25/11

Designing of analog filter:-

Digital filter

IIR (Infinite impulse response filter)

h(n)

$H(z) = \frac{\cancel{Y}(z)}{X(z)}$

DTLTI s/m

FIR (Finite impulse response)

* Directly FIR filter can be designed.

Analog → H(s)

Digital → H(z)

Analog filter:-

$H_a(s) = \frac{Y_a(s)}{X_a(s)}$

→ IIR filter can't be designed directly.

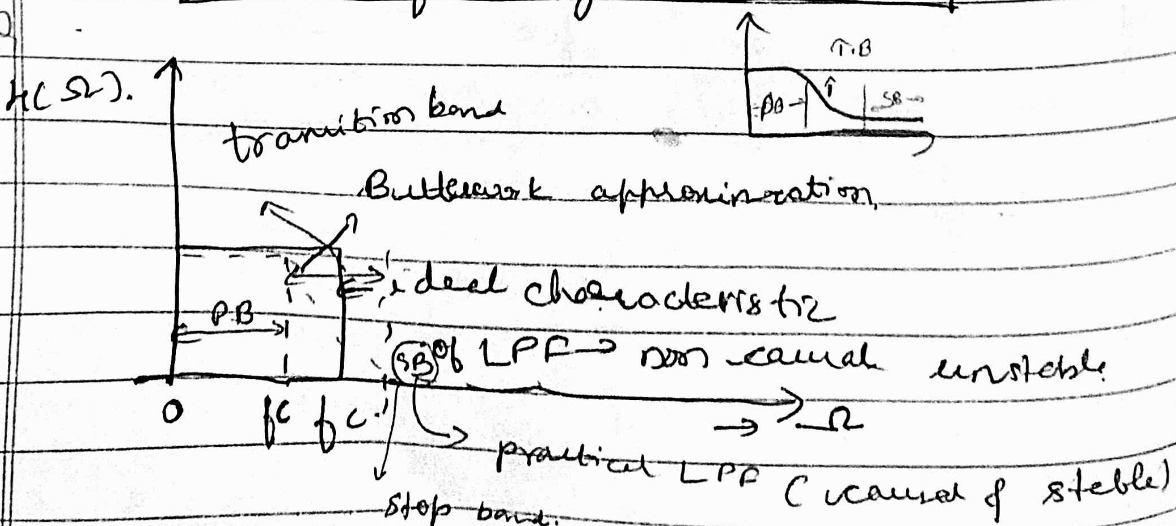
→ From analog filter the equivalent digital filter is derived.

Types of filter:-

1. Low pass filter.
2. High pass filter.
3. Band pass filter.
4. Band stop (rejection) filter.

$$\boxed{\begin{array}{l} \Omega = 2\pi f \rightarrow \text{Analog. rad/sec} \\ \omega = 2\pi f \rightarrow \text{Digital} \end{array}}$$

MLQ



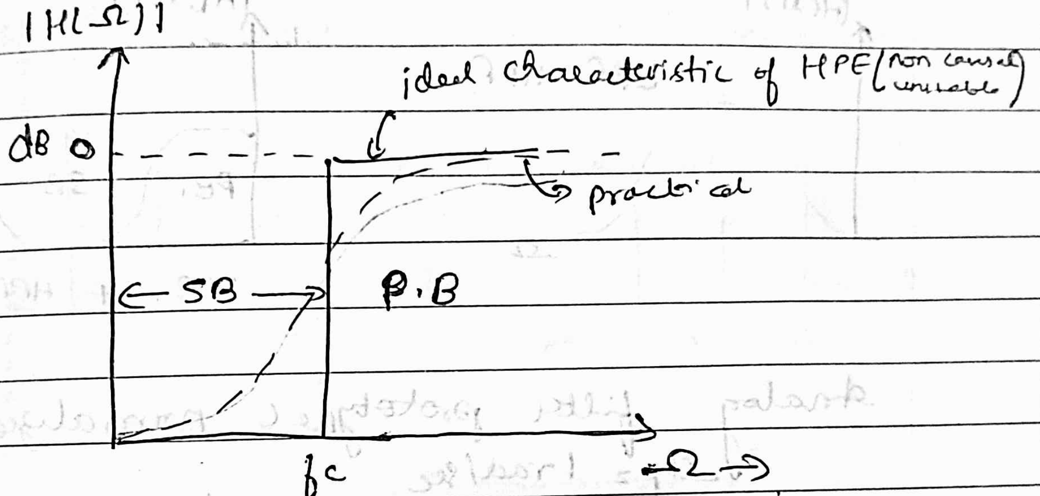
Butterworth approx (causal & stable)

Stop band :- Band of frequencies stopped by filter.

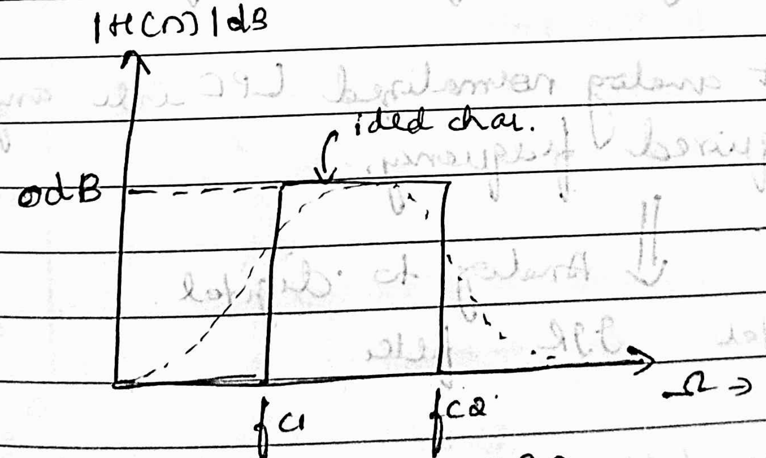
Transition :- Band of frequencies having transition from pass to stop.

→ Smaller transition band with increase the order 'N'.

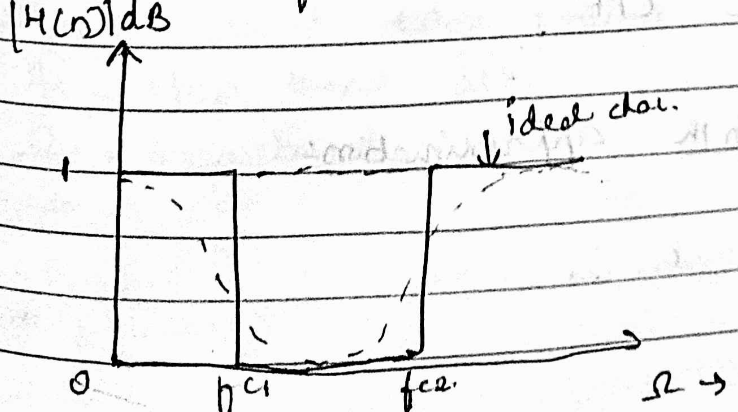
HPF:-



BPF :-



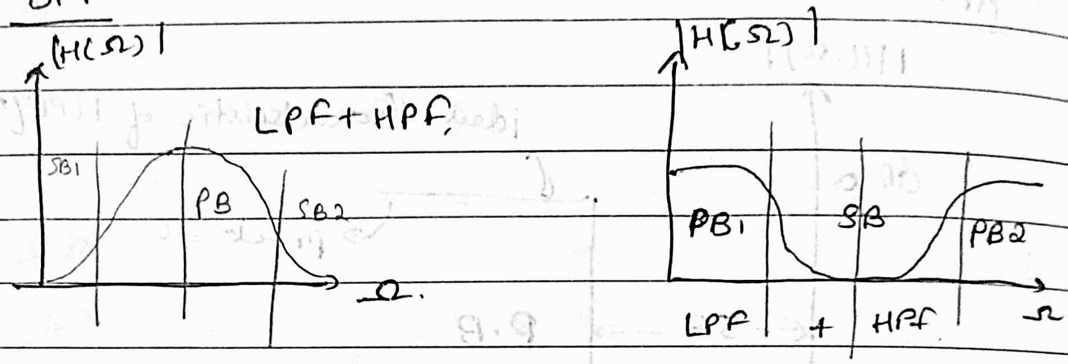
Band stop filter or BRF :-



Butterworth approximation → smooth transition
→ Pass band & stop band will be monotonic (or)
→ Smooth PB & smooth SB.

→ Finite IRF:- If the response is finite.
→ Infinite " infinite.

BPF:-



Analog filter prototype (normalized) LPF.
$\Omega_p = 1$ rad/sec

⇓ Analog to analog trans

Convert analog normalized LPF into any filter of required frequency.

⇓ Analog to digital.

Digital IIR filter

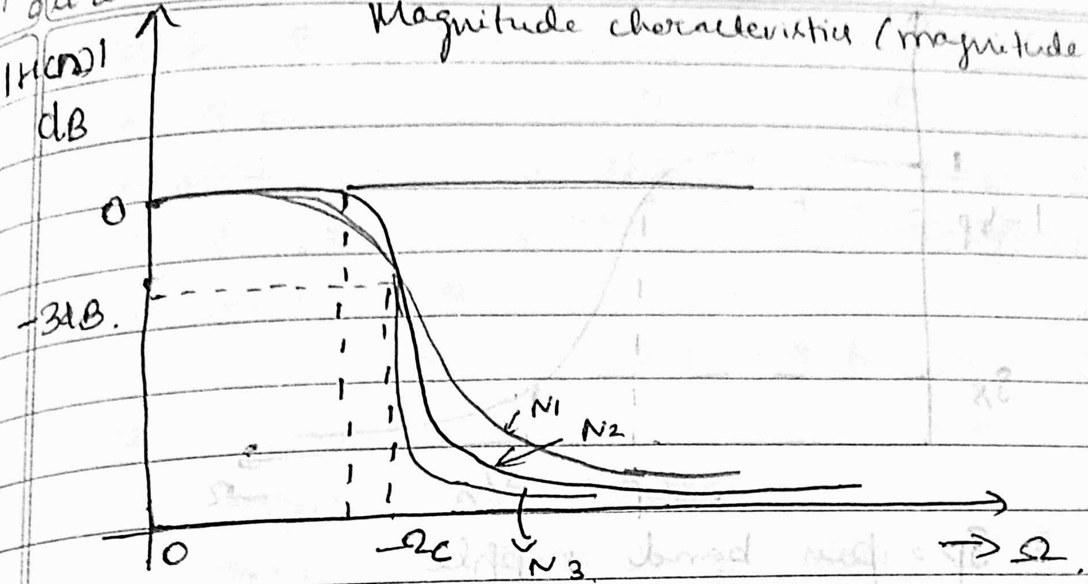
Analog filter
① Prototype LPF.

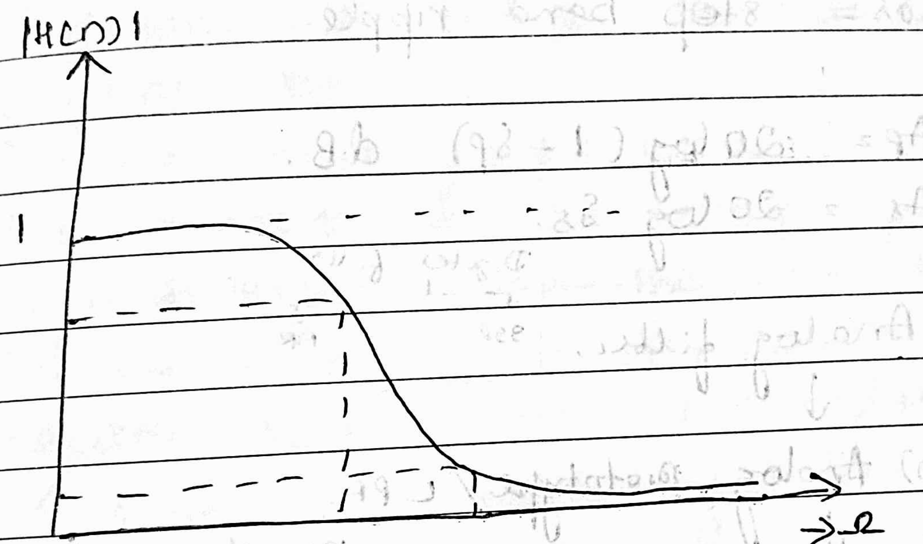
V.V. Imp.
Butterworth approximation:-

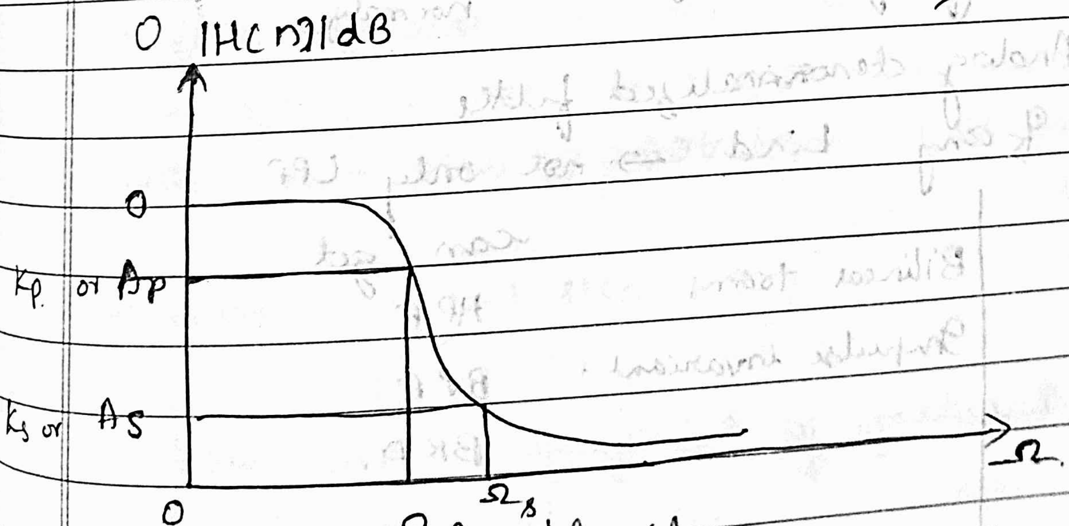
Magnitude characteristic

anyil dil aan

Magnitude characteristics / magnitude response

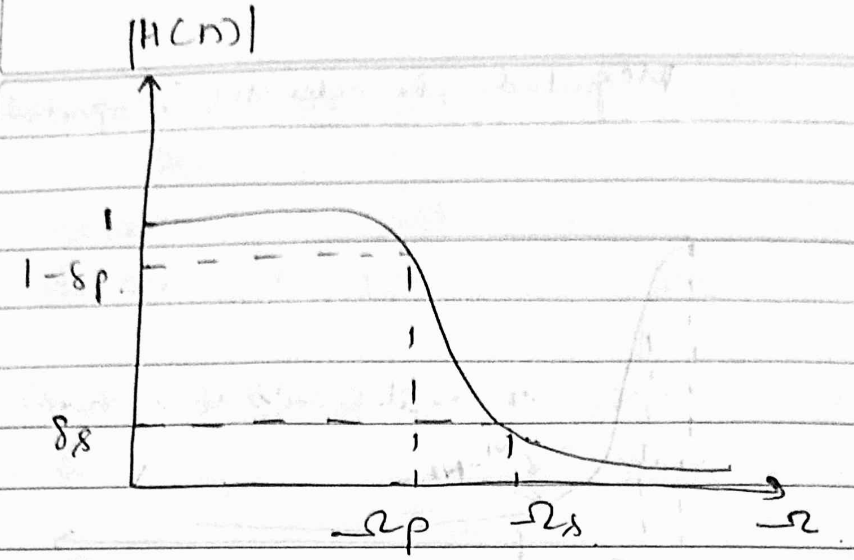






Aanam ~~ondganam ettem~~ Ω_p ~~problem'il.~~

- A_p = pass band gain.
- Ω_p = pass band edge frequency.
- A_s = stop band attenuation.
- Ω_s = stop band frequency.



δp = pass band ripple.
δs = stop band ripple.

$A_p = 20\log(1-\delta p)$ dB.
$A_s = 20\log \delta s$.

16/11/2021
IMP
MCQ.

Analog filter. | Digital filter: IIR, FIR |

↓

($H_a(s)$) Analog prototype / LPF
normalized

↓

Analog denormalized filter of any kind → not only LPF
can get
HPF
BPF
BRF.

Bilinear trans
Impulse invariant.

↓

Digital IIR filter.
($H(z)$)

Analog filter design:-

→ Design rules are to design only analog prototype / normalized LPF.

... Ω.

→ Though the given specifications are of any type of filter (LPF, HPF, BPF, BSF) of any required frequency the design rules are framed only to design a prototype/normalized (Ωp = 1 rad/sec).

