



POLITECNICO DI MILANO



A Brief Review of Mono-dimensional Heat Transfer: Conduction and Convection

Ref: Y. A. Cengel, Heat Transfer, a practical approach

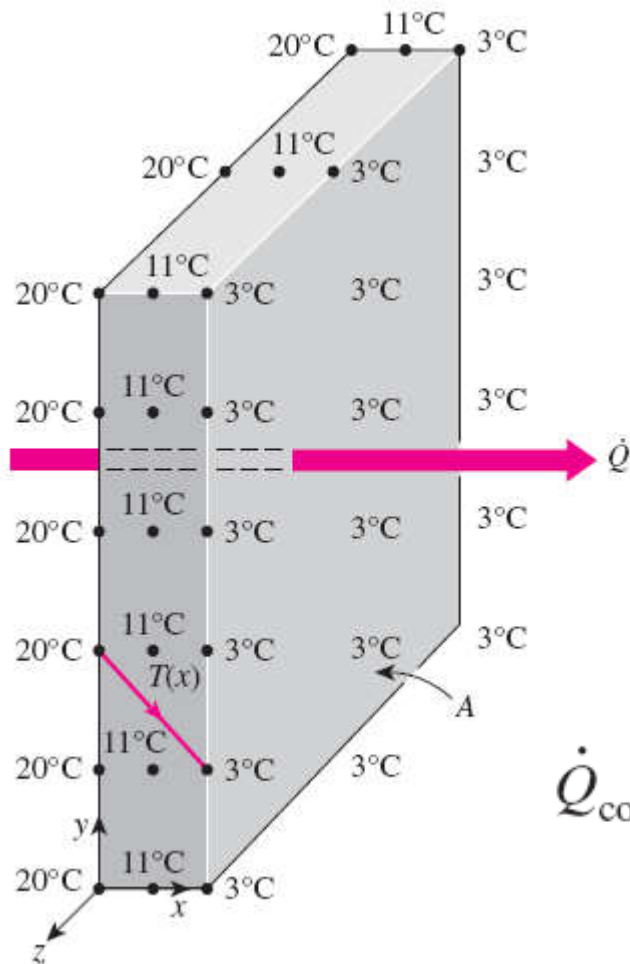
Technical Environmental Systems
Piacenza Campus

B. Najafi



Steady State heat conduction in Plane wall

- Heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional*.
- The temperature of the wall in this case depends on one direction only (say the x -direction) and can be expressed as $T(x)$.



$$\left(\begin{array}{c} \text{Rate of} \\ \text{heat transfer} \\ \text{into the wall} \end{array} \right) - \left(\begin{array}{c} \text{Rate of} \\ \text{heat transfer} \\ \text{out of the wall} \end{array} \right) = \left(\begin{array}{c} \text{Rate of change} \\ \text{of the energy} \\ \text{of the wall} \end{array} \right)$$

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt} \quad \text{for steady operation}$$
$$dE_{\text{wall}}/dt = 0$$

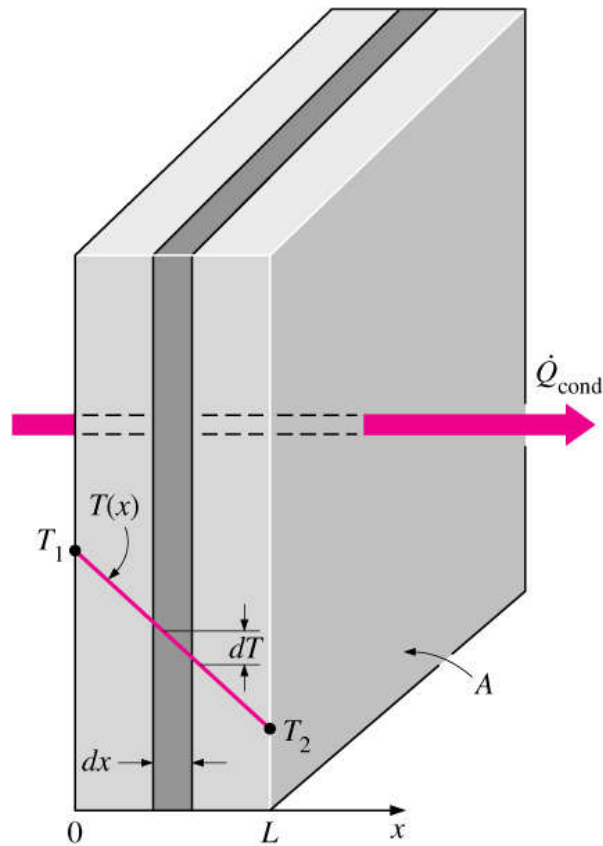
✓ In steady operation, the rate of heat transfer through the wall is constant.

