DEPARTMENT
OF
ELECTRONICS & COMMUNICATION ENGINEERING

Communication Theory
18EC4DCCOT

(Theory Notes)

Autonomous Course

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Module – 1 Contents
Amplitude modulation: Elements of communication system, AM modulation techniques: DSBFC Time and frequency domain description, modulation index, Spectral analysis, Generation of AM using square law modulator, Detection of AM using Envelope Detector. DSBSC Time and frequency domain description, Generation of DSBSC using Balanced Modulator, Coherent detection of DSBSC, SSBSC with frequency domain description, VSB with frequency domain description, Frequency translation, FDM

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UNIT-I → AMPLITUDE MODULATION

INTRODUCTION:

Modulation is a process of varying one of the characteristics of high frequency sinusoidal (the carrier) in accordance with the instantaneous values of the modulating (the information or audio) signal.

The high frequency carrier signal is represented by eqn ①

$$c(t) = A_c \cos(2\pi f_c t) \quad \text{--- ①}$$

Where

$c(t)$ → Instaneous values of the cosine wave

A_c → Its maximum value

f_c → carrier frequency

Any three parameters of a high frequency carrier can be varied by the modulating signal, giving rise to amplitude, frequency or phase modulation respectively.

1.1 Need for modulation:

* Reduces the height of antenna:- Height of antenna is a function of wavelength λ. The minimum height of antenna is given by $\lambda/4$. i.e, $[c/4f]$. As the transmitting frequency is increased, height of the antenna is decreased.
* Avoids mixing of signals:- All audio signals ranges from 20Hz to 20KHz. In order to separate the various signals, it is necessary to translate them all to different portions of channel; each must be given its own different carrier frequency.
* Increase the range of communication:- Low frequency signals have poor radiation and they get highly attenuated. Modulation increases the frequency of the signal & thus they can be transmitted over long distances.
* Allows multiplexing of signals:- Modulation allows the multiplexing to be used. Multiplexing means transmission of two or more signals simultaneously over the same communication channel.

* Allows adjustments in the bandwidth:- Bandwidth of a modulated signal may be made smaller or larger.
* Improves quality of reception:- Modulation techniques like frequency modulation, pulse code modulation reduces the effect of noise to great extent. Reduction of noise improves quality of reception.

2 AMPLITUDE MODULATION:

Amplitude modulation is defined as the process in which the amplitude of the carrier wave is varied in proportion to the instantaneous amplitude of the modulating signal.

Let A_m = Amplitude of modulating signal
A_c = Amplitude of carrier signal.

The degree of modulation is reflected by the modulation index, i.e the ratio of maximum value of the modulating signal to the maximum value of the carrier.

$$m = \frac{A_m}{A_c}$$

1.1 Time domain description [Standard form OR General expression]

Modulating signal is given by $m(t)$.

The carrier signal is given by $c(t) = A_c \cos 2\pi f_c t$ --- (2)

Let A be the amplitude of the modulated signal & is given by

$$A = A_c + m(t) \quad \text{--- (3)}$$

The instantaneous value of AM wave is given by

$$s(t) = A \cos 2\pi f_c t.$$

Substitute eqn (3) in above equation,

$$s(t) = [A_c + m(t)] \cos 2\pi f_c t$$

$$= A_c [1 + \tfrac{1}{A_c} m(t)] \cos 2\pi f_c t].$$

$$= A_c [1 + K_a m(t)] \cos 2\pi f_c t \quad \text{--- (4)}$$

Where K_a is a constant called the amplitude sensitivity of the modulator.

The amplitude of the time function multiplying $\cos 2\pi f_c t$ in equation (4) is called the envelope of the AM wave $s(t)$.

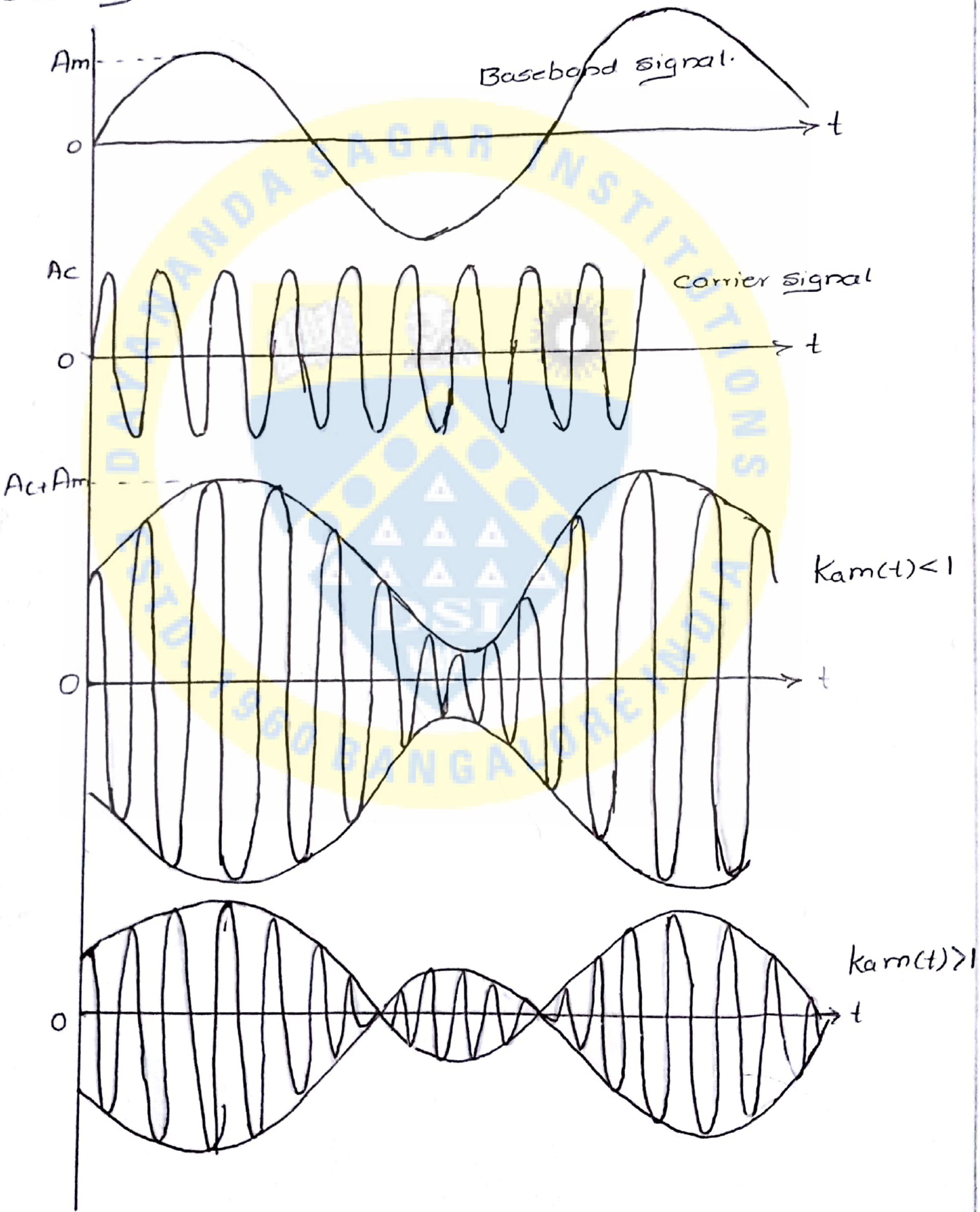
Using $a(t)$ to denote the envelope,

$$a(t) = A_c\,[1 + k_a m(t)] \quad \text{---} (5)$$

When $|k_a m(t)| \le 1$ for all t, the term $1 + k_a m(t)$ is always non-negative. Then the envelope of the AM wave can be written as

$$a(t) = A_c\,[1 + K_a m(t)] \quad \text{for all } t$$

When $|k_a m(t)| > 1$ for some t, then equation (5) is used for evaluating the envelope of the AM wave. The maximum absolute value of $k_a m(t)$ multiplied by 100 is referred to as the percentage modulation.



Simplifying equation (4),

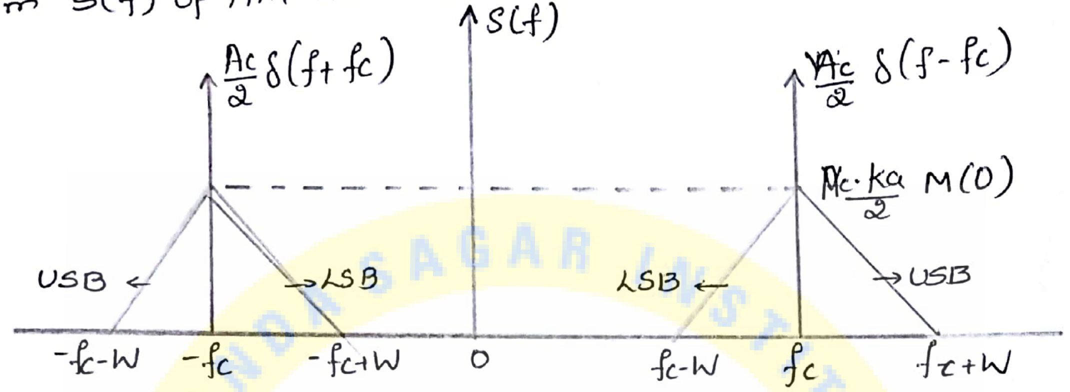
$$s(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t \quad \text{---} (5)$$

1.2 Frequency-Domain Description

Taking F.T of equation (5)

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] + \frac{A_c k_a}{2}\left[M(f-f_c)+M(f+f_c)\right] \quad \text{--} (6)$$

Spectrum S(f) of AM wave is as shown in figure below.



We can observe that

* The message spectrum centered at $f=0$ & extending from $-W$ to W gets translated to ~~α~~ f_c.

* On either sides of $\pm f_c$, sidebands known as the upper and lower side bands are present. Frequencies $f_c<|f|<f_c+W$ constitutes upper side band (USB) & frequencies $f_c-W<|f|$ constitutes lower side band (LSB).

* The width of the spectrum S(f) for the positive frequencies defines the transmission bandwidth B_T for AM wave and is given by

$$f_c + W - (f_c - W)$$

$$\cancel{f_c} + W - \cancel{f_c} + W$$

$$\Rightarrow B.W = 2W \Rightarrow 2f_m.$$

1.3 Single-tone modulation:

Time domain description

Consider a modulating wave m(t) that consists of a single tone or frequency component i.e,

$$m(t) = A_m \cos 2\pi f_m t \quad \text{---} (7)$$

Substitute equation (7) in equation (4),

$$s(t) = A_c\left[1 + k_a A_m \cos 2\pi f_m t\right]\cos 2\pi f_c t \quad \text{---(8)}$$

where $k_a A_m = m$ or μ $\left[\because k_a = 1/A_c \Rightarrow 1/A_c \cdot A_m = m\right]$

$$s(t) = A_c\left[1 + m\cos 2\pi f_m t\right]\cos 2\pi f_c t$$

$$= A_c\cos 2\pi f_c t + m A_c \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

W.K.T $\cos a \cdot \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)$

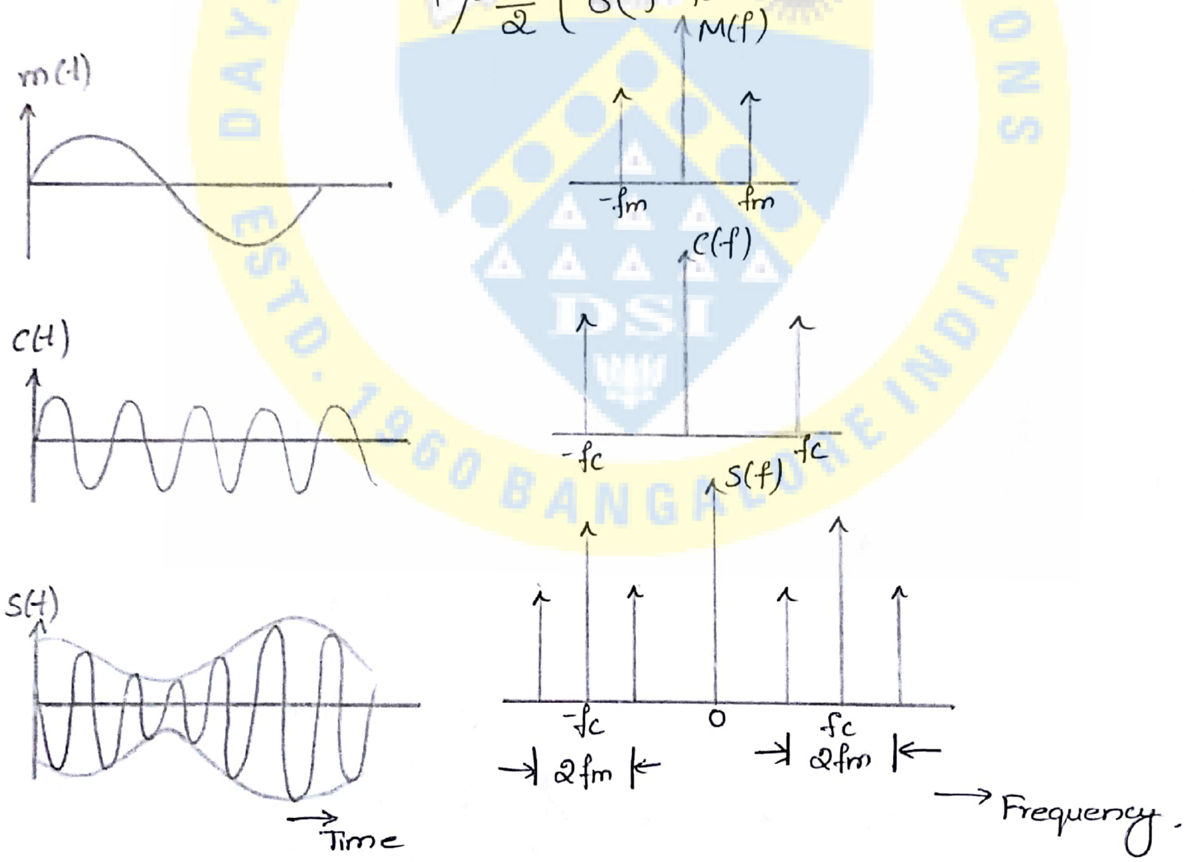
$$\therefore s(t) = A_c\cos 2\pi f_c t + \frac{\mu A_c}{2}\left[\cos(2\pi f_c - 2\pi f_m)t + \cos 2\pi(f_c+f_m)t\right]$$

$$= A_c\cos 2\pi f_c t + \frac{\mu A_c}{2}\left[\cos 2\pi(f_c - f_m)t + \cos 2\pi(f_c+f_m)t\right] \quad \text{---(9)}$$

1.4 Frequency domain description:

The fourier transform of equation (9).

