DEPARTMENT
OF
ELECTRONICS & COMMUNICATION ENGINEERING

Communication Theory
[19EC4DCOT]

(Theory Notes)

Autonomous Course

Prepared by

Sapna P J

Module – 2 Contents
Angle modulation: Basic concepts, Relationship between FM and PM .Single tone FM, Spectral analysis of Sinusoidal FM, Types of FM: NBFM and WBFM, Transmission bandwidth of FM waves, Generation of FM: Indirect FM and Direct FM, Zero crossing detector

Dayananda Sagar College of Engineering
Shavige Malleshwara Hills, Kumaraswamy Layout,
Banashankari, Bangalore-560078, Karnataka
Tel : +91 80 26662226 26661104 Extn : 2731 Fax : +90 80 2666 0789
Web - http://www.dayanandasagar.edu Email : hod-ece@dayanandasagar.edu
(An Autonomous Institute Affiliated to VTU, Approved by AICTE & ISO 9001:2008 Certified)
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UNIT 2

INTRODUCTION:

Amplitude modulation methods are also called linear modulation signal method. Another class of modulation is called angle modulation. In which either the phase or frequency of the carrier wave is varied according to the message signal. In this method of modulation the amplitude of the carrier wave is maintained constant.

The most important feature of angle modulation is that it can provide better discrimination against noise and interference than AM. This improvement is achieved at the expense of increased transmission band width.

Expressing the modulated wave in general form.

$$S(t) = Ac \cos [\theta i(t)] \text{--------1}$$

Where $\theta i(t)$ denote the angle of a modulated sinusoidal carrier which is the function of the message. Ac is the carrier amplitude

The instantaneous frequency of the angle modulated wave S(t) by

$$fi\ (t) = \frac{1}{2\pi} \frac{d}{dt} [\theta i(t)] \text{ ---------2}$$

According to equation (1), we may interpret the angle modulated wave S (t) as a rotating phasor of length A_c and angle $\theta i(t)$

The angular velocity of such a phasor is $\frac{d}{dt}[\theta i(t)]$, in accordance with eqn 2

In case of an unmodulated carrier , the angle $\theta i(t)$ is

$$\theta i(t) = 2\pi f_c t + \emptyset_C \text{ --------3}$$

And the corresponding phasor rotates with a constant angular velocity equal to $2\pi f_c$. The constant $\emptyset_C$ is the value of $\theta i(t)$ at t=0

There are an infinite number of ways in which the angle $\theta i(t)$ may be varied in some manner with the baseband signal. Two commonly used methods are phase modulation and frequency modulation.

Phase Modulation:

Here the angle $\theta i(t)$ is varied linearly with the base band signal m(t)

$\theta i(t) = 2\pi f_c t + K_P\, m(t)$--------4

$2\pi f_c t$ represents the angle of the unmodulated carrier

K_P is the phase sensitivity of the modulator expressed in radians per volt

The phase modulated wave S(t) is thus described in time domain by

Sub 4 in 1

$S(t) = Ac \cos[2\pi f_c t + K_P\, m(t)]$ ---------5

Frequency modulation:

Here the instantaneous frequency fi(t) is varied linearly with the message signal m(t) as shown by

$fi\,(t) = f_c(t) + K_f\, m(t)$

where f_c represents the frequency of the unmodulated carrier

Kf frequency sensitivity of the modulator -Hz/V

Integrating 2 w.r.t time and sub 6

w.k.t $\theta i(t) = 2\pi \int_0^t fi\,(t)\,dt$

$= 2\pi \int_0^t [fc + K_f\, m(t)]dt$

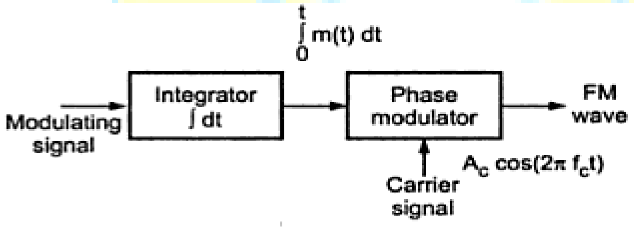
$= 2\pi f_c t + 2\pi K_f \int_0^t m(t)dt$---------7