$$\dot{Q}_{\text{cond, wall}} = -kA \frac{dT}{dx} \quad (\text{W})$$

✓ Fourier's law of heat conduction



Steady State heat conduction in Plane wall



- ✓ Under steady conditions, the temperature distribution in a plane wall is a straight line: $dT/dx = \text{const.}$

$$\dot{Q}_{\text{cond, wall}} = -kA \frac{dT}{dx}$$

$$\int_{x=0}^L \dot{Q}_{\text{cond, wall}} dx = - \int_{T=T_1}^{T_2} kA dT$$

$$\dot{Q}_{\text{cond, wall}} = kA \frac{T_1 - T_2}{L} \quad (\text{W})$$

The rate of heat conduction through a plane wall:

- ✓ is proportional to the average thermal conductivity, the wall area, and the temperature difference
- ✓ but is inversely proportional to the wall thickness.
- ✓ Once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 by T , and L by x .



Thermal Resistance Concept

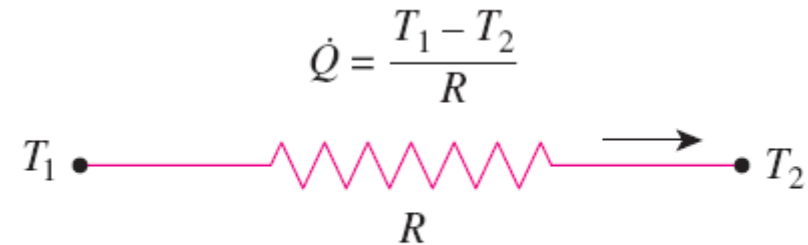
$$\dot{Q}_{\text{cond, wall}} = kA \frac{T_1 - T_2}{L}$$

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \quad (\text{W})$$

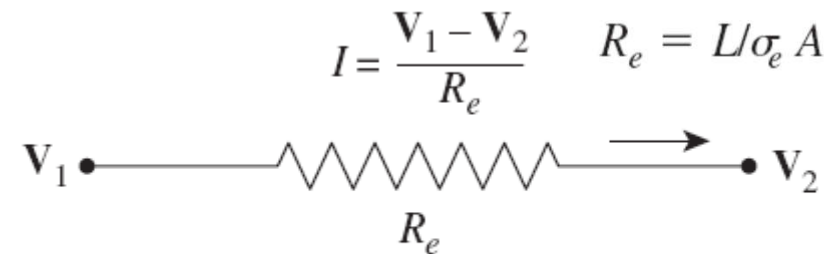
$$R_{\text{wall}} = \frac{L}{kA} \quad (^\circ\text{C/W})$$

✓ Conduction resistance of the wall:
Thermal resistance of the wall against heat conduction.

✓ Thermal resistance of a medium depends on the geometry and the thermal properties of the medium.



(a) Heat flow



(b) Electric current flow

Analogy between thermal and electrical resistance concepts:

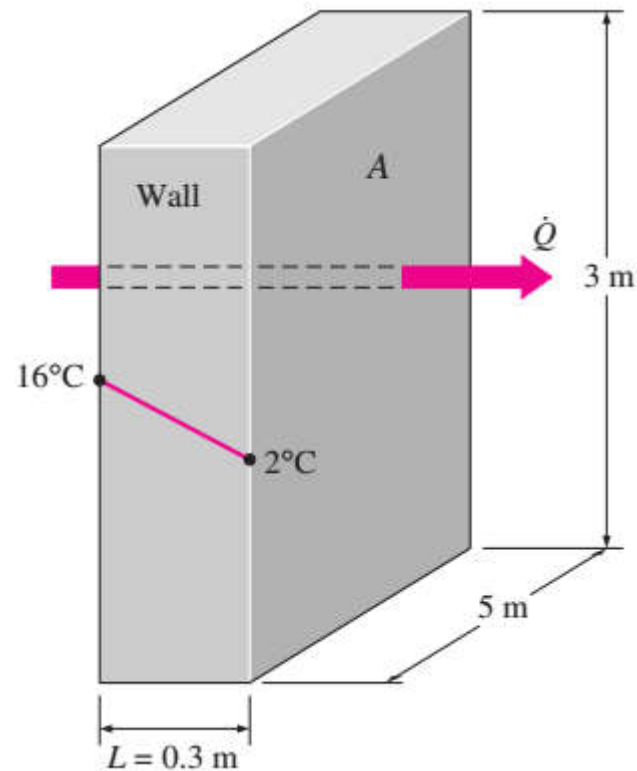
rate of heat transfer → electric current