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] + \frac{\mu A_c}{4}\left[\delta(f-f_c+f_m)+\delta(f+f_c-f_m)\right] + \frac{\mu A_c}{2}\left[\delta(f-f_c-f_m)+\delta(f_c+f_c+f_m)\right] \quad \text{---(10)}$$



Thus the spectrum of an AM wave for the special case of sinusoidal modulation, consists of delta functions at $\pm f_c$, $f_c \pm f_m$ & $-f_c \pm f_m$ as shown in above figures.

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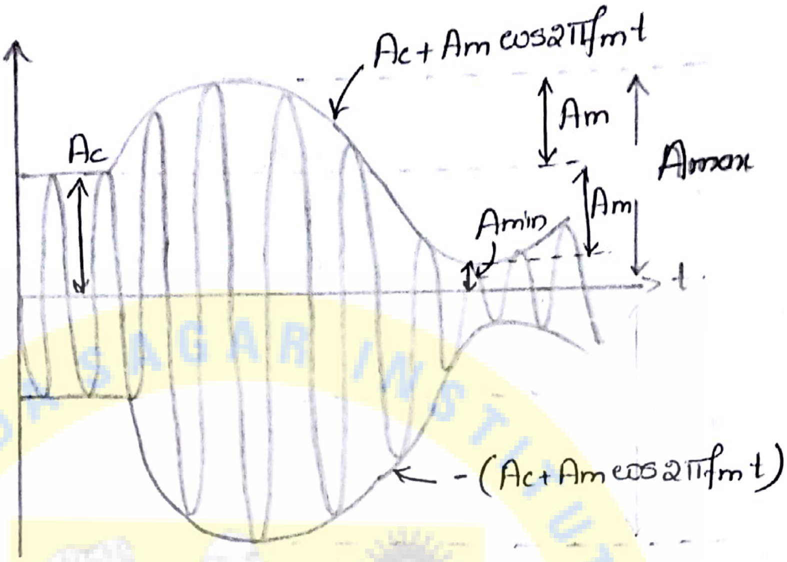
From equation (9), It is clear that modulated wave contains three terms. The first term represents the unmodulated carrier. The second term is the lower sideband & the third term is the upper side band.

Expression for modulation index interms of Amax & Amin.



From figure,

$$A_m = \frac{A_{max} - A_{min}}{2}$$

$$A_c = A_{max} - A_m$$

$$= A_{max} - \left[\frac{A_{max} - A_{min}}{2}\right]$$

$$A_c = \frac{A_{max} + A_{min}}{2}$$

$|||^{ly}$ $A_m = \frac{A_{max} - A_{min}}{2}$.

W.K.T m or $\mu = A_m / A_c$

$$\Rightarrow \frac{A_m}{A_c} = \frac{\frac{A_{max} - A_{min}}{2}}{\frac{A_{max} + A_{min}}{2}}$$

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = m. \quad ---(11)$$

Power relations in the AM wave (Average power)

- It is seen that the carrier component of the modulated wave has the same amplitude as the unmodulated wave along with two sidebands. So the modulated wave contains more power than the carrier had, before the modulation was done.
- The amplitude of the sidebands depends on the modulation index 'm' or 'μ' and thus the total power of the modulated signal is also dependent on 'm'.
- Therefore total power is sum of carrier power and power in two sidebands P_{USB} & P_{LSB}

$$P_t = P_c + P_{USB} + P_{LSB} \qquad \text{--- (12)}$$

$$= \frac{A_{carrier}^2}{R} + \frac{A_{USB}^2}{R} + \frac{A_{LSB}^2}{R}$$

Where all the voltages are rms values and R is the resistance in which the power is dissipated.

i.e, $$P_c = \frac{A_{carrier}^2}{R} = \frac{(A_c/\sqrt{2})^2}{R} = \frac{A_c^2}{2R}$$

$$P_{LSB} = P_{USB} = P_{SB} = \frac{V_{SB}^2}{R}$$

$$= \frac{\left(\frac{m \cdot A_c}{2}/\sqrt{2}\right)^2}{R} = \frac{m^2 A_c^2}{8R} = \frac{m^2}{4}\left[\frac{A_c^2}{2R}\right] = \frac{m^2}{4} \cdot P_c$$

Substitute in equation (12)

$$P_t = \frac{A_c^2}{2R} + \frac{m^2 A_c^2}{8R} + \frac{m^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R} + \frac{m^2}{4}\left[\frac{A_c^2}{2R} + \frac{A_c^2}{2R}\right]$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{m^2}{4} + \frac{m^2}{4}\right]$$

$$P_t = P_c\left[1 + \frac{m^2}{2}\right] \qquad \text{--- (13)}$$

The ratio of total sideband power to the total power in the modulated wave is given by

$$\frac{P_{SB}}{P_t} = \frac{P_c(m^2/2)}{P_c(1 + m^2/2)}$$

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$$\frac{P_{SB}}{P_t} = \frac{m^2}{2+m^2} = \eta$$

The ratio is called the efficiency of AM system. The transmission efficiency is defined as the ratio of the power carried by the sideband to the total power. And it take maximum value of 33% at $m=1$.

If $m=1$ in eqn (13), that is 100% modulation is used, the total power in the two side-frequencies of the resulting AM wave is only one third of the total power in the modulated wave.

i.e., $P_t = P_c\left[1+\frac{1}{2}\right] = \underline{1.5P_c}$

Current Calculation for an AM wave

Let I_c = rms unmodulated current

I_t = Total r.m.s modulated current

R = Resistance in which above currents flow.

The ratio of total power to carrier power is given by,

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = I_t^2/I_c^2$$

w.k.T $P_t = \left(1+m^2/2\right)P_c$

$$P_t/P_c = 1+m^2/2$$

$$\therefore I_t^2/I_c^2 = 1+m^2/2$$

$$I_t = \sqrt{1+m^2/2}\cdot I_c.$$

Modulation by Several sine waves

Several sinusoidal waves are simultaneously used to modulate the carrier.

Therefore amplitude of modulated wave will be

$s(t) = A_c + m_1(t) + m_2(t) + \ldots\ldots$

$= (A_c + A_{m_1}\cos W_{m_1}t + A_{m_2}\cos W_{m_2}t + \ldots\ldots)$

$= (A_c + A_{m_1}\cos W_{m_1}t + A_{m_2}\cos \omega_{m_2}t + \ldots\ldots)\cos W_c t$

$s(t) = (A_c + A_{m_1}\cos W_{m_1}t + A_{m_2}\cos \omega_{m_2}t$

By applying trignometric expansion

$$s(t) = A_c\cos W_c t + \frac{m_1 A_c}{2}\cos(\omega_c - \omega_{m_1}t) + \frac{m_1 A_c}{2}\cos(\omega_c + \omega_{m_1}t) +$$
$$\frac{m_2 A_c}{2}\cos(W_c - \omega_{m_2}t) + \frac{m_2 A_c}{2}\cos(\omega_c + W_{m_2})t + \ldots$$

Equation for total power is given by

$$P_t = P_c + P_{LSB1} + P_{LSB2} + P_{USB1} + P_{USB2} + \cdots$$

$$= \frac{A_c^2}{2R} + \frac{m_1^2 A_c^2}{8R} + \frac{m_1^2 A_c^2}{8R} + \frac{m_2^2 A_c^2}{8R} + \frac{m_2^2 A_c^2}{8R} + \cdots$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \cdots\right]$$

$$P_t = P_c\left[1 + \frac{m_t^2}{2}\right] \quad \text{where} \quad m_t^2 = m_1^2 + m_2^2 + \cdots$$

Prob (1) A 220W unmodulated carrier is modulated to a depth of 65%. Calculate the total power in the modulated wave.

Soln: Given $P_c = 220W$, $m = 0.65$

$$P_t/P_c = 1 + m^2/2$$

$$P_t = P_c(1 + m^2/2)$$

$$= 220\left(1 + \frac{0.65^2}{2}\right)$$

$$= \underline{266.5W}$$

(2) An audio frequency signal $5\sin 2\pi(1000)t$ is used to amplitude modulate a carrier of $100\sin 2\pi 10^6 t$. Assume modulation index as 0.4. Find
i) sideband frequencies
ii) Amplitude of each sideband
iii) Bandwidth required.
iv) Total power delivered to a load of 100Ω.

Soln: Given $m(t) = A_m \sin 2\pi f_m t$ $\qquad c(t) = A_m \sin 2\pi f_c t$

$= 5\sin 2\pi(1000)t \qquad = 100 \sin 2\pi 10^6 t$.

$\therefore A_m = 5$, $f_m = 1000Hz$, $m = 0.4$

$A_c = 100$, $f_c = 10^6 Hz$, $R = 100\Omega$

(i) $f_{LSB} = f_c - f_m = 10^6 - 1000 = \underline{999KHz}$

$f_{USB} = f_c + f_m = 10^6 + 1000 = \underline{1.001MHz}$

(ii) Amplitude of each sideband $\frac{mA_c}{2} = \frac{0.4 \times 100}{2} = \underline{\underline{20V}}$

(iii) Bandwidth $= 2f_m = 2000Hz$.

(iv) $P_t = P_c(1 + m^2/2)$

$$= \frac{A_c^2}{2R}(1 + m^2/2) = \frac{100^2}{2 \times 100}\left[1 + \frac{0.4^2}{2}\right] = \underline{\underline{54W}}$$

③ A carrier wave of frequency 10MHz & peak value 10V is amplitude modulated by a 5KHz sine wave of amplitude 6V. Determine the modulation index & draw the spectrum.

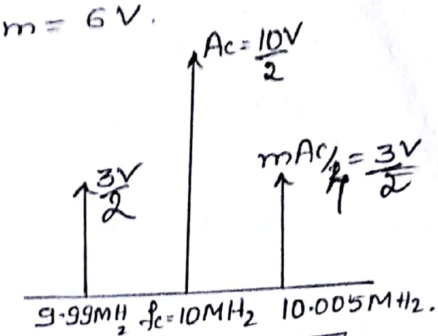
Soln Given $f_c = 10MHz$, $A_c = 10V$, $f_m = 5KHz$, $A_m = 6V$.

Modulation index, $m = \frac{A_m}{A_c} = \frac{6}{10} = 0.6$

LSB, $f_c - f_m = 10M - 5k = 9.99MHz$

USB, $f_c + f_m = 10M + 5K = 10.005MHz$

Amplitude $\frac{mA_c}{2} = \frac{0.6 \times 10}{2} = 3V$



④ The output current of 60% modulated AM signal is 1.5A. To what value will this current rise if the generator is modulated additionally by another audio wave, whose modulation index is 0.7.

Soln: Carrier current is, $I_c = \frac{I_t}{\sqrt{1+m^2/2}} = \frac{1.5}{\sqrt{1+\frac{0.6^2}{2}}} = 1.38A$

$m_t = \sqrt{m_1^2 + m_2^2} = \sqrt{0.6^2 + 0.7^2} = 0.922$

Current rise with $m_t = 0.922$ is

$I_t = I_c\sqrt{1+m_t^2/2}$

$= 1.38\sqrt{1+\frac{0.922^2}{2}} = 1.647A$

⑤ One input to AM is 500KHz f_c with A_c of 32V. Second I/p is 12KHz f_m, $A_m = 14V$. Determine i) USB, LSB frequencies. ii) Modulation index(m), & %m iii) Amax & Amin. iv) Draw o/p envelope v) Draw o/p frequency spectrum.

