

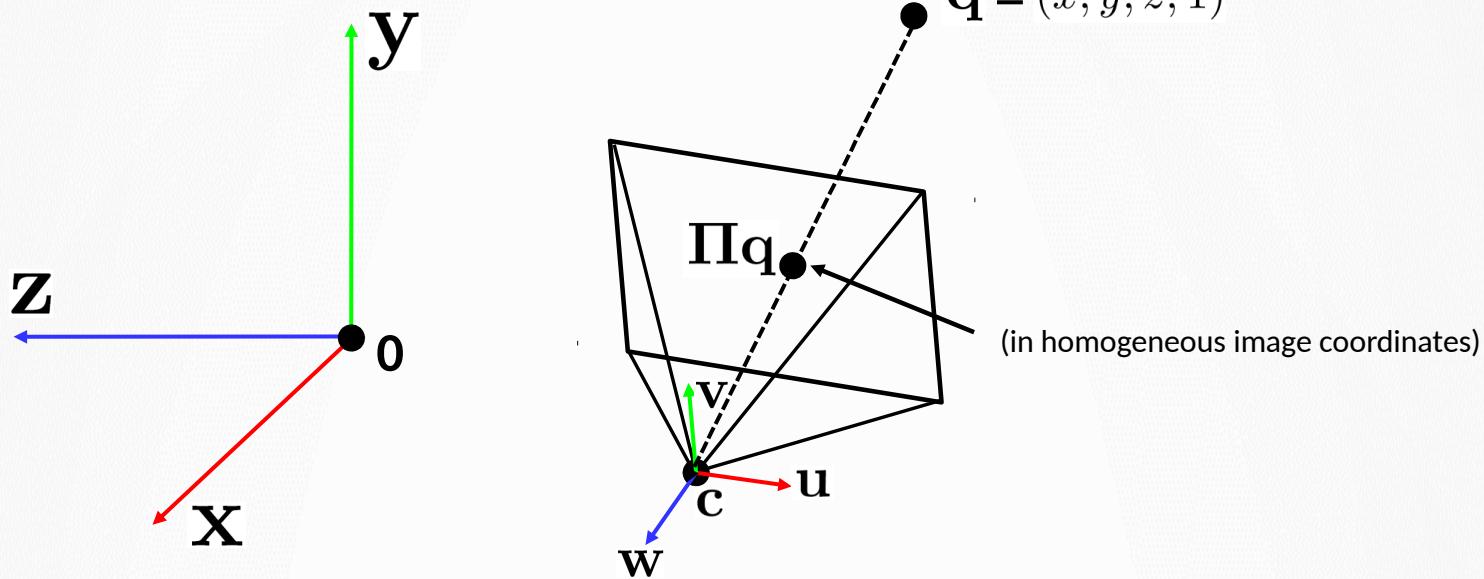
COMPUTER VISION

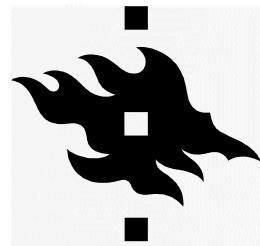
LECTURE 14 18.10.2019

Laura Ruotsalainen, Associate Professor
Department of Computer Science



Camera projection matrix: recap





Projection matrix

$$\Pi = \mathbf{K}_{\text{intrinsics}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{R} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3 \times 3} & -\mathbf{c} \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{translation}}$$

The \mathbf{K} matrix converts 3D rays in the camera's coordinate system to 2D image points in image (pixel) coordinates.

This part converts 3D points in world coordinates to 3D rays in the camera's coordinate system. There are 6 parameters represented (3 for position/translation, 3 for rotation).



Projection matrix



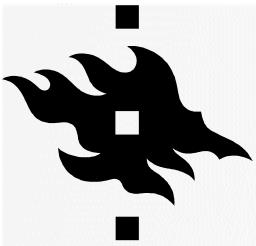
$$\Pi = \mathbf{K} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{intrinsics}} \underbrace{\begin{bmatrix} \mathbf{R} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3 \times 3} & -\mathbf{c} \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{translation}}$$

$$\left[\begin{array}{c|c} \mathbf{R} & -\mathbf{R}\mathbf{c} \end{array} \right]$$



(\mathbf{t} in book's notation)

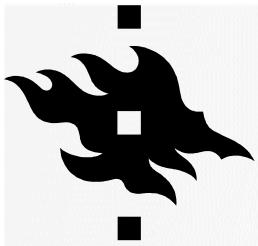
$$\Pi = \mathbf{K} \left[\begin{array}{c|c} \mathbf{R} & -\mathbf{R}\mathbf{c} \end{array} \right]$$



Typical intrinsics matrix

$$\mathbf{K} = \begin{bmatrix} -f & 0 & x_c \\ 0 & -f & y_c \\ 0 & 0 & 1 \end{bmatrix}$$

- **2D affine transform** corresponding to a scale by f (focal length) and a translation by (x_c, y_c) (principal point)



Perspective distortion



- Problem for architectural photography:
converging verticals

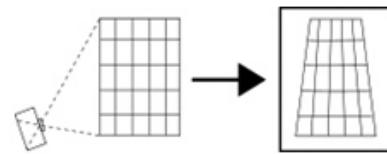


Source: F. Durand

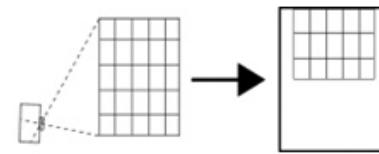


Perspective distortion

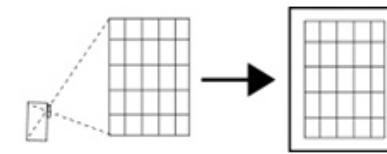
- Problem for architectural photography: converging verticals



Tilting the camera upwards results in converging verticals

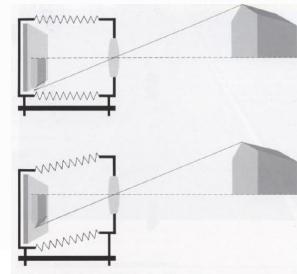


Keeping the camera level, with an ordinary lens, captures only the bottom portion of the building

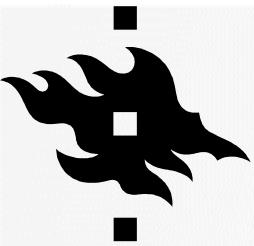


Shifting the lens upwards results in a picture of the entire subject

- Solution: view camera (lens shifted w.r.t

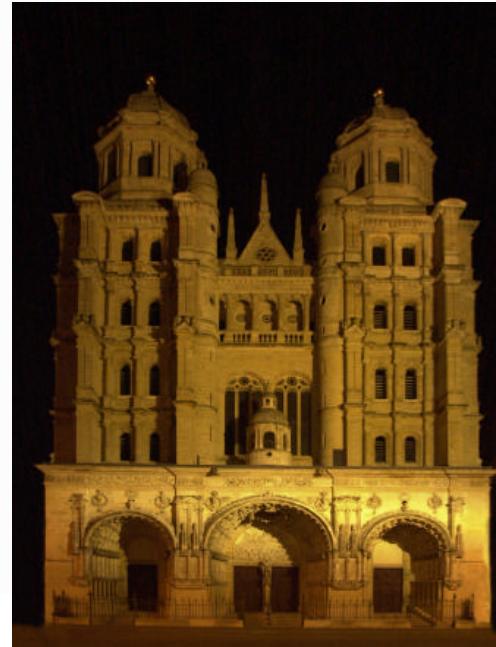


(Corresponds
to shifting the
principal point)

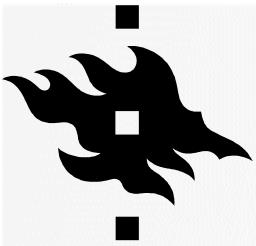


Perspective distortion

- Problem for architectural photography:
converging verticals
- Result:

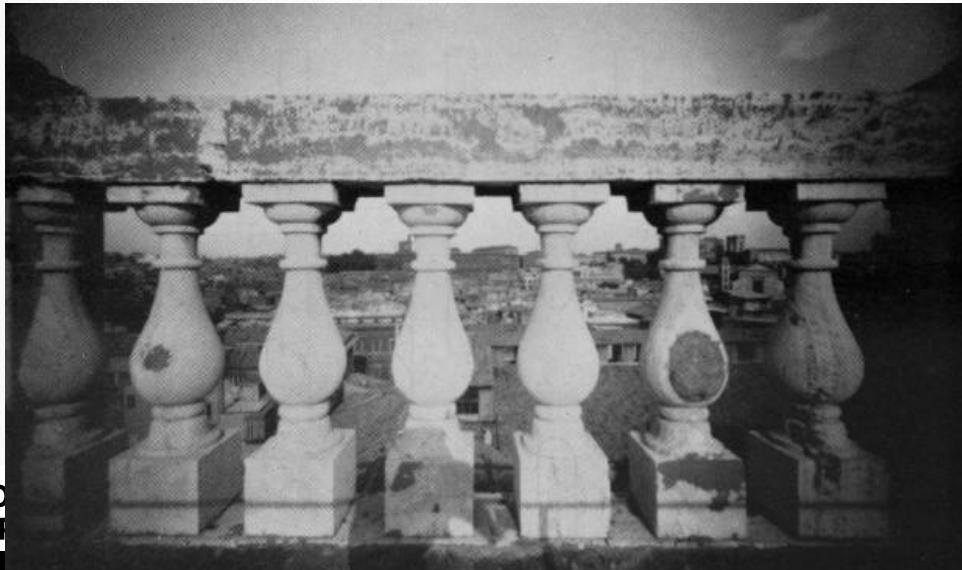


Source: F. Durand

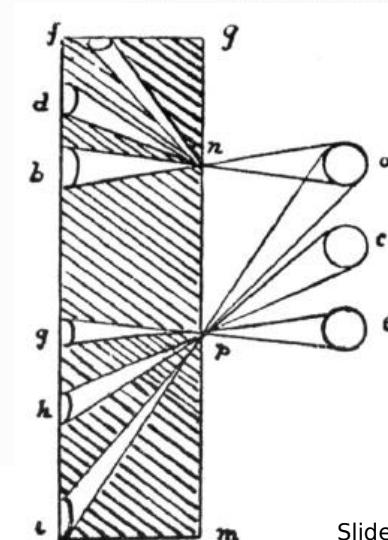


Perspective distortion

- The exterior columns appear bigger
- The distortion is not due to lens flaws
- Problem pointed out by Da Vinci



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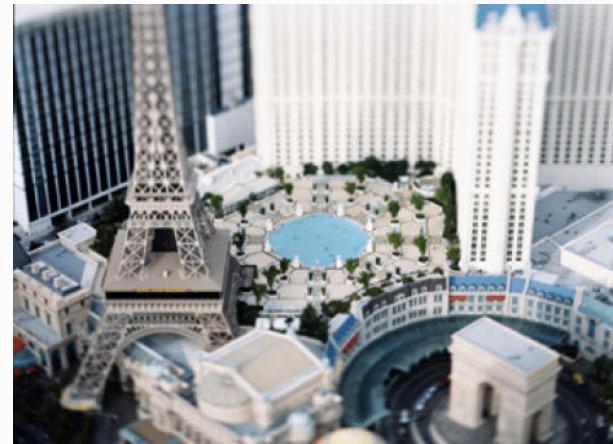
Slide by F. Durand



Tilt-shift



http://www.northlight-images.co.uk/article_pages/tilt_and_shift_ts-e.html



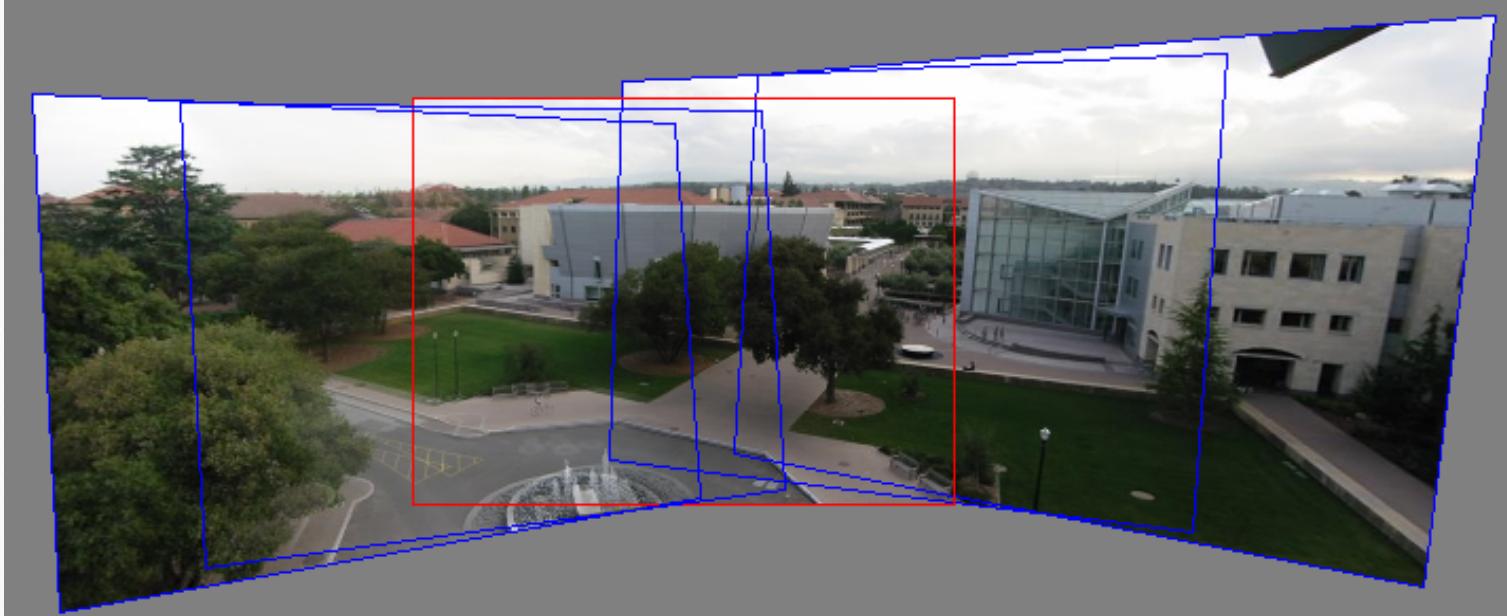
Tilt-shift images from [Olivo Barbieri](#) and Photoshop [imitations](#)

