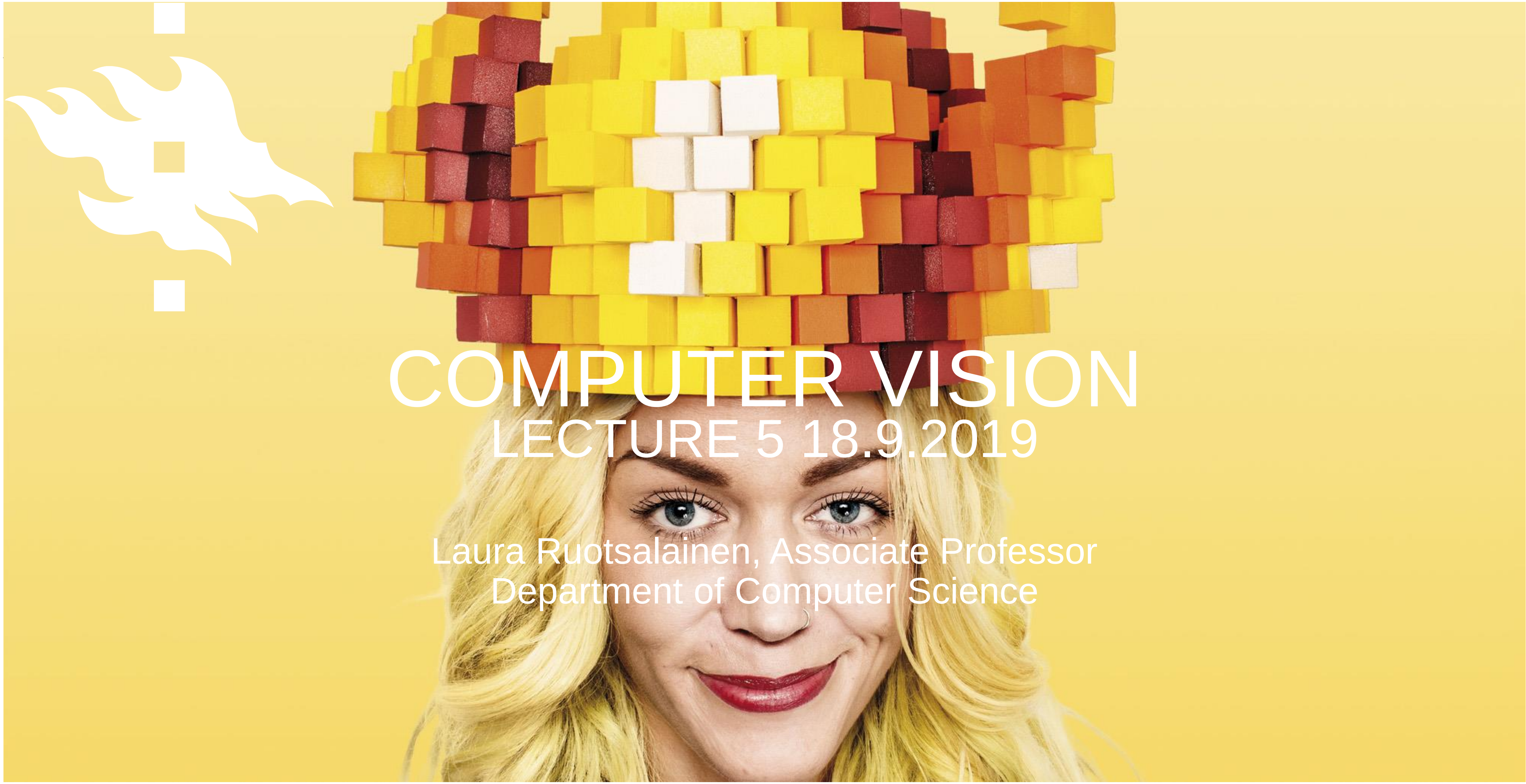

COMPUTER VISION

LECTURE 5 18.9.2019

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UNIVERSITY OF HELSINKI

ESTIMATION

- Estimation is a very important procedure in computer vision for
  - Image to image mapping (2D homography)
  - Camera projection (3D to 2D)
  - Computing a Fundamental matrix (for resolving camera matrix)
  - Computing a trifocal tensor relating points or lines in three views
- Problems related, here we'll concentrate on the first one

Lecture 7

Szeliski: Section 6.1
Hartley & Zissermann: Sections 2 & 4

PANORAMAS FROM IMAGE STITCHING

1. Capture multiple image from different viewpoints.

2. Stitch them together into a virtual wide-angle image.

HOW DO WE STITCH IMAGES FROM DIFFERENT VIEWPOINTS?

Use image homographies.

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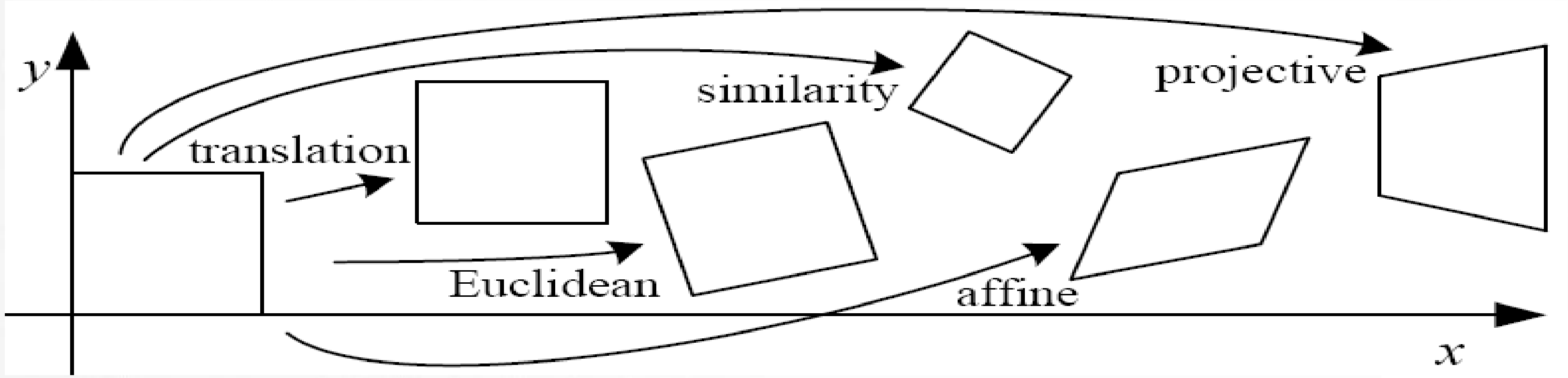
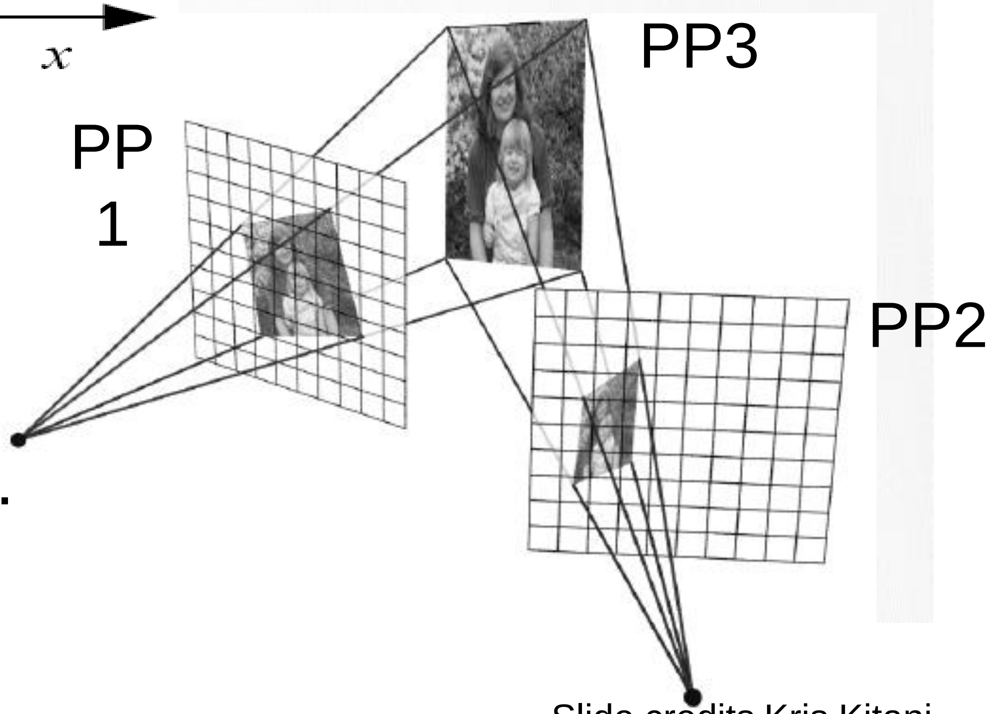
CLASSIFICATION OF 2D TRANSFORMATIONS





Which kind of transformation is needed to warp projective plane 1 into projective plane 2?

- A projective transformation (a.k.a. a homography).