Assuming carrier wave is zero at t=0

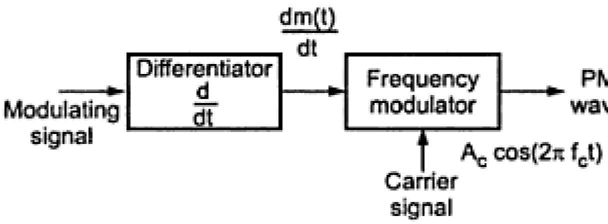
The frequency modulated wave is described in time domain by Sub 7 in 1

$$S(t) = Ac \cos [\, 2\pi f_c t + 2\pi K_f \int_0^t m(t)dt] \text{ -----8}$$

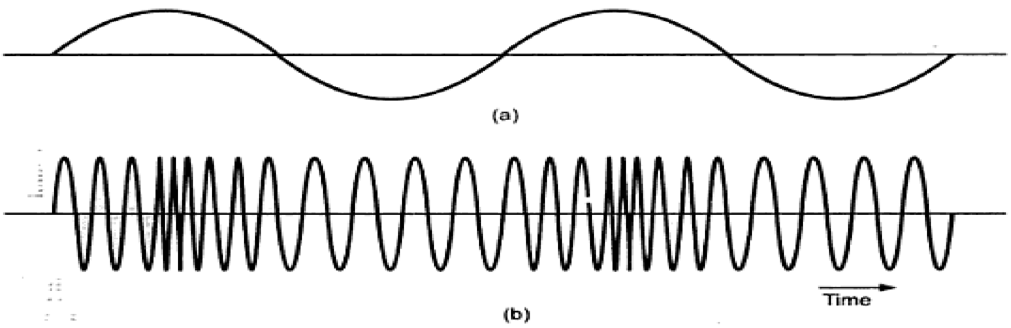
Comparing 5 and 8 reveals that FM wave may be regarded as a PM wave in which the modulating wave is $\int_0^t m(t)$ in place of m (t). This means that an FM wave can be generated by first integrating m (t) and then using the result as the input to a phase modulator as shown in fig below



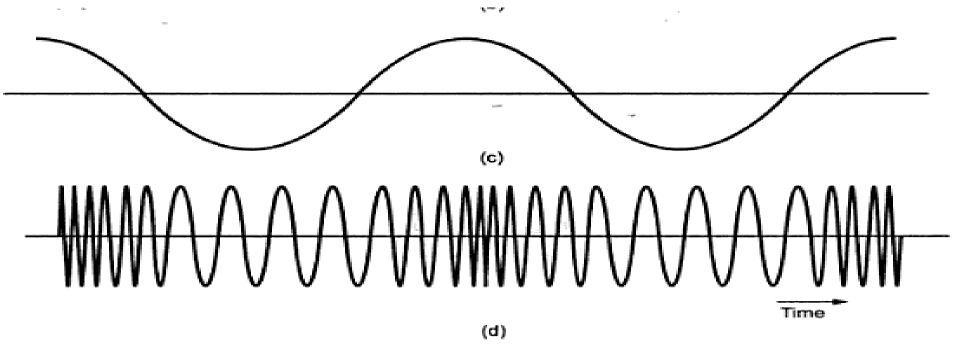
Conversely a PM wave can be generated by first differentiating m(t) and then using the result as the input to a frequency modulator as in fig below



Frequency modulated wave:



Phase modulated wave:



FREQUENCY MODULATION [FREQUENCY DEVIATION AND MODULATION INDEX]

The FM wave S (t) defined by eqn 8 is a nonlinear function of the modulating wave. Hence FM is a non-linear modulation process

Let us consider single-tone FM

Consider a sinusoidal modulating wave is defined by

$$m(t) = Am\ Cos\ 2\pi f_m t \text{ -------a}$$

The instantaneous frequency of the resulting FM wave is

$$fi\ (t) = f_c(t) + K_f\ m(t)$$

$$= f_c(t) + K_f\ Am\ Cos\ 2\pi f_m t$$

$$= f_c + \Delta f\ Cos\ 2\pi f_m t$$

Where $\Delta f = K_f\ Am$

Which is called the frequency deviation, representing the maximum departure of the instantaneous frequency of the FM wave from the carrier frequency fc. Δf is proportional to the amplitude of the modulating wave and is independent of the modulation frequency.

w.k.t $\theta i(t) = 2\pi \int_0^t fi\ (t)\ dt$

$$= 2\pi \int_0^t [fc + \Delta f \text{ Cos } 2\pi f_m t]dt$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \text{Sin } 2\pi f_m t$$

The ratio of the frequency deviation Δf to the modulation frequency f_m is called the modulation index of the FM wave.

$$\theta i(t) = 2\pi f_c t + \beta \text{ Sin } 2\pi f_m t$$

where $\beta = \frac{\Delta f}{f_m}$

The FM wave is given by

$$S(t) = Ac \text{ Cos } [2\pi f_c t + \beta \text{ Sin } 2\pi f_m t]$$

Problem:

1. A Sinusoidal modulating waveform of amplitude 5V and a frequency of 1KHz is applied to an FM generator that has the frequency sensitivity of 40 Hz/V

a) What is the frequency deviation

b) what is the modulation index

- Given Am = 5V; fm = 1KHz; Kf = 40Hz/V

a) Frequency deviation Δf = Kf Am

= 40 x5 = 200Hz

b) Modulation Index $\beta = \frac{\Delta f}{f_m}$

= 200/1000 = 0.2

Depending on the value of the modulation index β, there are 2 cases

1] Narrow band FM for which β is small compared to one radian

2] Wide band FM, for which β is large compared to one radian

Narrow Band Frequency Modulation:

For small values of the modulation index β compared to one radian, the FM wave assumes a narrow band form consisting essentially of a carrier an upper side frequency component and a lower side frequency component.

w.k.t FM signal is

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \text{ ------A}$$

Expanding this relation , we get

$$S(t) = A_c \cos 2\pi f_c t \cos(\beta \sin 2\pi f_m t) - A_c \sin 2\pi f_c t \sin(\beta \sin 2\pi f_m t) \text{-----B}$$

In case of narrow band , β is small therefore we approximate

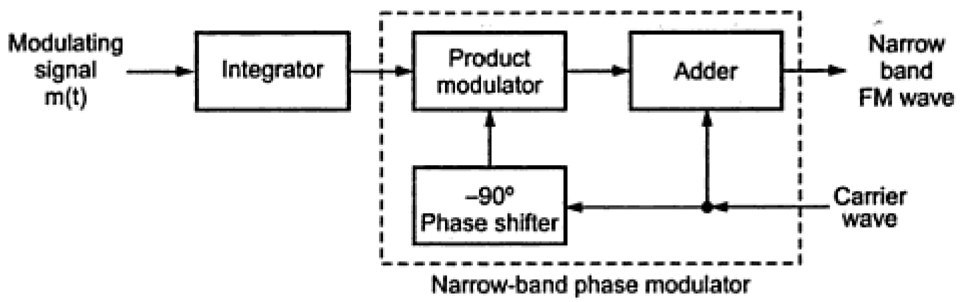
$$\cos(\beta \sin 2\pi f_m t) \approx 1$$

$$\& \sin(\beta \sin 2\pi f_m t) \approx \beta \sin 2\pi f_m t$$

Eqn B becomes

$$S(t) = A_c \cos 2\pi f_c t - \beta A_c \sin 2\pi f_c t \sin 2\pi f_m t \text{ ------c}$$

From this representation we show the block diagram of a method for generating a narrow band FM signal.



The modulator involves splitting the carrier wave $A_c \cos 2\pi f_c t$ into two paths. One path is direct, the other path contains a -90° phase shifting network and a product modulator, the combination of which generates a DSB modulated signal. The difference between these two signals produces a narrow band FM signal.

The Eqn C can be expressed as

$$S(t) = Ac \cos 2\pi f_c t + \frac{\beta\, Ac}{2} \cos 2\pi(f_c + f_m)t - \frac{\beta\, Ac}{2} \cos 2\pi(f_c - f_m)t \text{ -------D}$$

WIDE BAND FREQUENCY MODULATION [WBFM]

Now let us determine the spectrum of the single tone FM signal of eqn A for an arbitrary value of the modulation index β i.e specifically we assume that the carrier frequency fc is large. Then it is possible to obtain the spectrum of a wide band FM signal by expanding the FM wave as a Fourier series

The FM wave is

$$S(t) = Ac \cos [2\pi fct + \beta \sin 2\pi fmt] \text{ ----1}$$

w.k.t general form of FM

$$S(t) = Ac \cos [\theta i(t)] \text{-----2}$$

We can also write $S(t) = \mathrm{Re}[A_c e^{j\theta}]$----3

Comparing 1 and 2

$$\theta = 2\pi fct + \beta \sin 2\pi fmt \quad \text{----4}$$

Sub 4 in 3

$$S(t) = \mathrm{Re}[A_c e^{j(2\pi fct + \beta \sin 2\pi fmt)}]$$

$$= \mathrm{Re}[Ac\, e^{j2\pi fct}\, e^{j\beta \sin 2\pi fmt}]$$

$$= \mathrm{Re}[\tilde{s}(t)\, e^{j2\pi fct}] \text{----------------5}$$

Where $\tilde{s}(t)$ is known as complex envelope of FM

$$\tilde{s}(t) = Ac\, e^{j\beta \sin 2\pi fmt} \text{--------------6}$$

$\tilde{s}(t)$ can be expanded in the form of complex fourier series as

$$\tilde{s}(t) = \sum_{-\infty}^{\infty} C_n e^{j2\pi nfmt} \text{--------------7}$$

Where the complex Fourier coefficient C_n equals

$$C_n = fm \int_{\frac{-1}{2fm}}^{\frac{1}{2fm}} \tilde{s}(t)\, e^{-j2\pi nfmt}\, dt \text{ ------8}$$

