$\frac{2n!}{(n+1)!\cdot n!}$

4

HW 5 CSCI 104

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① Assuming each element in a subset can be unique, and there are only 5 unique letters in the 7 letter word "unusual" (the 3 u's are the same letter), then there is only 1 unique subset of 5 letters of the word "unusual": {u, n, s, a, l}.

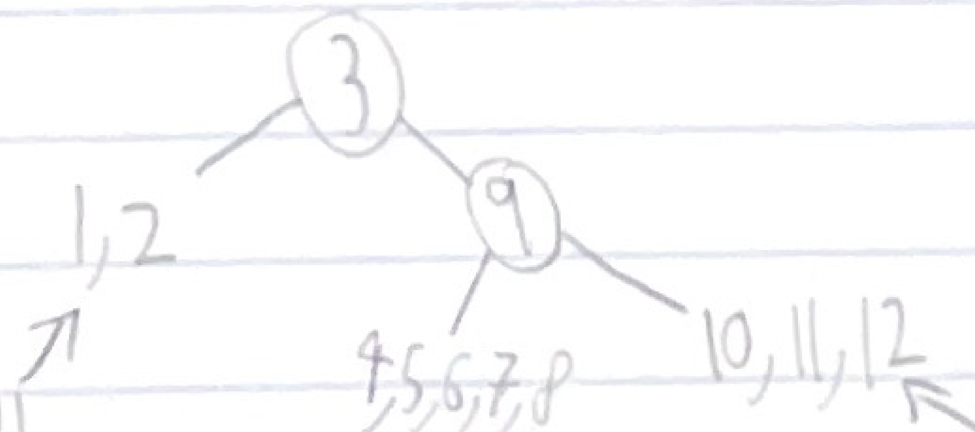
There would be $\frac{5!}{3!} = 20$ number of different strings because although order now matters, since there are 3 of the same letter and there can't be repetitions then the formula is $\binom{7}{3}\cdot\binom{4}{1}\cdot\binom{3}{1}\cdot\binom{2}{1}\cdot\binom{1}{1}$

② For this problem you need to think about the order of narrowing down the types of cards you get. To start you need to choose out of 13 ranks (Ace to King) for the 2 different pairs and the 3rd card that is different from both pairs $\binom{13}{3}$. Then out of those 3 ranks we only want 2 to be pairs so $\binom{3}{2}$. Out of those 2 diff pairs we need them to be 2 suits out of the 4 available, but we need that twice for the 2 diff pairs $\binom{4}{2}\cdot\binom{4}{2}$. Finally for the last card it needs to be 1 suit out of the 4 available so you add a $\binom{4}{1}$

$$\frac{\binom{13}{3}\cdot\binom{3}{2}\binom{4}{2}\binom{4}{2}\binom{4}{1}}{\binom{52}{5}} = \frac{13!}{\cancel{3!}(10!)}\cdot\frac{\cancel{3!}}{2!(1!)}\cdot\frac{4!}{2!(2!)}\cdot\frac{4!}{2!(2!)}\cdot\frac{4!}{1!(3!)} = 123{,}552$$

③ $${}_{16}C_1 \cdot {}_{15}C_6 = \frac{16!}{1!(15!)}\cdot\frac{15!}{6!\cdot 9!} = 80{,}080$$

④



formula: $\frac{2n!}{(n+1)!\cdot n!}$

$\frac{2(2)!}{3!\cdot 2!} = 2$ $\quad \frac{2(5)!}{6!\cdot 5!} = \frac{10!}{6!\cdot 5!} = 42$ $\quad \frac{2(3)!}{4!\cdot 3!} = 5$