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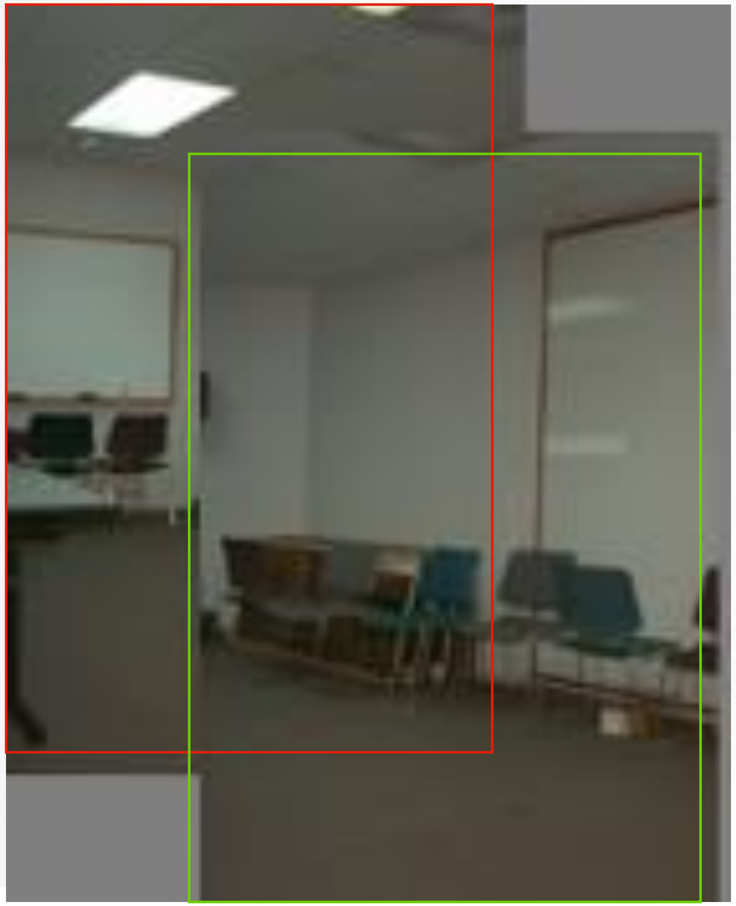
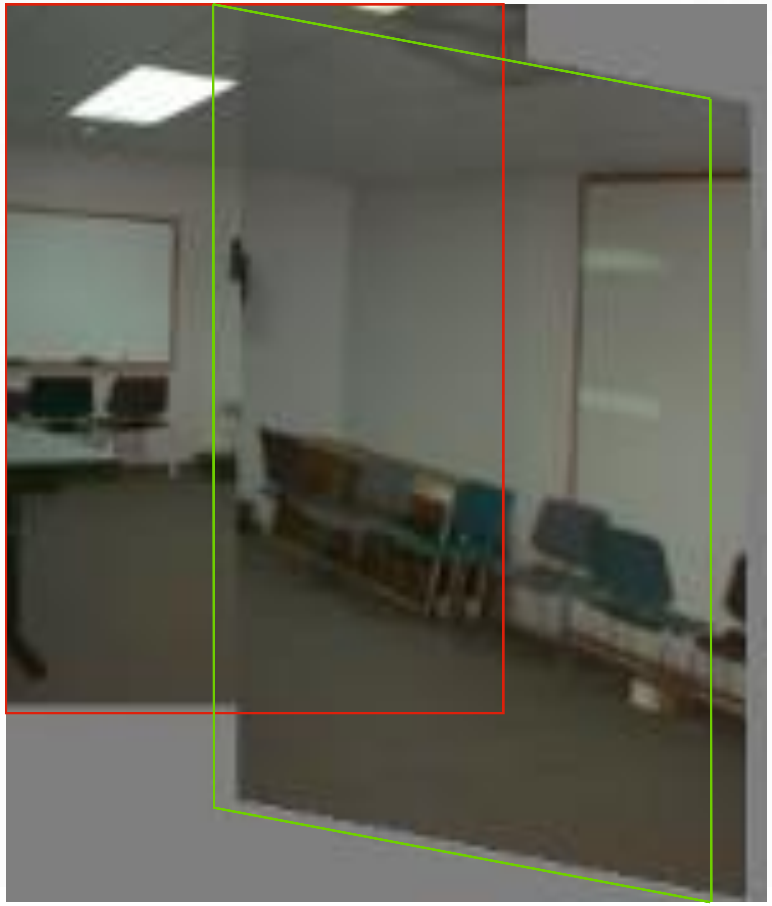
WARPING WITH DIFFERENT TRANSFORMATIONS

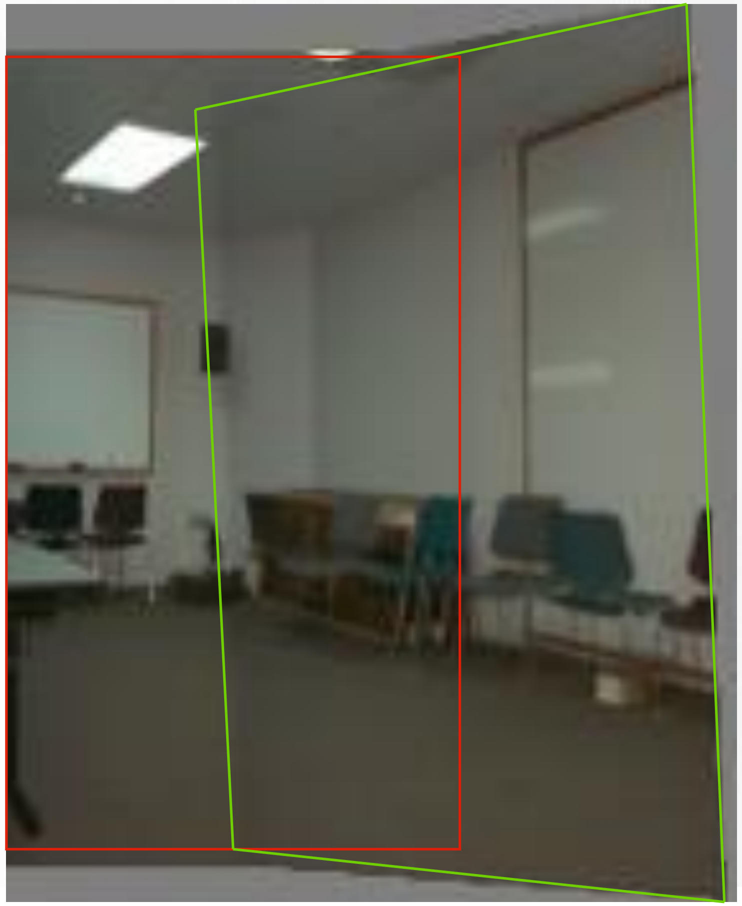
translation



affine



Projective (homography)



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HOMOGRAPHY

- May be used when
  - the scene is planar (a wall, a floor, …)
  - the scene is very far or has small (relative) depth variation → scene is approximately planar
  - camera is experiencing only rotation (no translation or pose change)

APPLYING A HOMOGRAPHY

1. Convert to homogeneous coordinates:

$$x = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow x = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

What is the size of the homography matrix?

$$x' = H \cdot \mathbf{x}$$

2. Multiply by the homography matrix:

3. Convert back to heterogeneous coordinates:

$$x' = \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} \Rightarrow x' = \begin{bmatrix} x'/w' \\ y'/w' \end{bmatrix}$$

APPLYING A HOMOGRAPHY

What is the size of the homography matrix (H) ?

How many degrees of freedom does the homography matrix have?

$$x' = H \cdot \mathrm{x}$$

How many matching points do we need to resolve the homography?

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ESTIMATION => FITTING

- When dealing with images we suffer from
  - Noisy data
  - Outliers
  - Missing data
- Four point correspondences => minimal solution for H

- Usually more correspondences obtained => solution not compatible with any projective transformation
- Find transformation H that minimizes a cost function

ESTIMATION => FITTING

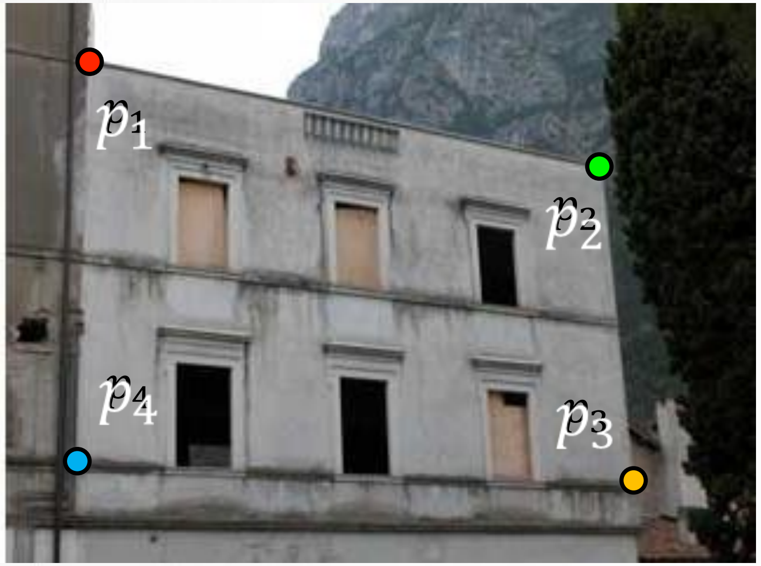
- Least squares methods
  - No outliers, noise Gaussian distributed
- RANSAC
  - Outliers
- Hough Transform (this we learned already)
- Expectation Maximization (EM)

THE DIRECT LINEAR TRANSFORM (DLT) FOR COMPUTING THE HOMOGRAPHY MATRIX

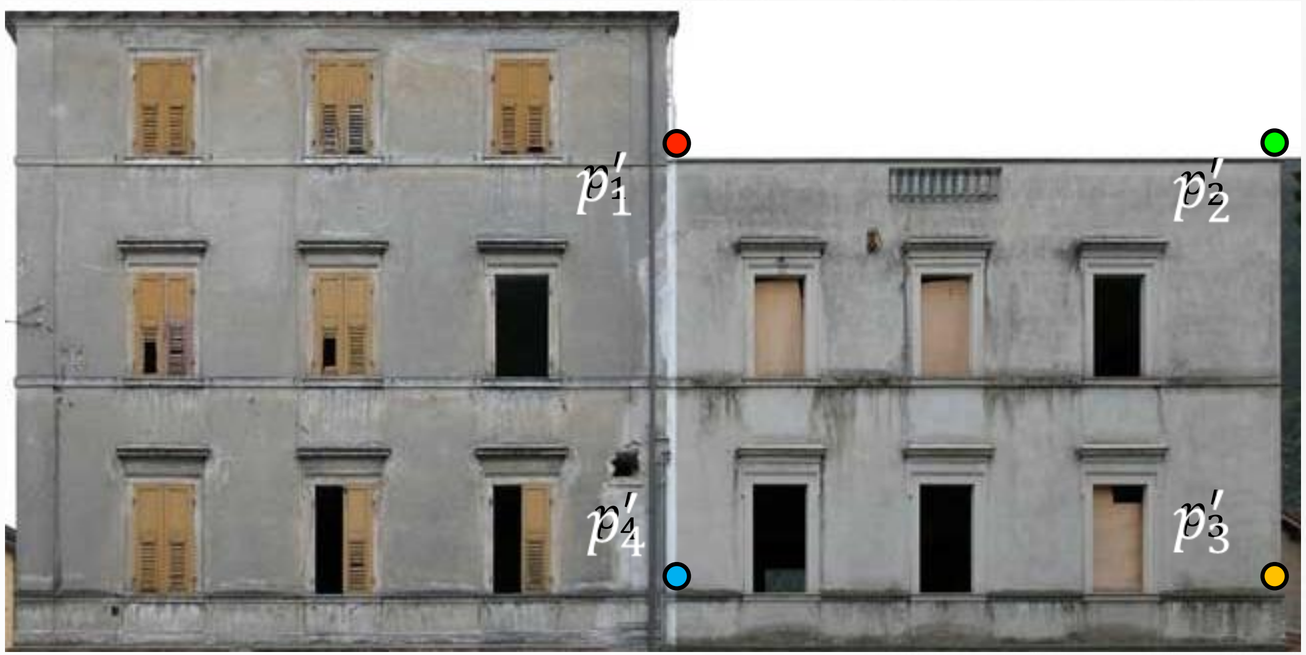
CREATE POINT CORRESPONDENCES

Given a set of matched feature points *x, x'* find the best estimate of H

such that $x' = H \cdot x$



original image



target image

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DETERMINING THE HOMOGRAPHY MATRIX