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slide credit Noah Snavely

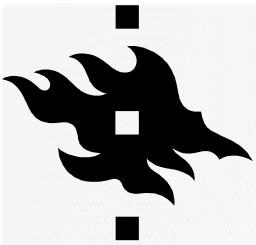


Panoramas

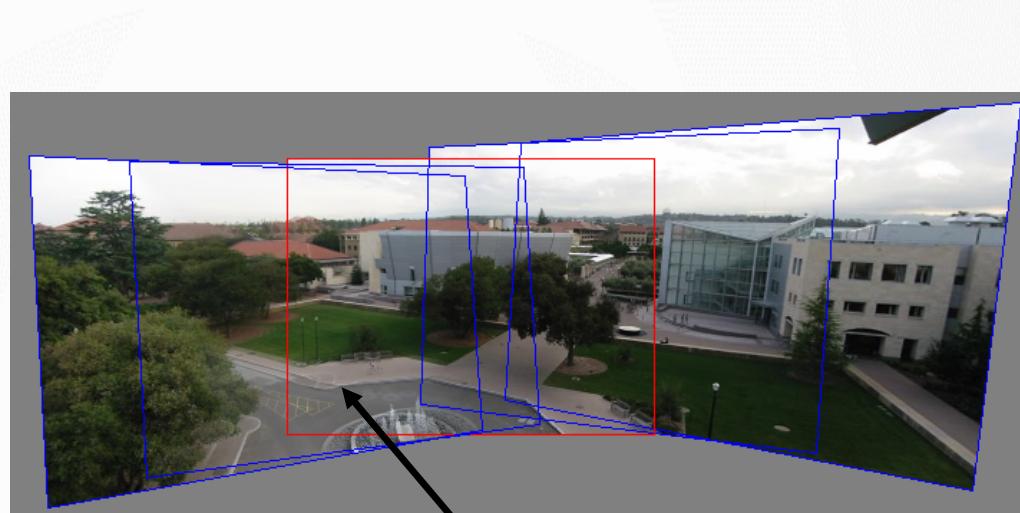
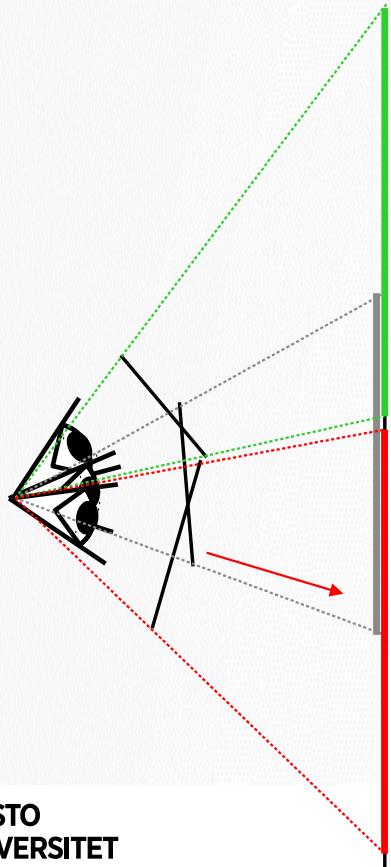


Can we use homographies to create a 360 panorama?

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Idea: projecting images onto a common plane



each image is warped
with a homography H

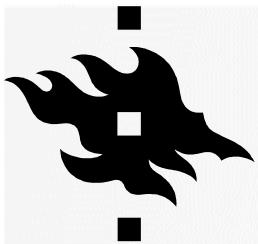
Can't create a 360 panorama this way... we'll fix
this shortly

mosaic projection plane

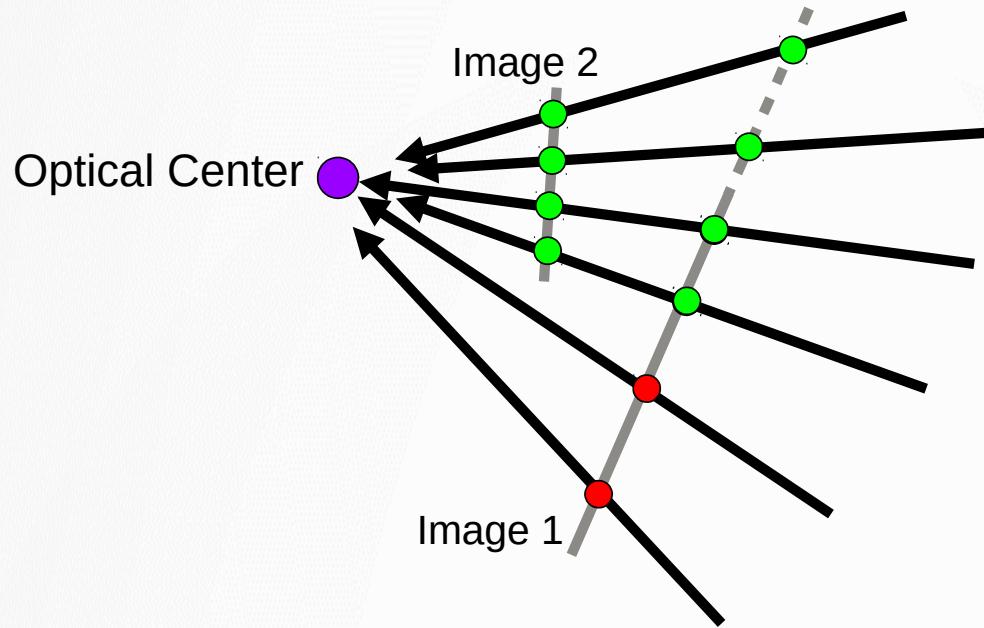


Creating a panorama

- Basic Procedure
 - Take a sequence of images from the same position
 - Rotate the camera about its optical center
 - Compute transformation between second image and first
 - Transform the second image to overlap with the first
 - Blend the two together to create a mosaic
 - If there are more images, repeat



Geometric Interpretation of Mosaics



- If we capture all 360° of rays, we can create a 360° panorama
- The basic operation is *projecting* an image from one plane to another
- The projective transformation is **scene-INDEPENDENT**

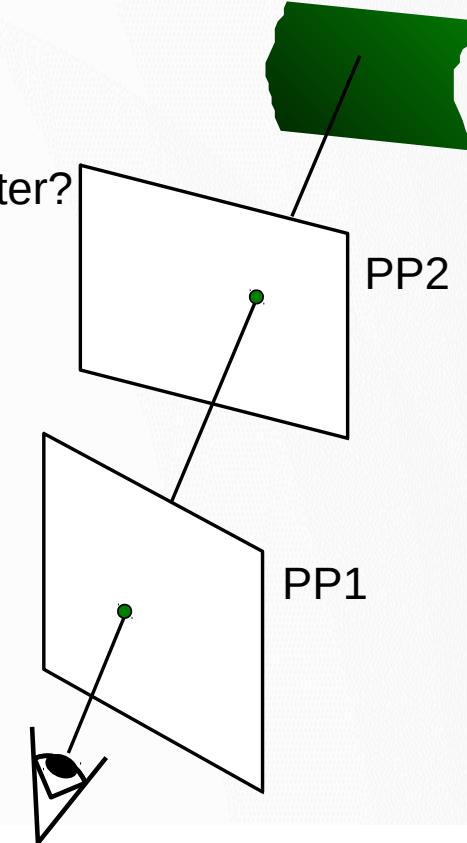


Image reprojection

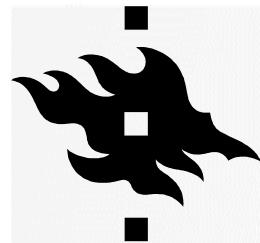
- Basic question
 - How to relate two images from the same camera center?
 - how to map a pixel from PP1 to PP2

Answer

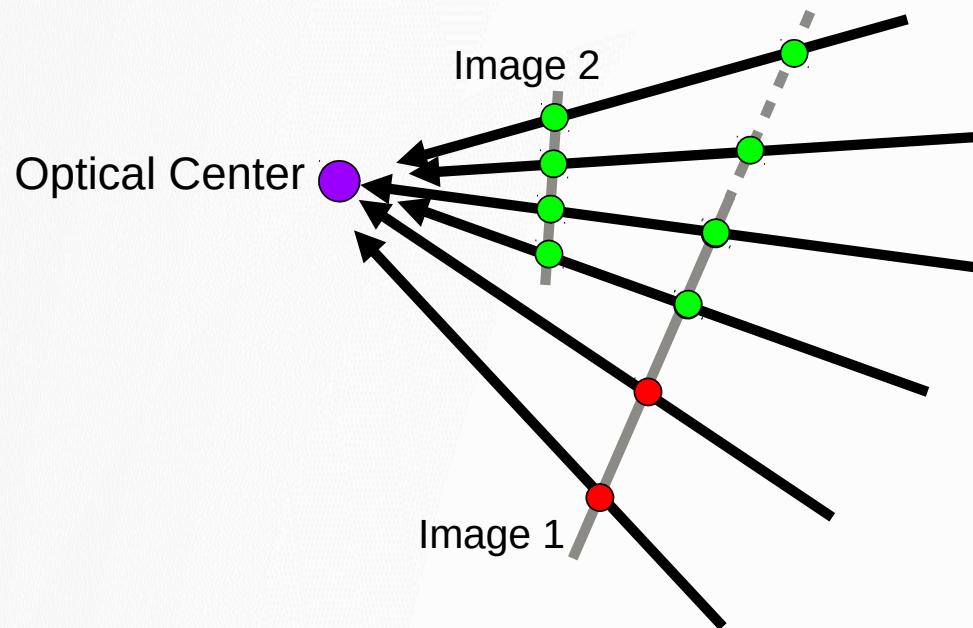
- Cast a ray through each pixel in PP1
- Draw the pixel where that ray intersects PP2



slide credit Noah Snavely



What is the transformation?



How do we map points in image 2 into image 1?

$$\text{intrinsics} \quad \text{image 1} \quad \mathbf{K}_1$$

$$\text{extrinsics} \quad \mathbf{R}_1 = \mathbf{I}_{3 \times 3}$$

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$$\text{image 2} \quad \mathbf{K}_2$$

$$\mathbf{R}_2$$

Step 1: Convert pixels in image 2 to rays in camera 2's coordinate system.

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \mathbf{K}_2^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

Step 2: Convert rays in camera 2's coordinates to rays in camera 1's coordinates.