thermal resistance → electrical resistance

temperature difference → voltage difference



Example A: Steady State heat conduction



❖ Find the rate of heat loss through the wall

✓ $K = 0.9 \text{ W/m C}$



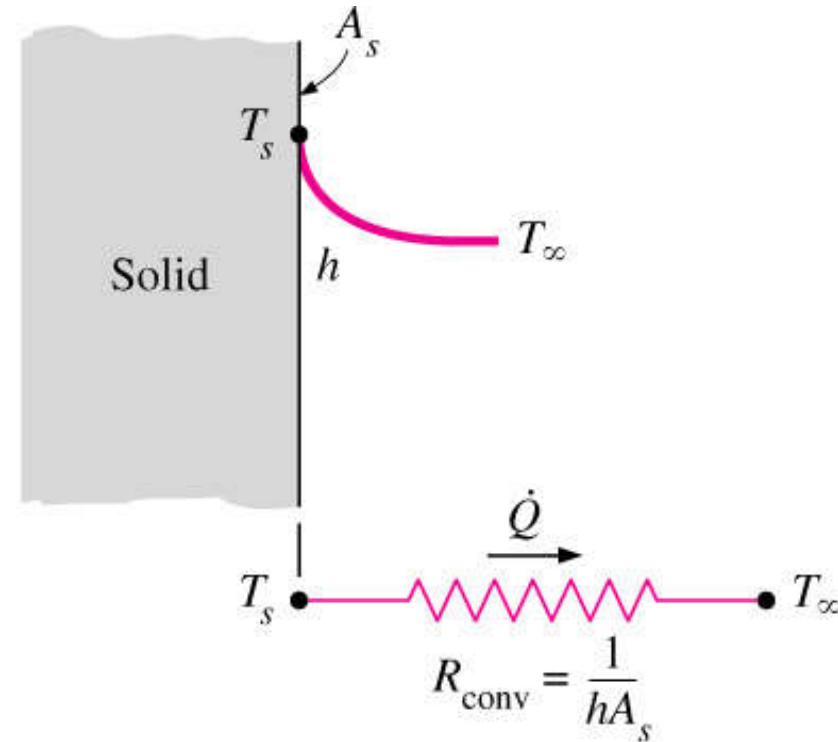
Convection: Newton's law of cooling

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \quad (\text{W})$$

$$R_{\text{conv}} = \frac{1}{hA_s} \quad (^\circ\text{C}/\text{W})$$

Convection resistance of the surface:
Thermal resistance of the surface against
heat convection.



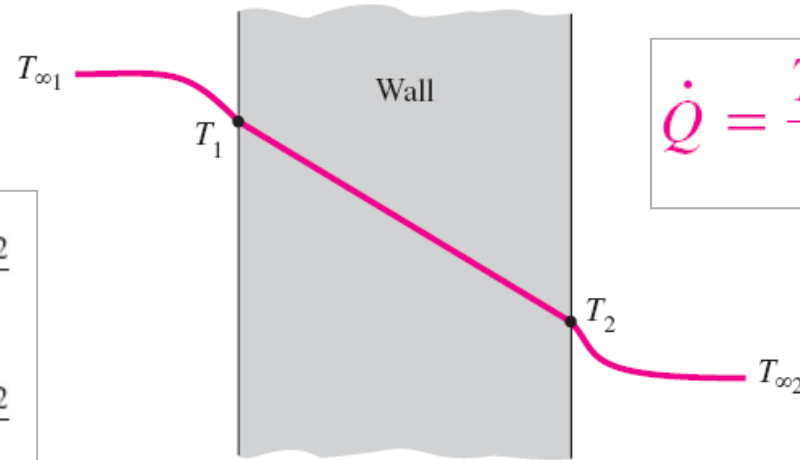
- ✓ When the convection heat transfer coefficient is very large ($h \rightarrow \infty$), the convection resistance becomes *zero* and $T_s \approx T$.
- ✓ That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process.
- ✓ This situation is approached in practice at surfaces where boiling and condensation occur.