Soln: i) $f_{USB} = f_c + f_m = 500K + 12K = 512KHz$

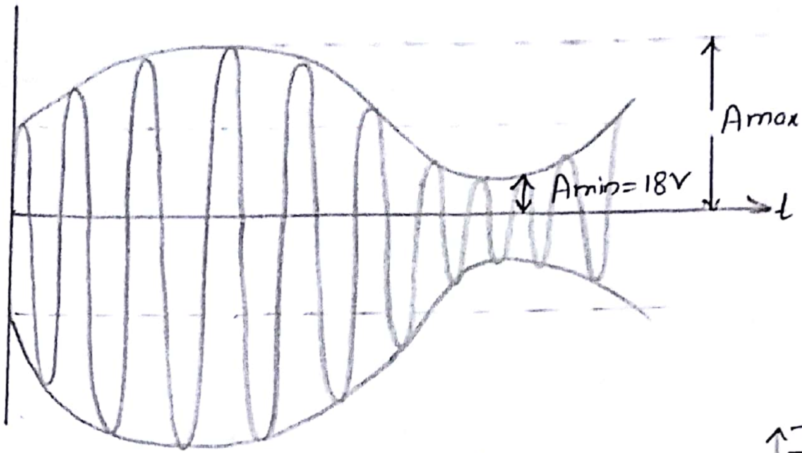
$f_{LSB} = f_c - f_m = 500K - 12K = 488KHz$

ii) $m = A_m/A_c = 14/32 = 0.4375$, & $\%m = 43.75\%$

iii) $A_{max} = A_c(1+m) = 32(1+0.4375) = 46V$ $[A_{max} = A_c + A_m]$

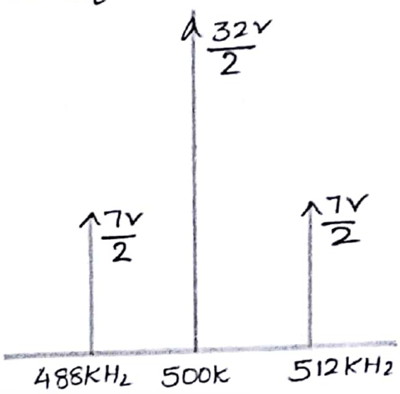
$A_{min} = A_c(1-m) = 32(1-0.4375) = 18V$ $[A_{min} = A_c - A_m]$

iv)



v) Amplitude of LSB = USB = $\frac{mA_c}{2}$

$= \underline{\underline{7V}}$



(6) A standard AM transmission, sinusoidally modulated to a depth of 40% produces sideband frequencies of 6.824 & 6.854 MHz. Amplitude of each sideband frequency is 50V. Determine amplitude & frequency of carrier.

Soln: Given $f_c + f_m = 6.854\,MHz$

$f_c - f_m = 6.824\,MHz$

$\Rightarrow 2f_c = 13.678\,MHz$

$f_c = \underline{\underline{6.839\,MHz}}$

Amplitude of sideband is given by $= \frac{mA_c}{2}$

$50 = \frac{mA_c}{2}$

$A_c = \frac{50\times2}{m} = \frac{50\times2}{0.4} = \underline{\underline{250V}}$

(7) A carrier wave with amplitude 12V & $f_c = 10MHz$ is amplitude modulated to 50% level in $f_m = 1KHz$. Write down equation of the above wave & sketch waveform in frequency domain.

Soln: $s(t) = A_c(1 + m\cos 2\pi f_m t)\cos 2\pi f_c t$

Given $A_c = 12V$, $m = 0.5$, $f_m = 1KHz$, $f_c = 10MHz$

$2\pi f_m = 2\times\pi\times1\times10^3 = 6283.2$

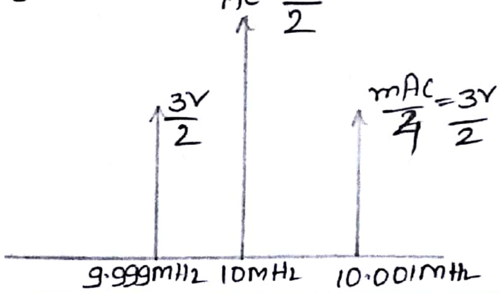
$2\pi f_c = 2\times\pi\times10\times10^6 = 62.83\times10^6.$

$\therefore s(t) = 12(1 + \underline{0.5\cos 6283.2t})\cos 62.83\times10^6 t$

$f_{USB} = f_c + f_m = 10.001\times10^6$

$f_{LSB} = f_c - f_m = 9.999\times10^6$

Amplitude of sideband $= \frac{mA_c}{2} = \underline{\underline{3V}}$



3 GENERATION OF AM:

Transmitters are catagorized according to where the modulation takes place. The two basic catagories are high level & low level modulation. It is rare to find both types of modulation within one transmitter.

Low-level Modulation:

Here generation of AM wave takes place in the initial stage of amplification. i.e, at a lower level. The generated AM signal is then amplified using number of amplifier stages.

High-level Modulation:

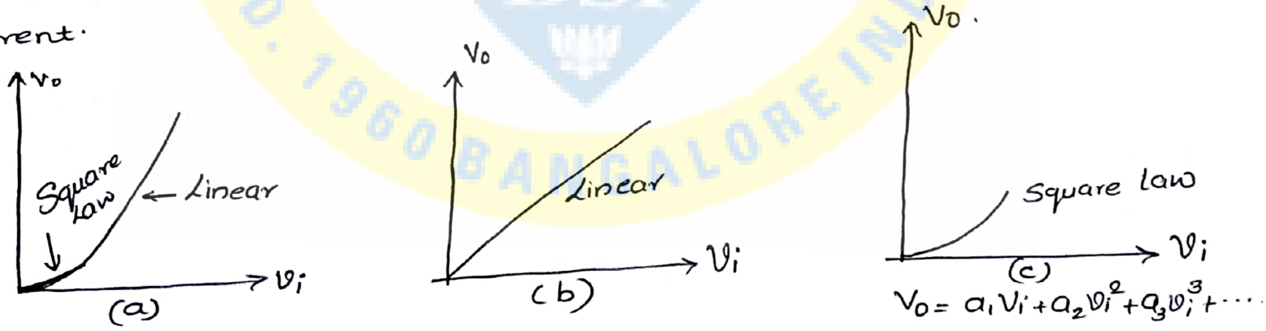
Here modulation takes place in the final stage of amplification & therefore more circuitry has to handle high power. For ex:- If transmitter power is 1500W & modulation index 1, then modulation power is 500W (33% of transmitted power). The modulator circuitry must be able to deliver such a high power.

Low-level Modulators:- Square-law modulator & the switching modulator use non-linear element for their implementation.

3.1 Square Law Modulator

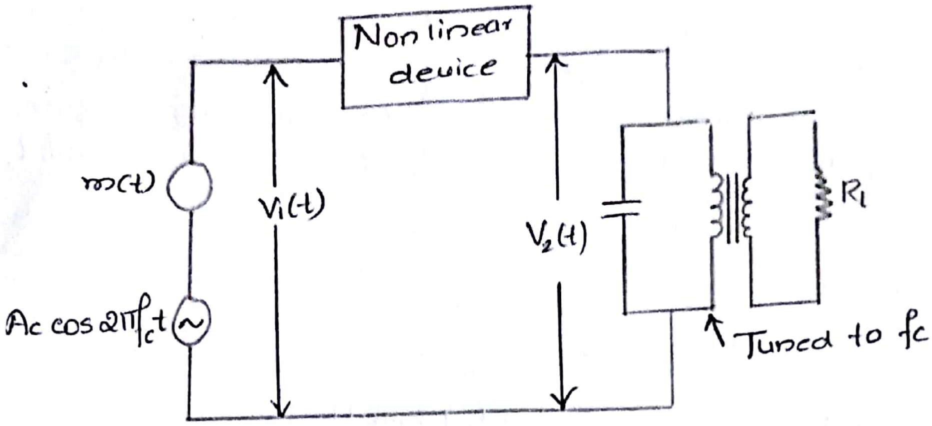
In this circuit a square law devices like diode is used therefore it is known as square law modulator.

A square law device is one that produces an output voltage or current that is proportional to the square of its input voltage or current.



$V_o = a_1 V_i + a_2 V_i^2 + a_3 V_i^3 + \cdots$

- For lower input voltage, device is act as a square law & for high voltage - linear characteristics as shown in fig (a).
- We know that the strength of practical signal is much smaller and if we generate carrier with little strength, the diode can be operate in square law characteristic region.
- A square low modulators requires three features: a means of
  - Summing the carrier & modulating waves
  - Nonlinear element
  - Band-pass filter for extracting the desired modulation product



Semiconductor diodes and transistors are the most common nonlinear devices used for implementing square law modulators. When a nonlinear element such as diode is suitably biased & operated in a restricted portion of its characteristic curve, that is the signal applied to the diode is relatively weak, we find that the transfer characteristics of diode load resistor combination can be represented closely by a square law:

$$V_2(t) = a_1 v_1(t) + a_2 v_1^2(t) + a_3 v_1^3(t) + \cdots$$

For simplicity consider first two terms of above equation,

$$V_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \quad \text{---} (14)$$

where a_1 & a_2 are constants. The input voltage $v_1(t)$ consists of the carrier wave plus the modulating wave, that is,

$$v_1(t) = A_c \cos 2\pi f_c t + m(t) \quad \text{---} (15)$$

Substituting equation (15) in (14),

$$V_2(t) = a_1\left[A_c \cos 2\pi f_c t + m(t)\right] + a_2\left[A_c \cos 2\pi f_c t + m(t)\right]^2$$

$$= a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2\left[A_c^2 \cos^2 2\pi f_c t + m^2(t) + 2A_c \cos 2\pi f_c t \cdot m(t)\right]$$

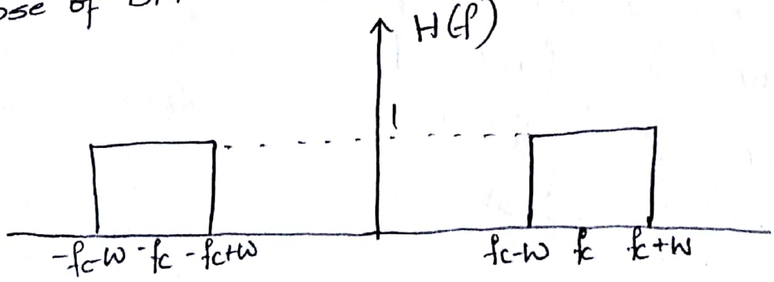
$$= a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t) + a_2 \cdot 2 A_c \cos 2\pi f_c t \cdot m(t).$$

$$= a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t + 2 \underline{a_2 A_c \cos 2\pi f_c t \cdot m(t)} \quad \text{---} (16)$$

$$= a_1 \underline{A_c \cos 2\pi f_c t} + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t \quad \text{---} (17)$$

$$= a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t$$

Output of diode ($V_2(t)$) is applied in BPF.
The response of BPF should be as shown below.



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Bandpass filter output will be the final modulated signal. First and fifth term in eqn (16) passes in BPF, since their frequency is f_c. Second & third term consist of only $m(t)$ & $m^2(t)$ and their signal is very less compared to f_c, therefore BPF blocks these terms. Finally the third term $\cos^2 2\pi f_c t = \frac{1+\cos 2\times 2\pi f_c t}{2} = \cos 4\pi f_c t$, which is higher than f_c. Therefore even this term also get block by BPF.

Final output of BPF is given by

$$v(t) = a_1 A_c \cos 2\pi f_c t + 2a_2 A_c \cos 2\pi f_c t\, m(t)$$

$$= a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t \rightarrow \text{AM wave}$$

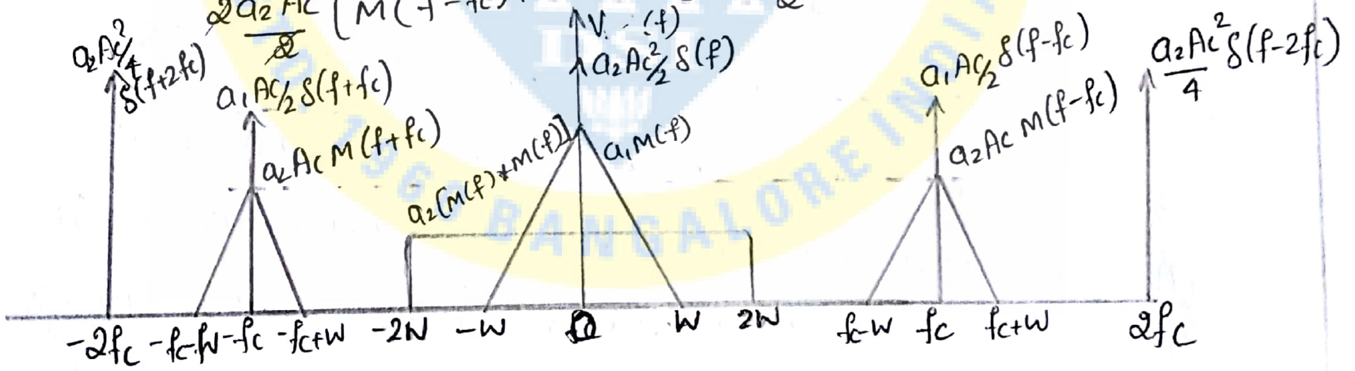
OR.

