DEPARTMENT
OF
ELECTRONICS & COMMUNICATION ENGINEERING

Communication Theory
18EC4DCCOT

(Theory Notes)

Autonomous Course

Prepared by

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Module – 4 Contents
Sampling process and Pulse Modulation techniques: Advantages of digital transmission, Analog to Digital Converter block diagram., Sampling Theorem, Reconstruction of message from its sample, Signal distortion in sampling, Practical aspects of sampling and signal recovery, PAM-PWM-PPM: modulator and demodulator circuit

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UNIT- 4 Sampling Process and Pulse Modulation Techniques

OBJECTIVE OF THE LECTURE:

1. To know the purpose of communication
2. Factors that led to the growth of digital commⁿ.
3. Sources & Signals
4. Conversion of Analog to Digital signal.

COURSE OUTCOMES:

1. CO61.1 : Able to understand digital communication, ways to represent digital data and types of digital modulation techniques

NOTES:

* The purpose of communication system is to transport an information-bearing signal from a source to a user destination via a communication channel.

* A communication system can be either analog or digital type.

1. ANALOG COMMUNICATION SYSTEM: Here, the information-bearing signal is continuously varying in both amplitude & time. It is used to directly modify some characteristic of a sinusoidal carrier wave, such as amplitude, phase or frequency.
2. DIGITAL COMMUNICATION SYSTEM: Here, the information bearing signal is processed so that it can be represented by a sequence of discrete

messages.

* The growth of digital communications is largely due to:

i. The impact of computers, not only as a source of data but also as a tool for communication.

ii. It offers flexibility & compatibility, in adoption of a common digital format makes it possible for transmission system to sustain many different sources of information.

iii. Reliability is increased.

iv. The availability of wide-band channels, provided by geo-stationary satellites, optical fibres & co-axial cables.

v. The availability of integrated solid-state electronic technology, which has helped to increase the system complexity in a cost-effective manner.

* SOURCES AND SIGNALS

A source of information generates a message.

Ex: human voice, television picture, teletype data, atm. temp. & pressure.

In these examples, the message is not electrical in nature, hence a transducer is used to convert it into an electrical waveform called as the message signal. The waveform is also referred (message signal) as a baseband signal, the term base-band used to designate the band of frequencies presenting the message signal generated at the source.

* The message signal can be analog or digital

i **ANALOG SIGNAL**: Both amplitudes & time vary continuously over their respective internals.
Ex: A speech signal, a TV signal, a signal representing atmospheric temp. or pressure at some locations.

ii **DIGITAL SIGNAL**: Both amplitude & time take on discrete values.
Ex: computer data, telegraph signals.

* ANALOG TO DIGITAL CONVERSION

An analog signal can always be converted into digital form by combining 3 basic operations

1. **SAMPLING**: Only sample values of the analog signal at uniformly spaced discrete instants of time are retained.
2. **QUANTIZATION**: Each sample value is approximated by the nearest level in a finite set of discrete levels.
3. **ENCODING**: The selected level is represented by a code-word that consists of a prescribed number of code elements.

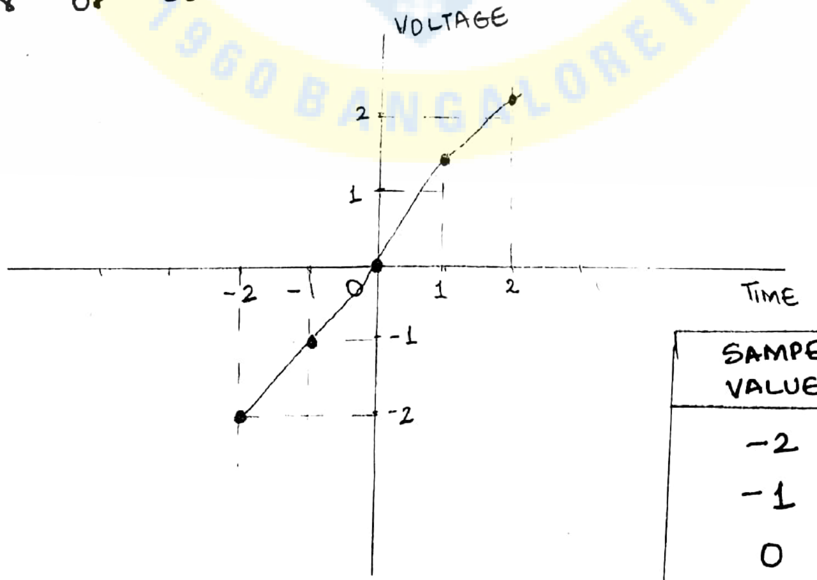


FIG 1.1 (a) ANALOG WAVEFORM

SAMPE VALUE	QUANTIZED LEVEL
−2	−2
−1	−1
0	0
1.7	2
2.1	2

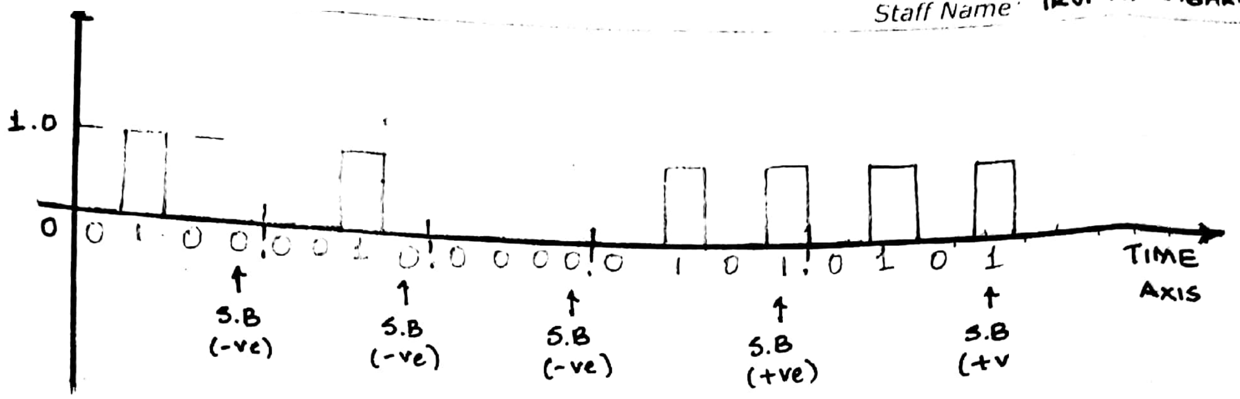


FIG. 1.1 (b) DIGITAL REPRESENTATION

ANALOG SIGNAL → SAMPLING → QUANTIZING → ENCODING → DIGITAL SIGNAL

FIG. 1.2 ANALOG TO DIGITAL CONVERSION

Fig. 1.1 (a) shows a segment of analog signal.
Fig. 1.1 (b) shows the corresponding digital waveform.

Symbols 0 & 1 of the binary code are represented by 0V & 1V resply. The code-word consists of four binary digits (bits), with the last bit assigned the role of a sign bit that signifies whether the sample value in question is +ve or -ve.

The remaining 3 bits are chosen to provide a numerical representation for the absolute value of a sample in accordance with Table.

TABLE: BINARY REPRESENTATION OF QUANTIZED LEVELS

Ordinal No. of Quantized Level	Level Number Expressed as SOP	Binary Number
0		000
1	2^0	001
2	2^1	010
3	2^1+2^0	011
4	2^2	100
5	2^2+2^0	101
6	2^2+2^1	110
7	$2^2+2^1+2^0$	111

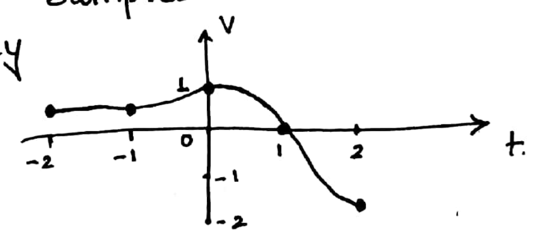
As a result of sampling & Quantization operations, errors are introduced into the digital signal. These errors are non-reversible in that, it is not possible to produce an exact replica of the original analog signal from its digital representation. But by proper selection of sampling rate & code-word length, the errors due to sampling & quantizing can be made so small that the difference betwn the analog signal & its digital representation is not discernible by a human observer.

POINTS TO REMEMBMER

- A communication system can be either analog or digital
- Transducer is used to convert non-electrical signal to electrical signal
- A message signal can be analog or digital
- 3 Basic Operations to convert analog signal to digital
  - ★ SAMPLING ★ QUANTIZATION ★ ENCODING
- Selection of sampling rate & code-word length properly can reduce errors due to sampling & quantization.

ASSIGNMENT QUESTIONS

1. Discuss the growth of digital communications.
2. Give the classification for signals with an example for each.
3. Briefly explain the 3 basic operations in the conversion of Analog signal to Digital form.
4. Consider the analog signal given below. Using appropriate quantization values & samples at t = -2, -1, 0, 1, 2, represent it digitally



REFERENCES

[1] DIGITAL COMMUNICATIONS, SIMON HAYKIN, John Wiley India Pvt Ltd, 2008.

OBJECTIVE OF THE LECTURE :

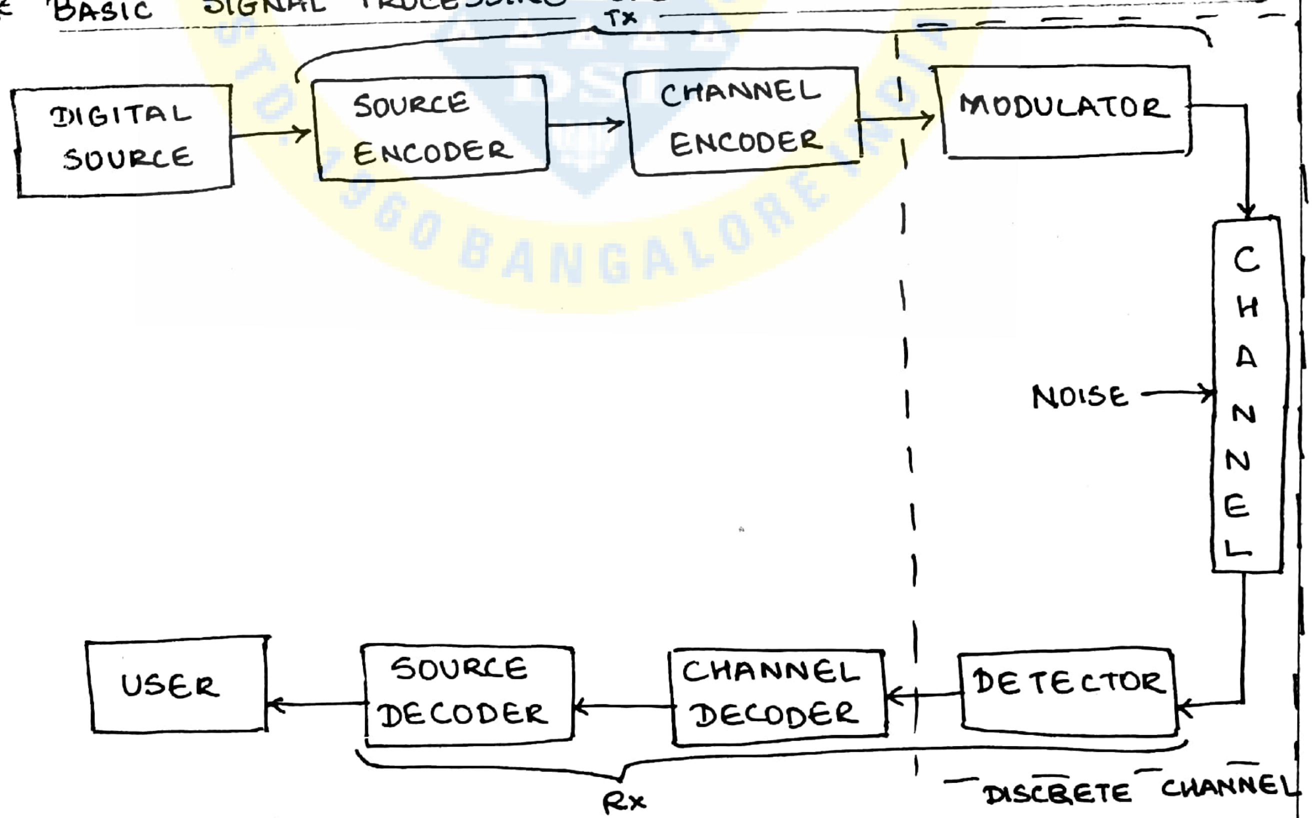
1. To know the basic signal processing operations in Digital communication.
2. Understand the sampling process, prerequistes.
3. To know the channels for Digital communication.

COURSE OUTCOMES :

1. CO61.1 : Able to understand digital communication, ways to represent digital data & types of digital modulation techniques.

