Valuation and Risk Management in the Norwegian Electricity Market*

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Abstract The purpose of this paper is twofold: Firstly, we analyse the option value approximation of traded options in the presence of a volatility term structure. The options are identified as: (a) "European" (written on the forward price of a future flow delivery); and (b) "Asian". Both types are in fact written on (arithmetic) price averages. Secondly, adopting a 3-factor model for market risk which is compatible with the valuation results, we discuss risk management in the electricity market within the Value at Risk concept. The analysis is illustrated by numerical cases from the Norwegian electricity derivatives market.

1 Introduction

Historical time series, implicit volatilities of quoted option prices, as well as the experience of professional traders and brokers, clearly indicate the presence of a volatility term structure in the Norwegian electricity derivatives market. The purpose of this paper is to analyse the implications of this volatility term structure for: (a) valuation of the most frequently traded options; and (b) market risk management.

Our starting point is to represent the electricity forward market at date t by a forward price function $f(t, T)$, which may be interpreted as the forward price at date t of a hypothetical contract with delivery at date T (i.e., with an infinitesemal delivery period). In the electricity forward market, the underlying quantity is delivered as a flow during a specific future time period. This contract may be interpreted as a portfolio of hypothetical single-delivery contracts, hence the forward price follows from the function $f(t, T)$ by no-arbitrage.

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*This chapter is a corrected version of Bjerksund, Rasmussen, and Stensland (2000).

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Assuming lognormality, we represent the uncertainty in the forward market at date t by a volatility function $\sigma(\tau - t, T - t)$, which corresponds to the Black'76 implicit volatility of a European option with time to exercise $\tau - t$ written on the future forward price $f(t, T)$ with time to delivery $T - t$.

However, the traded "European" electricity option is written on the forward price of a contract with delivery as a constant flow during a specific future time period. Following Kemna and Vorst (1990), we adopt the Black'76 concept for approximating the option value and obtain the theoretical forward price as well as an approximated plug-in volatility.

The traded Asian option is written on the average spot price observed during a specific period. The exercise date of the option typically coincides with the last observation date. We obtain the theoretical forward price and the Black'76 plug-in volatility.

Next, we turn to risk management within the Value at Risk concept. The idea of Value at Risk is to quantify the downside risk of the future market value of a given portfolio at a chosen horizon date. We represent the market risk by a 3-factor model, which is compatible with our forward price dynamics assumption. We use Monte Carlo simulation in order to generate the probability distribution of the future portfolio market price.

The advantage of integrating valuation and risk management is: (a) the market risk exposure of a future position is consistent with the current forward and option prices; and (b) we may use our option valuation approximation results to calculate conditional future option values.

2 The Model

2.1 The Forward Market

Research on valuation of commodity derivatives and management of commodity market risk has been an expanding area within finance during the last decade. At the same time, the use of various bilateral OTC arrangements in the industry has increased, and new commodity derivatives have been introduced in the financial market place.

For many commodities, the forward prices indicate a non-constant convenience yield (e.g., seasonal pattern). Moreover, the commodity option market prices clearly indicate that the constant volatility assumption of Black'76 is violated for most commodities. Typically, the implicit volatility is a decreasing and convex function of time to maturity.

Gibson and Schwartz (1990) develop a two-factor model for oil derivatives, where the commodity spot price is geometric Brownian, and the instantaneous convenience yield rate follows a mean-reverting Ornstein–Uhlenbeck process. Within this model, closed form solutions exist for the forward price as well as European calls (see Bjerksund (1991) and Jamshidian and Fein (1990)). Hilliard and Reis (1998) investigate several alternative models, including the case where the spot price is a mixed jump-diffusion process. For a survey on alternative models for valuation and hedging, see Schwartz (1997).

Models where assumptions on spot price and convenience yield dynamics are starting points will typically predict forward prices which are different from the ones observed in the market. Using the general Heath, Jarrow, and Morton (1992) approach, Miltersen and Schwartz (1998) develop a general framework for commodity derivatives valuation and risk management with stochastic interest rates as well as stochastic convenience yield. This model can be calibrated to the current forward market. In their Gaussian special case, the call option value essentially boils down to a generalised version of Black'76.

Our model assumptions may be considered as a special case of the gaussian Miltersen–Schwartz model. Complicating the picture in the case of electricity derivatives, however, is the fact that the physical "underlying asset" is a constant flow received during a specific time period, rather than one "bulk" delivery at a specific date.

Turning to our model, we represent the forward market at date t by a continuous forward price function, where $f(t, T)$ denotes the forward price at date t on a contract with delivery at date $T \geq t$. Consider a forward contract with delivery date T, and assume the following forward price dynamics at date $t \leq T$ (with respect to the risk-adjusted martingale probability measure)

$$\frac{df(t,T)}{f(t,T)} = \left(\frac{a}{T-t+b} + c\right) dW^*(t), \tag{1}$$

where a, b, and c are positive constants, and $dW^*(t)$ is the increment of a standard Brownian motion with expectation $E_t^*[dW^*(t)] = 0$ and $\mathrm{Var}_t^*[dW^*(t)] = dt$. By construction, the expectation of (1) is zero with respect to the martingale measure.

The above corresponds to the forward price of this contract at the future date $\tau \in [t, T]$ being lognormal, and given by the following stochastic integral

$$f(\tau,T) = f(t,T)$$
$$\exp\left\{\int_t^\tau \left(\frac{a}{T-s+b} + c\right) dW^*(s) - \frac{1}{2}\int_t^\tau \left(\frac{a}{T-s+b} + c\right)^2 ds\right\}.$$

Observe that $E_t^*[1_\tau f(\tau, T)] = f(t, T)$, which confirms that the forward price is a martingale with respect to the $*$-probability measure.

Now, consider a hypothetical European call option with time to exercise $\tau - t$, written on the future forward price $f(\tau, T)$ on a contract with time to delivery $T - t$. It follows from the literature (see, e.g., Harrison and Kreps (1979) and Harrison and Pliska (1981)) that the market value of the option can be represented by the expected (using the martingale measure) discounted (using the riskless rate) future pay-off. With the future forward price being lognormal, the call value is given by the Black'76 formula

$$\begin{aligned} V_t\left[1_\tau\left(f(\tau,T)-K\right)^+\right] &= E_t^*\left[e^{-r(\tau-t)}\left(f(\tau,T)-K\right)^+\right] \\ &= e^{-r(\tau-t)}\left\{f(t,T)N(d_1)-KN(d_2)\right\}, \end{aligned} \tag{2}$$

where $N(\cdot)$ is the standard normal cumulative probability function,

$$d_1 \equiv \frac{\ln(f(t,T)/K)+\frac{1}{2}\sigma^2(\tau-t)}{\sigma\sqrt{\tau-t}}, \tag{3}$$

$$d_2 \equiv d_1-\sigma\sqrt{\tau-t}, \tag{4}$$

$$\sigma \equiv \sqrt{\mathrm{Var}_t^*\left[\ln\left(\frac{f(\tau,T)}{f(t,T)}\right)\right]\Big/(\tau-t)}. \tag{5}$$

Observe that the key input of Black'76 is: (a) the forward price at date t of the underlying asset $f(t,T)$; and (b) the uncertainty of the underlying asset, represented by the volatility σ.