First term in eqn (17) is the AM with amplitude sensitivity $k_a = \frac{2a_2}{a_1}$. The remaining three terms are unwanted & can be removed by filtering. To remove unwanted terms apply F.T to equation (16).

$$V(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2 m^2(t) + 2a_2 A_c m(t) \cos 2\pi f_c t + \frac{a_2 A_c^2}{2}\left[1 + \cos 4\pi f_c t\right].$$

Frequency spectrum,

$$V(f) = \frac{a_1 A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + a_1 M(f) + a_2\left(M(f) * M(f)\right) + \frac{2a_2 A_c}{2}\left(M(f-f_c) + M(f+f_c)\right) + \frac{a_2 A_c^2}{2}\delta(f) + \frac{a_2 A_c^2}{4}\left[\delta(f-2f_c) + \delta(f+2f_c)\right]$$



Specification of filter:

Mid band frequency $= f_c$

Bandwidth $= 2f_m$

frequency specification of BPF should be

$f_c - W > 2W$ & $f_c + W < 2f_c$

$f_c > 3W$ & $f_c > W$.

$\therefore$ $f_c > 3W$

The term $\frac{1}{2}a_2 A_c^2 K_a m(t)$ represents desired signal. This term can be extracted with the help of low-pass filter.
Another term $\frac{1}{2}a_2 A_c^2 K_a^2 m^2(t)$ represents frequency components which is similar to desired frequency. They may give rise distortion.

The ratio of wanted signal to distortion is equal to

$$\frac{a_2 A_c^2 K_a m(t)}{\frac{1}{2} a_2 A_c^2 K_a^2 m^2(t)} = \frac{2}{K_a m(t)} = \frac{2}{\mu} = S/N$$

In order to make the ratio large, the product $|K_a m(t)|$ should be kept small compared to unity for all 't'. This means that the percentage modulation should be limited for minimising the distortion.

Signal to noise ratio [S/N] should be high for perfect reconstruction. That means μ should be very low.

Ex: Consider $S/N = 10$,

$$S/N = 2/\mu$$

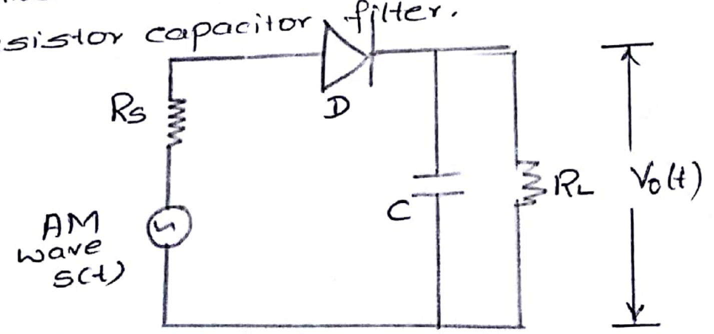
$$10 = 2/\mu \Rightarrow \mu = 0.2$$

$$\text{Efficiency}, \eta = \frac{\mu^2}{2+\mu^2} = \frac{0.2^2}{2+0.2^2} \cong 2\% \quad [98\% \text{ power wasted}].$$

If μ is low, efficiency will be very low. But for efficient power distibution η should be high so that square law demodulator is not preffered for AM demodulation.

4.2 Envelope Detector:

An envelope detector is a simple & highly effective device that is well suited for the demodulation of narrow band AM wave [i.e., carrier frequency is large compared with the modulating signal bandwidth] for which the percentage modulation is less than 100%. In an envelope detector, the output of detector follows the envelope of the modulated signal. Figure shows the envelope detector circuit. It consists of a diode & a resistor capacitor filter.



On the +ve half cycle of the input signal, the diode is forward biased and the capacitor charges up rapidly to the peak of the input signal.

When the input signal falls below this value, the diode becomes reverse biased, & the capacitor C discharges through the load resistor R_L. The discharge process continuous until the next +ve half cycle.

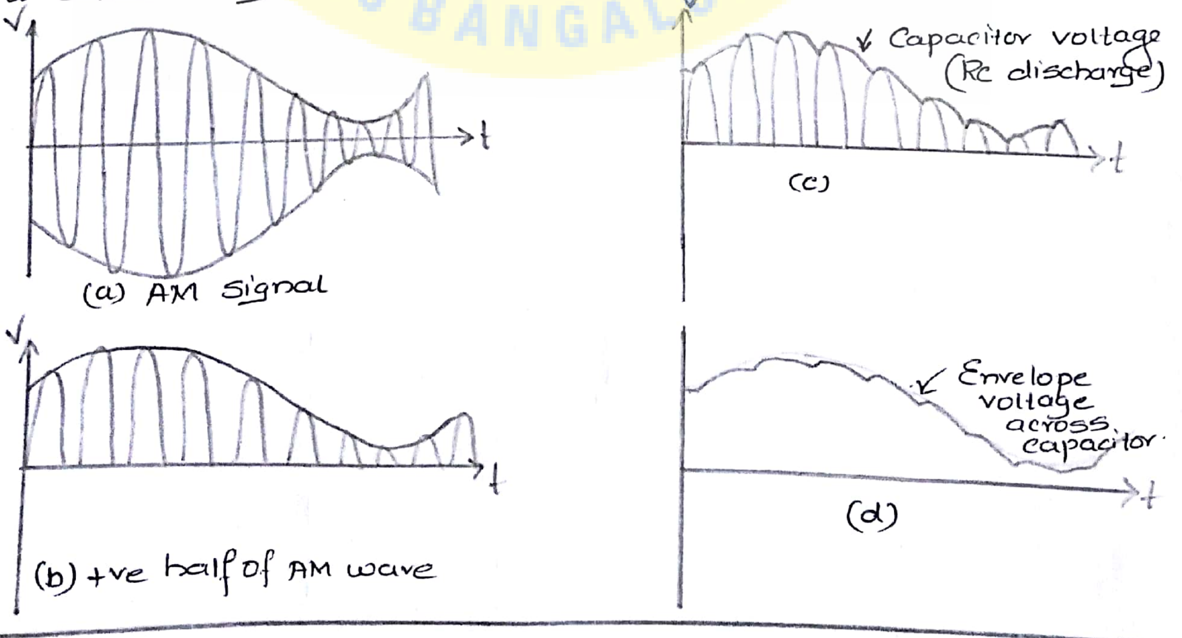
When the input signal becomes greater than the voltage across the capacitor, the diode conducts again & the process is repeated.

We assume the diode to be ideal & also assume that the AM wave has been applied to the envelope detector from a voltage source having series resistance R_s. The charging time constant R_sC must be short compared with the carrier time period. $R_sC \ll 1/f_c$

So that the capacitor charges rapidly & thereby follows the applied voltage when the diode is conducting.

On the other hand the discharging time constant R_LC must be long enough to ensure that the capacitor discharge slowly through the resistor R_L between +ve peaks of the carrier wave, but not so long that capacitor voltage will not discharge at the maximum rate of charge of the modulating wave i.e, $\frac{1}{f_c} \ll R_LC \ll \frac{1}{W}$ where W = message B.W.

The result is the capacitor voltage which is the detector o/p is very nearly same as the envelope of the AM wave as shown in fig below. In the detected output some ripples may be present which may be finally removed by using low pass filter.



(a) AM Signal

(b) +ve half of AM wave

(c)

(d)

Limitations & Modifications of AM

- AM is the oldest method of performing modulation.
- AM is cheap to build & simple.

Limitations:

- AM is wasteful of power: The carrier wave c(t) is completely independent of the base band signal m(t). The transmission of the carrier wave therefore represents waste of power i.e; only a fraction of the total transmitted power is actually affected by m(t).
- AM is wasteful of Bandwidth: The upper & lower side band of an AM wave are uniquely related to each other. This means that only one sideband is necessary to recover the information signal. Therefore modulation is wasteful of B.W.

To overcome these limitations, three forms of AM are used.

① Double-side band-suppressed carrier modulation (DSBSC)

The modulated wave consists of only the upper & lower sidebands. Transmitted power is saved through the suppression of the carrier wave, but the channel bandwidth requirement is the same as before. i.e, 2W.

② Single Side Band Suppressed Carrier [SSBSC] modulation:

The modulated wave consists of only the upper side band or lower sideband. It is suited for transmission of voice signals. It is an optimum form of modulation in that it requires the minimum transmission power & minimum channel bandwidth. Disadvantage is increased cost & complexity.

③ Vestigial Side Band [VSB] modulation:

In which one side band is passed almost completely & just a trace or vestige of the other side band is retained. The required bandwidth is therefore in excess of the message bandwidth by an amount equal to the width of the vestigial side band. This method is suitable for the transmission of wide band signals.

5 DOUBLE SIDE BAND SUPPRESSED CARRIER MODULATION [DSBSC]

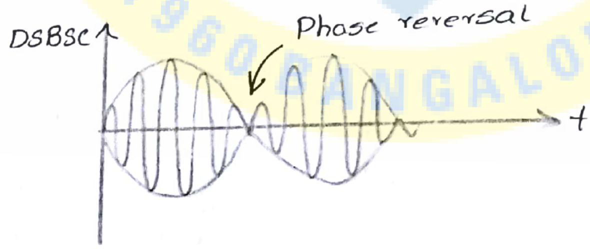
When the carrier is amplitude modulated by a single sine wave, the resulting signal consists of three frequencies i.e; original carrier (f_c) & two sidebands ($f_c \pm f_m$). In this standard form of amplitude modulation, both side bands & carrier are transmitted and carrier wave c(t) is completely independent of the message signal m(t) which means the transmission of the carrier wave represents a waste of power i.e. for 100% modulation, about 67% of the total power is required for transmitting the carrier, which does not contain any information. To overcome this short coming, the carrier is suppressed before transmission which reduces the overall power required. When the carrier is removed, the remaining signal contains simply upper & lower sidebands. Such a signal is referred to as a double sideband suppressed carrier [DSBSC] signal. With this signal, no power is wasted on the carrier & the saved power can be put into the sidebands for stronger signals over longer distances.

5.1 Time Domain Description

DSB-SC modulation consists of product of message signal $m(t)$ & the carrier $c(t)$

$$S(t) = c(t)\, m(t)$$
$$= A_c \cos 2\pi f_c t \cdot m(t) \quad \text{--- (21)}$$

This modulated wave undergoes a phase reversal whenever the message signal $m(t)$ crosses zero unlike amplitude modulation, the envelope of a DSBSC modulated wave is different from the message signal.



5.2 Frequency-Domain Description:

Taking F.T of both sides of eqn (21),

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

When the message signal $m(t)$ is limited to the interval $-W \leq f < W$ as in fig (a). The spectrum as in fig (b) we can see that this modulation process simply translates the base band spectrum by $\pm f_c$. The suppression of the carrier from the modulated wave is well understood by examing spectrum. The transmission bandwidth is 2W, same as that for standard AM

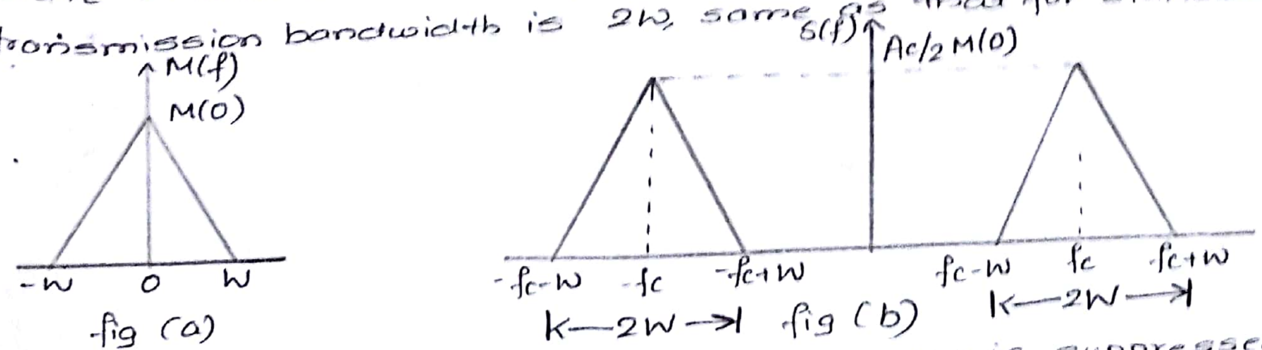


fig (a) fig (b)

Here the dotted line indicates that the carrier is suppressed.