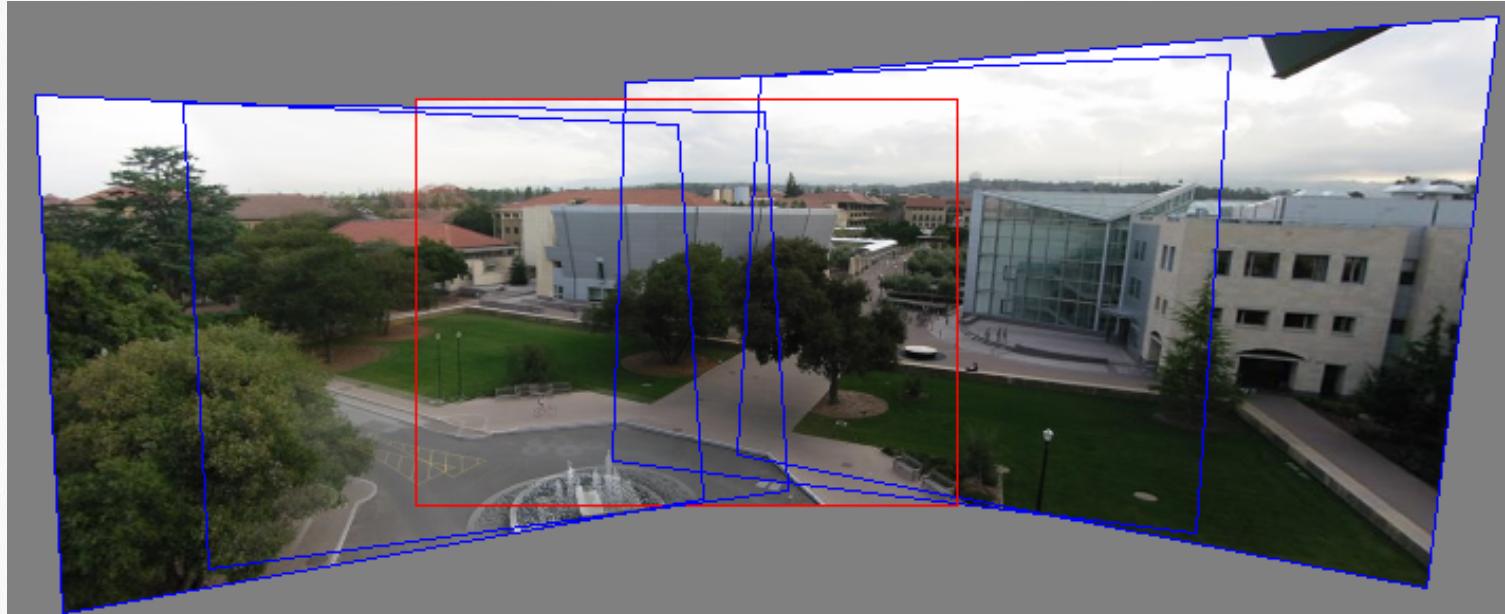
$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \mathbf{R}_2^T \mathbf{K}_2^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

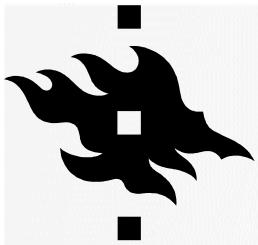
Step 3: Convert rays in camera 1's coordinates to pixels in image 1's coordinates.

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \sim \underbrace{\mathbf{K}_1 \mathbf{R}_2^T \mathbf{K}_2^{-1}}_{\text{3x3 homography}} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$



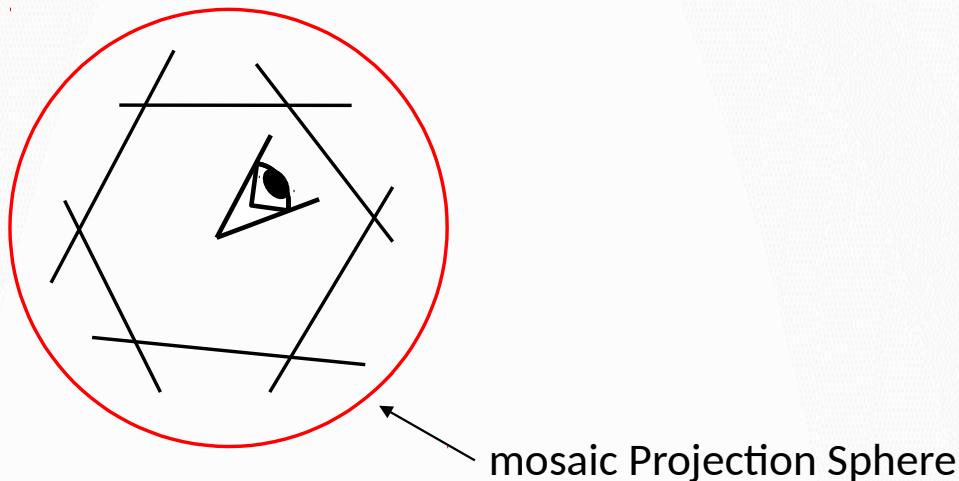
Can we use homography to create a 360 panorama?

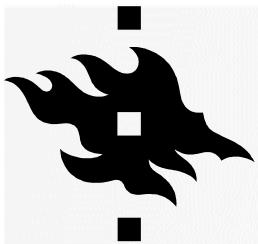




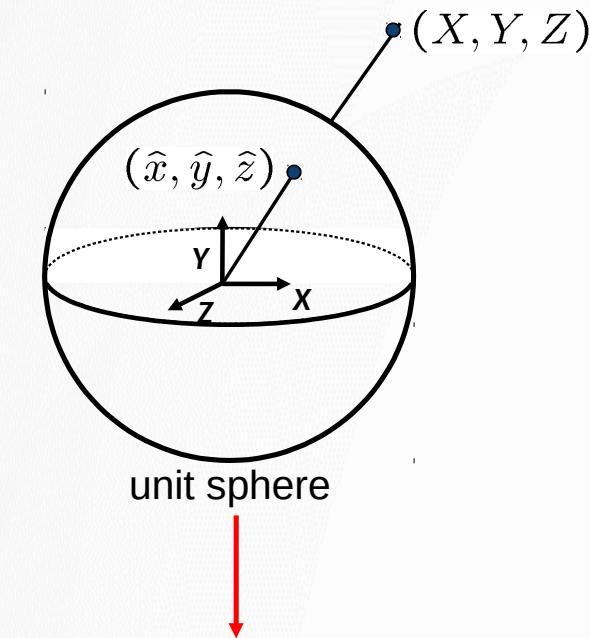
Panoramas

- What if you want a 360° field of view?





Spherical projection



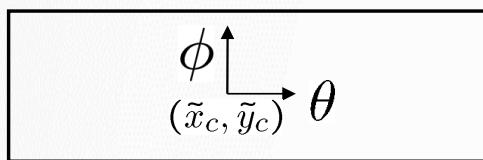
- Map 3D point (X, Y, Z) onto sphere

$$(\hat{x}, \hat{y}, \hat{z}) = \frac{1}{\sqrt{X^2+Y^2+Z^2}}(X, Y, Z)$$

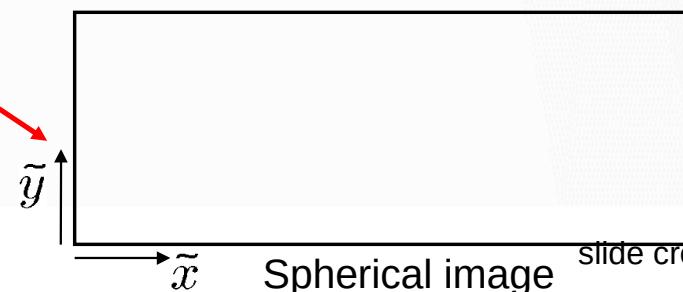
- Convert to spherical coordinates
 $(\sin\theta\cos\phi, \sin\phi, \cos\theta\cos\phi) = (\hat{x}, \hat{y}, \hat{z})$
- Convert to spherical image coordinates

$$(\tilde{x}, \tilde{y}) = (s\theta, s\phi) + (\tilde{x}_c, \tilde{y}_c)$$

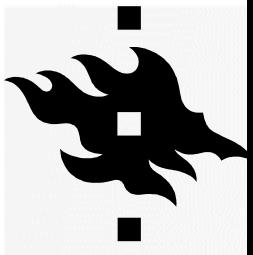
- s defines size of the final image
 - » often convenient to set $s = \text{camera focal length}$



unwrapped sphere

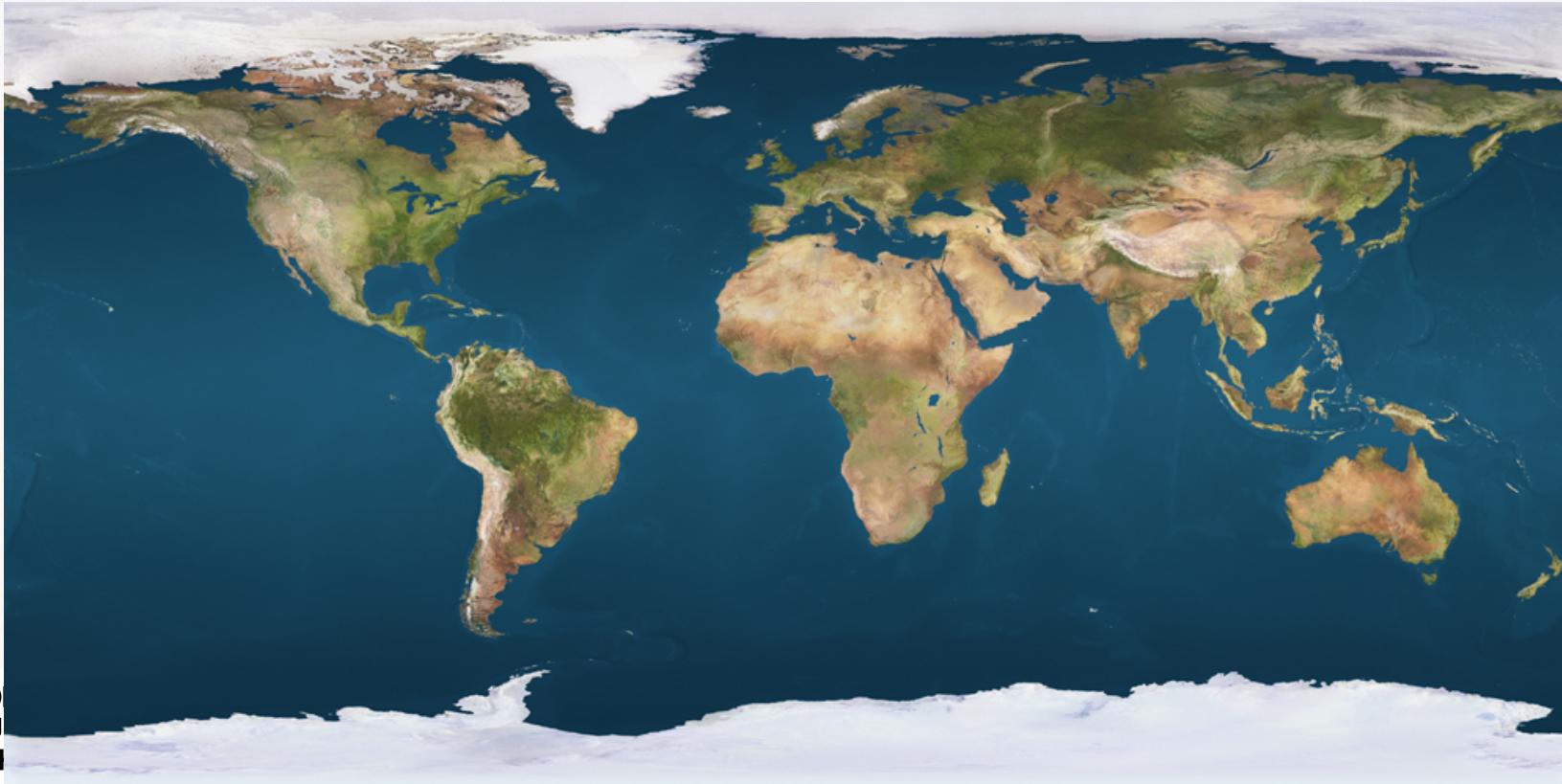


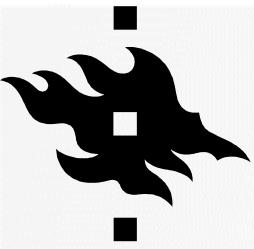
Spherical image



Unwrapping a sphere

Credit: JHT's Planetary Pixel Emporium





Spherical reprojection



input



$f = 200$ (pixels)

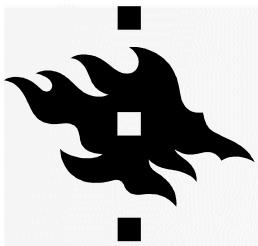


$f = 400$

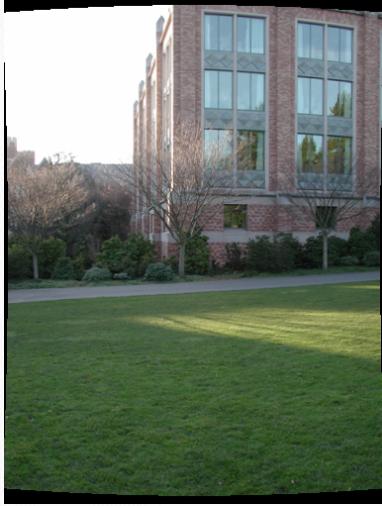


$f = 800$

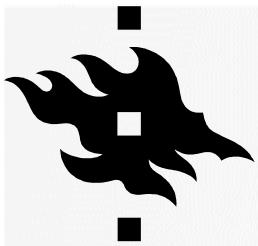
- Map image to spherical coordinates
 - need to know the focal length



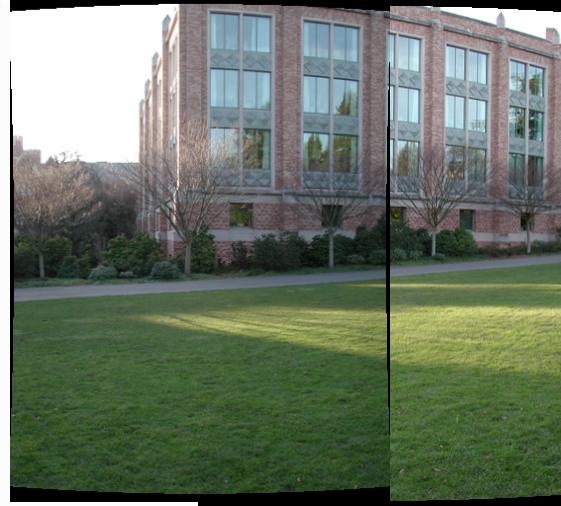
Aligning spherical images



- Suppose we rotate the camera by θ about the vertical axis
 - How does this change the spherical image?



