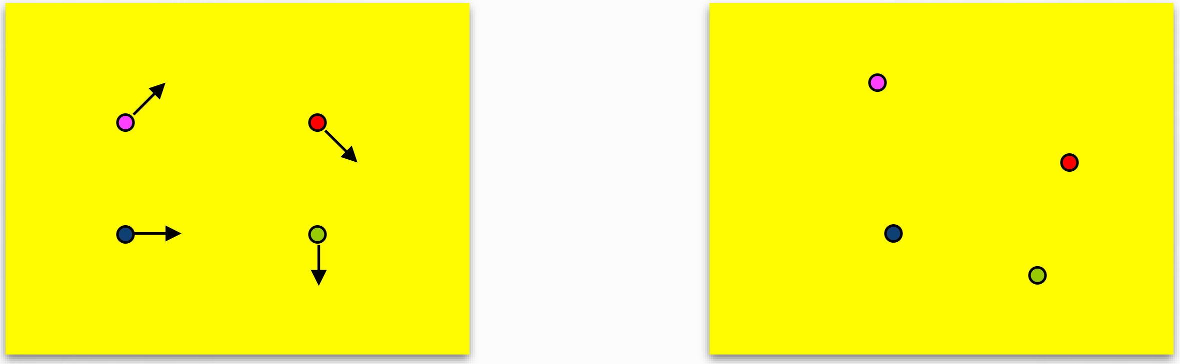
OPTICAL FLOW (PROBLEM DEFINITION)



$$I(x, y, t) \qquad\qquad I(x, y, t')$$

Estimate the motion
(flow) between these
two consecutive images

How is this different from estimating a 2D transform?

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KEY ASSUMPTIONS
(UNIQUE TO OPTICAL FLOW)

Color Constancy

(Brightness constancy for intensity images)

Implication: allows for pixel to pixel comparison
(not image features)

Small Motion

(pixels only move a little bit)

Implication: linearization of the brightness
constancy constraint

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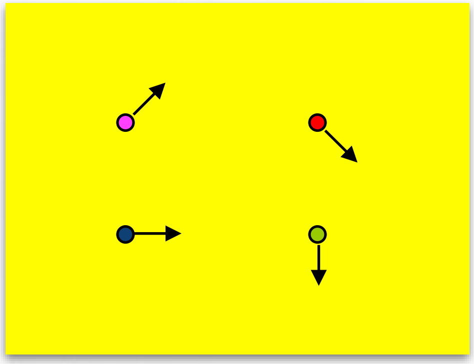
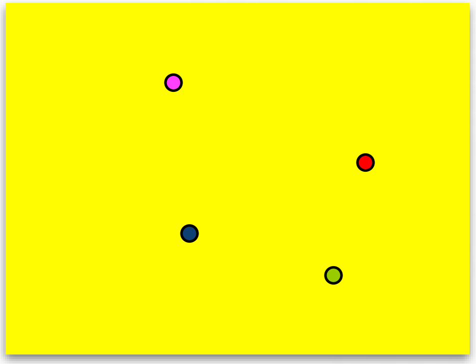
APPROACH



$$I(x, y, t)$$



$$I(x, y, t')$$

Look for nearby pixels with the same color

(small motion) (color constancy)

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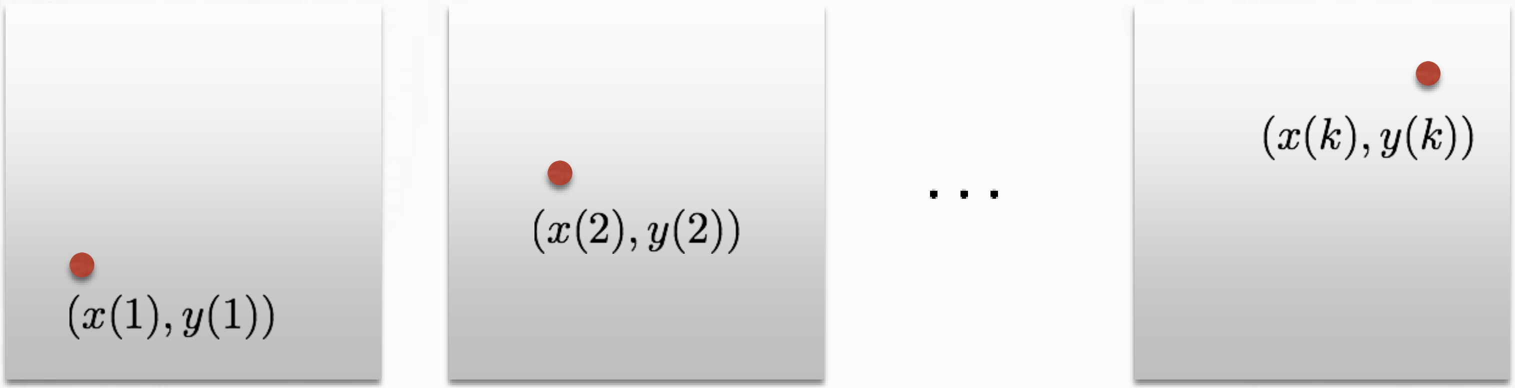
BRIGHTNESS CONSTANCY

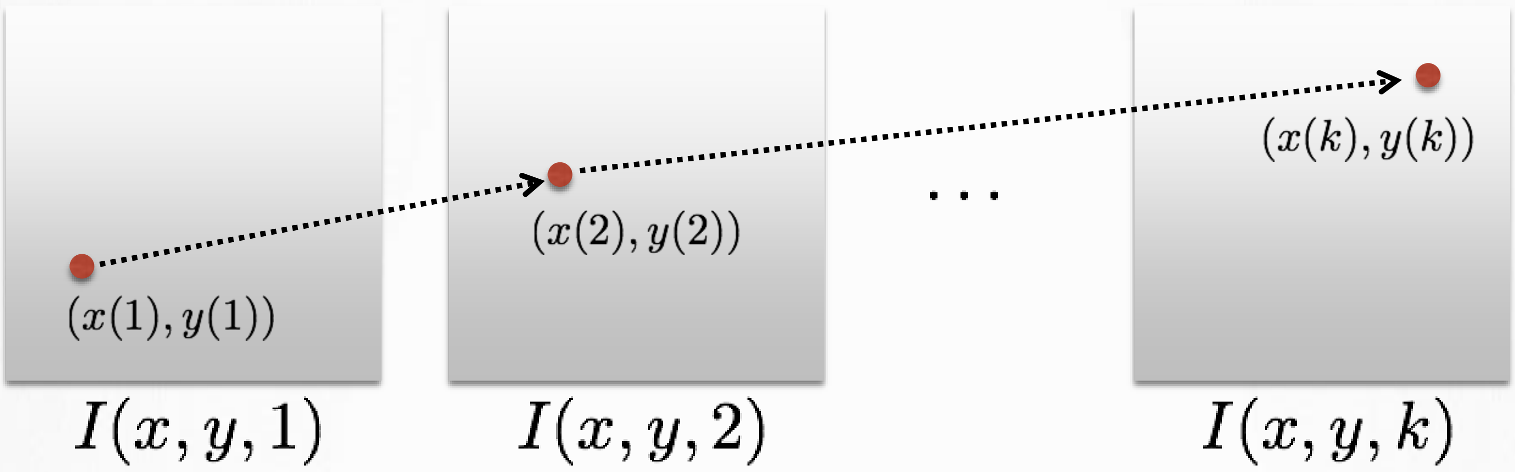
Scene point moving through image sequence