The assumed dynamics translates into the volatility σ being a function of time to exercise (of the option), $\tau-t$, and time to delivery (of the underlying forward), $T-t$, and given by

$$\begin{aligned} \sigma &= \sigma(\tau-t,T-t) \\ &= \sqrt{\mathrm{Var}_t^*\left[\ln\left(\frac{f(\tau,T)}{f(t,T)}\right)\right]\Big/(\tau-t)}, \end{aligned} \tag{6}$$

where[1]

$$\begin{aligned} \mathrm{Var}_t^*\left[\ln\left(\frac{f(\tau,T)}{f(t,T)}\right)\right] &= \mathrm{Var}_t^*\left[\int_{s=t}^{s=\tau}\frac{df(s,T)}{f(s,T)}\right] \\ &= \left[\frac{a^2}{T-s+b}-2ac\ln(T-s+b)+c^2s\right]_{s=t}^{s=\tau}. \end{aligned} \tag{7}$$

In the following, we represent the forward market at date t by the forward price function $f(t,T)$ and the volatility function $\sigma(\tau-t,T-t)$.

[1] To establish the first equality, apply Ito's lemma

$$\mathrm{Var}_t^*\left[\ln\left(\frac{f(\tau,T)}{f(t,T)}\right)\right] = \mathrm{Var}_t^*\left[\int_{s=t}^{s=\tau}\left(\frac{df(s,T)}{f(s,T)}\right)-\int_{s=t}^{s=\tau}\frac{1}{2}\left(\frac{df(s,T)}{f(s,T)}\right)^2\right],$$

insert the assumed forward price dynamics, and observe that the second integral is deterministic as of date t. The second equality follows from the fact that Brownian motions have independent increments across time.

3 European Option

3.1 Forward on a Flow Delivery

In the electricity forward market, the underlying physical commodity is delivered during a specific time period $[T_1, T_2]$ as a constant flow (at a rate of $(T_2 - T_1)^{-1}$ units per year). We observe delivery periods on contracts ranging from one day to one year, depending on the remaining time to delivery of the contract.

We represent the forward market at date t by the forward price function $f(t, s)$, $t \leq s \leq T$. By value additivity, the market value at date t of receiving one unit of the commodity from dates T_1 to T_2 (at a rate of $1/(T_2 - T_1)$) is simply

$$V_t \left[\int_{T_1}^{T_2} 1_s \frac{f(s,s)}{T_2 - T_1} ds \right] = \int_{T_1}^{T_2} e^{-r(s-t)} \frac{f(t,s)}{T_2 - T_1} ds, \tag{8}$$

where $t \leq T_1 < T_2$. In a rational market, the forward price $F(t, T_1, T_2)$ is determined such that the market value at date t of the payments equals the right-hand side of the equation just above. Indeed, in the hypothetical case of up-front payment at date t, the forward price would coincide with the right-hand side just above.

Now, suppose that the forward price is paid as a constant cash flow stream during the delivery period (at a rate of $F(t, T_1, T_2)/(T_2 - T_1)$ per time unit). At date t, the net market value of entering the contract is zero, leading to the following forward price

$$F(t, T_1, T_2) = \int_{T_1}^{T_2} w(s; r) f(t, s) ds, \tag{9}$$

where

$$w(s; r) = \frac{e^{-rs}}{\int_{T_1}^{T_2} e^{-rs} ds}. \tag{10}$$

Consequently, the forward price $F(t, T_1, T_2)$ may be interpreted as the average of the forward prices $f(t, s)$ over the delivery period $[T_1, T_2]$, with respect to the weight function[2], which reflects the time value of money.

3.2 Call Option Valuation

The European calls which are traded in the electricity derivatives market are typically written on a forward price. In particular, consider a European call option written on the pay-off $F(\tau, T_1, T_2)$ with strike K and exercise date $\tau \leq T_1$. Observe that the exercise date of the option precedes the delivery period of the underlying forward contract.

[2] Observe that $w(s; r) > 0 \forall s \in [T_1, T_2]$ and $\int_{T_1}^{T_2} w(s; r) ds = 1$.

Following Kemna and Vorst (op.cit.), we approximate the option value within the Black'76 framework. We have already obtained the theoretical forward price of the underlying uncertain pay-off, $F(t, T_1, T_2)$. In addition, we need an approximated volatility parameter. Approximate the forward price dynamics for $t \leq T_1$ by[3]

$$\begin{aligned}\frac{dF(t,T_1,T_2)}{F(t,T_1,T_2)} &\approx \int_{s=T_1}^{s=T_2} \frac{1}{T_2-T_1} \frac{df(t,s)}{f(t,s)} ds \\ &= \left\{ \frac{a}{T_2-T_1} \ln\left(\frac{T_2-t+b}{T_1-t+b}\right) + c \right\} dW^*(t). \end{aligned} \tag{11}$$

Next, obtain the approximated variance

$$\begin{aligned}\mathrm{Var}_t^*\left[\ln\left(\frac{F(\tau,T_1,T_2)}{F(t,T_1,T_2}\right)\right] &= \mathrm{Var}_t^*\left[\int_t^\tau \frac{dF(s,T_1,T_2)}{F(s,T_1,T_2)} ds\right] \\ &= \left(\frac{a}{T_2-T_1}\right)^2 \int_t^\tau \left(\ln\frac{T_2-s+b}{T_1-s+b}\right)^2 ds \\ &+ \frac{2ac}{T_2-T_1}\int_t^\tau \ln\frac{T_2-s+b}{T_1-s+b} ds + c^2 \int_t^\tau ds, \end{aligned} \tag{12}$$

where the first and the second integrals are[4]

$$\begin{aligned}\int_t^\tau \left(\ln\frac{T_2-s+b}{T_1-s+b}\right)^2 ds = \Big[&(x+\alpha)\left(\ln(x+\alpha)\right)^2 \\ &-2(x+\alpha)\ln(x+\alpha)\ln(x-\alpha) \\ &+4\alpha\ln(2\alpha)\ln\left(\frac{x-\alpha}{2\alpha}\right) \\ &-4\alpha \mathrm{dilog}\left(\frac{x+\alpha}{2\alpha}\right) \\ &+(x-\alpha)\left(\ln(x-\alpha)\right)^2\Big]_{X(\tau)}^{X(t)}, \end{aligned} \tag{13}$$

$$\begin{aligned}\int_t^\tau \ln\frac{T_2-s+b}{T_1-s+b} ds = [&(x+\alpha)\ln(x+\alpha) \\ &-(x-\alpha)\ln(x-\alpha)]_{X(\tau)}^{X(t)}, \end{aligned} \tag{14}$$