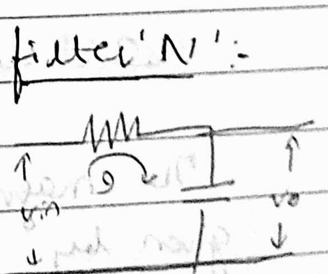
→ Ω_p = 1 rad/sec ondayil nathokaan direct aayt design cheyanam

The design steps involves:-

1. Determination of order of filter N.
2. Determination of the 3 dB cutoff frequency Ω_c.
3. Determination of poles - S_K
4. Determination of the transfer function of the normalized/prototype LPF ($H_n(s)$).
5. Determination of transfer function of required type of filter (LPF, HPF, BPF, BSP) with the frequency required ($H_a(s)$).
6. Before proceeding with above steps the given specifications need to be converted into normalized/prototype LPF specifications.

Nb: Derivation of each step → Imp (V.V.V.)

1. Determination of order of filter 'N':



$$V_{in} = I\left(R + \frac{1}{j\Omega C}\right) = I\left(\frac{Rj\Omega C + 1}{j\Omega C}\right)$$

$$V_o = I\left(\frac{1}{j\Omega C}\right)$$

$V_{in} = I(R - jX_c)$

$V_o = I(jX_c)$

$X_c = \frac{1}{2\pi f c} = \frac{1}{\omega c}$

$$\frac{V_o}{V_{in}} = \frac{jj\Omega C}{(1 + j\Omega RC)\Omega C}$$

wkt $\Omega_c = \frac{1}{RC}$

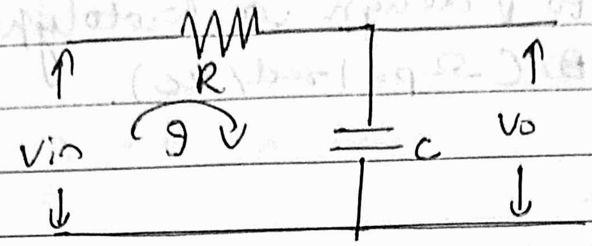
$\Rightarrow RC = \frac{1}{\Omega_c}$

analog ayadhu $X_c = \frac{1}{\Omega c}$.

magnitude consider cheyande

$$\frac{V_o}{V_{in}} = \frac{1}{\ldots} = |H_a(j\Omega)|$$

$$|H_a(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^2}}$$



$x_c = 1/\Omega C$

$\Omega_c = \frac{1}{RC}$

$RC = 1/\Omega_c$

$$V_{in} = i(R - j x_c)$$

$$= i\left(R + \frac{1}{j\Omega c}\right)$$

$$= i\left(\frac{j\Omega RC + 1}{j\Omega c}\right)$$

$$V_o = i(j x_c) = i\left(\frac{-j}{\Omega c}\right).$$

$$\frac{V_o}{V_{in}} = \frac{jj\,\Omega c}{(1+j\Omega Rc)\Omega c}$$

$$\frac{V_o}{V_{in}} = \frac{1}{\left(1+j\frac{\Omega}{\Omega c}\right)} = |H_a(j\Omega)|$$

$$|H_a(j\Omega)| = \frac{1}{\sqrt{1+(\Omega/\Omega_c)^2}}$$ for 1^{st} order

Ω_c = cut off frequency.

The magnitude response of N filter is given by N^{th} order.

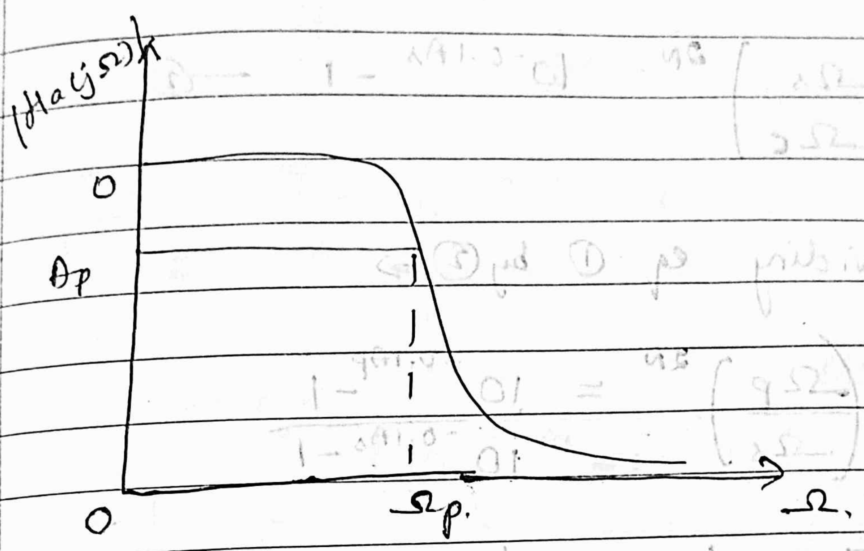
$$|H_a(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}$$

Taking 20 log on both sides

$$20\log|H_a(j\Omega)| = 20\log\left(1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right)$$

$$20\log|H_a(j\Omega)| = -10\log\left(1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right)$$

WKT at $\Omega = \Omega_p$



$\cancel{20\log_a}$ $20\log|H(j\Omega)| = A_p$ & substituting in above eqn.

$$\boxed{A_p = -10\log\left(1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}\right)}$$

Similarly at $\Omega = \Omega_s$ the $|H(j\Omega)|$ is

$H(j\Omega) = A_s$.

$$\boxed{A_s = -10\log\left(1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}\right)}$$

$$A_p = -10\log\left(1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}\right)$$

Analog filter — normalized $\Omega_p = 1$ rad/sec

LPF, HPF, BPF, BSF — $\Omega_p = 1$ rad/sec, prototype LPF (or) normalized LPF.

denormalized LPF, HPF, BSF — Ω_p can be any value

Taking antilog,

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = 10^{-0.1A_p} - 1 \quad \text{--- (1)}$$

$$A_s = -10\log\left(1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}\right)$$

$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = 10^{-0.1A_s} - 1 \quad \text{--- (2)}$$

Dividing eq (1) by (2) $\Rightarrow$

$$\left(\frac{\Omega_p}{\Omega_s}\right)^{2N} = \frac{10^{-0.1A_p}-1}{10^{-0.1A_s}-1}$$

Taking log on both sides,

$$N \times 2\log\left(\frac{\Omega_p}{\Omega_s}\right) = \log\left[\frac{10^{-0.1A_p}-1}{10^{-0.1A_s}-1}\right]$$

Fraction of order is always integer

Butterworth filter

$$\boxed{N \geq \frac{\log\left\{(10^{-0.1A_p}-1)/(10^{-0.1A_s}-1)\right\}}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}}$$

$N = 1.2 \Rightarrow 2$ rounding up (OK) $(10^{-0.1A_s}-1)/(10^{-0.1A_p}-1)$, $2\log(\Omega_s/\Omega_p)$ without sharp

$1.02 \Rightarrow 1$; $1.1 \Rightarrow 2$

(2) Determination of Ω_c (cutoff frequency).

(1) Cutoff frequency can be determined from (1). (3dB)

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = 10^{-0.1A_p} - 1$$

Analog filter

LPF → idh nalliam cheyd
HPF
BPF
BSF

prototype - basic

LPF → prototype LPF → Normalized

$$(\Omega_c)^{2N} = \frac{(\Omega_p)^{2N}}{10^{-0.1A_p} - 1}$$

$$\Omega_c = \frac{(\Omega_p)^{2N \times 1/2N}}{(10^{-0.1A_p} - 1)^{1/2N}}$$

$$\boxed{\Omega_{c_1} = \frac{\Omega_p}{(10^{-0.1A_p} - 1)^{1/2N}}}$$ ✓ theory

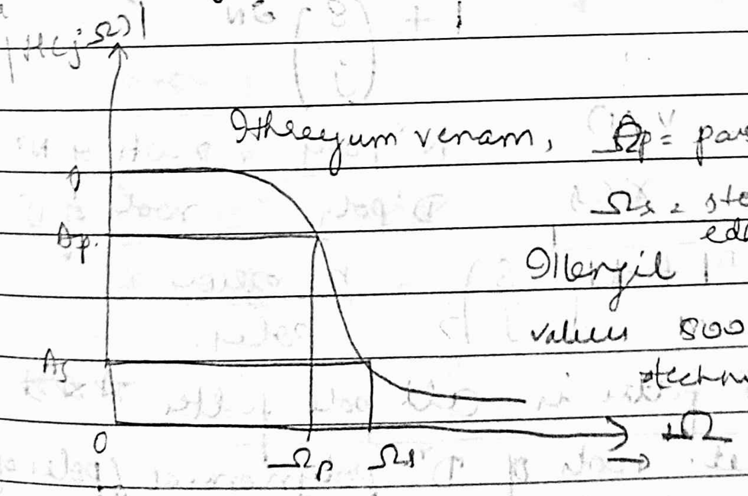
$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = 10^{-0.1A_s} - 1$$

$$\boxed{\Omega_{c_2} = \frac{\Omega_s}{(10^{-0.1A_s} - 1)^{1/2N}}}$$ MATLABil idha use cheyuane.

$$\boxed{\Omega_c = \frac{\Omega_{c1} + \Omega_{c2}}{2}}$$ Cheladhil idhum use cheyum.

17/11

Analog → digital & 99V



Itheeyum venam, Ω_p = pass band
Ω_s = stop band
edheyilum assume cheyth. Illengil values soochipik(k) munt technique only like Butterworth allel Chebyshev.

A_p = & A_s relative value
A_p = 0, -1, -2, -3, -4.

→ Ω_p angane thanne, Ω_s (stop bandil compromise) pakshe MUTILAB THIRICH.

③ Poles :-
Denoted by S_k.

Determination of poles:-

WKT the squared magnitude response is

$$|H_a(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}$$

N^r polynomial = zeros

Roots of N^r = Z
Roots of D^r ⇒ poles
Numerator = $\frac{Z}{P}$
D^r

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}.$$

$S = \sigma + j\Omega$. (stable again to left side)
↓ Real.

Wkt. $S = j\Omega$

$$\Omega = \frac{S}{j}$$

$$|H(S)\cdot H(-S)| = \frac{1}{1+\left(\frac{S}{j}\right)^{2N}}, \quad \Omega_c = 1\text{rad/sec}$$

$H(S) = \frac{Y(S)}{X(S)} = \frac{N^r \text{ poly}}{D^r \text{ poly}} = \frac{\text{roots of } N^r}{\text{roots of } D^r} = \frac{\text{zeros}}{\text{poles}}$

$\frac{1}{1+\left(\frac{S}{j}\right)} = \frac{\text{no zeroes}}{\text{poles.}}$

∴ IIR filter is all pole filter ☆☆☆

To det. roots of D^r polynomial / poles of system

$$1+\left(\frac{S}{j}\right)^{2N} = 0.$$

$$\left(\frac{S}{j}\right)^{2N} = -1$$

$$(S) = j(-1)^{1/2N}$$

1 06

Wkt $j = e^{j\pi/2}$

$(-1) = e^{\cancel{j\pi(2k+1)}\; j\pi(2k+1)}$

$$S = e^{j\pi/2} \cdot e^{\frac{j\pi(2k+1)}{2N}}$$

$$\boxed{S_k = e^{j\pi/2} \cdot e^{\frac{j\pi k}{N}} \cdot e^{j\pi/2N}} \quad ; \; k = 0, 1, 2, \ldots, 2N-1$$

Set of all the poles are this

Both left half plane poles & Right half poles.

LHP poles are chosen for stable system.

Only LHP poles need to be considered.

$$\boxed{S_K = e^{j\pi/2} \cdot e^{j\pi k/N} \cdot e^{j\pi/2N} \; ; \; 0 \le K \le N-1}$$

① Determine the poles involved in 1st order filters.

Soln:-

$N = 1$

$$S_K = e^{j\pi/2} \cdot e^{\frac{j\pi k}{N}} \cdot e^{j\pi/2N} \; ; 0 \le K \le 2N-1$$

$k = 0, 1.$

$K = 0$

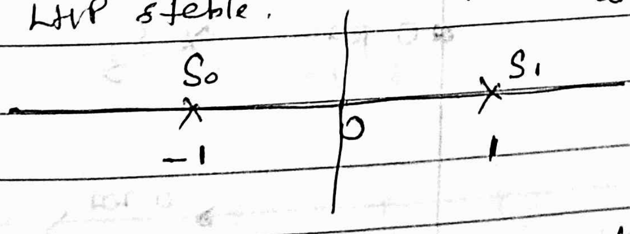
$$S_0 = e^{j\pi/2} \cdot e^{j\pi(0)/1} \cdot e^{j\pi/2}$$

$\boxed{S_0 = -1}$

$$S_1 = e^{j\pi/2} \cdot e^{j\pi(1)/1} \cdot e^{j\pi/2}$$

$\boxed{S_1 = 1}$

LHP stable. RHP unstable



② Determine the poles involved in the designing of a 2nd order filter:-

Soln:-

$N = 2.$

all poles → $2N-1$

LHP → $N-1$

$$S_k = e^{j\pi/2} \cdot e^{j\pi k/N} \cdot e^{j\pi/2N} , \quad 0 \le k \le 2N-1$$

$k = 0, 1, 2, 3$

$k = 0$.

$$S_0 = e^{j\pi/2} (1) \; e^{j\pi/2(2)}$$

$$= e^{j\pi/2} \cdot e^{j\pi/4}$$

$$= (j) \left[0.707 + 0.707j\right)$$

$$S_0 = -0.707 + j\,0.707$$

$$S_1 = e^{j\pi/2} \; e^{j\pi/4} \cdot e^{j\pi/2}$$

$$= (j) \left[0.707 + 0.707j\right]\left[j\right]$$

$$= 0.707 \cdot 2 - 0.707j$$

$0.707 - j(0.707)$ $\qquad$ $S_3 = 0.707 + j(0.707)$

$$S_2 = e^{j\pi/2} \; e^{j\pi} \cdot e^{j\pi/4}$$

$$= j \, [(-1)] \left[0.707 + 0.707j\right]$$

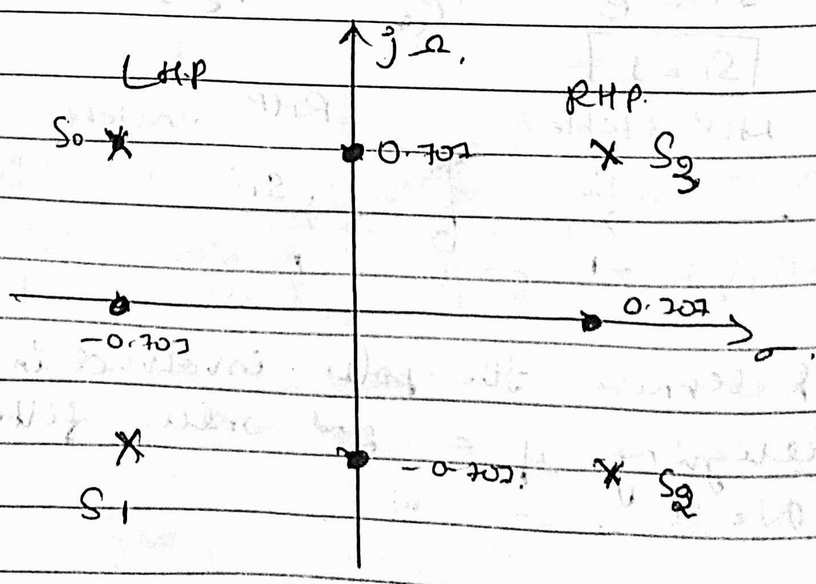
$=$

$$S_2 = 0.707 - j\,(0.707)$$

$$S_3 = e^{j\pi/2} \cdot e^{j3\pi/2} \cdot e^{j\pi/4}$$

$$= j \cdot [-j] \left[0.707 + 0.707j\right]$$

$$= 1\left[0.707 + 0.707j\right]$$



only S_0 & S_1 is used.

ethee poles ordegum atheem poles metheem LHP.

(i) Determination of transfer function of normalized analog filter.

$H_n(S)$

$\Omega_p = 1$ rad/sec.

LPF.

Derivation:

$$H_n(S) = \frac{1}{\text{product of } \prod (S - S_k)} \quad \text{or} \quad \frac{1}{B_N(S)}$$

$B_N(S) \Rightarrow$ Butterwork polynomial

$N = 1;$

$$H_n(S) = \frac{1}{(S - S_0)} = \frac{1}{S - (-1)} = \frac{1}{S+1}$$

$$\boxed{H_n(S) = \frac{1}{S+1}} \qquad \boxed{B_1(S) = S+1}$$

$N = 2$

$$H_n(S) = \frac{1}{(S - S_0)(S - S_1)} = \frac{1}{(S - (-0.707 + j(0.707)))(S - (-0.707 - j0.707))}$$

$(a-b)(a-c)$

$= a^2 - ac - ab + bc.$

$$= \frac{1}{s^2 - (0.707 - j\,0.707)s - (0.707 + j0.707)s + (0.707 + j(0.707))(0.707 - j\,0.707))}$$

$1.414 = \sqrt{2}$

$$H_n(s) = \frac{1}{s^2 + 1.414s + 1}$$

$$\boxed{B_{2s} = s^2 + 1.414s + 1}$$

order (3).