Assumption 1

BRIGHTNESS CONSTANCY

Scene point moving through image sequence



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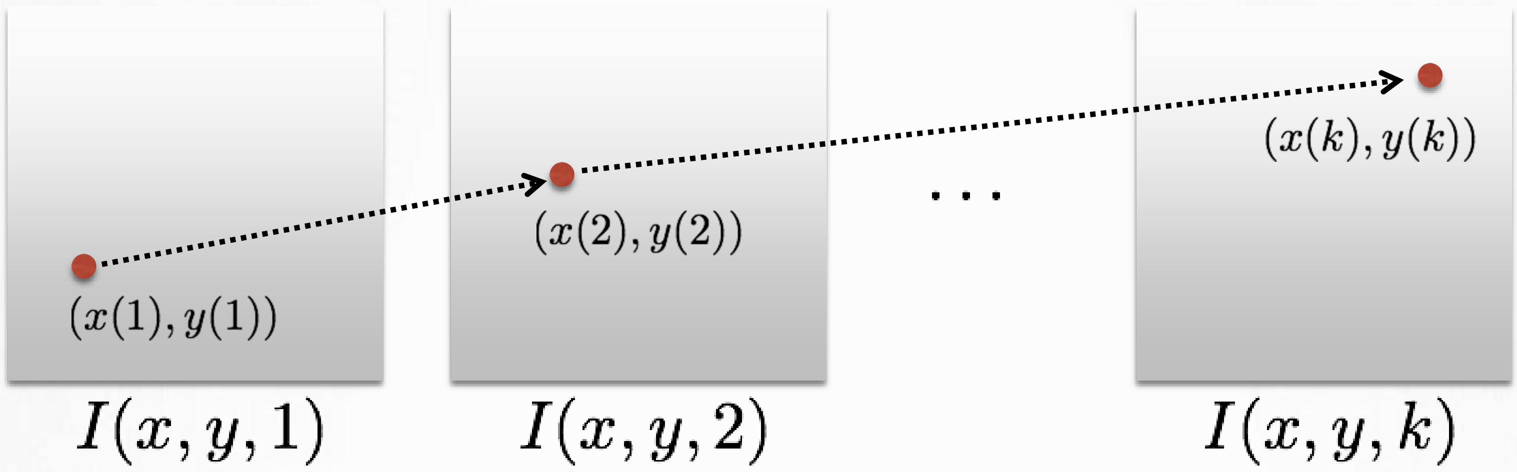
Assumption 1

BRIGHTNESS CONSTANCY

Scene point moving through image sequence



Assumption:Brightness of the point will remain the same

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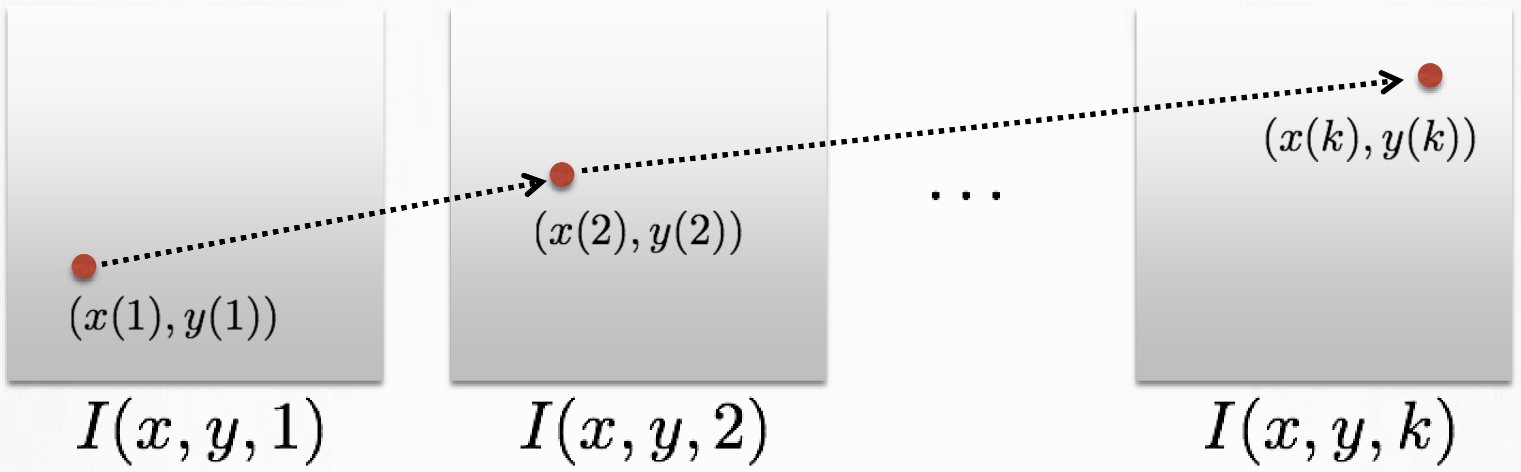
Assumption 1

BRIGHTNESS CONSTANCY

Scene point moving through image sequence



Assumption:Brightness of the point will remain the same

$$I(x(t), y(t), t) = \underbrace{C}_{\text{constant}}$$

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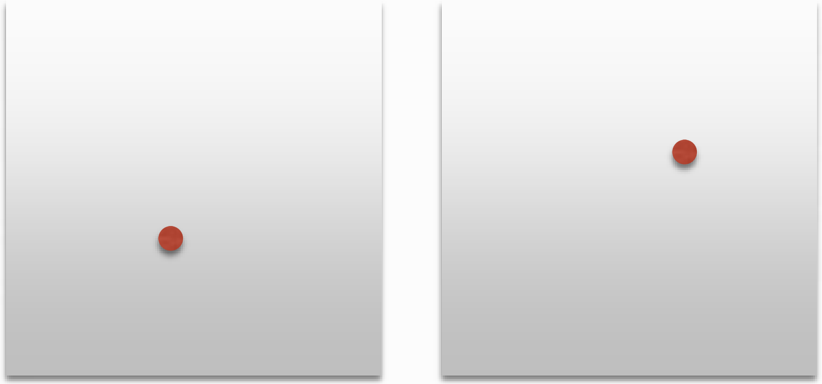
Assumption 2

SMALL MOTION



$I(x, y, t)$ $I(x, y, t + \delta t)$

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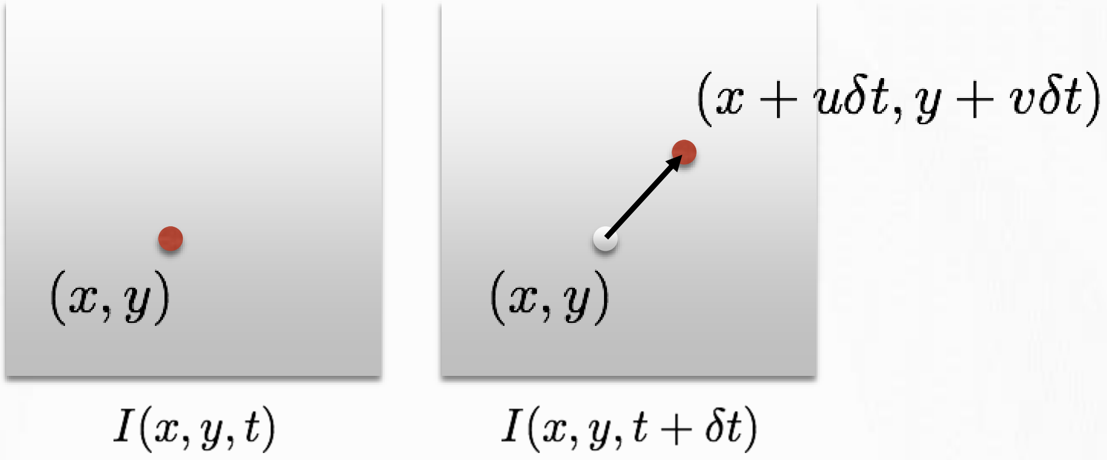
Assumption 2

SMALL MOTION



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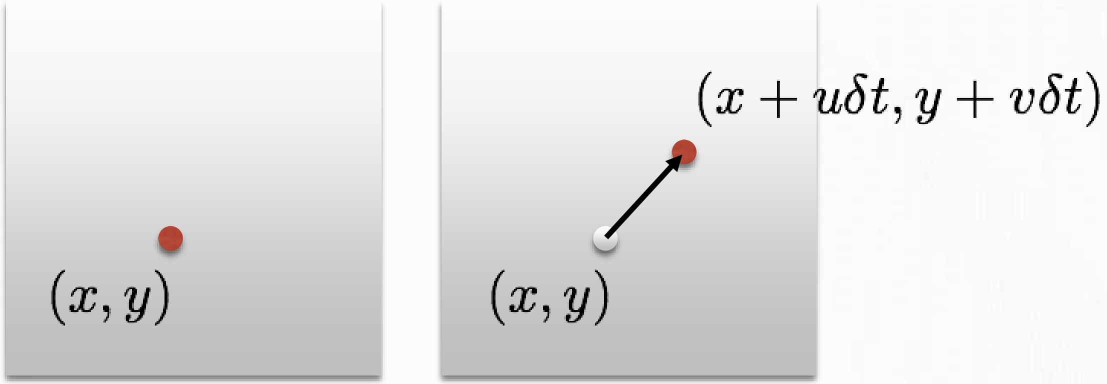
SMALL MOTION



