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Unit - 3

Time Response of Control System

Most of the control Systems use time as its independent variable, so it is imp to analyse the response given by the System for the applied excitation (input) which is function of time.

Analysis of response means to see the variation of output with respect to time. The evalution of s/m is based on the ~~~~ analysis of such response.

This output behaviour w.r.to time should be within specified limits to have satisfactory performance of the s/m. The complete base of stability analysis lies in the time response analysis. The s/m stability, s/m accuracy & complete evalution is always based on the time response analysis & corresponding results

Time Response: means how o/p behaves w.r.t time

Definition:- The response given by the s/m which is function of the time, to the applied excitation is called time response of a control s/m.

There are 2 types of Time response of a control s/m

1) Transient response
2) Steady state response

Transient response

The output variation during the time, it takes to achieve its final value is called as transient response. The time required to achieve the final value is called transient period.

* The transient response may be exponential or oscillatory in nature. It is denoted as $C_t(t)$.

Transient response must vanish after some time to get the final value closer to the desired value.
Such systems in which transient response dies out after some time are called stable systems.

Mathematically for stable operating systems

$$\lim_{t \to \infty} C_t(t) = 0$$

Steady State Response :- It is that part of the time response which remains after complete transient response vanishes from the system output.
It tells us how far away the actual output is from its desired value.
The steady state response indicates the accuracy of the system. The symbol for steady state response is C_{ss}.

Hence total time response $c(t)$ can be written as

$$\boxed{c(t) = C_{ss} + C_t(t)}$$

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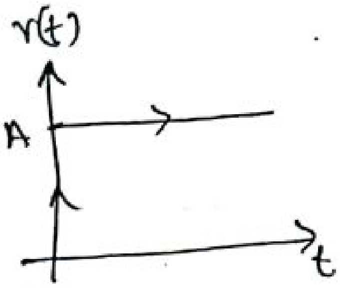
Standard Test Signals (inputs)

1) Step input (position function) → sudden change.

A step signal is a signal whose value changes from one level to another level in zero time.

It is the sudden changes of the input at a specified time.

Mathematically it can be described as



$$r(t) = \begin{cases} A; & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

If $A=1$ then it is called unit step function & denoted by $u(t)$

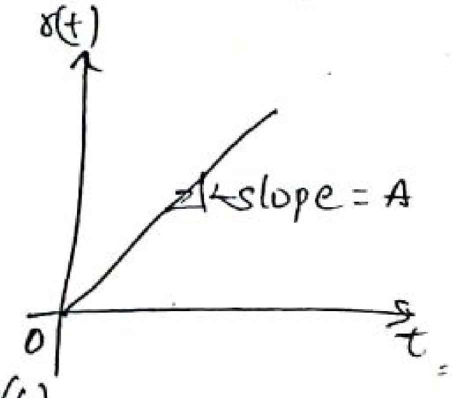
Laplace transform of such input is → $\frac{A}{S}$

ii) Ramp Signal (velocity function) → constant velocity ~~shock~~

The Ramp signals is a signal which changes constantly with time

It is constant rate of change in input.

Mathematically it is denoted as



$$r(t) = \begin{cases} = At; & \text{for } t \geq 0 \\ = 0; & \text{for } t < 0 \end{cases}$$

If $A=1$. It is called unit Ramp input. It is denoted as $r(t)$

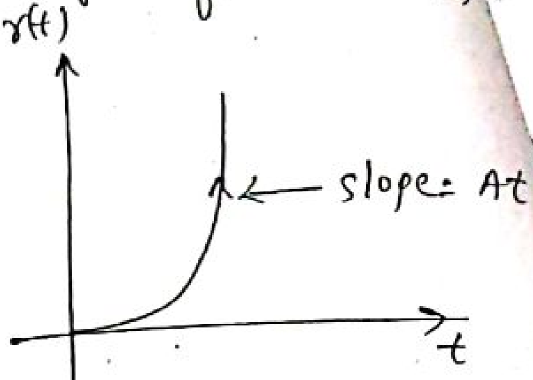
Laplace transform is $\frac{A}{s^2}$

iii) Parabolic Signal (Acceleration function) → constant Acceleration

This is the input which is one degree faster than a ramp type of signals

Mathematically denoted as

$$r(t) \begin{cases} = \frac{At^2}{2} & \text{for } t \geq 0 \\ = 0 & \text{for } t < 0 \end{cases}$$



where A is called magnitude of the parabolic input

If A = 1. $r(t) = \frac{t^2}{2}$ it is called unit parabolic input

Laplace transform is $\frac{A}{s^3}$

iv) Impulse input (shock)

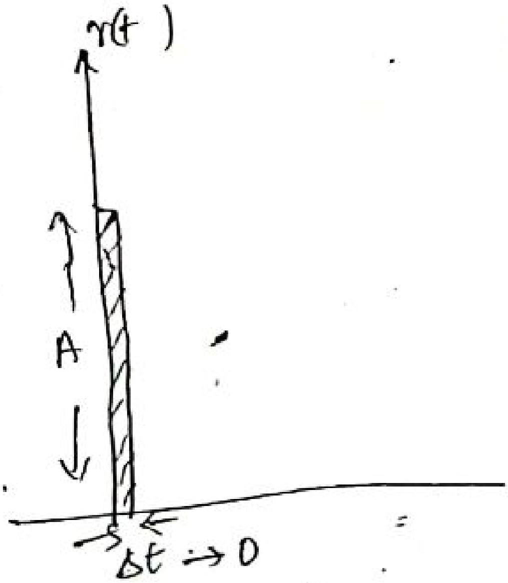
It is the pulse whose magnitude is infinite while its width tends to zero i.e., $t \to 0$.

If its area is unity it is called unit impulse input

denoted as $\delta(t)$

Mathematically expressed as

$$r(t) \begin{cases} = A & \text{for } t = 0 \\ = 0 & \text{for } t \neq 0 \end{cases}$$



The laplace transform of unit impulse input is always 1.

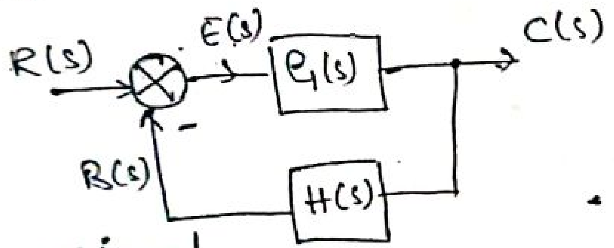
r(t)	Symbol	R(s)
Unit step	$u(t)$	$1/s$
Unit ramp	$r(t)$	$1/s^2$
Unit parabolic	—	$1/s^3$
Unit impulse	$\delta(t)$	1

- **Steady state error (e_{ss})**

It is the difference between the actual output & the desired output.

Derivation of steady state Error:-

Consider a simple closed loop system using negative feedback



$E(s) \rightarrow$ Error signal

$B(s) \rightarrow$ Feedback signal

$$E(s) = R(s) - B(s)$$

$$B(s) = C(s)H(s)$$

$$E(s) = R(s) - C(s)H(s)$$

$$C(s) = E(s)G(s)$$

$$E(s) = R(s) - E(s)G(s)H(s)$$

$$E(s) + E(s)G(s)H(s) = R(s)$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)} \quad -(1)$$

For non unity feedback.

$$E(s) = \frac{R(s)}{1+G(s)} \quad \text{for unity feed back}$$

This $E(s)$ is the error in Laplace domain & its expressed in 's' domain. if we want to calculate the error value in time domain corresponding error will be $e(t)$. Now steady state of the s/m is that state which remains on $t \to \infty$.

$$\boxed{\therefore \text{ Steady state error } e_{ss} = \lim_{t\to\infty} e(t)}$$

Now we can relate this in Laplace domain by using final value theorem which states that

$$\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \quad \text{where } F(s) = L\{f(t)\}}$$

$$\therefore e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

Substitute $E(s)$ from in eqn (1) we get.

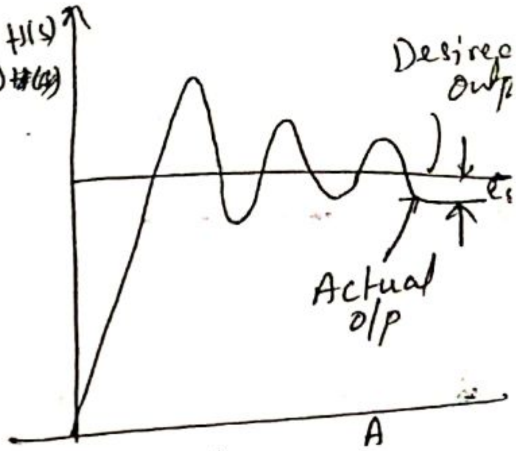
$$\boxed{e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}}$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} \quad \text{for unity feed back}$$

Static Error Constants (Static Error coefficients)

Error constants or coefficients are the measure of steady state errors & gives an idea in to how steady state error can be reduced or totally eliminated.

Consider a s/m having open loop T.F $G(s)H(s)$



$$e_{ss} = \frac{A}{1+K_p}$$

- Reference input is step input.

The LT of step o/p is

$$R(s) = \frac{A}{s}$$

W.K.T

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \cdot A/\cancel{s}}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{A}{1+G(s)H(s)}$$

$$e_{ss} = \frac{A}{1+\lim_{s \to 0} G(s)H(s)}$$

For a s/m $\lim_{s \to 0} G(s)H(s)$ is constant & called positional Error co-efficient of the s/m denoted as K_p

$K_p = \lim_{s \to 0} G(s)H(s)$ = positional error coefficient

For unit step input $A=1$

$$\boxed{e_{ss} = \frac{A}{1+K_p}} \qquad \boxed{e_{ss} = \frac{1}{1+K_p}}$$

b) Reference input is ramp of magnitude 'A'

velocity error constant

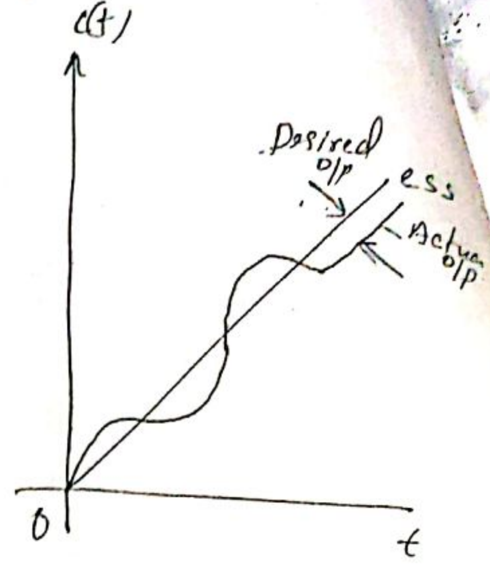
$$R(s) = \frac{A}{s^2}$$

$$e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s) H(s)}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \frac{A}{s^{\cancel{2}}}}{1 + G(s) H(s)}$$

$$= \lim_{s \to 0} \frac{A}{s[1 + G(s) H(s)]}$$

$$= \lim_{s \to 0} \frac{A}{s + s G(s) H(s)}$$



$$e_{ss} = \frac{A}{\lim_{s \to 0} s G(s) H(s)}$$

$\lim_{s \to 0} s G(s) H(s)$ is constant is called velocity error coefficient. denoted as K_v

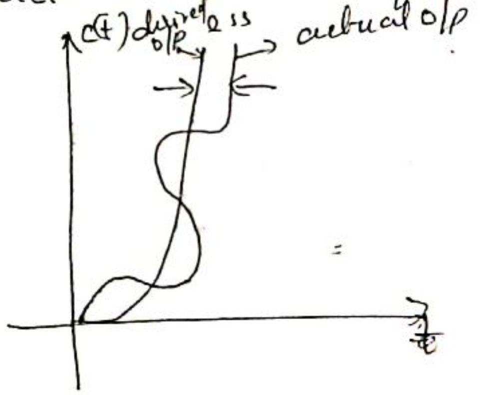
$K_v = \lim_{s \to 0} s G(s) H(s)$ = velocity error coefficient

$$\boxed{e_{ss} = \frac{A}{K_v}}$$

For unit ramp input

$$\boxed{e_{ss} = \frac{1}{K_v}}$$

c) Reference input is parabolic (Acceleration error coefficient



$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s) H(s)}$$

$$= \lim_{s \to 0} \frac{s \frac{A}{s^3}}{1 + G(s) H(s)}$$

$$\lim_{s \to 0} \frac{A}{s^2(1+G(s)H(s))}$$

$$\lim_{s \to 0} \frac{A}{s^2 + s^2 G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{A}{s^2 G(s)H(s)}$$

$K_a = \lim_{s \to 0} s^2 G(s)H(s)$ is constant is called Acceleration Error coefficient

$$e_{ss} = \frac{A}{K_a}$$

For unit parabolic input

$$\boxed{e_{ss} = \frac{1}{K_a}}$$

Static Error coefficient	e_{ss}
$K_p = \lim_{s \to 0} G(s)H(s)$	$\frac{A}{1+K_p}$
$K_v = \lim_{s \to 0} sG(s)H(s)$	$\frac{A}{K_v}$
$K_a = \lim_{s \to 0} s^2 G(s)H(s)$	$\frac{A}{K_a}$

Effect of change in $G(s)H(s)$ on steady state error

(Type of a System)

This is the way of designing a control system. & is different from that of the order of a system.