[3] The approximation proceeds in the following two steps

$$\frac{dF(t,T_1,T_2)}{F(t,T_1,T_2)} \approx \int_{s=T_1}^{s=T_2} w(s;r)\frac{df(t,s)}{f(t,s)} ds \approx \int_{s=T_1}^{s=T_2} w(s;0)\frac{df(t,s)}{f(t,s)} ds.$$

[4] We have corrected the typos in a previous version of this paper pointed out in Lindell and Raab (2008).

where we define

$$\alpha \equiv \tfrac{1}{2}(T_2 - T_1), \tag{15}$$
$$X(s) \equiv b + \tfrac{1}{2}(T_2 + T_1) - s, \tag{16}$$

and where the dilogarithm function is defined by[5]

$$\text{dilog}\,(x) = \int_1^x \frac{\ln(s)}{1-s} ds \text{ where } x \geq 0 \tag{17}$$

see, for example, Abramowitz and Stegun (1972).

Now, consider a European call option with exercise date τ written on the forward price $F(\tau, T_1, T_2)$, where $t < \tau \leq T_1 < T_2$ The option value at date t can now be approximated by Black'76, using the forward price $F(t, T_1, T_2)$ above and the volatility parameter v_E

$$\begin{aligned} v_E &\equiv v_E(\tau - t, T_1 - t, T_2 - t) \\ &= \sqrt{\text{Var}_t^* \left[\ln\left(\frac{F(\tau, T_1, T_2)}{F(t, T_1, T_2)} \right) \right] \Big/ (\tau - t)} \end{aligned} \tag{18}$$

The volatility parameter v_E associated with the European option is a function of the time to maturity of the option $(\tau - t)$, the time to start of delivery $(T_1 - t)$, and the time to stop of delivery $(T_2 - t)$.

4 Asian Option

Asian options are written on the *average* spot price observed during a specific period $[T_1, T_2]$, with exercise date $\tau \geq T_2$. With continuous sampling, the (arithmetic) average of the spot prices $f(s, s)$ observed from T_1 to T_2 is defined by

$$A(T_1, T_2) \equiv \int_{T_1}^{T_2} \frac{1}{T_2 - T_1} f(s, s) ds. \tag{19}$$

We are interested in evaluating a call option with strike K and exercise date T_2, written on the arithmetic average $A(T_1, T_2)$. For simplicity, we deal with the case of

[5] The function is approximated numerically by

$$\text{dilog}\,(x) = \begin{cases} \sum_{k=1}^n \frac{(x-1)^k}{k^2} & \text{for } 0 \leq x \leq 1 \\ -\frac{1}{2}(\ln(x))^2 - \sum_{k=1}^n \frac{((1/x)-1)^k}{k^2} & \text{for } x > 1 \end{cases}$$

where n is a sufficiently large positive integer.

$t \leq T_1$ first. With the future spot prices being lognormal, there is no known probability distribution for the arithmetic average. Within the Black'76 framework, the option value approximation problem boils down to finding the theoretical forward price and a reasonable volatility parameter.

Now, it follows from the martingale property of forward prices that the forward price on a contract written on (the cash equivalent of) $A(T_1, T_2)$ with delivery at date T_2 is

$$\begin{aligned} F_t[A(T_1,T_2)] &= E_t^*\left[\int_{T_1}^{T_2} \frac{1}{T_2-T_1} f(s,s)ds\right] \\ &= \int_{T_1}^{T_2} \frac{1}{T_2-T_1} f(t,s)ds. \end{aligned} \tag{20}$$

Observe that the forward price $F_t[A(T_1, T_2)]$ simply is the (equally weighted) arithmetic average of the current forward prices over the sampling period $[T_1, T_2]$. This forward price may be interpreted as the cost replicating this contract in the market.[6]

Turning to the Black'76 volatility parameter, approximate the dynamics of the underlying forward price at date $\tau \in [t, T_2]$ by

$$\begin{aligned} &\frac{dF_\tau[A(T_1,T_2)]}{F_\tau[A(T_1,T_2)]} \approx \int_{s=\max\{t,T_1\}}^{s=T_2} \frac{1}{T_2-T_1}\frac{df(\tau,s)}{f(\tau,s)}ds \\ &= \begin{cases} \left\{\dfrac{a}{T_2-T_1}\ln\left(\dfrac{T_2-\tau+b}{T_1-\tau+b}\right)+c\right\} dW^*(\tau) & \text{when } \tau \leq T_1 \\ \left\{\dfrac{a}{T_2-T_1}\ln\left(\dfrac{T_2-\tau+b}{b}\right)+\dfrac{T_2-\tau}{T_2-T_1}c\right\} dW^*(\tau) & \text{when } \tau > T_1 \end{cases} \end{aligned} \tag{21}$$

Obtain the approximated variance by

$$\begin{aligned} &\mathrm{Var}_t^*\left[\ln\left(\frac{A(T_1,T_2)}{F_t[A(T_1,T_2)]}\right)\right] \\ &= \mathrm{Var}_t^*\left[\int_{\tau=t}^{\tau=T_2} \frac{dF_\tau[A(T_1,T_2)]}{F_\tau[A(T_1,T_2)]}ds\right] \\ &= \left(\frac{a}{T_2-T_1}\right)^2 \int_t^{T_1}\left(\ln\frac{T_2-\tau+b}{T_1-\tau+b}\right)^2 d\tau \end{aligned}$$

[6] Assume for the moment a discrete time model where the delivery period $[T_1, T_2]$ is divided into n time intervals of time length Δt. Consider the following strategy: At the evaluation date t, buy $e^{-r(T_2-(T_1+i\cdot\Delta t))}(1/n)$ units forward for each delivery $T_1 + i \cdot \Delta t$, $i = 1, \ldots, n$. As time passes and the contracts are settled, invest (or finance) the proceeds at the riskless interest rate r. At the delivery date $\tau \geq T_2$, the pay-off from the strategy is $\sum_{i=1}^{n}(1/n)f(T_1 + i \cdot \Delta t, T_1 + i \cdot \Delta t) - \sum_{i=1}^{n}(1/n)f(t, T_1 + i \cdot \Delta t)$, where the first term represents the desired spot price, and the second (riskless) term may be interpreted as the forward price as of date t.