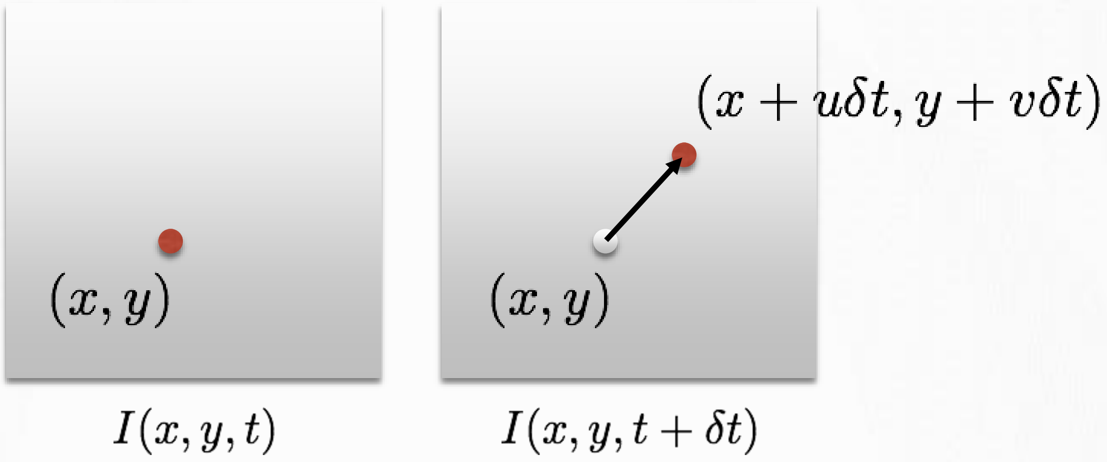
$I(x, y, t)$ $\qquad$ $I(x, y, t + \delta t)$

Optical flow (velocities): (u, v) $\qquad$ Displacement: $(\delta x, \delta y) = (u\delta t, v\delta t)$

Slide credits: Kris Kitani

Assumption 2

SMALL MOTION



Optical flow (velocities): (u, v) Displacement: $(\delta x, \delta y) = (u\delta t, v\delta t)$

For a *really small space-time step*…

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

… the brightness between two consecutive image frames is the same

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These assumptions yield the …

Brightness Constancy Equation

$$\frac{dI}{dt} = \frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

total derivative

partial derivative

Equation is not obvious. Where does this come from?

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$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

For small space-time step, brightness of a point is the same

Slide credits: Kris Kitani

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

For small space-time step, brightness of a point is the same

Insight:
If the time step is really small,
we can *linearize* the intensity function

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

Multivariable Taylor Series Expansion

(First order approximation, two variables)

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) - f_y(a,b)(y-b)$$

Slide credits: Kris Kitani

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

Multivariable Taylor Series Expansion

(First order approximation, two variables)

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) - f_y(a,b)(y-b)$$

$$I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = I(x,y,t)$$

assuming small motion

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$$I(x+u\delta t, y+v\delta t, t+\delta t) = I(x,y,t)$$

Multivariable Taylor Series Expansion

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partial derivative

$$I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = I(x,y,t)$$

fixed point

assuming small motion

cancel terms

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$$I(x+u\delta t, y+v\delta t, t+\delta t) = I(x,y,t)$$

Multivariable Taylor Series Expansion

(First order approximation, two variables)

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) - f_y(a,b)(y-b)$$

$$I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = I(x,y,t)$$

assuming small motion

$$\frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = 0$$

cancel terms

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$$I(x+u\delta t, y+v\delta t, t+\delta t) = I(x,y,t)$$

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$$I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = I(x,y,t)$$

assuming small motion

$$\frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = 0$$

divide by δt

take limit $\delta t \to 0$

$$I(x + u\delta t, y + v\delta t, t + \delta t) = I(x, y, t)$$

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$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

Slide credits: Kris Kitani

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(First order approximation, two variables)

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$$I(x,y,t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = I(x,y,t)$$

assuming small motion

$$\frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t = 0$$

divide by δt

take limit $\delta t \to 0$

$$\boxed{\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0}$$

Brightness Constancy Equation

Slide credits: Kris Kitani

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

Brightness Constancy Equation

$$I_x u + I_y v + I_t = 0$$

(x-flow) (y-flow)

shorthand notation

$$\nabla I^\top \boldsymbol{v} + I_t = 0$$

(1 x 2) (2 x 1)

vector form

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(putting the math aside for a second…)

What do the term of the
brightness constancy equation represent?

$$I_x u + I_y v + I_t = 0$$

Slide credits: Kris Kitani

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Image gradients
(at a point p)

Slide credits: Kris Kitani

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brightness constancy equation represent?

flow velocities

$$I_x u + I_y v + I_t = 0$$

Image gradients
(at a point p)

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(putting the math aside for a second…)

What do the term of the
brightness constancy equation represent?

flow velocities

$$I_x u + I_y v + I_t = 0$$

Image gradients
(at a point p)

temporal gradient

How do you compute these terms?

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$$I_x u + I_y v + I_t = 0$$

How do you compute …

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

spatial derivative

Slide credits: Kris Kitani

$$I_x u + I_y v + I_t = 0$$

How do you compute …

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

spatial derivative

Forward difference
Sobel filter
Scharr filter
…

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Slide credits: Kris Kitani

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spatial derivative

$$I_t = \frac{\partial I}{\partial t}$$

temporal derivative

Forward difference
Sobel filter
Scharr filter
…

frame differencing

FRAME DIFFERENCING

t

1	1	1	1	1
1	1	1	1	1
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10
1	10	10	10	10

-

$t + 1$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	10	10	10
1	1	10	10	10
1	1	10	10	10

=

$$I_t = \frac{\partial I}{\partial t}$$

0	0	0	0	0
0	0	0	0	0
0	9	9	9	9
0	9	0	0	0
0	9	0	0	0
0	9	0	0	0

(example of a forward difference)

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Example:

t

3 x 3 patch

1	1	1	1	1
1	1	1	1	1
1	**10**	**10**	**10**	**10**
1	**10**	**10**	**10**	**10**
1	**10**	**10**	**10**	**10**
1	**10**	**10**	**10**	**10**

$t+1$

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	**10**	**10**	**10**
1	1	**10**	**10**	**10**
1	1	**10**	**10**	**10**

$$I_x = \frac{\partial I}{\partial x}$$

-	0	0	0	-
-	0	0	0	-
-	**9**	0	0	-
-	**9**	0	0	-
-	**9**	0	0	-
-	**9**	0	0	-

$$I_y = \frac{\partial I}{\partial y}$$

-	-	-	-	-
0	0	0	0	0
0	**9**	**9**	**9**	**9**
-	0	0	0	0
0	0	0	0	0
-	-	-	-	-

$$I_t = \frac{\partial I}{\partial t}$$

0	0	0	0	0
0	0	0	0	0
0	**9**	**9**	**9**	**9**
0	**9**	0	0	0
0	**9**	0	0	0
0	**9**	0	0	0

-1 0 1

-1
0
1

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$$I_x u + I_y v + I_t = 0$$

How do you compute …

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

spatial derivative

$$u = \frac{dx}{dt} \quad v = \frac{dy}{dt}$$

optical flow

$$I_t = \frac{\partial I}{\partial t}$$

temporal derivative

Forward difference
Sobel filter
Scharr filter
…

How do you compute this?

frame differencing

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$$I_x u + I_y v + I_t = 0$$

How do you compute …

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

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$$I_t = \frac{\partial I}{\partial t}$$

temporal derivative

Forward difference
Sobel filter
Scharr filter
…

We need to solve for this!
(this is the unknown in the optical flow problem)

frame differencing

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$$I_x u + I_y v + I_t = 0$$

How do you compute …

$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$ **spatial derivative**	$u = \frac{dx}{dt} \quad v = \frac{dy}{dt}$ **optical flow**	$I_t = \frac{\partial I}{\partial t}$ **temporal derivative**
Forward difference Sobel filter Scharr filter …	(u, v) Solution lies on a line Cannot be found uniquely with a single constraint	frame differencing