The System type refers to the number of poles of Open-loop transfer function, $G(s).H(s)$ lying at the origin

If N is no of poles at the origin, then the s/m
type - N s/m.

The open-loop transfer-function can be expressed as

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s) \cdots}{s^j (1+T_a s)(1+T_b s) \cdots}$$

where K = Resultant system gain.
J = Type of the System → No. of poles at origin of O.L.T.F G(s)H(s).

'TYPE' of the System

if j = 0, TYPE zero s/m
j = 1, TYPE one s/m
j = 2; - " - two s/m
⋮
J = n TYPE 'n' s/m.

Analysis of TYPE 0, 1 & 2 System

Type 0 : $$G(s)H(s) = \frac{K(s+1) \cdots}{(s+2)(s+3) \cdots}$$

Type 1 $$G(s)H(s) = \frac{K(s+1) \cdots}{s(s+2)(s+3) \cdots}$$

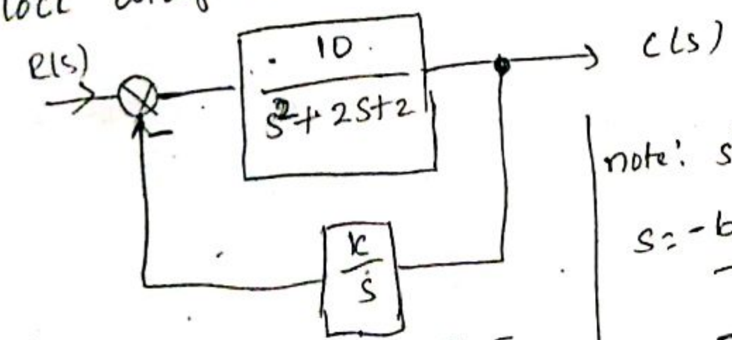
Type 2 : $$G(s)H(s) = \frac{K(s+1) \cdots}{s^2 (s+2)(s+3) \cdots}$$

Thus type is the property of O.L.T.F & order is the property of C.L.T.F compilation of steady state error

Inputs / Type of s/m	Steady state error formula	Type 0	Error e_{ss} for Type 1	Type 2
Step i/p	$\frac{1}{1+K_p}$	$\frac{1}{1+K_p}$	$K_v = \infty$ ∞, $e_{ss} = 0$	$e_{ss} = 0$
Ramp	$\frac{1}{K_v}$	∞	$\frac{1}{K_v}$	$e_{ss} = 0$
Parabolic	$\frac{1}{K_a}$	∞	∞	$\frac{1}{K_a}$

First order Systems

Find the type & order of the s/m for a given block diagram



consider the open loop T.F.

$$G(s)H(s) = \frac{10}{s^2+2s+2} \cdot \frac{k}{s}$$

$$= \frac{10k}{s(s^2+2s+2)}$$

$$G(s)H(s) = \frac{10K}{s(s+1+j)(s+1-j)}$$

note: s^2+2s+2 $(a=1, b=2, c=2)$

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2j}{2} \quad (\because \sqrt{-4} = 2j)$$

$$= \frac{\cancel{2}(-1 \pm j)}{\cancel{2}} = -1 \pm j$$

$s = -1 \pm j$

$s + 1 \pm j = 0$

$\therefore (s+1+j)(s+1-j)$

Since there is only one pole at the origin ∴ s/m is type-1 &

From the fig. closed loop T.F. can be written as

$$\frac{C(s)}{R(s)} = \frac{G_1(s)}{1+G_1(s)H(s)}$$

$$= \frac{\frac{10}{s^2+2s+2}}{1+\left[\frac{10}{s^2+2s+2}\right]\cdot\left[\frac{k}{s}\right]}$$

$$= \frac{\frac{10}{\cancel{s^2+2s+2}}}{\frac{s(s^2+2s+2)+10k}{\cancel{s^2+2s+2}}} = \frac{10}{\frac{s^3+2s^2+2s+10k}{s}}$$

$$\frac{C(s)}{R(s)} = \frac{10s}{s^3+2s^2+2s+10k}$$

since the highest power of s in the denominator is 3

The order of the s/m is 3

Order of s/m

The order of the s/m is given by the order of the differential equation governing the s/m

Order of s/m is the highest power of 's' in the denominator of a closed loop transfer function.

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Analysis of Type 0, 1, & 2 Slms.

error co-efficients are K_p, K_v, & K_a

Inorder to find these error constants the s/m must be stable

The concept of K_p, K_v, & K_a is applicable only if

i) slm is represented in its simple form

ii) only if the slm is stable.

- Consider the selected input is step input.

For type '0'

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\;---}{s^0(1+T_a s)(1+T_b s)\;---}$$

For a step o/p $K_p = \lim_{s\to 0} G(s)H(s) = K$

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+K}$$

- For type 1

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\;---}{s(1+T_a s)(1+T_b s)\;--}$$

As step input

$$K_p = \lim_{s\to 0} G(s)H(s) = \infty$$

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{\infty} = 0$$

For Type 2

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s^2(1+T_a s)(1+T_b s)\ldots}$$

As step input $K_p = \lim_{s\to 0} G(s)H(s) = \infty$

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{\infty} = 0$$

For Ramp input

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{(1+T_a s)(1+T_b s)\ldots}$$

$$\boxed{K_v = \lim_{s\to 0} s\,G(s)H(s)}$$

For type zero s/m

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s^0(1+T_a s)(1+T_b s)\ldots}$$

$$K_v = \lim_{s\to 0} sG(s)H(s) = 0$$

$$\therefore e_{ss} = \frac{A}{K_v} = \frac{A}{0} = \infty$$

For Type one

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s(1+T_a s)(1+T_b s)\ldots}$$

$$K_v = \lim_{s\to 0} \cancel{sG(s)}\, sG(s)H(s) = K$$

$$\therefore e_{ss} = \frac{A}{K_v} = \frac{A}{K}$$

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Type 2

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s^2(1+T_a s)(1+T_b s)\ldots}$$

$$K_v = \lim_{s\to 0} s\,G(s)H(s) = \infty$$

$$\therefore e_{ss} = \frac{A}{K_v} = \frac{A}{\infty} = \underline{0}$$

For a Parabolic input

consider Type 0

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{(1+T_a s)(1+T_b s)\ldots}$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s) = 0 \qquad \therefore e_{ss} = \frac{A}{K_a} = \frac{A}{0} = \infty$$

For type one s/m

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s(1+T_a s)(1+T_b s)\ldots}$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s) = 0$$

$$\therefore e_{ss} = \frac{A}{0} = \infty$$

For Type two s/m

$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s^2(1+T_a s)(1+T_b s)\ldots}$$

$$K_a = \lim_{s\to 0} s^2 G(s).H(s) = K \quad \therefore e_{ss} = \frac{A}{K_a} = \frac{A}{K}$$

Type of the s/m	Error coefficients K_p	K_v	K_a	step i/p	Ramp i/p	Parabolic i/p
0	K	0	0	$\frac{A}{1+K}$	∞	∞
1	∞	K	0	0	A/K	∞
2	∞	∞	K	0	0	A/K

Problems

Find the static error coefficient for the unity feedback control s/m whose open loop T.F are

(1) $G(s) = \frac{K}{s(s^2+4s+200)}$

2) $G(s) = \frac{K(1+2s)(1+4s)}{s^2(s^2+2s+10)}$

& also find the error for unit step, unit ramp & unit parabolic inputs. & Determine the type & order of the s/m.

Sol^n

i) $G(s) = \frac{K}{s(s^2+4s+200)}$

$K_p = \lim_{s \to 0} G(s) H(s)$

$= \lim_{s \to 0} \frac{K}{s(s^2+4s+200)}$

$= \infty$

$e_{ss} = \frac{A}{1+K} = \frac{A}{\infty} = 0$

(9)

ii) $K_v = \lim_{s \to 0} s\, G(s) H(s)$

$= \lim_{s \to 0} \frac{K s}{s(s^2+4s+200)}$

$= \frac{K}{200}$

$K_v = \frac{K}{200}$

$E_{ss} = \frac{A}{K_v} = \frac{200A}{K} = \frac{200A}{K}$

iii) $K_a = \lim_{s \to 0} s^2 G(s) H(s)$

$= \lim_{s \to 0} \frac{s^{\cancel{2}} K}{\cancel{s}(s^2+4s+200)}$

$= 0$

$E_{ss}(s) = \frac{A}{K_a} = \frac{A}{0} = \infty$

If $r(t) = 1$ $\quad R(s) = \frac{1}{s}$

i) $E_{ss}(s) = \frac{1}{1+K_p} = \frac{1}{\infty} = 0$

if $r(t) = \frac{1}{t}$ $\quad R(s) = \frac{1}{s^2}$

$E_{ss}(s) = \frac{1}{K_v} = \frac{1}{K/200} = \frac{200}{K}$

if ~~r(t)~~ = $R(s) = \frac{1}{s^3}$

$K_a = 0$

$E_{ss}(s) = \frac{1}{K_a} = \infty$

since $J=1$ type one s/m

To find order of the s/m consider closed loop s/m

$$1+G(s)H(s)=0$$

$$1+\frac{K}{s^3+4s^2+200s}=0$$

$$s^3+4s^2+200s+K=0$$

Order = 3

(2) $$G(s)=\frac{K(1+2s)(1+4s)}{s^2(s^2+2s+10)}$$

$$K_p=\lim_{s\to 0} G(s)H(s)$$

$$=\lim_{s\to 0}\frac{K(1+2s)(1+4s)}{s^2(s^2+2s+10)}$$

$$=\frac{K}{0}$$

$$e_{ss}=K_p=\infty$$

$$K_v=\lim_{s\to 0}\frac{\cancel{s}\,K(1+2s)(1+4s)}{s^{\cancel{2}}(s^2+2s+10)}$$

$$=\frac{K}{0}$$

$$=\infty$$

e_{ss}

$$K_a = \lim_{s \to 0} \frac{\cancel{s^2} k(1+2s)(1+4s)}{\cancel{s^2}(s^2+2s+10)}$$

$$= \frac{k}{10}$$

if $r(t) = 1$ $\quad R(s) = \frac{1}{s}$.