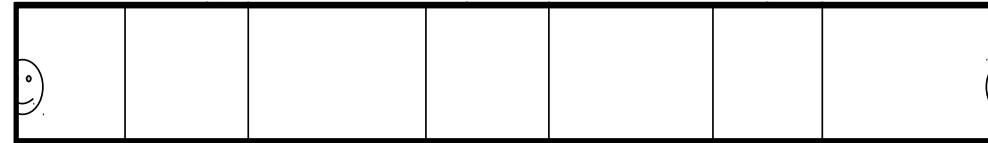
Aligning spherical images



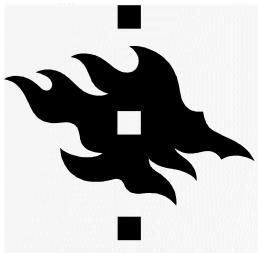
- Suppose we rotate the camera by θ about the vertical axis
 - How does this change the spherical image?
 - Translation by θ
 - This means that we can align spherical images by translation



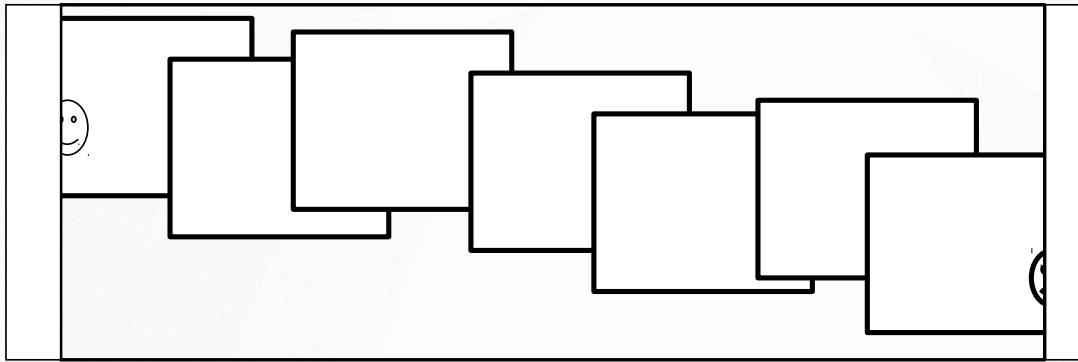
Assembling the panorama



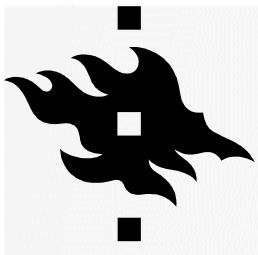
- Stitch pairs together, blend, then crop



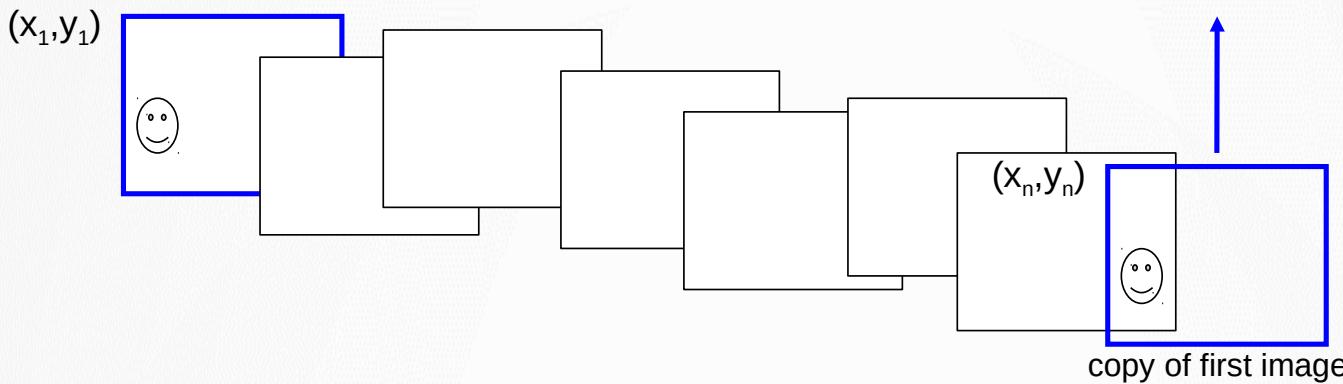
Problem: Drift



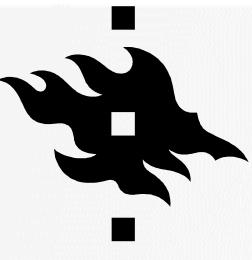
- Error accumulation
 - small errors accumulate over time



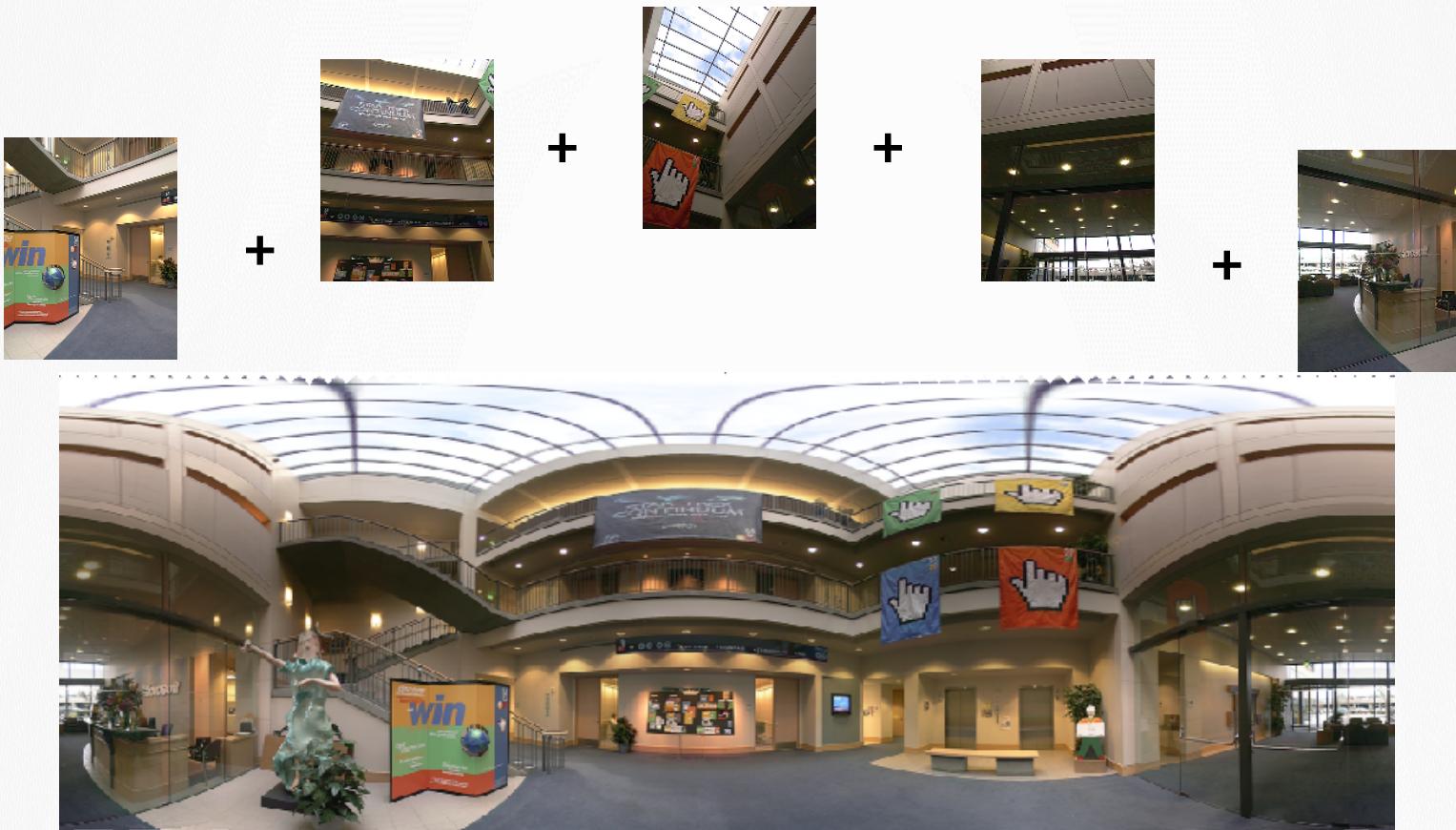
Problem: Drift



- Solution
 - add another copy of first image at the end
 - this gives a constraint: $y_n = y_1$
 - there are a bunch of ways to solve this problem
 - add displacement of $(y_1 - y_n)/(n - 1)$ to each image after the first
 - **apply an affine warp:** $y' = y + ax$
 - run a big optimization problem, incorporating this constraint
 - best solution, but more complicated



Spherical panoramas

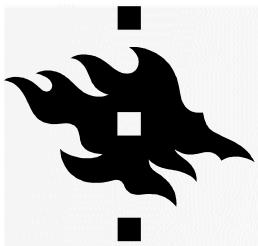




Different projections are possible



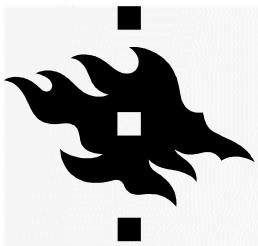
Cube-map



Blending

- We've aligned the images – now what?





Blending

- Want to seamlessly blend them together



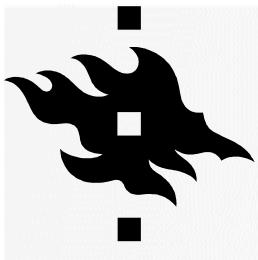
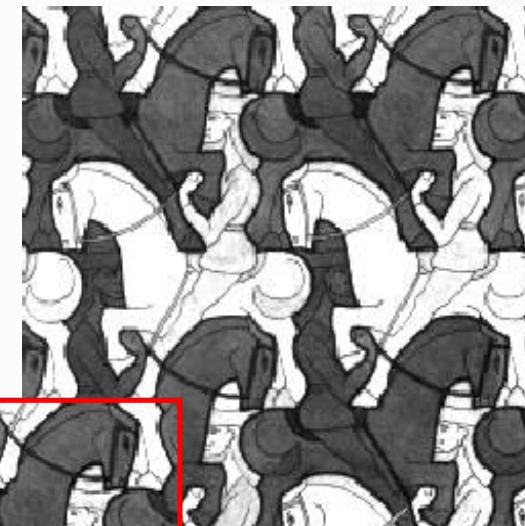
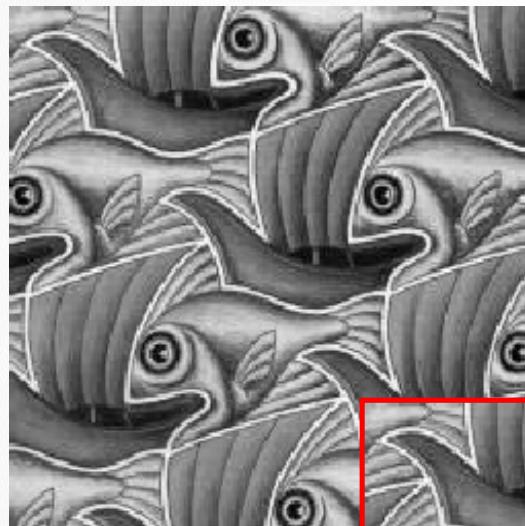
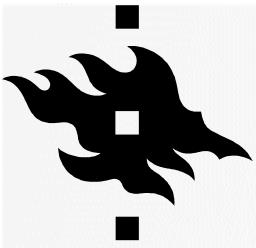


Image Blending

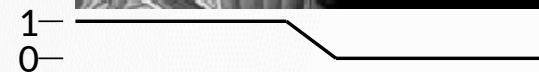
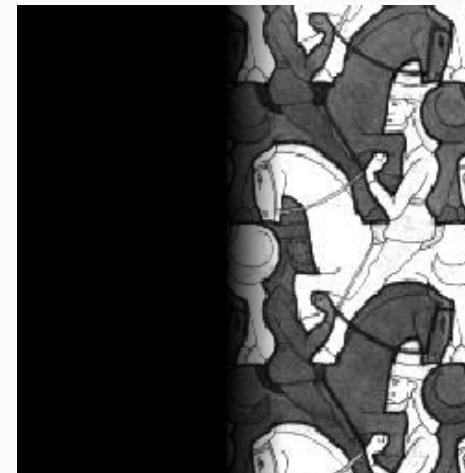