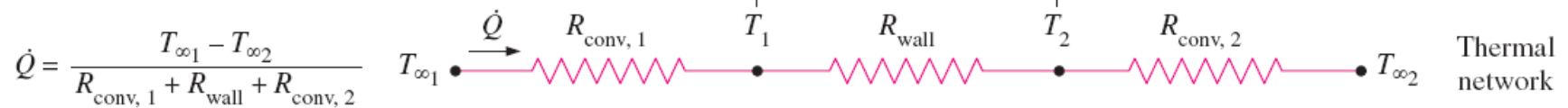
Thermal Resistance Network

$$\left(\begin{array}{c} \text{Rate of} \\ \text{heat convection} \\ \text{into the wall} \end{array} \right) = \left(\begin{array}{c} \text{Rate of} \\ \text{heat conduction} \\ \text{through the wall} \end{array} \right) = \left(\begin{array}{c} \text{Rate of} \\ \text{heat convection} \\ \text{from the wall} \end{array} \right)$$

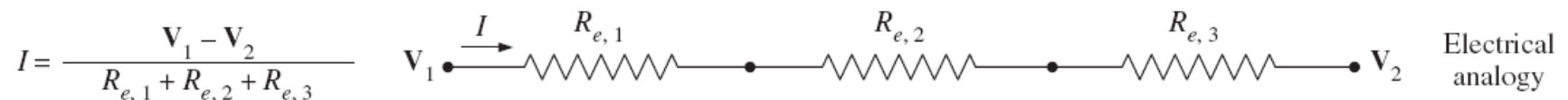
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$
$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv}, 2}}$$



$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$



Thermal network



Electrical analogy

The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

$$R_{\text{total}} = R_{\text{conv}, 1} + R_{\text{wall}} + R_{\text{conv}, 2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad (^\circ\text{C/W})$$



Temperature Drop

$$\Delta T = \dot{Q} R \quad (^\circ\text{C})$$

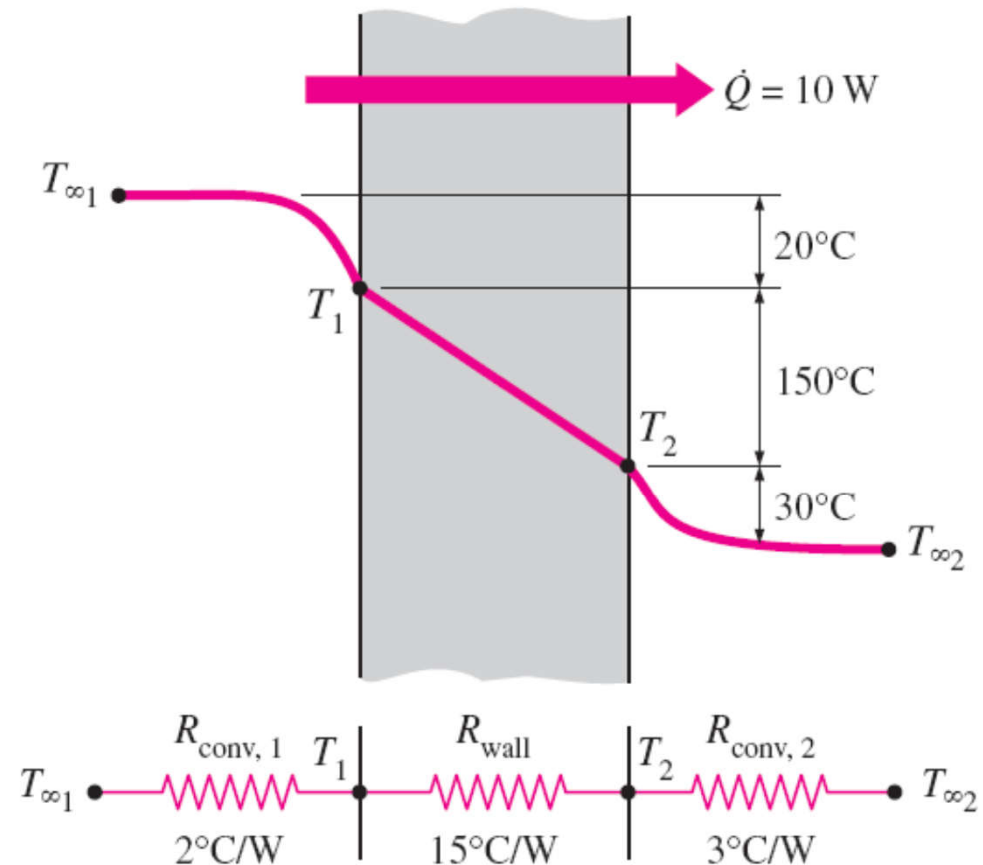
$$\dot{Q} = UA \Delta T \quad (\text{W})$$