$$E_{ss} = \frac{1}{1+k_p} = \frac{1}{1+\infty} = 0$$

if $r(t) = \frac{1}{t}$ $\quad R(s) = \frac{1}{s}$

$$E_{ss}(s) = \frac{1}{k_v} = \frac{1}{\infty} = 0$$

Since $j = 2$ type of the s/m is <u>two</u>

To find the order of the s/m

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k(1+2s)(1+4s)}{s^2(s^2+2s+10)} = 0$$

$$s^4 + 2s^3 + 10s^2 + k + 2ks(1+4s) = 0$$

$$s^4 + 2s^3 + 10s^2 + k + 2ks + 4ks + 8ks^2 = 0$$

$$s^4 + 2s^3 + 10s^2 + 8ks^2 + 6ks + k = 0$$

$$s^4 + 2s^3 + 10s^2 + 8ks^2$$

<u>order is 4</u>

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(2) consider a unity feed back control s/m whose open loop T. F is $\frac{100}{s(0.1s+1)}$ determine the steady state error when the input is $r(t) = 1 + t + at^2$, $a > 0$

Sol^n

$$G(s) = \frac{100}{s(0.1s+1)}$$

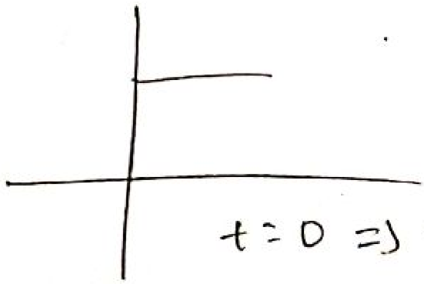
$$K_p = \lim_{s \to 0} G(s)H(s) = \infty$$

$$K_v = \lim_{s \to 0} sG(s)H(s) = 100$$

$$K_a = \lim_{s \to 0} s^2 G(s)H(s) = 0$$

i) for $r(t) = 1$

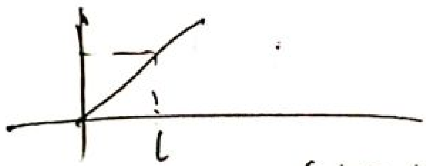
$$E_{ss} = \frac{A}{1+K_p} = \frac{1}{1+\infty} = 0$$



$t = 0 \Rightarrow r(t)$

ii) For $r(t) = t$

$$E_{ss} = \frac{A}{K_v} = \frac{1}{100}$$



$t = 1$ $r(t) = 1$

$r(t) =$

iii) For $r(t) = \frac{2at^2}{2}$

$$E_{ss} = \frac{A}{K_a} = \frac{2a}{2} = \infty$$

Hence Total steady state error is

$$E_{ss} = 0 + \frac{1}{100} + \infty$$

$$= \infty$$

③ Find K_p, K_v & K_a & steady state error for a s/m with open ~~op.~~ loop T. F. as. (4)

$$G(s)H(s) = \frac{10(s+2)(s+3)}{s(s+1)(s+5)(s+4)}$$

Where the input is

$$r(t) = \cancel{3+t+4t^2} \quad 3+t+t^2$$

Soln

- $K_p = \lim_{s \to 0} G(s)H(s) = \infty$

$K_v = \lim_{s \to 0} sG(s)H(s) = 3$

$$G(s)H(s) = \frac{\cancel{s}\,10(s+2)(s+3)}{\cancel{s}(s+1)(s+5)(s+4)} = \frac{6\cancel{0}}{2\cancel{0}} =$$

- $K_a = \lim_{s \to 0} s^2 G(s)H(s)$

$$\frac{s^{\cancel{2}}\,10(s+2)(s+3)}{\cancel{s}(s+1)(s+5)(s+4)} = \frac{0}{20} = 0$$

~~Now o/p is~~

$$\boxed{\begin{array}{cccc} e_{ss\,total} & \text{step} & \text{ramp} & \text{parabolic} \\ \cancel{r(t)} = & e_{ss_1} + & e_{ss_2} + & e_{ss_3} \end{array}}$$

$$r(t) = 3+t+t^2 = 3+t+2\frac{t^2}{2}$$

The i/p is combination of three standard inputs

$A_1 = 3$ step of 3 for step $r(t) = A$

$A_2 = 1$ ramp of 1 for ramp $r(t) = At$

$A_3 = 2$ parabolic input $g_2\, r(t) = \frac{At^2}{2}$

The parabolic input must be expressed as $\frac{At^2}{2}$

a) Step of 3 the error is

$$e_{ss1} = \frac{A_1}{1+K_p} = \frac{3}{1+\infty} = 0$$

b) ramp inp $A_2 = 1$

$$e_{ss2} = \frac{A_2}{K_v} = \frac{1}{3} = \frac{1}{3}$$

c) For parabolic $A_3 = 2$

$$e_{ss3} = \frac{A}{K_a} = \frac{2}{0} = \infty$$

Hence steady state error is

$$e_{ss} = e_{ss1} + e_{ss2} + e_{ss3} = 0 + \frac{1}{3} + \infty$$

$$= \infty$$

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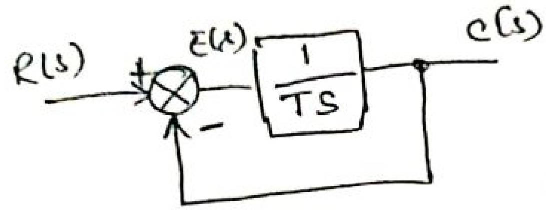
Analysis of First-Order System

Consider the first-order s/m with unity feed back

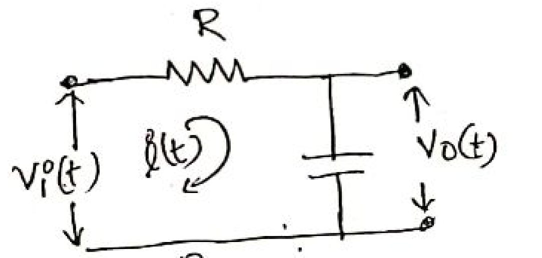


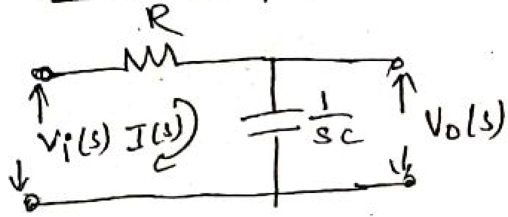
closed loop transfer function is given by

$$\frac{C(s)}{R(s)} = \cancel{\frac{T(s)}{1+T(s)}} \quad \cancel{\frac{T(s)}{1+T(s)}} = \cancel{A}$$

$$= \frac{\frac{1}{Ts}}{1+\frac{1}{Ts}} = \frac{1}{1+Ts} \quad \text{—} \ (1)$$

consider a simple s/m





$$V_i(s) = I(s)R + \frac{1}{sC} I(s)$$

$$V_o(s) = \frac{1}{sC} I(s)$$

$$T.F = \frac{V_o(s)}{V_i(s)} = \frac{1}{1+sRC} \quad \text{—} \ (2)$$

comparing eqn (1) & (2)

$$T = RC = \text{Time constant}$$

Why RC is called as time constant

$T = RC \uparrow \rightarrow$ speed

$$RC = \frac{V}{I} \times \frac{Q}{V} = \frac{Q}{I} = \frac{Q}{Q/t} \Rightarrow t$$

$T = RC \downarrow \rightarrow$ speed

So it is called time constant.

Unit step response of first-order Order System

Transfer function of first-order slm is given by

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts} \quad -①$$

The o/p response is given by

$$C(s) = R(s)\left[\frac{1}{1+Ts}\right] \quad -②$$

For the unit step input W.K.T $R(s) = \frac{1}{s}$ then

$$C(s) = \frac{1}{s}\left[\frac{1}{1+Ts}\right]$$

(multiply num & den by $\frac{1}{T}$ so that pole has no term with it i.e., T) It should be standard format)

$$= \frac{1/T}{s\left(\frac{Ts}{T} + \frac{1}{T}\right)} = \frac{1/T}{s\left(s + \frac{1}{T}\right)} = \frac{1/T}{s\left(s+\frac{1}{T}\right)} \quad -③$$

Expanding C(s) into partial fractions

let $C(s) = \frac{A_1}{s} + \frac{A_2}{s+\frac{1}{T}}$

A_1 is obtained by multiplying $C(s)$ of eqn (3) by s & letting $s=0$

$$C(s) = \frac{1}{s} \cdot \frac{1}{Ts+1}$$

To find partial fraction co-efficients.

$$C(s) = \frac{A}{s} + \frac{B}{1+Ts}$$

$$1 = A(1+Ts) + B(s)$$

Put $s=0$

$$1 = A(1+T(0)) + B(0)$$

$$\boxed{A = 1}$$

Put $s = -\frac{1}{T}$

$$1 = A\left(1+T\left(\frac{1}{-T}\right)\right) + B\left(-\frac{1}{T}\right)$$

$$1 = A(0) + B\left(-\frac{1}{T}\right)$$

$$B = -T$$

$$C(s) = \frac{1}{s} - \frac{T}{Ts+1}$$

$$C(s) = \frac{1}{s} - \frac{T}{Ts+1}$$

Taking the inverse laplace transform we get.

$$L^{-1}(C(s)) = L^{-1}\left[\frac{1}{s} - \frac{T}{Ts+1}\right]$$

$$= L^{-1}\left[\frac{1}{s} - \frac{T/T}{s+\frac{1}{T}}\right]$$

$$L^{-1}\left[\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right]$$

WKT

$$L^{-1}\left[\frac{1}{s+a}\right] = e^{-at}$$

$$\therefore L^{-1}\left[\frac{1}{s+\frac{1}{T}}\right] = e^{-t/T}$$

$$\boxed{c(t) = 1 - e^{-t/T} \ ; \text{ for } t \geq 0}$$

$L^{-1}\left(\frac{1}{s}\right) = \underline{1}$

This eqn states that initially the o/p c(t) is zero & finally it becomes unity

One imp characteristic of such an exponential response curve c(t) is that at $t = T$ the value of c(t) is 0.632 or the response c(t) has reached 63.2% of its total charge

let $t = T$ the c(t) is

$$c(t) = 1 - e^{-t/T}$$

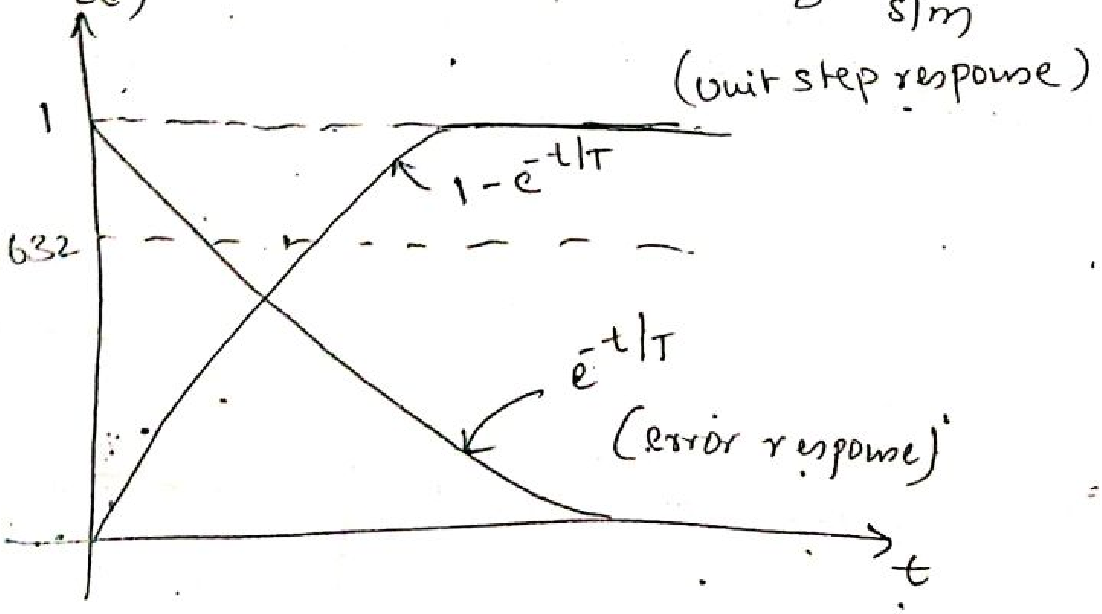
$$= 1 - e^{-T/T}$$

$$= 1 - e^{-1}$$

$$= 1 - 0.3679$$

$$= 0.632$$

Fig(1) shows the unit step response of a first order s/m



Steady state error (at t tends to ∞ what is the error)