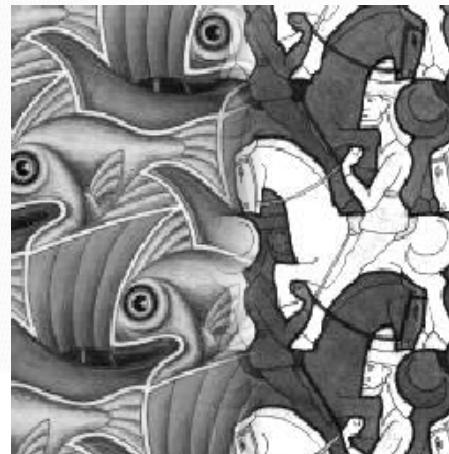
Feathering



+

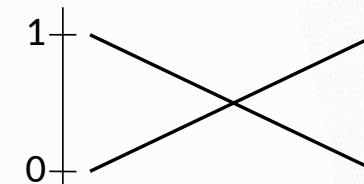
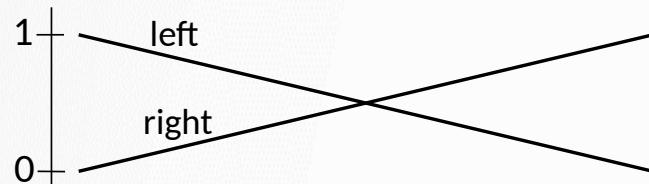


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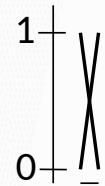


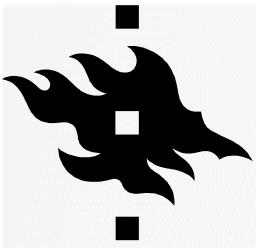
Effect of window size



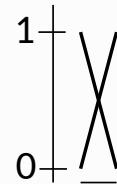


Effect of window size





Good window size



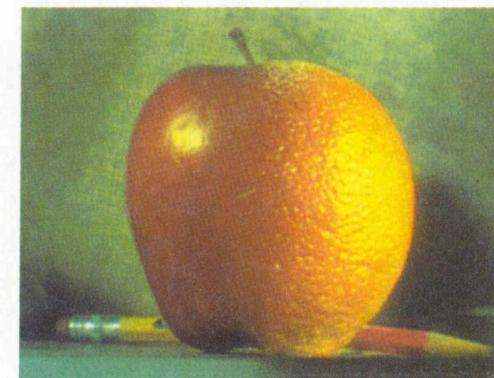
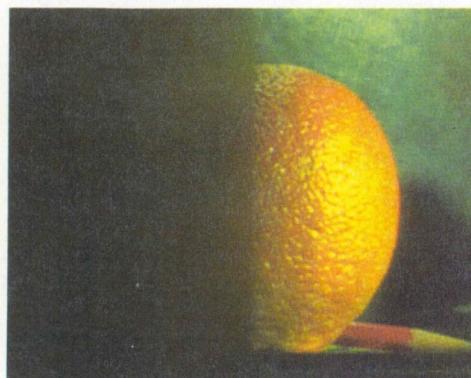
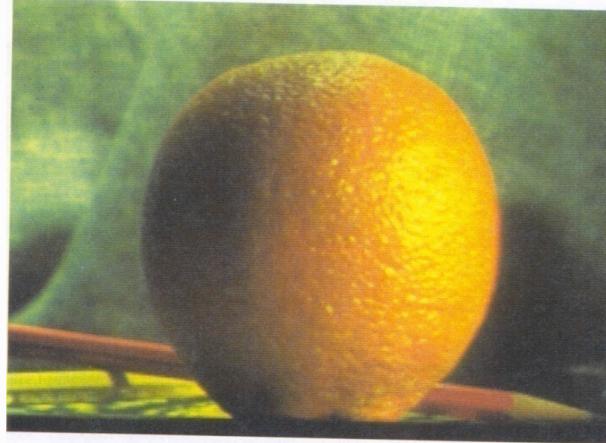
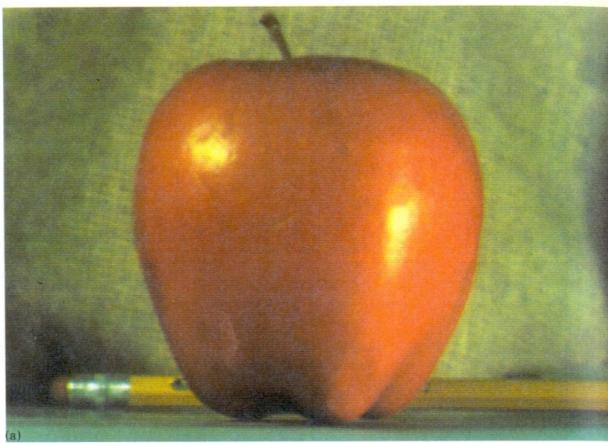
“Optimal” window: smooth but not ghosted

- Doesn't always work...

slide credit Noah Snavely



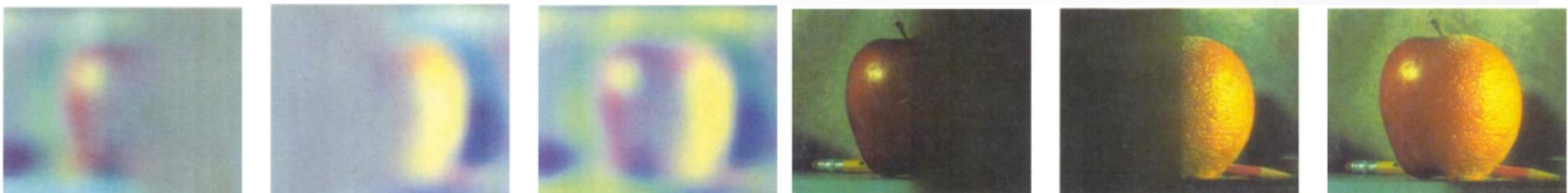
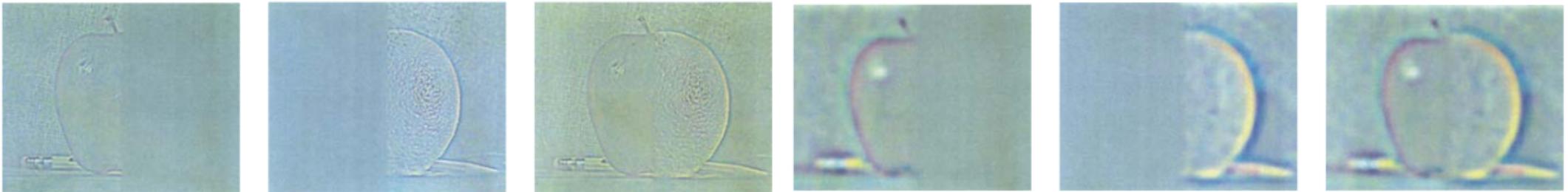
Pyramid blending



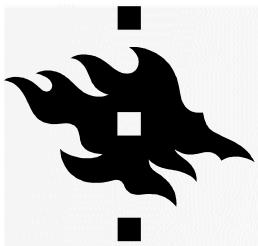
Create a Laplacian pyramid, blend each level



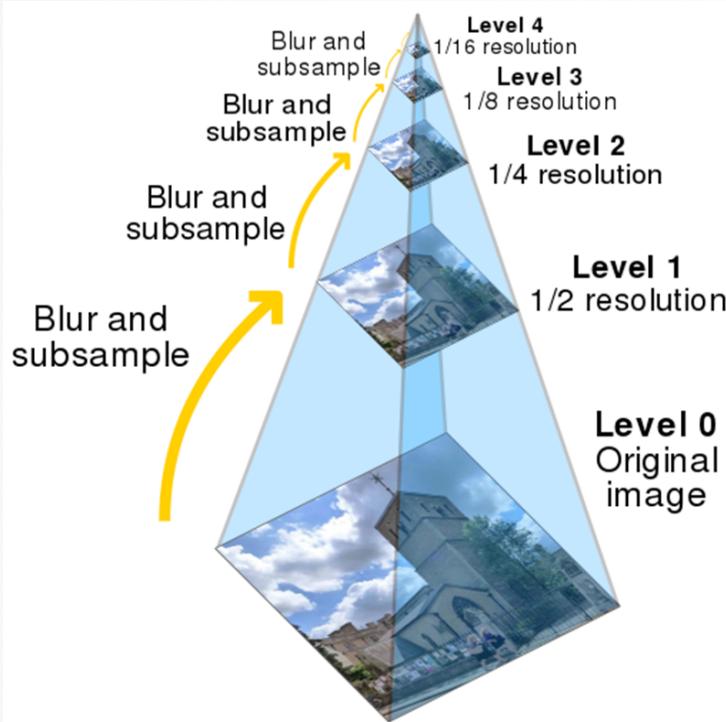
IMAGE BLENDING



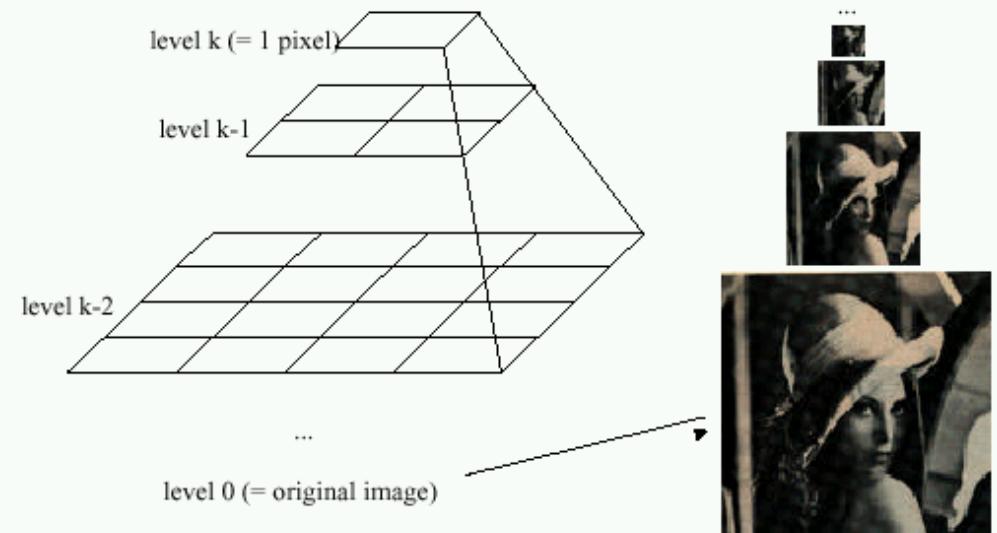
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RECAP (LECT 2) - GAUSSIAN PYRAMIDS



Idea: Represent $N \times N$ image as a “pyramid” of $1 \times 1, 2 \times 2, 4 \times 4, \dots, 2^k \times 2^k$ images (assuming $N=2^k$)

