Operational Matrix Methods - 6th Sem

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Motivation

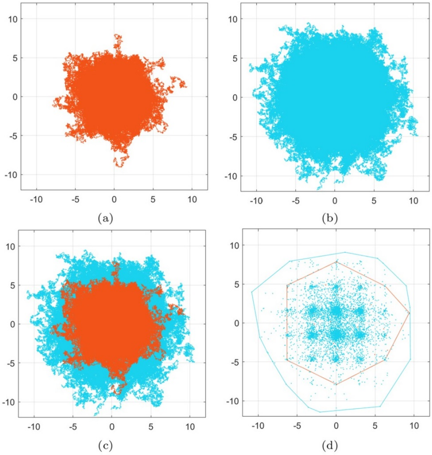

Figure 1: Modelling of cancer cell migration and tissue invasion.

Problem Statement

$$
\begin{aligned}
{}_{0}^{CF}D_t^{\alpha} N(x,t) &= D_N \frac{\partial^2 N}{\partial x^2} - a_3(1-e^{-U})N - c_4 NT - \omega_1 N^2 + \lambda_1 N,\\
{}_{0}^{CF}D_t^{\beta} T(x,t) &= D_t \frac{\partial^2 T}{\partial x^2} - a_2(1-e^{-U})T - c_3 NT - c_2 IT - \omega_2 T^2 + \lambda_2 T,\\
{}_{0}^{CF}D_t^{\gamma} I(x,t) &= D_I \frac{\partial^2 I}{\partial x^2} + a_1(e^{-U})I - c_1 IT - \lambda_1 I + \frac{\rho IT}{\mu + T} + \epsilon,\\
{}_{0}^{CF}D_t^{\zeta} U(x,t) &= D_U \frac{\partial^2 U}{\partial x^2} - d_2 U + \vartheta(t).
\end{aligned}
$$

Figure 2: Modified Parabolic PDE System

Linear Second Order PDE's

$$au_{xx} + bu_{xy} + cu_{yy} + du_x + eu_y + fu = 0$$

$b^2 - 4ac < 0$: Laplace Equation (Elliptic)

Steady-State Heat & Wave Equations

$$\nabla^2 U = U_{xx} + U_{yy} = 0$$

$b^2 - 4ac = 0$: Heat Equation (Parabolic)

Temperature of a body without a heat source.

$$U_t - U_{xx} = 0$$

$b^2 - 4ac > 0$: Wave Equation (Hyperbolic)

Displacement from rest situation.

$$U_{tt} - U_{xx} = 0$$

1D Heat Equation

Consider the Heat Equation:

$$\begin{cases} u_t(t,x) - u_{xx}(t,x) = 0 \\ u(0,x) = g(x) \end{cases} \qquad (t,x) \in (0,\infty) \times \mathbb{R}$$

Fundamental solution, **Heat Kernel**:

$$\Phi(t,x) = \frac{1}{\sqrt{4\pi t}} e^{-x^2/4t}$$

General solution is the convolution:

$$u_0(t,x) = (\Phi(t,\cdot) * g)(x) = \int_{-\infty}^{\infty} \Phi(t,x-y)g(y)dy$$

Intuition

To Obtain Computable Methods

- Closed solution obtained by rigorous, manual analysis.
- Idea: Approximate Solution space via suitable Vector Space.
- Express Differentiation as a Linear Transformation

Necessary and Sufficient Conditions

- $span(\mathrm{D}(\mathcal{B})) \subseteq span(\mathcal{B})$; D must be non-singular

Examples

- $span\{xsinx, xcosx, cosx, sinx\}$
- $span\{e^x sinx, e^x cosx, xe^x sinx, xe^x cosx,\}$
- Our Focus: **Orthogonal Polynomials**

Reduction to Convex Optimization

- Method of Separation of Variables
- Based on Initial Conditions choose P-family
- Compute or approximate Operational Matrix
- Convert to System of Non-Linear Equations

Fractional Time-Differentiation

- Riemann-Liouville
- **Caputo-Fabrizio**

Polynomial Families

- Legendre, Jacobi, Laguerre
- **Shifted Chebyshev, Fibonacci Wavelets**

Closed Form

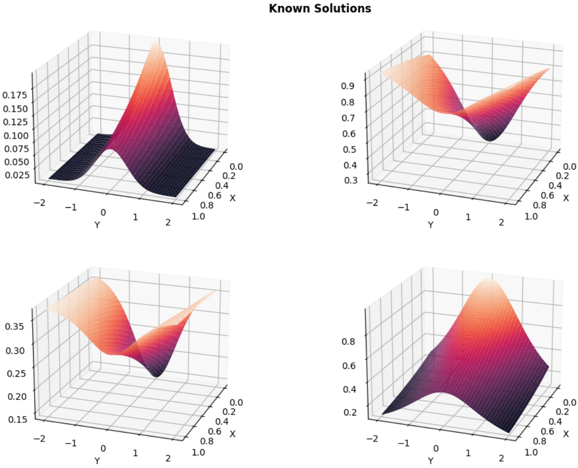

Figure 3: Solutions Computed By Hand

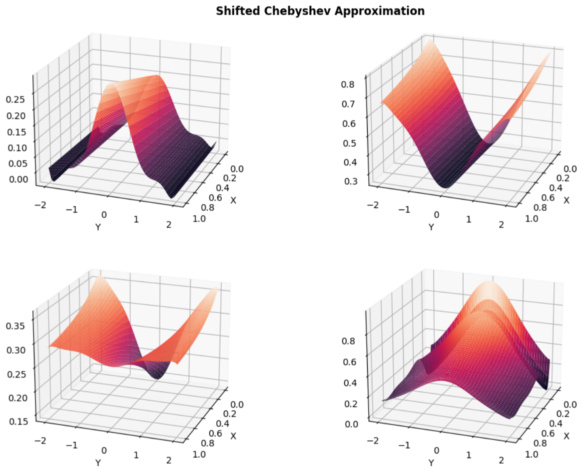

Figure 4: Shifted Chebyshev Approximation

Ordinary + FW

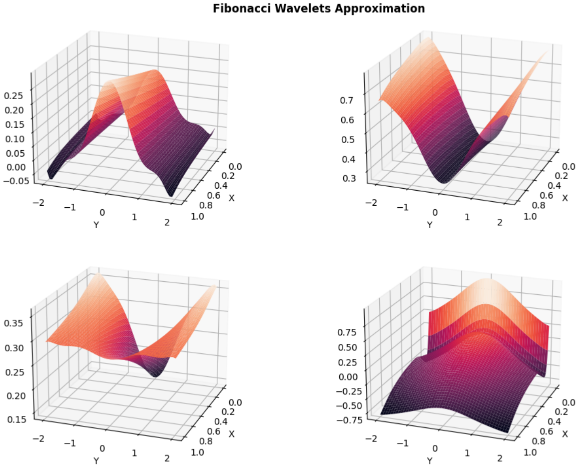

Figure 5: Fibonacci Wavelets Approximation

THANK YOU!

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