$$UA = \frac{1}{R_{\text{total}}} \quad (^\circ\text{C}/\text{K})$$

U : overall heat transfer coefficient

Once \dot{Q} is evaluated, the surface temperature T_1 can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$

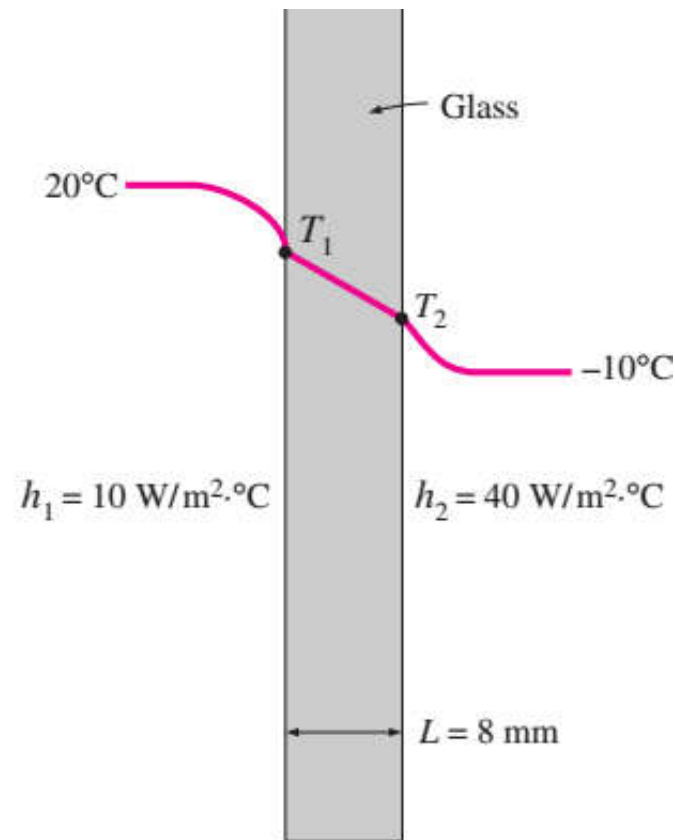


$$\Delta T = \dot{Q} R$$

The temperature drop across a layer is proportional to its thermal resistance.



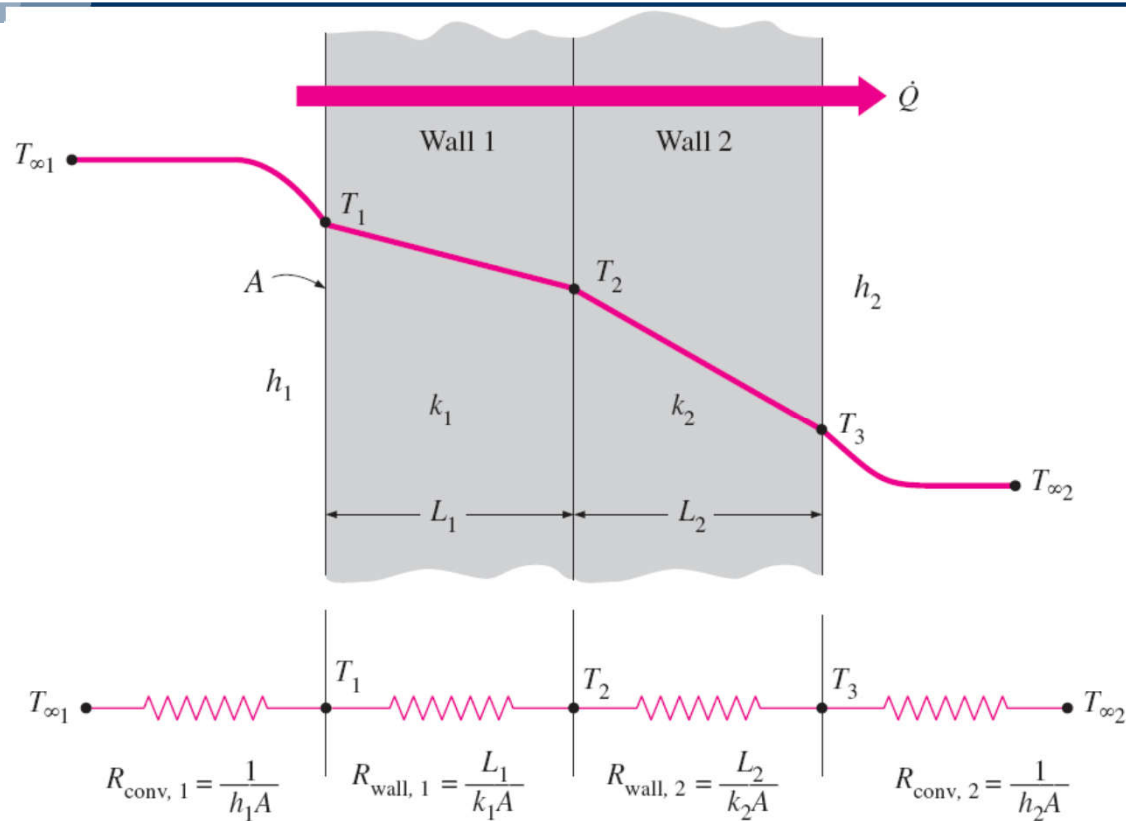
Example B: Heat loss through a single pane window



- ❖ Consider a 0.8 m high and 1.5 m wide glass window, shown above with a thermal conductivity of $k = 0.78 \text{ W/m}\cdot^\circ\text{C}$. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface



Multi-layer Plane walls



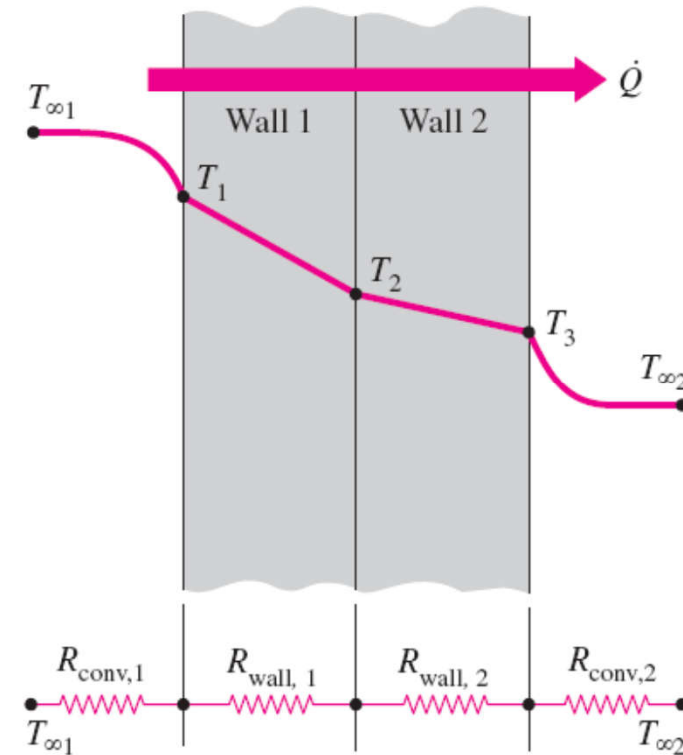
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv}, 1} + R_{\text{wall}, 1} + R_{\text{wall}, 2} + R_{\text{conv}, 2} \\ &= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \end{aligned}$$



$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total}, i-j}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv}, 1} + R_{\text{wall}, 1}} = \frac{T_{\infty 1} - T_2}{\frac{1}{h_1 A} + \frac{L_1}{k_1 A}}$$



The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and \dot{Q} is calculated.

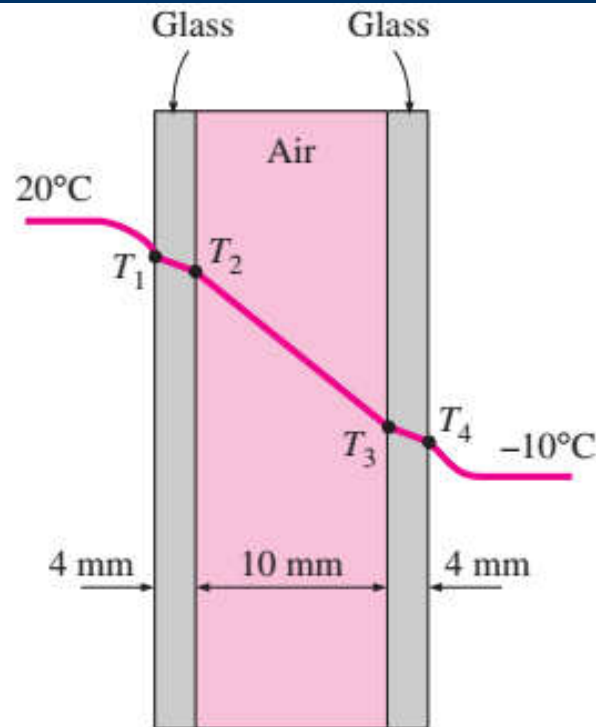
$$\text{To find } T_1: \dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}}$$

$$\text{To find } T_2: \dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv}, 1} + R_{\text{wall}, 1}}$$

$$\text{To find } T_3: \dot{Q} = \frac{T_3 - T_{\infty 2}}{R_{\text{conv}, 2}}$$



Example C: Heat loss through a double pane window



❖ Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass ($k = 0.78 \text{ W/m} \cdot ^\circ\text{C}$) separated by a 10-mm-wide stagnant air space ($k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface.

✓ Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_2 = 40 \text{ W/m}^2 \cdot ^\circ\text{C}$, which includes the effects of radiation.



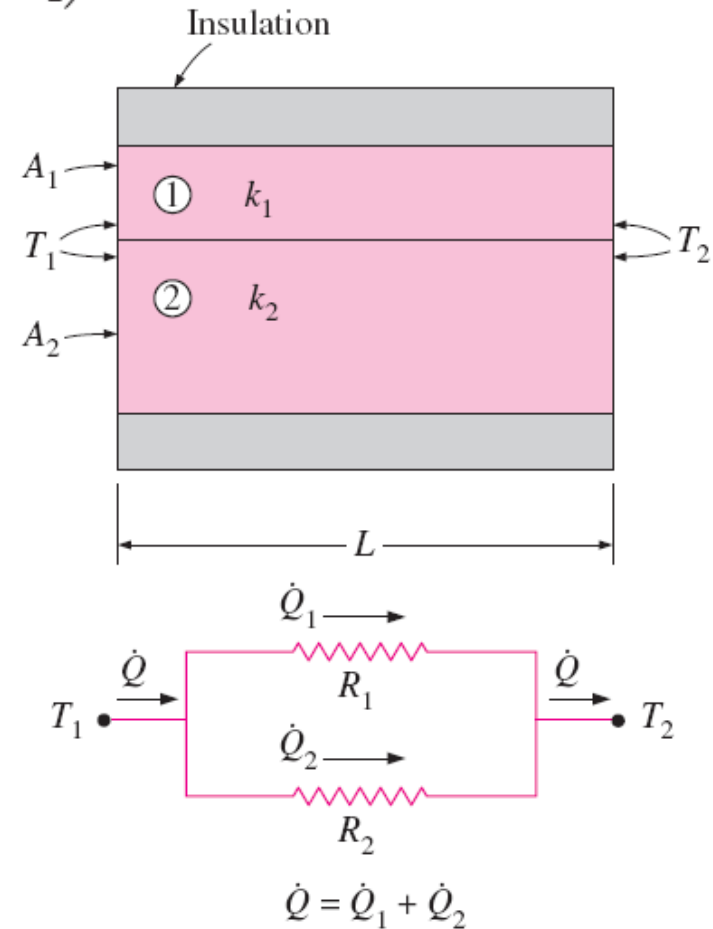
Generalized Thermal Resistance Networks

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}}$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$

Thermal resistance network for two parallel layers.