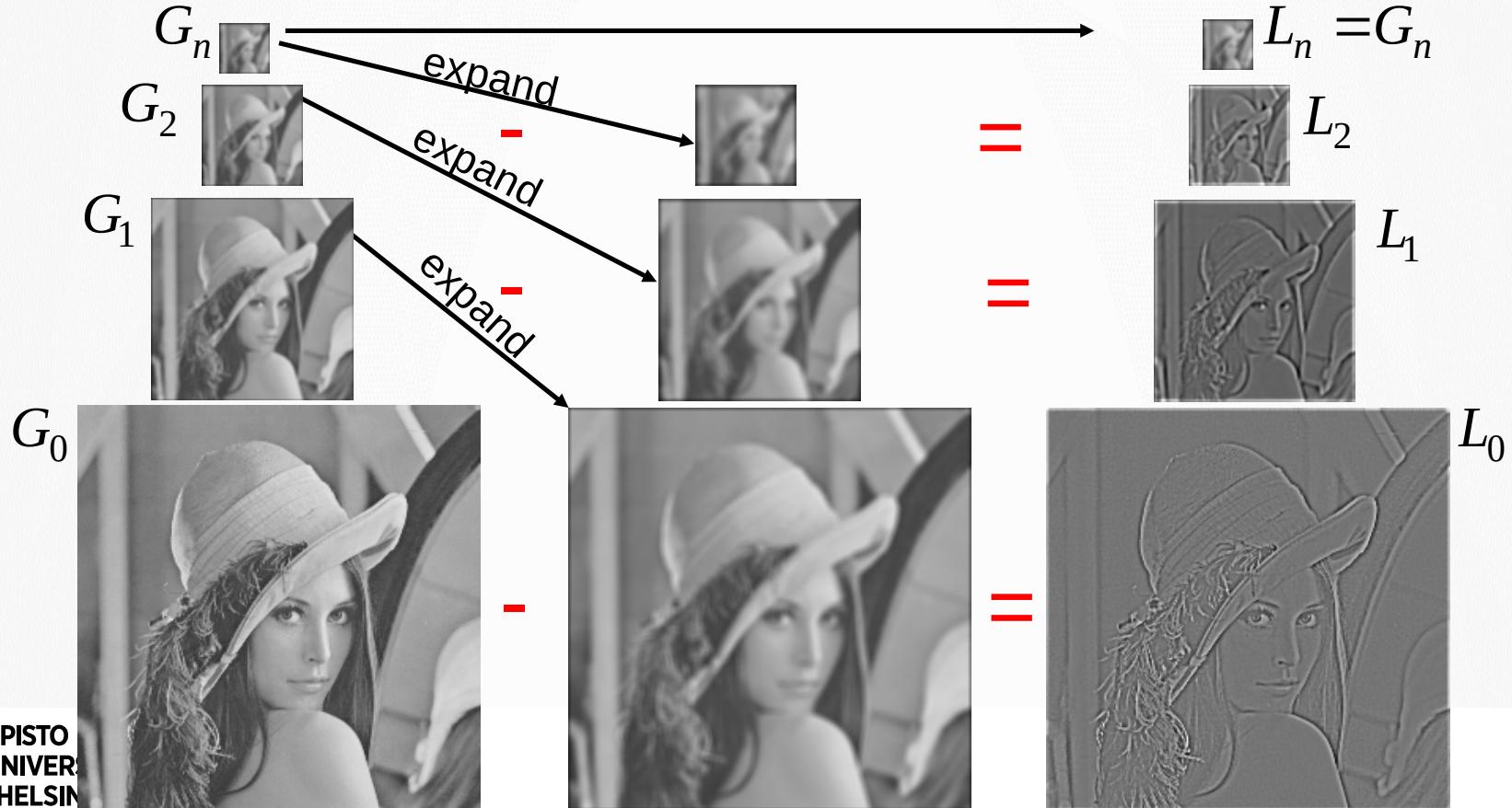


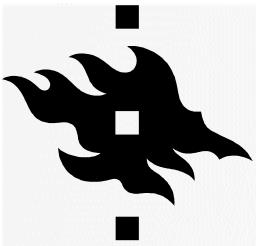


The Laplacian Pyramid

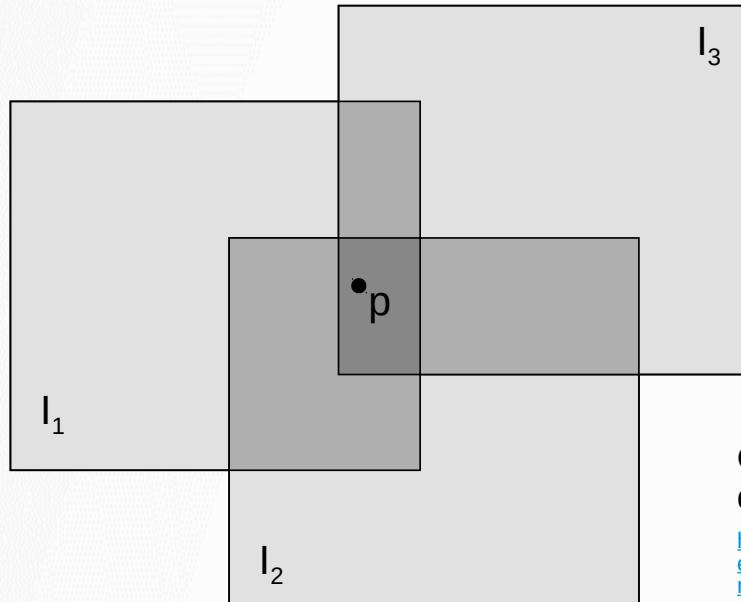
$$L_i = G_i - \text{expand}(G_{i+1})$$

Gaussian Pyramid $G_i = L_i + \text{expand}(G_{i+1})$ Laplacian Pyramid





Alpha Blending



Optional: see Blinn (CGA, 1994) for details:

<http://ieeexplore.ieee.org/iel1/38/7531/00310740.pdf?isNumber=7531&prod=JNL&arnumber=310740&arSt=83&ared=87&arAuthor=Blinn%2C+J.F>

Encoding blend weights: $I(x,y) = (\alpha R, \alpha G, \alpha B, \alpha)$

$$\text{color at } p = \frac{(\alpha_1 R_1, \alpha_1 G_1, \alpha_1 B_1) + (\alpha_2 R_2, \alpha_2 G_2, \alpha_2 B_2) + (\alpha_3 R_3, \alpha_3 G_3, \alpha_3 B_3)}{\alpha_1 + \alpha_2 + \alpha_3}$$

Implement this in two steps:

1. accumulate: add up the (α premultiplied) RGB α values at each pixel
2. normalize: divide each pixel's accumulated RGB by its α value

Q: what if $\alpha = 0$?

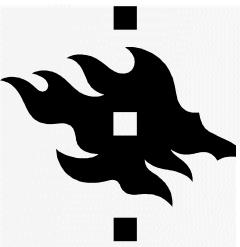
slide credit Noah Snavely



Some panorama examples

- Every image on Google Streetview





Magic: ghost removal



M. Uyttendaele, A. Eden, and R. Szeliski.

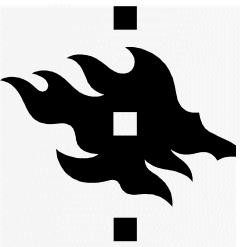
Eliminating ghosting and exposure artifacts in image mosaics.

In Proceedings of the International Conference on Computer Vision and Pattern Recognition,

HELSINKIN YLIOPISTO volume 2, pages 509--516, Kauai, Hawaii, December 2001.

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Magic: ghost removal



M. Uyttendaele, A. Eden, and R. Szeliski.

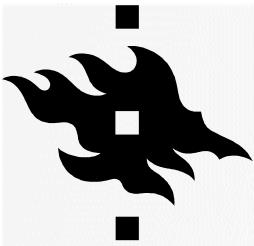
Eliminating ghosting and exposure artifacts in image mosaics.

In Proceedings of the International Conference on Computer Vision and Pattern Recognition,

HELSINKIN YLIOPISTO volume 2, pages 509--516, Kauai, Hawaii, December 2001.

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CHAPTER 3: TRACKING

Augmented Reality – Principles and Practice

<http://www.augmentedrealitybook.org>

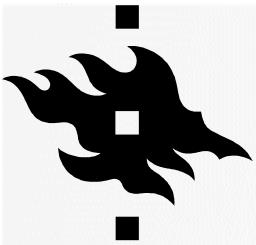
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www.augmentedrealitybook.org

PRINCIPLES AND PRACTICE



Dieter **SCHMALSTIEG**
Tobias **HÖLLERER**



TRACKING, CALIBRATION, AND REGISTRATION

Registration = alignment of spatial properties

Calibration = offline adjustment of measurements

Spatial calibration yields static registration

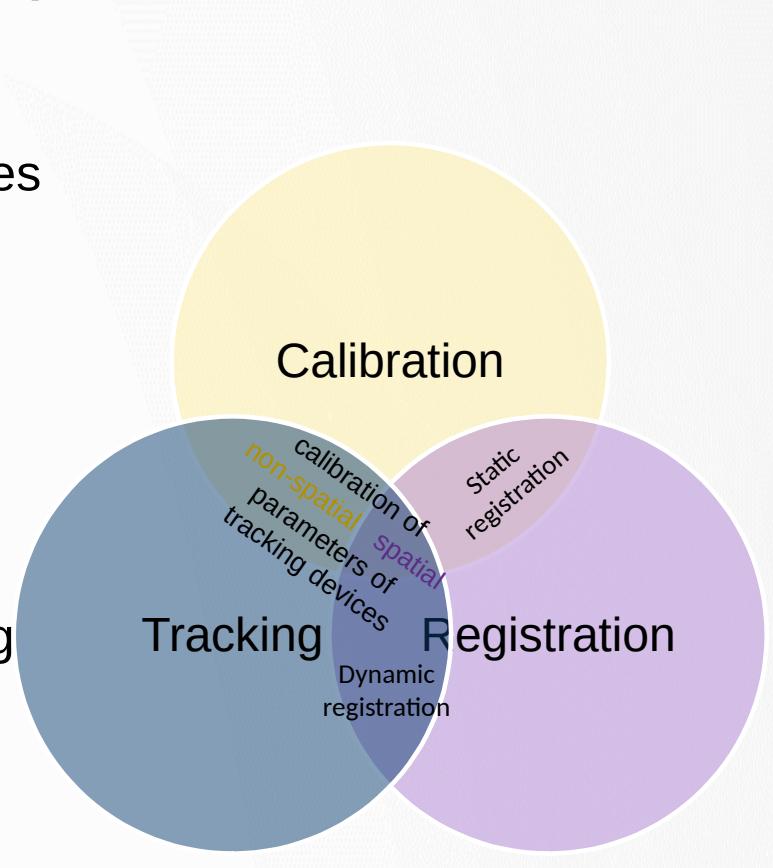
Offline: once in lifetime or once at startup

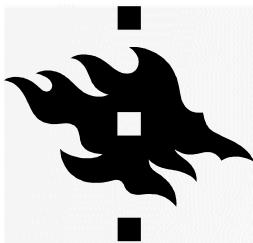
Alternative: autocalibration

Tracking = dynamic sensing and measuring of spatial properties

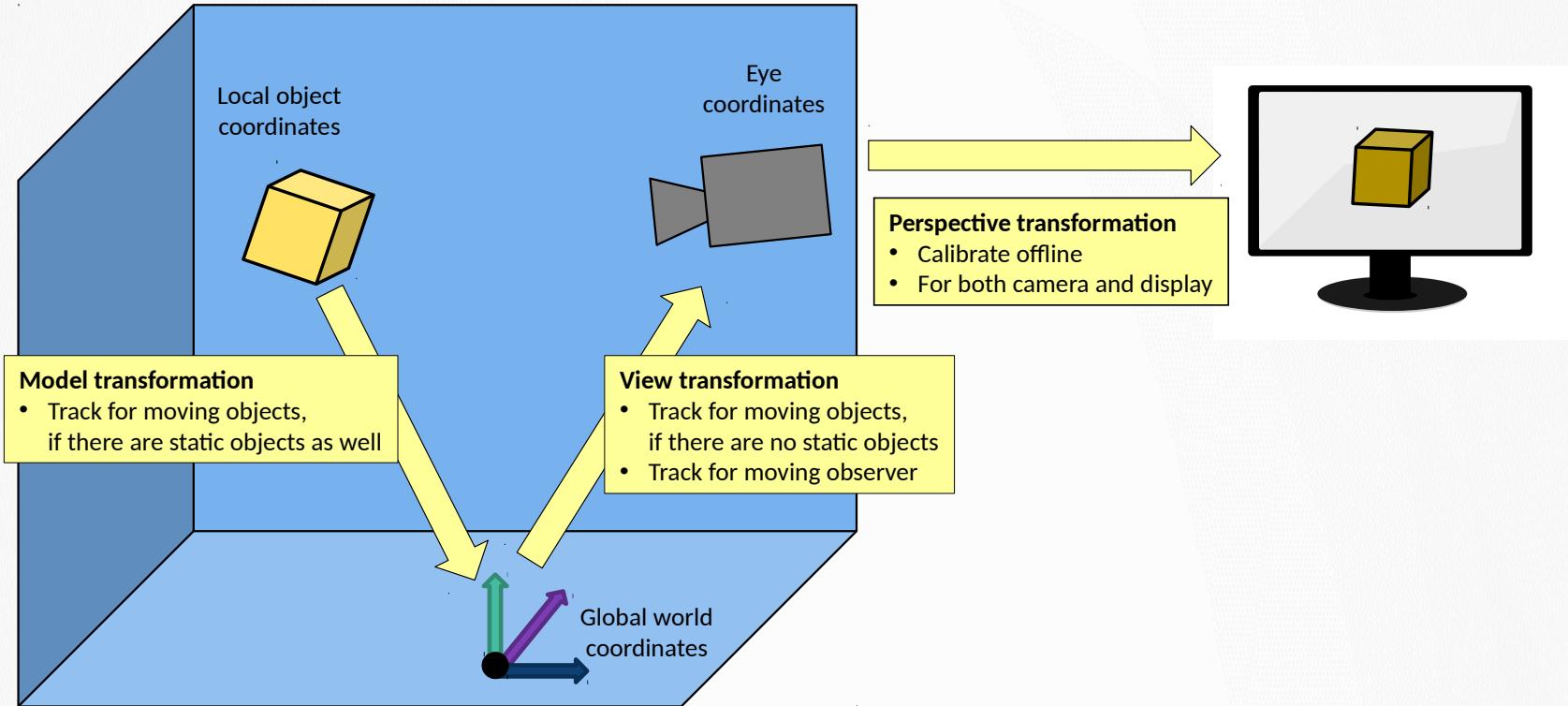
Tracking yields dynamic registration

Tracking in AR/VR always means “in 3D”!