NOTES :

* BASIC SIGNAL PROCESSING OPERATIONS IN DIGITAL COMM^{N}



In this diagram, 3 basic signal processing operations are identified:

1. Source Coding
2. Channel Coding
3. Modulation

It is assumed that source of information is digital by nature or converted into it by design.

I] SOURCE Coding : In source coding, the encoder maps the digital signal generated at the source output into another signal in digital form. The mapping is one-to-one & the objective is to eliminate or reduce redundancy so as to provide an efficient representation of the source output.

* Source coding reduces the bandwidth requirement.

II] CHANNEL Coding : In channel coding, the objective is for the encoder to map the incoming digital signal into a channel input & for the decoder to map the channel output into an output digital signal in such a way that the effect of channel noise is minimized.

Hence, the combined role of the channel encoder & decoder is to provide for reliable communication over a noisy channel.

* In source coding, we remove redundancy whereas in channel coding, we introduce controlled redundancy.

Here we introduce redundancy in a prescribed fashion in the channel encoder & exploit it in the decoder to reconstruct the original encoder input as accurately as possible.

III] MODULATION : It is performed for the efficient transmission of the signal over the channel. The modulator operates by keying shifts in the amplitude, frequency or phase of a sinusoidal carrier wave to the channel encoder output. The digital modulation techniques are referred to as amplitude-shift keying, frequency-shift keying, or phase-shift keying resply.

The detector performs demodulation, thereby producing a signal that follows the time variations in the channel encoder output.

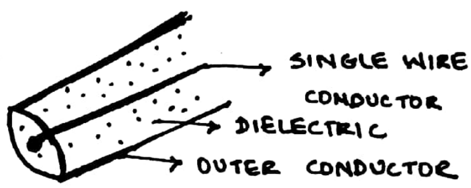
* The modulator, channel & detector form a discrete channel (bcoz, both its i/p & o/p signals are in discrete form).

CHANNELS FOR DIGITAL COMMUNICATION

The details of modulation & coding used in digital communication system depend on the characteristics of the channel & the application of interest. Bandwidth, power, linearity & interference are the channel characteristics.

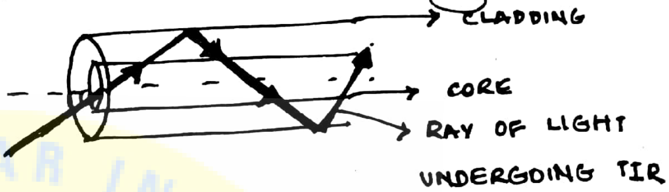
1. TELEPHONE CHANNEL: Designed to provide voice-grade communication. The channel has a band-pass characteristic occupying the freq. range 300-3400 Hz, a high SNR = 50 dB & linear response.

2. A COAXIAL CABLE: Consists of a single-wire conductor centered inside an outer conductor, which are insulated from each other by means of a dielectric material. The 2 primary advantages of coaxial cables are : 1. WIDE BANDWIDTH
2 FREEDOM from INTERFERENCE



3. AN OPTICAL FIBRE : Consists of a very fine inner core made of silica glass, surrounded by a concentric layer called cladding that is also made of glass. The R.I of core is slightly higher than that of cladding. The difference between RI helps guide the propogation of the ray of light.

* Smaller, higher transmission BW's & larger repeater separations.



4. A MICROWAVE RADIO :

Operates on a line of sight link, consists of a transmitter & receiver that are equipped with antennas of their own. The antennas are placed on towers at sufficient height to have the transmitter & receiver in LOS of each other.

* Operating freq. range : 1 to 30 GHz.

5. A SATELLITE CHANNEL :

Consists of a satellite in geostationary orbit, an uplink from a ground station & a downlink at microwave frequencies with uplink frequency being higher. Thus, the satellite channel may be viewed as a repeater in the sky, permitting communication over long distances at high B.W's & relatively low cost.

* The non-linear nature of the channel restricts its use to constant envelope modulation techniques.

POINTS TO REMEMBER

1. The 3 basic signal processing operations in digital communication are
   - i. Source coding
   - ii. Channel coding
   - iii. Modulation

2. Source coding reduces the B.W requirement.
3. In source coding, we remove redundancy whereas in channel coding, we introduce controlled redundancy.
4. The modulator, channel & detector form a discrete channel.
5. Coaxial cable requires closely spaced repeaters, Optical fibre requires longer repeater separations.
6. A Microwave radio works on LOS & optical fibre works on TIR principle.

ASSIGNMENT QUESTIONS

1. With a neat block diagram, explain the basic signal processing operations in digital communication.
2. Discuss on channels for Digital communication.

REFERENCES

[1]. DIGITAL COMMUNICATIONS, SIMON HAYKIN, John Wiley India Pvt Ltd, 2008

OBJECTIVE OF THE LECTURE :

1. To understand the Sampling Process
2. To prove Sampling Theorem.

COURSE OUTCOMES :

1. CO61.1 : Able to understand digital communication, ways to represent digital data & types of digital modulation techniques.

NOTES :

SAMPLING PROCESS

A msg signal may originate from a digital source or analog source. The sampling process is the first process performed in analog-to-digital conversion. Two other processes, quantizing & encoding are also involved in the conversion.

In sampling process, a continuous-time signal is converted into a discrete-time signal at periodic instants of time.

SAMPLING THEOREM

Consider an analog signal $g(t)$ that is continuous in both time & amplitude. We assume that $g(t)$ has infinite duration but finite energy.

A segment of signal is depicted in fig (a). Let the sample values of the signal $g(t)$ at times

$t = 0, \pm T_s, \pm 2T_s, \ldots;$ be denoted by the series $\{g(nT_s), n = 0, \pm 1, \pm 2, \ldots\}$.

We refer to T_s as the sampling period.

& $f_s = 1/T_s$ as the sampling rate.

We define the discrete-time signal, $g_\delta(t)$; that results from the sampling process as

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \cdot \delta(t - nT_s) \rightarrow (1)$$

where $\delta(t - nT_s)$ is a Dirac delta function located at time $t = nT_s$.

fig (b) illustrates the construction of $g_\delta(t)$ from sample values of $g(t)$.

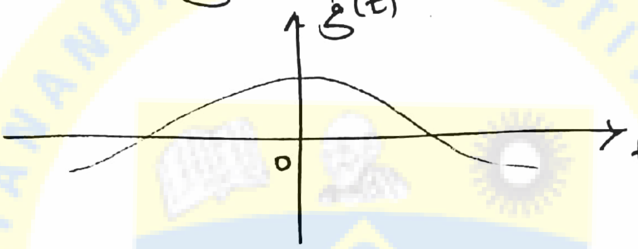


fig (a)

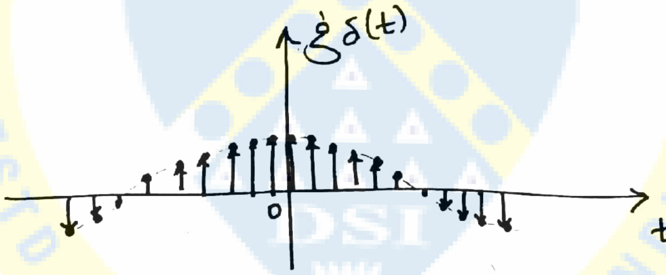


fig (b)

From the definition of a delta function, we have

$$g(nT_s)\, \delta(t - nT_s) = g(t) \cdot \delta(t - nT_s)$$

Hence, we may rewrite eqn (1) in the equivalent form

$$g_\delta(t) = g(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$= g(t)\, \delta_{T_s}(t) \rightarrow (2)$$

where $\delta_{T_s}(t)$ is the Dirac comb or ideal sampling function.

From eqn (2) we may view the discrete-time signal $g_\delta(t)$ as the o/p of an impulse modulator, which operates with $g(t)$ as the modulating wave & $\delta_{Ts}(t)$ as the carrier wave. This operation is viewed in the fig (c).

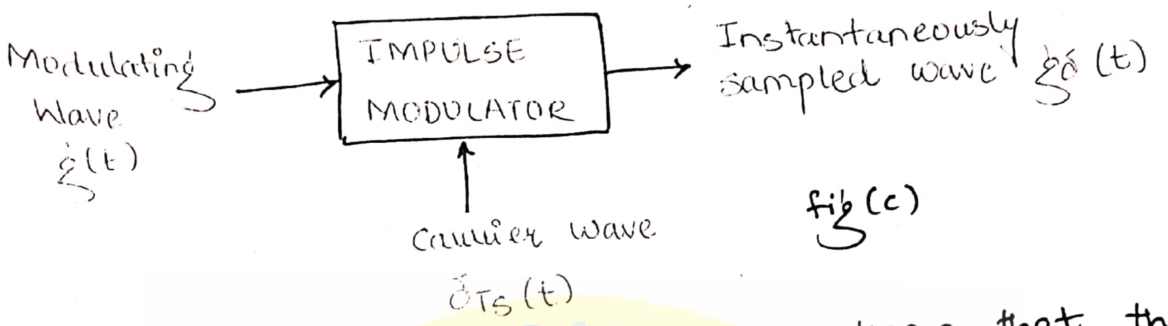


fig (c)

From the properties of F.T, we know that the multiplication of 2 time functions, is equivalent to the convolution of their respective F.T's.

Let $G(f)$ & $G_\delta(f)$ denote the F.Ts of $g(t)$ & $g_\delta(t)$ resply, for the fourier transform of $\delta_{Ts}(t)$, We have

$$F[\delta_{Ts}(t)] = f_s \sum_{m=-\infty}^{\infty} \delta(f-mf_s) \rightarrow (3)$$

where $F[\cdot]$ signifies the F.T operation

$f_s \rightarrow$ sampling rate

$$G_\delta(f) = G(f) \star \left[f_s \sum_{m=-\infty}^{\infty} \delta(f-mf_s)\right] \rightarrow (4)$$

$$G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f) \star \delta(f-mf_s) \rightarrow (5)$$

WKT, from the properties of convolution

$$G_\delta(f) = f_s \sum_{m=-\infty}^{\infty} G(f-mf_s) \rightarrow (6)$$

From (6), we see that $G_\delta(f)$ represents a spectrum that is periodic in freq. f with period f_s, but not necessarily continuous.

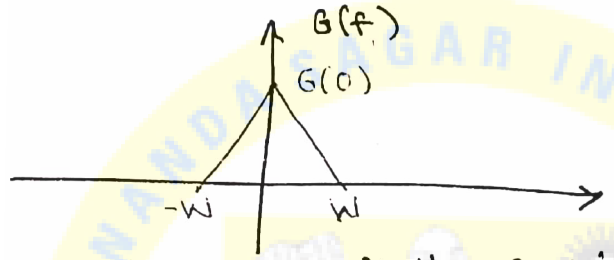
Thus, the process of uniformly sampling a signal in time domain results in a periodic spectrum in the freq. domain with a period equal to the sampling rate.

$$F[\delta(t-nT_s)] = \exp(-j2\pi n f T_s)$$

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \cdot \exp(-j2\pi n f T_s) \rightarrow (7)$$

The relations derived can be applied to any continuous time signal $g(t)$ of finite energy & infinite duration.

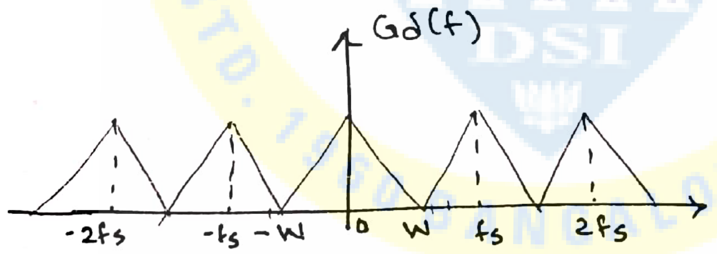
Suppose, the signal is strictly bandlimited, with no. freq. components higher than W Hz. The FT $G(f)$ of the signal $g(t)$ has the property that $G(f)$ is zero for $|f| \geq W$.



The signal $g(t)$ has finite energy means that the area under the curve of the energy spectral density $|G(f)|^2$ is likewise finite.

Suppose the sampling period is $T_s = 1/2W$

Then,



Putting $T_s = 1/2W$ in eq^n (7)

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(n/2W) \exp(-j\pi n f/W) \rightarrow (8)$$

Putting $f_s = 2W$ in eq^n (6)

$$G_\delta(f) = 2W \sum_{m=-\infty}^{\infty} G(f - m2W)$$

$$G(f) = \frac{1}{2W} G_\delta(f) \qquad -W < f < W \rightarrow (9)$$

Also from eq^n (8),

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \cdot \exp\left(\frac{-j\pi n f}{W}\right) \quad -W < f < W \rightarrow (10)$$

If the sample values of $g(n/2W)$ of the signal $g(t)$ are specified for all time, then the F.T $G(f)$ of the signal is uniquely determined by using the fourier series of eq^n (10).

Because $g(t)$ is related to $G(f)$ by the inverse F.T, it follows that the signal $g(t)$ is itself uniquely determined by the sample values $g(n/2W)$ for $-\infty \le n \le \infty$. Hence the sample values $g(n/2W)$ contains all of the information of $g(t)$.