$$
\begin{aligned}
&+ \frac{2ac}{T_2 - T_1}\int_t^{T_1} \ln \frac{T_2 - \tau + b}{T_1 - \tau + b} d\tau + c^2 \int_t^{T_1} d\tau \\
&+ \left(\frac{a}{T_2 - T_1}\right)^2 \int_{T_1}^{T_2} \left(\ln \frac{T_2 - \tau + b}{b}\right)^2 d\tau \\
&+ \frac{2ac}{T_2 - T_1}\int_{T_1}^{T_2} \ln \frac{T_2 - \tau + b}{b} \frac{T_2 - \tau}{T_2 - T_1} d\tau + c^2 \int_{T_1}^{T_2} \left(\frac{T_2 - \tau}{T_2 - T_1}\right)^2 d\tau,
\end{aligned} \tag{22}
$$

where the first and the second integrals are evaluated by inserting $\tau = T_1$ in (13)–(14) above, and the fourth and the fifth integrals are

$$
\int_{T_1}^{T_2} \left(\ln \frac{T_2 - \tau + b}{b}\right)^2 d\tau = b\left[y\,(ln(y))^2 - 2y\ln(y) + 2y\right]_1^{\overline{y}} \tag{23}
$$

$$
\int_{T_1}^{T_2} \ln\left(\frac{T_2 - \tau + b}{b}\right)\frac{T_2 - \tau}{T_2 - T_1} d\tau = \frac{b^2\left[\frac{1}{2}y^2\ln(y) - y\ln(y) + y - \frac{1}{4}y^2\right]_1^{\overline{y}}}{T_2 - T_1} \tag{24}
$$

where

$$
\overline{y} = \frac{T_2 - T_1 + b}{b}. \tag{25}
$$

The Black'76 volatility parameter v_A is now found by

$$
\begin{aligned}
v_A &\equiv v_A(T_1 - t, T_2 - t) \\
&= \sqrt{\mathrm{Var}_t^*\left[\ln\left(\frac{A(T_1, T_2)}{F_t[A(T_1, T_2)]}\right)\right] \Big/ (T_2 - t)}.
\end{aligned} \tag{26}
$$

Observe that the volatility parameter v_A is a function of time to the first sampling date, $T_1 - t$, and time to the last sampling date, $T_2 - t$, where the latter coincides with the time to exercise of the option.

Next, consider the case where the option is evaluated within the sampling period, that is, $T_1 < t \leq T_2$. It follows immediately from the definition of the arithmetic average that

$$
A(T_1, T_2) = \frac{t - T_1}{T_2 - T_1} A(T_1, t) + \frac{T_2 - t}{T_2 - T_1} A(t, T_2). \tag{27}
$$

Consequently, with $T_1 < t \leq T_2$, the call option problem is equivalent to

$$
V_t\left[1_{T_2}\left(A(T_1, T_2) - K\right)^+\right] = \frac{T_2 - t}{T_2 - T_1} V_t\left[1_{T_2}\left(A(t, T_2) - K'\right)^+\right], \tag{28}
$$

where

$$K' \equiv \frac{T_2 - T_1}{T_2 - t} K - \frac{t - T_1}{T_2 - t} A(T_1, t), \tag{29}$$

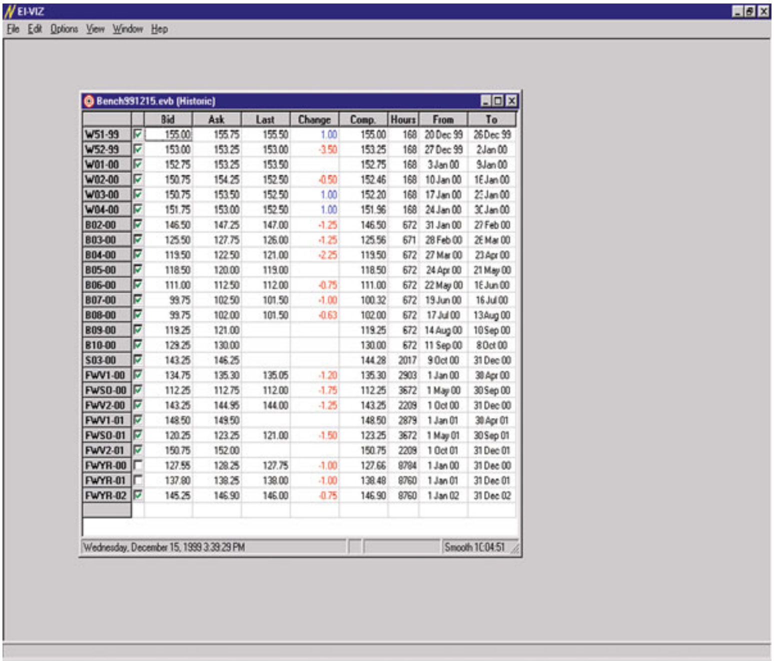
that is, a portfolio of $\frac{T_2 - t}{T_2 - T_1}$ call options, each written on the average over the remaining sampling period $[t, T_2]$ where the strike is adjusted for the already observed prices. In the non-trivial case of $K' > 0$, the value of the adjusted option can be evaluated by inserting $T_1 = t$ and $K = K'$ in the evaluation procedure above. In the degenerate case of $K' \leq 0$, it will always be optimal to exercise the call, which reduces the adjusted option to a forward with the current value

$$V_t \left[1_{T_2} \left(A(t, T_2) - K' \right)^+ \right] = e^{-r(T_2 - t)} \left((T_2 - t)^{-1} \int_t^{T_2} f(t, s) ds - K' \right). \tag{30}$$

5 Valuation: An Example

5.1 Current Term Structure

The Nordic electricity market NORDPOOL consists of several forward and futures contracts. The traded contract and their market prices at December 15, 1999 are found in Fig. 1.



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		Bid	Ask	Last	Change	Comp.	Hours	From	To
W51-99	☑	155.00	155.75	155.50	1.00	155.00	168	20 Dec 99	26 Dec 99
W52-99	☑	153.00	153.25	153.00	-3.50	153.25	168	27 Dec 99	2 Jan 00
W01-00	☑	152.75	153.25	153.50		152.75	168	3 Jan 00	9 Jan 00
W02-00	☑	150.75	154.25	152.50	-0.50	152.46	168	10 Jan 00	16 Jan 00
W03-00	☑	150.75	153.50	152.50	1.00	152.20	168	17 Jan 00	23 Jan 00
W04-00	☑	151.75	153.00	152.50	1.00	151.96	168	24 Jan 00	30 Jan 00
B02-00	☑	146.50	147.25	147.00	-1.25	146.50	672	31 Jan 00	27 Feb 00
B03-00	☑	125.50	127.75	126.00	-1.25	125.56	671	28 Feb 00	26 Mar 00
B04-00	☑	119.50	122.50	121.00	-2.25	119.50	672	27 Mar 00	23 Apr 00
B05-00	☑	118.50	120.00	119.00		118.50	672	24 Apr 00	21 May 00
B06-00	☑	111.00	112.50	112.00	-0.75	111.00	672	22 May 00	18 Jun 00
B07-00	☑	99.75	102.50	101.50	-1.00	100.32	672	19 Jun 00	16 Jul 00
B08-00	☑	99.75	102.00	101.50	-0.63	102.00	672	17 Jul 00	13 Aug 00
B09-00	☑	119.25	121.00			119.25	672	14 Aug 00	10 Sep 00
B10-00	☑	129.25	130.00			130.00	672	11 Sep 00	8 Oct 00
S03-00	☑	143.25	146.25			144.28	2017	9 Oct 00	31 Dec 00
FWV1-00	☑	134.75	135.30	135.05	-1.20	135.30	2903	1 Jan 00	30 Apr 00
FWSO-00	☑	112.25	112.75	112.00	-1.75	112.25	3672	1 May 00	30 Sep 00
FWV2-00	☑	143.25	144.95	144.00	-1.25	143.25	2209	1 Oct 00	31 Dec 00
FWV1-01	☑	148.50	149.50			148.50	2879	1 Jan 01	30 Apr 01
FWSO-01	☑	120.25	123.25	121.00	-1.50	123.25	3672	1 May 01	30 Sep 01
FWV2-01	☑	150.75	152.00			150.75	2209	1 Oct 01	31 Dec 01
FWYR-00	☐	127.55	128.25	127.75	-1.00	127.66	8784	1 Jan 00	31 Dec 00
FWYR-01	☐	137.80	138.25	138.00	-1.00	138.48	8760	1 Jan 01	31 Dec 01
FWYR-02	☑	145.25	146.90	146.00	-0.75	146.90	8760	1 Jan 02	31 Dec 02