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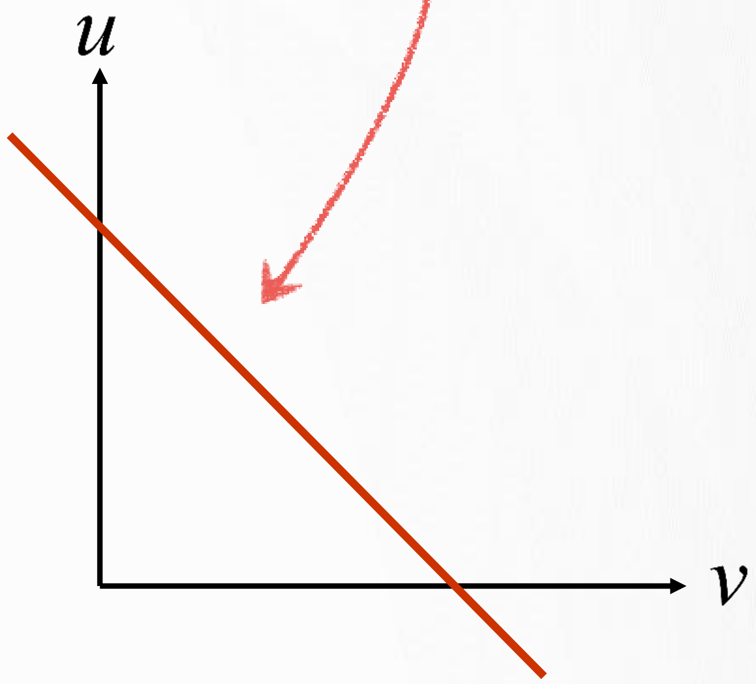
Solution lies on a straight line

$$\boxed{I_x u + I_y v + I_t = 0}$$

many combinations of u and v will satisfy the equality



The solution cannot be determined uniquely with a single constraint (a single pixel)

$$I_x u + I_y v + I_t = 0$$

$$I_x = \frac{\partial I}{\partial x} \quad I_y = \frac{\partial I}{\partial y}$$

spatial derivative

$$u = \frac{dx}{dt} \quad v = \frac{dy}{dt}$$

optical flow

$$I_t = \frac{\partial I}{\partial t}$$

temporal derivative

How can we use the brightness constancy equation to estimate the optical flow?

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unknown

$$I_x u + I_y v + I_t = 0$$

known

We need at least ____ equations to solve for 2 unknowns.

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unknown

$$I_x u + I_y v + I_t = 0$$

known

Where do we get more equations (constraints)?

Horn-Schunck Optical Flow (1981)

brightness constancy

small motion

‘smooth’ flow

(flow can vary from pixel to pixel)

global method
(dense)

Lucas-Kanade Optical Flow (1981)

method of differences

‘constant’ flow

(flow is constant for all pixels)

local method
(sparse)

CONSTANT FLOW:

LUCAS-KANADE OPTICAL FLOW

Where do we get more equations (constraints)?

$$I_x u + I_y v + I_t = 0$$

Assume that the surrounding patch (say 5x5) has **'constant flow'**

Assumptions:

Flow is locally smooth

Neighboring pixels have same displacement

Using a 5 x 5 image patch, gives us [illegible] equations

Assumptions:

Flow is locally smooth

Neighboring pixels have same displacement

Using a 5 x 5 image patch, gives us 25 equations

$$
\begin{gathered}
I_x(\boldsymbol{p}_1)u + I_y(\boldsymbol{p}_1)v = -I_t(\boldsymbol{p}_1) \\
I_x(\boldsymbol{p}_2)u + I_y(\boldsymbol{p}_2)v = -I_t(\boldsymbol{p}_2) \\
\vdots \\
I_x(\boldsymbol{p}_{25})u + I_y(\boldsymbol{p}_{25})v = -I_t(\boldsymbol{p}_{25})
\end{gathered}
$$

Assumptions:

Flow is locally smooth

Neighboring pixels have same displacement

Using a 5 x 5 image patch, gives us 25 equations

$$\begin{bmatrix} I_x(\boldsymbol{p}_1) & I_y(\boldsymbol{p}_1) \\ I_x(\boldsymbol{p}_2) & I_y(\boldsymbol{p}_2) \\ \vdots & \vdots \\ I_x(\boldsymbol{p}_{25}) & I_y(\boldsymbol{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\boldsymbol{p}_1) \\ I_t(\boldsymbol{p}_2) \\ \vdots \\ I_t(\boldsymbol{p}_{25}) \end{bmatrix}$$

Matrix form

Assumptions:

Flow is locally smooth

Neighboring pixels have same displacement

Using a 5 x 5 image patch, gives us 25 equations

$$\begin{bmatrix} I_x(\boldsymbol{p}_1) & I_y(\boldsymbol{p}_1) \\ I_x(\boldsymbol{p}_2) & I_y(\boldsymbol{p}_2) \\ \vdots & \vdots \\ I_x(\boldsymbol{p}_{25}) & I_y(\boldsymbol{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\boldsymbol{p}_1) \\ I_t(\boldsymbol{p}_2) \\ \vdots \\ I_t(\boldsymbol{p}_{25}) \end{bmatrix}$$

$$\underset{25 \times 2}{A} \qquad \underset{2 \times 1}{x} \qquad \underset{25 \times 1}{b}$$

How many equations? How many unknowns? How do we solve this?

Least squares approximation

$\hat{x} = \arg\min_x ||Ax - b||^2$ is equivalent to solving $A^\top A\hat{x} = A^\top b$

Least squares approximation

$$\hat{x} = \arg\min_x ||Ax - b||^2$$ is equivalent to solving $$A^\top A\hat{x} = A^\top b$$

To obtain the least squares solution solve:

$$\begin{array}{ccc} A^\top A & \hat{x} & A^\top b \\ \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix} & \begin{bmatrix} u \\ v \end{bmatrix} = - & \begin{bmatrix} \sum_{p \in P} I_x I_t \\ \sum_{p \in P} I_y I_t \end{bmatrix} \end{array}$$

where the summation is over each pixel $\boldsymbol{p}$ in patch $\boldsymbol{P}$

$$x = (A^\top A)^{-1} A^\top b$$

Least squares approximation

$$\hat{x} = \arg\min_x ||Ax - b||^2$$ is equivalent to solving $$A^\top A\hat{x} = A^\top b$$

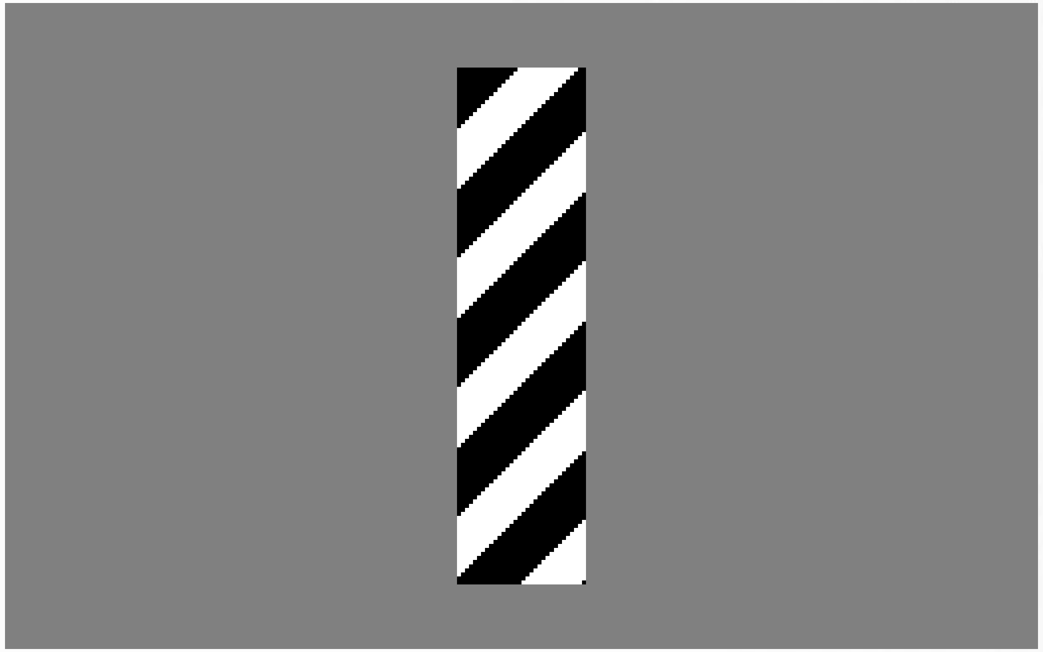
To obtain the least squares solution solve:

$$\begin{array}{ccc} A^\top A & \hat{x} & A^\top b \\ \begin{bmatrix} \sum_{p\in P} I_x I_x & \sum_{p\in P} I_x I_y \\ \sum_{p\in P} I_y I_x & \sum_{p\in P} I_y I_y \end{bmatrix} & \begin{bmatrix} u \\ v \end{bmatrix} = - & \begin{bmatrix} \sum_{p\in P} I_x I_t \\ \sum_{p\in P} I_y I_t \end{bmatrix} \end{array}$$

where the summation is over each pixel $\boldsymbol{p}$ in patch $\boldsymbol{P}$

You want to compute optical flow.
What happens if the image patch contains only a line?