Reducing the above eqn by sub 6 in 8

$$C_n = \frac{Ac}{2\pi}\int_{-\pi}^{\pi} \exp[j(\ \beta \mathrm{Sin}x - nx)]dx \text{------}9$$

$$C_n = Ac\ Jn(\beta)$$

The integral on the right hand side of the above equation is the nth order Bessel function of the first kind and argument β denoted by Jn(β)

$$Jn(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp[j(\ \beta \mathrm{Sin}x - nx)]dx \text{----}10$$

$$\tilde{s}(t) = Ac \sum_{-\infty}^{\infty} Jn(\beta)\ \exp\ (j2\pi nfmt) \text{----}11$$

Sub 11 in 5

$$S(t) = Ac \sum_{-\infty}^{\infty} Jn(\beta)\ \mathrm{Cos}\ [2\pi(fc + nfm)] \text{------}12$$

This is the desired form for the Fourier series representation of the single tone FM.

The discrete spectrum of S(t) is obtained by taking the Fourier transforms of both sides of eqn 12

$$S(f) = \frac{Ac}{2}\sum_{-\infty}^{\infty} Jn(\beta)[\delta(f - fc - nfm) + \ \delta(f + fc + nfm)] \text{-----}13$$

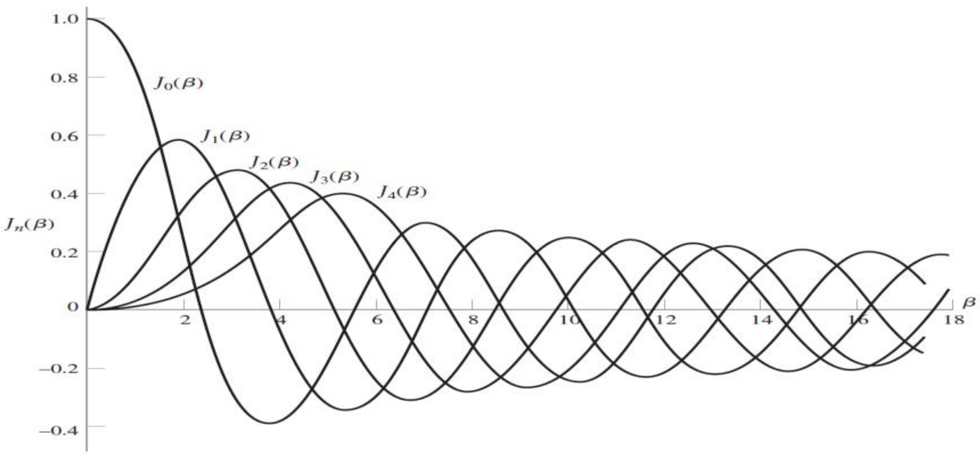


FIGURE 4.6 Plots of the Bessel function of the first kind, $J_n(\beta)$, for varying order n.

1. For different integer (positive and negative) values of n, we have

$$J_n(\beta) = J_{-n}(\beta), \qquad \text{for } n \text{ even}$$

and

$$J_n(\beta) = -J_{-n}(\beta), \qquad \text{for } n \text{ odd}$$

2. For small values of the modulation index β, we have

$$\left.\begin{aligned} J_0(\beta) &\approx 1, \\ J_1(\beta) &\approx \frac{\beta}{2}, \\ J_n(\beta) &\approx 0, \qquad n > 2 \end{aligned}\right\}$$

3. The equality

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

holds exactly for arbitrary β.

The average power dissipated by S(t) is given by $P = Ac^2/2R$ considering equation

$$P = \frac{Ac^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

TRANSMISSION BANDWIDTH OF FM SIGNALS:

FM signal contains an infinite number of side frequencies so that the bandwidth required to transmit such a signal is similarly infinite in extent. In practice however, we find that the FM signal is effectively limited to a finite number of significant side frequencies.

Consider the case of an FM signal generated by a single tone modulating wave of frequency fm. Here the side frequencies that are separated from the carrier frequency fc by an amount greater than the frequency deviation Δf decrease rapidly toward zero, therefore the bandwidth always exceeds the total frequency excursion for large values of modulation index β, the bandwidth is slightly greater than, the total frequency excursion $2\Delta f$.