Consider next the problem of reconstruction of the signal $g(t)$ from the sequence of sample values $\{g(n/2W)\}$.

∴ On performing reverse or Inverse Fourier Transform, & defining $g(t)$ in terms of $G(f)$

$$g(t) = \int_{-\infty}^{\infty} G(f).\exp(j2\pi ft).df$$

$$= \int_{-\infty}^{\infty} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi nf}{W}\right) \exp(j2\pi ft).df$$

$-\infty$ to $+\infty$ change to $-W$ to $+W$ because $g(t)$ is Bandlimited signal

$$= \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi nf}{W}\right) \exp(j2\pi ft).df$$

On Interchanging the order of summation & integration:

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \int_{-W}^{W} \exp\left[j2\pi f\left(t - \frac{n}{2W}\right)\right].df \rightarrow (11)$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) . \frac{\sin(2\pi Wt - n\pi)}{(2\pi Wt - n\pi)} \rightarrow (12)$$

We have,

$$\text{Sinc } x = \frac{\sin(\pi x)}{\pi x} \rightarrow (13)$$

where x is an independent variable.
The sinc function exhibits an important property known as the interpolatory property, which is described as

$$\text{sinc}\, x = \begin{cases} 1 & , x = 0 \\ 0 & , x = \pm 1, \pm 2, \dots \end{cases} \rightarrow (14)$$

Hence, Eqn (12) can be written as

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \cdot \text{sinc}(2Wt - n) \rightarrow (15)$$

Hence Eqn (15) provides an interpolation formula for reconstructing the original signal $g(t)$ from the sequence of sample values $\{g(n/2W)\}$, with the sinc function sinc$(2Wt)$ playing the role of an interpolation function.

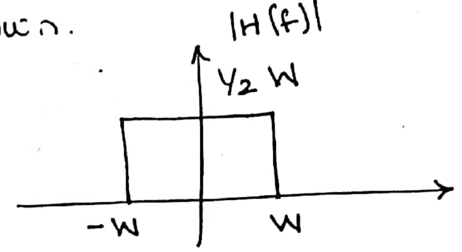
Each sample is delayed by the interpolation function & all the waveforms are added to obtain $g(t)$.

Eqn (15) also represents the response of an ideal low-pass filter of B.W W & zero transmission delay, which is produced by an i/p signal consisting of the sequence of samples $\{g(n/2W)\}$ for $-\infty \le n \le \infty$

from fig (b) we see that the original signal $g(t)$ may be recovered exactly from the sequence of samples $g(n/2W)$ by passing it through an ideal L.P.F of B.W 'W'.

Seq. of samples $g(nT_s)$ → Ideal LPF → Analog signal $g(t)$

The ideal amplitude response of the reconstruction filter is shown.



POINTS TO REMEMBER

1. $\delta(t-nT_s) \rightarrow$ Dirac delta function
2. $\delta_{T_s}(t) \rightarrow$ Dirac comb or ideal sampling function.
3. $F[\delta_{T_s}(t)] = f_s \sum_{m=-\infty}^{\infty} \delta(f-mf_s)$
4. $F[\delta(t-nT_s)] = \exp(-j2\pi n f T_s)$
5. $\operatorname{sinc} x = \frac{\sin \pi x}{\pi x}$, $\operatorname{sinc} x = \begin{cases} 1, & x=0 \\ 0, & x=\pm 1, \pm 2, \ldots \end{cases}$

REFERENCE

[1]. DIGITAL COMMUNICATIONS, SIMON HAYKIN, John Wiley India, Pvt Ltd, 2008.

ASSIGNMENT QUESTIONS

1. Explain sampling Process
2. Derive the interpolation formula for reconstructing the original signal $g(t)$.
3. State & Prove : Sampling Theorem.

OBJECTIVE OF THE LECTURE:

1. To develop imp. interpretation by using the property that the function Sinc(2Wt−n) is one of the family of shifted sinc functions, that are mutually orthogonal.
2. To understand the statement of Sampling Theorem.

COURSE OUTCOMES:

1. CO61.1 : Able to understand digital communication, ways to represent digital data and types of digital modulation techniques.

NOTES:

(1) SIGNAL SPACE INTERPRETATION

From eqn (15), we may develop another important interpretation, by using the property that the function Sinc(2Wt−n), where n is an integer, is one of the family of shifted sinc functions that are mutually orthogonal.

To prove this property, we use the formula

$$\int_{-\infty}^{\infty} g_1(t) \cdot g_2^*(t) \cdot dt = \int_{-\infty}^{\infty} G_1(f)\, G_2^*(f) \cdot df \rightarrow (1)$$

[Rayleigh's energy theorem]

F.T pair

$g_1(t) \Longleftrightarrow G_1(f)$

$g_2(t) \Longleftrightarrow G_2(f)$

Put $g_1(t) = \text{Sinc}(2Wt-n) = \text{Sinc}\left[2W\left(t - \frac{n}{2W}\right)\right]$

& $g_2(t) = \text{sinc}(2Wt - m) = \text{sinc}\left[2W\left(t - \frac{m}{2W}\right)\right]$

for $\text{sinc}(2Wt) \rightleftharpoons \frac{1}{2W} \cdot \text{rect}\left(\frac{f}{2W}\right)$

where $\text{rect}(x) = \begin{cases} 1 & , \quad -\frac{1}{2} < x < \frac{1}{2} \\ 0 & , \quad |x| > \frac{1}{2} \end{cases}$

The function $g_1(t)$ & $g_2(t)$ are time-shifted versions of the sinc pulse $\text{sinc}(2Wt)$.

Using the time-shift property of F.T, we may express the F.T of $g_1(t)$ & $g_2(t)$, as follows,

* $x(t - t_0) \rightleftharpoons e^{-j2\pi f t_0} \, x(f)$

$$G_1(f) = \frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right)$$

$$G_2(f) = \frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right) \exp\left(-\frac{j\pi m f}{W}\right)$$

$\therefore$ Eqn ① becomes

$$\int_{-\infty}^{\infty} \text{sinc}(2Wt - n)\, \text{sinc}(2Wt - m). \, dt = \left(\frac{1}{2W}\right)^2 \int_{-W}^{W} \exp\left(-\frac{j\pi f}{W}(n-m)\right). df$$

$$= \frac{\sin[\pi(n-m)]}{2W \; \pi \, (n-m)}$$

$$= \frac{1}{2W} \; \text{sinc}(n-m)$$

$\left\{ \begin{array}{l} n = m \\ = \left(\frac{1}{2W}\right)^2 \int_{-W}^{W} e^0 . df \\ = \frac{1}{(2W)^2} \cdot 2W \\ = \frac{1}{2W} \end{array} \right.$

when $n = m$, $= \frac{1}{2W}$

$n \neq m$, $= 0$

$$\therefore \int_{-\infty}^{\infty} \text{sinc}(2Wt - n). \, \text{sinc}(2Wt - m). \, dt$$

$$= \begin{cases} 1/2W & n = m \\ 0 & n \neq m \end{cases}$$

we already had

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right). \, \text{sinc}(2Wt - n)$$

it represents the expansion of the signal $g(t)$ as an infinite sum of orthogonal functions with the co-effs of expansion, $g(n/2W)$ defined by

$$g\left(\frac{n}{2W}\right) = 2W \int_{-\infty}^{\infty} g(t) \cdot \text{Sinc}(2Wt-n) \cdot dt$$

* In this space, each signal corresponds uniquely to one pt. & each pt. to one signal

(2) Statement of the SAMPLING THEOREM :

The sampling theorem for band-limited signals of finite energy can be stated in 2 separate parts :

a) If a finite-energy signal $g(t)$ contains no frequencies higher than W Hz, it is completely determined by specifying its ordinates at a sequence of points spaced $1/2W$ secs apart.

b) If a finite energy signal $g(t)$ contains no frequencies higher than W Hz, it may be completely recovered from its ordinates at a sequence of points spaced $1/2W$ secs aparts.

Part 1 is a restatement of Eq (9) &

Part 2 is a restatement of Eq (15).

The min. sampling rate of 2W samples per second, for a signal B.W of W Hz, is called the Nyquist rate. Correspondingly, the reciprocal $1/2W$ is called the Nyquist interval. The sampling theorem serves as the basis for the interchangeability of analog signal & digital sequences.

POINTS TO REMEMBER

1. The function $\text{Sinc}(2Wt-n)$, is one of the family of shifted sinc functions that are mutually orthogonal.

2. $\text{rect}(x) = \begin{cases} 1 & -1/2 < x < 1/2 \\ 0 & |x| > 1/2 \end{cases}$

3. $x(t-t_0) \Leftrightarrow e^{-j2\pi f t_0} \cdot X(f)$.

4. $\int_{-\infty}^{\infty} \text{sinc}(2wt-n) \cdot \text{sinc}(2wt-m) \cdot dt = \begin{cases} 1/2w & n=m \\ 0 & n \neq m \end{cases}$

5. Statements of Sampling Theorem.

ASSIGNMENT QUESTIONS

1. Discuss the Signal Space Interpretation.
2. State Sampling Theorem.

REFERENCES

[1] DIGITAL COMMUNICATIONS, SIMON HAYKIN, John Wiley India Pvt Ltd, 2008.

FACULTY NAME: TRUPTI TAGARE

DATE:

LECTURE HOUR:

OBJECTIVE OF THE LECTURE:

1. To understand Quadrature Sampling of Band-Pass signals.
2. Practical aspects of sampling & signal recovery.

COURSE OUTCOMES:

1. CO61.1: Able to understand digital communication, ways to represent digital data & types of digital modulation techniques

NOTES:

QUADRATURE SAMPLING OF BAND-PASS SIGNALS

Till now, we have discussed the sampling process, focussing on the signals as low pass. However, in practice we always come across band-pass signals.

In this scheme, called Quadrature sampling for uniform sampling of Band-Pass signals is considered in this section.

Firstly, we represent the band pass signal in terms of its in-phase & quadrature components, each of which may then be sampled separately.

Consider a band-pass signal $g(t)$ whose spectrum is limited to a B.W $2W$, centered around the freq. f_c, as illustrated in fig 1(a). It is assumed that $f_c > W$.

Let $g_I(t)$ denote the in-phase component. $g_Q(t)$ denote the quadrature component.

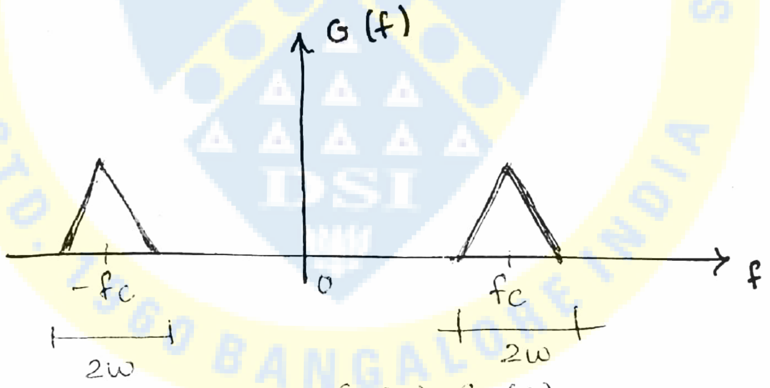
$\therefore\ g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t) \longrightarrow$ (A)

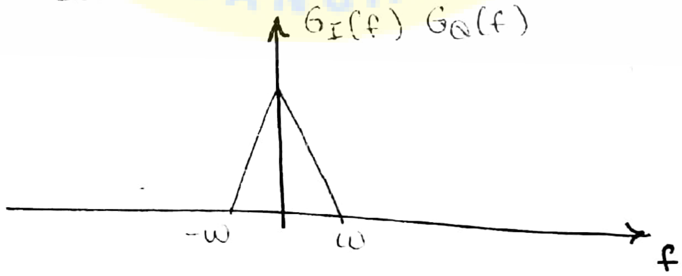
$g(t) = \mathrm{Re}[\tilde{g}(t)\exp(j2\pi f_c t)]$

$\quad = g_I(t) + j\,g_Q(t)$

$\exp(j2\pi f_c t) = \cos(2\pi f_c t) + j\sin 2\pi f_c t$

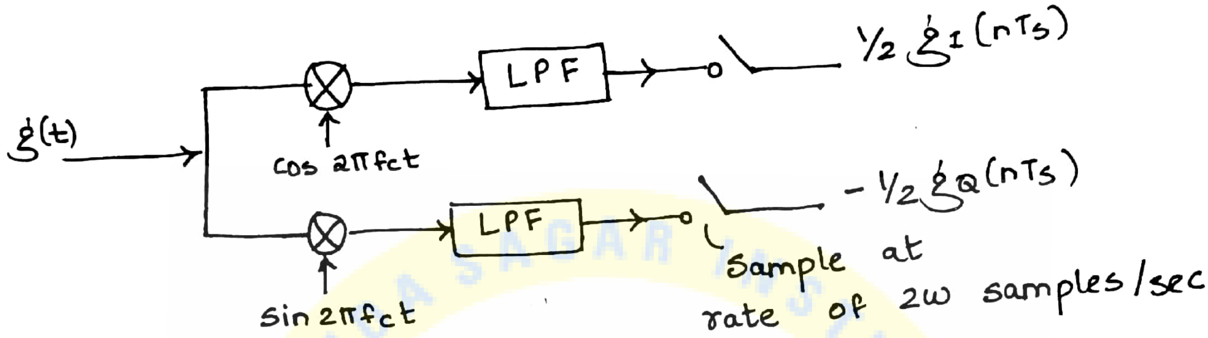
Hence $g_I(t)$ & $g_Q(t)$ components may be obtained by multiplying the band-pass signal $g(t)$ by $\cos(2\pi f_c t)$ & $\sin(2\pi f_c t)$, resply. & then suppressing the sum-freq. components by means of appropriate L.P.F. Under the assumption that $f_c > w$, we find $g_I(t)$ & $g_Q(t)$ are both-low pass signals limited to $-W < f < W$, as shown in fig 1(b). Accordingly, each component may be sampled at the rate of $2W$ samples per second. This form of sampling is called quadrature sampling.