$$e_{ss} = \lim_{t\to\infty} e(t)$$

W.K.T $e(t) = r(t) - c(t)$

$$e_{ss} = \lim_{t\to\infty} \left(r(t) - c(t)\right)$$

$$e_{ss} = \lim_{t\to\infty} \left(1 - (1 - e^{-t/T})\right)$$

$G(t) H(t) = 1$

$$e_{ss} = \lim_{t\to\infty} e^{-t/T}$$

at $t = \infty$

$e^{-\infty} = 0$

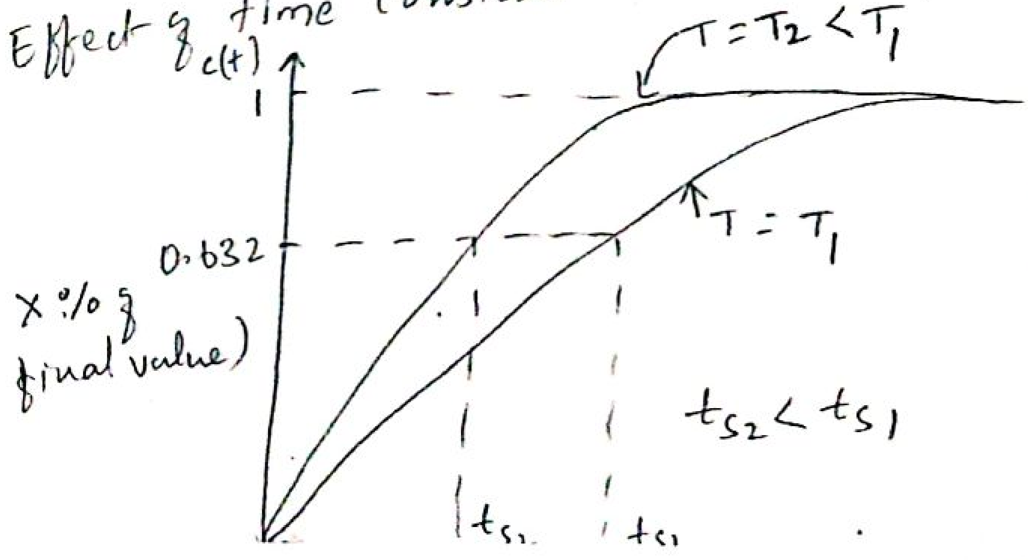
$$\boxed{e_{ss} = 0}$$

Note :- Since $e_{ss} = 0$ First order s/m is capable of responding to step input without any steady state error.

The time constant is the time, it takes for the step response to rise to 63.2% of its final value. The time constant can be considered as a transient response specification of a first order s/m.

The time constant is related to the speed of response of the s/m. The time constant is indicative of how fast the s/m tends to reach the final value.

Effect of time constant on s/m response



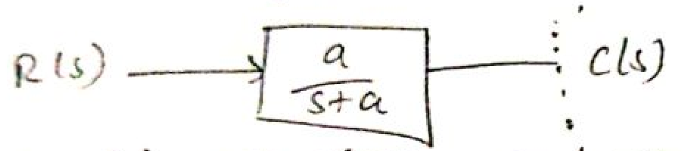
A large time constant → Slower response

a small time constant corresponds to a fast response

The speed of response depends upon the time constant (T) Smaller the time constant speed of response is more

One more example for first order s/m

$$G(s) = \frac{a}{s+a}$$



if the input is unit step we have $R(s) = \frac{1}{s}$ then

$$C(s) = R(s)\,G(s)$$

$$= \frac{1}{s} \cdot \frac{a}{s+a}$$

$$C(s) = \frac{1}{s} + \frac{1}{s+a}$$

Taking inverse LT we get.

$$c(t) = 1 - e^{-at} \qquad t \geq 0$$

when $t = \frac{1}{a}$ we get e^{-a} let $\frac{1}{a} = 0.37$

$$c(t) = 1 - e^{-1} = 0.63$$

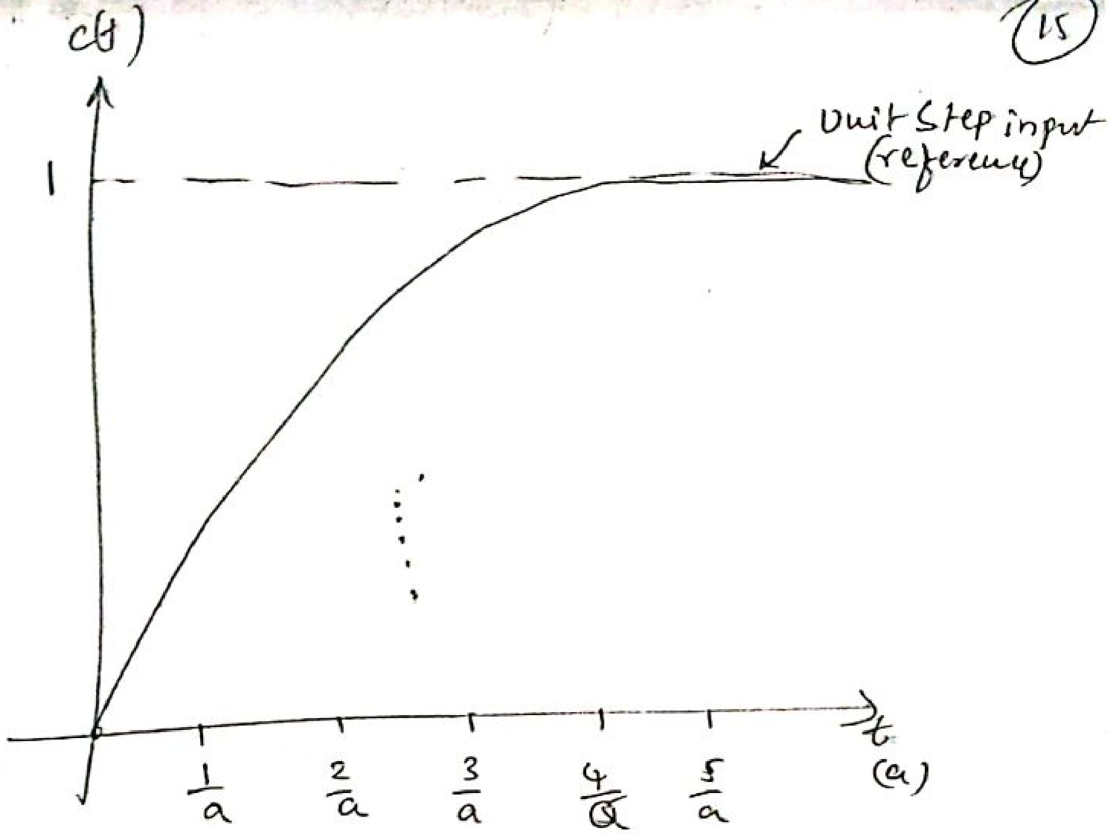
when $t = 2/a$ $c(t) = 0.86$

$t = 3/a$ $c(t) = 0.95$

$t = 4/a$ $c(t) = 0.982$

$t = 5/a$ $c(t) = 0.993$

$t = \infty$ $c(t) = 1$



· Time response analysis of first order slm to a ramp i/p / Input = unit ramp input = t

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

$R(s) = \frac{1}{s^2}$

· For a first order slm. find out the o/p of the slm when the o/p applied to the slm is unit ramp o/p sketch the r(t) & c(t) & shown the steady state error.

$$\frac{C(s)}{R(s)} = \frac{1}{s+T}$$

$r(t)$ = unit ramp input = t

$R(s) = \frac{1}{s^2}$

$C(s) = R(s)\frac{1}{s+T}$

$C(s) = \frac{1}{s^2} \cdot \frac{1}{s+T}$

$$C(s) = \frac{1}{s^2(s+T)}$$

Expanding C(s) into partial fractions

$$C(s) = \frac{1}{s^2(s+T)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+T}$$

$\therefore A(s+T) + Bs(s+T) + Cs^2 = 1$

$(B+C)s^2 + (A+BT)s + AT = 1$

$AT = 1 \qquad \therefore A = \frac{1}{T}$

$A+BT = 0 \qquad \therefore B = -\frac{1}{T^2}$

$B+C = 0 \qquad \therefore C = \frac{1}{T^2}$

$$C(s) = \frac{\frac{1}{T}}{s^2} - \frac{1/T^2}{s} + \frac{1/T^2}{s+T}$$

Taking inverse Laplace Transform

$$c(t) = \left(\frac{1}{T}\right)t - \frac{1}{T^2} + \frac{1}{T^2}e^{-Tt}$$

$L[t^n] =$

$L[$

$\frac{1}{s^2(s+T)} = \frac{A}{s^2}$

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Time response analysis of first order s/m to unit ~~impulse~~ ramp input

$$C.L\ T.F = \frac{1}{1+Ts}$$

Multiply num & den by $\frac{1}{T}$ so that pole has no term with

$$\frac{C(s)}{R(s)} = \frac{1/T}{\left(s+\frac{1}{T}\right)}$$

$$C(s) = \frac{1/T}{s^2\left(\frac{Ts}{T}+\frac{1}{T}\right)}$$

Expanding into partial fractions

$$C(s) = \frac{1/T}{s^2\left(s+\frac{1}{T}\right)}$$

$$\frac{1/T}{s^2\left(s+\frac{1}{T}\right)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+T}$$

$$\frac{1/T}{s^2\left(s+\frac{1}{T}\right)} = \frac{B}{s^2}$$

$$B = s^2\left[\frac{\frac{1}{T}}{s^2\left(s+\frac{1}{T}\right)}\right]\Bigg|_{s=0}$$

$$= \frac{1/T}{s+1/T}\Bigg|_{s=0}$$

$$= \frac{1/T}{1/T} = 1$$

$$\frac{1/T}{s^2\left(s+\frac{1}{T}\right)} = \frac{A}{s}$$

$$A = s\left[\frac{1/T}{s^2(s+1/T)}\right]\Bigg|_{s=0}$$

$$= \frac{1}{1!}\frac{d}{ds}\left[\frac{\frac{1}{T}}{s+1/T}\right]\Bigg|_{s=0}$$

$$A = -T$$

$$= \frac{1/T}{s^2(s+\frac{1}{T})} = \frac{C}{(s+1/T)}$$

let $s = -1/T$

$(\text{C3})\quad C = T$

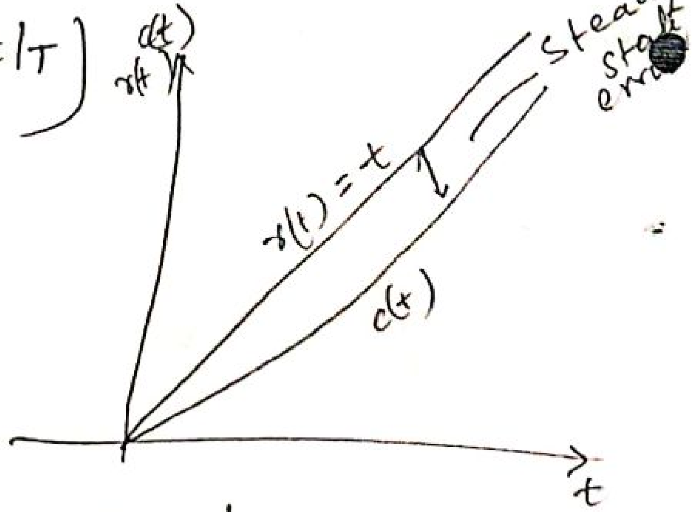
$$\therefore C(s) = -\frac{T}{s} + \frac{1}{s^2} + \frac{T}{s+1/T}$$

$$= \frac{1}{s^2} - T/s + \frac{T^2}{Ts+1}$$

Taking the inverse laplace transform

ramp o/p $r(t) = t\,u(t)$

$$c(t) = t - T\left(1 - e^{-t/T}\right)$$



The error signal is

$$e(t) = r(t) - c(t)$$
$$= T\left(1 - e^{-t/T}\right)$$

The steady state error is given by

$$e_{ss} = \lim_{t \to \infty} e(t) \quad = \underline{T}$$

The first order slm under consideration will track the unit ramp o/p with a steady state error T, which is equal to the time constant T. of the slm unit

Time response analysis of first order slm to impulse input

$$CLTF = \frac{1}{1+TS}$$

$$\frac{C(s)}{R(s)} = \frac{1/T}{S + \frac{1}{T}}$$

Unit impulse function $R(s) = 1$

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$$C(s) = R(s) \frac{1/T}{s + 1/T}$$

$$C(s) = 1 \times \frac{1/T}{s + 1/T}$$

$$C(s) = \frac{1/T}{s + 1/T}$$

taking inverse laplace transform of C(s)

$$c(t) = 1 - e^{-t/T}$$

$$e(t) = r(t) - c(t)$$

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$= \lim_{t \to \infty} (r(t) - c(t))$$

$$e_{ss} = \lim_{t \to \infty} r(t) - \lim_{t \to \infty} c(t)$$

$$e_{ss} = 0$$

Time response or Analysis of Seconde order s/m.