BARBER'S POLE ILLUSION



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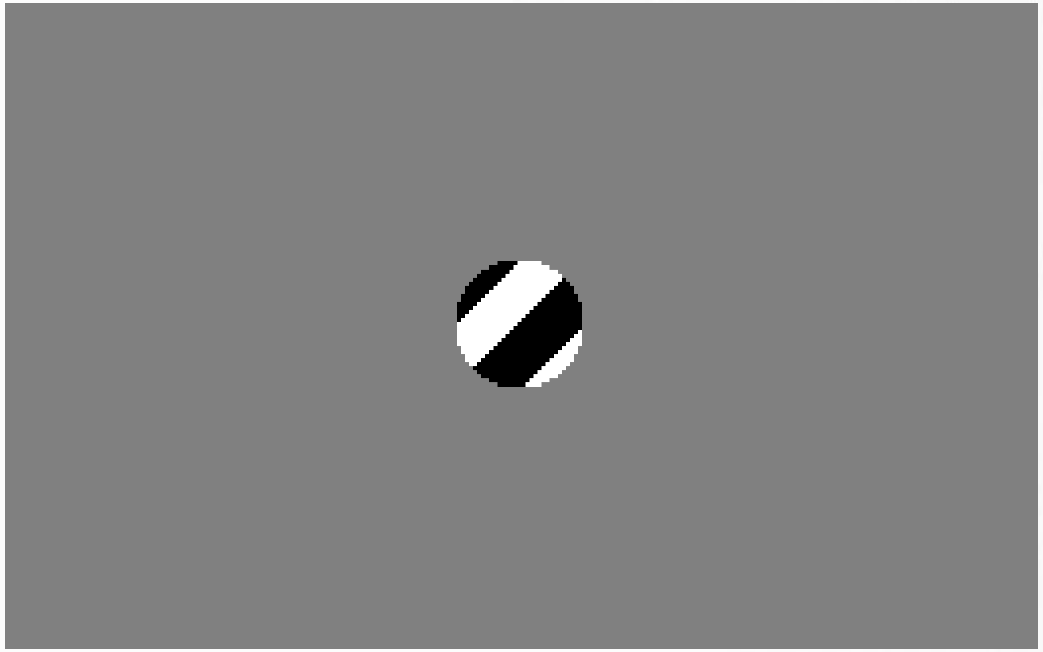
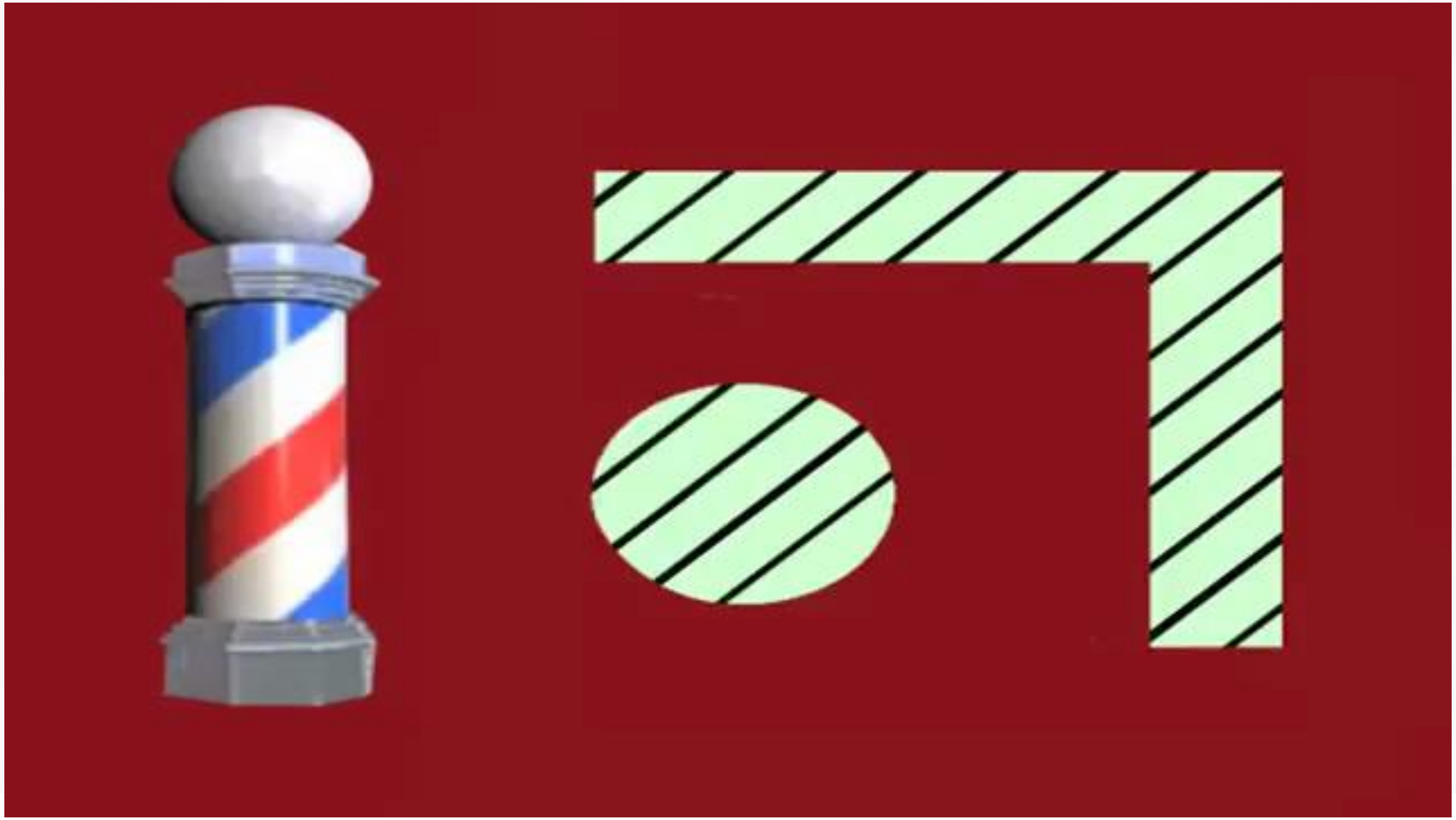
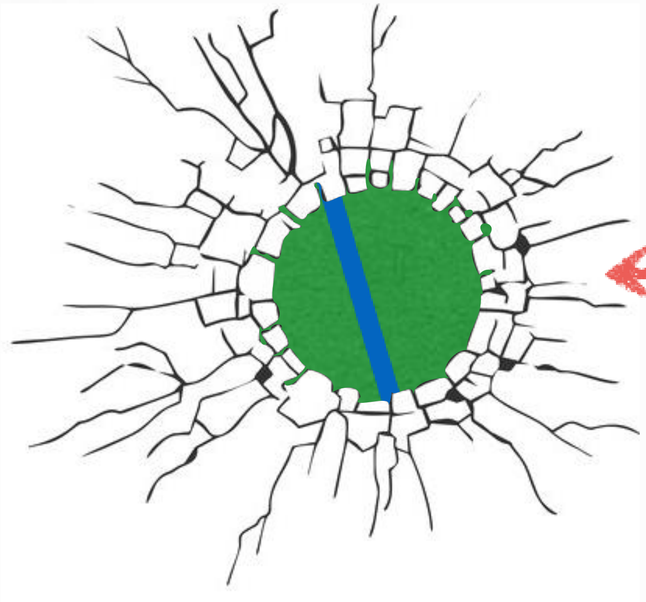


BARBER’S POLE ILLUSION



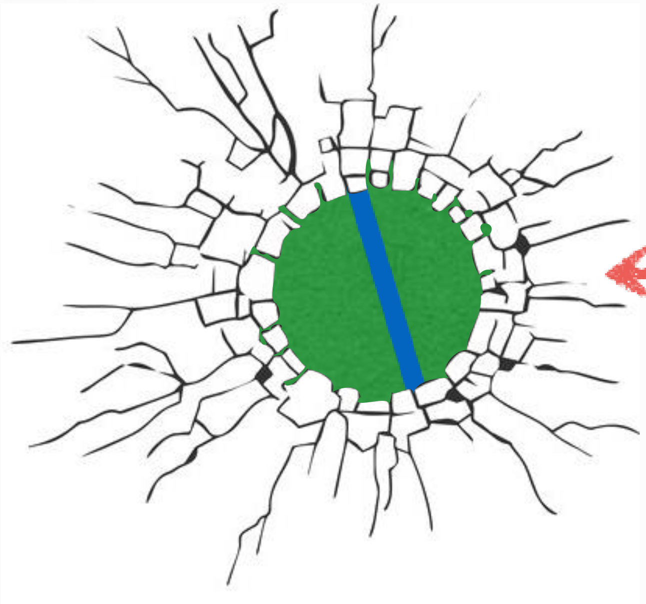
APERTURE PROBLEM



small visible image patch

In which direction is the line moving?