$2\cdot 42\cdot 5 = \boxed{420 \text{ possible combinations}}$

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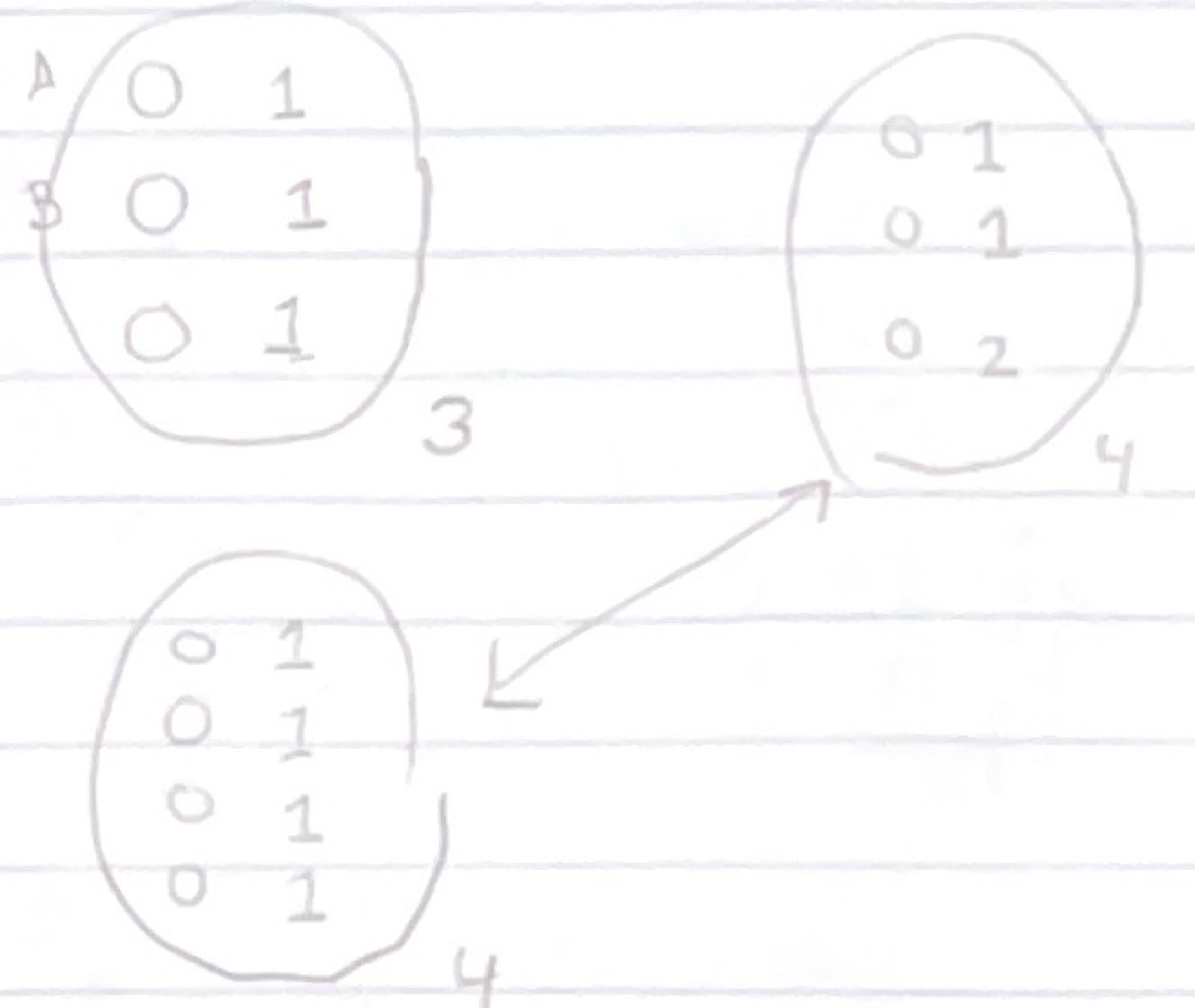
5



Taking the question at face value that we're only interested in "number of patients" then there will only be 8 combinations. The special case is that if one nurse is missing then only 3 patients will be served, other than that they are able to serve paitents 4-10 in any order. The case of 3 nurses serving 4 patients with 1 nurse sees 2 patients is the same as 4 nurses serving 1 patient because the outcome of serving 4 patients is the same.