For small values of β, the spectrum of the FM signal is limited to the carrier frequency fc and one pair of side frequencies at fc $\pm$ f_m. so that the bandwidth approaches 2fm. Therefore to find the practical bandwidth a rule of thumb i.e carson's rule is used. It states that the bandwidth of FM wave is twice the sum of the deviation and the highest modulating frequency

$$B \cong 2\Delta f + 2f_m$$

$$= 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

Deviation ratio

The modulating signal m(t) with its highest frequency component denoted by W. the bandwidth required to transmit an FM signal generated by this modulating signal is given by deviation ratio D. It is defined as the ratio of the frequency deviation Δf , which corresponds to the maximum possible amplitude of the modulating signal m(t) to the highest modulation frequency W.

$$D = \frac{\Delta f}{W}$$

By replacing β by D and fm with W the bandwidth is given by

$$B = 2W\,(D+1)$$

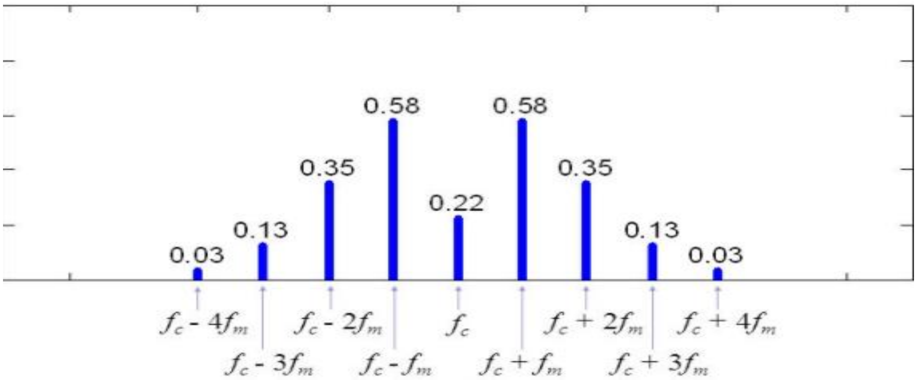
Universal rule for evaluating bandwidth;

Universal rule states that " the transmission bandwidth of an FM wave as the separation between the two frequencies beyond which none of the side frequencies is greater than 1 percent of the carrier amplitude obtained when the modulation is removed i.e transmission bandwidth is

$$2n_{max}\, f_m$$

f_m = modulating frequency

n_{max} = largest value of the integer n that satisfies $|J_n(\beta)| > 0.01$



Here n_{max} is 4 therefore B= 2x4xfm=8fm same as [fc+4fm-(fc-4fm)]

GENERATION OF FM WAVES:

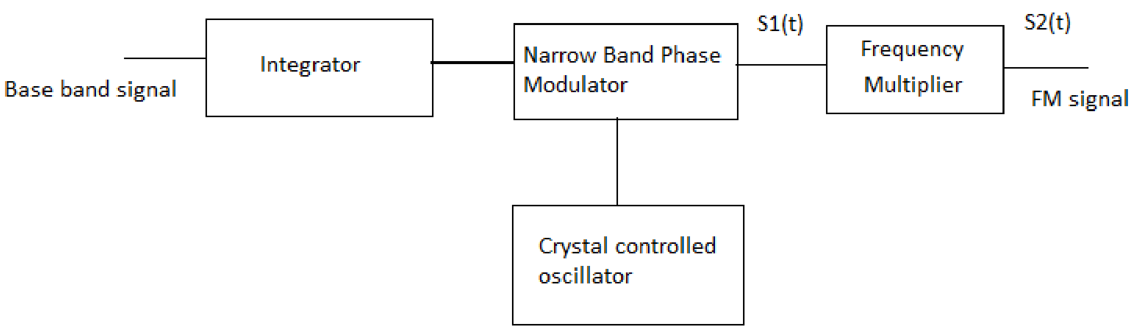
There are two basic methods of generating frequency modulated signals

1] Indirect method

2] Direct method

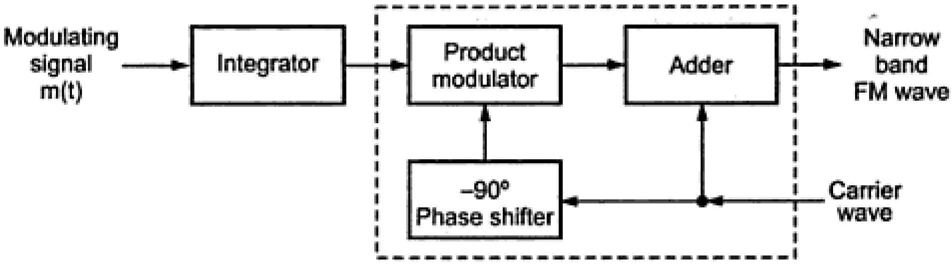
Indirect FM:

A simplified block diagram of an indirect FM is shown in fig. the message signal m(t) is first integrated and then used to phase modulate a crystal controlled oscillator.



The use of crystal control provides frequency stability. In order to minimize the distortion inherent in the phase modulator, the maximum phase deviation or modulation index β is kept small thereby resulting in a narrow band FM signal.

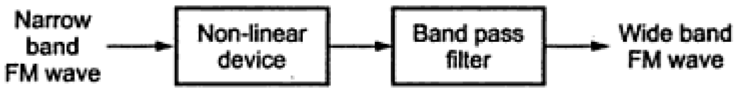
Implementation of the narrow band phase modulator is as shown in fig below by the dotted line.