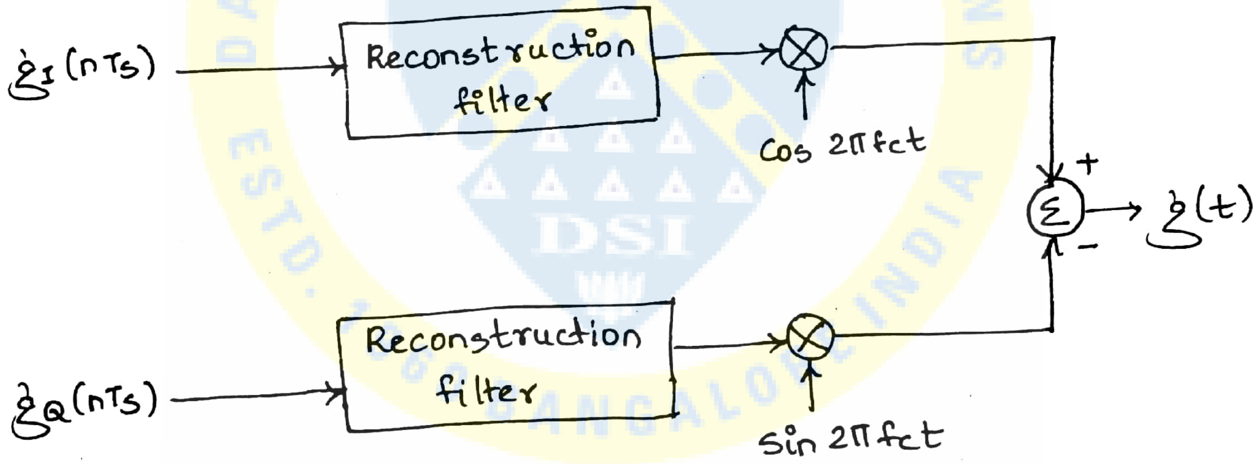
To reconstruct the original band-pass signal from its quadrature-sampled version, we first construct the in-phase component $g_I(t)$ & the quadrature component $g_Q(t)$ from their respective samples, multiply them resply by $\cos(2\pi f_c t)$ & $\sin(2\pi f_c t)$ & then add the results.

The operations that constitute quadrature sampling are as shown



(a)

The operations pertaining to the reconstruction process are depicted as



PRACTICAL ASPECTS OF SAMPLING & SIGNAL RECOVERY

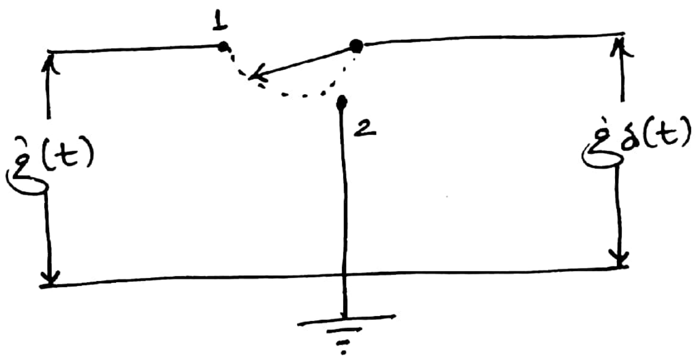
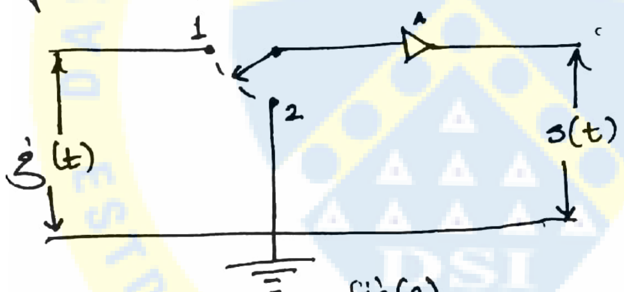


fig. 1.

This ckt. illustrates the concept of ideal sampling ie., sampling of a strictly bandlimited signal $g(t)$ using Dirac comb. The switch remains in position 1 for a time equal to almost zero & then moves to position 2 & remains in this position for a period equal to Ts. This type of switch cannot be realised using high-speed switching transistor circuits.

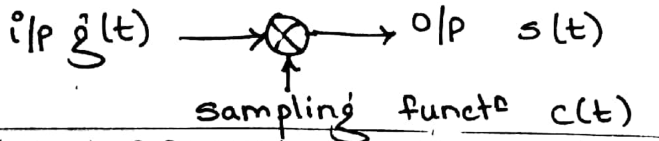
The problem is to find a switch that remains in position 1 for a length of time equal to zero. Hence, in practice, we select a switching circuit in which the switch remains in position 1 for a breif interval of time = T secs. In instrumentation or transmission we usually tend to

(1) Ordinary samples of finite duration [Natural sampling]



fig(a)

The principle of natural sampling can be explained using the switching sampler as shown in the fig(a). Here, the switch moves from position 1 to position 2 at a rate $f_s = 1/T_s$ secs & remains in position 1 for a brief interval of time equal to T seconds & on grounded contact for the remainder of each sampling period. The o/p of the sampler s(t) consists of segments of $g(t)$, & s(t) can be visualized as the product of $g(t)$ & c(t) as shown.



$$s(t) = g(t) \cdot c(t) \rightarrow (1)$$

where $c(t)$ is the sampling or switching func^n.

Two questions need to be answered are:

(1) Are the sampled segments known as natural samples sufficient to describe the original signal $g(t)$?

(2) If so, how can we reconstruct the original signal $g(t)$ from the sampled signal $s(t)$?

POINTS TO REMEMBER

1. Writing of in-phase component & quadrature phase component by multiplying the B.P. signal $g(t)$ by $\cos(2\pi f_c t)$ & $\sin(2\pi f_c t)$ resply.
2. $g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$
3. Natural sampling: switch remains in position 1 = T secs & on grounded contact for $(T_s - T)$ secs

ASSIGNMENT QUESTIONS

1. Write a short note on: Quadrature Sampling of Band Pass signals.
2. Discuss: Practical Aspects of Sampling & Signal Recovery.

REFERENCE

[1] DIGITAL COMMUNICATIONS, SIMON HAYKIN, John Wiley India Pvt Ltd, 2008.

OBJECTIVE OF THE LECTURE :

1. To understand Natural sampling ; sampling & reconstruction of the signal.

COURSE OUTCOMES :

1. CO61.1 : Able to understand digital communication, ways to represent digital data & types of digital modulation techniques.

NOTES :

Firstly, we construct the spectrum $s(t)$ using the rectangular pulse train $c(t)$. It has complex Fourier series representation given as

$$c(t) = f_s TA \sum_{n=-\infty}^{\infty} \text{sinc}(nf_sT)\, e^{j2\pi f_s n t} \rightarrow (2)$$

where $T_s = 1/f_s$ defines the sampling period

Substituting eq^n (2) in (1) we get

$$s(t) = f_s TA \sum_{n=-\infty}^{\infty} \text{sinc}(nf_sT) \cdot e^{j2\pi f_s n t} \cdot g(t) \rightarrow (3)$$

From freq. shift property

$$x(t)\, e^{j2\pi\beta t} \xleftrightarrow{F.T} X(f-\beta)$$

Taking F.T on both sides & applying the freq. shift property to the above eq^n we get

$$S(f) = f_s TA \sum_{n=-\infty}^{\infty} \text{sinc}(nf_sT)\, G(f-nf_s) \rightarrow (4)$$

The spectrum $S(f)$ for the assumed spectrum $G(f)$

is shown in the fig. (b), for $f_s > 2W$

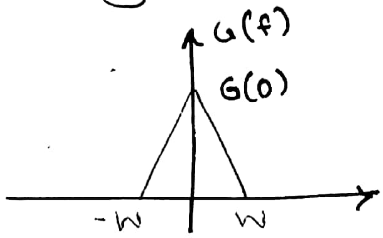
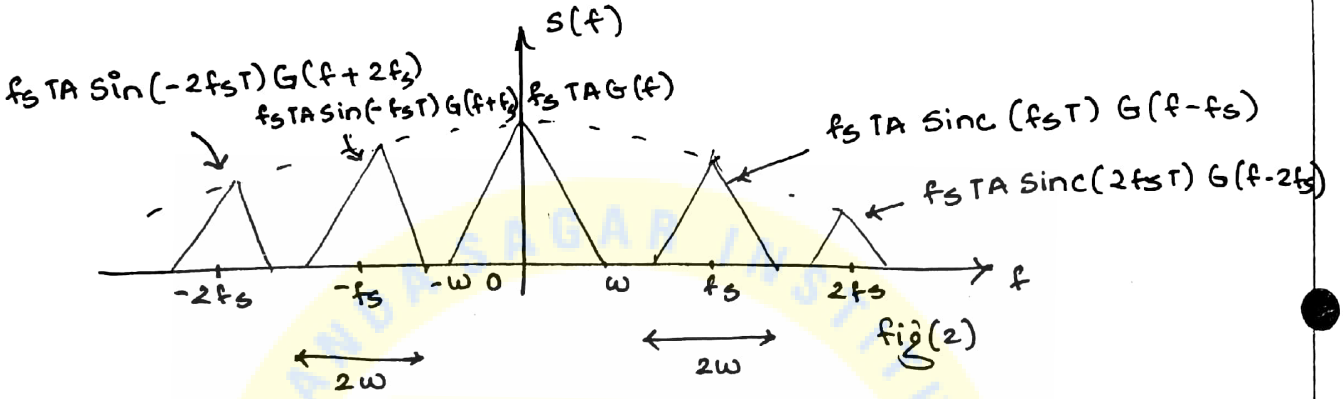


fig (1)



fig(2)

From fig (2), it is evident that the sampling operation leaves the message spectrum intact, but it simply repeats periodically in the freq. domain with a period equal to f_s. Also we observe that the msg spectrum centered at $f=0$ is attenuated by $f_s TA$

Since the sampling operated has not altered the message spectrum, it should be possible to reconstruct $g(t)$ from $s(t)$. from fig (2) it is also seen $G(f)$ can be separated from $S(f)$ by L.P. filtering. If we can filter $G(f)$ from $s(f)$, then we have recovered $g(t)$. This is possible only when $g(t)$ is strictly band-limited & $f_s > 2W$.

When $f_s > 2W$, $g(t)$ can be recovered from $s(t)$ by passing $s(t)$ thro' an ideal LPF with B.W B. where $W \le B \le f_s - W$

We know from fundamentals that a pulse which is even symmetric about vertical axis & having unit area gives an impulse function of unit strength as its width $T \to 0$. Hence, with $A = \frac{1}{T}$ & as $T \to 0$, the rectangular pulse train becomes the unit impulse train. Consequently, natural sampling becomes ideal sampling.

POINTS TO REMEMBER

1. $c(t) = f_s T A \sum_{n=-\infty}^{\infty} \text{Sinc}(n f_s T) . e^{j2\pi f_s n t}$

2. Rectangular pulse train $\longrightarrow$ Impulse train

$A = \frac{1}{T}$ & as $T \to 0$

ASSIGNMENT QUESTIONS

1. With the help of ckt. diagram & spectrum for G(f) & S(f) for signal g(t) explain Natural sampling.
2. Discuss: Natural sampling becomes ideal sampling.

REFERENCES

[1] DIGITAL COMMUNICATIONS, SIMON HAYKIN, John Wiley India Pvt Ltd, 2008.

OBJECTIVE OF THE LECTURE:

1. To understand Flat-Top sampling.
2. The concept of aperture effect & how to remove it.

COURSE ~~OBJECTIVE~~ OUTCOME:

1. CO61.1 : Able to understand digital communication, ways to represent digital data & types of digital modulation techniques.