24/11 Designing of analog filter:-

length finite $\rightarrow$ FIR

dB pinne normally eppolum nok. Ithazhe do de
numbol ve ayiritum.

$Z(h(n)) = H(z) \leftarrow$ digital filter.

$\downarrow$

Length of $h(n)$ → finite (FIR filter)
→ infinite (IIR filter).

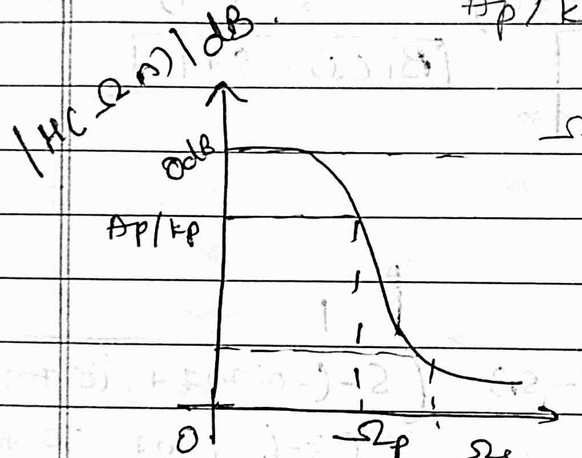
<u>Designing of analog filter:</u>

$H_{n}(s) \rightarrow$ prototype / normalised.

$H_{co}(s) =$ any Ω_p & any type (LPF, HPF, BPF, BRF).

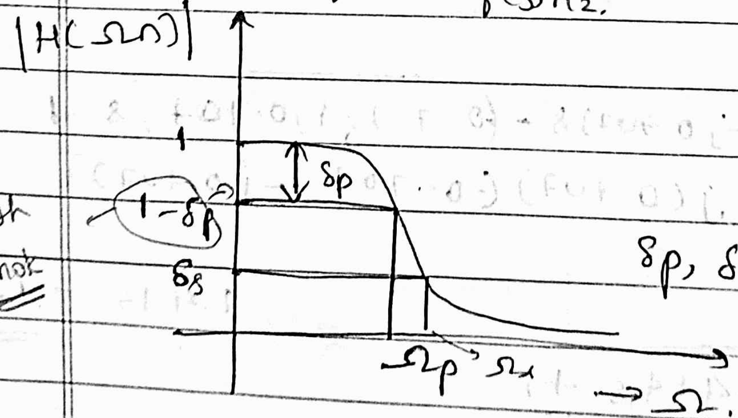
$A_p / k_p \Rightarrow$ pass band gain. Ith eppidum -ve aan

$\Omega_p =$ pass band frequency



$\Omega \rightarrow r/s$.

f_p (Hz) $f(s)$ Hz.



ith nok

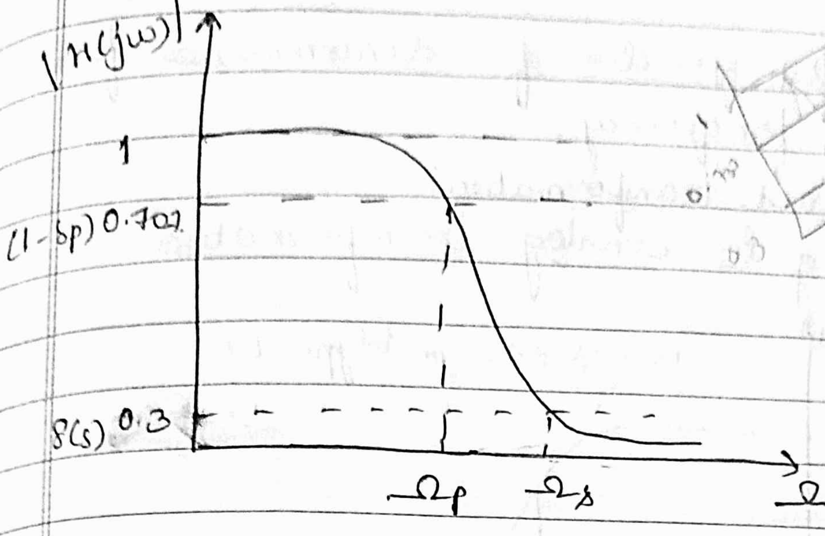
$\delta_p, \delta_s \rightarrow$ ripple.

$\rightarrow \Omega$.

f_p Hertz anengil then $\Omega_p = 2\pi f_p$

$$\boxed{\begin{aligned} A_p &= 20\log(1-\delta_p) \\ A_s &= 20\log(\delta_s) \end{aligned}}$$

$0.707 \le |H(j\omega)| \le 1$ → ith dB-il allo ethi
$|H(j\omega)| \ge 0.3$ maximal 1 aan maximum



Designing of filter :-
order of filter eppolum integer aayirom

① $N \ge \dfrac{\log\left((10^{-0.1A_P}-1)/(10^{-0.1A_S}-1)\right)}{2\log\left(\frac{\Omega_P}{\Omega_S}\right)}$

② 3 dB cut off frequency.

in MATLAB inbuilt fn is written using this formula

$$\Omega_C = \frac{\Omega_P}{\left(10^{-0.1A_P}-1\right)^{1/2N}} \quad \text{or} \quad \frac{\Omega_S}{\left(10^{-0.1A_S}-1\right)^{1/2N}} \quad \text{or}$$

MATLAB

Average of both

③ poles $S_k = e^{j\pi/2 + \pi k/n + \frac{\pi(2k+1)}{2N}}$; K=0 to 2N-1

ith eppolum varila nokivck. e eppolum. (both LHP & RHP poles).

ennalum N=2, 4 poles and 2 on edath, 2 on valadh

N=3, 6 poles (3 on edath & 3 on valadh ; k=0 to N-1

Left) eppolum. Idh ingane ayirikkum (only LHP poles)

(poles and) → To show the no. of poles

k=0 to 2N-1

④ Transfer function of normalized /prototype LPF ($H_n(s)$)

ithil nalla formula adhayath oru 8 label pevanam

$$H_n(s) = \frac{1}{\prod (s - s_k)} \quad \text{or} \quad \frac{1}{B_N(s)}.$$ max order 4

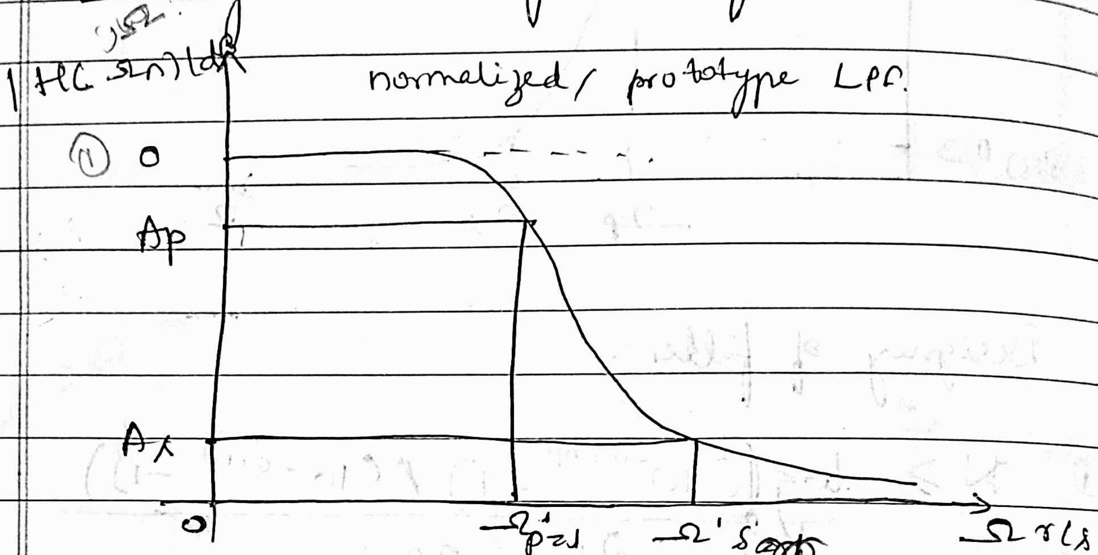
(LPF, HPF, BPF, BRF)

⑤ Transfer function of denormalized filter of any frequency.

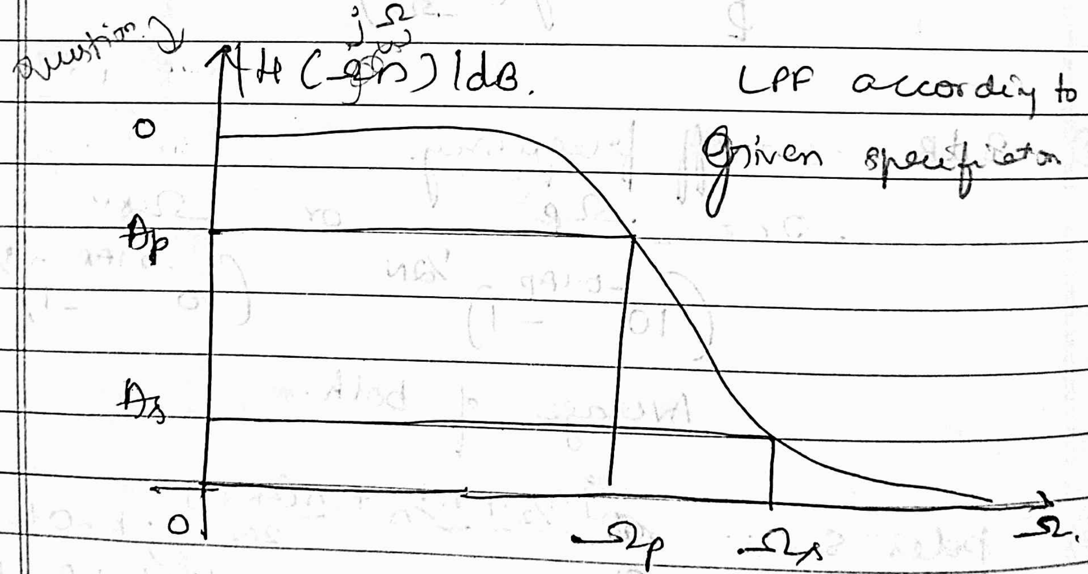
Illustration

Backward transformation:-

⑥ Analog to analog transformation:-



normalized / prototype LPF.

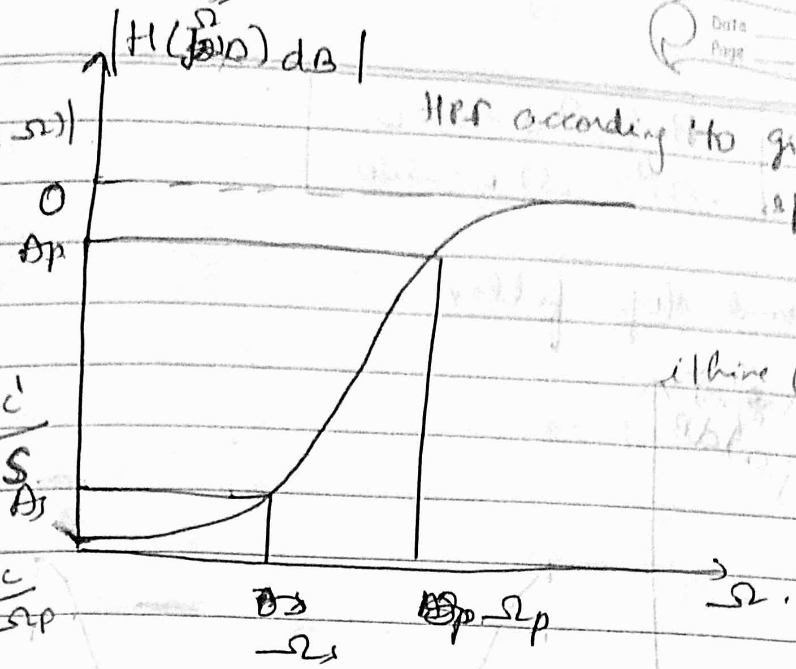


LPF according to Given specification

$$\boxed{\Omega_s' = \frac{\Omega_s}{\Omega_p}}$$

Ω_s' is found out by dividing the specification in question

$$N \geq \frac{\log\left(\left(10^{-0.1A_p} - 1\right) / \left(10^{-0.1A_s} - 1\right)\right)}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}.$$



HPF according to given specification

ithine ① il lot aakoram

$s \to \frac{\Omega_c'}{s}$

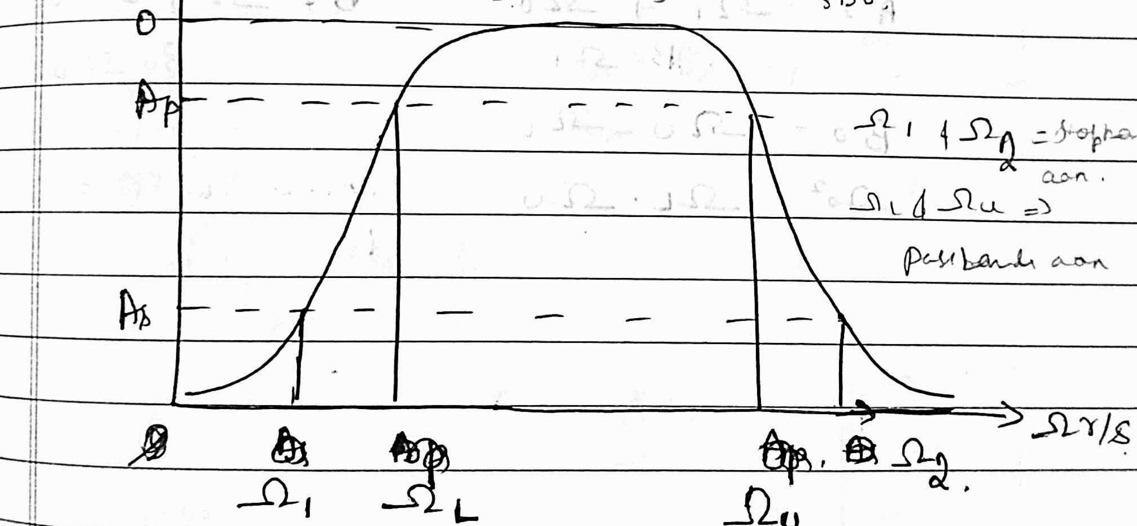
$\Omega_c' = \frac{\Omega_c}{\Omega_p}$

$$\boxed{\Omega_s' = \frac{\Omega_p}{\Omega_s}}$$

$$\boxed{\Omega_s' = \frac{\Omega_p}{\Omega_s}}$$

$|H(j\Omega)|dB$, note

$$s \to \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)} = \frac{s^2 + \Omega_0^2}{sB_0}$$



Ω_1 & Ω_2 = stopband aan.

Ω_l & Ω_u ⇒ passband aan

ithine ① il lot nadam.