Wednesday, December 15, 1999 3:39:29 PM Smooth 10:04:51

Fig. 1 Market prices at December 15, 1999

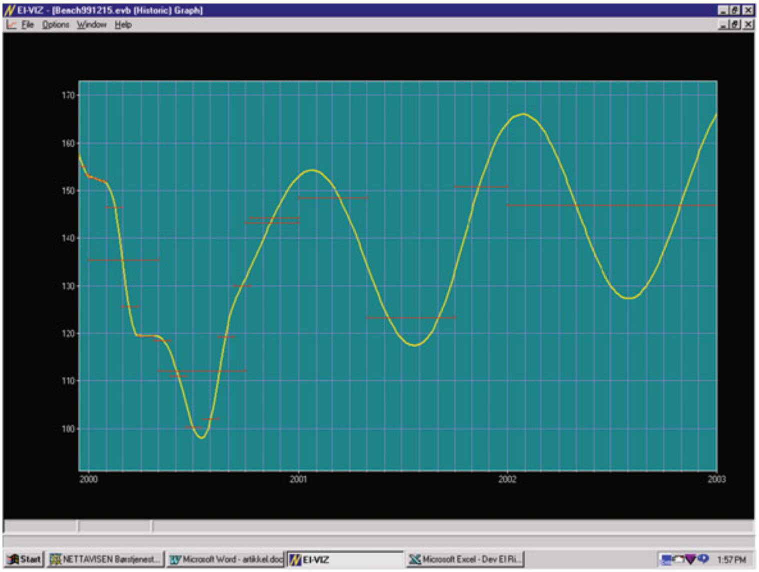


Fig. 2 Forward prices

On the Basis of the bid/ask prices, we construct a continuous forward price function. The forward function is given by the smoothest function that prices all traded contracts on NORDPOOL within the bid/ask spread. The forward price function on December 15, 1999 is represented by the continuous yellow curve in Fig. 2. The red horizontal lines in Fig. 2 correspond to the quoted forward price of each traded contract.

5.2 Volatility

The volatility in forward prices falls rapidly in this market. The volatility on a single day delivery starting in one week might be 80%, whereas a similar delivery starting in 6 months will typically have less than 20% immediate volatility.

Figure 3 shows the forward price function and the volatility curve at December 15, 1999 for the following calendar year (i.e., 2000).

5.3 Contract Valuation

In the following, we consider three valuation cases as of December 15, 1999. The first case corresponds to the contract “FWYR-2000 Asian/M”, see the first line in Fig. 4. The strike of the option is 120 and the contract expires at December 31, 2000.

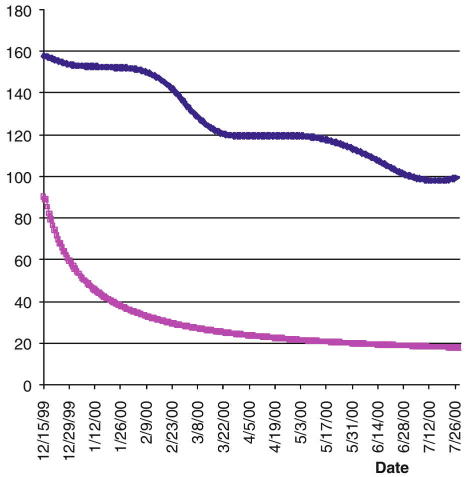
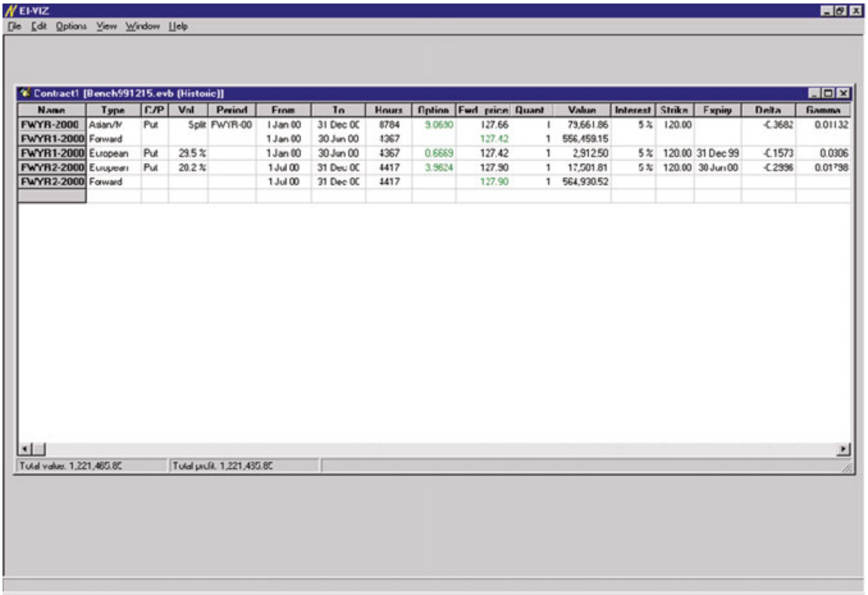