NOTES:

FLAT TOP SAMPLING

Ideal sampling is done by multiplying the original signal $g(t)$ by a train of impulses that are separated by T_s seconds.

Mathematically,

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s) \rightarrow (1)$$

where $g(nT_s)$ represents the weight of the impulses that are separated in time by T_s seconds.

Now, Let the impulses be replaced by rectangular pulses having an amplitude $g(nT_s)$ & width equal to T secs. This is as though the impulse of zero width are stretched to rectangular pulses of length T.

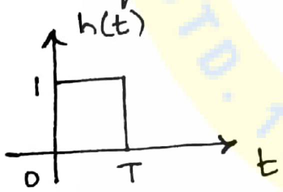
Let $s(t)$ denote the sequence of samples generated by flat-top sampling as shown in fig.

Flat - Top samples

∴ we can write

$$s(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, h(t-nT_s) \rightarrow (2)$$

From eqn (1), if the pulses of zero width are replaced by pulses of duration T seconds, then we get eqn (2). Hence, h(t) is a rectangular pulse of unit amplitude & duration equal to T seconds.



we have,

$$\text{rect}(x) = \begin{cases} 1, & -1/2 < x < 1/2 \\ 0, & |x| > 1/2 \end{cases}$$

Hence,

$$\text{rect}\left(\frac{t}{T} - \frac{1}{2}\right) = \begin{cases} 1, & -1/2 < (t/T - 1/2) < 1/2 \\ 0, & \left|\frac{t}{T}\right| > 1 \end{cases}$$

$$\text{rect}\left(\frac{t}{T} - \frac{1}{2}\right) = \begin{cases} 1, & 0 < t/T < 1 \\ 0, & |t/T| > 1 \end{cases}$$

Thus the mathematical description of $h(t)$ is

$$h(t) = \text{rect}\left(\frac{t}{T} - \frac{1}{2}\right) \rightarrow (3)$$

Now, convolving $g_\delta(t)$ with the pulse $h(t)$, we get

$$g_\delta(t) * h(t) \triangleq \int_{-\infty}^{\infty} g_\delta(\tau)\, h(t-\tau).\, \delta\tau \rightarrow (4)$$

Substituting eqn (1) in (4) we get,

$$g_\delta(t) * h(t) = \int_{\tau=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g(nT_s)\, \delta(\tau - nT_s).\, h(t-\tau).\, \delta\tau.$$

Interchanging the order of summation & integration, we get

$$g_\delta(t) * h(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \int_{\tau=-\infty}^{\infty} \delta(\tau - nT_s).\, h(t-\tau).\, \delta\tau \rightarrow (5)$$

Recall the shifting property:

$$\int_{-\infty}^{\infty} x(t).\, \delta(t - t_0)\, dt = x(t)\Big|_{t=t_0} = x(t_0)$$

∴ Applying shifting property,

$$g_\delta(t) * h(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, h(t-\tau)\Big|_{\tau = nT_s}$$

$$= \sum_{n=-\infty}^{\infty} g(nT_s)\, h(t - nT_s) \rightarrow (6)$$

Comparing eqns (2) & (6) we find,

$$s(t) = g_\delta(t) * h(t) \rightarrow (7)$$

The above eqn means that flat-top samples could be obtained by convoluting the o/p of an ideal sampler with a rectangular pulse of unit amplitude & duration equal to T seconds.

Taking F.T on both sides of eqn (7),

$$S(f) = G_\delta(f)\, H(f) \rightarrow (8)$$

Put $G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$ in eq^n (8),
we get,

$$S(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s) \text{ (in eq}^n \text{ (8))} . H(f).$$

$$S(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s) . H(f) \rightarrow (9)$$

Next we find the F.T of $h(t)$

$$H(f) \triangleq \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi ft} . dt$$

$$= \int_0^T 1 \times e^{-j2\pi ft} . dt$$

$$= \left. \frac{e^{-j2\pi ft}}{-j2\pi f} \right|_{t=0}^{T}$$

$$= -\frac{1}{j2\pi f} \left[e^{-j2\pi fT} - 1 \right]$$

$$= T \operatorname{sinc}(fT)\, e^{-j2\pi fT}$$

Substituting $H(f)$ in eq^n (9) we get

$$S(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s) . T \operatorname{sinc}(fT)\, e^{-j2\pi fT}$$

$$= f_s T \operatorname{sinc}(fT)\, e^{-j2\pi fT} \sum_{n=-\infty}^{\infty} G(f - nf_s) \rightarrow (10)$$

The magnitude & phase plots of the rectangular pulse $h(t)$ are shown

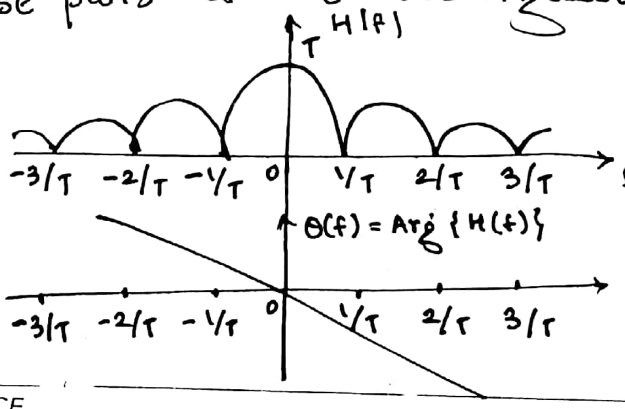


FIG: MAGNITUDE & PHASE SPECTRA OF THE RECT PULSE h(t)

Since $G_\delta(f)$ is multiplied by $\text{sinc}(fT)$ as evident from eqn (11), the spectral replicates in $G_\delta(f)$ centered at $\pm f_s$, $\pm 2f_s, \ldots$ are attenuated & thus helps the reconstruction filter in recovering $g(t)$ from $g_\delta(t)$.

The sinc function in the expression for $H(f)$ suggests that it acts like a LPF & attenuates the upper portion of the message spectrum centered at $f=0$. This loss of high freq. content is called <u>aperture effect</u>. The larger the pulse duration T, the larger the aperture effect. Aperture effect can be compensated by using an equalizer having the amplitude response $1/|H(f)|$ in cascade with the reconstruction filter.

POINTS TO REMEMBER:

1. $\text{rect}(x) = \begin{cases} 1, & -1/2 < x < 1/2 \\ 0, & |x| > 1/2 \end{cases}$

2. Flat top sampling : $s(t) = g_\delta(t) * h(t)$

3. $\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) \cdot dt = x(t_0)$: Shifting Property.

4. Aperture effect can be removed by using an equalizer having amplitude response $1/|H(f)|$ in cascade with reconstruction filter.

ASSIGNMENT QUESTIONS

1. Explain Flat-Top Sampling, with relevant waveforms & equations.
2. What is Aperture Effect? How do you overcome it?

REFERENCES

[1] DIGITAL COMMUNICATIONS, SIMON HAYKINS, John Wiley, India Pvt Ltd, 2008

OBJECTIVE OF THE LECTURE:

1. To understand Practical sample & Hold Circuit.
2. To solve numericals on Sampling.

COURSE OUTCOMES:

1 CO61.1 : Able to understand digital communication, ways to represent digital data and types of digital modulation techniques

NOTES:

PRACTICAL SAMPLE & HOLD CIRCUIT

A more popular method of obtaining flat-top samples from a continuous-time signal $g(t)$ is known as Sample & Hold circuit / technique.

An elementary sample & hold circuit consists of 2 FET switches & a capacitor as shown in fig. 1.

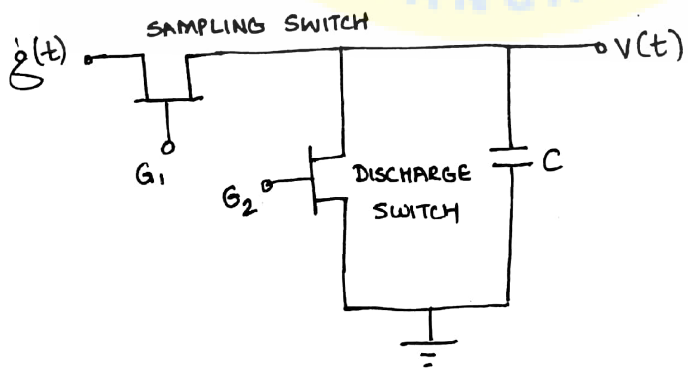


FIG 1. SAMPLE & HOLD CIRCUIT

A gate pulse applied at G_1 briefly closes the sampling switch & the capacitor holds the sampled voltage until discharged by a pulse applied to G_2.

Periodic gating of the sample & hold circuit generates the sampled wave

$$v(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\, h(t - nT_s) \rightarrow (1)$$

where $h(t)$ represents the impulse response of the sample & hold circuit, ie.,

$$h(t) = \begin{cases} 1, & 0 < t < T_s \\ 0, & \text{otherwise} \end{cases}$$

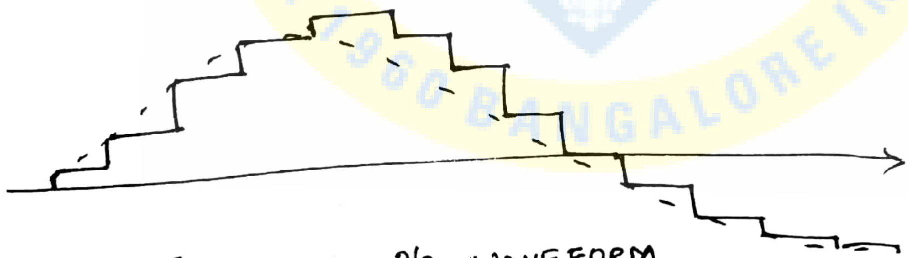
Accordingly, the transfer function of the sample & hold circuit is

$$H(f) \triangleq \int_{-\infty}^{\infty} h(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_{0}^{T_s} 1 \cdot e^{-j2\pi ft} \cdot dt$$

$$H(f) = T_s \operatorname{sinc}(fT_s)\, e^{-j\pi fT_s} \rightarrow (2)$$

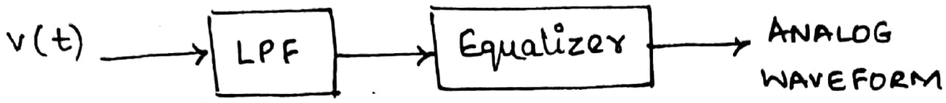
The spectrum of the output signal is



IDEALIZED O/P WAVEFORM OF THE CIRCUIT

$$\& \quad V(f) = f_s \sum_{n=-\infty}^{\infty} H(f)\, G(f - nf_s) \rightarrow (3)$$

The original signal $g(t)$ can be recovered from flat-top samples $v(t)$ by passing the flat-top samples through the various blocks as shown



The main aim of LPF is to suppress the spectra of flat-top samples present at integer multiples of f_s & allows the message spectrum centred at $f=0$ freely. The aperture effect is removed by using an equalizer whose amplitude response equals $1/|H(f)|$.

POINTS TO REMEMBER

1. Sample & Hold Circuit is one of the methods to obtain Flat-Top samples

2. $$h(t) = \begin{cases} 1, & 0 < t < T_s \\ 0, & \text{otherwise} \end{cases}$$

ASSIGNMENT QUESTIONS

1. Explain Sample & Hold Circuit, with a neat circuit diagram & waveform.

REFERENCES

[1] DIGITAL COMMUNICATIONS, SIMON HAYKIN, John Wiley India Pvt Ltd, 2008

[2] DIGITAL COMMUNICATIONS, Dr. K.N. Hari Bhat, Dr. D. Ganesh Rao, 2nd Edition, Sanguine Technical Publishers, Bangalore India, 2005

NUMERICALS

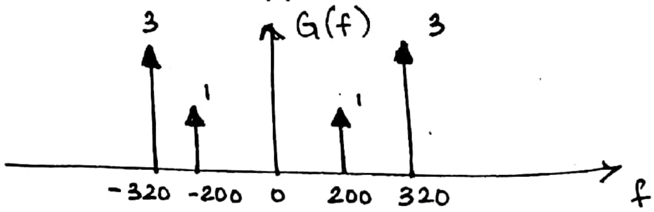
1. A signal $g(t) = 2\cos 400\pi t + 6\cos 640\pi t$ is ideally sampled at $f_s = 500\,Hz$. If the sampled signal is passed through an ideal lowpass filter with a cutoff freq. of $400\,Hz$, what freq. components will appear in the filter output?

→ Give $g(t) = 2\cos 400\pi t + 6\cos 640\pi t$

Taking FT on both sides of the above eqn, we get

$$G(f) = [\delta(f-200) + \delta(f+200)] + 3[\delta(f-320) + \delta(f+320)]$$

Using the above equation, the spectrum of the signal $g(t)$ is drawn & it appears as shown in fig.