Every practical s/m takes finite time to reach to its steady state & during this period, it oscillates or incre exponentially. The behaviour of s/m gets decided by type of closed loop poles & locations of closed loop pole in s-plane. Every s/m has a tendency to oppose the oscillatory behaviour of the s/m which is called damping

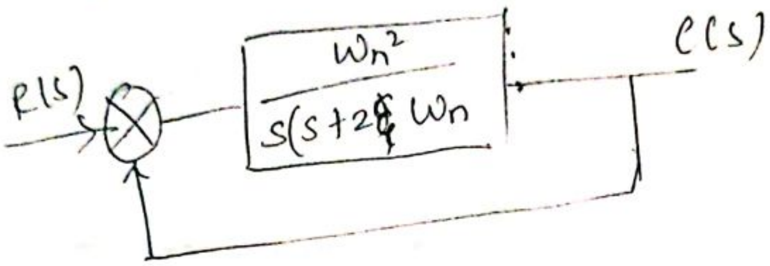
This damping is measured by a factor or a ratio called damping ratio of the s/m.
This factor explains us, how much dominant the opposition from the s/m is to the oscillations in the o/p.
This damping ratio is denoted by ξ (zeta)

The natural frequency (ω_n) is the freq of oscillation of the s/m without damping. It is denoted by ω_n rad/sec.

If there is no opposition from the s/m, s/m naturally & freely oscillates under $\xi = 0$ condition. hence this freq of oscillations is called natural freq of oscillations of the s/m. denoted by (ω_n)

The response $c(t)$ of the second-order s/m depends on the value of 'ξ'

Now consider a standard second-order s/m as shown.



The closed loop transfer function

$$CLTF = \frac{C(s)}{R(s)} = \frac{\omega_n^2 / s(s+2\xi\omega_n)}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$ — standard 2nd order s/m.

where ω_n = undamped natural freq of osillations

ξ = damping ratio

Effect of ξ on Second order slm performance.

consider o/p applied to the standard 2nd order slm is unit step.

$$R(s) = \frac{1}{s}$$

$$CLTF = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Denominator $s^2 + 2\xi\omega_n s + \omega_n^2 \rightarrow$ characteristic polynomial.

When
To finding the roots of the equation equate this eqn to zero

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$ This is called charct eqn

$1 + G(s)H(s) = 0$

ξ

$\frac{C(s)}{R(s)}$ = roots of the charct eqn.

Two roots of the charct equations are

$$s_1, s_2 = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$a = 1$
$b = 2\xi\omega_n$
$c = \omega_n^2$

$$s_1 \,\&\, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$ (Taking 2 common

$$\boxed{s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{\left(s+\xi\omega_n+\omega_n\sqrt{\xi^2-1}\right)\left(s+\xi\omega_n-\omega_n\sqrt{\xi^2-1}\right)}$$

Nature of the three roots is dependent on damping ratio ξ. consider the following cases.

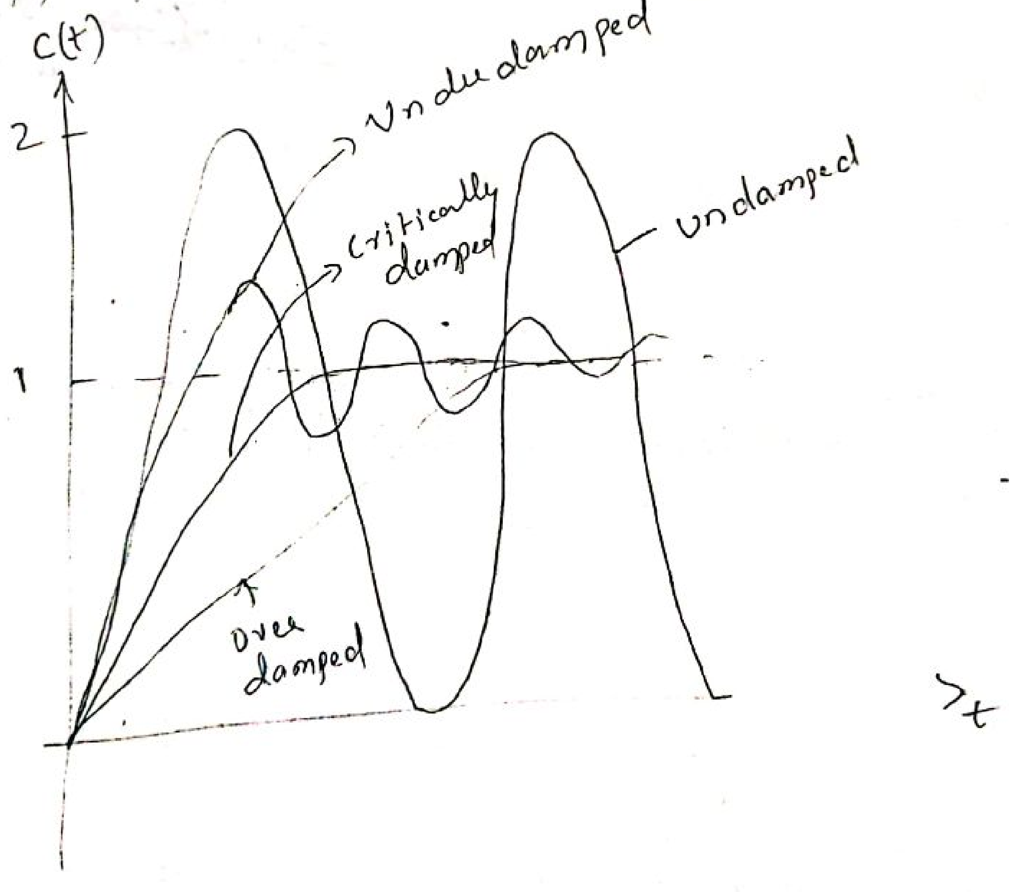
1) Over damped $(\xi > 1)$

1) un damped $(\xi = 0$

2) Under damped $(0 < \xi < 1)$

3) critically damped $(\xi = 1)$

4) Over damped $(\xi > 1)$



case(1) ~~Over damped~~ Undamped $(\xi = 0)$

The roots are

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$s_1, s_2 = 0 \pm \omega_n\sqrt{0-1}$$

$$\boxed{s_1, s_2 = \pm j\omega_n}$$ The roots are complex conjugate

complex conjugates with zero real part i.e, purely imaginary.

The response is ~~purely~~ only oscillations with constant freq & amplitude & no damping. This type of slms are ~~It is~~ called Undamped slms.

$$\frac{\omega_n^2}{s(s+j\omega_n)(s-j\omega_n)} = \frac{\omega_n^2}{s(s^2+\omega_n^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+\omega_n^2}$$

$$C_{ss} + K \cdot \sin(\omega_n t + \theta)$$

case ii) Underdamped slm $(\xi < 1)$

The roots are

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

As $\xi < 1$ the term $\sqrt{\xi^2-1}$ is written as $j\sqrt{1-\xi^2}$

$$\boxed{s_1, s_2 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}}$$

Hence roots are complex conjugate with negative real part

This response is oscillatory with oscillating freq $\omega_n\sqrt{1-\xi^2}$ but decreasing amplitude.

$$C_{ss} + K \cdot e^{-\xi\omega_n t}\sin(\omega_n\sqrt{1-\xi^2}\,t + \theta)$$

Such osillations are called Damped Osillations (20)

& freq of such osillations is called damped

freq of oscillation (ω_d)

$$\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}}$$

Case iii) $\xi = 1$ (critically damped)

The roots are

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$\xi = 1$

$$s_1, s_2 = -\omega_n \pm \omega_n\sqrt{1-1}$$

$$s_1, s_2 = -\omega_n$$

real term with negative sign

is called critically critically damped.

Since there is no imaginary part no oscillations

only damped

Case (iv) over damped ($\xi > 1$)

roots are $s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$

For our understanding ~~(Example)~~ assume $\xi = 2$

$$s_1, s_2 = -2\omega_n \pm \omega_n\sqrt{3}$$

$= -2\omega_n \pm 1.73\omega_n$

$S_1 = -3.73\omega_n$ $\quad S_2 = -0.27\omega_n$

The roots are distinct & real.

Summary

range of ξ	Types of CL poles	Nature of response	s/m classification
$\xi = 0$	~~Pure~~ only imaginary	oscillations with constant freq & Amplitude	Undamped
$0 < \xi < 1$	Complex conjugate with -ve real part	Damped oscillations	Underdamped
$\xi = 1$	Real part & -ve	Critical & pure exponential	Critically damped
$1 < \xi < \infty$	Real & unequal & -ve	purely exponential Slow and sluggish	Over damped.

Time response analysis of 2nd order s/m for unit step input

or

Derivation of unit step response of a 2nd order s/m (underdamped case)

CLTF of 2nd order s/m is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

For unit step o/p $r(t) = u(t)$

$$R(s) = \frac{1}{s}$$

$$C(s) = R(s)\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

$$= \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$C(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n^2 = A(s^2 + 2\xi\omega_n s + \omega_n^2) + (Bs + C)s$$

Put $s = 0$

$$\omega_n^2 = A(0 + 0 + \omega_n^2) + 0$$

$$\omega_n^2 = A(\omega_n^2)$$

$$\boxed{A = 1}$$

Equating of s^2 terms,
(equating coefficients of s^2)

$$A + B = 0$$

$$A = -B$$

$$\boxed{B = -1}$$

Equating of s term

$$A\,2\xi\omega_n + C = 0$$

$$C = -A\,2\xi\omega_n$$

$$\boxed{C = -2\xi\omega_n}$$

splitting it as

$$C(s) = \frac{1}{s} - \frac{-1s - 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$2\xi\omega_n = \xi\omega_n + \xi\omega_n$

$$\cancel{C(s) = \frac{1}{s} - \frac{(s + 2\xi\omega_n)}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$ - splitting it as

$$\cancel{C(s) = \frac{1}{s} - \frac{(s + \xi\omega_n)}{\quad}}$$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$ (Taking -ve sign outside)

$= \left(\frac{1}{s} - \frac{s+\xi\omega_n+\xi\omega_n}{\cancel{}}\right.$

Splitting $2\xi\omega_n = \xi\omega_n + \xi\omega_n$ into 2

$$= \frac{1}{s} - \frac{s+\xi\omega_n+\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2+\xi^2\omega_n^2-\xi^2\omega_n^2}$$

$\boxed{\text{adjusting the denominator as}}$

$$= \frac{1}{s} - \frac{(s+\xi\omega_n)+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)}$$

