MATRICES

Symmetric - if A = A transpose

eg. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Finding A^{-1} using elementary operations

If $|A| = 0$
Then A → singular matrix.

$\boxed{\text{Skew symmetric}}$ if A = - A transpose (eg) $\begin{bmatrix} 1 & x \\ 5 & 2 \end{bmatrix}$

$\boxed{\text{MINORS}}$

- Find MINORS
- Find CO-FACTORS
- COFACTOR MATRIX
- TRANSVERSE OF COFACTOR MATRIX
- TRANSVERSE OF COFACTOR MATRIX = ADJOINT OF A
- $A^{-1} = \dfrac{\text{Adj}\,A}{|A|}$

MATRIX METHOD - MARTINS

- $|A| \neq 0$
  A^{-1} exists
  [uniqe soln]
- $|A| = 0$
  - $(\text{Adj}\,A)B = 0$
    (infinite no of soln)
  - $(\text{Adj}\,A)B \neq 0$
    (no soln)

CRAMERS RULE

- $D \neq 0$
  $\boxed{\text{UNIQUE SOLN}}$
- $D = 0$
  Dy (or) Dz (or) Dx
  is non-zero
  $\boxed{\text{No solution}}$
- $D = 0,\ D_y = D_x = D_z = 0$
  Put z = k
  $\boxed{\text{Elimination}}$
  - $\boxed{\text{SOLVED, INFINITE NO OF SOLN}}$
  - $\boxed{\text{CANNOT BE SOLVED NO SOLUTION}}$

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APPLICATION OF DERIVATIVES

INCREASING OR DECREASING FUNCTIONS

(i) Increasing decreasing function
(ii) Absolute Maximum & Minimum
(iii) Stationery pt, Critical pt, turning pt
(iv) Local max, Local min.

1. Rate of change
2. Tangents & normals
3. Approximation
4. Inc. dec. functions
5. Word probs in maxima & minima

INCREASING DECREASING FUNCTION

Given $f(x)$

i) $f'(x) \geq 0$ => $f(x)$ is increasing
ii) $f'(x) > 0$ => $f(x)$ is strictly increasing
iii) $f'(x) \leq 0$ => $f(x)$ is decreasing
iv) $f'(x) < 0$ => $f(x)$ is strictly decreasing.

Given $f(x)$

$a \leq x \leq b$

(i) CRITICAL NUMBERS:

find the first derivation

$f'(x) = 0$

Critical nos are c_1 & c_2
(The no should be witin the boundary)

(ii) STATIONERY POINTS:

$[c_1, F(c_1)]$
$[c_2, F(c_2)]$

(iii) ABSOLUTE MAX & ABSOLUTE MIN:

$\left.\begin{array}{l} \bullet f(c_1) \\ \bullet f(c_2) \end{array}\right\}$ CRITICAL NO.S

$\left.\begin{array}{l} \bullet f(c_3) \\ \bullet f(c_4) \end{array}\right\}$ BOUNDARY NO.S

Abs. max - the max value among the 4
Abs. min - the min value among the 4

(iv) LOCAL MAX & LOCAL MIN

find 2nd derivative

At critical no's:-

Find:-

(i) $f''(c_1)$

(ii) $f''(c_2)$

If $f''(c)<0$ then local max is at c

If $f''(c)>0$ then local min is at c.

PROPERTIES OF DETERMINANTS (1(or) 2 marks)

1. The value of determinant is unaltered by interchanging rows and columns (transpose)
2. If any 2 rows or coloumnsare interchanged the determinant changes its sign but numerical value remains unchanged.
3. If any 2 rows or columns are identical then the value of the determinant is zero.

Condition for 3 pts to be collinear. Area=0

DETERMIANANTS

1) Proper ties of derminants
2) Finding A^{-1} using Matrix method
3) Solving simultaneous eqn using - Martins rule
4) Solving simultaneous eqn using - Cramers rule or determinant method
5) Area of triangle with vertices

Area $A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

RELATIONS & FUNCTIONS

TYPES OF PROBLEMS

TYPE 1 : Find $a * b$

TYPE 2 : Check whether $G, *$ is a group (or) Abelian group

TYPE 3 : Break the problem

TYPE 4 : Binary composition table

[Note: $a^{-1} \neq \frac{1}{a}$]

Binary operation $*$ on S is

$*: S \times S \longrightarrow S$ by $(a,b) \longrightarrow a * b$

A non empty set G, together with an operation $*$ (ie) $(G *)$ is said to be a group if

(i) CLOSURE AXIOM :

$a, b \in G \Rightarrow a * b \in G$

(ii) ASSOCIATIVE AXIOM :