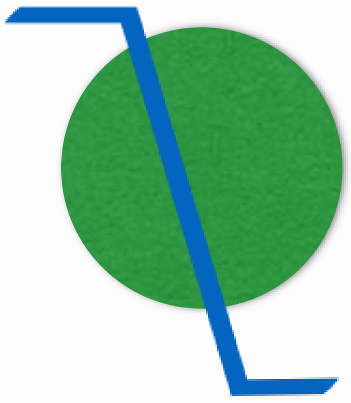
APERTURE PROBLEM



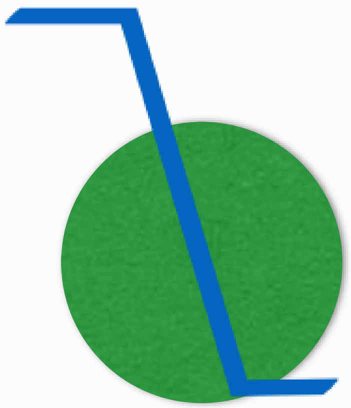
small visible image patch

In which direction is the line moving?

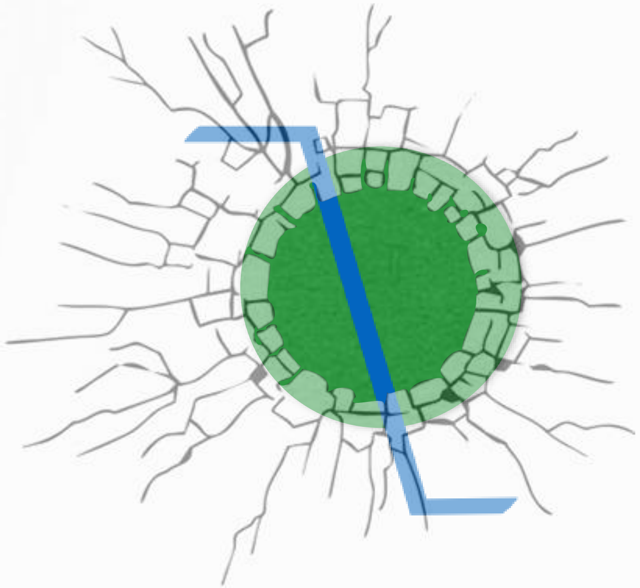
APERTURE PROBLEM



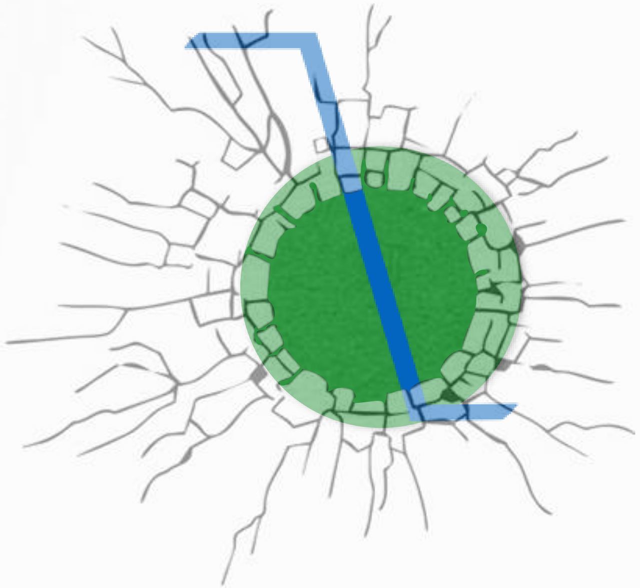
APERTURE PROBLEM

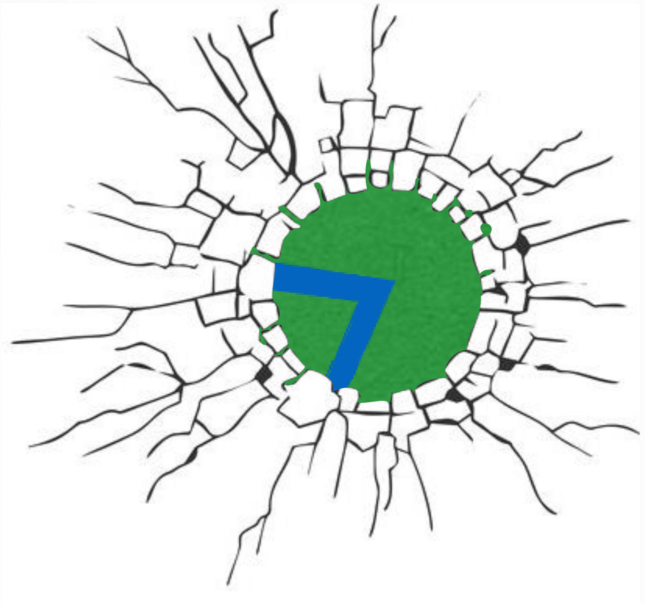


APERTURE PROBLEM

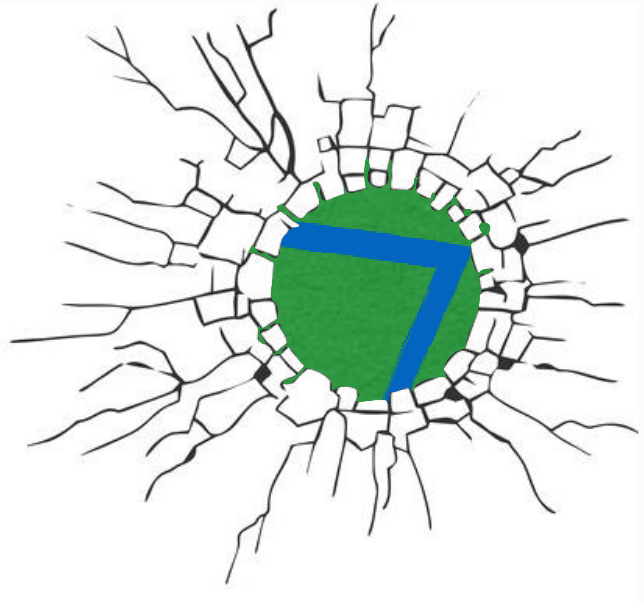


APERTURE PROBLEM





Want patches with different gradients to the avoid aperture problem



Want patches with different gradients to the avoid aperture problem

H(x,y) = y

1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5

x
y

optical flow: (1,1)

I(x,y)

-	-	-	-
-	1	1	1
-	2	2	2
-	3	3	3
-	4	4	4

x
y

$$I_x u + I_y v + I_t = 0$$

Compute gradients

$$I_x(3,3) = 0$$
$$I_y(3,3) = 1$$
$$I_t(3,3) = I(3,3) - H(3,3) = -1$$

Solution:

$$v = 1$$

We recover the v of the optical flow but not the u.

This is the aperture problem.