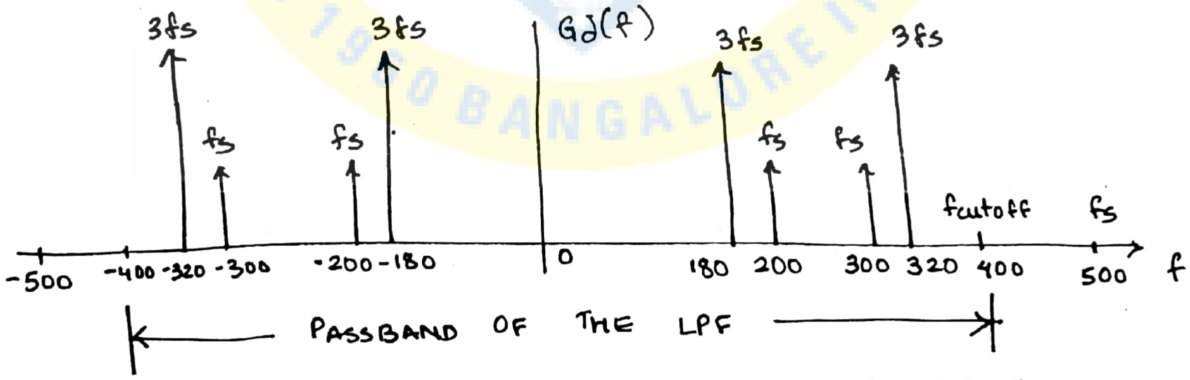
The FT of the sampled signal $g_\delta(t)$ is drawn using the expression given below

$$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$$

$$= f_s \sum_{n=-\infty}^{\infty} G(f - 500n)$$

$$\Rightarrow \quad G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} [\delta(f-500n-200) + \delta(f-500n+200)] + 3f_s \sum_{n=-\infty}^{\infty} [\delta(f-500n-320) + \delta(f-500n+320)]$$

The spectrum of the sampled signal $g_\delta(t)$ is drawn using the above equation & it appears as shown in Fig.



It should be noted that the spectrum $G_\delta(f)$ is periodic with a period f_s.

The frequencies that appear in the output of the LPF are 180 Hz, 200 Hz, 300 Hz & 320 Hz.

2. A signal $g(t) = 10\cos(20\pi t)\cos(200\pi t)$ is sampled at the rate of 250 samples / second.

(a) Sketch the spectrum of the sampled signal.

(b) Specify the cutoff ideal reconstruction filter so as to recover $g(t)$ from $g_\delta(t)$.

(c) Specify the Nyquist rate for the signal $g(t)$.

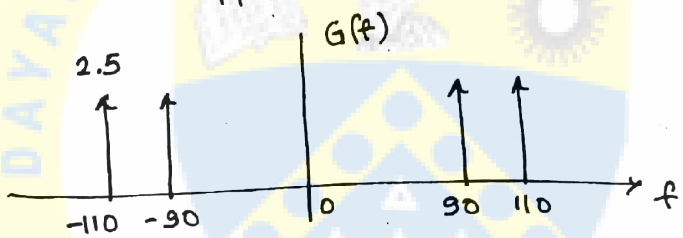
→ Given $g(t) = 10\cos(20\pi t)\cos(200\pi t)$

$$\Rightarrow g(t) = 5\cos(220\pi t) + 5\cos(180\pi t)$$

Taking FT on both sides of the above equation, we get

$$G(f) = 2.5[\delta(f-110) + \delta(f+110) + \delta(f-90) + \delta(f+90)]$$

The spectrum of the signal $g(t)$ is drawn using the above expression & it appears as shown in fig.



Recall: $G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - nf_s)$

$$G_\delta(f) = f_s \sum_{n=-\infty}^{\infty} G(f - 250n)$$

$$= 2.5 f_s \sum_{n=-\infty}^{\infty} [\delta(f - 250n - 110) + \delta(f - 250n + 110)$$

$$+ \delta(f - 250n - 90) + \delta(f - 250n + 90)]$$

Using the above expression, the spectrum of the sampled signal is drawn & it appears as shown in fig. It should be noted that $G_\delta(f)$ is symmetric about vertical axis & is periodic with a period equal to f_s.

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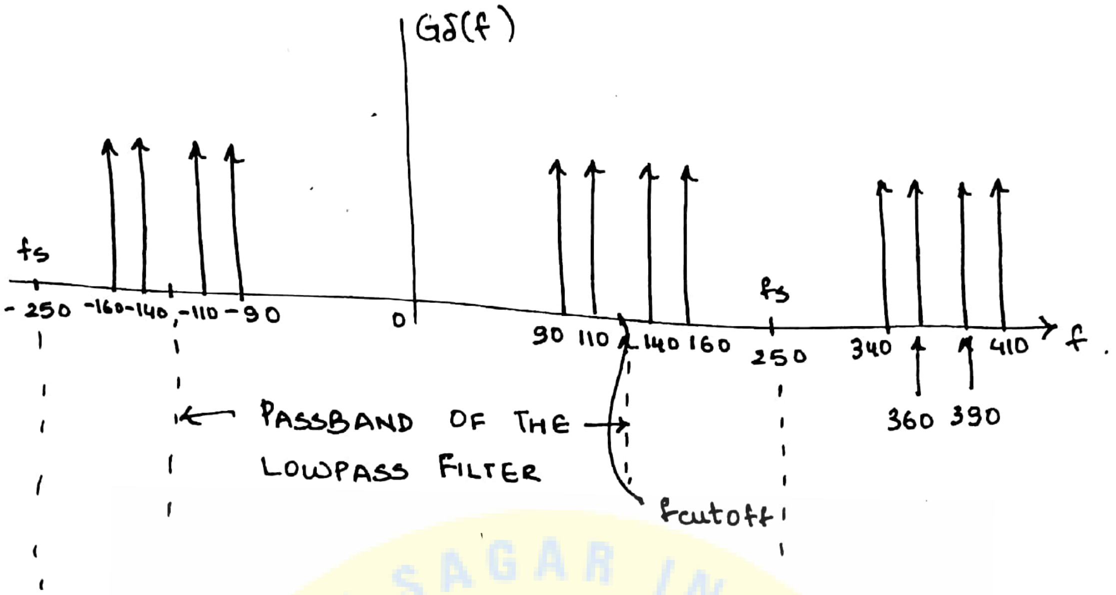
a)



b) The cutoff freq. of the ideal LPF should be greater than 110 Hz & less than 140 Hz for recovering $g(t)$ from $g_\delta(t)$

$* \quad W < B < f_s - W$

c) Nyquist rate, $f_s = 2W$, where W is the highest frequency present in $g(t)$. Thus, $f_s = 2 \times 110 = 220$ Hz.

OBJECTIVE OF THE LECTURE :

1. To solve numericals on Sampling.

COURSE OUTCOMES :

1. CO61.1 : Able to understand digital communication, ways to represent digital data and types of digital modulation techniques.

NOTES :

3. If E denotes the energy of a strictly band-limited signal $g(t)$, then prove that

$$E = \frac{1}{2W} \sum_{n=-\infty}^{\infty} \left| g\left(\frac{n}{2W}\right)\right|^2$$

where W is the highest frequency component of $g(t)$.

→ If $g(t)$ is band-limited to $-W \le f \le W$, we may write $g(t)$ as

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) . \operatorname{sinc}(2Wt - n) \longrightarrow (1)$$

Recall:

$$E = \int_{-\infty}^{\infty} |g(t)|^2 . dt$$

$$= \int_{-\infty}^{\infty} g(t) . g^*(t) . dt \longrightarrow (2)$$

Substituting eq^n (1) in (2) we get,

$$E = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt - n) \sum_{k=-\infty}^{\infty} g^*\left(\frac{k}{2W}\right) \operatorname{sinc}(2Wt-k). dt$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} g\left(\frac{n}{2W}\right) . g^*\left(\frac{k}{2W}\right) \int_{-\infty}^{\infty} \operatorname{sinc}(2Wt - n) \operatorname{sinc}(2Wt-k). dt \longrightarrow (3)$$

But, $\int_{-\infty}^{\infty} \text{sinc}(2Wt-n)\,\text{sinc}(2Wt-k)\,dt = \begin{cases} \frac{1}{2W}, & k=n \\ 0, & k\neq n \end{cases}$

Hence, Eqn (3) becomes

$$E = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \cdot g^*\left(\frac{n}{2W}\right)$$

$$= \frac{1}{2W} \sum_{n=-\infty}^{\infty} \left| g\left(\frac{n}{2W}\right)\right|^2$$

4. Determine the Nyquist rate & the Nyquist sampling interval for the following signals:

a) $g_1(t) = \text{sinc}(100\pi t)$

b) $g_2(t) = \text{sinc}^2(100\pi t)$

c) $g_3(t) = \text{sinc}(100\pi t) + \text{sinc}(50\pi t)$

→ a) Given $g_1(t) = \text{sinc}(100\pi t)$

Recall:

1) The spectrum of a sinc function is rectangular & is of finite width.

2) The spectrum of a rectangular pulse is a sinc function.

From the spectrum of $g_1(t)$ shown in Fig. (a), we find that the B.W of the signal $g_1(t)$ is $W = 50$ Hz. Hence, the Nyquist rate $= 2W = 100$ Hz & Nyquist interval $= 1/100 = 0.01$ sec

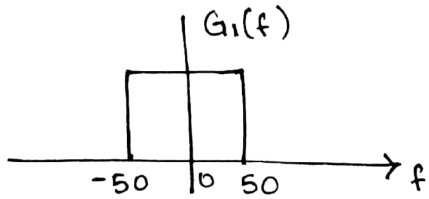
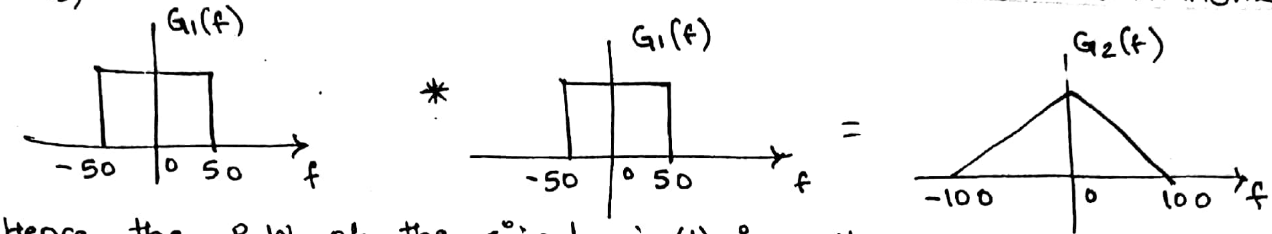


FIG. (a)

b) Given $g_2(t) = \text{sinc}^2(100\pi t)$.

The spectrum of $g_2(t)$ is a triangular pulse as shown in Fig. (b).

FIG. (b)



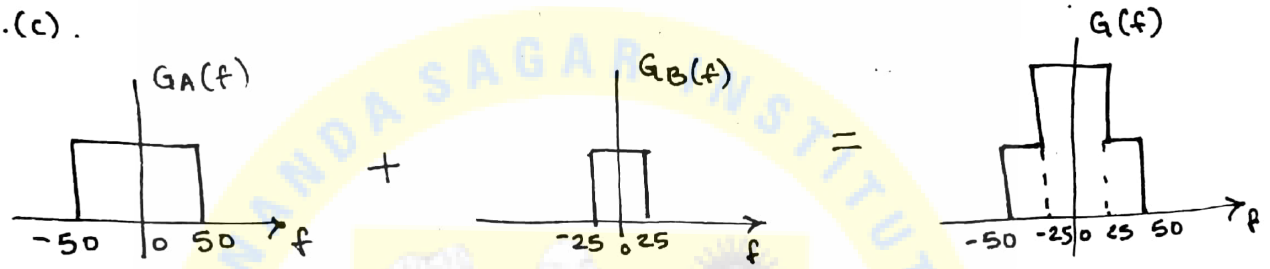
Hence, the B.W of the signal $g_2(t)$ is 100Hz.

Hence, Nyquist rate = $2 \times 100 = 200$ Hz & Nyquist interval = $1/200$ sec.

c) Given $g_3(t) = \text{sinc}(100\pi t) + \text{sinc}(50\pi t) = g_A(t) + g_B(t)$

The spectrum of $g_3(t)$ is as shown in fig. (c)

FIG. (c).



From fig (c), we find the B.W of the signal $g_3(t)$ is 50 Hz.

$\therefore$ Nyquist rate $= 2 \times 50 = 100$ Hz & Nyquist interval $= 1/100 = 0.1$ sec.

5. A signal $g(t)$ consists of two frequency components $f_1 = 3.9$ kHz & $f_2 = 4.1$ kHz in such a relationship they cancel out each other when $g(t)$ is sampled at the instants $t = 0, T, 2T, \ldots$ where $T = 125$ μsec. The signal $g(t)$ is defined by

$$g(t) = \cos\left[2\pi f_1 t + \pi/2\right] + A\cos\left[2\pi f_2 t + \phi\right]$$

Find the values of A & ϕ of the second freq. component.