COORDINATE SYSTEMS





FRAMES OF REFERENCE

Word-stabilized

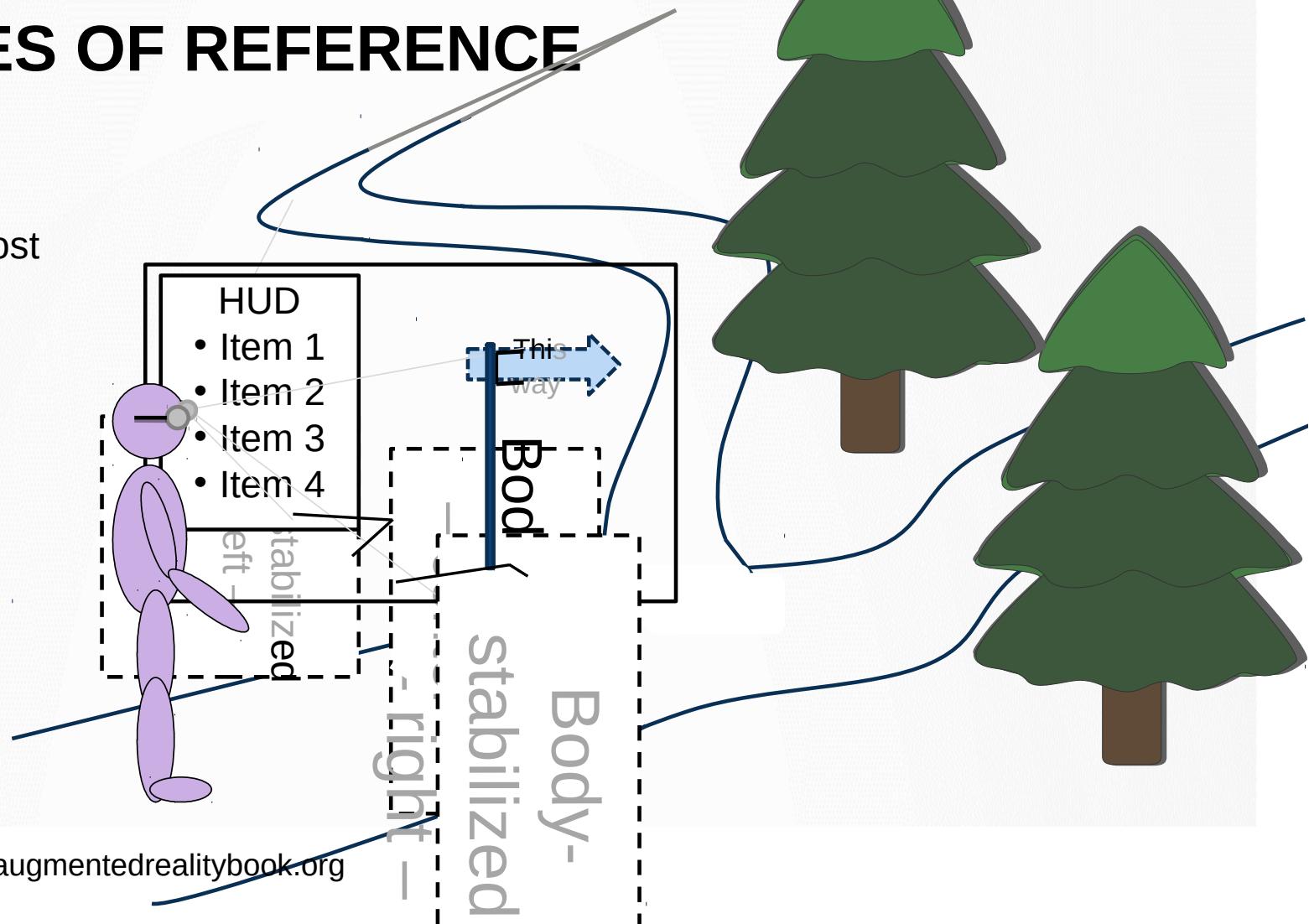
E.g., billboard or signpost

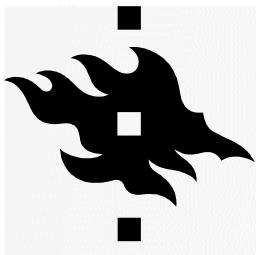
Body-stabilized

E.g., virtual tool-belt

Screen-stabilized

Heads-up display





SENSOR GROUP ARRANGEMENT

Outside-in

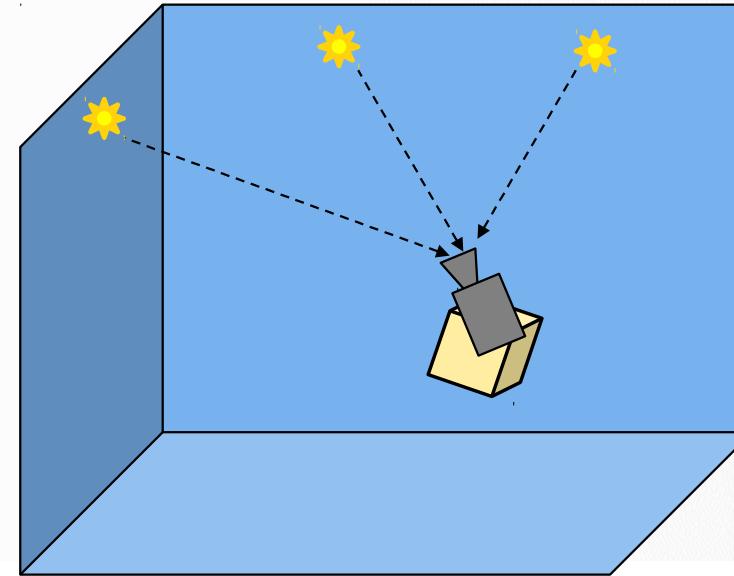
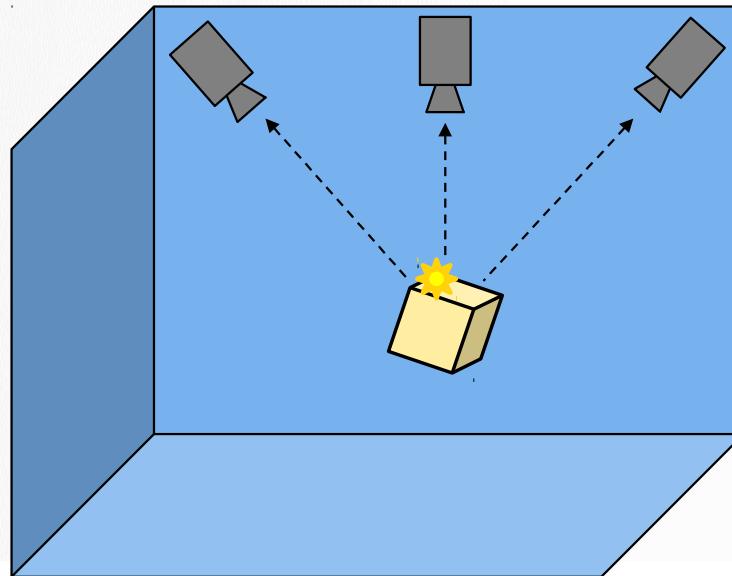
Stationary mounted sensors

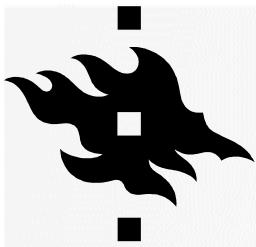
Good position, poor orientation

Inside-out

Mobile sensor(s)

Good orientation, poor position





OPTICAL SENSORS

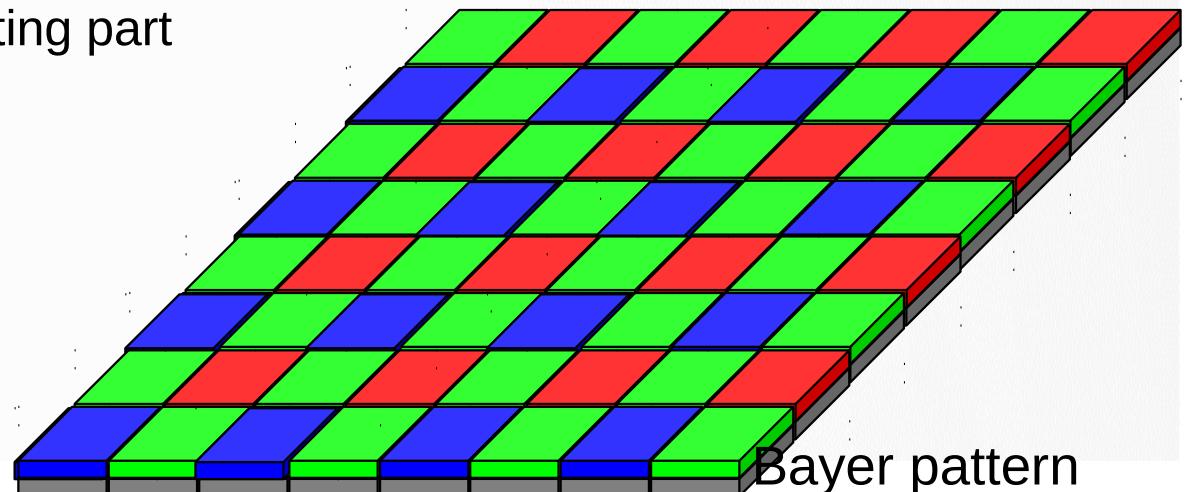
Digital cameras are cheap and powerful

CCD (charge coupled devices) – professional photography

CMOS (complementary metal oxide semiconductor) – fast and cheap

Computer vision techniques improve with Moore's law

Lenses are becoming the most limiting part





MODEL-BASED VERSUS MODEL-FREE TRACKING

Model-based

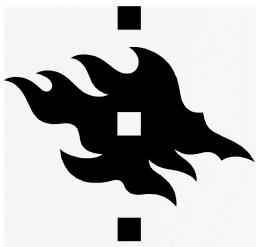
A tracking model representing the 3D world is available

Compare the model to observations in the images

Model-free

At start-up, no tracking model is available
Most build a temporary tracking model while tracking

Measurements only relative to starting point

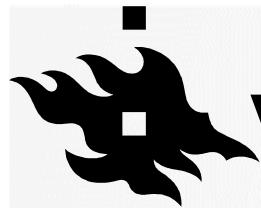


LEAP MOTION

2 cameras, 3 infrared LEDs

Short-distance reflection
of the hands





VALVE/HTC VIVE

“Lighthouses” = two scanning infrared lasers
Photodiodes on head pick up lasers





MARKERS VS NATURAL FEATURES

Fiducials markers

Artificial tracking targets

Square shapes yield 4 points (enough for pose)

Circular shapes yield only 1 point

Digital marker model exists first,
marker manufactured second (e.g., printing)

Natural feature tracking

Existing visual features in the environment

Physical features exist first,
tracking model reconstructed second



Image: Daniel Wagner



Image: Andrei State, UNC Chapel Hill



NATURAL FEATURES

Detect salient interest points in image

Must be easily found

Location in image should remain stable
when viewpoint changes

Requires textured surfaces

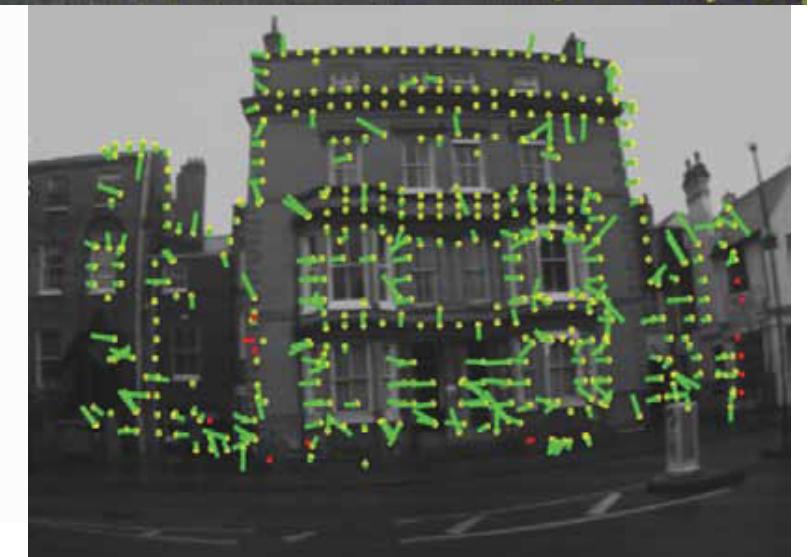
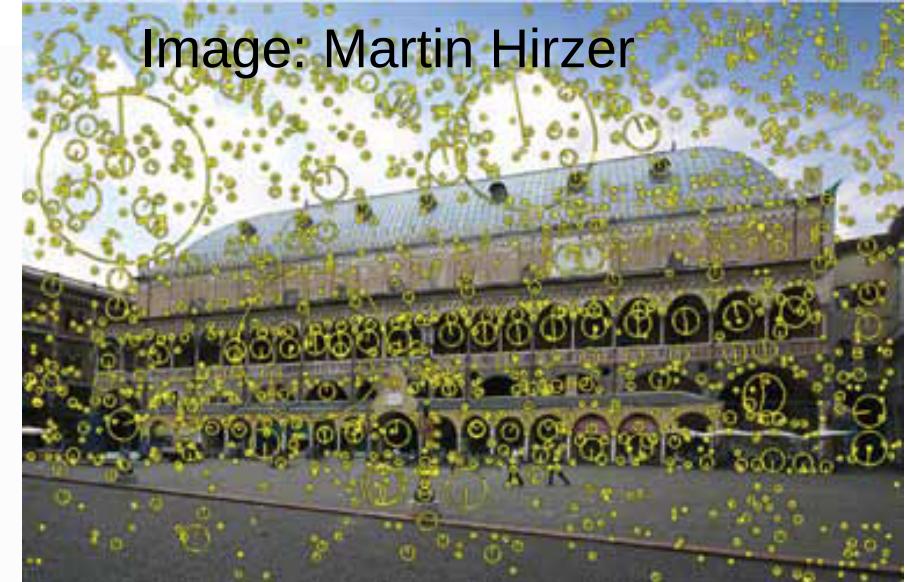
Alternative: can use edge features (less discriminative)