Write out linear equation for each correspondence:

$$x' = H \cdot x \quad \text{or} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Slide credits Kris Kitani

DETERMINING THE HOMOGRAPHY MATRIX

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$$x' = H \cdot x \quad \text{or} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Expand matrix multiplication:

$$x' = \alpha(h_1 x + h_2 y + h_3)$$
$$y' = \alpha(h_4 x + h_5 y + h_6)$$
$$1 = \alpha(h_7 x + h_8 y + h_9)$$

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DETERMINING THE HOMOGRAPHY MATRIX

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Expand matrix multiplication:

$$x' = \alpha(h_1 x + h_2 y + h_3)$$
$$y' = \alpha(h_4 x + h_5 y + h_6)$$
$$1 = \alpha(h_7 x + h_8 y + h_9)$$

Divide out unknown scale factor:

$$x'(h_7 x + h_8 y + h_9) = (h_1 x + h_2 y + h_3)$$
$$y'(h_7 x + h_8 y + h_9) = (h_4 x + h_5 y + h_6)$$

How do you rearrange terms to make it a linear system?

$$x'(h_7x + h_8y + h_9) = (h_1x + h_2y + h_3)$$
$$y'(h_7x + h_8y + h_9) = (h_4x + h_5y + h_6)$$

Just rearrange the terms

$$\boxed{\begin{aligned} h_7xx' + h_8yx' + h_9x' - h_1x - h_2y - h_3 = 0 \\ h_7xy' + h_8yy' + h_9y' - h_4x - h_5y - h_6 = 0 \end{aligned}}$$

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DETERMINING THE HOMOGRAPHY MATRIX

Re-arrange terms:
$$h_7xx' + h_8yx' + h_9x' - h_1x - h_2y - h_3 = 0$$
$$h_7xy' + h_8yy' + h_9y' - h_4x - h_5y - h_6 = 0$$

Re-write in matrix form:

$$\mathbf{A}_i \boldsymbol{h} = \mathbf{0}$$

$$\mathbf{A}_i = \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix}$$

$$\boldsymbol{h} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 & h_9 \end{bmatrix}^\top$$

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DETERMINING THE HOMOGRAPHY MATRIX

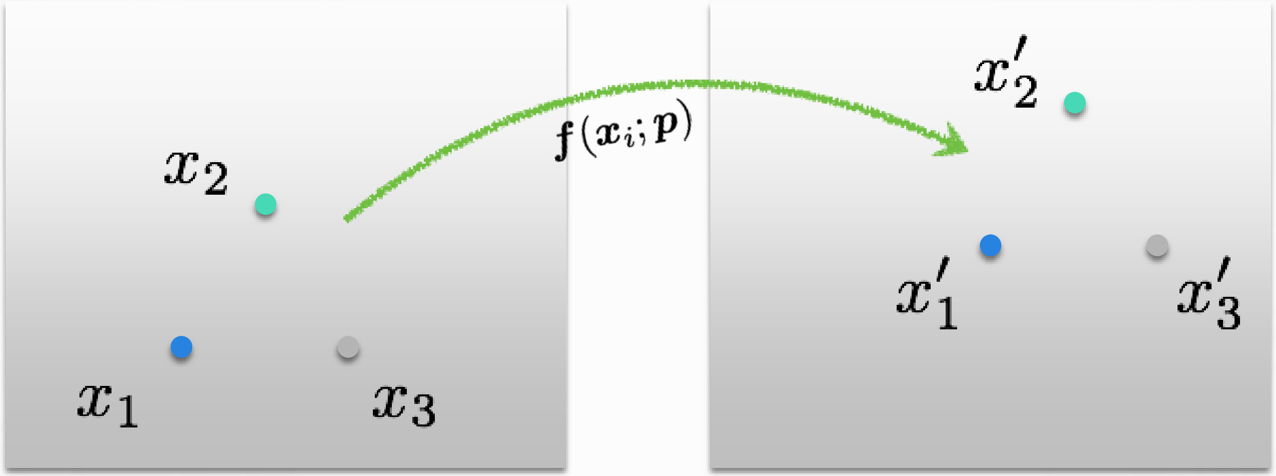
- Stack together constraints from multiple point correspondences:

$$\mathbf{A}\boldsymbol{h} = \mathbf{0}$$

$$\begin{array}{c}
\begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \\
\begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \\
\vdots \\
\begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix}
\end{array}
\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Homogeneous linear least squares problem

Slide credits Kris Kitani



Least Squares Error

$$E_{\mathrm{LS}} = \sum_i \| f(x_i; p) - x'_i \|^2$$

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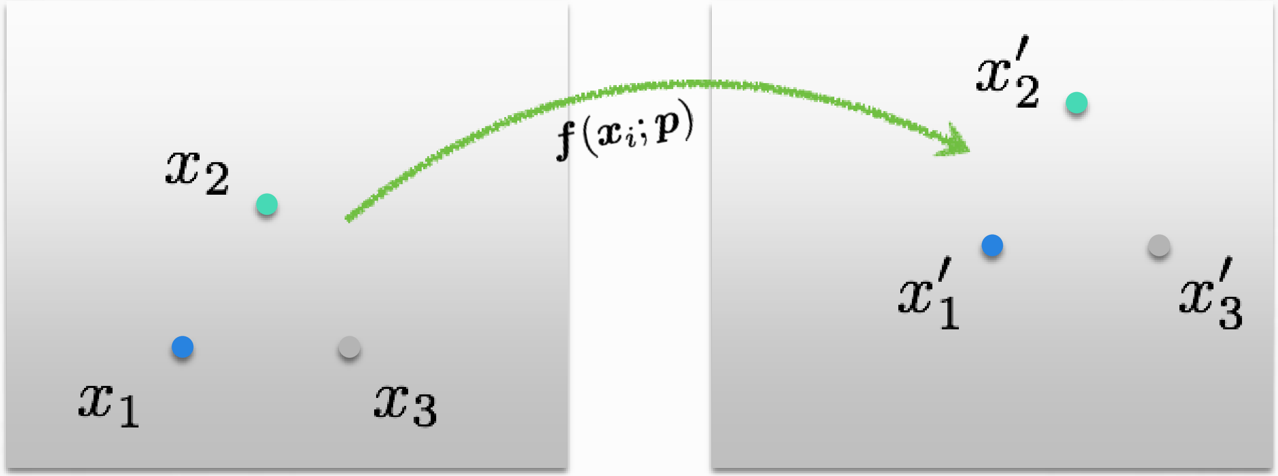

Least Squares Error

What is this?

$$E_{\mathrm{LS}} = \sum_i \| f(x_i; p) - x'_i \|^2$$

What is this? *What is this?*

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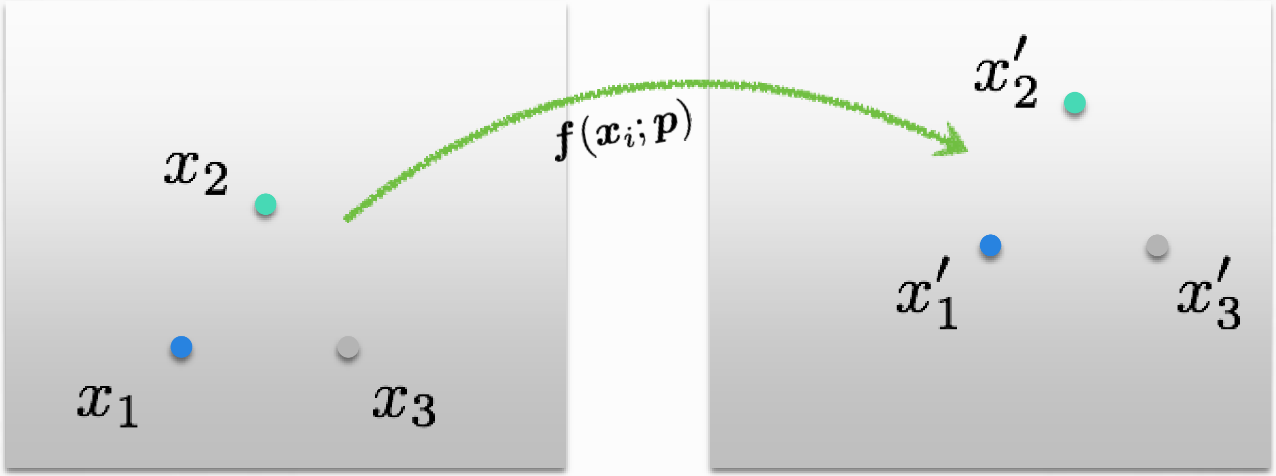

$$||x|| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

Least Squares Error

Euclidean
(L2) norm

$$E_{\mathrm{LS}} = \sum_i ||f(x_i; p) - x'_i||^2$$

squared!