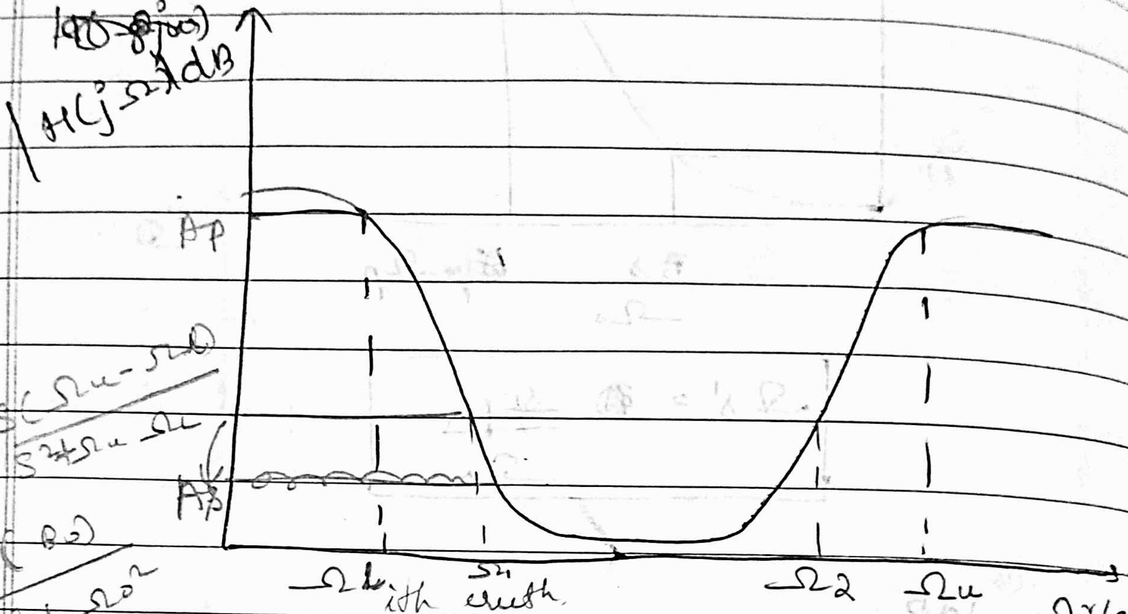
$$\boxed{\Omega_s' = \min\{|A|, |B|\}}$$

$$\boxed{A = \frac{-\Omega_1^2 + \Omega_0^2}{B_0 \Omega_1}}$$

$$\boxed{B = \frac{\Omega_2^2 - \Omega_0^2}{B_0 \Omega_2}}$$

$$\boxed{\begin{aligned} B_0 &= \Omega_U - \Omega_L \\ \Omega_0^2 &= \Omega_L \cdot \Omega_U \end{aligned}}$$

Band stop filter:- ↑ Same formula



$S \Rightarrow \frac{S(\Omega_u - \Omega_L)}{S^2 + \Omega_u \Omega_L}$

$S \Rightarrow \frac{S(B_0)}{S^2 + \Omega_0^2}$

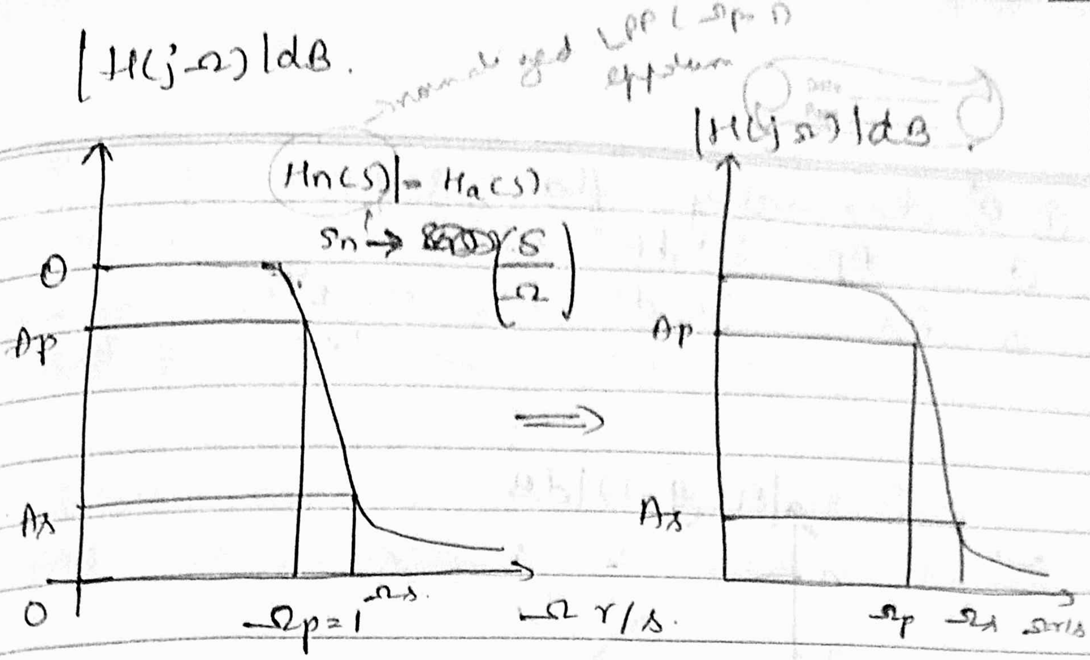
$\Omega'_s = \min\{|A|, |B|\}$

$A = \frac{-\Omega_1^2 + \Omega_0^2}{B_0 \Omega_1}$ $\qquad$ $B = \frac{\Omega_2^2 - \Omega_0^2}{B_0 \Omega_2}$.

$B_0 = \Omega_U - \Omega_L$

$\Omega_0^2 = \Omega_L \cdot \Omega_U$

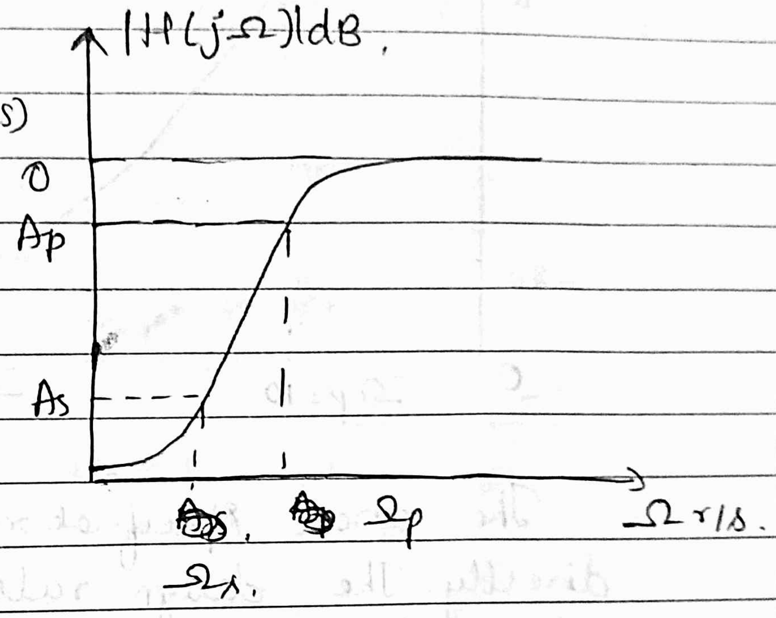
Analog to analog transformation:-



$$H_n(s)\Big|_{S_n \to \left(\frac{S}{\Omega}\right)} = H_a(s)$$

$$H_n(s) = H_a(s)$$

$$\boxed{S_n \to \frac{\Omega}{S}}$$



BPF:-

$$H_n(s)\Big| = H_a(s)$$

$$\boxed{S_n \to \frac{s^2+\Omega_0^2}{B_0 s}}$$

$$H_n(s)\Big|_{S_n \to \frac{s^2+\Omega_0^2}{B_0 s}} = H_a(s)$$

BSF:-

$$H_n(s)\Big|_{2} = H_a(s)$$

note.

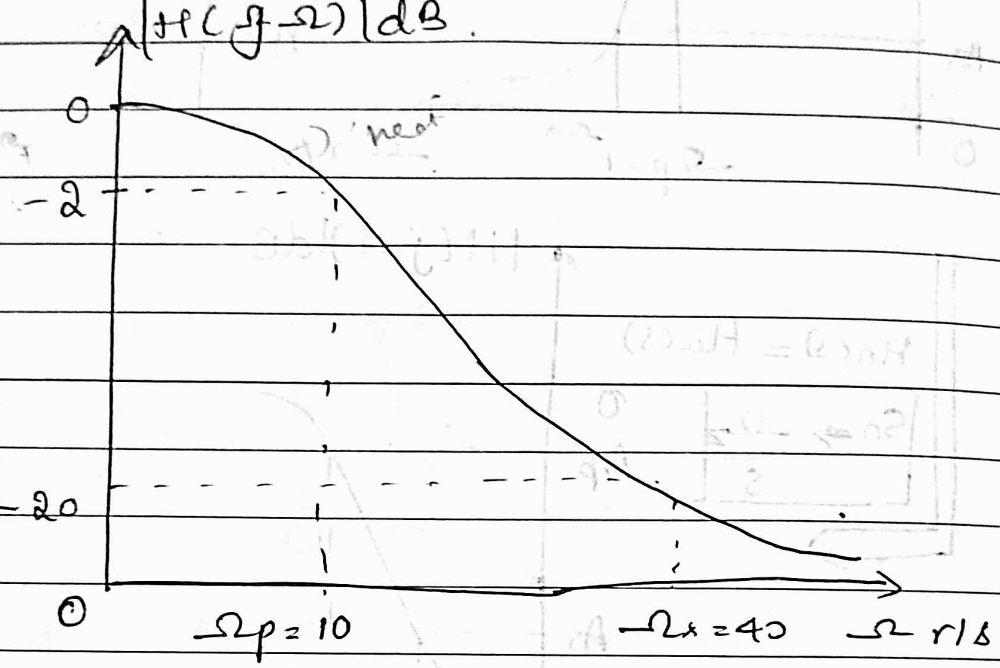
$$\boxed{S_n \to \frac{B_0{}^2}{s^2+\Omega_0{}^2}}$$

① Design an analog filter to satisfy the following specification.

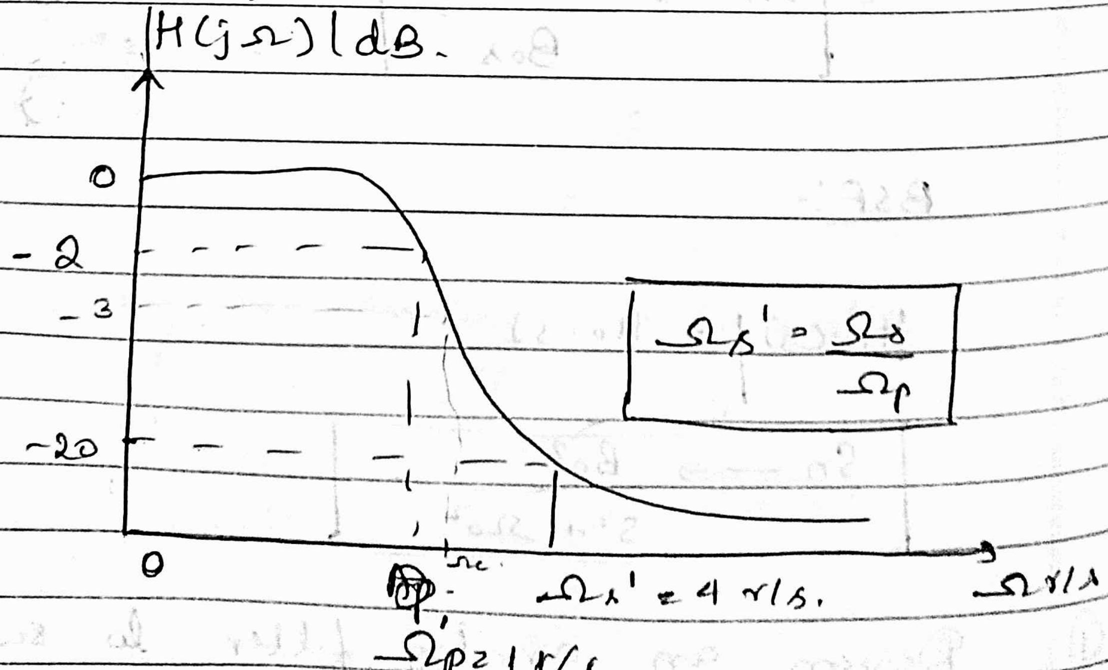
① Maximally flat response.
② $A_p = -2$ dB at 10 r/s.
③ $A_s = -20$ dB at 40 r/s.

→ Type of filter is not explicitly described in question. So draw the specification & know

Soln:



The above specification can't be designed directly. The design rules are available only to design a normalized LPF means that the pass band edge frequency is 1 rad/sec.



Since Ω_s' of normalized LPF is not known, it should be calculated from the given specifications.

For the LPF the $\boxed{\Omega_s' = \frac{\Omega_s}{\Omega_p}}$

where Ω_p & Ω_s are the given analog pass band & stop band edge frequencies.

$$\Omega_s' = \frac{\Omega_s}{\Omega_p} = \frac{40}{10} = 4 \, r/s$$

narrow transition width → order more.

① $$N \geq \frac{\log\left((10^{-0.1 A_P} - 1) / (10^{-0.1 A_s} - 1)\right)}{2\log\left(\frac{\Omega_p'}{\Omega_s'}\right)}$$

$$N \geq \frac{\log\left((10^{-0.1(-2)} - 1) / 10^2 - 1\right)}{2\log\left(\frac{1}{4}\right)}$$

$$\therefore 10^{-0.1(A_P)} - 1 \Rightarrow 10^{-0.1(-2)} - 1 = 10^{0.2} - 1,$$

$$N \geq \frac{\log(0.584/99)}{2\log(1/4)} = 1.85$$

~~calculator~~ eppolum radian il alljirikyanam. Sookshik

$N \geq 1.851$

$\boxed{N = 2}$

② $$\Omega_c = \frac{\Omega_p'}{(10^{-0.1 A_P} - 1)^{1/2N}}$$

$$= \frac{1}{(10^{-0.1(-2)} - 1)^{1/2\times 2}}$$

$$= \frac{1}{(0.584)^{1/4}} = \underline{\underline{1.143 \, r/s}}$$

Nok 1.14 agumbol. ath melevalik graph appol ath 2 inde adulh values.

$N = 2$ $H_n(s) = \dfrac{1}{S_n^2 + 1.414 S_n + 1}$

③ Determination of poles.

For $S_k = e^{j(\pi/2 + \pi k/N + \pi/2N)}$; $0 \le K \le N-1$

this changes $1 \le k \le N$

stable

For the designing of filter only the LHP poles are required therefore to exclude RHP poles the k values can range from 0 to N-1, where N = order of filter.

$$S_0 = e^{j(\pi/2 + \pi(0)/2 + \pi/4)}$$

$$\boxed{S_0 = -0.707 + j(0.707)}$$

$$S_1 = e^{j(\pi/2 + \pi/2 + \pi/4)}$$

$$\boxed{S_1 = -0.707 - j(0.707)}$$ not.

④ $$H_n(s) = \frac{1}{\prod(S - S_k)} = \frac{1}{(S - S_0)(S - S_1)}$$

$$= \frac{1}{(S - (-0.707 + j(0.707)))(S - (-0.707 - j0.707))}$$

$$\cancel{= \frac{1}{(S+1)(S + 0.707 + 0.707j)}}$$

$$= \frac{1}{(S + 0.707 - j0.707)(S + 0.707 + 0.707j)}$$

$$= \frac{1}{S^2 + 0.707S + 0.707js + 0.707s + 0.499 + 0.499sj - 0.707s}$$

$$\boxed{H_n(s) = \frac{1}{S_n^2 + 1.414 S_n + 1}}$$

(5) To design the LPF of $\Omega_p = 10\ r/s$ & $\Omega_s = 40\ r/s$, the analog transformation need to be performed on the $H_n(s)$ as mentioned below.

$$H_a(s) = H_n(s) \Big|_{S_n \to \frac{S}{\Omega_c'}}$$

; 1R, 3dB also

For low pass,

$\Omega_c' = \Omega_p \times \Omega_c$ ← given in question

normalized $\Omega_p' = 1$

$\Omega_c' = 10 \times 1.14$

ippad normal

$\Omega_c' = 11.4$

denormalize cheyrnbal divide cheyum.

$$H_a(s) = \frac{1}{\left(\frac{S}{11.4}\right)^2 + 1.414\left(\frac{S}{11.4}\right) + 1}$$

$$H_a(s) = \frac{(11.4)^2}{s^2 + (1.414)(11.4\,s) + (11.4)^2}$$

$$\boxed{H_a(s) = \frac{129.96}{S^2 + 16.1196s + 129.96}}$$

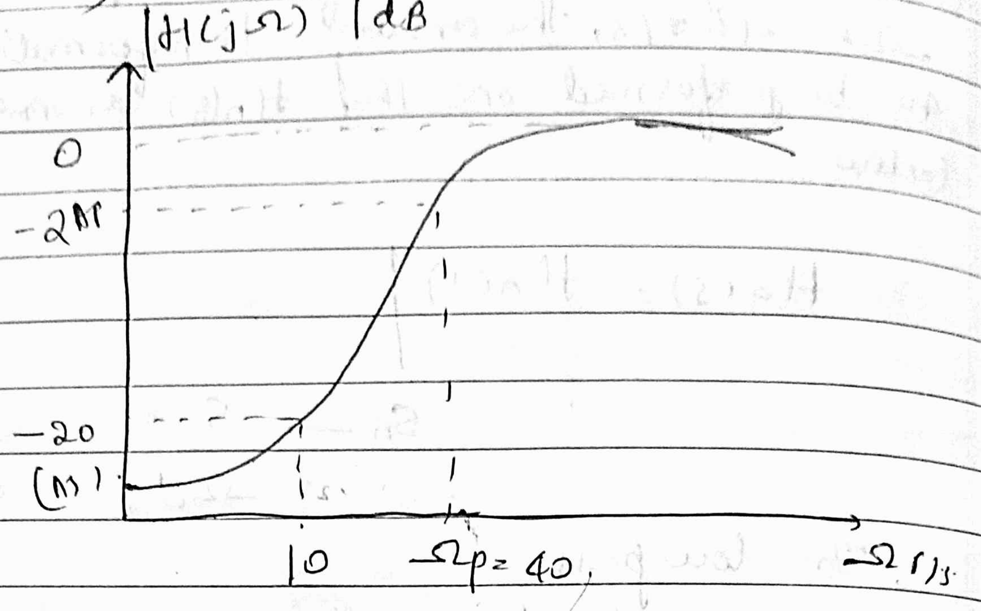
(6) Design monotonic or Butterworth approx an analog HPF to satisfy the following specification.