Narrow-band phase modulator

The narrow band FM signal is next multiplied in frequency by means of a frequency multiplier so as to produce desired wide band FM signal.

A frequency multiplier consists of a memory-less non-linear device followed by a band pass filter. Memory-less non-linear device implies that it has no energy storage elements



The input-output relation of such a device may be expressed in a general form

$V(t) = a_1 S(t) + a_2 S^2(t) + \text{---------} a_n S^n(t)$------1

Where $a_1, a_2,$ --------- a_n are constant coefficients determined by the operating point of the device.

The input S(t) is a FM signal defined by

$S(t) = A_c \cos [2\pi f_c t + 2\pi K_f \int_0^t m(t)dt]$------2

Whose instantaneous frequency is

$f_i(t) = f_c(t) + K_f m(t)$ -----3

The mid-band frequency of BPF is set equal to nf_c. the BPF is designed to have a bandwidth equal to n times the transmission bandwidth of S(t)

After band pass filtering of the non-linear device's output V(t). we have new FM signal defined by

$$s'(t) = Ac' \cos [2\pi n f_c t + 2\pi n K_f \int_0^t m(t)dt]------4$$

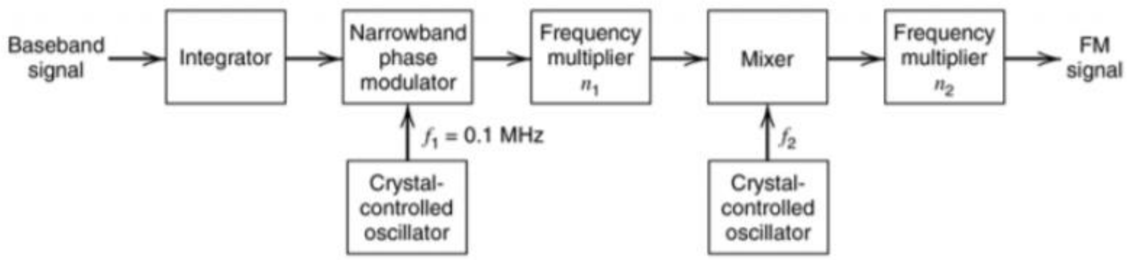
whose instantaneous frequency is

$$fi'(t) = n f_c + n K_f m(t)-----5$$

comparing 3 and 5 we see that nonlinear processing circuits acts as a frequency multiplier.

Problem:

Figure shows the block diagram of a FM transmitter used to transmit audio signals containing frequencies in the range 100Hz to 15KHz. The narrow band phase modulator is supplied with a carrier wave of frequency f_1 = 0.1MHz by a crystal controlled oscillator. Second carrier frequency f_2=9.5MHz.The desired FM wave at the transmitter output has a carrier frequency fc =100MHz and frequency deviation Δf =75KHz. β is less than 0.3 radian (assume β=0.2)



Given fm = 100Hz to 15KHz

w.k.t $\beta = \frac{\Delta f}{fm}$

The lowest modulating frequency produces a frequency deviation of $\Delta f_1 = \beta$ fm

= 0.2 x 100 = 20 Hz

Let n_1 and n_2 denote the respective frequency multiplication ratios

$$n_1 n_2 = \frac{\Delta f}{\Delta f1} = 3750 -------1$$

$$f_2 - n1f1 = fc/n_2$$

$$9.5 - 0.1n_1 = 100/n_2 -------2$$

Solving 1 and 2, Sub $n_1=3750/n_2$ in 2

$n_2 = 50$ therefore $n_1= 75$

Direct FM:

In a direct FM system, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal by means of a device known as voltage controlled oscillator. One way of implementing such a device is to use a sinusoidal oscillator having a highly resonant network in which the capacitance will vary in accordance with modulating signal. The capacitive component in the frequency selective network consists of a fixed capacitor in parallel with a voltage variable capacitor. The resultant capacitance is represented by C(t). The voltage variable capacitor called varicap or varactor is one whose capacitance depends on the voltage applied across its electrodes. The capacitance of a reverse biased varacter diodes depends on the voltage applied across its pn junction. The larger the reverse voltage applied to such a diode, the smaller will be its transition capacitance. The frequency of oscillation of oscillator is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L1+L2)C(t)}} \quad ---1$$

where C(t) is the total capacitance of fixed capacitor and the variable voltage capacitor and L1 and L2 arethe two inductances in the frequency determining network of the oscillator.