$\rightarrow$ Given

$$g(t) = \cos\left[2\pi f_1 t + \pi/2\right] + A\cos\left[2\pi f_2 t + \phi\right]$$

Also given in the problem that,

$$g(nT) = 0$$

Hence, $\cos[2\pi f_1 nT + \pi/2] + A\cos[2\pi f_2 nT + \phi] = 0$

$$\Rightarrow \cos\left[2\pi \times 3.9 \times 10^{3} \times 125 \times 10^{-6} \times n + \pi/2\right] + A\cos\left[2\pi \times 4.1 \times 10^{3} \times 125 \times 10^{-6} \times n + \phi\right] = 0$$

$$\Rightarrow \cos[0.975\pi n + \pi/2] + A\cos[1.025\pi n + \phi] = 0 \quad \rightarrow (1)$$

i. Let $n = 0$

Then, Eqn (1) becomes

$$\cos(\pi/2) + A\cos\phi = 0$$

Since, $A \neq 0$, we get $\phi = \pm \pi/2$

ii. Let $n = 1$,

Equation (1) becomes,

$$\cos[0.975\pi + \pi/2] + A\cos[1.025\pi \pm \pi/2] = 0$$

$$\Rightarrow -\sin(0.975\pi) \mp A\sin(1.025\pi) = 0$$

$$\Rightarrow -\sin(\pi - 0.025\pi) \mp A\sin(\pi + 0.025\pi) = 0$$

$$\Rightarrow -\sin(0.025\pi) \pm A\sin(0.025\pi) = 0$$

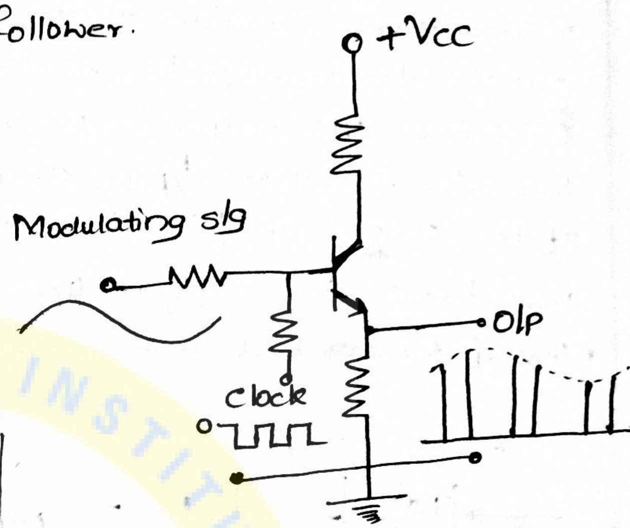
The above eqn is satisfied only when $A = 1$ & +ve sign for the second term is taken. The +ve sign can appear in the above equation, only when $\phi = +\pi/2$ in eqn (1).

Thus $A = 1$ & $\phi = +\pi/2$

PAM, PWM & PPM MODULATOR AND DEMODULATOR CIRCUIT

PAM MODULATOR CIRCUIT:

* This circuit is a simple emitter follower.
* The modulating signal is applied as the input signal. Another input to the base of the transistor is the clock signal.
* The frequency of the clock signal is made equal to the desired carrier pulse train frequency.
* The amplitude of the clock s/g is so chosen that the high level is at ground (or) and low level is at some -ve voltage which is sufficient to bring the transistor in the cut-off region.



* When the clock signal is high, the circuit behaves as an emitter follower, and the output follows the input modulating s/g. When the clock signal is low, the transistor is cut-off & the output is zero.

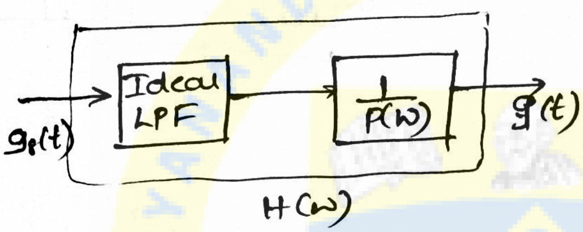
DEMODULATION OF PAM SIGNALS:

* Demodulation of natural sampled signal can be done with the help of an ideal LPF with cut-off frequency ω_M. But for this, the pulse top shape is to be maintained after transmission. This is very difficult due to transmitter & receiver noise. Therefore normally flat-top sampling is preffered over natural sampling.
* There are two demodulation methods for the flat top sampled signal.

(i) Using an Equalizer
(ii) Using Holding circuit.

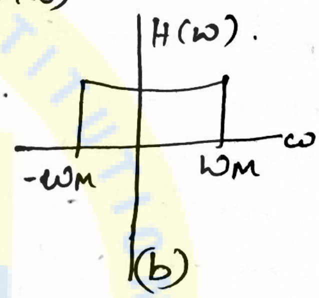
(i) Using an Equalizer:

* If the flat top sampled signal is passed through an ideal LPF, the spectrum of the output will be $G(\omega)\cdot P(\omega)$.
* The time function of the output is somewhat distorted due to the multiplying factor $P(\omega)$.
* If the LPF output is passed through a filter having a transfer function $1/P(\omega)$ over the range $0-\omega_M$, the spectrum at the output of this filter will be $G(\omega) \times P(\omega) \times \frac{1}{P(\omega)} = G(\omega)$ and hence the original time function $g(t)$ will be recovered.
* The filter with a transfer function $\frac{1}{P(\omega)}$ is known as equalizer.



(a)

Composite filter with T.F $H(\omega)$



(b)

The Desired characteristic of $H(\omega)$

Demodulation of Flat top sampled PAM using an equalizer.

* The combination of ideal LPF and equalizer is known as composite filter. The transfer function (T.F) of this filter is shown in above fig (b).

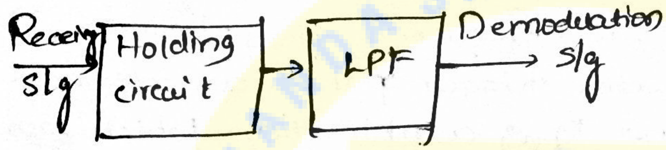
$$H(\omega) = \begin{cases} \frac{1}{P(\omega)}, & |\omega| < \omega_m \\ 0, & \text{otherwise} \end{cases}$$

(ii) Using Holding Circuit:

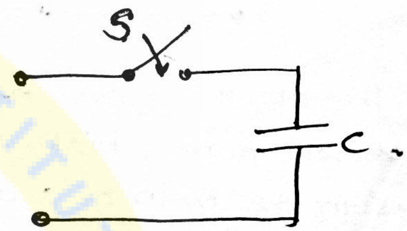
* In this method, the received signal is passed through a holding circuit and a LPF shown in fig below (a).
* Fig (b) shows a simple holding circuit. The switch "S" closes after the arrival of the pulse and it opens at the end of the pulse.
* The capacitor C gets charged to the pulse amplitude value and it holds this value during the interval b/w the 2 pulses. The sampled values are held as shown in fig (c).
* The holding circuit o/p is smoothened in LPF as shown in fig (d)

* Some distortion is introduced because of the holding circuit. The circuit of fig (b) is known as zero order holding circuit which considers only the previous sample to decide the value between the two pulses. The first order holding circuit considers the previous 2 samples; the second order holding circuit considers the previous 3 samples & so on.

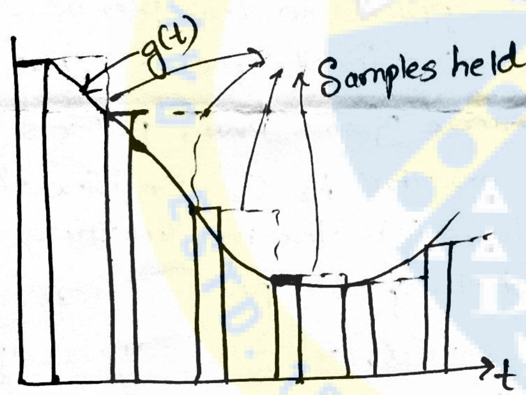
* As the order of the holding circuit increases, the distortion decreases at the cost of the circuit complexity. The amount of permissible distortion decides the order of the holding circuit.



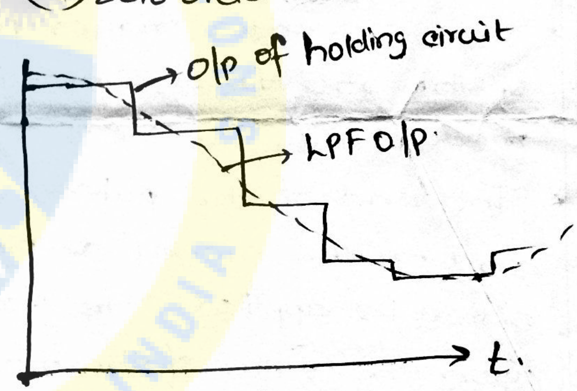
(a): Building block of demodulator



(b) Zero order holding circuit



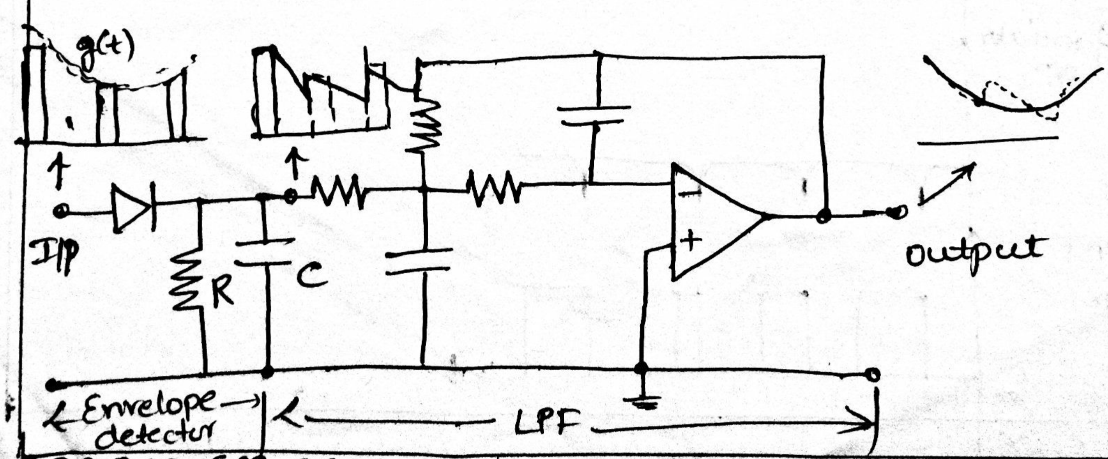
(c) output of holding circuit



(d) output of LPF

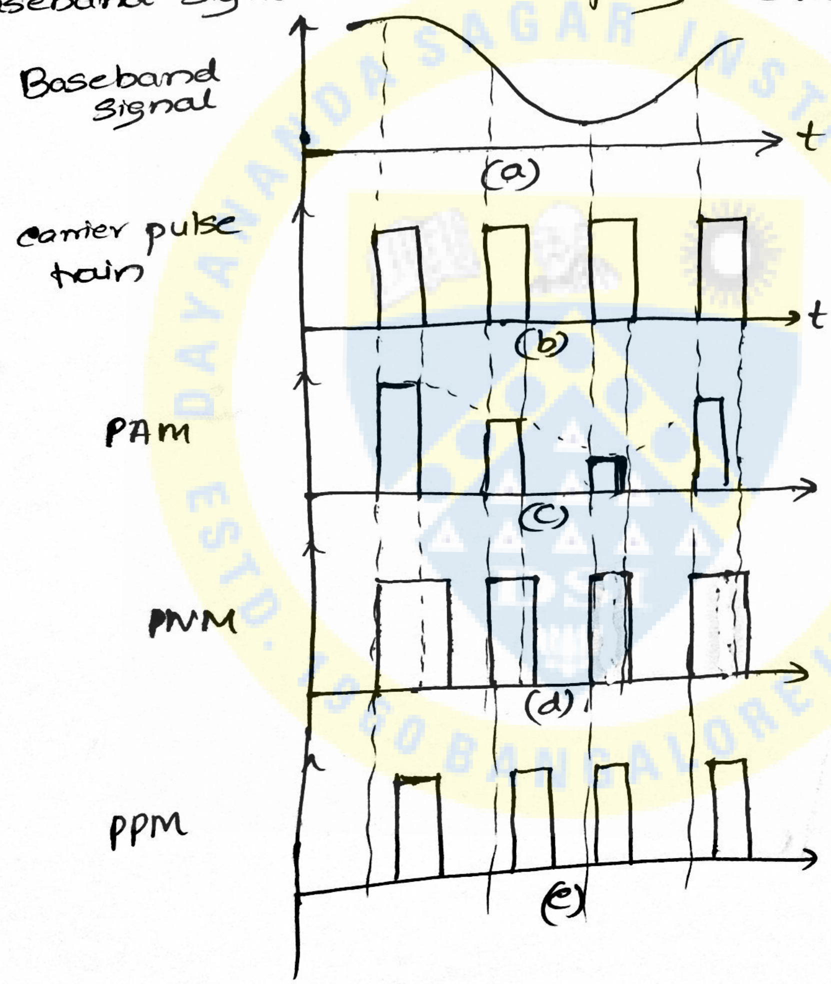
PAM Demodulator circuit

It is just an envelope detector followed by a LPF. The diode & R-C combination work as the envelope detector. This is followed by a 2nd order OP-AMP LPF to have a good filtering characteristic.

