Andrea Foppiani // Week 5 ASSIGNMENT

T 1 → Summary

a) EMISSIVITY (ε) is a way to measure how close an object (surface) is to a blackbody in terms of the emitted radiation

↳ ratio between $\dfrac{\text{emitted radiation by a surface (at a given temperature)}}{\text{emitted radiation by a black body (at the same T)}}$

for this reason:

$\boxed{0 \leq \varepsilon \leq 1}$ where $\varepsilon = 1$ is a blackbody

- ε_λ spectral emissivity: ε at a certain wavelenght (λ) ← (ε_λ constant ↳ GREY SURFACE)
- ε_θ angular emissivity: ε in a specified direction (θ) ← given by the angle ← (ε_θ constant ↳ DIFFUSE SURFACE)

b) ABSORPTIVITY (α) is a coefficient used to measure (and calculate) how much of the incident radiation is absorbed by the surface (Knowing G)

$$\boxed{\alpha = \frac{G_{absorbed}}{G}}$$

↳ $G_{absorbed} = \alpha \cdot G$

c) REFLECTIVITY (ρ) is a coefficient used to calculate how much of the incident radiation (G) is reflected by a surface (Knowing the value of G)

$$\boxed{\rho = \frac{G_{reflected}}{G}}$$

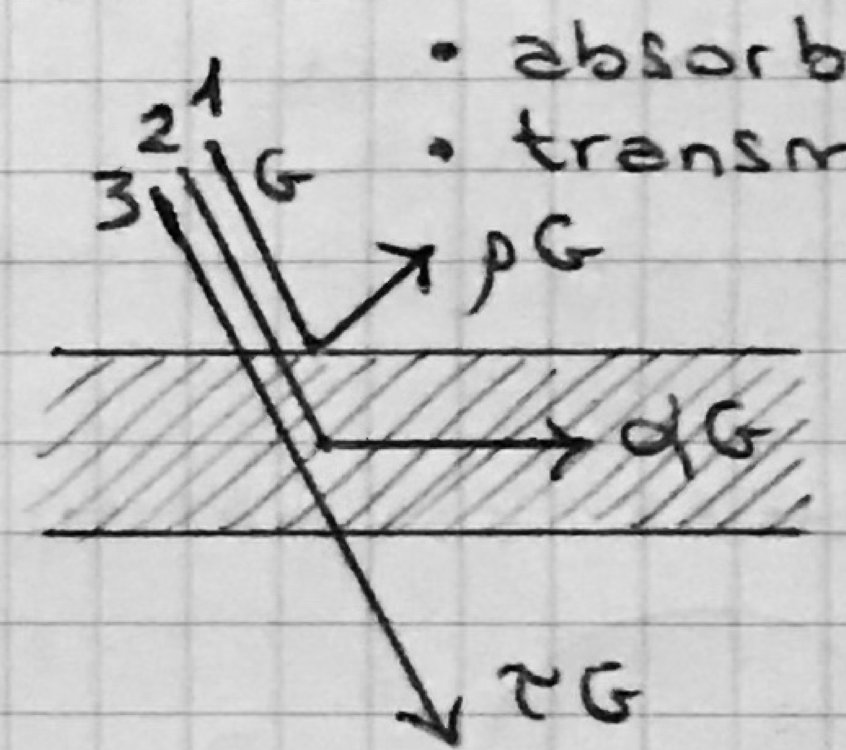
↳ $G_{reflected} = \rho \cdot G$

d) TRANSMISSIVITY (τ) is a coefficient used to calculate how much of the incident radiation (G) is transmitted by a surface (given G)

$$\boxed{\tau = \frac{G_{transmitted}}{G}}$$

↳ $G_{transmitted} = \tau \cdot G$

THE INCIDENT RADIATION (G) ON A SEMITRANSPARENT S. can be
- refleted (1)
- absorbed (2)
- transmitted (3)



$$\boxed{G_{abs} + G_{ref} + G_{tran} = G}$$

$(\alpha + \rho + \tau = 1)$

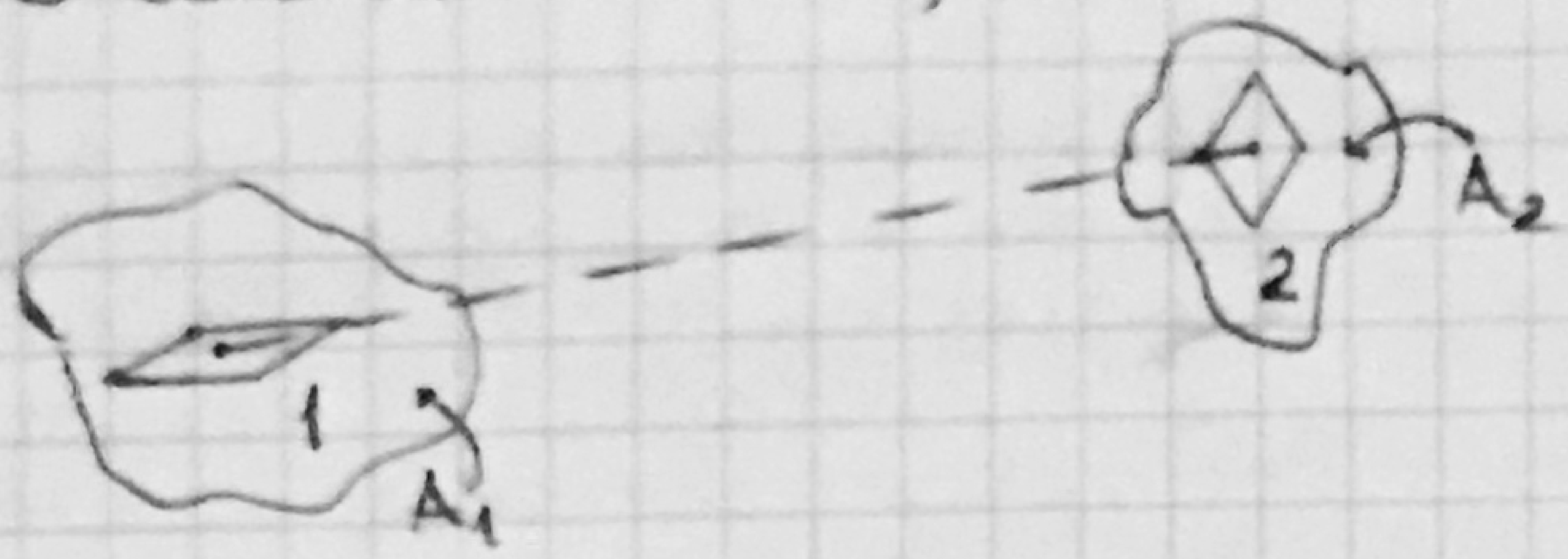
($\tau = 0$ for opaque surfaces)

! • by KIRCHOFF'S LAW we know that $\varepsilon_{\lambda,\theta}(T) = \alpha_{\lambda,\theta}(T)$

[emissivity (at a certain temperature with a certain λ and θ) equals to asborptivity of a surface (")

e) VIEW FACTOR (F_{12}) is the fraction (the partial quantity) of radiation leaving SURFACE 1 that reaches (and is intercepted) by a second surface, SURFACE 2.

! (F_{12} doesn't depend on the properties of SURF. 1 or SURF. 2)



↳ by the RECIPROCITY LAW, we know that

$$* \quad A_1 \cdot F_{12} = A_2 \cdot F_{21}$$

$$F_{12} = \frac{\text{radiation emitted by S1 and received by S2}}{\text{radiation emitted by S1}} = \frac{\dot{q}_{12}}{\dot{q}_1}$$

$$\boxed{\dot{q}_{12} = F_{12} \cdot \dot{q}_1}$$

(how much of the radiation from S1 reaches S2? IT DEPENDS ON HOW THE TWO SURFACES "SEE" EACH OTHER (commonly it's a low quantity))

emissive radiation

$$[E_{b\lambda}(T) = \sigma \cdot T^4]$$

f) HEAT EXCHANGE BETWEEN 2 BLACK SURFACES:

$\dot{Q}_{1\to2} = ?$

$$\dot{q}_{12} = F_{12} \cdot \dot{q}_1$$ (radiation emitted by S1 and received by S2)

$$\dot{q}_{12} = F_{12} \cdot \sigma \cdot T_1^4 \quad \left(\frac{W}{m^2}\right)$$

$$\dot{Q}_{12} = A_1 \cdot F_{12} \cdot \sigma T_1^4 \quad (W)$$ radiation emitted by S1 received by S2

in the same way: $\dot{Q}_{21} = A_2 \cdot F_{21} \cdot \sigma T_2^4 \quad (W)$ radiation emitted by S2 received by S1

$\Rightarrow \dot{Q}_{1\to2}$ = NET EXCHANGE BETWEEN S1 AND S2

$$= \dot{Q}_{12} - \dot{Q}_{21} = A_1 \cdot F_{12} \cdot \sigma T_1^4 - A_2 \cdot F_{21} \cdot \sigma T_2^4$$

(* given that $A_1 \cdot F_{12} = A_2 \cdot F_{21}$)

$$\boxed{\dot{Q}_{1\to2} = A_1 \cdot F_{12} \cdot \sigma \cdot (T_1^4 - T_2^4)}$$ FOR BLACKBODIES

8) HEAT EXCHANGE BETWEEN 2 GREY SURFACES:
(+ DIFFUSE)

given that, in this case the radiation leaving a surface is defined by RADIOSITY (J)

where: $J = \underbrace{\sigma T^4 \cdot \varepsilon}_{\text{emissive power}} + \underbrace{\rho \cdot G}_{\text{reflected radiation}}$

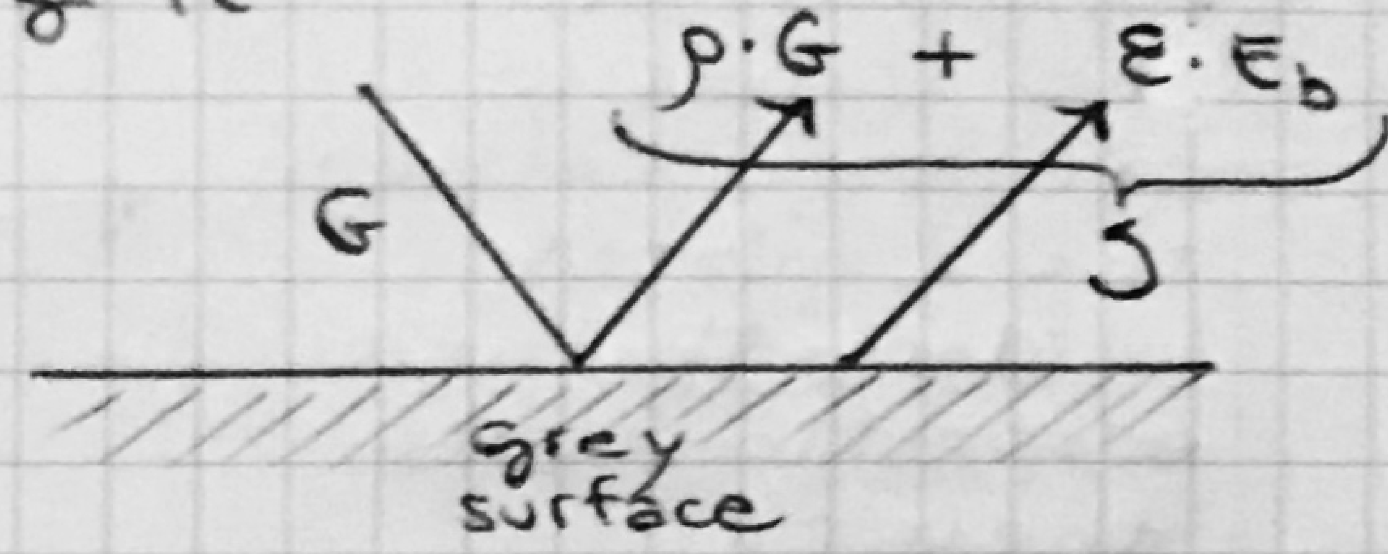
$\begin{cases} \rho = 1 - \alpha & \text{for opaque surfaces} \\ \alpha = \varepsilon \end{cases}$

which is the same of: $J = \sigma T^4 \cdot \varepsilon + (1-\varepsilon) G$

So a greybody is receiving an incident radiation G while a radiosity J is leaving it

↳ $\dot{q} = J - G \quad \left(\frac{W}{m^2}\right)$

$\dot{Q} = (J - G) \cdot A$



Knowing that $J = \overbrace{\sigma T^4}^{E_b} \cdot \varepsilon + (1-\varepsilon) \cdot G$
we can get the expression of $G \longrightarrow G = \dfrac{J - E_b \cdot \varepsilon}{1-\varepsilon}$

and then replace the found G in the formula written above for $\dot{Q}$

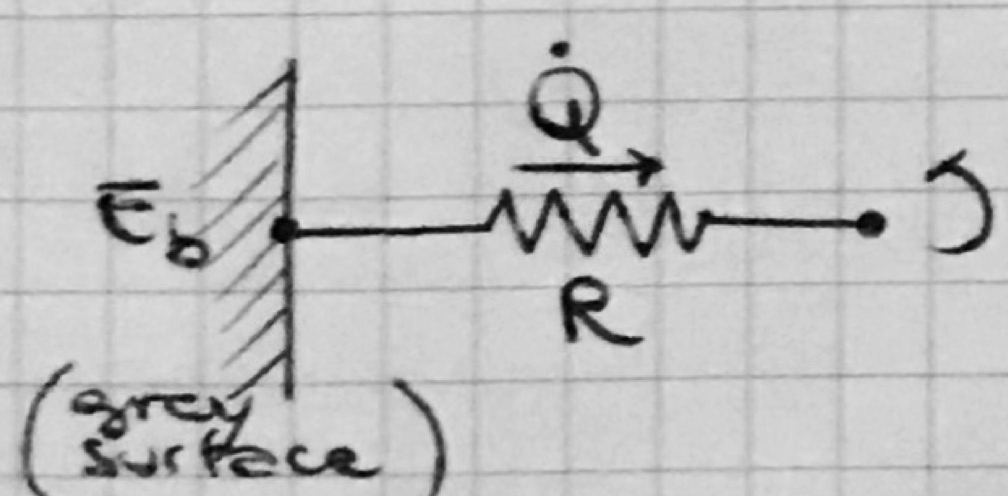
...

finding that: $\boxed{\dot{Q} = \dfrac{A \cdot \varepsilon}{1-\varepsilon}(E_b - J)}$ → heat actually leaving the surface of a greybody

using this formula we can get to the following conclusion:

there is an ELECTRICAL ANALOGY
(using the concept of electrical resistence) $R = \dfrac{1-\varepsilon}{A \cdot \varepsilon}$

$\dfrac{A \cdot \varepsilon}{1-\varepsilon} = \dfrac{1}{R} \Rightarrow \dot{Q} = \dfrac{E_b - J}{R}$

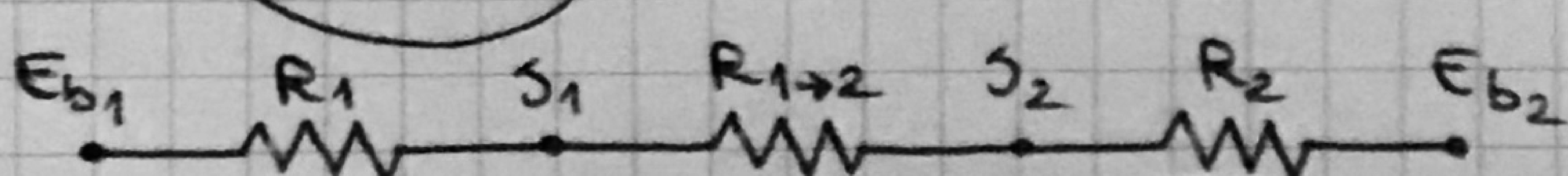


$\dot{Q}_{1\to2} = ?$

(What if we have two surfaces exchanging heat?)

$\dot{Q}_{1\to2} = \dfrac{J_1 - J_2}{R_{1\to2}}$ where $R_{1\to2} = \dfrac{1}{A_1 F_{12}}$

$A_1 \cdot J_1 \cdot F_{12} - A_2 \cdot J_2 \cdot F_{21}$
(with $A_1 F_{12} = A_2 F_{21}$)

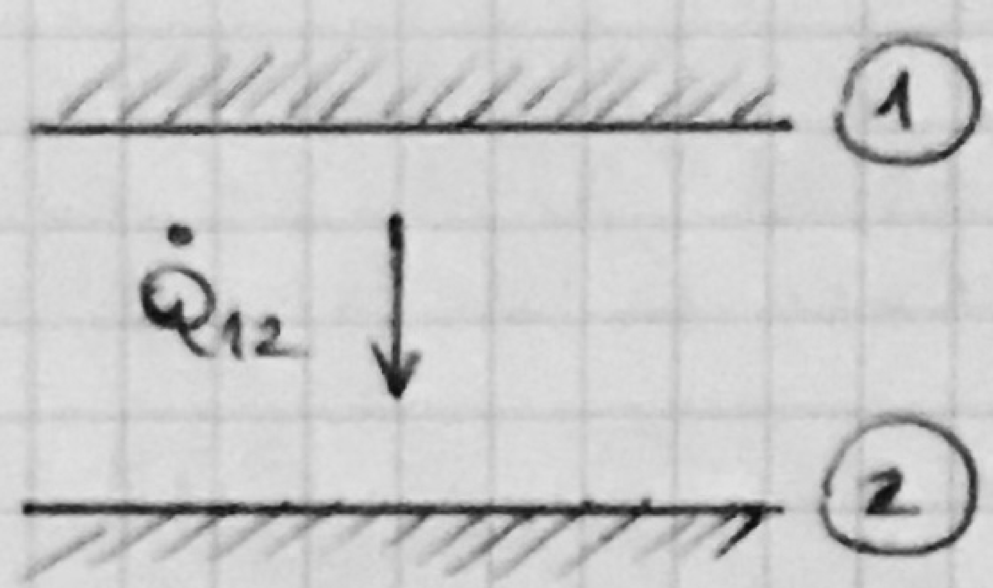


$$\boxed{\dot{Q}_{1\to2} = \dfrac{\sigma (T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1 \varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2 \varepsilon_2}}}$$

FOR GREYBODIES

T 2 → EXERCISE

Radiative exchange between two parallel surfaces (plates)



with: $\varepsilon_1 = \varepsilon_2 = 0,1$

$$\begin{pmatrix} T_1 = 800\,K \\ T_2 = 500\,K \end{pmatrix}$$

$$\dot{Q}_{1\to 2} = \frac{\sigma\,(T_1^4 - T_2^4)\cdot A}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} =$$

$$= A \cdot \left(\frac{800^4 - 500^4}{\frac{1}{0,1} + \frac{1}{0,1} - 1} \cdot 5,67 \cdot 10^{-8} \right) \frac{W}{m^2} =$$

$$= A \cdot 1035,82\ \frac{W}{m^2}$$

• OBSERVATION:
the radiative exchange between the 2 plates is lower if their emissivity ε is lower.