Assume that for a sinusoidal modulating wave of frequency fm, the capacitance C(t) is expressed as

$$C(t) = C_o + \Delta C \cos 2\pi f_m t \quad ----2$$

Co is the capacitance in the absence of modulation and ΔC is the maximum change in capacitance

Sub 2 in1

$$fi\ (t) = \frac{1}{2\pi\sqrt{(L1+L2)(Co + \Delta C \cos 2\pi fmt)}}$$

$$= \frac{1}{2\pi\sqrt{(L1+L2)Co + (L1+L2)\Delta C \cos 2\pi fmt}}$$

$$= \frac{1}{2\pi\sqrt{(L1+L2)Co}} \frac{1}{\sqrt{1+\frac{(L1+L2)\Delta C \cos 2\pi fmt}{(L1+L2)Co}}}$$

$$fi\ (t) = fo[1+ \frac{\Delta C}{Co} \cos (2\pi fmt)]^{-0.5} \text{-------------A}$$

where fo is the unmodulated frequency of oscillation that is

$$fo = \frac{1}{2\pi\sqrt{(L1+L2)Co}}$$

Considering the max change in capacitance ΔC is small compared with the unmodulated capacitance Co . using binomial theorem eqn A becomes

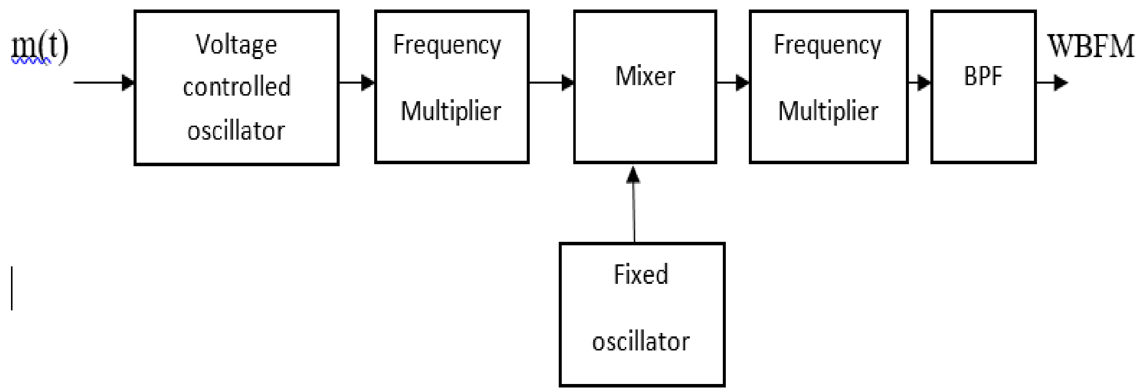
$$fi\ (t) = fo[1- \frac{\Delta C}{2Co} \cos (2\pi f_m t)]$$

let $\frac{\Delta C}{2Co} = -\frac{\Delta f}{fo}$

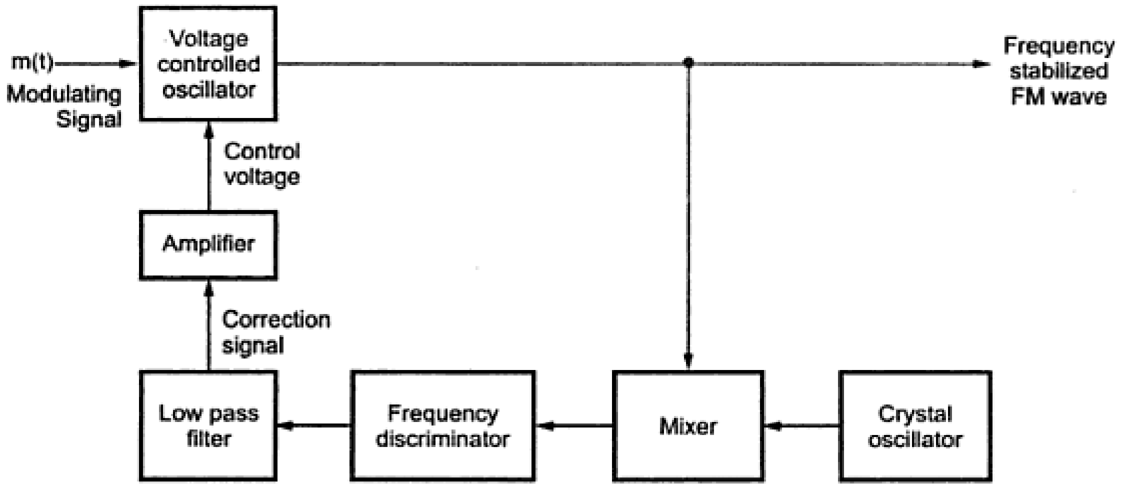
Hence the instantaneous frequency of the oscillator ,which is being frequency modulated by varying the capacitance of the frequency determining network is approximately given by

$$fi\ (t) = fo + \Delta f \cos (2\pi f_m t)$$

to generate a wide band FM wave with the required frequency deviation, the configuration as shown in fig consisting of a voltage controlled oscillator followed by a series of frequency multipliers and mixers is used.



