



## MINIMIZATION OF FINITE AUTOMATA

2017 Spring 1(b)

Solution:

Here, the given transition table is

$\delta / \Sigma$	0	1
$\rightarrow q_0$	$q_1$	$q_2$
* $q_1$	$q_2$	$q_3$
$q_2$	$q_2$	$q_2$
* $q_3$	$q_5$	$q_2$
* $q_4$	$q_4$	$q_2$
* $q_5$	$q_4$	$q_2$
$q_6$	$q_5$	$q_6$
$q_7$	$q_5$	$q_6$

Now, Zero equivalence classes are:

$$\Pi_0 = \{ \{ q_0, q_2, q_6, q_7 \}, \{ q_1, q_3, q_4, q_5 \} \}$$

1 equivalence tests:

$$q_0 \neq q_2$$

$$q_6 \equiv q_0$$

$$q_7 \equiv q_0$$

$$q_1 \neq q_3$$

$$q_1 \neq q_4$$

$$q_3 \equiv q_4$$

$$q_1 \neq q_5$$

$$q_3 \equiv q_5$$

$$\{ q_0, q_6, q_7 \}$$

$$\{ q_2 \}$$

$$\{ q_1 \}$$

$$\{ q_3, q_4, q_5 \}$$

Hence, 1 equivalence classes are:

$$\Pi_1 = \{ \{ q_0, q_6, q_7 \}, \{ q_2 \}, \{ q_1 \}, \{ q_3, q_4, q_5 \} \}$$

doesn't contain a null string. Provided that

2-equivalence tests:

$$\begin{aligned} q_0 &\neq q_6 \\ q_0 &\neq q_7 \\ q_6 &\equiv q_7 \end{aligned}$$

$$\begin{aligned} &\{q_0\} \\ &\{q_6, q_7\} \\ &\{q_2\} \{q_1\} \\ &\{q_3, q_4, q_5\} \end{aligned}$$

$$q_3 \equiv q_4$$

$$q_3 \equiv q_5$$

$$\therefore \Pi_2 = \{\{q_0\}, \{q_6, q_7\}, \{q_2\} \{q_1\}, \{q_3, q_4, q_5\}\}$$

3-equivalence tests:

$$q_6 \equiv q_7$$

$$\begin{aligned} q_3 &\equiv q_4 \\ q_3 &\equiv q_5 \end{aligned}$$

$$\begin{aligned} &\{q_0\} \\ &\{q_6, q_7\} \\ &\{q_2\} \\ &\{q_1\} \\ &\{q_3, q_4, q_5\} \end{aligned}$$

$$\therefore \Pi_3 = \{\{q_0\}, \{q_6, q_7\}, \{q_1\}, \{q_2\}, \{q_3, q_4, q_5\}\}$$

Since  $\Pi_3 = \Pi_2$ . The equivalence tests stops. The sets in  $\Pi_3$  are new states of minimized DFA.

Let us construct the transition table for minimized DFA

Q / $\Sigma$	0	1
$\rightarrow \{q_0\}$	$\{q_1\}$	$\{q_2\}$
* $\{q_1\}$	$\{q_1\}$	$\{q_3, q_4, q_5\}$
$\{q_2\}$	$\{q_2\}$	$\{q_2\}$
* $\{q_3, q_4, q_5\}$	$\{q_3, q_4, q_5\}$	$\{q_2\}$
$\{q_6, q_7\}$	$\{q_3, q_4, q_5\}$	$\{q_6, q_7\}$

more set

The transition diagram is:

