

Statistical Inference Course Project - Part 1

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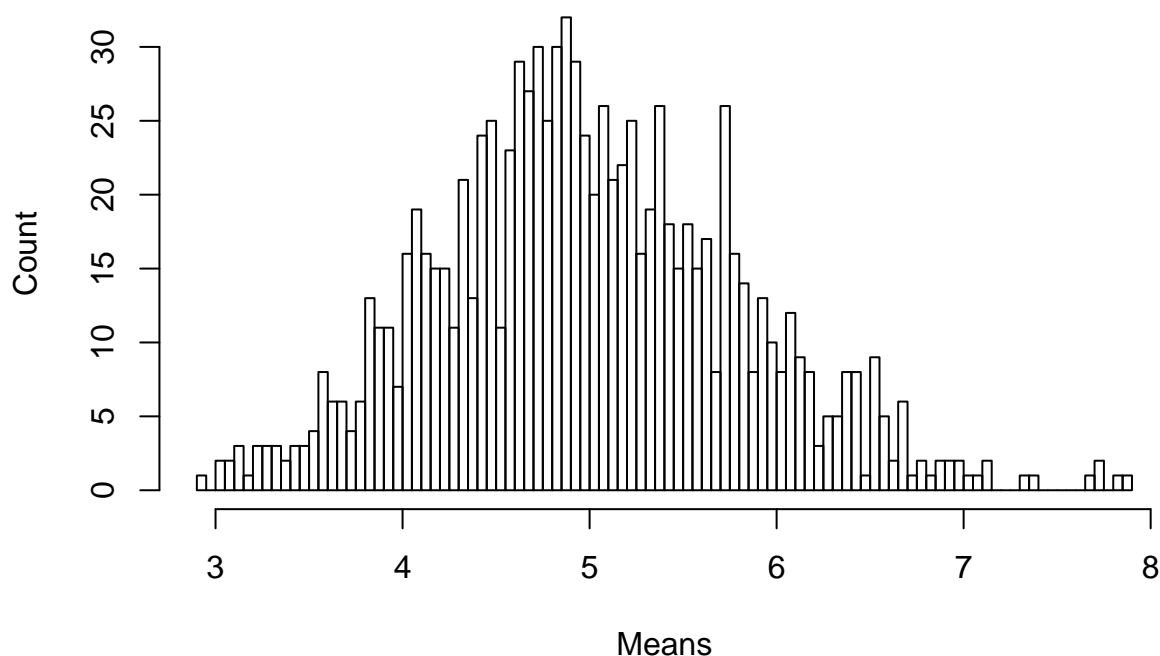
Introduction

In this project we will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. We will set $\lambda = 0.2$ for all of the simulations. Also, we will investigate the distribution of averages of 40 exponentials over a thousand simulations.

Simulations

```
lambda <- 0.2
n <- 40
numberOfSims <- 1000
set.seed(0) #for reproducibility
expDists <- matrix(data=rexp(n*numberOfSims, lambda), nrow = numberOfSims)
expDistMeans <- data.frame(means=apply(expDists, 1, mean))
```

Histogram of means



Sample and theoretical mean

The expected mean μ of an exponential distribution of rate λ is:

$$\mu = 1 / \lambda$$

```
mu <- 1/lambda
mu
```

```
## [1] 5
```

Then we can calculate the average of sample means for the simulations as

```
meanOfMeans <- mean(expDistMeans$means)
meanOfMeans
```

```
## [1] 4.989678
```

Conclusion: the mean of means converges to the expected mean for a large number of samples (1000).

Sample and Theoretical variance

The expected standard deviation σ of an exponential distribution rate λ is

$$\sigma = (1 / \lambda) / \sqrt{n}$$

```
sigma <- (1/lambda) / sqrt(n)
sigma
```

```
## [1] 0.7905694
```

The variance var of the standard deviation σ is

$$\text{var} = \sigma^2$$

```
var <- sigma ^ 2
var
```

```
## [1] 0.625
```

For the simulations, let var_x be the variance of the average sample mean of the 1,000 simulations of 40 randomly sampled exponential distributions and σ_x its standard deviation.

```
sigma_x <- sd(expDistMeans$means)
sigma_x
```

```
## [1] 0.8147837
```

```
var_x <- var(expDistMeans$means)
var_x
```

```
## [1] 0.6638725
```

Conclusion: the numbers are close to each other and should get closer if the number of simulations further increases.

Distribution and approximation to normal

The following chart shows the histogram of samples overlayed with a normal distribution with calculated values and the theoretical normal distribution. The theoretical and calculated distributions have a high degree of overlapping, allowing us to use the calculated one as a good approximation as the Central Limit Theorem states.

```
## Warning: package 'ggplot2' was built under R version 3.2.2
```