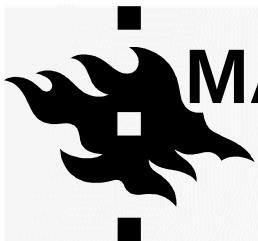
Match interest points to tracking model database

Database filled with results of 3D reconstruction

Matching entire (sub-)images is too costly

Typically interest points are compiled into “descriptors”





MARKER TARGET IDENTIFICATION

More targets or features \Rightarrow more easily confused

Must be as unique as possible

Square markers

2D barcodes with error correction

E.g., $6 \times 6 = 36$ bits (2 orientation, 6-12 payload, rest for error correction)

Marker tapestries

Spherical targets

5 spheres in different geometric configurations

Can distinguish 10-20 targets

Pulsed LEDs

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www.augmentedrealitybook.org

Image: Mark Fiala

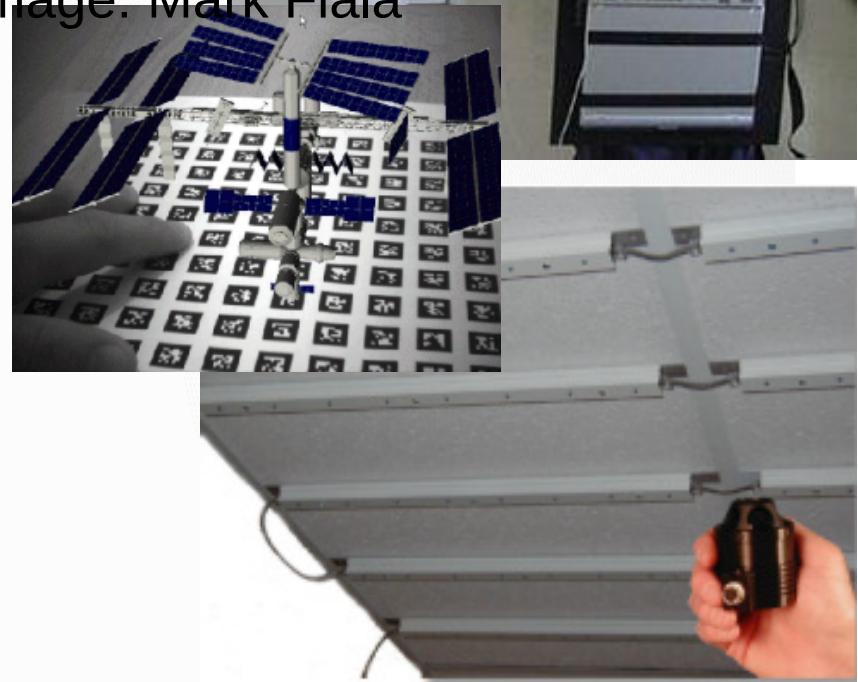


Image: Greg Welch, UNC Chapel Hill



NATURAL FEATURE TARGET IDENTIFICATION

Individual natural interest points
too easily confused

Rely on co-occurrence of interest
points for detection

Probabilistic search methods
used to deal with errors

Vocabulary trees

Random sampling consensus

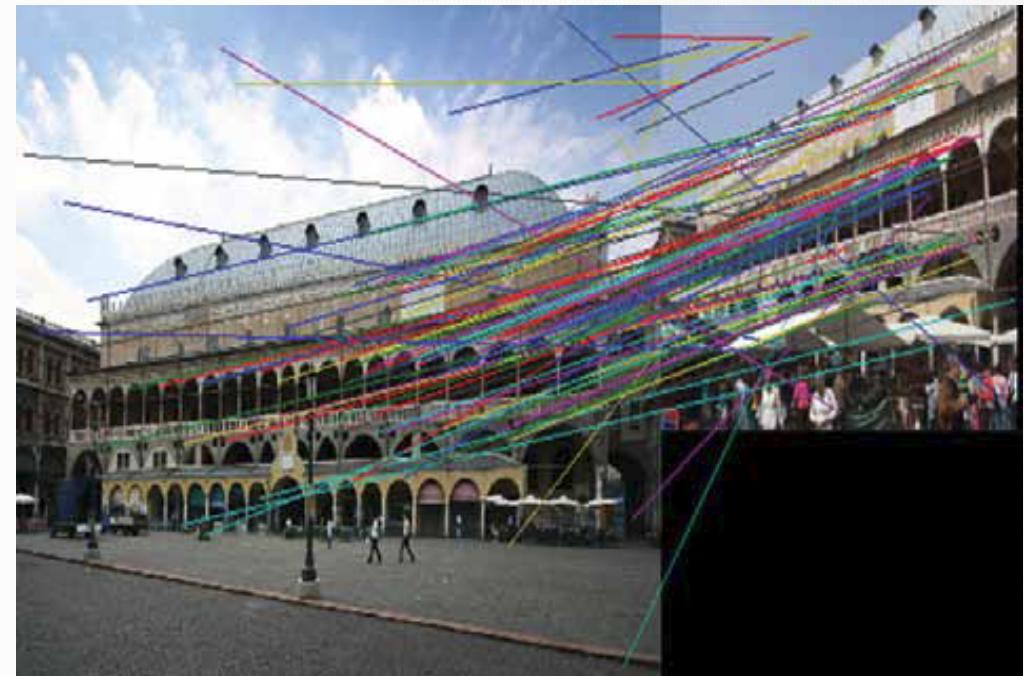


Image: Martin Hirzer



COOPERATIVE SENSOR FUSION

Primary sensor relies on information from secondary sensor to obtain its measurements

E.g., A-GPS combines celltower + GPS

Combination of inside-out + outside-in

Stereo cameras with known epipolar geometry

Non-overlapping cameras (e.g., 360°)

Indirect sensing (cont'd)



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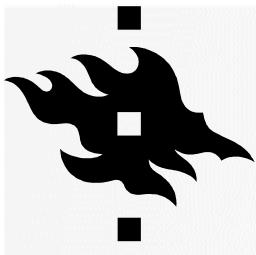
Image: Georg Klein

www.augmentedrealitybook.org



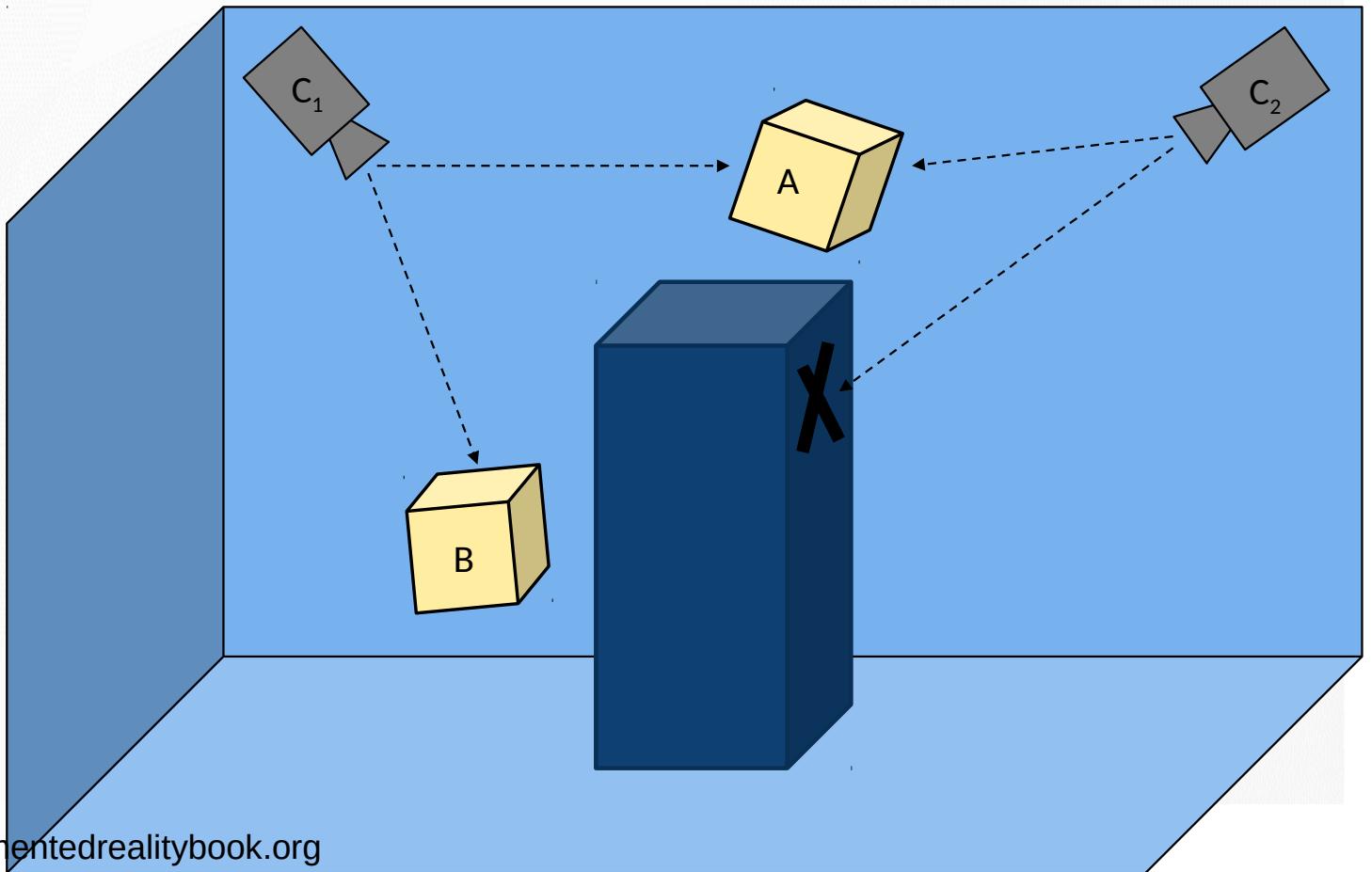
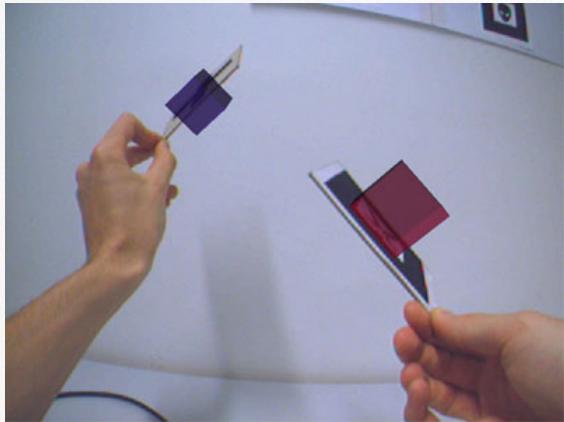
PointGrey LadyBug





COOPERATIVE SENSOR FUSION FOR INDIRECT SENSING

“Track around the corner”





EXERCISE 6 AND LECTURE 14

- Lecture 14 will be a summary lecture, but as communicated at the first lecture it will be given by the students
- 4 groups, 4 papers, 15 minutes presentation + 5 minutes discussion each
 - Describe the topic and innovation of the paper
 - Map the paper to the computer vision principles you have learned during the course, what were the principles and why were they relevant in the method discussed in the paper
- You may use exercise 6 for preparing the slides and asking about the things you did not understand (in the paper or in lectures)
- Papers will be found from Wiki
- I'll name all students who have participate in the course into a group. In case you want to change the group, agree that straight with someone in another group who is willing to change places with you
- Agree straight with your group members how you'll arrange the work. In case you can't reach them via email please contact me



60° 10 1.2 N, 24° 57 18 E

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