Fig. 3 Forward price and volatility

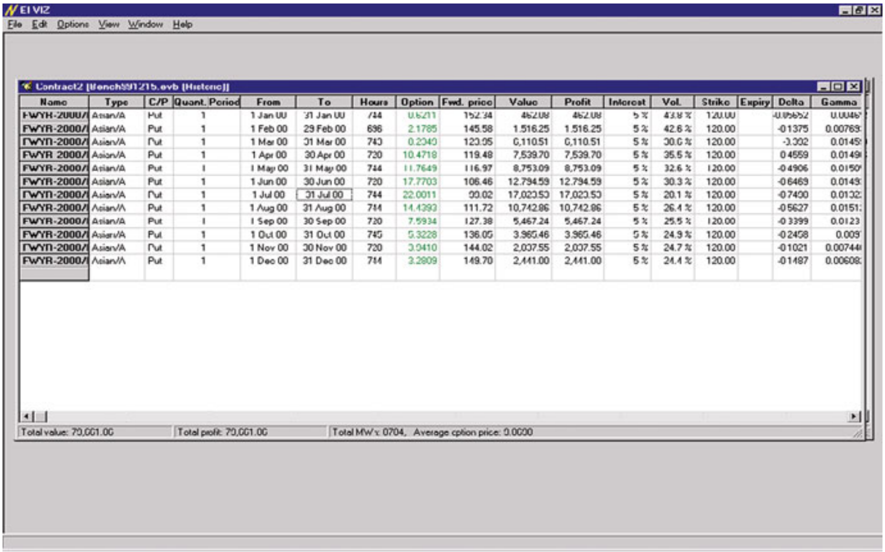


Name	Type	C/P	Vol	Period	From	To	Hours	Option	Fwd price	Quant	Value	Interest	Strike	Expiry	Delta	Gamma
FWYR-2000	Asian/M	Put	Split	FWYR-00	1 Jan 00	31 Dec 0C	8784	9.0690	127.66	1	79,661.86	5 %	120.00		-C.3682	0.01132
FWYR1-2000	Forward				1 Jan 00	30 Jun 00	4367		127.42	1	556,459.15					
FWYR1-2000	European	Put	29.5 %		1 Jan 00	30 Jun 00	4367	0.6669	127.42	1	2,912.50	5 %	120.00	31 Dec 99	-C.1573	0.0306
FWYR2-2000	European	Put	20.2 %		1 Jul 00	31 Dec 0C	4417	3.9624	127.90	1	17,501.81	5 %	120.00	30 Jun 00	-C.2996	0.01798
FWYR2-2000	Forward				1 Jul 00	31 Dec 0C	4417		127.90	1	564,930.52					

Fig. 4 Contract valuation

The contract is subject to “monthly settlements”, which means that the contract represents a portfolio 12 monthly Asian options, where each option is written on the monthly price average and settled at the end of the month.

The second case is a European put option with strike 120 and expiration date December 31, 1999, written on the forward price on the forward contract on delivery from January 1, 2000 to June 30, 2000. The value of the option and the underlying contract are found in lines 3 and 2 in Fig. 4.



Name	Type	C/P	Quant. Period	From	To	Hours	Option	Fwd. price	Value	Profit	Interest	Vol.	Strike	Expiry	Delta	Gamma
FWYR-2000/	Asian/A	Put	1	1 Jan 00	31 Jan 00	744	0.6211	152.34	462.08	462.08	5 %	43.8 %	120.00		-0.05652	0.0046
FWYR-2000/	Asian/A	Put	1	1 Feb 00	29 Feb 00	696	2.1785	145.58	1,516.25	1,516.25	5 %	42.6 %	120.00		-0.1375	0.00769
FWYR-2000/	Asian/A	Put	1	1 Mar 00	31 Mar 00	743	0.2340	123.05	6,110.51	6,110.51	5 %	30.6 %	120.00		-0.302	0.0145
FWYR-2000/	Asian/A	Put	1	1 Apr 00	30 Apr 00	720	10.4718	119.48	7,539.70	7,539.70	5 %	35.5 %	120.00		0.4559	0.0149
FWYR-2000/	Asian/A	Put	1	1 May 00	31 May 00	744	11.7649	116.97	8,753.09	8,753.09	5 %	32.6 %	120.00		-0.4906	0.0150
FWYR-2000/	Asian/A	Put	1	1 Jun 00	30 Jun 00	720	17.7703	106.46	12,794.59	12,794.59	5 %	30.3 %	120.00		-0.6469	0.0149
FWYR-2000/	Asian/A	Put	1	1 Jul 00	31 Jul 00	744	22.0011	90.02	17,023.53	17,023.53	5 %	20.1 %	120.00		-0.7400	0.0132
FWYR-2000/	Asian/A	Put	1	1 Aug 00	31 Aug 00	744	14.4393	111.72	10,742.86	10,742.86	5 %	26.4 %	120.00		-0.5627	0.0151
FWYR-2000/	Asian/A	Put	1	1 Sep 00	30 Sep 00	720	7.5934	127.38	5,467.24	5,467.24	5 %	25.5 %	120.00		-0.3399	0.0123
FWYR-2000/	Asian/A	Put	1	1 Oct 00	31 Oct 00	745	5.3228	136.05	3,965.46	3,965.46	5 %	24.9 %	120.00		-0.2458	0.009
FWYR-2000/	Asian/A	Put	1	1 Nov 00	30 Nov 00	720	3.9410	144.02	2,837.55	2,837.55	5 %	24.7 %	120.00		-0.1021	0.00744
FWYR-2000/	Asian/A	Put	1	1 Dec 00	31 Dec 00	744	3.2809	149.70	2,441.00	2,441.00	5 %	24.4 %	120.00		-0.1487	0.00608

Fig. 5 Split of Asian option

The third case is a European put option with strike 120 and expiration date June 30, 2000, written on the forward price on the forward contract on delivery from July 1, 2000 to December 31, 2000. The value of the option and the underlying contract are found in lines 4 and 5.

Figure 5 considers the first case in more detail. Each line corresponds to an Asian option with strike 120 written on a monthly price average with expiration at the end of the month. Observe that as seen from December 15, 1999, the volatility of the underlying monthly price average is a decreasing and convex function of the delivery month (e.g., January 43.8%; June 30.3%; December 24.4%). By value additivity, the value of each monthly option adds up to the value of the quoted contract (79,661.86 in Fig. 4).

6 Value at Risk

The idea of Value at Risk (VaR) is to focus on the downside market risk of a given portfolio at a future horizon date. For a discussion on VaR, see Hull (1998) and Jorion (1997).

Evidence suggests that even though a one-factor model may be adequate for valuation in a multi-factor environment, it typically performs poorly as a tool for risk management (e.g., dynamic hedging). In the following, we discuss a three-factor Value at Risk (VaR) model, which is consistent with the valuation and approximation results above, following from (1) above.