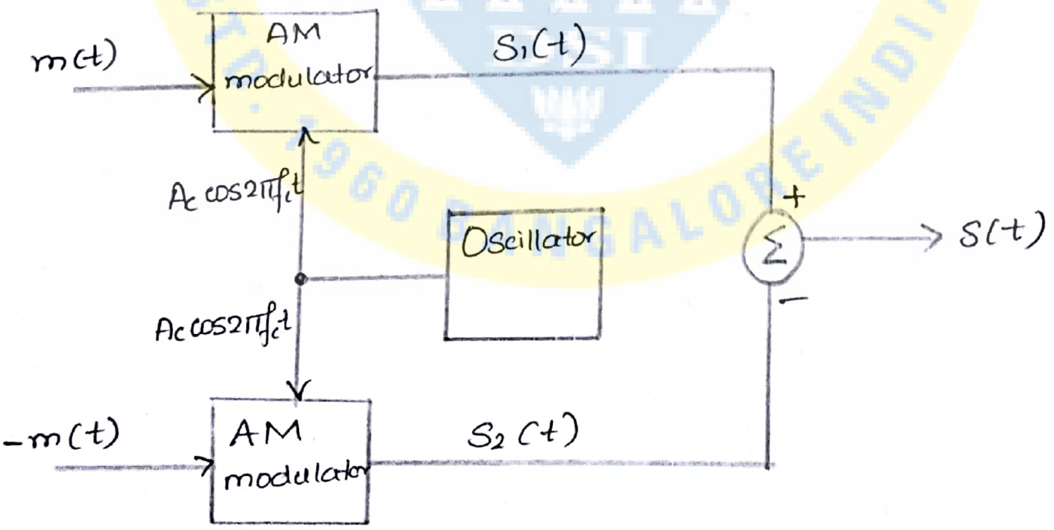
6 GENERATION OF DSBSC WAVES:

A DSB-sc signal is the product of the base band signal and the carrier wave. A single electronic device or component cannot generate this signal. A system for achieving this is called product modulator.

Two types of product modulators are : Balanced modulator
Ring modulator.

6.1 Balanced Modulator:

It consists of 2 AM modulators arranged in balanced configuration to suppress the carrier. as shown in fig below.



Assume that the 2 modulators are identical. One input to each modulator is from an oscillator which generates sinusoidal carrier, other input is the modulating wave. Baseband signal applied to one of the modulators has a sign reverse. The output of 2 AM modulators can be expressed as,

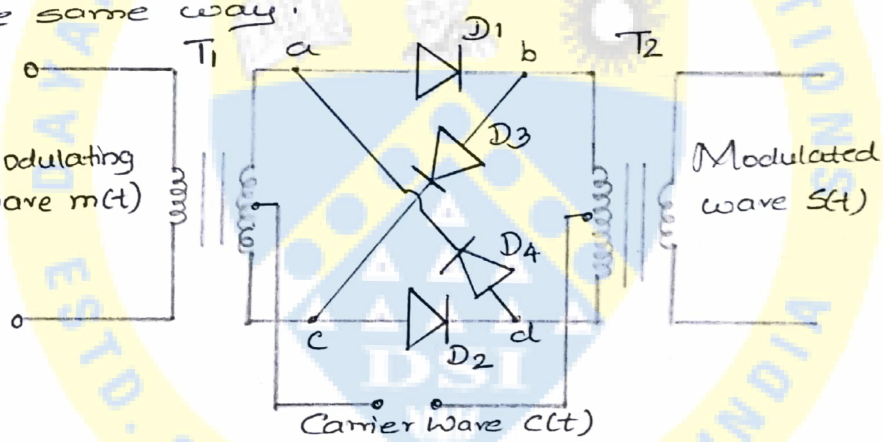
$$S_1(t) = A_c[1+k_a m(t)]\cos 2\pi f_c t$$
$$S_2(t) = A_c[1-k_a m(t)]\cos 2\pi f_c t$$

$$\therefore S(t) = S_1(t) - S_2(t)$$
$$= A_c[1+k_a m(t)]\cos 2\pi f_c t - A_c[1-k_a m(t)]\cos 2\pi f_c t$$
$$S(t) = 2A_c k_a m(t)\cos 2\pi f_c t$$

Therefore except for a scaling factor. $2k_a$, the balanced modulator output is equal to the product of the modulating wave & the carrier which is nothing but DSBSC signal.

6.2 Ring Modulator

One of the most useful product modulator that is well suited for generating a DSBSC modulated wave is the ring modulator. It is also known as a lattice or double balanced modulator. The four diodes in figure below form a ring in which they all point in the same way.



Square wave carrier signal c(t) is applied to the center taps of the input & output transformers and modulating signal is applied to the input transformer T_1. The output appears across the secondary of the transformer T_2.

First assume that the modulating input zero. In the positive half cycle of the carrier signal, diodes D_1 & D_2 are forward biased & D_3 & D_4 are reverse biased.

The current divides equally in the upper and lower portions of the ~~fo~~ primary winding of T_2. The current in the upper part of the winding produces a magnetic field that is equal & opposite to the magnetic field produced by the current in the lower half of the winding. Magnetic fields are equal & opposite are cancel each other producing no output at the secondary of T_2. Thus the carrier is suppressed.

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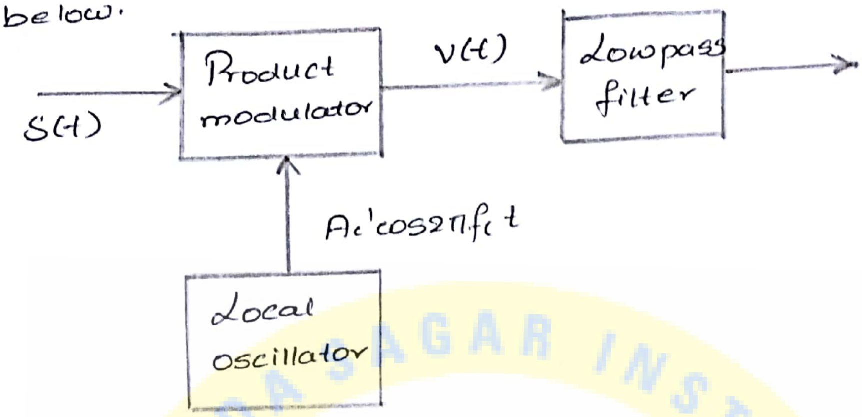
7. COHERENT DETECTION OF DSBSC MODULATED WAVES: [SYNCHRONOUS DETECTION]

- The message signal m(t) is recovered from a DSB-SC wave s(t) by first multiplying s(t) with a locally generated sinusoidal wave and then low-pass filtering the product as in figure below.



It is assumed that the local oscillator output is exactly coherent or synchronized in both frequency and phase with the carrier wave c(t) used in the product modulator to generate s(t). This method of demodulation is known as coherent detection.

$$v(t) = s(t) \cdot A_c' \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t \cdot m(t) \cdot \cos 2\pi f_c t \cdot A_c'$$

$$= A_c \cdot A_c' \cos^2 2\pi f_c t \cdot m(t)$$

$$= A_c \cdot A_c' \left[\frac{1 + \cos 4\pi f_c t}{2} \right] m(t)$$

$$= \underbrace{\frac{A_c \cdot A_c'}{2} m(t)}_{\text{Desired signal.}} + \frac{A_c \cdot A_c'}{2} \cos 4\pi f_c t$$

$$\therefore V_o(t) = \frac{A_c \cdot A_c'}{2} m(t)$$

The desired signal ~~about~~ centered about f = 0 is obtained by passing v(t) through an ideal LPF of B.W greater than W Hz but less than $2f_c - W$ Hz

Two possible errors

- Frequency error: Let the output of local oscillator be $\cos 2\pi f_1 t$ where $f_1 = f_c \pm \Delta f$ where Δf is the errors in freq.

$$v(t) = s(t) \cdot \cos 2\pi f_1 t$$

$$= A_c \cdot \cos 2\pi f_c t \cdot m(t) \cdot \cos 2\pi f_1 t$$

$$v(t) = \frac{A_c}{2} m(t) \left[\cos 2\pi (f_c - f_1)t + \cos 2\pi (f_c + f_1)t \right]$$

So we can see that there is no desired modulating signal. For small values of Δf, the baseband spectrum will overlap. However for speech signals $\Delta f = 10Hz$ is tolerable.

- **Phase error:** For recovery of modulating signal $m(t)$, the local oscillator output should be exactly of the same frequency but arbitrary phase difference φ is measured with respect to the carrier wave $c(t)$. Thus if the local oscillator o/p is $\cos(2\pi f_c t + \varphi)$, the output of product modulator is given by

$$v(t) = s(t) \cdot (\cos 2\pi f_c t + \varphi)$$

$$= A_c \cos 2\pi f_c t \cdot m(t) \cdot \cos(2\pi f_c t + \varphi)$$

$$= \frac{A_c \cdot m(t)}{2} \left[\cos(4\pi f_c t + \varphi) + \cos\varphi \right] \qquad \because \cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}$$

$$v(t) = \frac{A_c m(t)}{2} \left(\cos 4\pi f_c t + \varphi \right) + \frac{A_c m(t)}{2} \cos\varphi \qquad \text{--- (23)}$$

The first term in eqn (23) represents a DSBSC modulated signal with carrier freq $2f_c$, where the 2nd term is proportional to the message signal $m(t)$. The first term of eqn (23) is removed by LPF provided cut off frequency of filter is greater than W but less than $2f_c - W$. This is satisfied by choosing $f_c > W$. $\therefore$ Filter output is given by,

$$V_0(t) = \frac{A_c}{2} \cos\varphi \cdot m(t)$$

The demodulated signal $V_0(t)$ is therefore proportional to $m(t)$ when phase error φ is a constant. The amplitude of this demodulated is maximum when $\varphi = 0$ & is minimum when $\varphi = \pm \pi/2$. The zero demodulated signal which across for $\varphi = \pm\pi/2$ represents the quadrature null effect of the coherent detector.

Thus phase error φ in the local oscillator causes the detector o/p to be attenuated by a factor equal to $\cos\varphi$. As long as the phase error φ is constant, the detector o/p provides an undistorted version of the original base band signal $m(t)$. However phase error φ varies randomly due to random variations in the communication channel. Therefore provision must be made in the system to maintain the local oscillator

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In the receiver in perfect synchronism in both frequency & phase with the carrier wave used to generate the DSBSC signal in the transmitter.

Single tone Modulation

Consider the sinusoidal modulating signal,

$$m(t) = A_m \cos 2\pi f_m t.$$

The corresponding DSBSC modulated wave is given by,

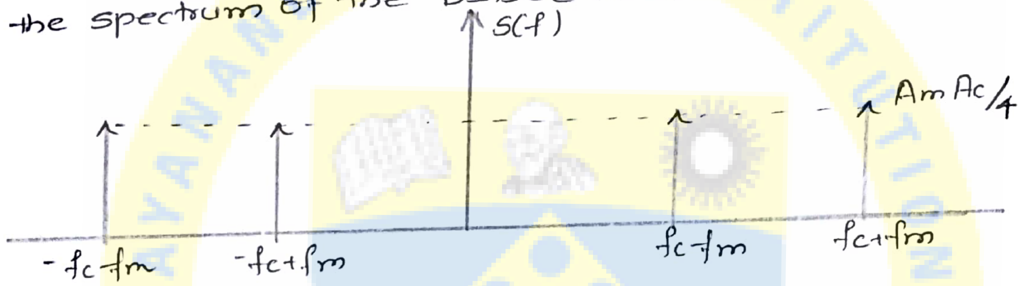
$$s(t) = A_c \cos 2\pi f_c t \cdot A_m \cos 2\pi f_m t.$$

$$= \frac{A_c \cdot A_m}{2} \cos[2\pi(f_c+f_m)t] + \frac{A_c A_m}{2} \cos[2\pi(f_c-f_m)t]$$

Taking Fourier transform,

$$S(f) = \frac{A_c \cdot A_m}{4}\left[\delta(f-f_c-f_m) + \delta(f+f_c+f_m) + \delta(f-f_c+f_m) + \delta(f+f_c-f_m)\right]$$

Thus the spectrum of the DSBSC modulated wave is,



Assuming perfect synchronism between local oscillator & the carrier wave, we find that the product modulator output is

$$v(t) = s(t) \cdot \cos 2\pi f_c t.$$

$$= \left\{\frac{A_m A_c}{2} \cos 2\pi(f_c+f_m)t + \frac{A_m A_c}{2} \cos[2\pi(f_c-f_m)t]\right\} \cos 2\pi f_c t$$

$$= \frac{A_m A_c}{2} \cos 2\pi(f_c+f_m)t \cos 2\pi f_c t + \frac{A_m A_c}{2} \cos(2\pi(f_c-f_m)t) \cos 2\pi f_c t$$

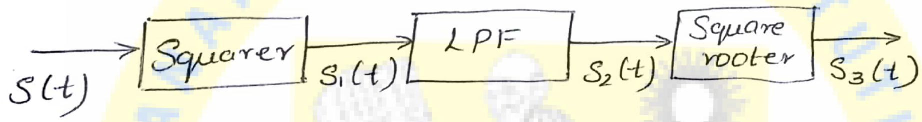
$$\Rightarrow \frac{A_m A_c}{2\times 2} \cos 2\pi(2f_c+f_m)t + \frac{A_m A_c}{2\times 2} \cos 2\pi f_m t + \frac{A_m A_c}{2\times 2} \cos 2\pi f_m t + \frac{A_m A_c}{2\times 2} \cos 2\pi(2f_c-f_m)t$$

$$v(t) = \frac{A_m A_c}{4} \cos 2\pi(2f_c+f_m)t + \underbrace{\frac{A_m A_c}{2} \cos 2\pi f_m t} + \frac{A_m A_c}{2} \cos 2\pi(2f_c-f_m)t$$

2nd term is the desired modulating signal. 1st & 3rd are unwanted terms which are removed using a LPF.

Therefore the coherent detector output reproduces original modulating wave.