predicted
location

measured
location

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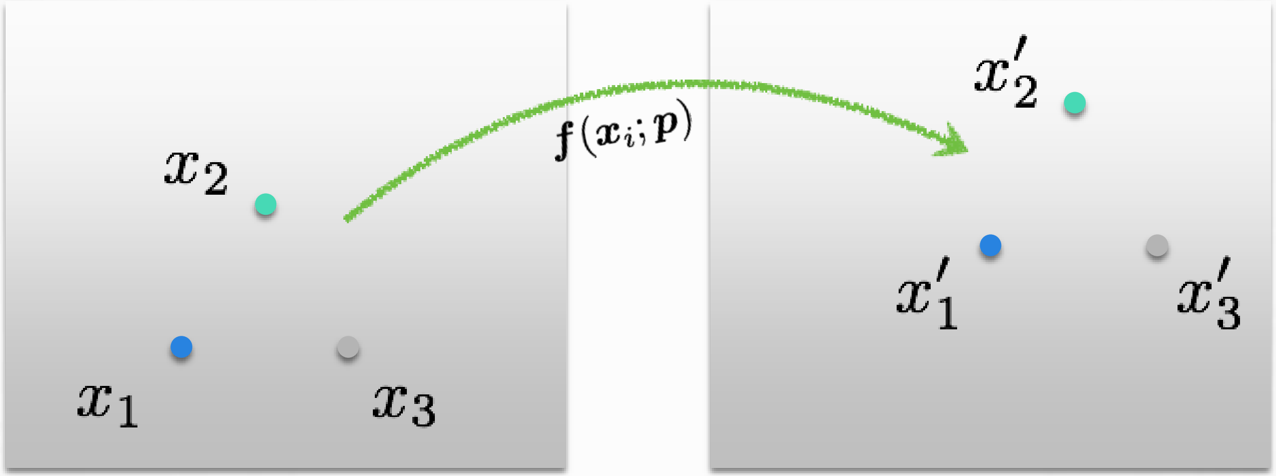

Least Squares Error

$$E_{\mathrm{LS}} = \sum_i \underbrace{\|f(x_i; p) - x'_i\|^2}$$

Residual
(projection error)

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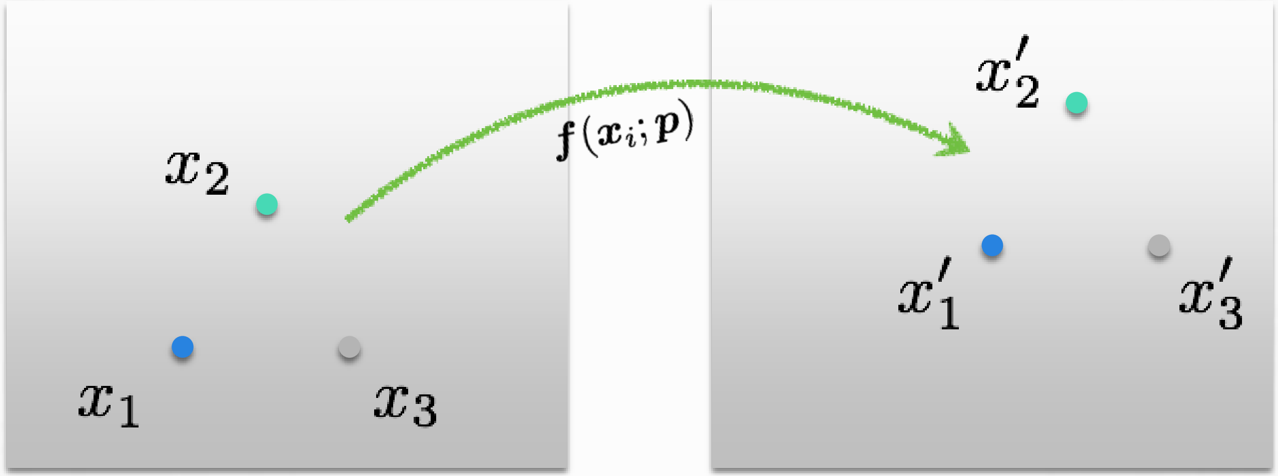

Least Squares Error

$$E_{\mathrm{LS}} = \sum_i \| f(x_i; p) - x'_i \|^2$$

What is the free variable?
What do we want to optimize?

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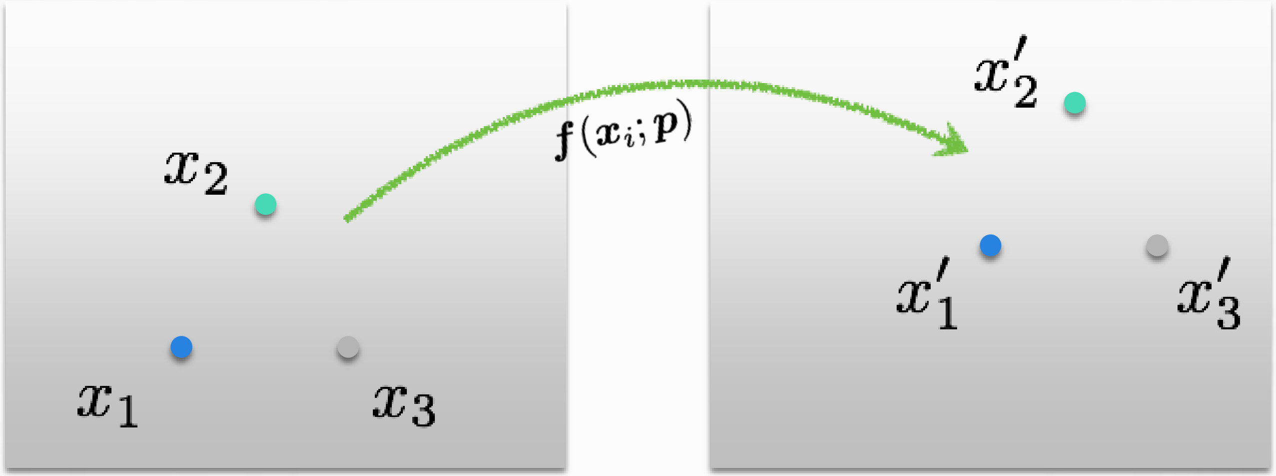

Find parameters that minimize squared error

$$\hat{\boldsymbol{p}} = \arg\min_{\boldsymbol{p}} \sum_i \|\boldsymbol{f}(\boldsymbol{x}_i; \boldsymbol{p}) - \boldsymbol{x}'_i\|^2$$

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Solving the linear system

Convert the system to a linear least-squares problem:

$$E_{\mathrm{LLS}} = \|\mathbf{A}\boldsymbol{x} - \boldsymbol{b}\|^2 \qquad\qquad E_{\mathrm{LLS}} = \sum_i |\boldsymbol{a}_i\boldsymbol{x} - \boldsymbol{b}_i|^2$$

Expand the error:

$$E_{\mathrm{LLS}} = \boldsymbol{x}^\top(\mathbf{A}^\top\mathbf{A})\boldsymbol{x} - 2\boldsymbol{x}^\top(\mathbf{A}^\top\boldsymbol{b}) + \|\boldsymbol{b}\|^2$$

Minimize the error:

Set derivative to 0 $\quad (\mathbf{A}^\top\mathbf{A})\boldsymbol{x} = \mathbf{A}^\top\boldsymbol{b}$

Solve for x $\quad \boldsymbol{x} = (\mathbf{A}^\top\mathbf{A})^{-1}\mathbf{A}^\top\boldsymbol{b} \longleftarrow$

Note: You almost <u>never</u> want to compute the inverse of a matrix.

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DETERMINING THE HOMOGRAPHY MATRIX

Stack together constraints from multiple point correspondences:

$$\mathbf{A}\boldsymbol{h} = \mathbf{0}$$

$$\begin{bmatrix} \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \\ \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \\ \vdots \\ \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Homogeneous linear least squares problem

- How do we solve this?

DETERMINING THE HOMOGRAPHY MATRIX

Stack together constraints from multiple point correspondences:

$$\mathbf{A}\boldsymbol{h} = \mathbf{0}$$

$$\begin{array}{c}
\begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & yy' & y' \end{bmatrix} \\
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\end{array}
\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Homogeneous linear least squares problem

- Solve with SVD

SINGULAR VALUE DECOMPOSITION

ortho-normal diagonal ortho-normal

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top$$

unit norm constraint

n x m n x n n x m m x m

$$= \sum_{i=1}^{9} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^\top$$

n x 1 1 x m

Each column of V represents a solution for $\mathbf{A}\boldsymbol{h} = \mathbf{0}$

where the singular value represents the reprojection error

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SOLVING FOR H USING DLT

Given $\{\boldsymbol{x}_i, \boldsymbol{x}'_i\}$ solve for H such that $\boldsymbol{x}' = \mathbf{H}\boldsymbol{x}$

1. For each correspondence, create 2x9 matrix $\mathbf{A}_i$

2. Concatenate into single 2n x 9 matrix $\mathbf{A}$

3. Compute SVD of $\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top$

4. Store singular vector of the smallest singular value $\boldsymbol{h} = \boldsymbol{v}_{\hat{i}}$

5. Reshape to get $\mathbf{H}$

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General form of total least squares

(**Warning:** change of notation. x is a vector of parameters!)

$$E_{\mathrm{TLS}} = \sum_i (\boldsymbol{a}_i \boldsymbol{x})^2$$

$$= \|\mathbf{A}\boldsymbol{x}\|^2 \qquad \text{(matrix form)}$$

$$\|\boldsymbol{x}\|^2 = 1 \qquad \text{constraint}$$

minimize $\|\mathbf{A}\boldsymbol{x}\|^2$

subject to $\|\boldsymbol{x}\|^2 = 1$

(Rayleigh quotient)

minimize $\dfrac{\|\mathbf{A}\boldsymbol{x}\|^2}{\|\boldsymbol{x}\|^2}$

Solution is the eigenvector corresponding to smallest eigenvalue of

$$\mathbf{A}^\top \mathbf{A}$$

(equivalent)

Solution is the column of **V** corresponding to smallest singular value

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top$$

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DATA NORMALIZATION

- Data normalization is an essential step in DLT
  1. Compute a similarity transformation T for image points x consisting of translation and scaling (centroid of points is the coordinate origin (0,0) ,average distance sqrt(2))
  2. Perform similar transformation for x'
  3. Apply DLT as presented before
  4. Denormalization $H = T'^{-1}HT$

DEGENERATED CASE

- What if three of the four points in image 1 are collinear?
- Is the homography sufficiently constrained if the corresponding three
  points in image 2 are also collinear?