In order to obtain a richer class of possible forward price functions, assume the following forward price dynamics (with respect to the martingale probability measure)

$$\frac{df(t,T)}{f(t,T)} = \frac{a}{T-t+b}dW_1^*(t) + \left(\frac{2ac}{T-t+b}\right)^{\frac{1}{2}} dW_2^*(t) + c\,dW_3^*(t), \tag{31}$$

where a, b, and c are the positive constants from (1) above, and $dW_1^*(t)$, $dW_2^*(t)$, and $dW_3^*(t)$ are increments of three uncorrelated standard Brownian motions. Observe that the instantaneous dynamics of (31) just above is normal with zero expectation and variance

$$\mathrm{Var}_t^*\left[\frac{df(t,T)}{f(t,T)}\right] = \left\{\left(\frac{a}{T-t+b}\right)^2 + \frac{2ac}{T-t+b} + c^2\right\} ds, \tag{32}$$

which is consistent with the dynamics of (1) above.

It follows that the forward price function $f(\tau,T)$ at the future date τ is the stochastic integral

$$\begin{aligned} f(\tau,T) = f(t,T) \exp&\left\{\int_t^\tau \frac{a}{T-s+b}dW_1^*(s) - \tfrac{1}{2}\int_t^\tau \left(\frac{a}{T-s+b}\right)^2 ds\right\} \\ \exp&\left\{\int_t^\tau \left(\frac{2ac}{T-s+b}\right)^{\frac{1}{2}} dW_2^*(s) - \tfrac{1}{2}\int_t^\tau \frac{2ac}{T-s+b}ds\right\} \\ \exp&\left\{\int_t^\tau c \quad dW_3^*(s) - \tfrac{1}{2}\int_t^\tau c^2 ds\right\}. \end{aligned} \tag{33}$$

In addition, the forward market at the future date τ is represented by the associated Black'76 implicit volatility function $\sigma(\theta-\tau, T-\tau)$, where $\theta \in [\tau, T]$ is the exercise date of the option, and $T \geq \theta$ is the delivery date of the underlying forward.

Consider a portfolio of electricity derivatives at the future date τ. The idea of VaR is to analyse the downside properties of the probability distribution of the future portfolio value. We apply the simulation methodology in order to generate this probability distribution, from which Value at Risk can be calculated. The procedure consists of the following steps (which are repeated): First, use a random generator to draw a possible realisation for the future forward price function consistent with (33) above. Second, use the above valuation and approximation results to calculate the associated market value of each position, conditional on the realised forward price function (as well as the future implicit Black'76 volatility function). Thirdly, calculate the conditional market value of the portfolio (which follows immediately from value additivity). Now, for a large number of iterations, we approximate the probability distribution of the future portfolio value by the histogram following from the simulation results.

7 Value at Risk: An Example

7.1 Price Path Simulations

Equation (33) describes how the future forward price function is simulated from current market information. The $f(t, T)$ function is the forward price at time t for delivery at time T. The parameters a, b, and c are inputs to the volatility function.

In order to simulate possible price paths, we use (33) repeatedly. In Fig. 6 we present 100 simulated week prices based on this model. In each simulated path, the following procedure is followed. First, the forward function next week is simulated, integrating this curve from zero to 7 days gives the first week price. Next, we use this new forward curve in combination with the volatility curve to obtain the forward curve in the next step and so on. In this way, we obtain the correct and large short-term volatility in prices in addition to the much smaller volatility in prices as seen from today. We observe that the simulation model gives a substantial mean reversion in prices. This is in accordance with empirical data. The advantage of this method is that current information about the volatility curve and the term structure of prices is sufficient to perform this simulation.

7.2 Value at Risk Calculation

In the following, we focus on the downside risk of a given financial portfolio of forwards and options. Assume that we want a probability distribution which represents

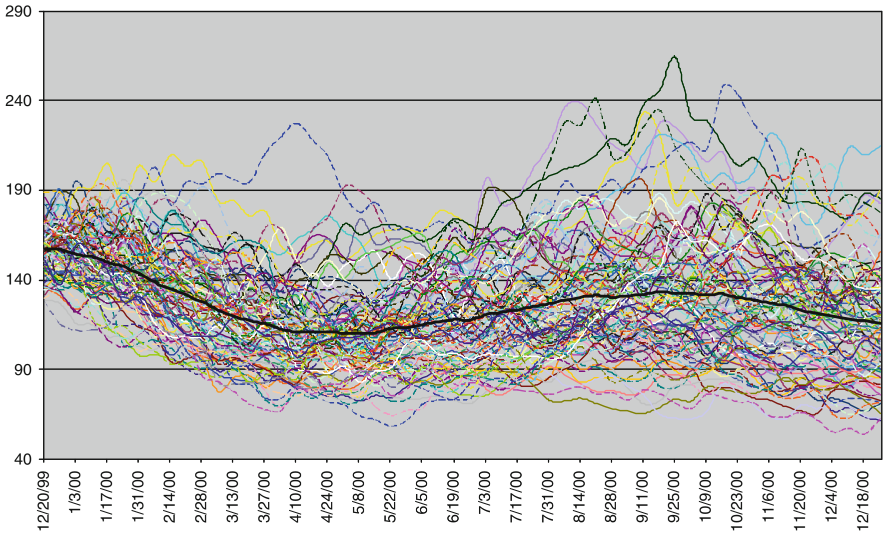


Fig. 6 Price path simulation

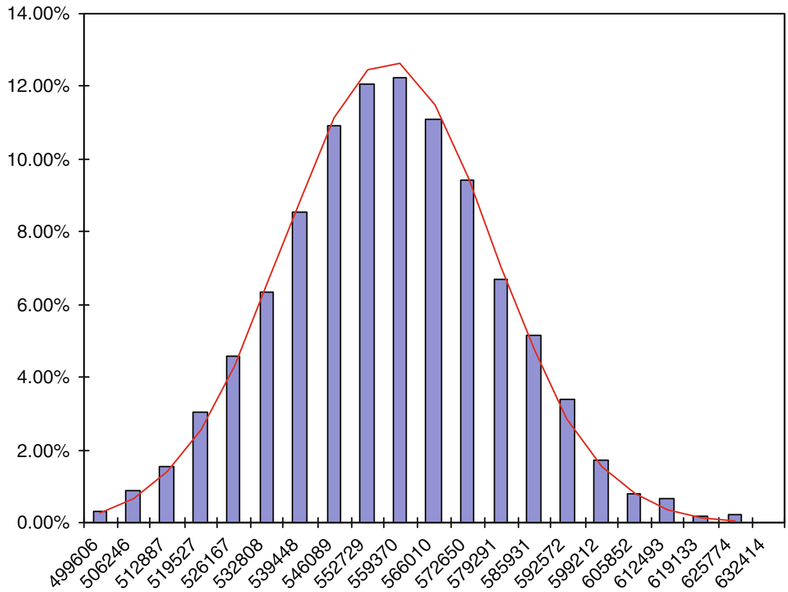


Fig. 7 Distribution for the value of the forward contract first half of 2000 in one week, NOK

the possible future market values of the portfolio in one week. First we simulate the term structure starting in one week using (33). For each simulation, we find the market value of all instruments in the portfolio. By assigning equal probability to each simulation, this gives a distribution of future market values.