Prob ⑧ The AM wave is given by $s(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$ is applied to the system shown in figure. Assume that the message signal $m(t)$ is limited to the interval $W \le f$ & that $f_c >> W$. Show that $m(t)$ can be obtained from the square rooter output.



o/p: The AM signal is

$$s(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$$

Squarer o/p is,

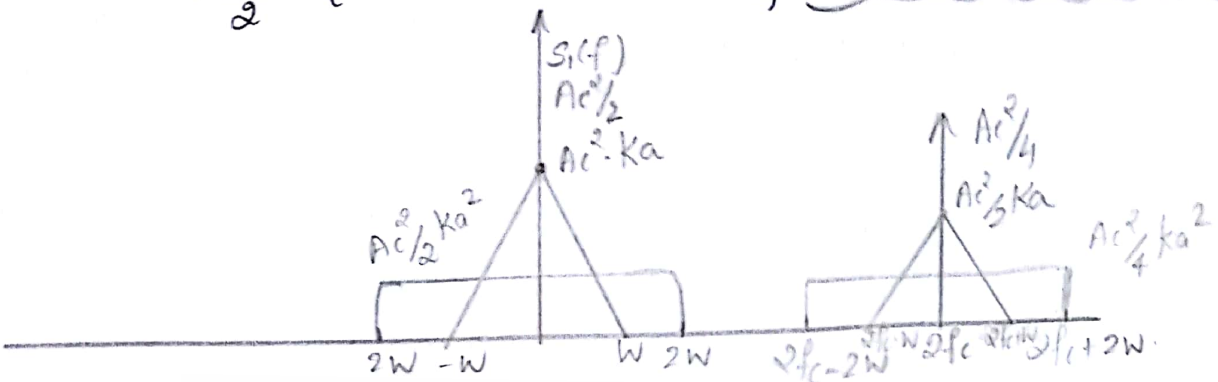
$$s_1(t) = A_c^2[1 + k_a m(t)]^2\cos^2 2\pi f_c t = \frac{A_c^2}{2}(1 + k_a m(t))^2(1 + \cos 4\pi f_c t)$$

$$= \frac{A_c^2}{2}\left[1 + k_a^2 m(t)^2 + 2k_a m(t)\right]\left[1 + \cos 4\pi f_c t\right]$$

$$= \frac{A_c^2}{2}\left[1 + k_a^2 m(t)^2 + 2k_a m(t) + \cos 4\pi f_c t + k_a^2 m(t)^2\cos 4\pi f_c t + 2k_a m(t)\cos 4\pi f_c t\right]$$

Fourier transform of this,

$$S_1(f) = \frac{A_c^2}{2}\delta(f) + A_c^2 k_a M(f) + \frac{A_c^2}{2}[M(f) * M(-f)]k_a^2 + \frac{A_c^2}{4}[\delta(f-2f_c)+\delta(f+2f_c)]$$
$$+ \frac{A_c^2}{2}k_a[M(f-2f_c)+M(f+2f_c)] + \frac{A_c^2}{4}(M(f) * M(f)\cos 4\pi f_1)k_a^2$$



For the LPF, $2f_c - 2W > 2W$

$$2f_c > 4W$$
$$f_c > 2W$$

$\therefore$ Choosing the cut-off frequency of the LPF greater than 2W but less than $2f_c - 2W$, we obtain

$$S_2(t) = \frac{A_c^2}{2}(1+k_a m(t))^2$$

Hence the square rooter o/p is

$$S_3(t) = \frac{A_c}{\sqrt{2}}[1+k_a m(t)] = \frac{A_c}{\sqrt{2}} + \frac{A_c}{\sqrt{2}}k_a m(t)$$

Expect for the dc component $A_c/\sqrt{2}$, $S_3(t)$ is proportional to message signal $m(t)$.

3) Consider the message signal $m(t) = 20\cos(2\pi t)$ V & carrier wave $c(t) = 50\cos(100\pi t)$ V.

(i) Sketch the resulting AM wave for 75% modulation.

(ii) Find the power developed across a load of 100Ω due to this AM wave.

Sol: Given $A_m = 20V$, $f_m = 1Hz$, $A_c = 50V$, $2\pi f_c t = 100\pi t$, $f_c = 50Hz$, $\mu = 0.75$, $R = 100\Omega$.

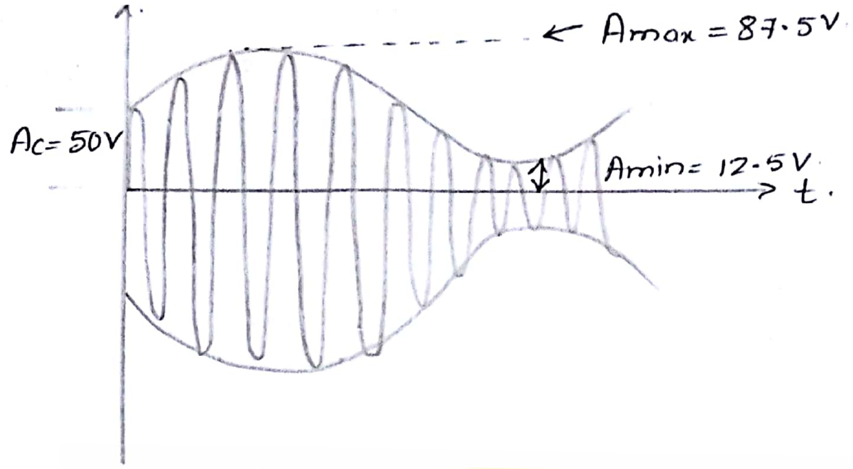
AM wave is given by

$$S(t) = A_c[1+\mu\cos 2\pi f_m t]\cos 2\pi f_c t$$

$$\therefore S(t) = 50[1+0.75\cos 2\pi t]\cos 2\pi(50)t$$

(i) $A_{max} = A_c(1+\mu) = 50(1+0.75) = 87.5V$

$A_{min} = A_c(1-\mu) = 50(1-0.75) = 12.5V$



ii) $P_T = P_c[1 + \mu^2/2]$

But $P_c = \frac{A_c^2}{2R} = \frac{50^2}{2\times100} = 12.5V$

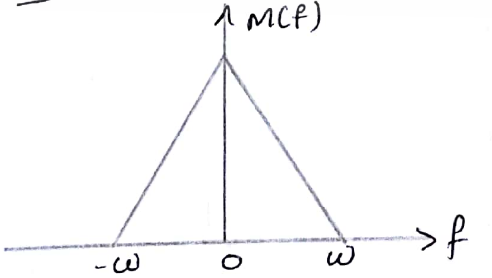
$\therefore P_T = 12.5[1+0.75^2/2]$

$\underline{P_T = 16.01W}$

10) Consider a message signal m(t) with a spectrum shown in figure. The message bandwidth W = 1KHz. This signal is applied to a product modulator together with a carrier wave $A_c\cos2\pi f_c t$ producing the DSB-SC modulated signal s(t). The modulated signal is next applied to a coherent detector. Assuming perfect synchronism between the carrier waves in the modulator and detector. Determine the spectrum of the detector output when

(i) $f_c = 1.25KHz$ ii) $f_c = 0.75KHz$.

What is the lowest carrier frequency for which each component of the modulated signal s(t) is uniquely determined by m(t).



Soln: Given $W = f_m = 1KHz$

(i) when $f_c = 1.25KHz$

$f_c + W = 1.25+1 = 2.25KHz$, $-f_c + W = -1.25+1 = -0.25KHz$

$f_c - W = 1.25-1 = 0.25KHz$, $-f_c - W = -1.25-1 = -2.25KHz$

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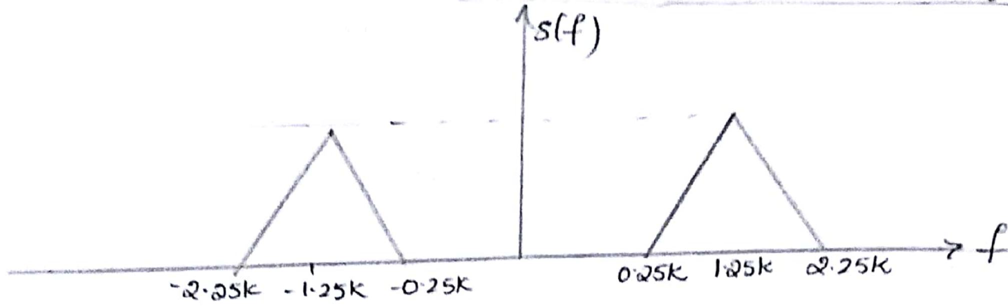
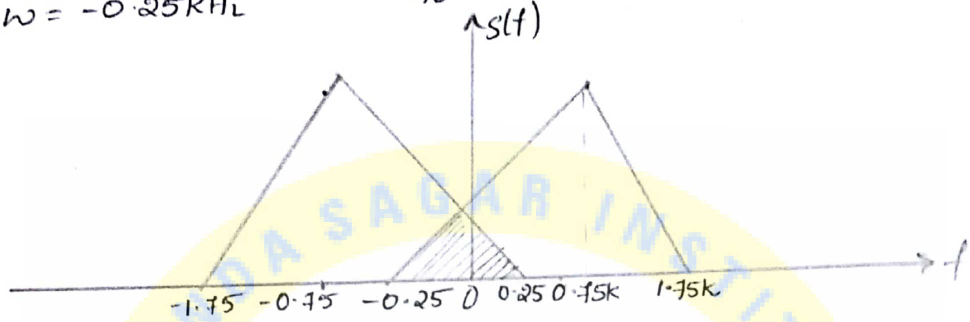


ii) $f_c = 0.75\,KHz$.

$f_c + W = 1.75\,KHz$ $\qquad -f_c + W = 0.25\,KHz$

$f_c - W = -0.25\,KHz$ $\qquad -f_c - W = -1.75\,KHz$

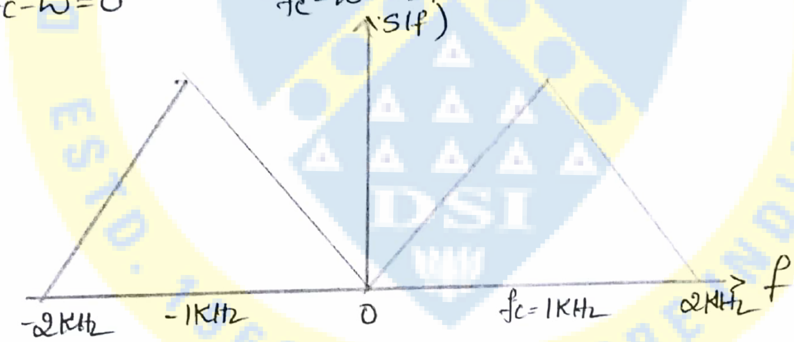


The lowest f_c, so that $m(t)$ can be properly determined from $s(t)$.

$f_c = 1\,KHz$, $\quad W = 1\,KHz$

$f_c + W = 2\,KHz$ $\qquad -f_c + W = 0$

$f_c - W = 0$ $\qquad -f_c - W = -2\,KHz$



(11) Using the message signal $m(t) = \frac{t}{1+t^2}$,

Determine & Sketch the modulated wave for amplitude modulation whose % modulation equals to

i) 50%, ii) 100%, c) 125%.

Soln: Let us consider different values of t,

t	0	0.1	0.2	0.4	0.5	0.6	0.8	1	2	3
$m(t) = \frac{t}{1+t^2}$	0	0.099	0.192	0.345	0.4	0.44	0.49	0.5	0.4	0.3

↳ Maximum value

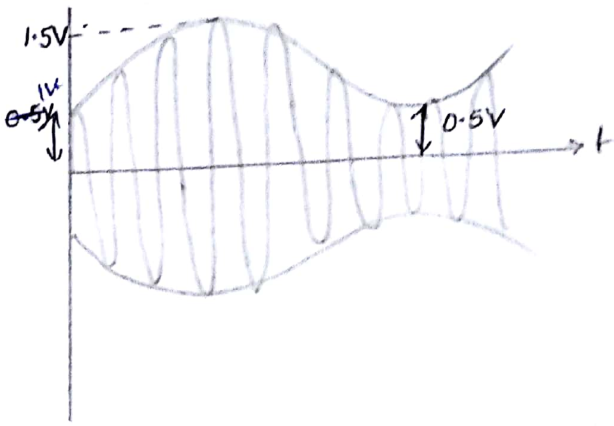
$\therefore$ Maximum amplitude of modulating signal $A_m = 0.5$