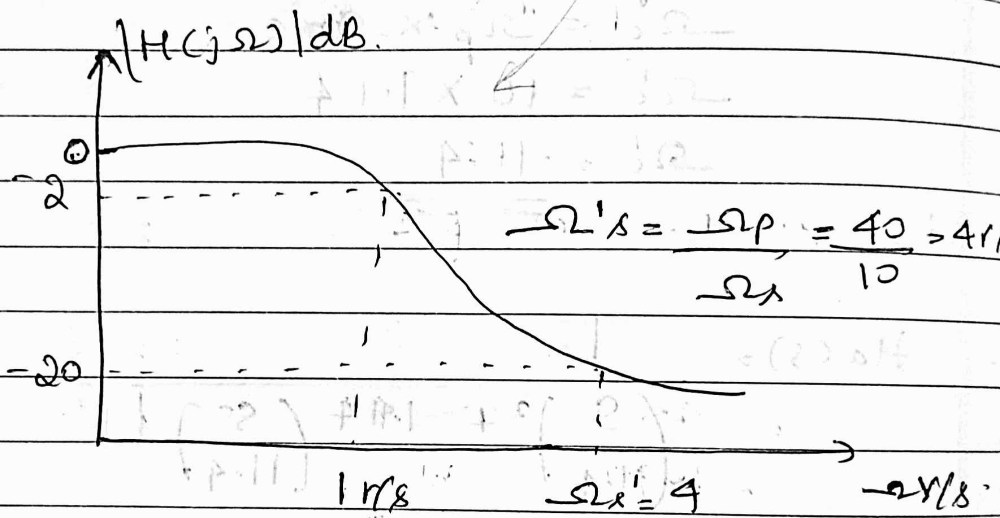
(1) maximally flat response

(2) $A_p = -2dB$ at $40\ r/s$.

$A_s = -20dB$ at $10\ r/s$.

Soln: Given,





$\Omega'_s = \frac{\Omega_p}{\Omega_s} = \frac{40}{10} = 4$ r/s

① N

② Ω_c

③ S_k

④ $H_n(s)$

⑤ $H_a(s) = H_n(s) \Big|_{s_n \to \frac{\Omega_c'}{s}}$ $\qquad \Omega_c' = \frac{\Omega_p}{\Omega_c}$ — given in specification

↳ 3dB calculated

① $$N \geq \frac{\log\left((10^{-0.1A_p} - 1)/(10^{-0.1A_s} - 1)\right)}{2\log\left(\frac{\Omega_p'}{\Omega_s'}\right)}$$

$$N \geq \frac{\log\left(\left(10^{-0.1(-2)} - 1\right) / \left(10^{-0.1(20)} - 1\right)\right)}{2\log\left(\frac{1}{4}\right)}$$

$N \geq 1.851$

$N = 2$

② $$\Omega_c = \frac{\Omega'_p}{\left(10^{-0.1A_P} - 1\right)^{1/2N}}$$

$$= \frac{1}{\left(10^{-0.1(-2)} - 1\right)^{1/4}}$$

$$= 1.1437 r/s$$

③ Determination of poles (LHS only)

$$S_k = e^{j\pi/2} \cdot e^{j\pi k/N} \cdot e^{j\pi/2N} \quad ; 0 \leq k \leq N-1$$

$k = 0, 1$

$$= e^{j\pi/2 + j\pi k/N + j\pi/2N}$$

$$= e^{j(\pi/2 + \frac{\pi(k)}{2} + \pi/4)}$$

$$S_0 = e^{j(\pi/2 + \pi/4)} = e^{j(3\pi/4)}$$

$$S_0 = -0.707 + 0.707j$$

$$S_1 = e^{j(\pi/2 + \pi/2 + \pi/4)}$$

$$= e^{j(\pi + \pi/4)} = e^{j(5\pi/4)}$$

$$= -0.7071 - 0.707j$$

④ $$H_n(s) = \frac{1}{\pi(S - S_k)}$$

$$= \frac{1}{(S - S_0)(S - S_1)}$$

$$= \frac{1}{(S - (-0.707 + 0.707j))(S - (-0.707 - 0.707j))}$$

$$= \frac{1}{S^2 - (-0.707 - 0.707j)S - (0.707S + 0.707j)S + 1}$$

$$H_n(s) = \frac{1}{S_n^2 + 1.414 S_n + 1}$$

(5) $H_a(s) = H_n(s) \Big|_{s \to \frac{\Omega_c'}{s}}$

$$\Omega_c' = \frac{\Omega_p}{\Omega_c}$$

$$= \frac{40}{1.143}$$

$$\Omega_c' = \underline{34.99 \ r/s.}$$

$$H_a(s) = \frac{1}{\left(\frac{35}{s}\right)^2 + 1.414\left(\frac{35}{s}\right) + 1}$$

$$\boxed{H_a(s) = \frac{s^2}{s^2 + 49.49s + 1225}}$$

(B) Determine an ~~analog~~ Butterworth analog HPF to meet following specification

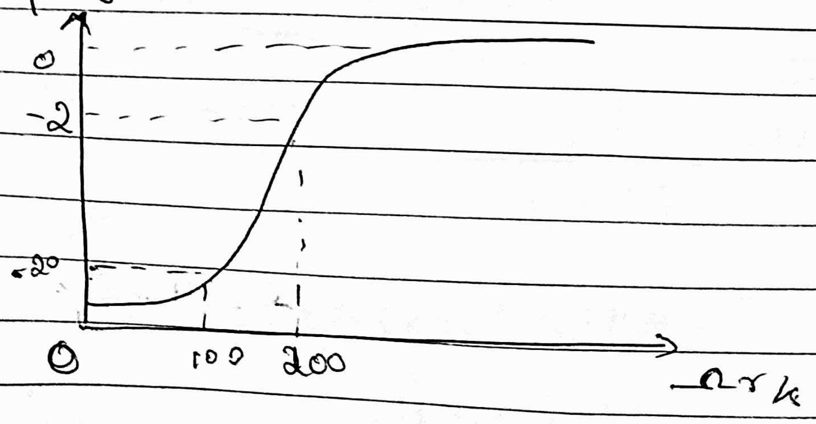
maximum pass band gain of $A_p = -2dB$ ~~at 200 r/s~~

$A_s = -20$ dB at 100 r/s.

(3) minimum stop band attenuation of $-20dB$

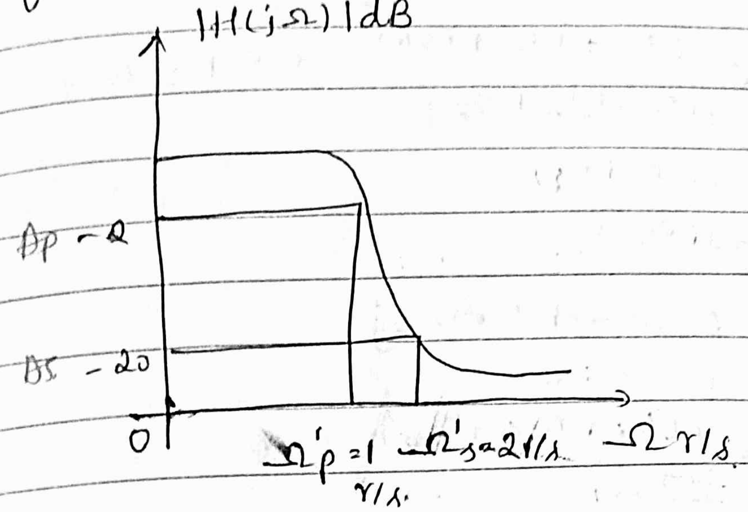
(4) stop band edge frequency of 100 r/s.

$|H(j\Omega)|dB.$



Date
Page

The above characteristics can't be designed directly since the design is availiable for normalized / prototype filter.



(1) $\Omega'_s = \dfrac{\Omega_p}{\Omega_s} = 2 r/s.$

(2) $N \geq \dfrac{\log\left[(10^{-0.1A_p} - 1) / (10^{-0.1A_s} - 1)\right]}{2\log\left(\dfrac{\Omega'_p}{\Omega'_s}\right)}.$

$N \geq \dfrac{\log\left[(10^{-0.1(-2)} - 1) / (10^{-0.1(-20)} - 1)\right]}{2\log\left(\frac{1}{2}\right)}.$

$N \geq \dfrac{\log(0.584/99)}{2\log(1/2)}.$

$N \geq 3.7$

$\boxed{N = 4}$

(3) 3 dB cut off frequency. $\Omega_c = 1.07 r/s$

$\Omega_c = \left(\dfrac{\Omega'_p}{(10^{-0.1A_p} - 1)}\right)^{1/2N}$

$$= \frac{1}{(0.584)^{1/8}}$$

$$= 1.07 \text{ r/s}$$

④ $S_k = e^{j(\pi/2 + \pi k/N + \pi/2N)}$; $0 \le k \le N-1$

$S_k = e^{j(\pi/2 + \pi k/4 + \pi/8)}$.

$S_0 = e^{j(\pi/2 + \pi/8)}$

$= e^{j(5\pi/8)}$

$= -0.387 + 0.923j$

$S_1 = e^{j(\pi/2 + \pi/4 + \pi/8)}$.

$= e^{j(7\pi/8)}$

$= -0.923 + 0.3827j$

$S_2 = e^{j(\pi/2 + \pi/2 + \pi/8)}$.

$= e^{j(9\pi/8)}$

$= -0.923 - 0.3827j$

$S_3 = e^{j(\pi/2 + 3\pi/4 + \pi/8)}$

$= e^{j(11\pi/8)}$

$= -0.3827 - 0.923j$

⑤ $H_n(s) = \dfrac{1}{\prod(s - s_k)}$.

$$= \frac{1}{(s-(-0.387+0.923j))(s-(-0.92+3.82j))(s-(-0.923-0.382j))(s-(-0.382-0.923j))}$$

$$= \frac{1}{(s^2 + 0.92s - 0.382js + 0.387s - 0.923js + 3.88 - 0.6j)}$$

$(s^2 + (1.3 - 1.3j)s + 3.88 - 0.6j)$

3kHz

Monotonic response.

$(s-(-0.923-0.38j))(s-(-0.382-0.92j))$

$= s^2+0.923s+0.38js+0.382s+0.92js+1$

$3.16+2.32i$

$\cancel{= s^2+1.74+}$

$=(s^2+(1.3+1.3j)s+3.16+2.32i)$

$=[(s^2+(1.3-1.3j)s+(3.88-0.6i))][s^2+(1.3+1.3j)s+[(3.16+2.32i)]]$

$= s^4+(1.3+1.3j)s^3+3.16s^2+2.32is^3+$
$1.3s^2-1.3js^2+-3.38js^2+$

$(s^4+2.613s^3+3.14s^2+2.583s+1$

$(s^4+1.8477s^3+s^2+0.7653s^3+1.414s^2+0.7653+s^2+1.8477s+1$

$$H_n(s)=\frac{1}{s^4+1.8477s^3+4.109s^2+2.613s+1}$$

$(s^2+0.76536s+1)(s^2+1.8477s+1)$

(b) $H_a(s)=H_n(s)\Big|_{s=\frac{\Omega_c'}{s}}$

where $\boxed{\Omega_c'=\frac{\Omega_p}{\Omega_c}}$

$\Omega_c'=\frac{200}{1.07}$

$=\cancel{186.93}\ 186.09$ r/s. $\quad 187.09$

$$H_a(s)=\frac{1}{\left(\frac{187.09}{s}\right)^4+(1.8477)\left(\frac{187.09}{s}\right)^3+4.109\left(\frac{187.09}{s}\right)^2+2.613\left(\frac{187.09}{s}\right)+1}$$

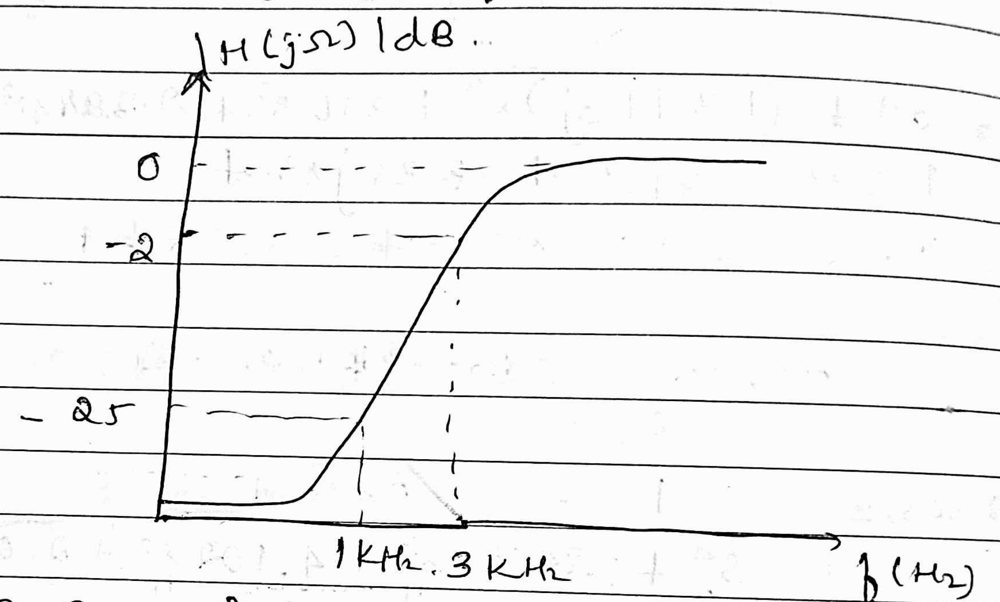
$$= \frac{84}{1.22\times10^{9}}$$

$$H_n(s) = \frac{1}{(s^2+0.07653s+1)(s^2+1.847s+1)}$$

(4) Design an analog Butterworth HPF with the pass band gain of $-2dB$ at 3 KHz & the minimum stop band attenuation of $-25dB$ at 1KHz.

angular meas.

Soln:



(1) $\Omega_p = 2\pi f_p$

$\Omega_s = 2\pi f_s$.

$\Omega_p = 2\pi(3) = 6\pi$ r/s.

$\Omega_s = 2\pi(1) = 2\pi$ r/s //

(1) High pass filter $\Omega'_s = \frac{\Omega_p}{\Omega_s} = \frac{6\pi}{2\pi} = 3r/s$

$\Omega'_p = 1r/s$ $\Omega'_s = 3r/s$

(2) $N \geq \frac{\log\{(10^{-0.1A_p}-1)/(10^{-0.1A_s}-1)\}}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$

$N \geq \frac{\log\{(10^{-0.1(-2)}-1)/(10^{-0.1(-25)}-1)\}}{2\log\left(\frac{1}{3}\right)}$

$$N \geq \frac{\log(0.584/315.22)}{-0.954}$$

$N \geq 2.86$

$\boxed{N \geq 3}$

(2) 3 dB cut off frequency,

$$\Omega_c = \frac{\Omega_p}{(10^{-0.1A_p}-1)^{1/2N}} = \frac{1}{(0.584)^{1/6}}$$

$$= 1.09 \ r/s$$

(3) For designing filters only LHP pole are required, so we exclude RHP pole from k values.