$(a+b)^2 = a^2+2ab+$
$a = s \quad b = \xi\omega$

let $\omega_d = \omega_n\sqrt{1-\xi^2}$ where ω_d = Damped freq of oscillation

$\omega_d = \omega_n\sqrt{1-\xi^2}$
$\omega_d^2 = \omega_n^2(1-$

$$c(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$c(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

Multiply & divided by $\sqrt{1-\xi^2}$ we get

$$c(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n\sqrt{1-\xi^2}}{\sqrt{1-\xi^2}\left((s+\xi\omega_n)^2+\omega_d^2\right)}$$

$$c(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi}{\sqrt{1-\xi^2}}\left(\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}\right)$$

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$\omega_d = \omega_n\sqrt{1-\xi^2}$

WKT Factor

$$L^{-1}\left[\frac{s+a}{(s+a)^2+b^2}\right] = e^{-at}\cos bt$$

$$L^{-1}\left[\frac{b}{(s+a)^2+b^2}\right] = e^{-at}\sin bt$$

$$L^{-1}\left[\frac{1}{s}\right] = 1$$

from eqn 1

where $a = \xi\omega_n$ $b = \omega_d$

Taking inverse LT of eqn 1 we get

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_d t$$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right)} \quad -(2)$$

WKT

$$A\cos\theta + B\sin\theta = \sqrt{A^2+B^2}\sin(\theta+\phi)$$

where $\phi = \tan^{-1}(A/B)$

comparing this with eqn 2 we get

$A = \sqrt{1-\xi^2}$ $B = \xi$ & $\theta = \omega_d t$

$$\boxed{c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)}$$

~~where $\phi = \tan^{-1}$~~

$$\boxed{\text{where } \phi = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}}$$ radius.

steady state error(e_{ss})

$e_{ss} = \lim_{t \to \infty} e(t)$ $e(t) = r(t) - c(t)$

$r(t) = 1$

$\lim_{t \to \infty} \left(1 - \left(1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)\right)\right)$ $c(t) = 1 - e^{-\xi \omega_n t} \frac{\sin(}{\sqrt{1-\xi^2}}$

put $t = \infty$

$$\boxed{e_{ss} = 0}$$

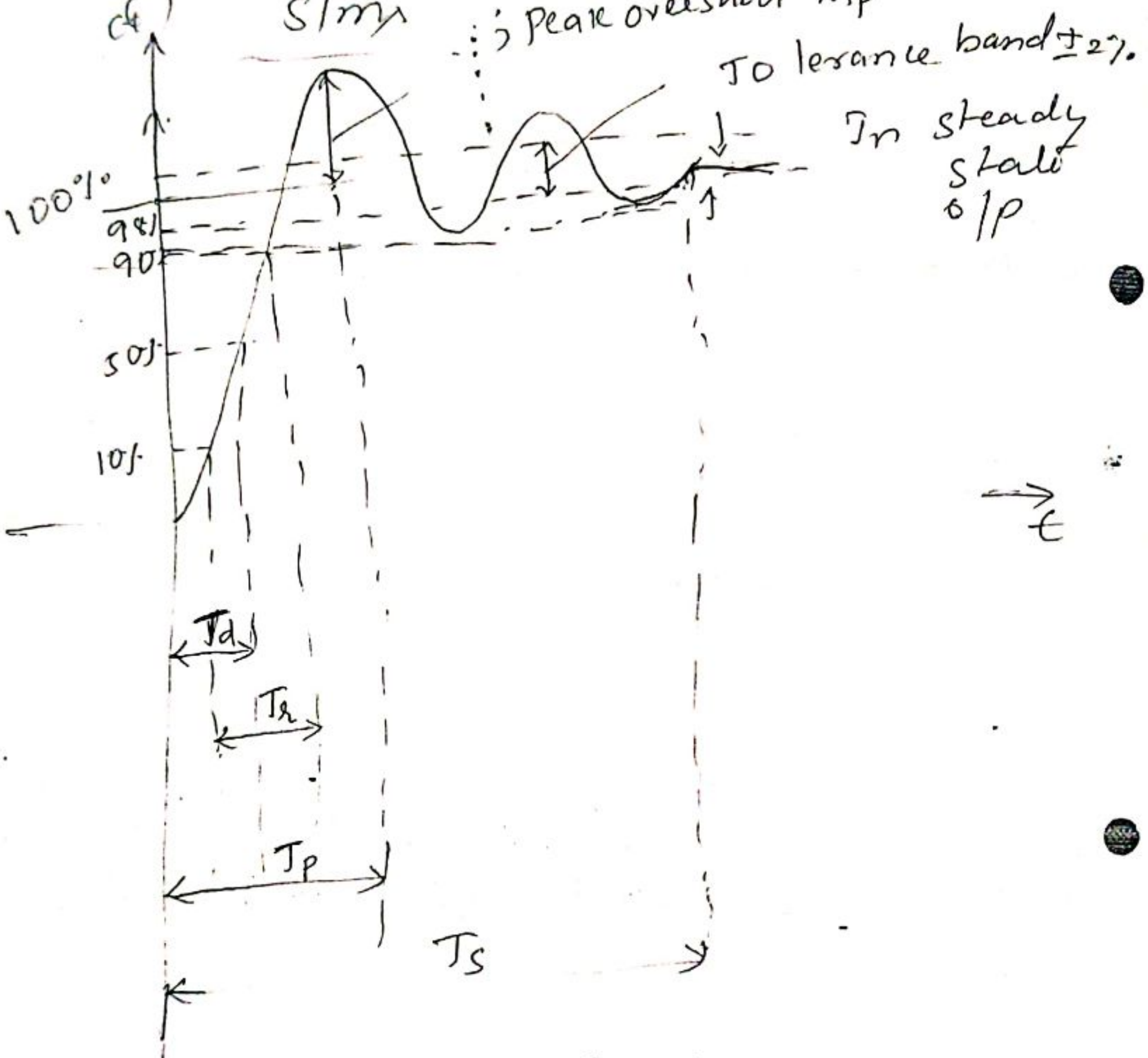
Since $e_{ss} = 0$ the 2nd order s/m is capable of responding for unit step input without any steady state error.

~~exponential de~~
The response is exponentially decaying sinusoidal waveform.

exponential decay depends on value of zeta(ξ)

larger value of zeta (ξ) the exponential decay is more & reaches steady state faster

Transient Response Specifications of 2nd order S/m



Transient response Specifications

The various time response specifications are.

1) Delay Time Td :- It is the time taken by the S/m response to reach 50% of the final value in the first attempt. It is given by

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Delay time :- It is the time required for the response to reach 50% of the final value in the first attempt. It is given by

$$\boxed{T_d = \frac{1+0.7\xi}{\omega_n}}$$

2) Rise time Tr :- It is the time required for the response to rise from 10% to 90% of the final value for overdamped s/ms & 0 to 100% of the final value for underdamped s/ms.

It is given by

$$\boxed{T_r = \frac{\pi - \theta}{\omega_d} \text{ sec}}$$ where θ must be in radians.

(3) Peak time Tp :- [It is the time required for the response to reach its peak value] It is also defined as the time at which response undergoes the first overshoot which is always peak overshoot

$$\boxed{T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ sec}}$$

4) <u>Peak over shoot M_p</u> :- It is the largest error b/w reference o/p & output during the transient period

$$M_p = \{ c(t) |_{t = T_p} \} - 1 \quad 1 \text{ for unit step o/p}$$

$$\% M_p = e^{-\pi \xi / \sqrt{1-\xi^2}} \times 100$$

5) <u>Settling Time T_s</u> :- This is defined as the time required for the response to decrease & stay within specified percentage of its final value (within tolerance band)

$$\text{Time constant of s/m} = \frac{1}{\xi \omega_n} = T$$

$$T_s = 4 \times \text{Time constant}$$

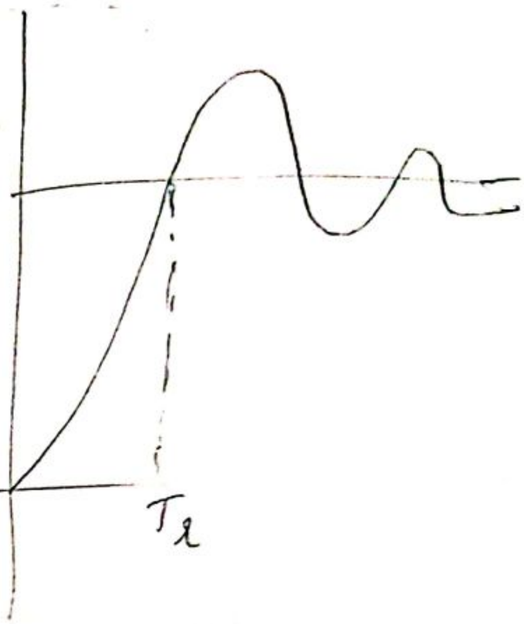
Practically T_s is assumed to be 4 times.

$$\boxed{T_s = \frac{4}{\xi \omega_n}}$$

Derivations of Time domain Specifications

Derivation of T_r (rise time)

Time required by o/p to achieve 100% of its final value starting from zero during the first attempt.



The transient response of 2nd order s/m underdamped s/m is given by

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

at $t = t_r$ $c(t_r) = 1$

$\{ c(t) |_{t = T_r} \} = 1$ for unit step i/p

$$\cancel{1} = \cancel{1} - \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)$$

$$- \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi) = 0$$

to becomes LHS zero $\left(- \frac{e^{\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \right)$ cannot be zero. exponential term cannot be equal to zero so

$$\sin(\omega_d t_r + \phi) = 0$$

$$\omega_d t_r + \phi = n\pi \quad \text{where } n = 1, 2 \text{ ---}$$

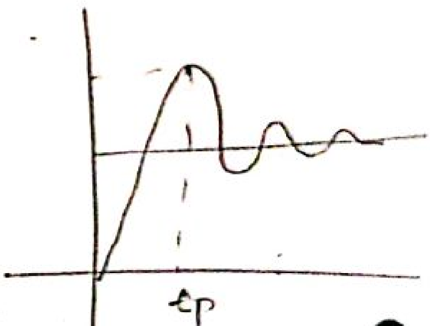
$$t_r = \frac{n\pi - \phi}{\omega_d} \quad \text{~~When~~ put } n = 1$$

$$\boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$ see

② Peak time t_p ~~or Maximum over shoot~~

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

where $\theta = \tan^{-1} \dfrac{\sqrt{1-\xi^2}}{\xi}$



As at $t = T_p$ $c(t)$ will achieve its maxima.