(i) For $\mu = 0.5$, $A_c = A_m/\mu = \underline{1}$

(ii) For $\mu = 1$, $A_c = A_m/\mu = \underline{0.5}$

(iii) $\mu = 1.25$, $A_c = \dfrac{A_m}{\mu} = \underline{0.4}$

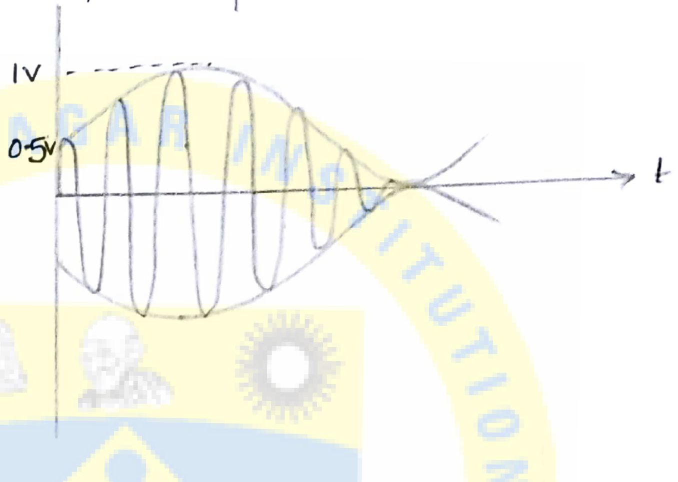
(i) $\mu = 0.5$, $A_c = 1$, $A_{max} = A_c(1+\mu) = 1.5V$.
$A_{min} = A_c[1-\mu] = \underline{0.5V}$



ii) $\mu = 1$, [100%], $A_c = 0.5$,

$A_{max} = A_c[1+\mu] = \underline{1V}$

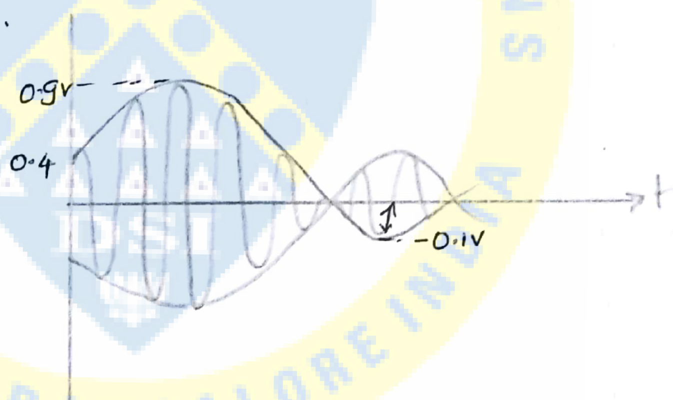
$A_{min} = A_c[1-\mu] = \underline{0}$



iii) $\mu = 1.25$ [125%], $A_c = 0.4$.

$A_{max} = A_c[1+\mu] = 0.4(1+1.25)$
$= \underline{0.9V}$

$A_{min} = A_c[1-\mu]$
$= 0.4(1-1.25)$
$= \underline{-0.1V}$



(12) Consider a composite wave is obtained by adding a non-coherent carrier $A_c\cos(2\pi f_c t + \varphi)$ to a DSB-SC wave $m(t)\cos 2\pi f_c t$. The composite wave is then applied to an envelope detector. Evaluate the detector output for $\varphi = 0$.

Soln: Input to the envelope detector is

$$v(t) = A_c\cos[2\pi f_c t + \varphi] + m(t)\cos 2\pi f_c t$$

W.K.T $\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

$$\therefore v(t) = A_c\cos 2\pi f_c t \cdot \cos\varphi - A_c \sin 2\pi f_c t \cdot \sin\varphi + m(t)\cos 2\pi f_c t$$
$$= [A_c\cos\varphi + m(t)]\cos 2\pi f_c t - A_c\sin\varphi \sin 2\pi f_c t$$

Envelope detector output is,

$$V_o(t) = \sqrt{\begin{pmatrix}\text{Inphase}\\ \text{component}\end{pmatrix}^2 + \begin{pmatrix}\text{Qudrature}\\ \text{component}\end{pmatrix}^2}$$

$$= \sqrt{[A_c\cos\varphi + m(t)]^2 + (A_c\sin\varphi)^2}$$

when $\varphi = 0$,

$$V_o(t) = \sqrt{[A_c + m(t)]^2 + 0} = \underline{A_c + m(t)}$$

$\therefore$ Output of envelope detector contains message signal $m(t)$

(13) An amplitude modulated signal is given by

$$s(t) = [10\cos(2\pi \times 10^6 t) + 5\cos(2\pi\times 10^6 t)(\cos 2\pi\times 10^3 t) + 2\cos(2\pi \times 10^6 t)(\cos(4\pi\times 10^3 t)] V$$

Find i) Total modulated power

ii) Sideband power

iii) Net modulation index.

Soln: W.K.T $s(t) = A_c[1 + \mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t]\cos 2\pi f_c t]$.

Given.

$$s(t) = 10\cos(2\pi\times 10^6 t) + 5\cos(2\pi\times 10^6 t)\cos(2\pi\times 10^3 t) + 2\cos(2\pi\times 10^6 t)\cos(4\pi\times 10^3 t)$$

$$s(t) = 10\cos(2\pi\times 10^6 t)[1 + 0.5\cos 2\pi\times 10^3 t + 0.2\cos(4\pi\times 10^3 t]$$

$A_c = 10$, $\mu_1 = 0.5$, $\mu_2 = 0.2$, $f_1 = 1\times 10^3 Hz$, $f_2 = 2\times 10^3 Hz$, $f_c = 1\times 10^6 Hz$.

(i) $P_T = P_c[1 + \mu_t^2/2]$.

But iii) $\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{0.5^2 + 0.2^2} = \underline{0.538}$

$$P_c = A_c^2/2R = \frac{10^2}{2\times 1} = \underline{50W}$$

(i) $\therefore$ $P_T = 50\left[1 + \frac{0.538^2}{2}\right] = \underline{57.25W}$

(ii) $P_{SB} = P_{USB} + P_{LSB}$

$$= P_c \cdot \mu_t^2/2 = \frac{0.538^2}{2} \times 50$$

$$= \underline{7.25W}$$

(4) An amplitude modulated voltage is given by
$$V = 50[1+0.2\cos 100t + 0.01\times\cos 3500t]\cos 10^6 t.$$ State all frequencies components presents in AM signal. Find the modulation index for each modulating voltage term. What is effective modulation index? Find the efficiency of modulation.

Soln: AM signal is given as
$$V(t) = A_c[1+\mu_1\cos 2\pi f_1 t + \mu_2\cos 2\pi f_2 t]\cos 2\pi f_c t$$

We have
$$V = 50[1+0.2\cos 100t + 0.01\times\cos 3500t]\cos 10^6 t.$$

Comparing those two equation, we get

$A_c = 50V$, $\mu_1 = 0.2$, $\mu_2 = 0.01$,

$$2\pi f_1 t = 100t$$
$$f_1 = \frac{100}{2\pi} = 15.92Hz$$

$$2\pi f_2 t = 3500t$$
$$f_2 = \frac{3500}{2\times\pi} = \underline{557.3Hz}$$

$$2\pi f_c t = 10^6 t$$
$$f_c = \frac{10^6}{2\pi}$$
$$f_c = \underline{159.2KHz}$$

Effective modulation index,
$$\mu_t = \sqrt{\mu_1^2+\mu_2^2} = \sqrt{0.2^2+0.01^2}$$
$$= \underline{0.2}$$

Efficiency of modulation $$\eta = \frac{\mu_t^2}{2+\mu_t^2} = \frac{0.2^2}{2+0.2^2} = 0.019 = \underline{1.9\%}$$

SINGLE SIDE BAND MODULATION:

Standard amplitude modulation and double side band suppressed carrier modulation are wasteful of bandwidth because they both require a transmission bandwidth equal to twice the message bandwidth. In either case, one half the transmission bandwidth is occupied by the upper side-band of the modulated wave, whereas the other half is occupied by the lower sideband. But the information contained in the USB is exactly identical to that carried by the LSB. Therefore we can transmit only one sideband LSB or USB without loss of information. It is possible to suppress the carrier and one side band completely by the modulation technique known as single side band Modulation.

FREQUENCY-DOMAIN DESCRIPTION OF SSB WAVE

The precise frequency domain description of a single side band modulated wave depends on which side band is transmitted.

Consider a message signal $m(t)$ with a spectrum $M(f)$ limited to the band $-W \le f \le W$ as in fig (a)

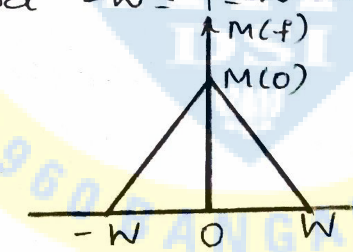


Fig (a): Message spectrum.

The spectrum of DSB-SC modulated wave is obtained by multipling $m(t)$ by the carrier wave $A_c \cos 2\pi f_c t$ is as in fig (b)

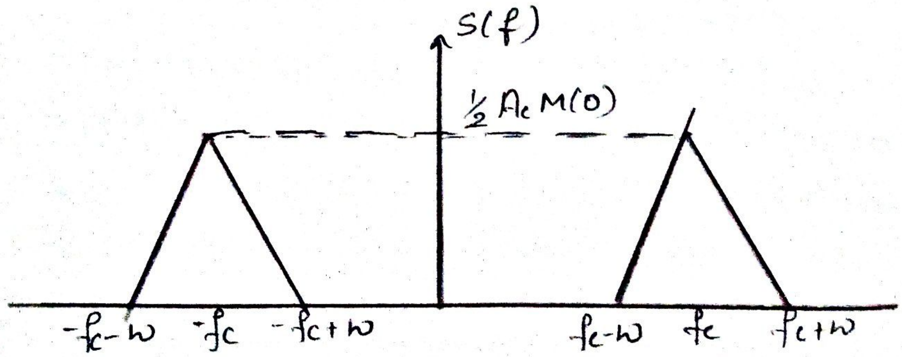


Fig (b): DSBSC spectrum.

The upper side band is represented by the frequencies above f_c & those below $-f_c$. When only USB is transmitted the resulting SSB modulated wave has the spectrum shown in fig (c).

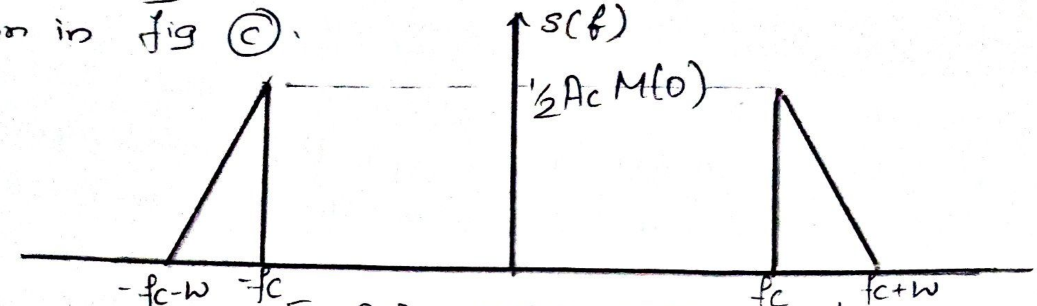


Fig (C): USB SSB spectrum.

Similarly the LSB is represented by frequencies below f_c & above $-f_c$ and when only LSB is transmitted, the resulting spectrum is as in fig (d)

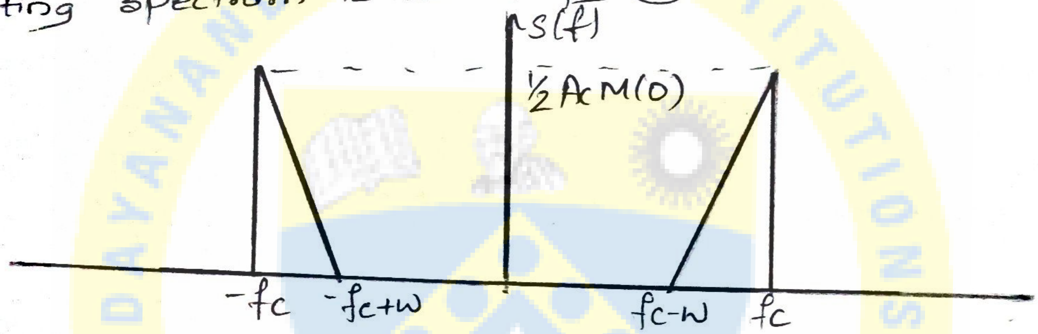


Fig (d): LSB SSB Spectrum

Advantages of SSB:-

* The spectrum space occupied by the SSB signal is f_m, which is only half that of AM and DSB signals. This reduction in frequency spectrum OR bandwidth allows more signals to transmit in the same frequency range without interfering each other.
* Due to suppression of high power carrier and one sideband power is saved and saved power can be used to produce a stronger signal that will carry further and it will be reliably received at greater distances.
* When bandwidth is less, the receiver circuits can be made with a narrower bandwidth, filtering out most of the noise.

Disadvantages of SSB:-

* High cost and complexity of its implementation.

Applications of SSB:

* Used to save power in applications where such a power saving is required i.e; Mobile Systems.

