Center-Pixel Model

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1 Introduction

This document details our Center-Pixel model. We consider the class of generalized linear models (GLM) i.e. a linear predictor with a link function which can be non-linear. This model reads an image and classifies it based on the intensity of the pixel at the center. As the center-pixel can have 0 to 255 range, we have 256 output nodes and the node corresponding to intensity is activated. Given that, we manually engineered the weights of the model, it is 100 percent accuracy on any test image with decision class probability always higher than 95 percent.

2 Center-Pixel Model

Let us assume that we just have one-pixel (center pixel) in the input image and we have to classify it based on its intensity value i.e. we want a model f which maps the center-pixel x to the space $\mathbb{R}^C$, where C is the number of output classes. In this case, the number of output nodes C is same as the number of intensity values i.e. 256 (0-255) (in case of 8-bit image). We choose to work with a linear model for simplicity where the model can be described as follows -

$$f(x) = Soft(wx + b)$$

where vector w, b belongs to $\mathbb{R}^C$ and $Soft(\cdot)$ refers to the softmax function.

We hand-designed the weight and bias vector in a way that our model f gives 100% accuracy and their general formulation is as follows -

w	$2*0$	$2*1$	-	-	-	$2*(C-1)$
b	$b_0 = C^2 + 1$	$b_1 = b_0 - 1^2$	-	-	-	$b_{C-1} = b_0 - (C-1)^2$

For this particular set of weight and bias vector, the model score can be seen in the following table for all the possible values of input pixel x (0-255).

f \ x	$f_{x,0}$	$f_{x,1}$	$f_{x,2}$	-	-	$f_{x,(C-1)}$
0	$f_{0,0} = b_0$	$f_{0,1} = f_{0,0} - 1^2$	-	-	-	$f_{0,C-1} = f_{0,0} - (C-1)^2$
1	$f_{1,0} = f_{1,1} - 1^2$	$f_{1,1}$	$f_{1,3} = f_{1,1} - 2^2$	-	-	$f_{1,C-1} = f_{1,1} - (C-2)^2$
2	$f_{2,0} = f_{2,2} - 2^2$	$f_{2,1} = f_{2,2} - 1^2$	$f_{2,3} = f_{2,2} - 1^2$	$f_{2,4} = f_{2,2} - 2^2$	-	$f_{2,C-1} = f_{2,2} - (C-3)^2$
-	-	-	-	-	-	-
-	-	-	-	-	-	-
-	-	-	-	-	-	-
$C-1$	$f_{C-1,0} = f_{C-1,C-1} - (C-1)^2$	-	-	$f_{C-1,C-2} = f_{C-1,C-1} - 2^2$	$f_{C-1,C-2} = f_{C-1,C-1} - 1^2$	$f_{C-1,C-1}$

where $f_{x,c}$ means the model score for the node/intensity c for the given pixel/image x.

Now, let us consider we have the entire image as input and we have to come up with a model whose output only depends on the center-pixel intensity. In this part, we feed an input image x of size $D \times D$ to our linear model ($f(x)$) with parameters, $W \in \mathbb{R}^{C \times N}$ and $B \in \mathbb{R}^C$. C is the no. of output classes or the number of color intensities a pixel can take i.e. 256 $(0-255)$ and N is the dimension of flattened version of the image i.e. $N = D * D$. Let $\hat{x} \in \mathbb{R}^N$ be the flattened image.

The model can be described as follows -

$$f(x) = Soft(W\hat{x} + b)$$

We again hand-designed the weight matrix and bias vector in a way that out model gives 100% accuracy and the general formulation is as follows -

$$\begin{bmatrix} \alpha & \beta & - & - & 2*0 & - & - & \gamma & \theta \\ \alpha & \beta & - & - & 2*1 & - & - & \gamma & \theta \\ - & - & - & - & 2*2 & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ \alpha & \beta & - & - & 2*(C-1) & - & - & \gamma & \theta \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ - \\ - \\ x_k \\ - \\ - \\ x_{N-1} \\ x_N \end{bmatrix} + \begin{bmatrix} b_0 = C^2 + 1 \\ b_1 = b_0 - 1^2 \\ b_2 = b_0 - 2^2 \\ - \\ - \\ - \\ b_{C-1} = b_0 - (C-1)^2 \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ - \\ - \\ - \\ f_{C-1} \end{bmatrix}$$

Here k in the index of centre pixel in the flattened image. α, β, γ and θ are some arbitrary numbers which can be chosen accordingly.

The speciality of this particular combination of weight matrix and bias vector is that it is equivalent to one-pixel model defined earlier and can be seen below -

$$\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ - \\ - \\ - \\ f_{C-1} \end{bmatrix} = \begin{bmatrix} \mathbb{Z} + 2 * 0 \times x_k + b_0 \\ \mathbb{Z} + 2 * 1 \times x_k + b_1 \\ \mathbb{Z} + 2 * 2 \times x_k + b_2 \\ - \\ - \\ - \\ \mathbb{Z} + 2 * (C-1) \times x_k + b_{C-1} \end{bmatrix}$$

where $\mathbb{Z}$ is weighted sum of all the pixels of the image except for the center pixel. It is a constant for all the output nodes and is given by $\alpha x_1 + \beta x_2 + ... + \gamma x_{N-1} + \theta x_N$.