According to Maxima theorem $\left.\dfrac{dc(t)}{dt}\right|_{t=T_p} = 0$

so diff w.r.to t we get

$$-\frac{e^{-\xi\omega_n t}(-\xi\omega_n)\sin(\omega_d t + \theta)}{\sqrt{1-\xi^2}} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\omega_d \cos(\omega_d t + \theta) = 0$$

Substitute $\omega_d = \omega_n\sqrt{1-\xi^2}$

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$$\frac{\xi \omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t+\theta) - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \omega_n \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0$$

$$\therefore \xi \sin(\omega_d t+\theta) - \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0$$

$$\therefore \tan(\omega_d t+\theta) = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} \qquad \frac{\sqrt{1-\xi^2}}{\xi} = \tan\theta$$

$$\therefore \tan(\omega_d t+\theta) = \tan\theta$$

$\tan(n\pi+\theta) = \tan\theta$ from trigonometric formula

$\omega_d t = n\pi$ where $n = 1, 2, 3$ - - -

T_p, time required for first peak overshoot $\therefore n=1$

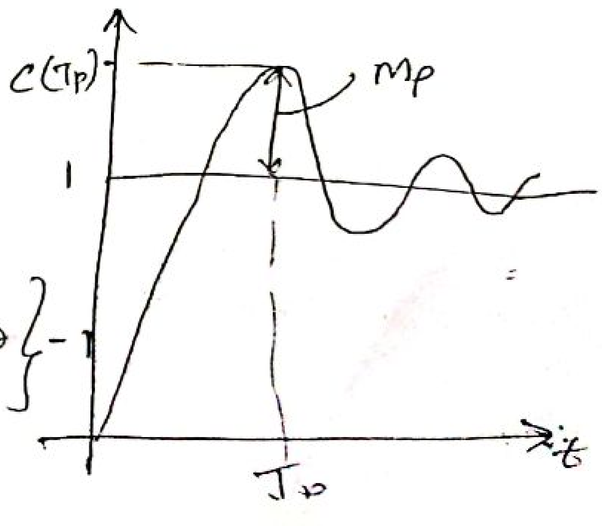
$$\omega_d T_p = \pi$$

$$\boxed{T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ sec}}$$

(3) Derivation of M_p

From fig $M_p = c(T_p) - 1$

$$M_p = \left\{1 - \frac{e^{-\xi\omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\omega_d T_p + \theta)\right\} - 1$$



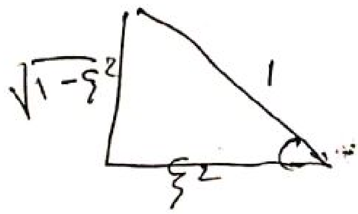
$$M_p = -\frac{e^{-\xi \omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\omega_d T_p + \theta)$$

$T_p = \frac{\pi}{\omega_d}$, Substituting

$$M_p = -\frac{e^{-\xi \omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\pi + \theta) -$$

Now $\sin(\pi + \theta) = -\sin(\theta)$

$$M_p = \frac{e^{-\xi \omega_n T_p}}{\sqrt{1-\xi^2}} \sin\theta$$



We know $\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$

$\sin\theta = \frac{\text{opp side}}{\text{hypotenuse}}$ hypotenuse $= \sqrt{(\sqrt{1-\xi^2})^2 + \xi^2} = 1$

$\sin\theta = \sqrt{1-\xi^2}$

$$M_p = \frac{e^{-\xi \omega_n T_p}}{\cancel{\sqrt{1-\xi^2}}} \cdot \cancel{\sqrt{1-\xi^2}}$$

$M_p = e^{-\xi \omega_n T_p}$ $\quad T_p = \frac{\pi}{\omega_d}$

$M_p = e^{-\xi \omega_n \frac{\pi}{\omega_n \sqrt{1-\xi^2}}}$ $\quad \omega_d = \omega_n \sqrt{1-\xi^2}$

$$\boxed{\% M_p \; e^{-\pi\xi / \sqrt{1-\xi^2}} \times 100}$$

Derivation of T_s

The settling time T_s is the time required by the o/p to settle down within $\pm 2\%$ of tolerance band. so T_s is the time when o/p becomes 98% of its final value & remains within the range of $\pm 2\%$ as $t \to \infty$

$\therefore \quad c(t) = \text{at } (t = T_s) = 0.98$

Now at $t = T_s$ the transient oscillatory term completely vanishes. The only term which controls the amplitude of the output within $\pm 2\%$ is $e^{-\xi \omega_n t}$

$$\therefore \quad c(t) \text{ at } (t = T_s) = 1 - e^{-\xi \omega_n T_s}$$

$$0.98 = 1 - e^{-\xi \omega_n T_s}$$

$$e^{-\xi \omega_n T_s} = 1 - 0.98$$

$$e^{-\xi \omega_n T_s} = 0.02$$

Taking ln

$$-\xi \omega_n T_s = \ln(0.02)$$

$$-\xi \omega_n T_s = -3.912$$

$$T_s = \frac{3.912}{\xi \omega_n}$$

In practice the T_s is assumed to be :

$$\boxed{T_s = \frac{4}{\xi \omega_n}}$$ for 2% Tolerance

11/1/4 for 5% of Tolerance band

$c(t)$ at $(t = T_s) = 0.95$

$$0.95 = 1 - e^{-\xi \omega_n T_s}$$

$$\boxed{T_s = \frac{2.995}{\xi \omega_n} = \frac{3}{\xi \omega_n}}$$

ω_d?

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$\boxed{T_d = \frac{1+0.7\xi}{\omega_n}}$$

$$T_r = \frac{\pi - \theta}{\omega_d}$$ θ : must be radians

$$\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ sec}$$

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$T_s = \dfrac{4}{\xi \omega_n}$ for 2% tolerance band

$= \dfrac{3}{\xi \omega_n}$ for 5% — " —

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Problems

(1) Determine the T_r, T_p, ξ, T_s for the second order system having $\xi = 0.6$ $\omega_n = 5\ rad/sec$ when selected to unit step input

Sol^n

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$\boxed{\omega_d = 4\ rad/sec}$

where $\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

- $T_r = \frac{1}{\omega_d}\left[\pi - \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$

$= \frac{1}{4}\left[\pi - \tan^{-1}\left[\frac{\sqrt{1-0.6^2}}{0.6}\right]\right]$

$\boxed{T_r = 0.5536\ sec}$

- $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{4} = 0.7854\ sec.$

$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$= 0.09478$

$T_s = \frac{4}{\xi\omega_n}$ for 2% Tolerance

$T_s = \frac{3}{\xi\omega_n}$ for 5% —"—

$T_s = \frac{3}{\sigma} = \frac{3}{\xi\omega_n}$ 5% Tolerance

$T_s = \frac{3}{0.6(5)} = 1s$

$T_s = \frac{4}{0.6(5)} = 1.33\ sec$

(3) A unity feed back system is characterised by $G(s) = \frac{K}{s(s+10)}$ (3)
find the value of K, so that the system will have damping ratio of $(\xi) = 0.5$. for this value of K determine T_s, M_p, T_p for unit step input

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+10)} = 0$$

$$s^2 + 10s + K = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = K$$

$$\omega_n = \sqrt{K}$$

$$2\xi\omega_n = 10$$

$$2\xi\sqrt{K} = 10$$

$$\sqrt{K} = \frac{10}{2(0.5)}$$

$$= 10$$

$$\sqrt{K} = 10$$

$$K = 10^2$$

$$\boxed{K = 100}$$

$$\omega_n^2 = 100$$

$$\omega_n = 10 \text{ rad/sec}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\boxed{T_p = 0.362 \text{ sec}}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$M_p = 0.163$$

$$\% M_p = 16.3\%$$

$$T_s = \frac{3}{\xi\omega_n} = \frac{3}{(0.5)(10)} = 0.6 \text{ for } 5\%$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{5}{5} = 0.8 \text{ for } 2\%$$

3) A s/m has 30% overshoot & settling time as 5 sec for unit step input, determine 1) CLTF, 2) peak time 3) output response (assume 2% Tolerance band for settling time)

Sol^n

$M_p = 0.3 \quad T_s = 5\,sec$

$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$

$0.3 = e^{-\xi\pi/\sqrt{1-\xi^2}}$

Taking ln log on both sides

$\log(0.3) = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$

$-1.2 = \frac{-\xi\pi}{\sqrt{1-\xi^2}}$

$\cancel{-}0.38 = \cancel{+}\frac{\xi}{\sqrt{1-\xi^2}}$

$(0.38)^2 = \frac{\xi^2}{(1-\xi^2)}$

$0.144 = \frac{\xi^2}{1-\xi^2}$

$0.144 - 0.144\xi^2 = \xi^2$

$\xi^2 = \frac{0.144}{1.144}$

$\xi^2 = 0.1$

$\boxed{\xi = 0.3568}$

0-3162

2% Tolerance.

$T_s = \frac{4}{\alpha} = \frac{4}{\xi\omega_n} = 2\%$

$\frac{4}{(0.3568)\omega_n} =$

$\boxed{\omega_n = 2.242\ rad/sec}$

$CLTF = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$= \frac{5.027}{s^2 + 1.599s + 5.02}$

$\boxed{CLTF = \frac{5.027}{s^2 + 1.6s + 5.027}}$

$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$\boxed{T_p = 1.5\,sec}$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = 1.2$$

$$\sqrt{1-\xi^2} = 0.932$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 2.1 \text{ rad/sec}$$

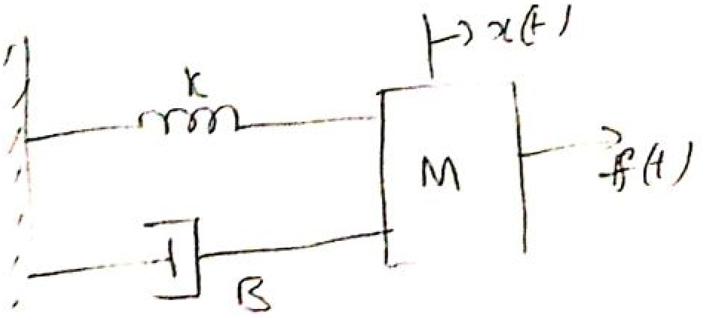
$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{0.932} \sin(2.1t + 1.2)$$

$$\boxed{c(t) = 1 - 1.07\,e^{-0.8t} \sin(2.1t + 1.2)}$$

(4) Refer to the figure & find the following

a) $T.F = \frac{X(s)}{F(s)}$ ii) ξ, ω_n, $\%M_p$, T_s & T_p

assume $K = 33$ N/M $B = 15$ N-sec/m $M = 3$ kg



Dynamic eqn of Motion

$$f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + K\,x(t)$$

While writing Dynamic eqn of K we should write K instead of $\frac{1}{K}$

Taking L.T

$$F(s) = Ms^2 X(s) + BsX(s) + kX(s)$$

$$F(s) = (Ms^2 + Bs + K)X(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}}$$

characteristic eq^n

$$Ms^2 + Bs + K = 0$$

$\div M$ $\quad s^2 + \frac{B}{M}s + \frac{K}{M} = 0$

WKT

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$2\xi\omega_n = \frac{B}{M} \qquad \xi = \frac{B}{2M\omega_n} = \frac{15}{2(3)(3.316)}$$

$$= 0.753$$

$$\omega_n^2 = \frac{K}{M}$$

$$\omega_n = \sqrt{K/M} = \frac{33}{3} = \sqrt{11}\ \text{rad/sec} = 3.316\ \text{rad/sec}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{\frac{-(0.75\pi)}{\sqrt{1-(0.75)^2}}} = 0.0283 = \boxed{2.74\%}$$

$$= 2.83\%$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{3.3166\sqrt{1-(0.75)^2}} = 1.140\ \text{sec}$$

$$1.43\ \text{sec}$$

5% Tolerance

$$T_s = \frac{3}{\cancel{\sigma}} = \frac{3}{\xi\omega_n} = \frac{3}{(0.75)(3.316)} = 1.206\ \text{sec} = \boxed{1.63\ \text{sec}}$$

$$4\ \ = 1.608\ \text{sec} = \boxed{1.201\ \text{sec}}$$

2% tolerance

(5) For a servomechanism system with $G(s) = \frac{K_1}{s^2}$ & $H(s) = 1+K_2 s$, determine the value of K_1 & K_2, so that the peak overshoot to a unit step response is 0.25 & peak time is 2 seconds.

Sol^n

Given $M_p = 0.25$ & $T_p = 2$ sec.