$\forall a, b, c \in G$

$(a * b) * c = a * (b * c)$

(iii) IDENTITY AXIOM :

$\exists e \in G \ni :$

$a * e = a \quad \forall a \in G$

(iv) INVERSE AXIOM :

$\forall a \in G \; \exists a^{-1} \in G \ni :$

$a^{-1} * a = e$

Abelian group :

Commutative :- A binary operation $*$ on a set S of

$a * b = b * a \quad \forall a, b \in S$

Semi Group :- Properties (i) & (ii)

(i) CLOSURE (ii) ASSOCIATIVE AXIOM

Monoid :-

(i) CLOSURE (ii) ASSOCIATIVE (iii) IDENTITY

TYPES OF RELATIONS

• REFLEXIVE

If a ~~reis~~ a related to itself then the relation is called

If $(a,a) \in R$
$\Rightarrow aRa$
$\Rightarrow R$ is reflexive

• SYMMETRIC

If aRb
$\Rightarrow bRa$
$\Rightarrow R$ is symmetric

• TRANSITIVE

If aRb & bRc
$\Rightarrow aRc$
$\Rightarrow R$ is transitive.

• EQUIVALENCE RELATION

If R is (i) Reflexive
(ii) Symmetric
(iii) Transitive

FUNCTIONS

- COMPOSITION FUNCTION
- INVERSE
- DOMAIN & RANGE

CONTINUITY

TYPE : 1

$$f(x) = \begin{cases} \dfrac{x^2-2}{x-4} & x \neq 2 \\ 4 & x = 2 \end{cases}$$

TYPE : 2

$$f(x) = \begin{cases} \dfrac{x^2}{2} & 0 \le x \le 1 \\ x & x > 1 \end{cases}$$

INTEGRATION

- Definite integrals
- Indefinite integrals.

Differentiation	Integration
1. $\frac{d}{dx}(x^n) = nx^{n-1}$	1. $\int x^n.dx = \frac{x^{n+1}}{n+1} + c$
2. $\frac{d}{dx}(1) = 0$	2. $\int 1.dx = 1x + c$
3. $\frac{d}{dx}(\sin x) = \cos x$	3. $\int \sin x.dx = -\cos x + c$
4. $\frac{d}{dx}(\cos x) = -\sin x$	4. $\int \cos x.dx = \sin x + c.$
5. $\frac{d}{dx}(e^x) = e^x$	5. $\int e^x.dx = e^x + c$
6. $\frac{d}{dx}(\tan x) = \sec^2 x$	6. $\int \tan x.dx = \log \sec x + c$ $\int \sec^2 x.dx = \tan x + c$
7. $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$	7. $\int \cot x.dx = \log \sin x + c$ $\int \mathrm{cosec}^2 x.dx = -\cot x + c$
8. $\frac{d}{dx}(\sec x) = \sec x.\tan x$	8. $\int \sec x.dx = \log(\sec x - \tan x) + c$ $\int \sec x.\tan x.dx = \sec x + c$
9. $\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x.\cot x$	9. $\int \mathrm{cosec}\, x.dx = \log(\mathrm{cosec}\, x - \cot x) + c$ $\int \mathrm{cosec}\, x.\cot x.dx = -\mathrm{cosec}\, x + c$
10. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$	10. $\int \frac{1}{\sqrt{1-x^2}}.dx = \sin^{-1}x + c$
11. $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$	11. $\int \frac{-dx}{\sqrt{1-x^2}} = \cos^{-1}x + c$
12. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{\sqrt{1+x^2}}$	12. $\int \frac{dx}{\sqrt{1+x^2}} = \tan^{-1} + c$
13. $\frac{d}{dx}(\log x) = \frac{1}{x}$	14. $\int \frac{1}{x}.dx = \log x + c$ 15. $\int a^x.dx = \frac{a^x}{\log a} + c$

SPECIAL 9

1. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right) + c$

2. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right) + c$

3. $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

4. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c$

5. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left(x+\sqrt{x^2-a^2}\right) + c$

6. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left(x+\sqrt{x^2+a^2}\right) + c$

7. $\int \sqrt{a^2-x^2}\,.dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + c$

8. $\int \sqrt{x^2-a^2}\,.dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \log\left(x+\sqrt{x^2-a^2}\right) + c$

9. $\int \sqrt{x^2+a^2}\,.dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \log\left(x+\sqrt{x^2+a^2}\right) + c$

~~SUMMARY~~

TRIGNOMETRICAL FORMULAS

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
3. $\sec^2\theta - \tan^2\theta = 1$

Set 2

1. $\sin 2x = 2\sin x.\cot x$
2. $\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