$\therefore$ k ranges from : $0 \leq K \leq N-1$. N = order of filter 0, 1, 2.

$$S_k = e^{j\pi/2} \cdot e^{j\pi/2N} \cdot e^{\frac{j\pi k}{N}}$$

$$S_0 = e^{j\pi/2} \, e^{j\pi/2 \cdot 3} \, e^{\frac{j\pi(0)}{3}}$$

$$= e^{j\pi/2} \, e^{j\pi/6} = e^{j(2/3\pi)} = -0.5 + 0.86j$$

$$S_1 = e^{j\pi/2} \, e^{j\pi/6} \, e^{\frac{j\pi(1)}{3}} = -1$$

$$S_2 = e^{j\pi/2} \, e^{j\pi/6} \, e^{j\pi(2)/3} = e^{j4\pi/3} = -0.5 - 0.866j$$

(4)
$$H_n(s) = \frac{1}{\pi(s-s_k)} = \frac{1}{(s-s_0)(s-s_1)(s-s_2)}$$

$$= \frac{1}{(s+(+0.5-0.86j))(s+1)(s+(+0.5+0.86j))}$$

$$= \frac{1}{(s+1)(s^2+0.5s+0.86js+0.5s-0.86js+1)}$$

$$= \frac{1}{(s+1)(s^2+1s+1)} = \frac{1}{s^3+s^2+s+s^2+s+1}$$

$$= \frac{1}{s_n^3+2s_n^2+2s_n+1}$$

(5)
$$H_a(s) = H_n(s)\Big|_{s \to \frac{\Omega_c'}{s}}$$

$$\Omega_c' = \frac{\Omega_p}{\Omega_c} = \frac{6\pi}{1.09} = 17.28 \ r/s$$

$$H_a(s) = H_n(s)\Big|_{s \to \frac{\Omega_c'}{s}}$$

$$= \frac{1}{\left(\frac{17.28}{s}\right)^3 + 2\left(\frac{17.28}{s}\right)^2 + 2\left(\frac{17.28}{s}\right) + 1}$$

$$= \frac{s^3}{(5159.78) + 597.19s + 34.56s^2 + s^3}$$

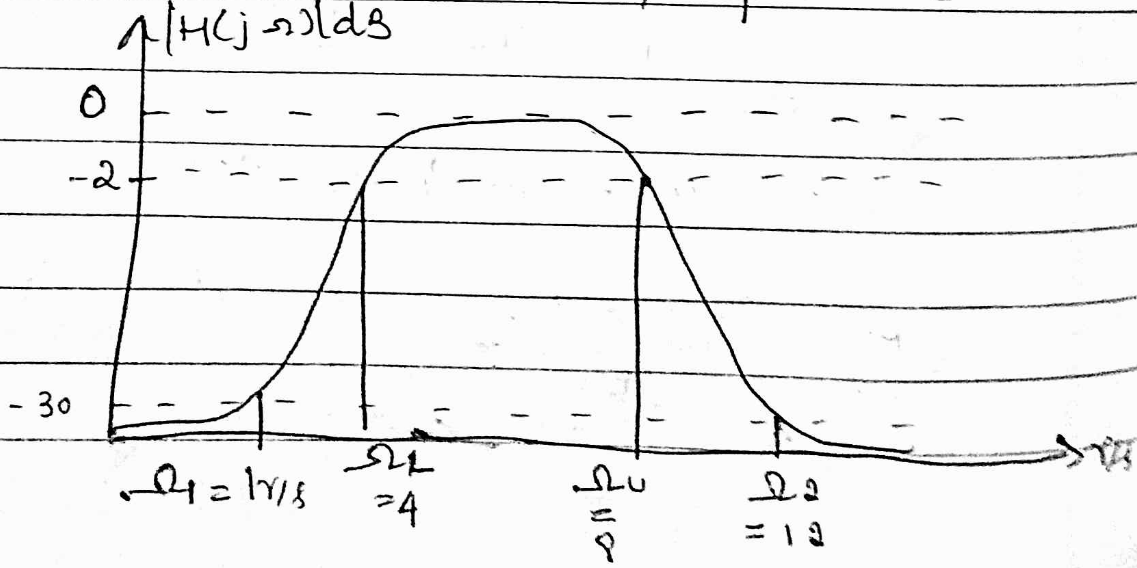
6. Design an analog Butterworth band pass filter to satisfy the following specification:

① Pass band gain of $-2\,dB$ at $4\,r/s$ & $8\,r/s$.

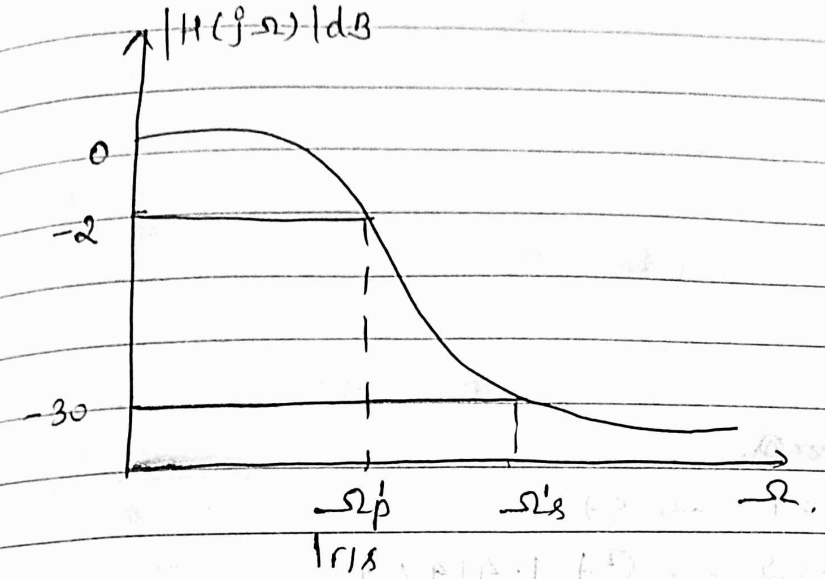
② min. stop band attenuation of $-30\,dB$ at $1\,r/s$ & $12\,r/s$.

Sol:- $A_p = -2\,dB$ at $4\,r/s$ & $8\,r/s$.

$A_s = -30\,dB$ at $1\,r/s$ & $12\,r/s$



This cant be designed directly as design equations are availiable only for normalized HPF.



$\Omega'_s = \min\{|A|, |B|\}$.

$$A = \frac{-\Omega_1^2 + \Omega_0^2}{B_0 \Omega_1}$$

$$B = \frac{\Omega_2^2 - \Omega_0^2}{B_0 \Omega_2}.$$

$\Omega_0^2 = \Omega_L \cdot \Omega_U = 32$ $\qquad B_0 = \Omega_U - \Omega_L = 4$

$$A = \frac{-1 + 32}{4} \qquad B = \frac{144 - 32}{4(12)}.$$

$$A = \frac{31}{4} \qquad B = 7/3$$

$$N \geq \frac{\log\{(10^{-0.1A_p} - 1)/(10^{-0.1A_s} - 1)\}}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}.$$

$\Omega'_s = \min\{|A|, |B|\} = 2.33$

$$= \frac{\log\{(10^{-0.1(-2)} - 1)/(10^{-0.1(-30)} - 1)\}}{2\log\left(\frac{1}{2.33}\right)}.$$

$$= \frac{\log\{0.0584/999\}}{2\log(1/2.33)} = 4.4$$

$N \geq 5$

3dB cutoff frequency

① $\Omega_c = \dfrac{\Omega_p}{(10^{-0.1\alpha_p} - 1)^{1/2N}}$

$$= \frac{1}{((0.584)^{1/10}} = 1.055 \text{ r/s}$$

③

$$S_k = e^{j\pi/2}\, e^{j\pi k/N}\, e^{j\pi/2N}.$$

~~Note~~.

$N=1$	$\rightarrow$	$s+1$
$N=2$	$\rightarrow$	$s^2+1.414s+1$
$N=3$	$\rightarrow$	s^3+2s^2+2s+1
$N=4$	$\rightarrow$	$(s^2+0.765s+1)(s^2+1.847s+1)$

Passband:-

① Design a Butterworth analog BPF that will meet the following specifications.

a) ~~Max. pass band~~ attenuation = 2dB.

b) ~~Passband edge frequency~~ = ~~200 rad/sec~~.

a) ~~Min stop b~~ -3.0103 dB upper & lower cutoff frequency of 50Hz & 20KHz.

b) a stop band attenuation of at least 20dB at 20Hz & 45KHz &.

c) a monotonic frequency respon.

Soln:- $\Omega_{a_1} = 125.6 \qquad \Omega_2 = 282.6 \times 10^3$

$\Omega_L = 314.15 \qquad \Omega_U = 125.6 \times 10^3$

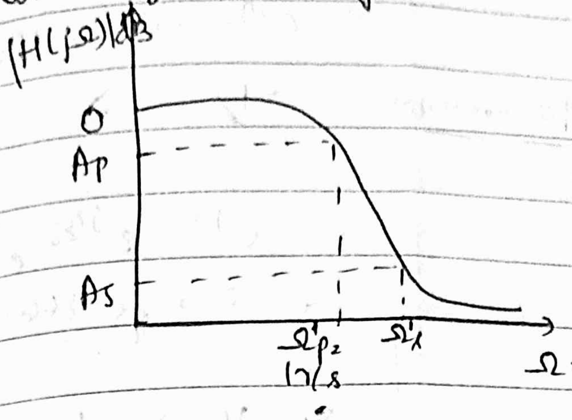
$B_0 = \Omega_U - \Omega_L = 125.6 \times 10^3 - 314.15$

$= 125.28 \times 10^3$

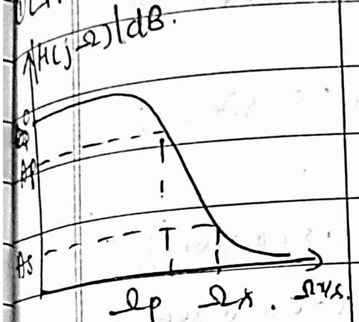
$y(n) = \{1, 9, 8, 8, 3, 0, 0, 0\}$

Analog to analog:

① Normalized / Prototype LPF.



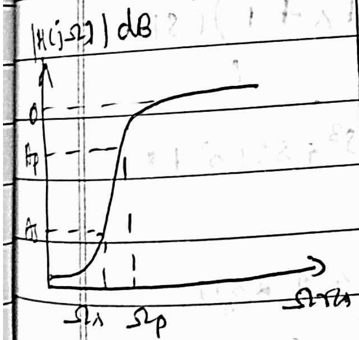
① LPF



① $\Omega'_s = \frac{\Omega_s}{\Omega_p}$ $\quad \Omega'_p = 1 \, r/s$

② $\boxed{S_n \to \frac{S}{\Omega'_c}}$ $\quad \Omega'_c = \Omega_p \times \Omega_c$

② HPF



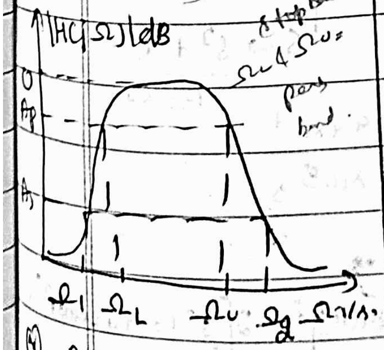
① $\Omega'_s = \frac{\Omega_p}{\Omega_s}$ $\quad \Omega'_p = 1 \, r/s$

② $\boxed{S_n = \frac{\Omega'_c}{S}}$ $\quad \Omega'_c = \frac{\Omega_p}{\Omega_c}$

③ BPF

Ω_1 & $\Omega_2 \to$ stop band

Ω_L & $\Omega_U \to$ pass band.



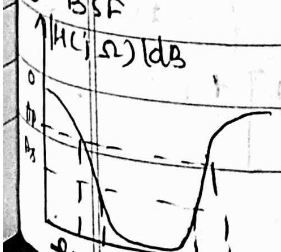
① $\Omega'_s = \min\{|A|, |B|\}$

$A = \frac{-\Omega_1^2 + \Omega_0^2}{B_0 \Omega_1}$ $\quad B = \frac{\Omega_2^2 - \Omega_0^2}{B_0 \Omega_2}$

$B_0 = \Omega_U - \Omega_L$ $\quad S_n \to \frac{S^2 + \Omega_0^2}{B_0 s}$.

$\Omega_0^2 = \Omega_L \cdot \Omega_U$

④ BSF



① $\Omega'_s = \min\{|A|, |B|\}$. $\quad B = \frac{\Omega_2^2 - \Omega_0^2}{B_0 \Omega_2}$

$A = \frac{-\Omega_1^2 + \Omega_0^2}{B_0 \Omega_1}$ $\quad \Omega_0^2 = \Omega_L \cdot \Omega_U$

$B_0 = \Omega_U - \Omega_L$

$S_n \to$

Band pass note

$\Omega_0^2 = \Omega_L \times \Omega_0$
$= 125.6\times10^3 \times 314.15$
$= 39457.24\times10^3$

$A = \dfrac{-\Omega_p^2 + \Omega_0^2}{B_0\,\Omega_1}$

$= \dfrac{15.775\times10^3 + 39457.24\times10^3}{15.73\times10^6}$

$= -2.51$

$B = \dfrac{\Omega_2^2 - \Omega_0^2}{B_0\,\Omega_2}$

$= \dfrac{\cancel{39457.24\times10^3} - 29823.31\times10^4}{}$
$= \dfrac{79862.76\times10^6 - 39457.24\times10^3}{125.286\times10^3 \times 282.6\times10^3}$

$= 2.25$

$\Omega' = \min\{|A|, |B|\}$
$= \{|-2.51|, 2.25\}$
$= 2.25$

$N \geq \dfrac{\log\{(10^{-0.1A_P}-1)/(10^{-0.1A_S}-1)\}}{2\log\left(\frac{\Omega'_p}{\Omega_s}\right)}$

$N \geq \dfrac{\log\{(10^{-0.1(3.0103)}-1)/(10^{-0.1(-20)}-1)\}}{2\log\left(\frac{1}{2.25}\right)}$

$N \geq \dfrac{\log(1/99)}{-0.7044}$

$N \geq 2.833$

$N = 3$

3dB cutoff frequency

$\Omega_c = \dfrac{\Omega'_p}{(10^{-0.1A_P}-1)^{1/2N}}$

$= \dfrac{1}{(1)^{1/6}} = 1\ \mathrm{r/s}$

$= 1.95\times10^{15}$

$s^6 +$

③ Poles

$s_k = e^{j\pi/2}\cdot e^{\frac{j\pi}{2N}}\cdot e^{\frac{j\pi k}{N}}$; $k = 0, 1, \ldots$

$s_0 = e^{j\pi/2}\cdot e^{j\pi/6}$
$= -0.5 + 0.866i$

$s_1 = e^{j\pi/2}\cdot e^{j\pi/6}\cdot e^{j\pi/3}$
$= -1$

$s_2 = e^{j\pi/2}\cdot e^{j\pi/6}\cdot e^{j2\pi/3}$
$= -0.5 - 0.866i$

$H_n(s) = \dfrac{1}{\prod (s - s_k)}$

$= \dfrac{1}{(s-s_0)(s-s_1)(s-s_2)}$

$= \dfrac{1}{(s-(-0.5+0.866i))(s-1)(s-(-0.5-0.866i))}$

$= \dfrac{1}{(s^2+1s+1)(s+1)}$

$= \dfrac{1}{s^3+s^2+s^2+s+s+1}$

$= \dfrac{1}{s_n^3+2s_n^2+2s_n+1}$

$H_a(s) \to H_n(s)$ | $s \to \dfrac{s^2+\Omega_0^2}{B_0 \times s}$

$s \to \dfrac{s^2+3.94\times10^7}{1.25\times10^5\, s}$

$= \dfrac{1}{\left(\dfrac{s^2+3.94\times10^7}{1.25\times10^5 s}\right)^3 + 2\left[\dfrac{s^2+3.947\times10^7}{1.25\times10^5 s}\right]^2 + 2\left[\dfrac{s^2+3.94\times10^7}{1.25\times10^5 s}\right] + 1}$

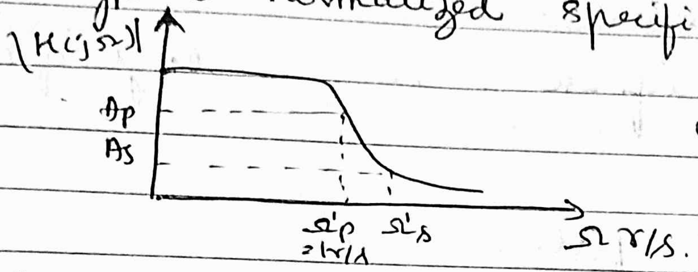
$2s^{2} + 15.76\times10^{14}$
3.10×10^{5}

30/11 **Analog filter : (all pole filter)**

$$H_a(s) = \frac{N^r\ poly}{D^r\ poly} = \frac{zeroes}{poles} = \frac{1}{(\ )(\ )}$$

Design rules / procedure is available only to design normalized / prototype LPF.

(1) Transform / convert the given specification into prototype or normalized specification $\Omega'_p = 1\,r/s$.



Normalized spec $\Omega'_p = 1r/s$.

LPF	HPF	BPF	BSF.
$\Omega'_s = \frac{\Omega_s}{\Omega_p}$	$\Omega'_s = \frac{\Omega_p}{\Omega_s}$	$\Omega'_s = \min\{\lvert A\rvert, \lvert B\rvert\}$	$\Omega'_s = \min\{\lvert A\rvert, \lvert B\rvert\}$.

(2) $N \geq \log\{(10^{-0.1A_p} - 1)/(10^{-0.1A_s} - 1)\} / 2\log(\Omega_p/\Omega_s)$.