DEGENERATED CASE

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  - No, there is a family of homographies defining the mapping
- What if they are not?

DEGENERATED CASE

- What if three of the four points in image 1 are collinear?
- Is the homography sufficiently constrained if the corresponding three points in image 2 are also collinear?
  - No, there is a family of homographies defining the mapping
- What if they are not?
  - There will be no transformation h, since projective transformation must preserve collinearity
- Unique solution not determined => degenerate
- Not restricted to minimal solution

- Linear least-squares estimation performs well when the transform function is linear
- Doesn't perform well when there are outliers

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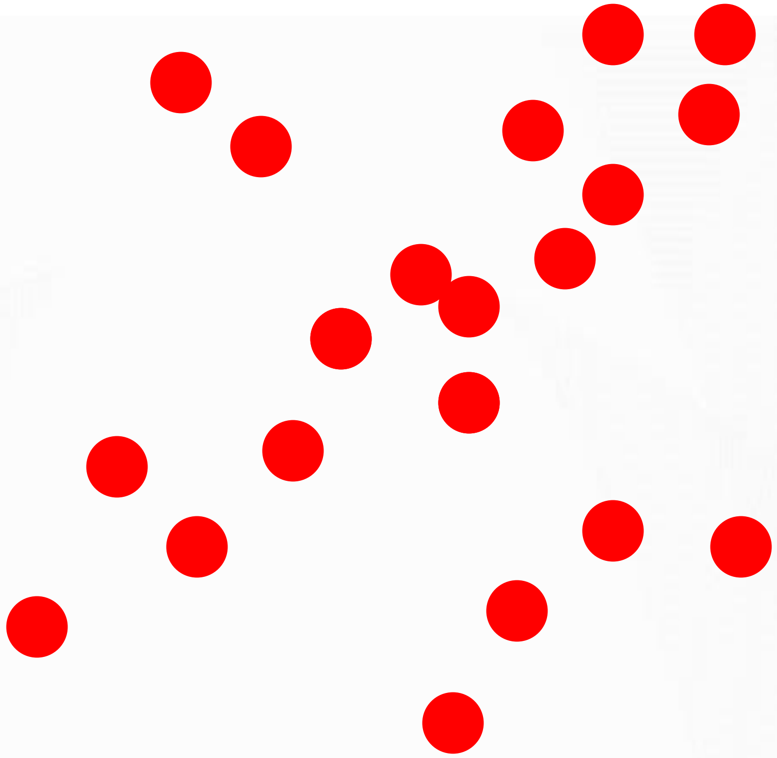
RANDOM SAMPLE CONSENSUS (RANSAC)

Fitting lines
(with outliers)



Algorithm:

1. Sample (randomly) the number of points required to fit the model
2. Solve for model parameters using samples
3. Score by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

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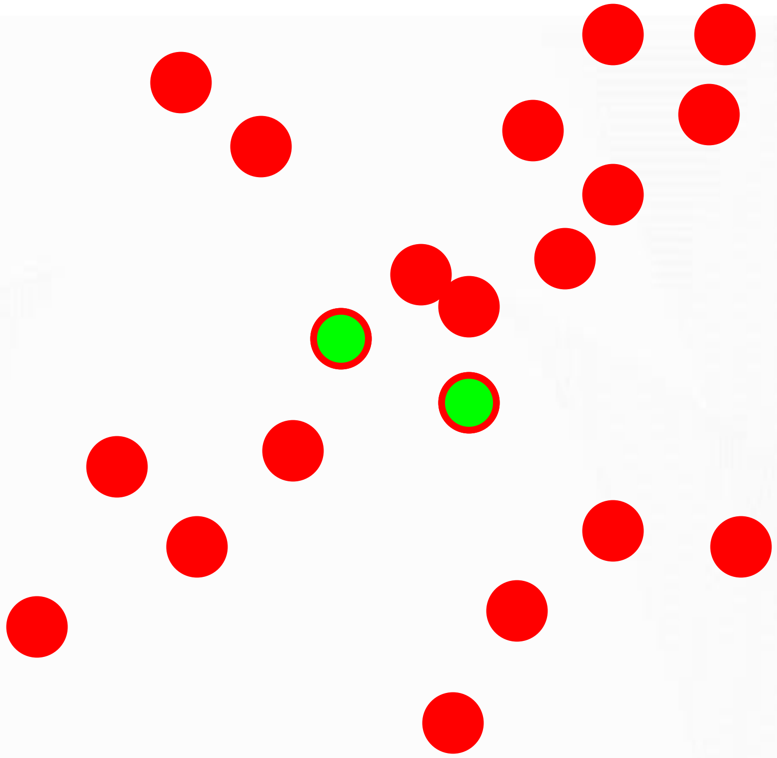
Fitting lines
(with outliers)



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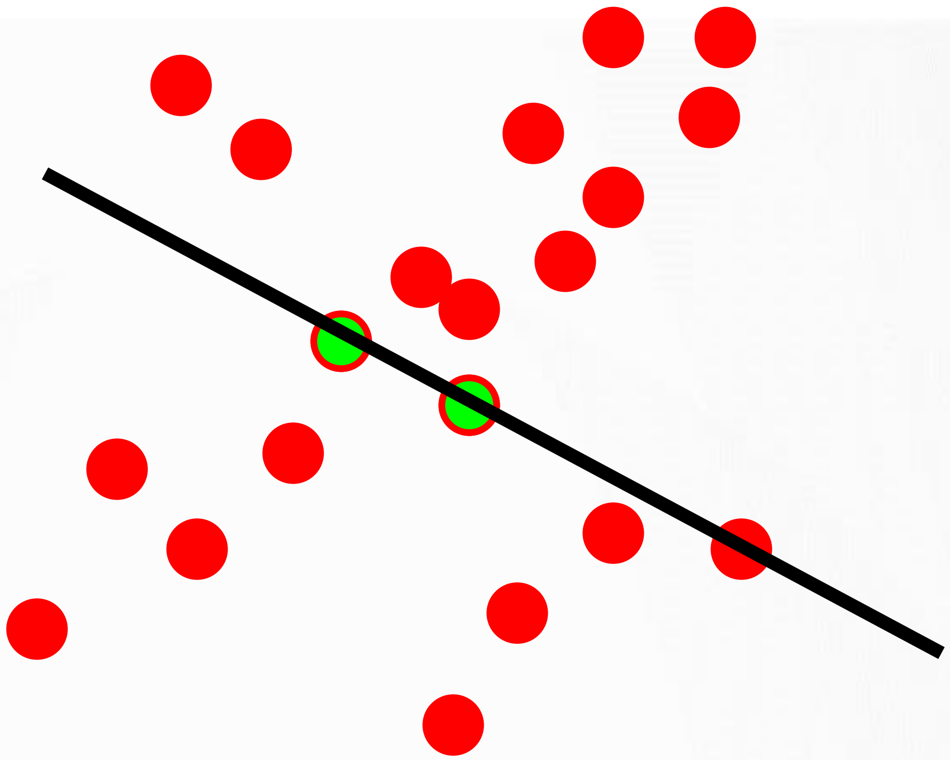
Fitting lines
(with outliers)



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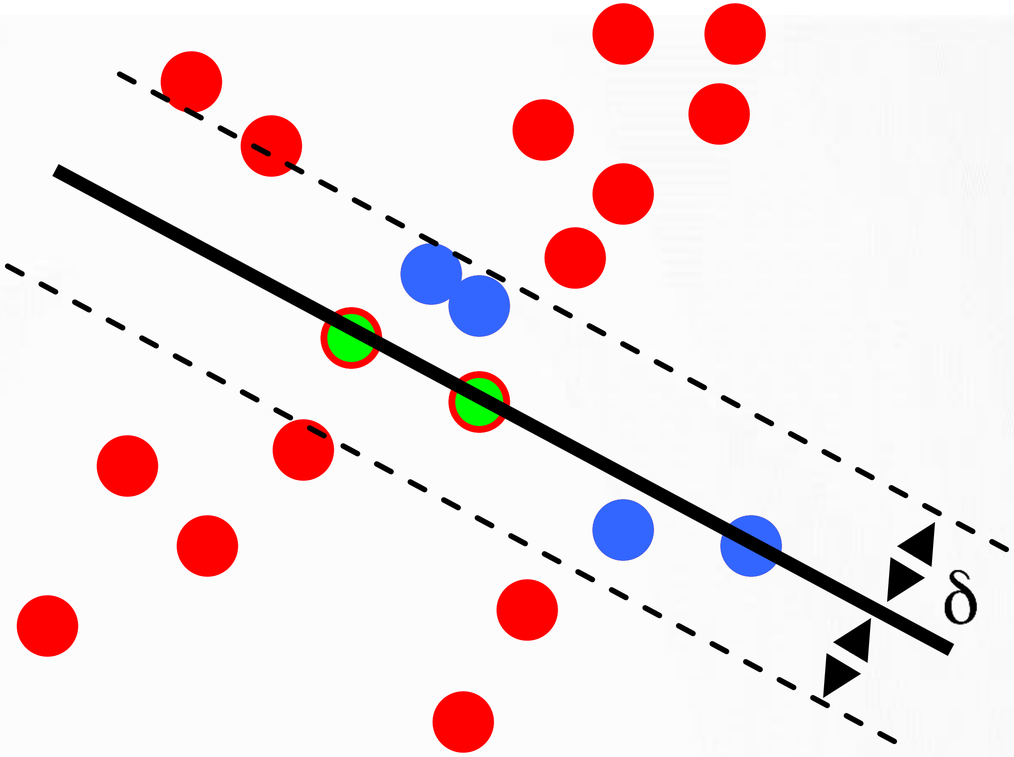


Fitting lines
(with outliers)