The FM transmitter described above has the disadvantage that the carrier frequency is not obtained from a highly stable oscillator. It is necessary to provide some auxiliary means by which a very stable frequency generated by a crystal will be able to control the carrier frequency. One method 0f frequency stabilization is shown in figure below



The output of the FM generator is applied to a mixer together with the output of a crystal controlled oscillator and the difference frequency term is extracted. The mixer output is next applied toa frequency discriminator and then low pass filtered.

A frequency discriminator is a device whose output voltage has an instantaneous amplitude that is proportional to the instantaneous frequency of the FM signal applied to its input.

When the FM transmitter has exactly the correct carrier frequency the LPF output is zero. However deviations of the transmitter carrier frequency from its assigned value will cause the frequency discriminator filter combination to develop a dc output voltage with a polarity determined by the sense of the transmitter frequency drift. This dc voltage after suitable amplification is applied to the voltage controlled oscillator of the FM transmitter.in such a way as to modify the frequency of the oscillator in a direction that tends to restore the carrier frequency to its correct value.

Problems:

1. At Low frequencies it may be possible to generate an FM wave by varying the capacitance of a parallel resonant circuit shown in fig. Show that the output S(t) of the tuned circuit shown below is an FM wave if the capacitance has the form C(t) =Co-km(t) & $\left|\frac{km(t)}{Co}\right| << 1$

$$fo = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{L(Co - km(t))}}$$

$$= \frac{1}{2\pi\sqrt{LCo\left(1 - \frac{Lkm(t)}{LCo}\right)}}$$

$$= \frac{1}{2\pi\sqrt{LCo}}\left(1 - \frac{Lkm(t)}{LCo}\right)^{-0.5}$$

$$= \frac{1}{2\pi\sqrt{LCo}}\left(1 + \frac{1}{2}\frac{km(t)}{Co}\right)$$

$$= \frac{1}{2\pi\sqrt{LCo}} + \frac{1}{4\pi}\frac{km(t)}{Co}\frac{1}{\sqrt{LCo}}$$

This is of the form

$$f_i(t) = f_c + k_f\, m(t)$$

Therefore is an FM wave

DEMODULATION OF FM

ZERO CROSSING DETECTOR

This detector exploits the property that the instantaneous frequency of the FM wave is approximately given by

$$f_i = \frac{1}{2\Delta t} \text{------}1$$

where Δt is the time difference between adjacent zero crossings of the FM wave as in fig. the interval T is chosen according to the following conditions.

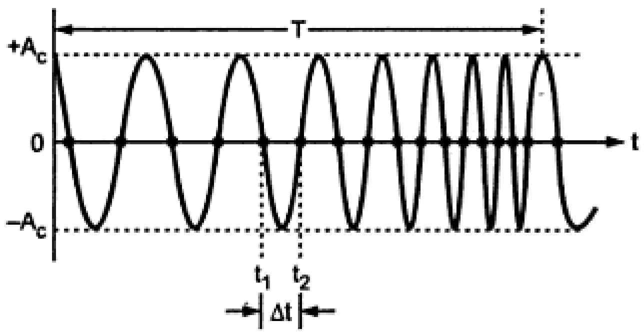
1] the interval T is small compared to the reciprocal of the message bandwidth W

2] the interval T is large compared to the reciprocal of the carrier frequency fc of the FM wave

Condition 1 a means that the message signal m(t) is essentially constant inside the interval T

Condition 2 ensures that a reasonable number of zero crossings of the FM wave occurs inside the interval T

These conditions are illustrated by the waveform



Dots indicate zero crossings

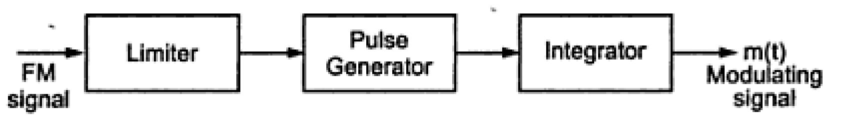
Let n_o denote the number of zerocrossings inside the interval T.

$$\Delta t = \frac{T}{n_o} \text{------------}2$$

Eqn 1 can be rewritten as

$$fi = \frac{n_o}{2T} \text{---------}3$$

since w.k.t the instantaneous frequency is linearly related to the message signal m(t). from eqn 3 we see that m(t) can be recovered from the knowledge of n_o. The simplified block diagram of ZCD is as in fig



The limiter produces a square wave version of the input FM wave. The pulse generator produces short pulses at the positive going as well as negative going edges of the limiter output. The integrator performs the averaging over the interval T as indicated in eqn 3 thereby reproducing the original message signal m(t) at its output.