We have chosen a very simple example portfolio. It consists of a forward contract for the first 6 months in year 2000 and a put option with exercise date at the last day of 1999, written on the same forward. The strike on the option is 120. Figure 7 gives the distribution in one week for the forward contract. Figure 8 gives similar information for the put option. In Fig. 9, we give the statistics for the total portfolio. The example illustrates the risk reduction effect from the option on the total portfolio.

8 Conclusions

The purpose of this paper is to derive a decision support model for professionals in the electricity market for valuation and risk management. The paper applies results and methods from finance, and incorporates the fact that electricity derivatives are written on a commodity flow rather than a bulk delivery.

The electricity derivatives market is represented by a forward price function, following from the quoted prices on traded contracts. The market uncertainty is modelled by a volatility function being a decreasing (and convex) function of time.

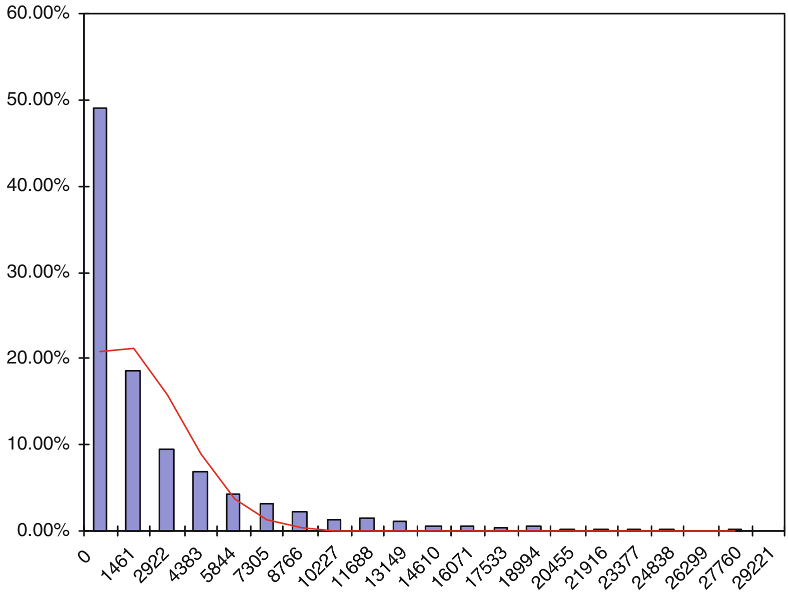


Fig. 8 Distribution for the value of a put option on the forward contract first half of 2000 in one week, strike equal 120, NOK

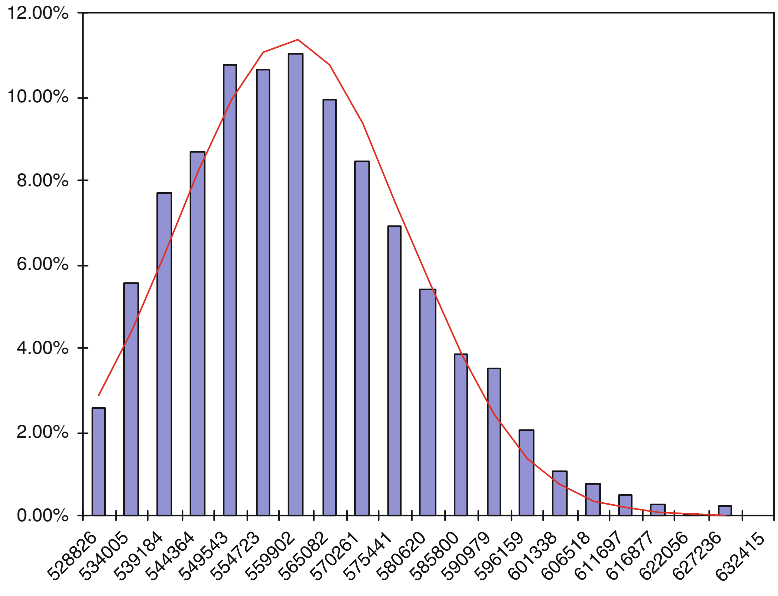


Fig. 9 Distribution for the value of a portfolio consisting of one forward and one put option, NOK

The paper presents value approximation results for "European" as well as Asian call options. The 3-factor market risk management model presented in the paper is compatible with these results and can be used for quantifying the future market risk of given portfolios (including VaR).

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Appendix

This appendix evaluates (12) above. Define the new integration variable $x = b + \frac{1}{2}(T_2 + T_1) - s$ and the constant $\alpha = \frac{1}{2}(T_2 - T_1)$, and write the integral as

$$\begin{aligned}\mathrm{Var}_t^*\left[\ln\left(\frac{F(\tau, T_1, T_2)}{F(t, T_1, T_2)}\right)\right] &= \left(\frac{a}{T_2 - T_1}\right)^2 \int_{X(\tau)}^{X(t)} \left(\ln\left(\frac{x+\alpha}{x-\alpha}\right)\right)^2 dx \\ &\quad + \frac{2ac}{T_2 - T_1} \int_{X(\tau)}^{X(t)} \ln\left(\frac{x+\alpha}{x-\alpha}\right) dx + c^2(\tau - t),\end{aligned}$$

where $X(t) = b + \frac{1}{2}(T_2 + T_1) - t$ and $X(\tau) \equiv b + \frac{1}{2}(T_2 + T_1) - \tau$.

Observe that with $b > 0$ and $t < \tau \leq T_1 < T_2$, we have $x + \alpha > 0$ and $x - \alpha > 0$ for $x \in [X(\tau), X(t)]$. Now, use the following two results:[7]

$$\begin{aligned}\int \left(\ln\left(\frac{x+\alpha}{x-\alpha}\right)\right)^2 dx &= (x+\alpha)\left(\ln(x+\alpha)\right)^2 \\ &\quad -2(x+\alpha)\ln(x+\alpha)\ln(x-\alpha) \\ &\quad +4\alpha\ln(2\alpha)\ln\left(\frac{x-\alpha}{2\alpha}\right) - 4\alpha \mathrm{dilog}\left(\frac{x+\alpha}{2\alpha}\right) \\ &\quad +(x-\alpha)\left(\ln(x-\alpha)\right)^2 - 4\alpha, \\ \int \ln\left(\frac{x+\alpha}{x-\alpha}\right) dx &= (x+\alpha)\ln(x+\alpha) - (x-\alpha)\ln(x-\alpha) - 2\alpha,\end{aligned}$$

where

$$\mathrm{dilog}(x) \equiv \int_1^x \frac{\ln(s)}{1-s} ds.$$

Substitute the results into the variance expression, to obtain the desired result.

[7] It is straightforward to verify these results using the fact that

$$\frac{\partial}{\partial x}\,\mathrm{dilog}\,(x) = \frac{\ln(x)}{1-x}.$$

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