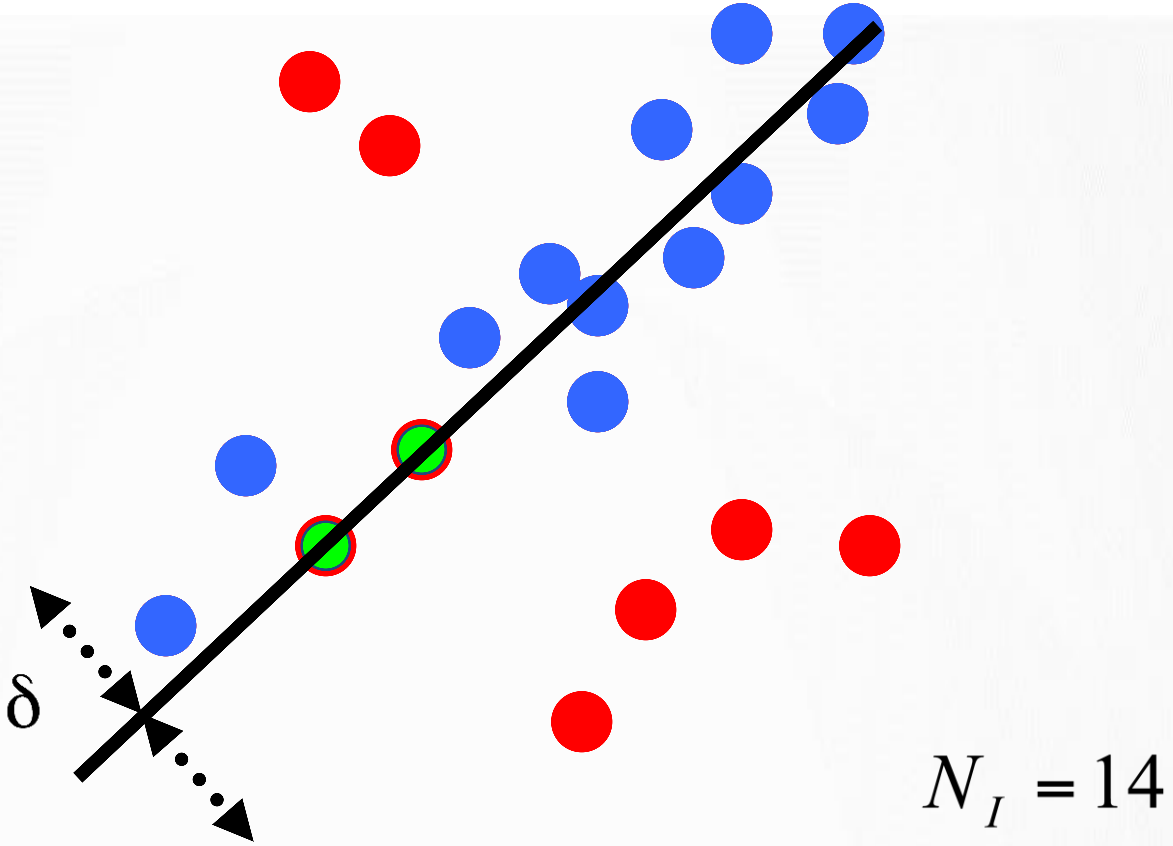
$$N_I = 6$$

Algorithm:

1. Sample (randomly) the number of points required to fit the model
2. Solve for model parameters using samples
3. **Score by the fraction of inliers within a preset threshold of the model**

Repeat 1-3 until the best model is found with high confidence

Fitting lines
(with outliers)



Algorithm:

1. Sample (randomly) the number of points required to fit the model
2. Solve for model parameters using samples
3. Score by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

HOW TO CHOOSE PARAMETERS?

- Number of samples N
  - Choose N so that, with probability p, at least one random sample is free from outliers (e.g. p=0.99) (outlier ratio: e)
- Number of sampled points s
  - Minimum number needed to fit the model
- Distance threshold δ
  - Choose δ so that a good point with noise is likely (e.g., prob=0.95) within threshold
  - Zero-mean Gaussian noise with std. dev. σ: $\delta^2 = 3.84\sigma^2$

$$N = \frac{\log(1-p)}{\log\left(1-(1-e)^s\right)}$$

	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

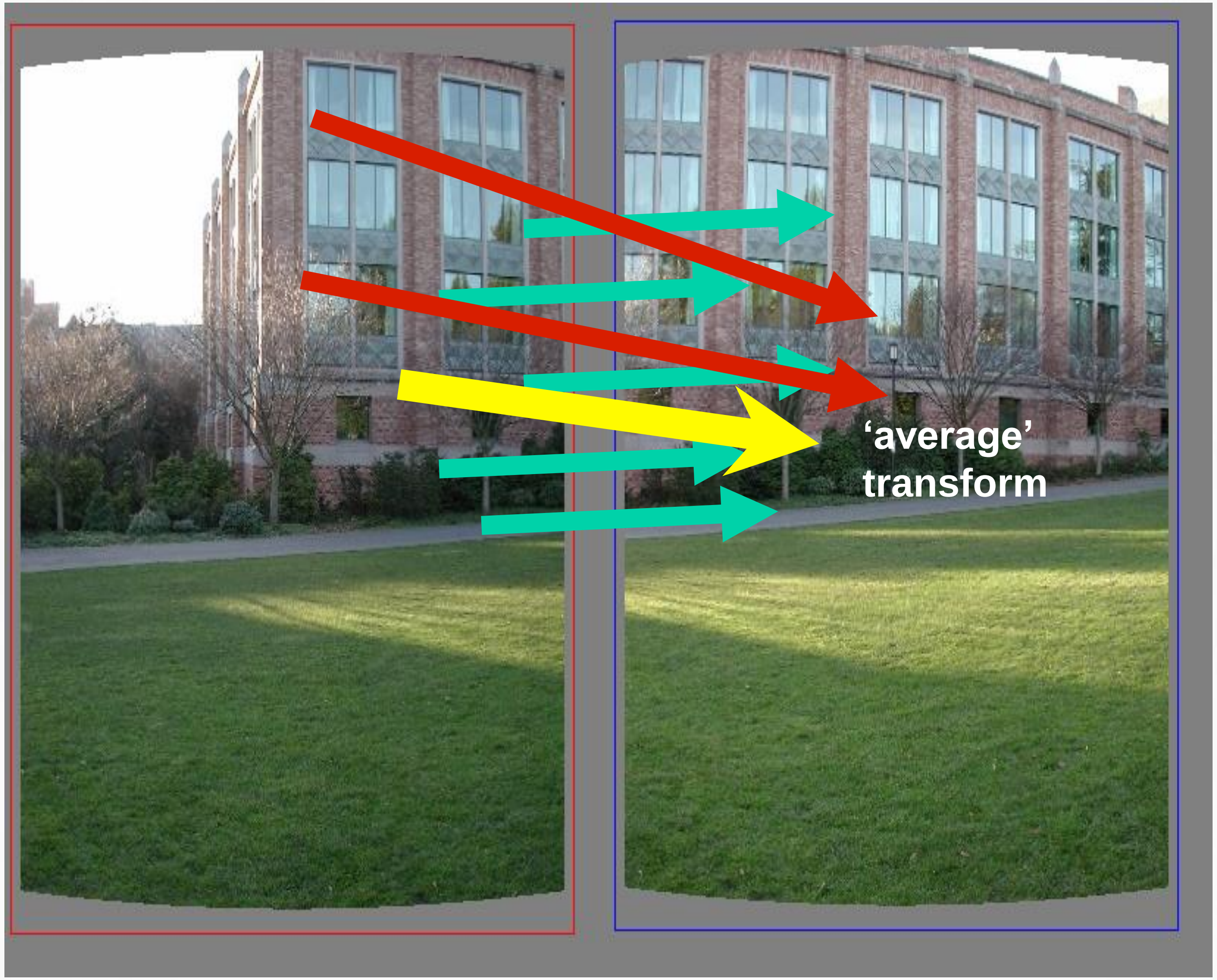
Given two images…

find matching features (e.g., SIFT) and a translation transform

Matched points will usually contain bad correspondences

how should we estimate the transform?

LLS will find the 'average' transform



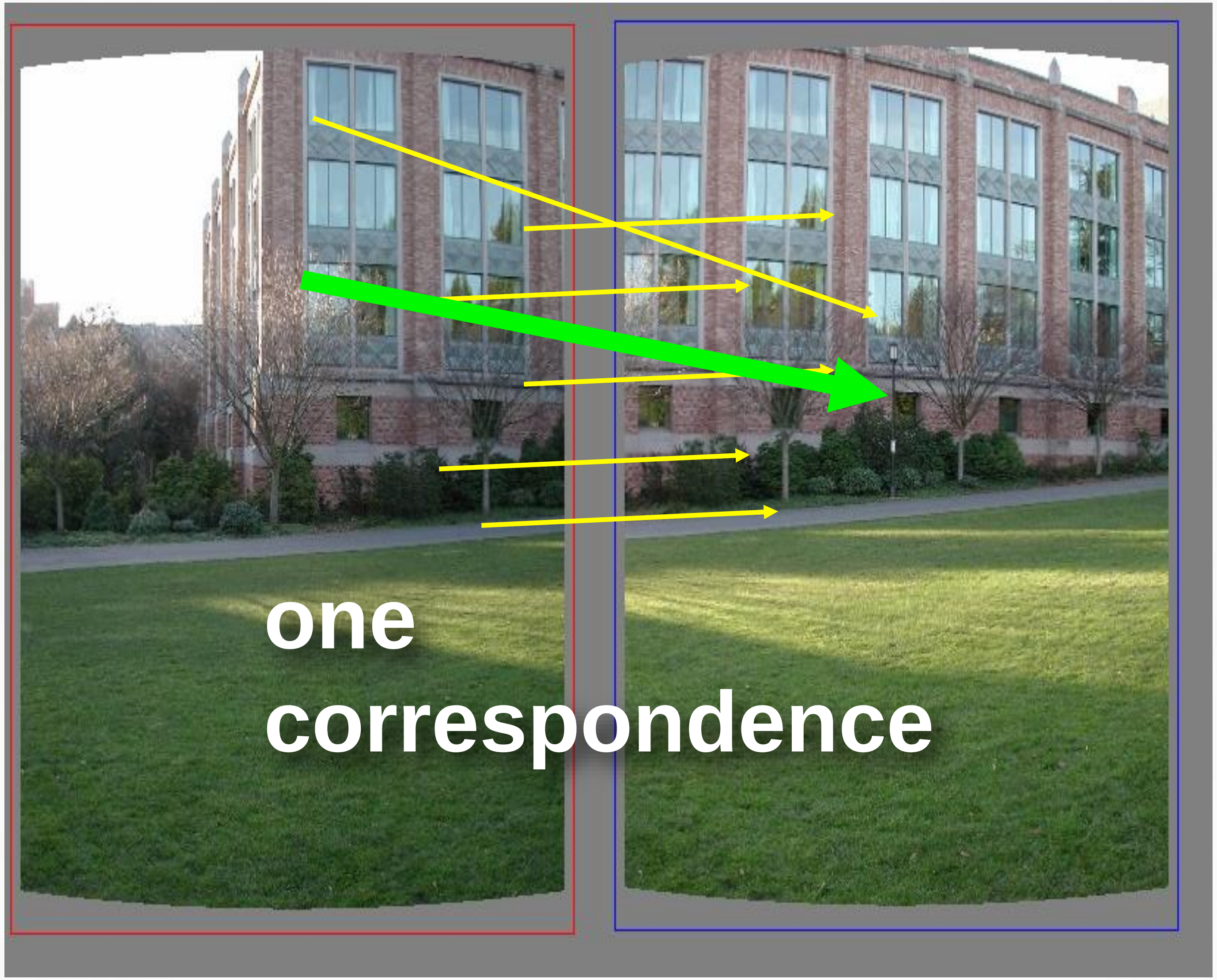
solution is corrupted by bad correspondences