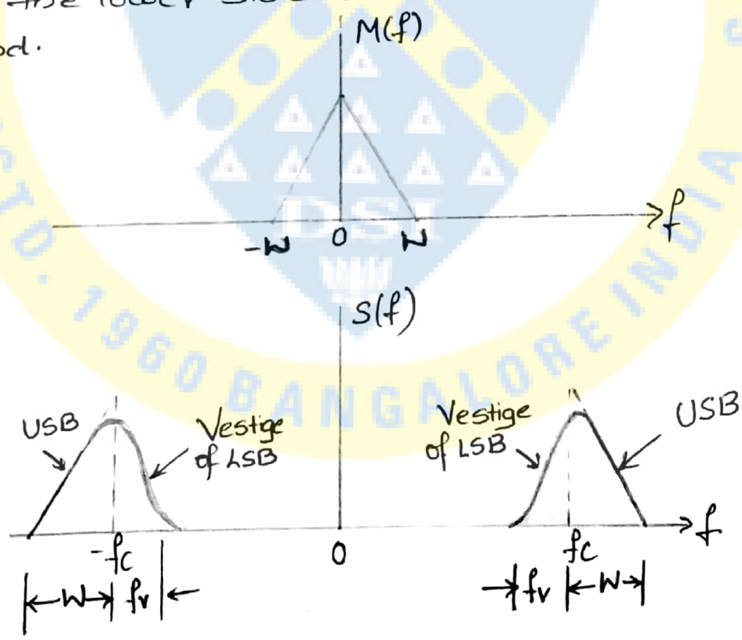
VESTIGIAL SIDE-BAND MODULATION [VSB]:

When the message signal contains significant components at extremely low frequency, the upper and lower sidebands meet at the carrier frequency. In such cases, the use of SSB modulation is inappropriate for the transmission of such message signals owing to the difficulty of isolating one sideband. To overcome this difficulty the modulation technique known as Vestigial sideband modulation is used. In this one sideband is passed almost completely whereas just a trace or vestige of the other sideband is retained. This is the compromise between SSB & DSBSC modulation.

The television signals contain significant components at extremely low frequencies & hence vestigial side band modulation is used in television transmission.

1. FREQUENCY DOMAIN DESCRIPTION

Figure below shows the spectrum of a vestigial sideband (VSB) modulated wave $s(t)$ in relation to that of the message signal $m(t)$ assuming that the lower sideband is modified into the vestigial sideband.



The transmission bandwidth required by the VSB modulated wave is given by $B = W + f_v$

where W is message bandwidth.
f_v is width of the vestigial sideband.

COMPARISON OF AMPLITUDE MODULATION TECHNIQUES:

SL No	Parameter	DSB-SC Standard AM	DSB-SC	SSB	VSB
1.	Power	High	Medium	Less	Less than DSB-SC but greater than SSB
2.	Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
3	Carrier suppression	No	Yes	Yes	~~No~~ Yes
4.	Sideband transmission	No	No	One sideband completely	One side band suppressed partly.
5.	Transmission efficiency	Minimum	Moderate	Maximum	Moderate
6	Receiver complexity	Simple	Complex	Complex	Simple
7.	Modulation type	Non-linear	Linear	Linear	Linear
8	Applications	Radio communication	Linear radio communication	Linear point to point mobile communication	Television

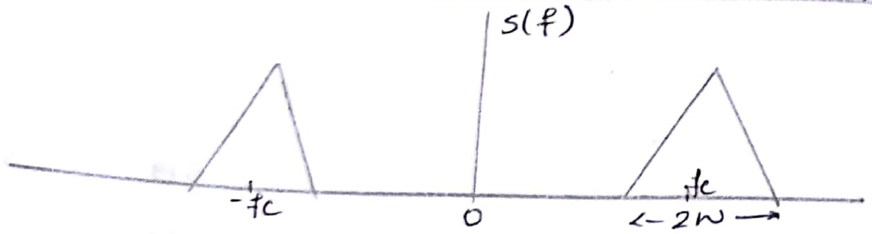
6 • **FREQUENCY TRANSLATION:**

In communication systems, it is necessary to translate the modulated wave upward or downward in frequency, so that it occupies a new frequency band. This frequency translation is accomplished by multiplication of the signal by a locally generated sine wave & subsequent filtering.

Consider the DSB-SC wave

$$s(t) = m(t)\cos 2\pi f_c t$$

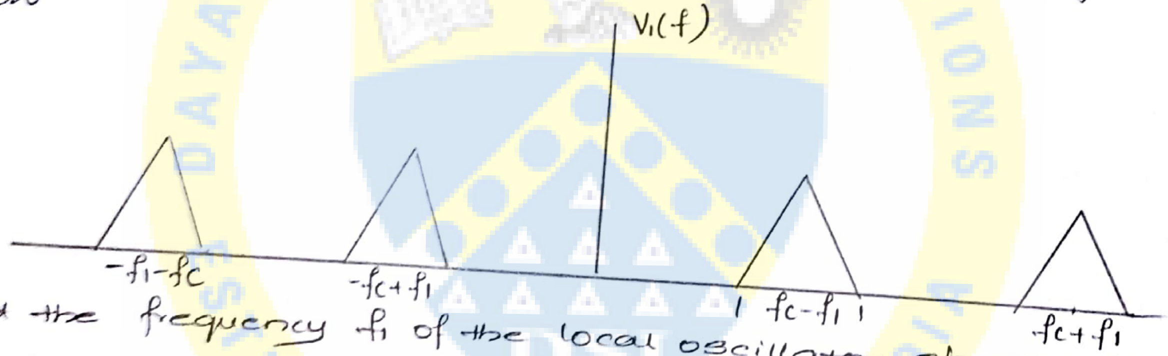
The modulating wave m(t) is limited to the frequency band $-W \le f \le W$. The spectrum of s(t) therefore occupies the bands $f_c - W \le f \le f_c + W$ & $-f_c - W \le f \le -f_c + W$ as in fig below



Suppose that it is required to translate this modulated wave downward in frequency, so that its carrier frequency is changed from fc to a new value 'fo', where $f_0 < f_c$. To do this, we must first multiplying the incoming modulated wave s(t) by a sinusoidal wave of frequency f_1 supplied by a local oscillator to obtain

$$v_1(t) = s(t) \cos 2\pi f_1 t$$
$$= m(t)(\cos 2\pi f_c t) \cos 2\pi f_1 t$$
$$= \frac{m(t)}{2}\left[\cos 2\pi (f_c - f_1) t + \cos 2\pi (f_c + f_1) t\right]$$

The multiplier output $v_1(t)$ consists of 2 DSBSC waves one with a carrier frequency of $f_c - f_1$ & other with a carrier frequency of $f_c + f_1$. The spectrum of $V_1(t)$ is therefore in fig below



Let the frequency f_1 of the local oscillator chosen so that

$$f_c - f_1 = f_0$$

Then from above figure we can see that the modulated wave with the desired carrier frequency 'fo' may be extracted by passing the multiplier output $V_1(t)$ through a bandpass filter of mid-band frequency f_0 & B.W of 2W, provided

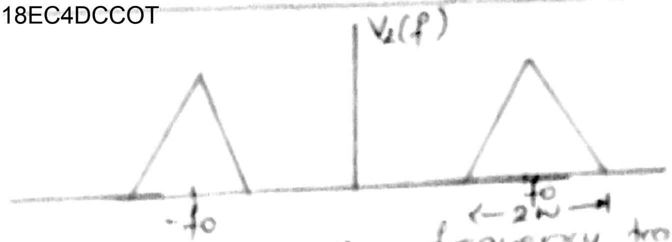
$$f_c + f_1 - W > f_c - f_1 + W$$

OR $\quad f_1 > W$

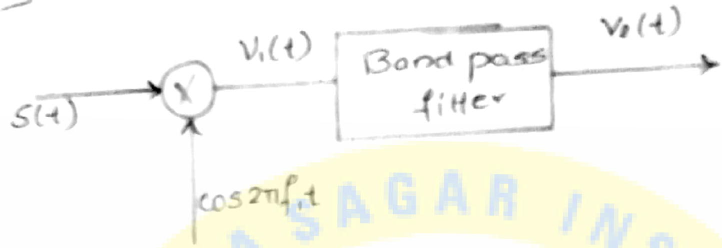
The filter output is therefore

$$v_2(t) = \frac{1}{2} m(t) \cos [2\pi (f_c - f_1) t]$$
$$= \frac{1}{2} m(t) \cos 2\pi f_0 t$$

This output is desired modulated wave, translated downward in frequency as shown in fig below



A device that carries out the frequency translation of a modulated wave is called a mixer. The operation itself is called mixing or heterodyning. For the implementation of a mixer, we use a multiplier and band pass filter as shown in fig



Ex:- Consider an incoming narrow band signal of BW 10kHz and midband frequency that may lie in the range 0.535 - 1.605MHz. It is required to translate this signal to a fixed frequency band centered at 0.455MHz. Determine the range of tuning that must be provided in the local oscillator

Let f_c denote the midband frequency of the incoming signal & f_1 denote the local oscillator frequency. Then we may write $0.535 < f_c < 1.605$ & $f_c - f_1 = 0.455$

where both f_c & f_1 are expressed in MHz that is $f_1 = f_c - 0.455$

when $f_c = 0.535MHz \rightarrow f_1 = 0.08MHz$

$f_c = 1.605MHz \rightarrow f_1 = 1.15MHz$

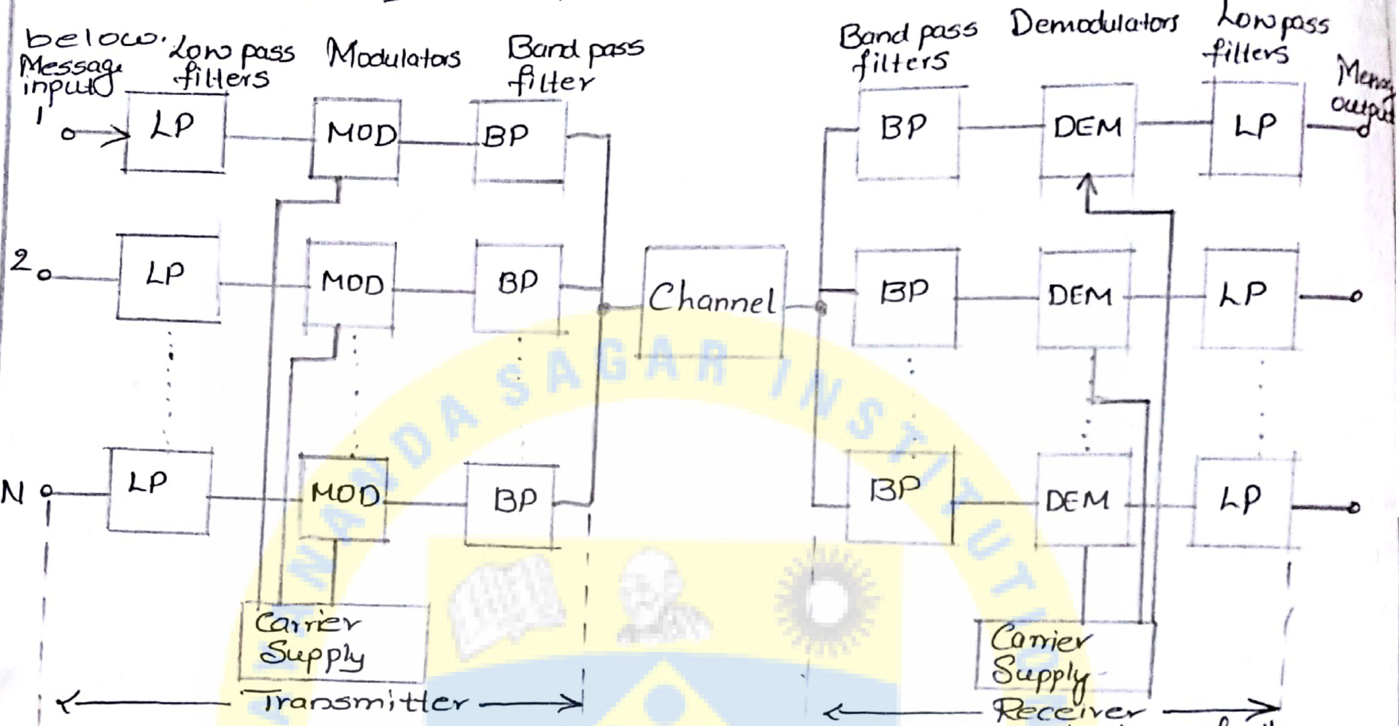
Thus the required range of tuning of the local oscillator is 0.08 - 1.15 MHz

7. FREQUENCY DIVISION MULTIPLEXING

Multiplexing is a technique whereby a number of independent signals can be combined into a composite signal suitable for transmission over a common channel. To transmit a number of signals over the same channel, the signal must be kept apart so that they do not interfere with each other & thus they can be separated at the receiving end.

This is accomplished by separating the signals either in frequency or time. The technique of separating the signals in frequency is refered to as frequency division multiplexing [FDM]. Whereas the technique of separating the signa in time is called as time division multiplexing [TDM].

The block diagram of an FDM system is as shown below.



The incoming message signals are assumed to be of the low pass type. Following each signal input, LPF is used which is designed to remove high-frequency components that donot contibute significantly to signal representation but are capable of disturbing other message signals that share the common channel. The filtered signals are applied to the modulators that shift the frequency ranges of the signals so as to occupy mutually exclusive frequency intervals. The necessary carrier frequencies needed to perform these frequency translations are obtained from a carrier supply. For the modulation any one of the method described is used. The most widely used method is single sideband modulation, which in the case of voice signals requires a B.W that is approximately equal to that of original voice signal. The band pass filters following the modulators are used to restrict the band of each modulated wave to its prescribed range. The resulting band pass filter outputs are next combined in parallel to form the input to the common channel.