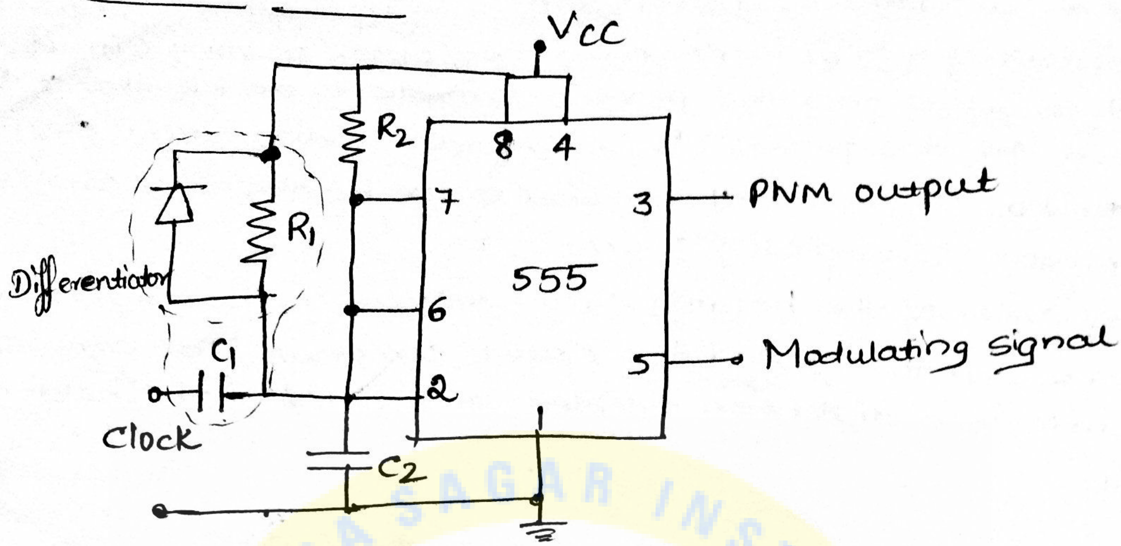


Pulse Modulation concepts:

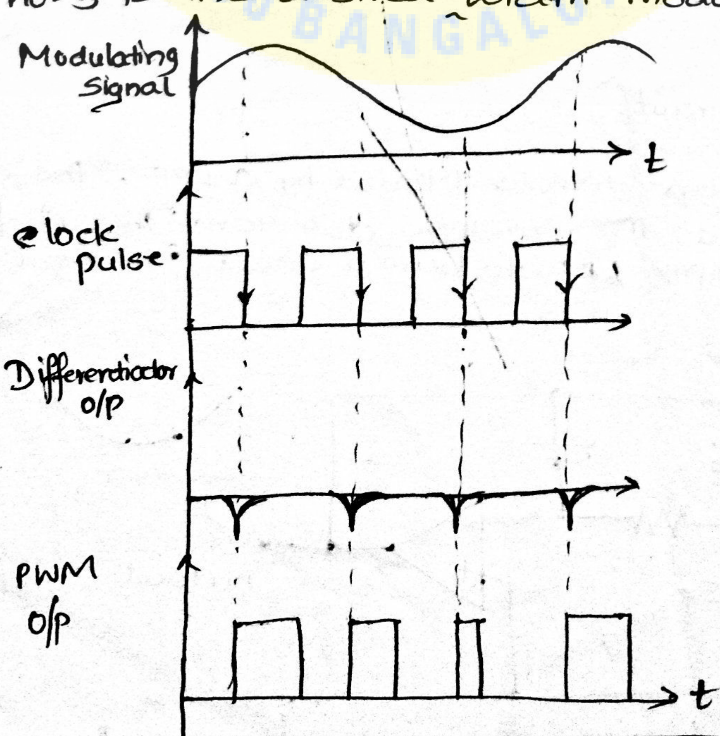
* Fig (a) is the baseband signal g(t), whereas fig (b) is the carrier pulse train. Fig (c) is the PAM signal. Fig (d) is the PWM signal where the width of each pulse depends on the instantaneous value of the baseband signal at sampling instant.
* Fig (e) is the PPM signal where the shift in the position of the each pulse depends on the instantaneous value of the baseband signal at the sampling instant.



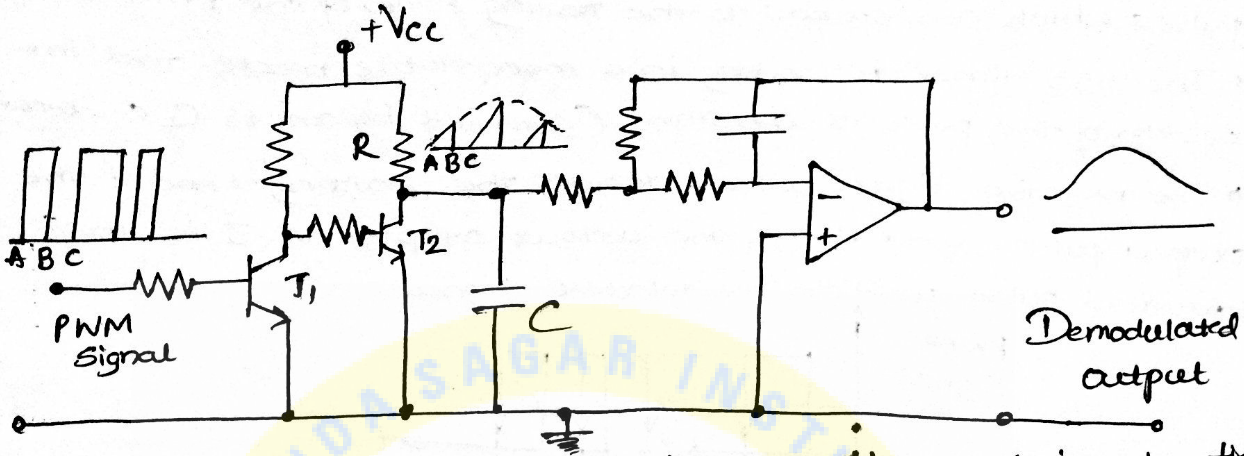
PWM MODULATOR CIRCUIT



* The clock signal of the desired frequency is applied as shown from which the negative trigger pulses are derived with the help of a diode and R_1-C_1 combination which works as a differentiator.
* These negative trigger pulses are applied to the pin no. 2 of the 555 timer which is put working in the monostable mode. They decide the starting time of the PWM pulses.
* The end of the pulses depends on the R_2-C_2 combination and on the signal at pin no. 5 to which the modulating signal is applied. Therefore, the width of the pulses depends upon the value of the modulating signal, and thus the output at pin no. 3 is the desired pulse width modulated signal.

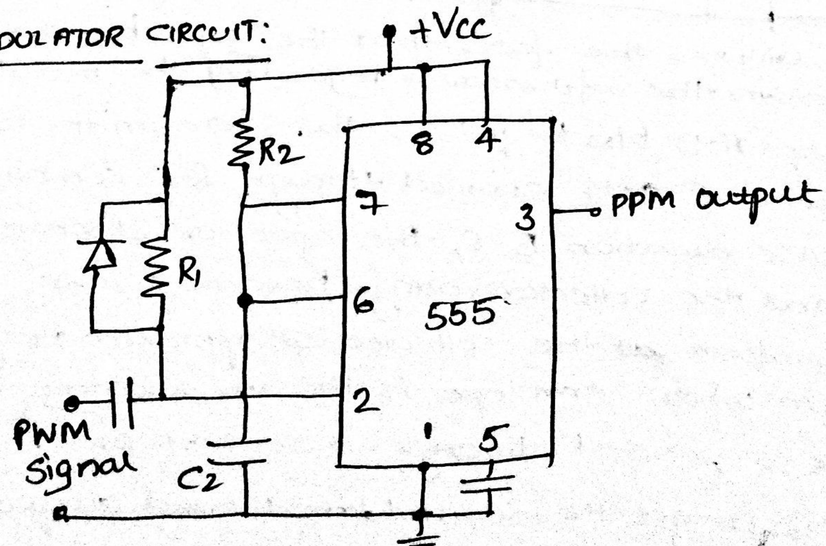


PWM DEMODULATOR CIRCUIT:

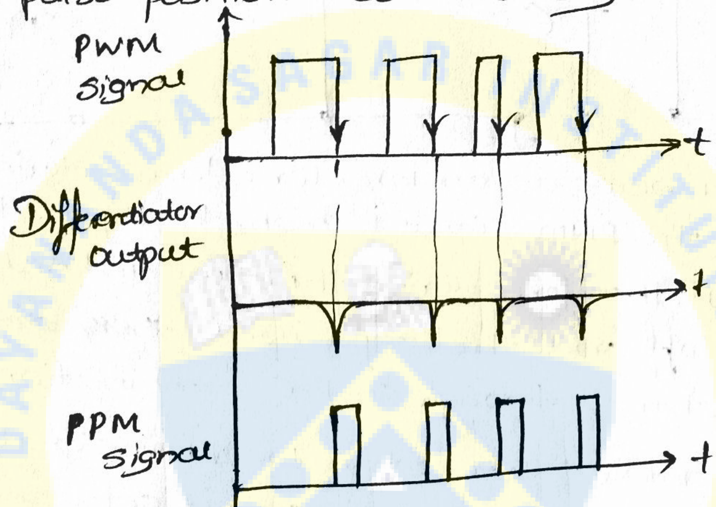


* The transistor T_1 works as an inverter. Hence, during the time interval A-B, when the PWM signal is high, the input to the transistor T_2 is low. Therefore during this time interval, the transistor T_2 is cut-off and the capacitor C gets charged through an R-C combination.
* During the time interval B-C when the PWM signal is low, the input to the transistor T_2 is high and it gets saturated. The capacitor C then discharges very rapidly through T_2.
* The collector voltage of T_2 during the interval B-C is then low. Thus the waveform at the collector of T_2 is more or less a sawtooth waveform whose envelope is the modulating signal.
* When this is passed through a second-order OP-AMP low pass filter, we get the desired demodulated output.

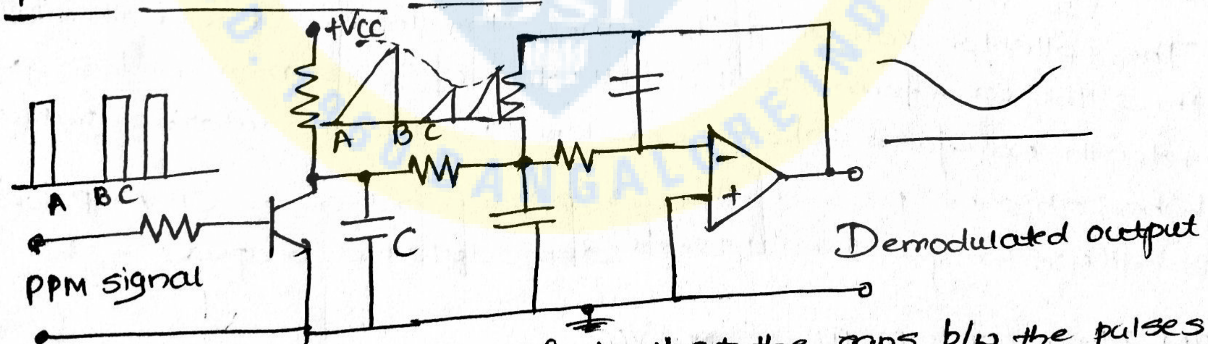
PPM MODULATOR CIRCUIT:



* The PWM signal is applied to pin no. 2 through the diode & R_1-C_1 combination. Thus the input to pin no. 2 is the negative trigger pulses which corresponds to the trailing edges of the PWM waveform.
* The 555 timer is working in a monostable mode and the width of the pulse is constant (governed by an R_2-C_2 combination)
* The negative trigger pulses decides the starting time of the output pulses and thus, the output at pin no. 3 is the desired pulse position modulated signal.



PPM DEMODULATOR CIRCUIT:



* This circuit utilizes the facts that the gaps b/w the pulses of a PPM s/g contain the information regarding the modulating s/g.
* During the gaps A-B b/w the pulses, the transmitter is cut off and the capacitor C gets charged through R-C combination.
* During the pulse duration B-C, the capacitor discharges through transmitter, and the collector voltage becomes low.
* Thus the waveform at the collector is approximately a saw-tooth waveform whose envelope is the modulating signal.
* When this is passed through a second order OP-AMP Low pass filter, we get the desired demodulated output.