WKT T.F $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K_1/s^2}{1+\left(\frac{K_1}{s^2}\right)(1+K_2 s)}$

$$= \frac{K_1}{s^2 + K_1 K_2 s + K_1}$$

$$\frac{K/s^2}{\cancel{s^2}\left(1+\frac{K_1 + K_1 K_2 s}{s^2}\right)}$$

$$\frac{K/\cancel{s^2}}{\frac{s^2 + K_1 + K_1 K_2}{\cancel{s^2}}}$$

$$\frac{K_1}{s^2 + K_1 K_2 s + K_1}$$

Comparing this with $s^2 + 2\xi\omega_n + \omega_n^2$ we get

$\omega_n^2 = K_1 \Rightarrow \omega_n = \sqrt{K_1}$

$2\xi\omega_n = K_1 K_2 \Rightarrow \xi = \frac{1}{2}\frac{K_1 K_2}{\omega_n}$

$$= \frac{1}{2}\frac{\cancel{\sqrt{K_1}}\sqrt{K_1}\,K_2}{\cancel{\sqrt{K_1}}}$$

$$= \frac{1}{2}\sqrt{K_1}\,K_2$$

Given

WKT $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$ $\therefore 0.25 = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$0.25 = e^{-\pi\xi/\sqrt{1-\xi^2}}$

Taking Natural log on both sides

$\therefore$ $\boxed{\xi = 0.4037}$

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WKT $T_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

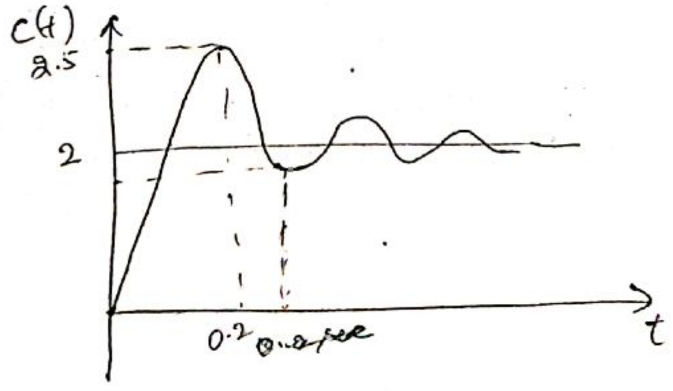
$$2 = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\omega_n = \frac{\pi}{2\sqrt{1-\xi^2}} = 1.7169 \text{ rad/sec}$$

$$K_1 = \omega_n^2 = 2.9478.$$

$$K_2 = \frac{2\xi\omega_n}{K_1} \quad \text{or} \quad \frac{2\xi}{\sqrt{K_1}} = \underline{\underline{0.4702}}$$

(b) Find the open loop transfer function & closed loop T.F of the equivalent prototype, single loop unity feedback s/m of second order, whose step response is as shown in fig.



Given $M_p = 2.5$ $\quad T_p = 0.2$ sec

W.K.T

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$2.5 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$\ln(2.5) = -\pi\xi/\sqrt{}$

$$\boxed{\xi = 0.30}$$

WKT. $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$ $\qquad$ $T_p = 0.2$ sec given

$$0.2 = \frac{\pi}{\omega_n\sqrt{1-(0.3)^2}}$$

$$\omega_n = \underline{16.46}$$

Open loop T.F $\quad G(s) = \dfrac{T(s)}{1-T(s)}$

$$\text{C.L.T.F} : \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$= \frac{(16.46)^2}{s^2+2\times 0.3\times 16.46s+(16.46)^2}$$

$$\boxed{T(s) = \text{CLTF} = \frac{C(s)}{R(s)} = \frac{270.93}{s^2+9.876s+270.93}}$$

$$\text{OLTF} \quad G(s) = \frac{T(s)}{1-T(s)} = \frac{\dfrac{270.93}{s^2+9.876s+270.93}}{1-\dfrac{270.93}{s^2+9.876s+270.93}}$$

$$\boxed{G(s) = \frac{270.93}{s^2+9.876s}}$$

7) The OLTF of a unity FBCS is given by

$$G(s) = \frac{K}{s(sT+1)}$$

i) By what factor the amp gain K should be multiplied so that damping ratio is increased from 0.2 to 0.8

ii) By what factor the time constant T should be multiplied so that the damping ratio is reduced from 0.6 to 0.3

Soln

Given $G(s) = \frac{K}{s(sT+1)}$ & $H(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(sT+1)}}{1+\frac{K}{s(sT+1)}}$$

$$= \frac{K}{Ts^2+s+K} = \frac{K/T}{s^2+\frac{s}{T}+\frac{K}{T}}$$

comparing with standard 2nd order s/m

$$\omega_n^2 = \frac{K}{T} \quad \& \quad 2\xi\omega_n = \frac{1}{T}$$

$$\omega_n = \sqrt{\frac{K}{T}} \quad \& \quad \xi = \frac{1}{2\sqrt{KT}}$$

case i) $\xi_1 = 0.2$ $K = K_1$ while $\xi_2 = 0.8$ $K_2 = K_2$ T = constant

$$\therefore 0.2 = \frac{1}{2\sqrt{K_1T}} \quad \& \quad 0.8 = \frac{1}{2\sqrt{K_2T}}$$

$$\frac{0.2}{0.8} = \frac{\frac{1}{2\sqrt{K_1T}}}{1/2\sqrt{K_2T}} = \frac{\sqrt{K_2}}{\sqrt{K_1}}$$ squaring both sides

$$\frac{1}{16} = \frac{K_2}{K_1}$$

$$\therefore K_2 = \frac{1}{16}K_1$$

So K must be multiplied by 1/16 to increase ξ from 0.2 to 0.8

case ii) $\xi_1 = 0.6$, $T = T_1$ while $\xi_8 = \xi_2$ $T = T_2$ K = constant

$\therefore 0.6 = \frac{1}{2\sqrt{K T_1}}$ & $0.3 = \frac{1}{2\sqrt{K T_2}}$

$\therefore \frac{0.6}{0.3} = \frac{\sqrt{T_2}}{\sqrt{T_1}}$ $\therefore 2 = \sqrt{\frac{T_2}{T_1}}$

$\therefore 4 = \frac{T_2}{T_1}$

$T_2 = 4T_1$

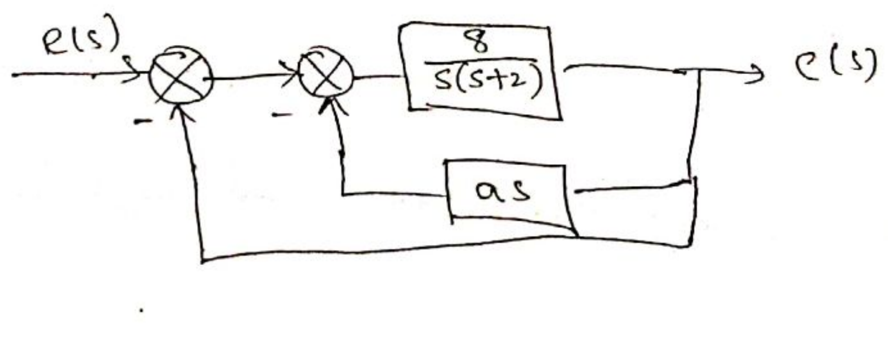
So T must be multiplied by 4 to reduce ξ from 0.6 to 0.3.

(8) The slm given in figure is a unity f/B slm with minor feedback loop.

i) In the absence of derivative feedback (a=0), determine the damping ratio ξ undamped natural freq.

ii) Determine the constant 'a' which will increase damping ratio to 0.7

iii) Find the overshoot in both the cases



case i) when $a = 0$

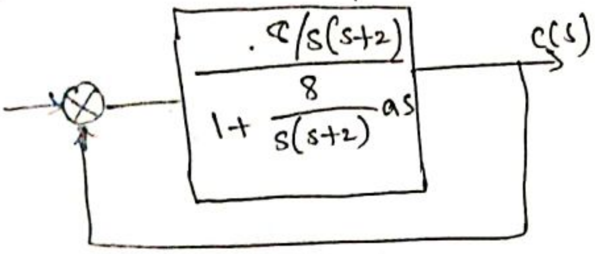
$$G(s) = \frac{8}{s(s+2)} \quad \& \quad H(s) = 1$$

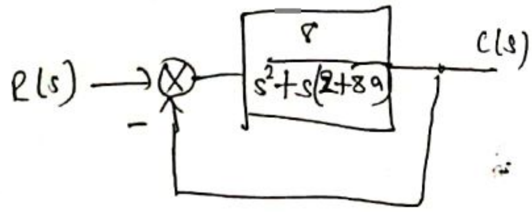
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{8}{s(s+2)}}{1+\frac{8}{s(s+2)}} = \frac{8}{s^2+2s+8}$$

$\omega_n^2 = 8$ $\qquad 2\xi\omega_n = 2$

$\omega_n = \sqrt{8}$ rad/sec $\qquad \xi = 0.3535$

ii) ~~at~~ with 'a' present, the s/m can be reduced as





$$\frac{C(s)}{R(s)} = \frac{\frac{8}{s^2+s(2+8a)}}{1+\frac{8}{s^2+s(2+8a)}} = \frac{8}{s^2+s(2+8a)+8}$$

$\omega_n^2 = 8 \quad \& \quad 2\xi\omega_n = 2+8a$

$\omega_n = \sqrt{8} \quad \& \quad \xi = \frac{2+8a}{2\sqrt{8}}$

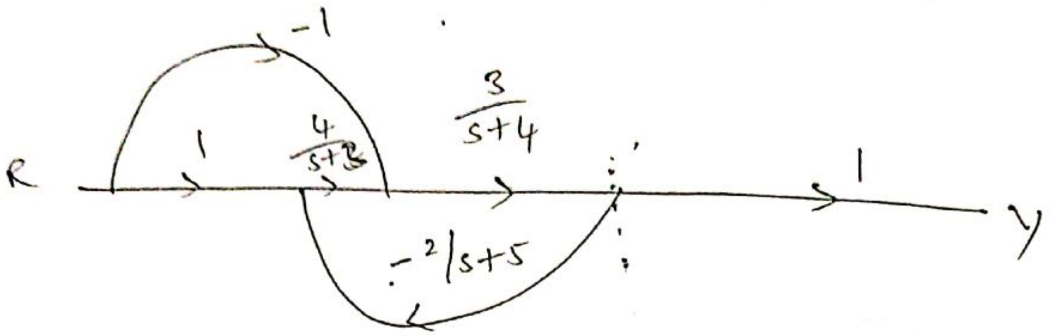
$$\therefore \frac{2+8a}{2\sqrt{8}} = 0.7$$

$a = 0.2449$ for $\xi = 0.7$

iii) $\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 30.507\%$ for $\xi = 0.3535$, $a = 0$

$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 4.59\%$

9) For the SFG shown in fig. Mention the Type number & order of the sm & determine the steady state errors for step & ramp inputs. $e(t) = r(t) - y(t)$



For the given SFG

$$P_1 = \frac{4}{s+3} \times \frac{3}{s+4} = \frac{12}{(s+3)(s+4)}$$

$$P_2 = -1 \times \frac{3}{s+4} = -\frac{3}{s+4}$$

$$L_1 = \frac{4}{s+3} \times \frac{3}{s+4} \times \frac{-2}{s+5} = \frac{-24}{(s+3)(s+4)(s+5)}$$

$$\Delta = 1 - L_1 = 1 + \frac{24}{(s+3)(s+4)(s+5)}$$

$\Delta_1 = \Delta_2 = 1$ as L_1 touches to both the forward paths

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{\frac{12}{(s+3)(s+4)} - \frac{3}{(s+4)}}{1 + \frac{24}{(s+3)(s+4)(s+5)}}$$

$$T(s) = \frac{\frac{12 - 3(s+3)}{(s+3)(s+4)(s+5) + 24}}{(s+3)(s+4)(s+5)} = \frac{3(1-s)(s+5)}{s^3 + 12s^2 + 47s + 84}$$

For unity feedback

$$T(s) = \frac{G(s)}{1 + G(s)} \quad \text{ie,} \quad [1 + G(s)]T(s) = G(s)$$

$G(s)$:-

$T(s) = G(s)[1 - T(s)]$

$$\boxed{G(s) = \frac{T(s)}{1 - T(s)} \quad \& \quad H(s) = 1}$$

$$G(s) = \frac{\dfrac{3(1-s)(s+5)}{s^3 + 12s^2 + 47s + 84}}{1 - \dfrac{3(1-s)(s+5)}{s^3 + 12s^2 + 47s + 84}} = \frac{-3s^2 - 12s + 15}{s^3 + 15s^2 + 59s + 69}$$

- Type = zero as no open loop pole at Origin.

Order = 3 ans

$$K_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{-3s^2 + 12s + 15}{s^3 + 15s^2 + 59s + 69} = \frac{15}{69}$$

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = 0$$

- $e_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + \frac{15}{69}} = \frac{69}{84} = 0.8214$. for step.

$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$ for ramp.