HORN-SCHUNCK OPTICAL FLOW

Horn-Schunck Optical Flow (1981)

Lucas-Kanade Optical Flow (1981)

Horn-Schunck Optical Flow (1981)	Lucas-Kanade Optical Flow (1981)
brightness constancy small motion	method of differences
'smooth' flow (flow can vary from pixel to pixel)	**'constant' flow** (flow is constant for all pixels)
global method (dense)	local method (sparse)

SMOOTHNESS

most objects in the world are rigid or deform elastically
moving together coherently

we expect optical flow fields to be smooth

KEY IDEA
(OF HORN-SCHUNCK OPTICAL FLOW)

Enforce

brightness constancy

Enforce

smooth flow field

to compute optical flow

KEY IDEA
(OF HORN-SCHUNCK OPTICAL FLOW)

Enforce
brightness constancy

Enforce
smooth flow field

to compute optical flow

ENFORCE BRIGHTNESS CONSTANCY

$$\boxed{I_x u + I_y v + I_t = 0}$$

For every pixel,

$$\min_{u,v} \left[I_x u_{ij} + I_y v_{ij} + I_t \right]^2$$

ENFORCE BRIGHTNESS CONSTANCY

$$\boxed{I_x u + I_y v + I_t = 0}$$

For every pixel,

$$\min_{u,v} \left[I_x u_{ij} + I_y v_{ij} + I_t \right]^2$$

lazy notation for $I_x(i,j)$

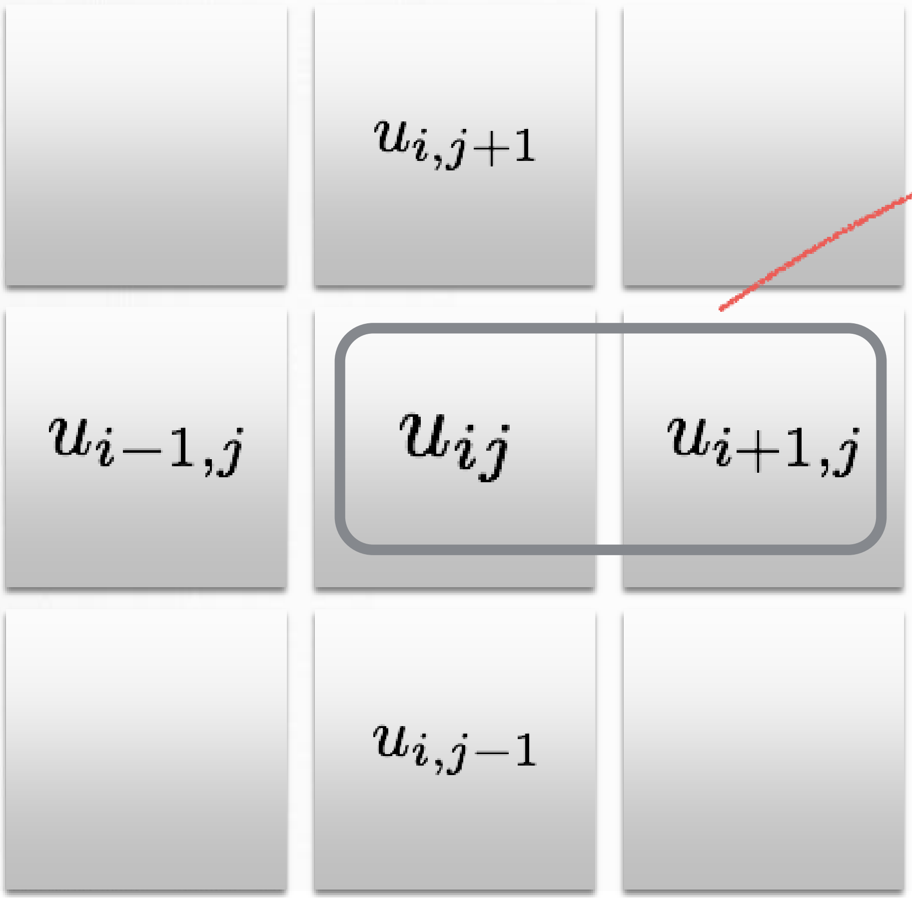
KEY IDEA
(OF HORN-SCHUNCK OPTICAL FLOW)

Enforce
brightness constancy

Enforce
smooth flow field

to compute optical flow

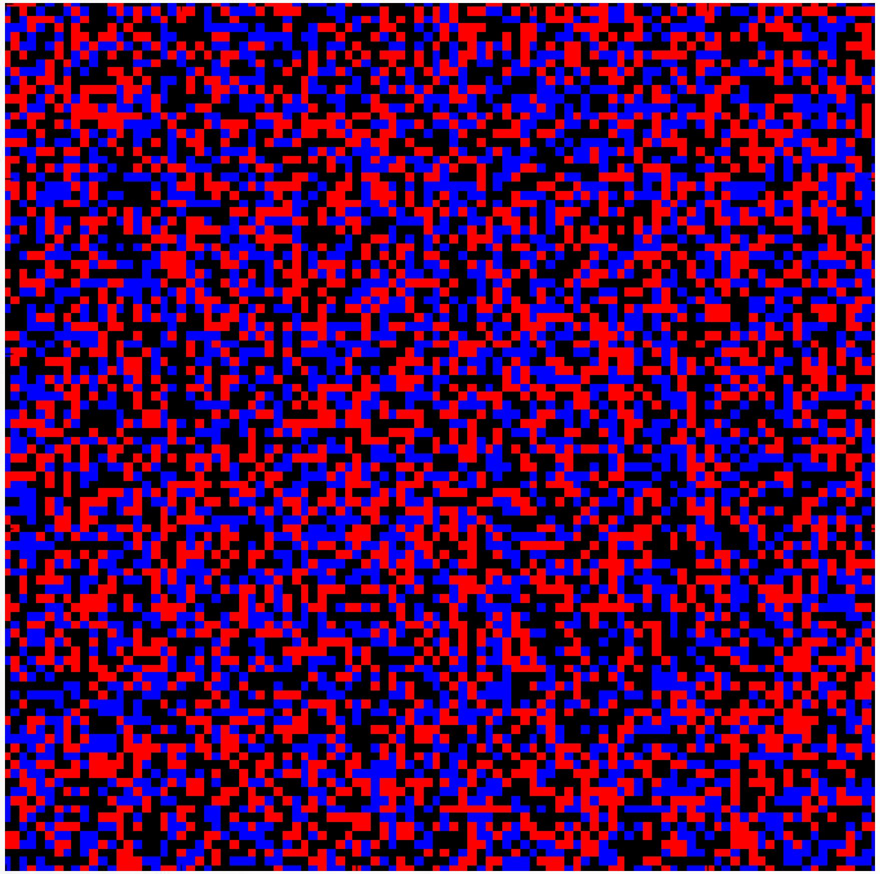
ENFORCE SMOOTH FLOW FIELD



$$\min_{\boldsymbol{u}} (u_{i,j} - u_{i+1,j})^2$$

u-component of flow

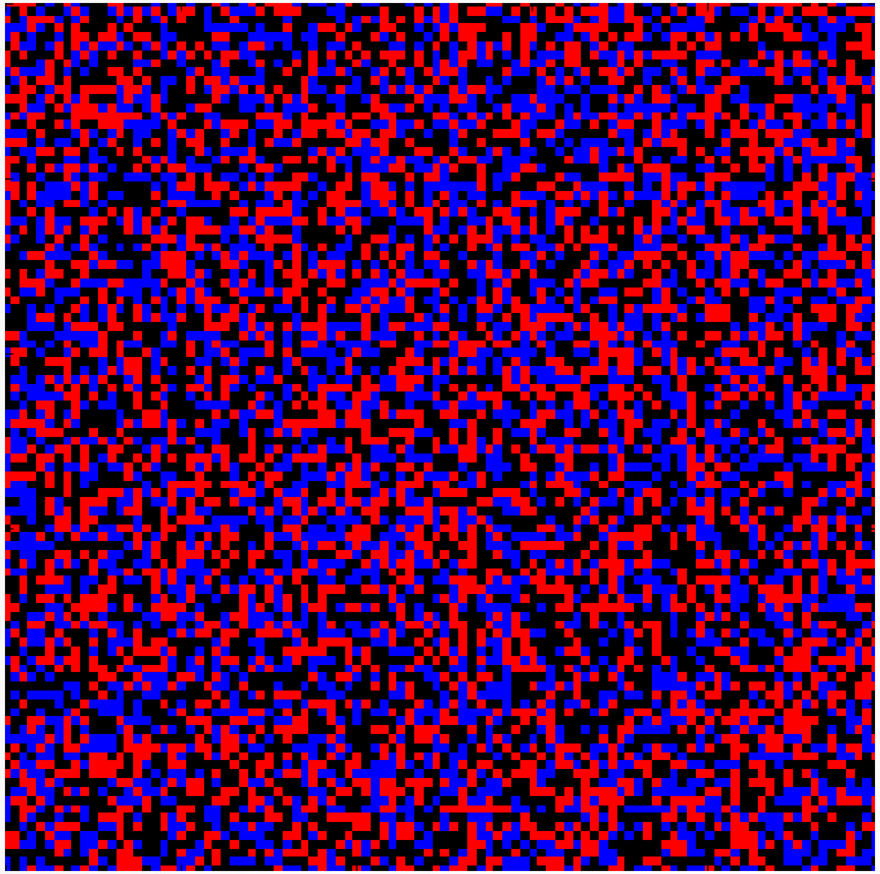
Which flow field optimizes the objective? $\min_{\boldsymbol{u}} (u_{i,j} - u_{i+1,j})^2$



$$\sum_{ij} (u_{ij} - u_{i+1,j})^2 \quad ? \quad \sum_{ij} (u_{ij} - u_{i+1,j})^2$$