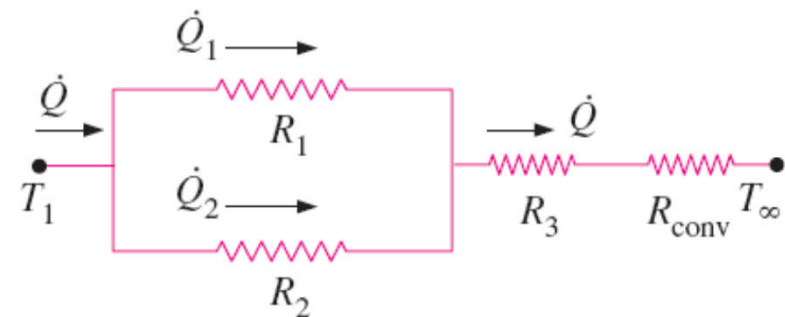
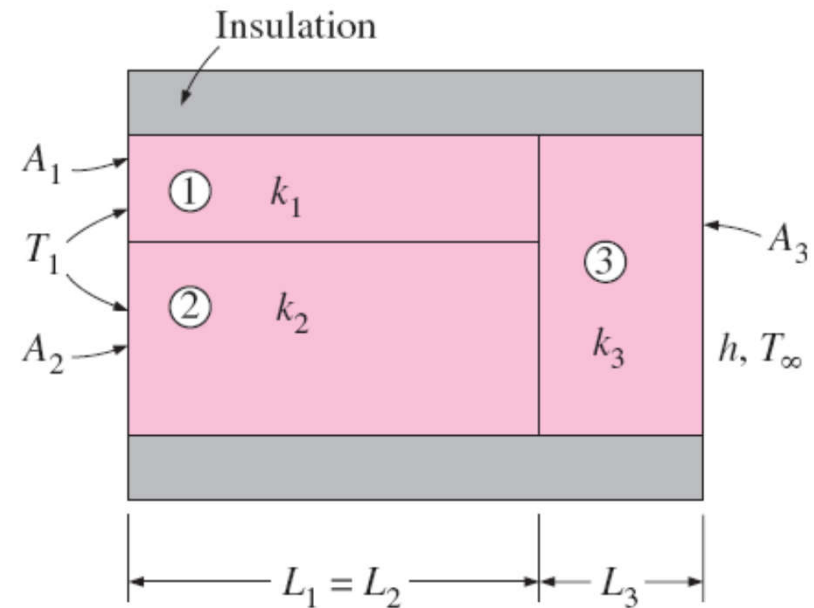




Generalized Thermal Resistance Networks

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}}$$
$$R_1 = \frac{L_1}{k_1 A_1} \quad R_2 = \frac{L_2}{k_2 A_2}$$
$$R_3 = \frac{L_3}{k_3 A_3} \quad R_{\text{conv}} = \frac{1}{h A_3}$$
$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}}$$

- ❖ Two possible assumptions in solving complex multidimensional heat transfer problems by treating them as one-dimensional using the thermal resistance network are :
 - ✓ any plane wall normal to the x -axis is *isothermal* (i.e., to assume the temperature to vary in the x -direction only)
 - ✓ any plane parallel to the x -axis is *adiabatic* (i.e., to assume heat transfer to occur in the x -direction only)



Thermal resistance network for combined series-parallel arrangement.



Example D: Heat loss through a composite wall

- ✓ A 3 m high and 5 m wide wall consists of long 16 cm 22 cm cross section horizontal bricks ($k = 0.72 \text{ W/m} \cdot ^\circ\text{C}$) separated by 3 cm thick plaster layers ($k = 0.22 \text{ W/m} \cdot ^\circ\text{C}$).
- ✓ There are also 2 cm thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$) on the inner side of the wall
- ✓ The indoor and the outdoor temperatures are 20°C and 10°C , and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_2 = 25 \text{ W/m}^2 \cdot ^\circ\text{C}$, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