(3) 3dB cut off frequency $\Omega_c = \frac{\Omega_p}{(10^{-0.1A_p} - 1)^{1/2N}}$ or $\frac{\Omega_s}{(10^{-0.1A_s} - 1)^{1/2N}}$ or $Avg\left\{\frac{\Omega_p}{(\ )} + \frac{\Omega_s}{(\ )}\right\}$.

(4) $S_k = e^{j(\pi/2 + \pi k/N + \pi/2N)}$, $0 \leq k \leq 2N-1 \Rightarrow$ LHP + RHP.

(5) $H_n(s)$ = normalized / prototype T.F.

$H_n(s) = \frac{1}{B_N(s)}$ --> Butterworth polynomial or $\frac{1}{\pi(s - s_0)}$

(6) $H_a(s) \rightarrow$ required type of filter with desired freq given in specification.

$H_a(s) = H_n(s)\Big|_{s_n \rightarrow \Theta}$

LPF	HPF	BPF.
$s_n \rightarrow s/\Omega'_c$	$s_n \rightarrow \frac{\Omega'_c}{s}$	
$\Omega'_c = \Omega_c \times \Omega_p$	$\Omega'_c = \frac{\Omega_p}{\Omega_c}$	

Analog filter of required type with desired freq = H_a

$\rightarrow$ 1.95×10^{15}

$(s^6 + 4.65\times10^5 s^4 + 11.28\times10^7 s^2 + 6.1\times10^{22}) + (2.5 s^5 + 9.85\times10^7 s^3 + 1.95\times10^{15} s^3)$

$(15.52\ s) + (312.5\ s^2) + 12.3125\ s^2$, 30/11/21

Module-3

Digital filter design { IIR, FIR

length of impulse response

Digital filter

impulse response → when impulse is given as i/p how the system performs

IIR: length of h(n) is infinite

FIR: length of h(n) is finite

H(Z)

Digital IIR filter:-

① All pole filter.

② It digital domain is designed / derived from analog filter because the processing of all the infinite length h(n) samples is not possible in digital domain.

① Backward transformation
② Matched z transform.
③ Impulse invariance (IIV)
④ Bilinear transformation (BLT).

} to convert $H_a(s)$ to $H(z)$.

Impulse invariant transformation:- (IIV).

① Using sampling of analog signal to get the digital is utilized in IIT (Impulse invariance)

② ∴ The impulse response h(n) corresponds to corresponds to digital filter H(z) is a sampled version of impulse response h(t) corresponding of $H_a(s)$.

③ This done to preserve the frequency response characteristics of analog filter.

④ The frequency response of digital filter will be identical with the frequency response corresponding to analog filter.

Date
Page

(5) If the sampling period P is selected sufficiently small to minimize the effect of aliasing is minimized or sampling frequency fs should be high to avoid completely the effect of alaising. → P must be small → fs should be high if aliasing reduce

$H_a(s)$

Inverse {H(n)} ⟹ $H(z)$
↓ ⇓ Inverse ZT{ H(z)}

$h(t)\Big|_{t=nT}$ = $h(n)$

T should be small to reduce aliasing.
f_s → should be high (5KHz ne kalum koodathe).

(6) Wkt, the T.F (transfer function) of analog filter is given by.

Transformation b/w S & Z

$$H_a(s) = \sum_{i=1}^{N} \frac{A_i}{S-P_i} = \frac{A_1}{S-P_1} + \frac{A_2}{S-P_2} + \frac{A_N}{S-P_N}$$

~~Analog & digital poles in IIR.~~

are $A_1, A_2, \ldots A_N$ gain corresponding to filter. $P_1, P_2, \ldots P_N$ poles corre ". to filter

(7) L 2. On taking inverse Laplace transform on H(s) we get h(t).

$$L^{-1}\{H_a(s)\} = h(t).$$

$$h(t) = \sum_{i=1}^{N} A_i \left(\frac{1}{S-P_i}\right).$$

wkt, $\left(\frac{1}{2-a}\right)$ Inverse L transform = $e^{-at} \cdot u(t)$

$$h(t) = \sum_{i=1}^{N} A_i\, e^{-P_i t} \cdot u(t)$$

If $h(t)$ is sampled at T to get $h(n)$.

$$h(n) = h(t)\Big|_{t=nT} = \sum_{i=1}^{N} A_i e^{-P_i nT} u(n)$$

on taking Z transform of $h(n)$,

$$Z\{h(n)\} = H(z)$$

$$H(z) = \sum_{i=1}^{N} A_i \left(\frac{1}{1-e^{-P_i T}z^{-1}}\right) \quad (*)$$

$$Z\{e^{-anT} u(n)\} = \frac{1}{z-a} \text{ or } \frac{1}{1 \pm z^{-1}a}$$

① T.F of $h(t)$ ② sampled transform $t = nT$ ③ Z transform

Transformation:- (* & **)

Imp

$$\boxed{\frac{1}{s-P_i} \Longleftrightarrow \frac{1}{1-e^{-P_i T} z^{-1}}}$$

poles. analog pole. Digital pole

V.V. Imp (CIA12M)

① The designed T.F of the filter should represent a stable & causal system.

② For stability & causality of analog filter Ha(s) should satisfy the following requirement.

① H(s) should be a rational fn of s & the coefficients should be real.

② The poles (P_i) should lie on LHP of s plane.

③ No. of zeroes should be less than or equal to the number of poles.

H(s) should be a rational fn of s & coefficient should be real.

→ For stability & causality of digital filter H(z) the following requirements should be met.

① H(z) should be a rational function of z & the coefficients must be real.
② Poles of H(z) should lie inside the unit circle.
③ No. of zeroes should be less than or equal to no. of poles.

∴ From the above pts, it is clear that for the proper mapping of an analog domain to equivalent digital domain the following properties must be satisfied.

① The $j\omega$ axis in the s plane must map onto the unit circle
② The LHP of s plane must map inside the unit circle.

30/11

→ Digital IIR filter can't be designed directly

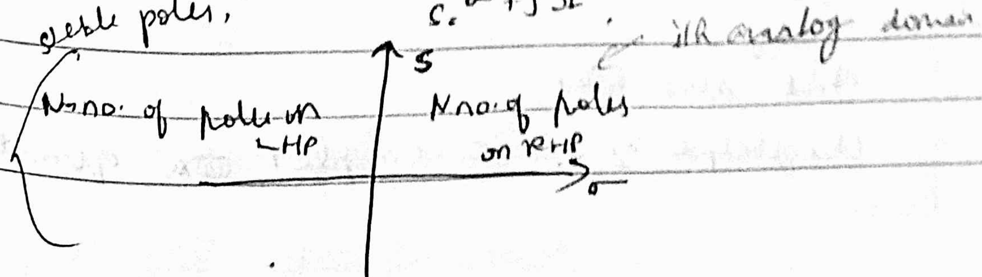
Transformations — IIV, BLT } IIR
$H_a(s) \Rightarrow H(z)$.

→ one-to-one mapping.
$\Omega \to \omega$.

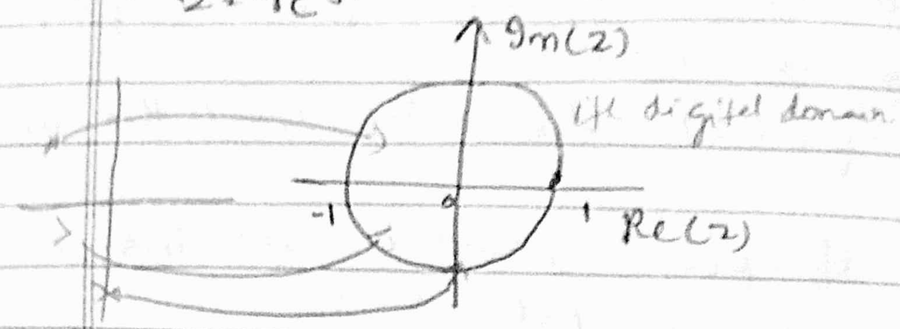
→ ① stable and causal system.
causal → i/p is countable (integrably).

$\sum_{n=-\infty}^{\infty} h(n) = \text{finite}$ stable.

causal.



$z = re^{j\omega}$



Of poles na ee circle lot match cheyeram. LHP will be inside unit circle.
Circle lode agath ente - system causal.

$$\boxed{\frac{1}{s-p_i} \Rightarrow \frac{1}{1-e^{-p_iT}z^{-1}}}$$

Imp (10M) Relation b/w analog & digital filter poles in impulse invariant transformation:- (3M)

The analog poles are of the form $s+p_i \Rightarrow s = -p_i$ and the digital poles
poles LHP ayathin $s+p_i$ edutha.
are of the form $1-e^{-p_iT}z^{-1}$ leading to

$z = e^{-p_iT}$

Considering the digital pole $z = e^{-p_iT}$ and substituting $-p_i = s_i$ the digital pole z

digital $\boxed{z = e^{s_iT}}$ analog

Wkt the digital pole $s_i = \sigma_i + j\Omega_i$

$$z = e^{(\sigma_i + j\Omega_i)T} = e^{\sigma_iT} \cdot e^{j\Omega_iT}$$

And also wkt,
digital pole z is a complex quantity

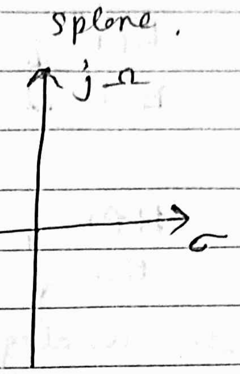
can be written as

$$\boxed{\begin{array}{l} |z| = e^{\sigma i T} \\ \angle z = e^{\Omega i T} \end{array}}$$

third wise

① If $\sigma < 0$, the poles lie on left half plane of s plane, similarly in the z plane

$\boxed{|z| < 1}$ inside of unit not appeal bottom.

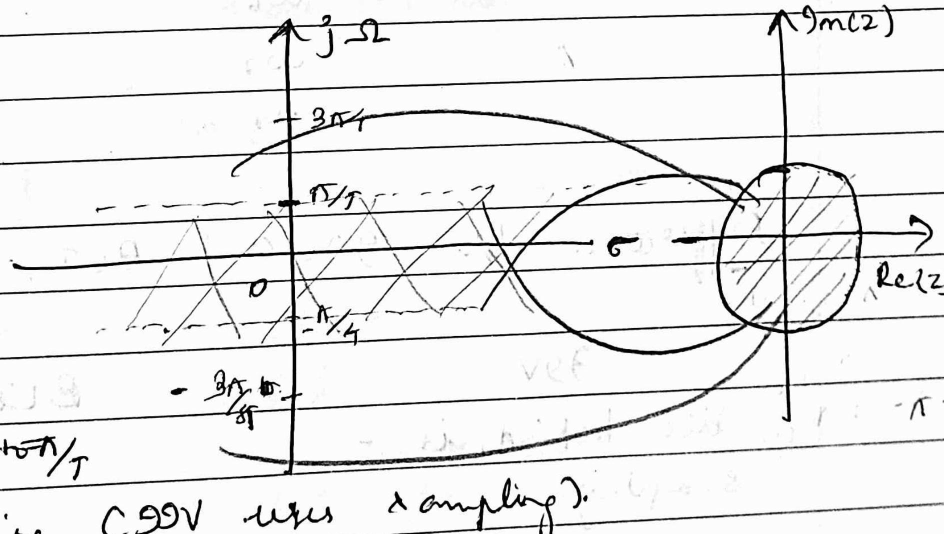


$s_i = \sigma + j\Omega$.
poles ($i = 0 \ldots n-1$).

② If $\sigma = 0$ the poles of the splane will lie on $j\Omega$ axis (imaginary axis).
& $\boxed{|z| = 1}$

③ If $\sigma > 0$ poles on the RHP of s plane &
$\boxed{|z| > 1}$

S-domain



→ Sampling (IIV uses sampling).
Impulse Invariant IIV many - to one mapping.

→ Impulse Invariant is many to one mapping.
To eliminate it make (to eliminate affect of aliasing
① $(T \ll)$ T less (sufficiently period,
② (Sampling frequency) >>> sampling photo (increase)

T = sampling period.

IIV	BLT
Idea ① sampling ② Many to one ③ work efficiently with LPF & ~~LBB~~ BPF. ④ not possible with HPF ~~BSF~~ & BSF	

LPF, BPF } $f_s >>>$

HPF, BSF } not at all possible.

→ Ω → analog , digital → ω.

Many to one.

$\Omega_1 - \omega_1$

$\Omega_2 - \omega_2$

$\Omega_3 - \omega_3$

$\Omega_4 - \omega_4$

ω_5

ω_6

ω_7

ω_8

⋮

Difference b/w IIV and BLT :-

IIV	BLT.
① Idea behind it - sampling.	
② Many to one mapping	
③ Works efficiently with LPF & BPF.	
④ Not possible with HPF & BSF.	

Imp

Relationship between analog frequency (Ω) and digital (ω) in IIV:

$$S + P_i \Rightarrow S = -P_i$$

$$1 - e^{-P_i T} z^{-1} \Rightarrow z = e^{-P_i T}$$

$$z = e^{-P_i T}$$

$$\boxed{z = e^{S_i T}}$$

$$S_i = \sigma_i + j\Omega_i$$

$$z = e^{(\sigma_i + j\Omega_i)T} = e^{\sigma_i T} \cdot e^{j\Omega_i T}$$

$$\boxed{\begin{array}{l} |z| = e^{\sigma_i T} \\ \angle z = e^{\Omega_i T} \end{array}}$$

Considering the phase angle on the both the side

$$r e^{j\omega} = e^{\Omega_i T}$$

$$\boxed{\omega = \Omega_i T} \quad \text{(equating phase)}$$

$$\Rightarrow \boxed{\Omega = \frac{\omega}{T}}$$

digital — digital

- IIV is not a proper transformation, many to one. aliasing undergoes.

Formula (Imp)

$$\frac{1}{S + P_i} \Rightarrow \frac{1}{1 - e^{-P_i T} z^{-1}} \quad (1)$$

$$\frac{S + a}{(S+a)^2 + b^2} \Rightarrow \frac{1 - e^{-aT}(\cos bT) z^{-1}}{1 - 2e^{-aT}(\cos bT) z^{-1} + e^{-2aT} z^{-2}}$$

(2)

$$\frac{b}{(s+a)^2+b^2} \Rightarrow \frac{1-e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$\rightarrow (2)$

Problem:-

$$\boxed{\Omega = \frac{\omega}{T}} \qquad \boxed{z = e^{sT}}$$

many to one.
LPF, BPF, poss.
BSF, HPF - not possible

① Transform the given analog filter into an equivalent digital filter using impulse invariance transformation for the value of

(1) $T = 1\,sec$ (2) $T = 0.5\,sec$.

Soln:

$$H(s) = \frac{2}{s^2+3s+2}$$

$$H(s) = \frac{2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$2 = A(s+2) + B(s+1)$$

Put $s = -2$.

$2 = -B$

$\boxed{B = -2}$

Put $s = -1$.

$\boxed{A = 2}$

$$H(s) = \frac{2}{(s+1)} + \frac{-2}{(s+2)}$$

It is of form eq ①.

$$\boxed{H(z) = \frac{2}{1-e^{-1T}z^{-1}} - \frac{2}{1-e^{-2T}z^{-1}}}$$

① $T = 1$,

$$H(z) = \frac{2}{1-e^{-1}z^{-1}} \quad \frac{-2}{1-e^{-2}z^{-1}}$$

$$= \frac{2}{\left(1 - \frac{1}{ez}\right)} - \frac{2}{1 - \frac{1}{e^2 z}}$$

$$= \frac{2ez}{ez - 1} - \frac{2e^2 z}{(e^2 z - 1)}$$

$$= \cancel{2ez(1 - 2e)}$$

$$= \cancel{\frac{2}{\left(1 - \frac{0.36^1}{(0.36)^2}\right)} - \frac{2}{1 - \left(\frac{1}{0.135z}\right)}}$$

$$= \cancel{\frac{0.36z}{(0.36z - 1)} - \frac{2(0.135z)}{(0.135z - 1)}}$$

$$= \cancel{\frac{0.36z(0.135z - 1) - 2(0.27z)}{0.048z^2 - 0.36z - 0.135z + 1}}$$

$$= \frac{2(1 - e^{-2}z^{-1}) - 2(1 - e^{-1}z^{-1})}{1 - e^{-2}z^{-1} - e^{-1}z^{-1} + e^{-3}z^{-2}}$$

$$= \frac{2 - 2e^{-2}z^{-1} - 2 + z^{-1}e\,2}{1 - e^{-3}z^{-1} + e^{-3}z^{-2}}$$

$$= \frac{0.465z^{-1}}{1 - 0.503z^{-1} + 0.049z^{-2}}$$

(2) $T = 0.5$.

$$H(z) = \frac{2(1 - e^{-2T}z^{-1}) - 2(1 - e^{-1T}z^{-1})}{(1 - e^{-T}z^{-1})(1 - e^{-2T}z^{-1})}$$

$$= \frac{2 - 2e^{-2T}z^{-1} - 2 + 2e^{-T}z^{-1}}{1 - e^{-2T}z^{-1} - e^{-T}z^{-1} + e^{-3T}z^{-2}}$$

$T = 0.5$.

$$= \frac{-2e^{-2(0.5)}z^{-1} + 2e^{-0.5}z^{-1}}{1 - e^{-1}z^{-1} - e^{-0.5}z^{-1} + e^{-1.5}z^{-2}}$$

$$H(z) = \frac{0.477z^{-1}}{1 - 0.974z^{-1} + 0.223z^{-2}}$$

20/11/21.