Set 3

1. $\sin 3x = 3\sin x - 4\sin^3 x$
2. $\cos 3x = 4\cos^3 x - 3\cos x$

Set 4

1. $\sin x = 2\sin\frac{x}{2}.\cos\frac{x}{2}$
2. $\cos x = 2\cos^2\frac{x}{2} - 1$
3. $\cos x = 1 - 2\cos\frac{x}{2}$

Set 5

1. $2\sin x\cos y = \sin(x+y) + \sin(x-y)$
2. $2\cos x.\sin y = \sin(x+y) - \sin(x-y)$
3. $2\cos x\cos y = \cos(x+y) + \cos(x-y)$
4. $2\sin x.\sin y = \cos(x-y) - \cos(x+y)$

SUMMARY

Set 1. standard form

Type 1: with respect to x

Type 2: To the power of x

Set 2: Trignometical formula

Set 3: Substitution [t-method]

Set 4: Integration by parts (u.dv method)

$u.dv = uv - \int v.du$

Set 5: Special q.

Type 1: Application of form

Type 2: $\int \frac{dx}{ax^2+bx+c}$ (or) $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

Type 3: $\int \frac{px+q}{ax^2+bx+c} \cdot dx$ (or) $\int \frac{px+q}{\sqrt{ax^2+bx+c}}$

Type 4: $\int \frac{1}{f(x)\sqrt{g(x)}}$

a) $g(x)$-L , $f(x)$-L

b) $g(x)$-L , $f(x)$-Q

c) $g(x)$-Q , $f(x)$-L

d) $g(x)$-Q , $f(x)$-Q

$g(x)$-Linear

(i) $f(x)$-L

(ii) $f(x)$-Q

same method

$t = \sqrt{g(x)}$

$g(x)$ - Quadratic

(iii) $f(x)$-L

$f(x) = \frac{1}{t}$

(iv) $f(x)$-Q

take x^2 as common term & put $t = \sqrt{g(x)}$ (new $g(x)$)

Type 5: $\int \frac{dx}{a+b\sin x}$

$\sin x = \frac{2t}{1+t^2}$

$\int \frac{dx}{a+b\cos x}$

$\cos x = \frac{1-t^2}{1+t^2}$

Short method:

① $\int \frac{f'(x)}{f(x)} \cdot dx = \log f(x)$

② $\int \frac{f'(x)}{\sqrt{f(x)}} \cdot dx = 2\sqrt{f(x)}$

③ $e^x (f(x)+f'(x)) = e^x . f(x)$

Type 6: $\int (px+q)\sqrt{quadratic}$

Set 6: ~~Type 7~~: $\int$ partial fractions.

Type 1.

Type 2

Type 3

Type 4

For Type 5.

1. put $t = \tan\frac{x}{2}$

$dt = \sec^2\frac{x}{2} . \frac{\pi}{2} . dx$

$dx = \frac{2dt}{\sec^2\frac{x}{2}} . dt$

$dx = \frac{2dt}{1-\tan^2\frac{x}{2}}$ => $dx = \frac{2\,dt}{1+t^2}$

Replace the ~~ste~~ trignometic function.

DIFFERENTIAL EQUATION

1. VARIABLE SEPERABLE

Seperate the variables & integrate it

2. HOMOGENEOUS METHOD

(degree should be same)

- Bring $\frac{dy}{dx}$ format
- $y = vx$

Put

$$\frac{dy}{dx} = v + x.\frac{dv}{dx}$$

- $y = v$, $x = 1$

3. LINEAR FORM

FORM : 1

(when poyer of $y = 1$)

$$\frac{dy}{dx} \mp Py = Q$$

Integrating factor = If

$$If = e^{\int p.dx}$$

Required soln:

$$If \cdot y = \int Q.If.dx + c$$

FORM : 2

(when power of $x = 1$)

$$\frac{dx}{dy} \mp Px = Q$$

$$If = e^{\int p.dy}$$

Required soln:

$$x.If = \int Q.If.dy \mp c.$$

PROBABLITY - I

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
- $P(A \cup B) = P(A-B) + P(B-A) + P(A \cap B)$
- $P(A-B) = P(A \cap B')$
- $P(B-A) = P(A' \cap B)$
- $P(A \cup B)' = P(A' \cap B')$
- $P(A)' = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B)$ [A & B are mutually exclusive]

* either - $\cup$
* both - $\cap$
* atleast - $\cup$

⇒ $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

A	B
To be found	Condition given

~~⇒ P(~~

PROBABLITY - II

1) 10^{th} Basics
2) Conditional probability
3) Multiplication probability
4) Replacement problems

PROBABLITY - III

5) TOTAL Probablity
6) Bayes Theorm
7) Probability distribution
8) Mean variance