Use RANSAC

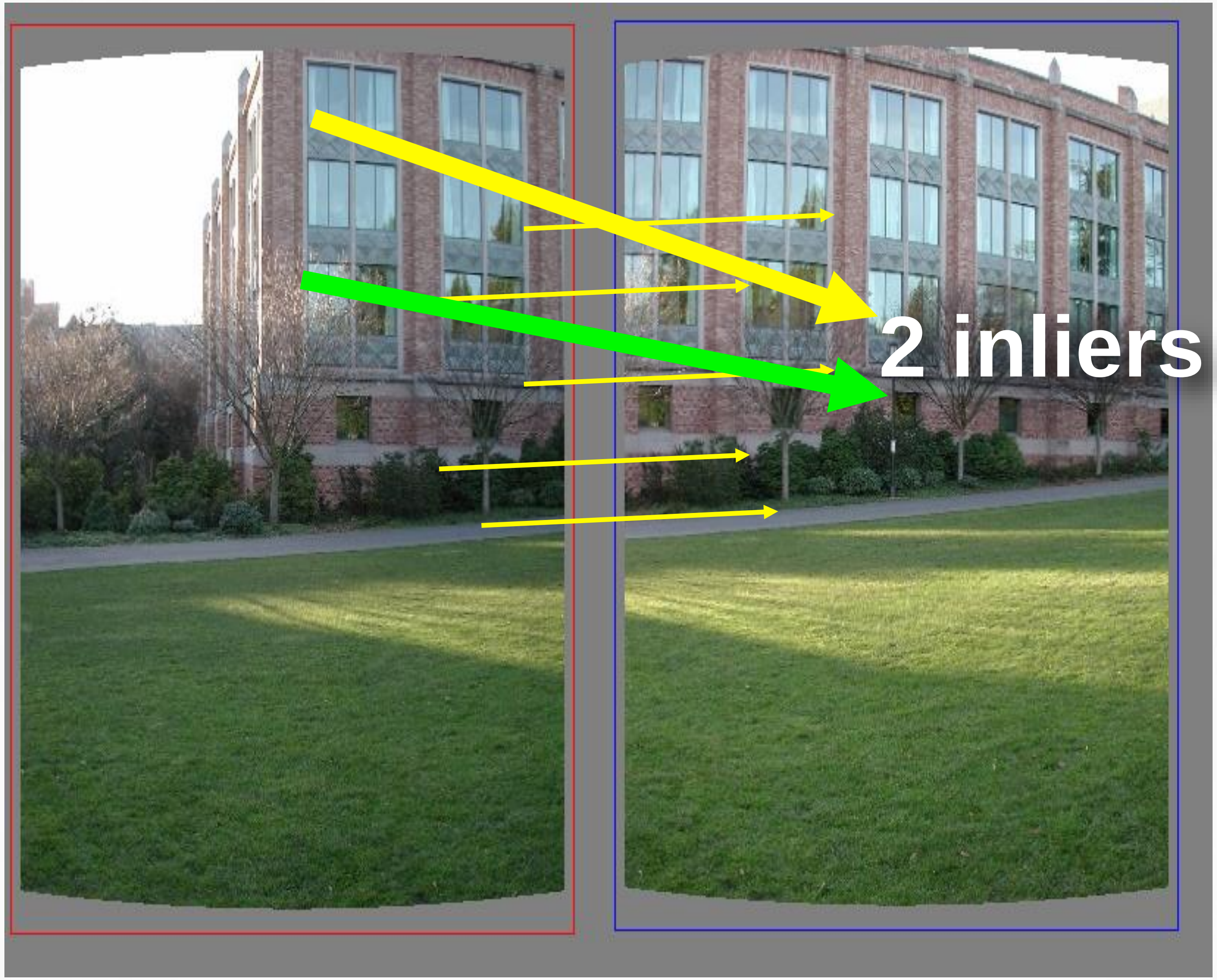
How many correspondences to compute translation transform?