Which flow field optimizes the objective?

$$\min_{\boldsymbol{u}} (u_{i,j} - u_{i+1,j})^2$$



big

small

KEY IDEA
(OF HORN-SCHUNCK OPTICAL FLOW)

Enforce
brightness constancy

Enforce
smooth flow field

to compute optical flow

bringing it all together…

HORN-SCHUNCK OPTICAL FLOW

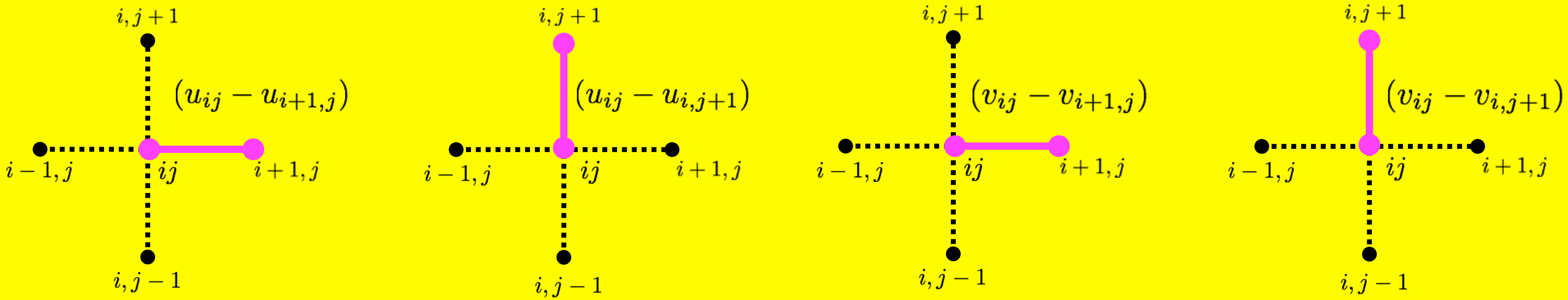
$$\min_{\boldsymbol{u},\boldsymbol{v}} \sum_{i,j} \left\{ \overbrace{E_s(i,j)}^{\text{smoothness}} + \underbrace{\lambda}_{\text{weight}} \overbrace{E_d(i,j)}^{\text{brightness constancy}} \right\}$$

HS OPTICAL FLOW OBJECTIVE FUNCTION

Brightness constancy $$E_d(i,j) = \left[I_x u_{ij} + I_y v_{ij} + I_t \right]^2$$

Smoothness

$$E_s(i,j) = \frac{1}{4}\left[(u_{ij} - u_{i+1,j})^2 + (u_{ij} - u_{i,j+1})^2 + (v_{ij} - v_{i+1,j})^2 + (v_{ij} - v_{i,j+1})^2 \right]$$



why not all four neighbors?

HOW DO WE SOLVE THIS MINIMIZATION PROBLEM ?

$$\min_{\boldsymbol{u},\boldsymbol{v}} \sum_{i,j} \left\{ E_s(i,j) + \lambda E_d(i,j) \right\}$$

Compute partial derivative, derive update equations

(gradient decent!)

HORN-SCHUNCK OPTICAL FLOW ALGORITHM

1. Precompute image gradients

   $I_y \quad I_x$

2. Precompute temporal gradients

   I_t

3. Initialize flow field

   $$\boldsymbol{u} = \mathbf{0}$$
   $$\boldsymbol{v} = \mathbf{0}$$

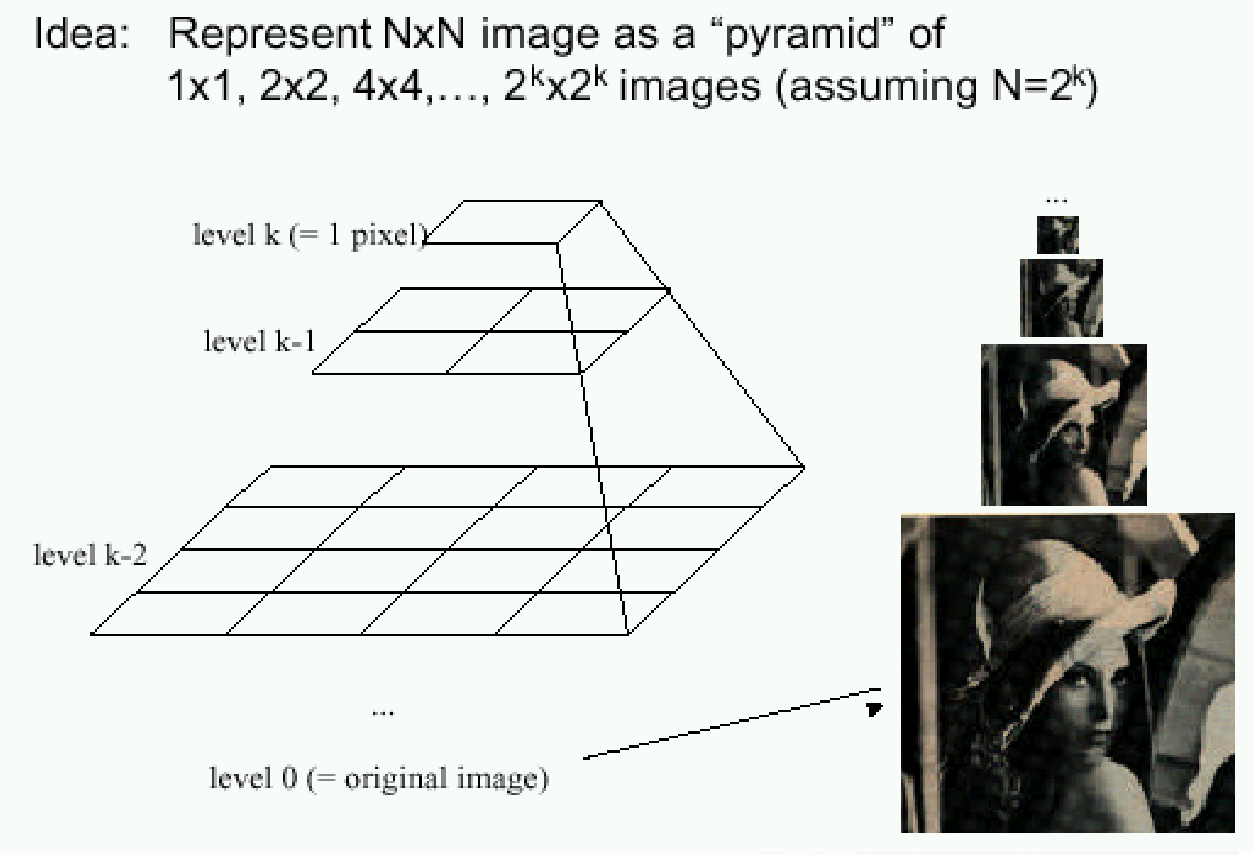
4. While not converged, compute flow field updates for each pixel:

$$\hat{u}_{kl} = \bar{u}_{kl} - \frac{I_x \bar{u}_{kl} + I_y \bar{v}_{kl} + I_t}{\lambda^{-1} + I_x^2 + I_y^2} I_x \qquad \hat{v}_{kl} = \bar{v}_{kl} - \frac{I_x \bar{u}_{kl} + I_y \bar{v}_{kl} + I_t}{\lambda^{-1} + I_x^2 + I_y^2} I_y$$

Just 8 lines of code!

SMALL MOTION ASSUMPTION

- If motion is not small enough, image resolution has to be reduced => motion will be smaller in pixels
- Lecture 3: Gaussian Pyramids:
  "have all sorts of applications in computer vision"




60˚ 10 1.2 N, 24˚ 57 18 E