Need only **one correspondence**, to find translation model

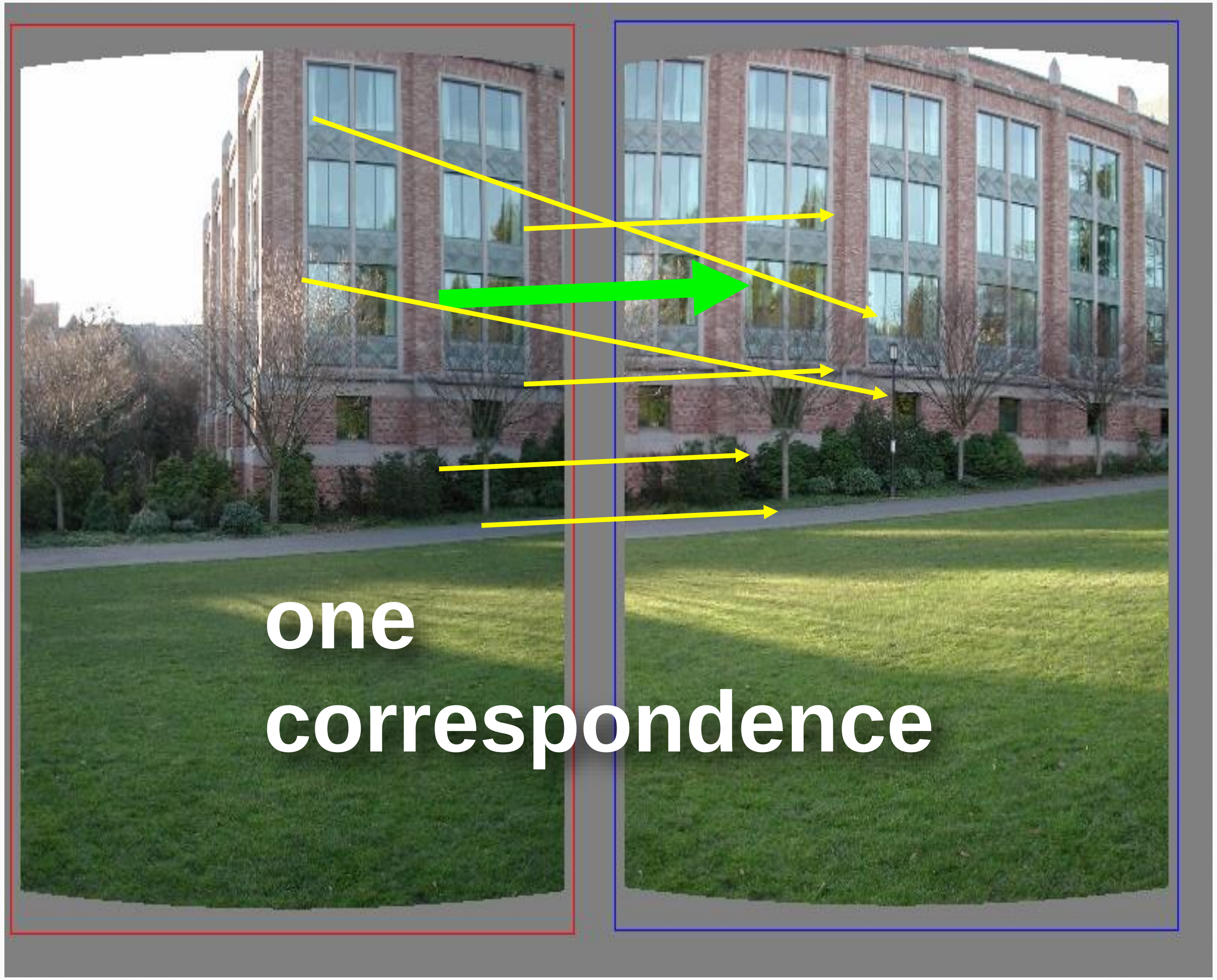
Pick one correspondence, count inliers



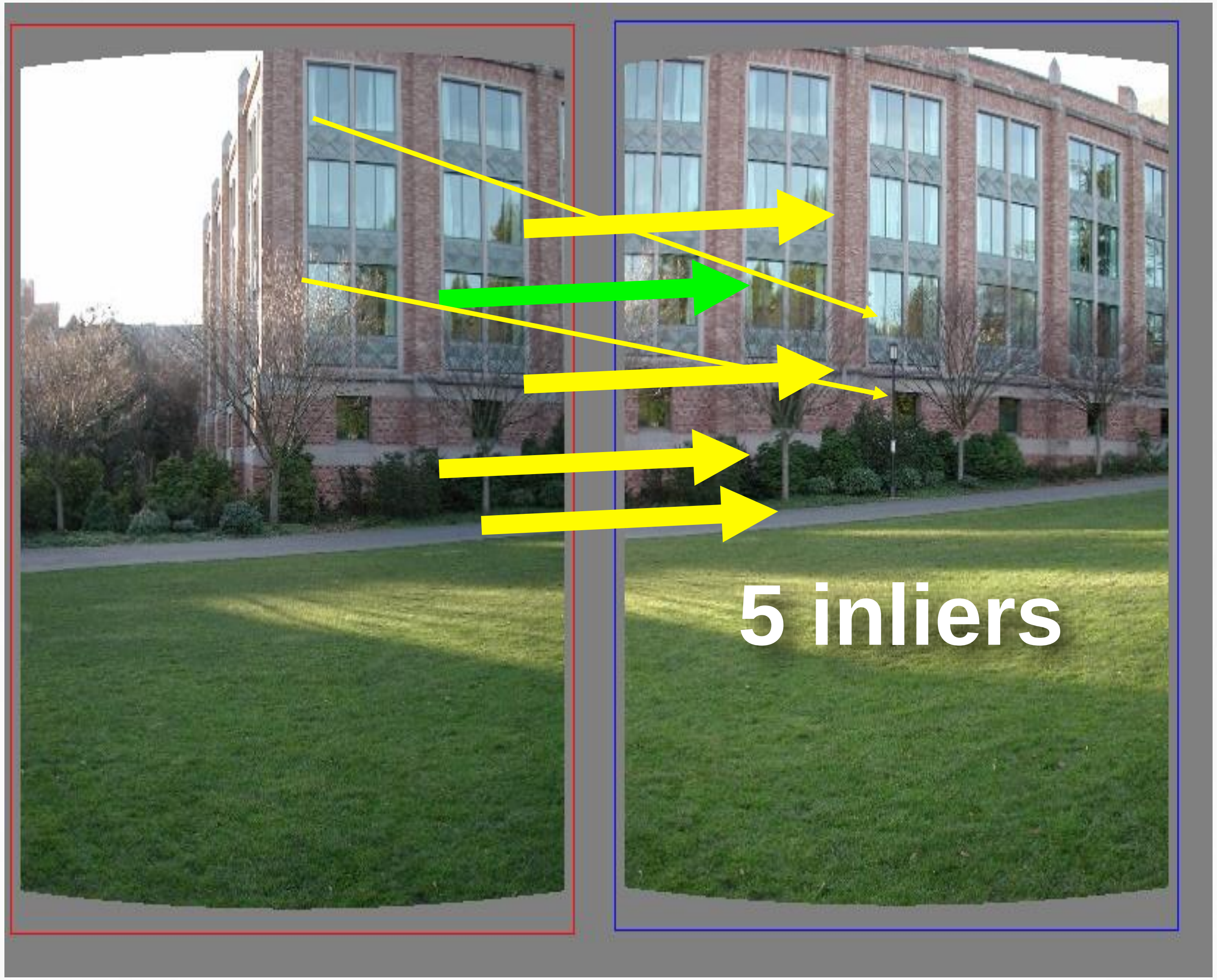
Pick one correspondence, count inliers



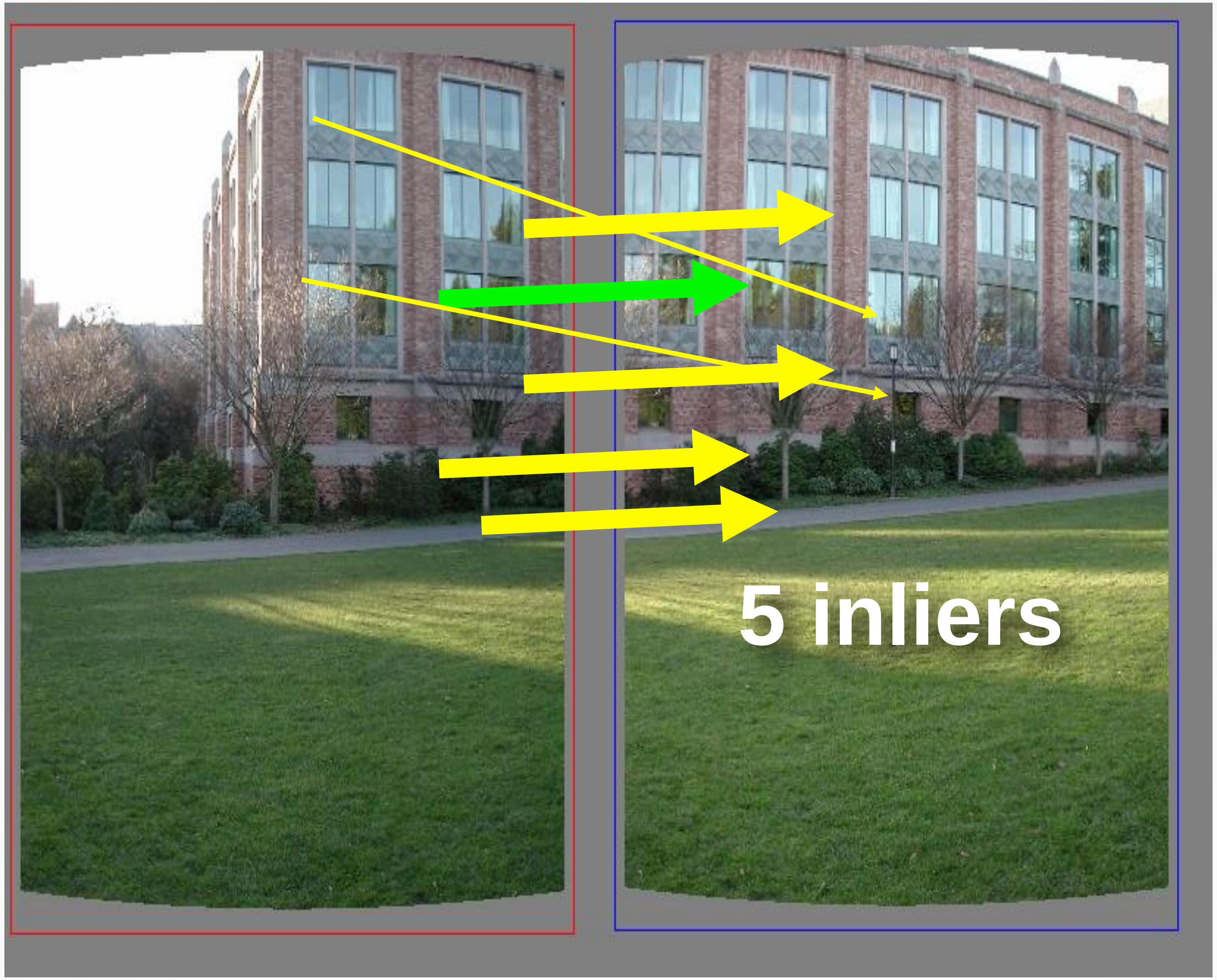
Pick one correspondence, count inliers



Pick one correspondence, count inliers



Pick one correspondence, count inliers



Pick the model with the highest number of inliers!

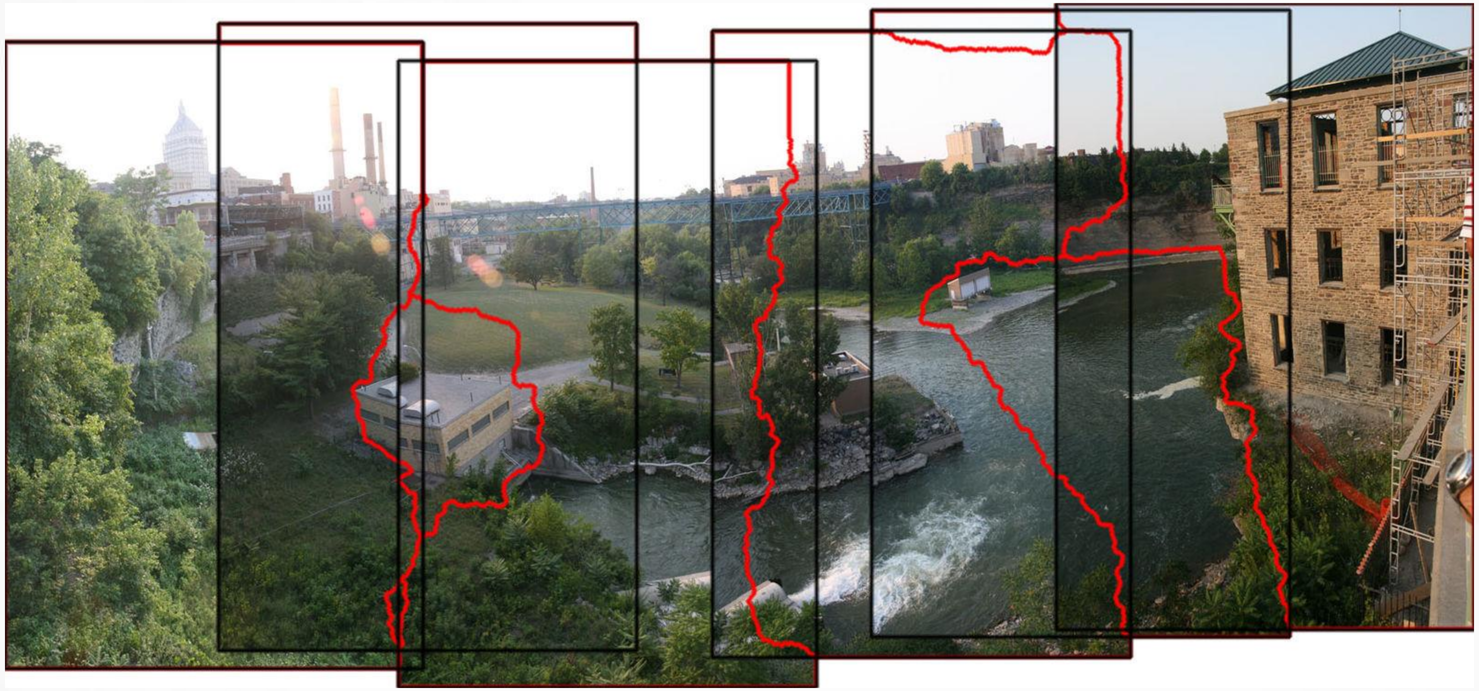
ESTIMATING HOMOGRAPHY USING RANSAC

RANSAC loop

1. Get four point correspondences (randomly)
2. Compute H using DLT
3. Count inliers
4. Keep H if largest number of inliers

Recompute H using all inliers

Useful for…



EXPECTATION-MAXIMIZATION (EM)

- ”The Expectation-Maximization algorithm is a general technique for finding maximum likelihood solutions for probabilistic models having latent variables” (Dempster et al., 1977; McLachlan and Krishnan, 1997)
- Used in computer vision for e.g. object detection, we’ll look at this later

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Slide credits Kris Kitani