2. Transform the given analog filter into the equivalent digital filter using IIV.

$$H(S) = \frac{S+0.1}{(S+0.1)^2+9}$$

I thomndille cyil T=1 sec eduk

Soln:- $T = 1\,sec.$ $a = 0.1$
$b = 3$

From formula (2).

$$\frac{S+a}{(S+a)^2+b^2} \Rightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-1}}$$

$$H(s) = \frac{s+0.1}{(S+0.1)^2+3^2}.$$

$$H(z) = \frac{1-e^{-0.1T}(\cos(3T))z^{-1}}{1-2e^{-0.1T}\cos(3T)z^{-1}+e^{-2\times0.1T}z^{-2}}.$$

$$= \frac{1-e^{-0.1}\cos(3)z^{-1}}{1-2e^{-0.1}\cos(3)z^{-1}+e^{-0.2}z^{-2}}.$$

$$= \frac{1+0.896z^{-1}}{1+1.79z^{-1}+0.819z^{-2}}$$

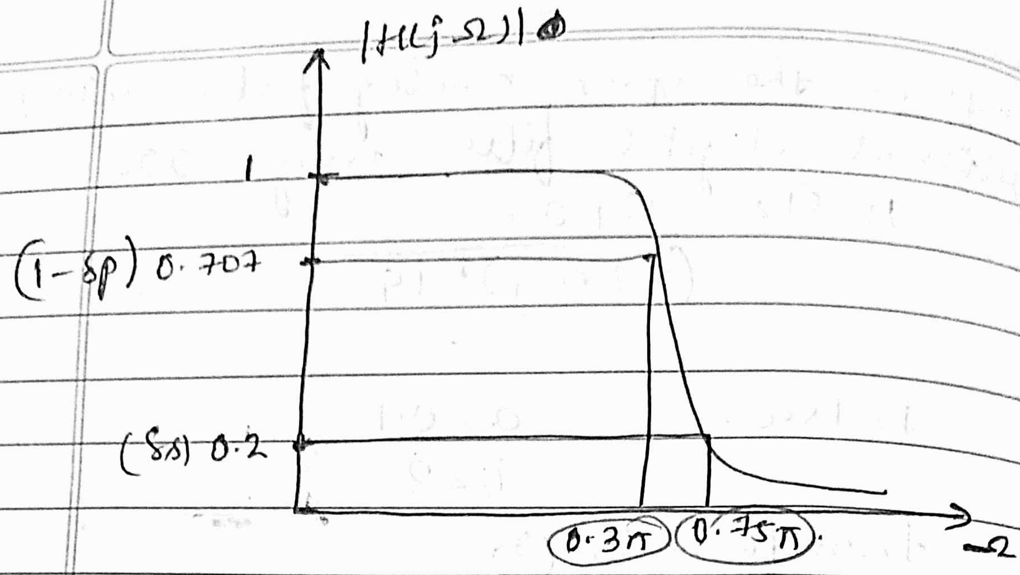
T = sampling period $\Rightarrow$ sampling frequency $= \frac{1}{T}$

Imp

3. Design a Butterworth digital IIR LPF using impulse IIV transformation by considering $T = 1\,sec$ (sampling period / sampling frequency) to satisfy the following specifications.

$0.707 \le |H(j\omega)| \le 1.0$ $\quad 0 \le \omega \le 0.3\pi$

$|H(j\omega)| \le 0.2,$ $\quad 0.75\pi \le \omega \le \pi.$



δ_p = pass band ripple.

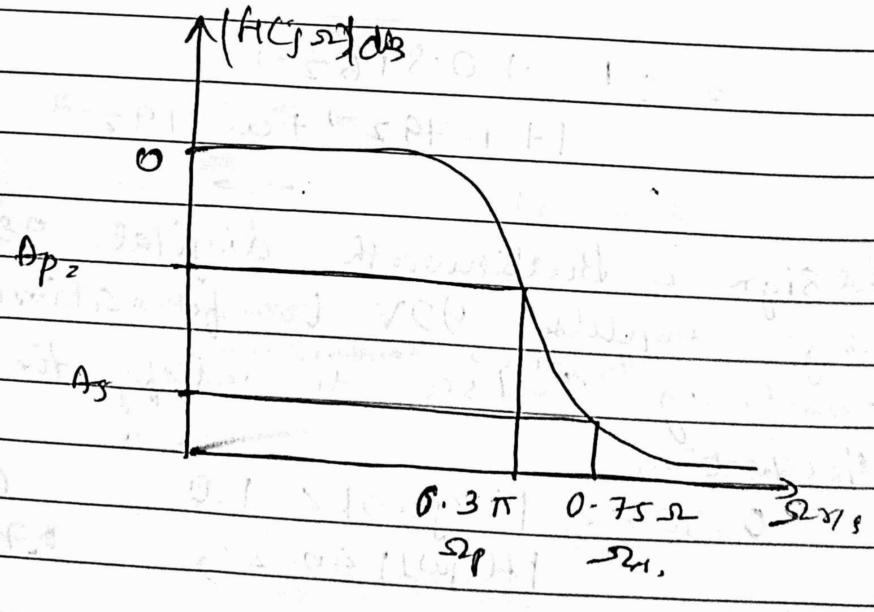
① Conversion of given digital ω into an equivalent analog Ω.
In the case of IIV the relation b/w analog & digital frequencies are given by

$$\Omega = \frac{\omega}{T}$$

$$\Omega_p = \frac{\omega_p}{T} \quad ; \quad \Omega_s = \frac{\omega_s}{T}$$

$\boxed{\Omega_p = 0.3\pi \text{ r/s}}$ $\boxed{\Omega_s = 0.75\pi \text{ r/s}}$

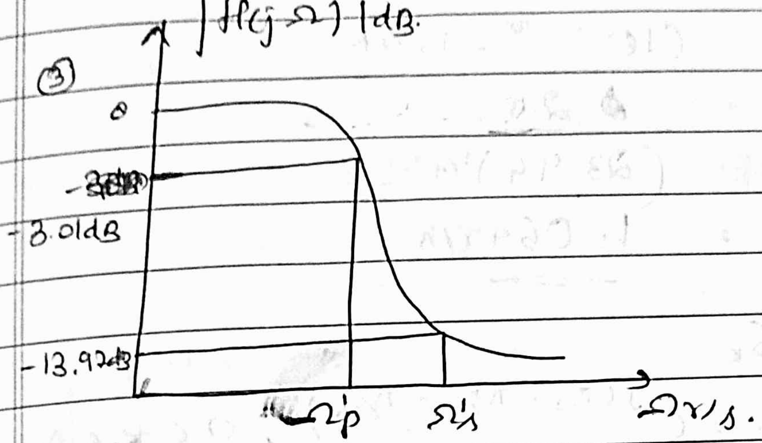
Normalized / prototype LPF:



$A_p = 20\log(1-\delta_p) = 20\log(0.707)$
$= -3.01\,dB$

$A_s = 20\log \delta_s = 20\log(0.2)$
$= -13.97\,dB$

③



$$\Omega'_s = \frac{\Omega_s}{\Omega_p} = \frac{0.75\Omega}{0.3\pi}$$

$$\Omega'_s = 2.5\,r/s$$

④ $$N \geq \frac{\log\{(10^{-0.1A_p}-1)/(10^{-0.1A_s}-1)\}}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$N \geq \frac{\log\{(10^{-0.1(-3.01)}-1)/(10^{-0.1(13.97)}-1)\}}{2\log(1/2.5)}$$

$$N \geq \frac{\log(0.999/23.94)}{-0.795}$$

$N \geq$ ~~0.08~~ 1.79

$\boxed{N = 2}$

⑤ 3dB cut off frequency

$$\Omega = \frac{\Omega_p}{(10^{-0.1A_p}-1)^{1/2N}}.$$

$$= \frac{1}{(0.99)^{1/2\times 2}}$$

$$= 1.0025\,r/s \approx 1.003\,r/s$$

$$\Omega_c = \frac{\Omega_s}{(10^{-0.1A_s} - 1)^{1/2N}}$$

$$= \frac{2.5}{(23.94)^{1/4}} \quad N=2$$

$$= 1.064\,r/s$$

(6) Poles S_K

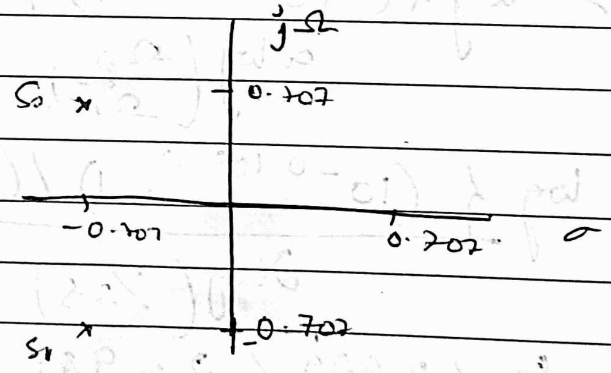
$$S_K = e^{j(\pi/2 + \pi k/N + \pi/2N)} \; ; \; 0 \le K \le N-1$$

$$S_0 = e^{j(\pi/2 + 0 + \pi/4)}$$

$$= -0.707 + j0.707j$$

$$S_1 = e^{j(\pi/2 + \pi/2 + \pi/4)}$$

$$= -0.707 - 0.707j$$



(7) $$H_N(s) = \frac{1}{\prod(s - s_k)} = \frac{1}{(s - s_0)(s - s_1)}$$

$$= \frac{1}{(s + 0.707 - 0.707j)(s + 0.707 + 0.707j)}$$

$$\boxed{H_N(s) = \frac{1}{s_n^2 + 1.414 s_n + 1}}$$

(8) $H_a(s) = H_N(s)\Big|_{s_n \to s/\Omega_c'}$

$\Omega_c' = \Omega_c \times \Omega_p = 1.1003 \times 0.3\pi$

$\Omega_c' = 0.944\ r/s.$

$$H_a(s) = \frac{1}{\frac{s^2}{(0.944)^2} + 1.414\left(\frac{s}{0.944}\right) + 1}$$

$$= \cancel{\frac{}{1.29 + 1.606s + 1}}$$

$$\cancel{H_a(s) = \frac{0.774}{s^2 + 1.244s + 0.774}}$$

$$H_a(s) = \frac{0.88}{s^2 + 1.329s + 0.888}$$

2/12/21

(9) In 3dB $\frac{1}{Dr}$ then $\Omega_c' = \Omega_c \times \Omega_p$ is necessary if not we cant substitute directly.

Bring it to $\frac{b}{(s+a)^2+b^2}$ form

$s^2 + 1.319s + 0.870$

$s^2 + 2as + a^2$

$2a \to 1.319$

$a \to \frac{1.319}{2}$

$a = 0.6595$

$$\cancel{H_a(s) = \frac{0.659}{(s+0.659)^2}}$$

$2as \to$ (a) | $2as \to 1.319$

$a = 0.659$

-0.434

$(s+0.659)^2$

$0.88 - 0.434$

$$H_{c(a)}(s) = \frac{0.88}{s^2 + 1.319s + 0.88}$$

$s^2 + as + (b)^2$

ithine nommal $(s+a)^2 + b^2 = s^2 + 2as + a^2 + b^2$ maattan

$s^2 + 1.319s + 0.88$

$a = 1.319s, \; b = 0.88$

ithine $s^2 + as + b$ ne maatam

So $s^2 + as + \left(\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 + b^2$

$\left(s + \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 + b^2$

$\left(s + \frac{1.319}{2}\right)^2 - \left(\frac{1.319}{2}\right)^2 + (0.88)$

$(s+0.66)^2 + 0.445$ ith b

Namak vendeth. puthiya b^2.

$(s+0.66)^2 + (0.667)^2$

$(s+a)^2 + (b)^2$

$$H_a(s) = \frac{0.667}{(s+0.66)^2 + (0.667)^2}$$ ivide nomek 0.88 venam. so $\frac{0.88}{0.667}$

$$H_a(s) = 1.319 \left[\frac{0.667}{(s+0.66)^2 + (0.667)^2}\right]$$

$$H(z) = \frac{1 - e^{-aT}(\sin bT)z^{-1}}{1 - 2e^{-aT}(\cos bT)z^{-1} + e^{-2aT}z^{-2}}$$

$$= \frac{1 - e^{-(0.66)\times 1}\sin(0.667)z^{-1}}{1 - 2e^{-0.66}\cos(0.667)z^{-1} + e^{-2(0.667)}z^{-2}}$$

$$= \left(\frac{1 - 0.319z^{-1}}{1 - 0.812z^{-1} + .0263z^{-2}}\right) \times 1.319$$

2. Design a Butterworth digital LPF using IIV transformation taking T = 1sec to satisfy the following specification.

$$0.7 \le |H(j\omega)| \le 1 \;;\; 0 \le \omega \le 0.35\pi.$$

$$|H(j\omega)| \le 0.275 \;;\; 0.75\pi \le \omega \le \pi.$$

Soln:- ① $(1-\delta p) = 0.707 \Rightarrow Ap = 20\log(0.707) = -3.01dB$

Sy LPF for IIV.

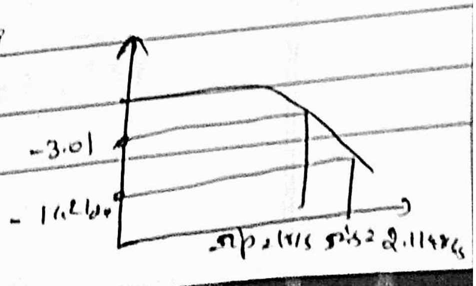
$\delta s = 0.275 \Rightarrow As = 20\log(0.275) = -11.21dB.$

② Analog PB & stop band edge frequency from the given PB / SB frequency.

$$\Omega p = \frac{\omega p}{T} \;;\; \Omega s = \frac{\omega s}{T}$$

$$\Omega p = 0.35\pi \, r/s \;;\; \Omega s = 0.75\pi \, r/s$$

③ $$\Omega' s = \frac{\Omega s}{\Omega p} = \frac{0.75\pi}{0.35\pi} = 2.14 \, r/s$$



④ $N \geq \dfrac{\log\{(10^{-0.1A_p}-1)/(10^{-0.1A_s}-1)\}}{2\log\left(\dfrac{\Omega_p'}{\Omega_s'}\right)}$.

$= \dfrac{\log\{(10^{-0.1(-3.01)}-1)/(10^{-0.1(-11.21)}-1)\}}{2\log\left(\dfrac{1}{2.14}\right)}$.

$= \dfrac{\log\{0.999/}{}$

≥ 1.638

$N = 2$

⑤ 3dB cutoff frequency

$\Omega = \dfrac{\Omega_p'}{(10^{-0.1A_p}-1)^{1/2N}}$.

$= \dfrac{1}{(10^{-0.1(-3.01)}-1)^{1/4}}$

$= 1.0002\ r/s$

need not determine poles; it already u know only here.

⑥ $H_N(s) = \dfrac{1}{B_N(s)} = \dfrac{1}{s_n^2+1.414s_n+1}$

⑦ $H_a(s) = H_N(s)\Big|_{s \to \frac{s}{\Omega_c'}}$

$\Omega_c' = \Omega_p \times \Omega_c$

$= 0.357 \times 1.0002$

$= 1.09\ r/s$

$$H_a(s) = \frac{1}{\left(\frac{s}{1.09}\right)^2 + 1.414\left(\frac{s}{1.09}\right) + 1}$$

$$= \frac{1.188}{s^2 + 1.541s + 1.188}$$

$$= \frac{1.188}{s^2 + 1.541s + \left(\frac{1.541}{2}\right)^2 - \left(\frac{1.541}{2}\right)^2 + 1.188}$$

b^2

$$= \frac{1.188}{(s + 0.77)^2 + 0.595}$$

$$= \frac{1.188}{(s+0.77)^2 + (0.771)^2}$$

$$= 1.54 \left[\frac{0.771}{(s+0.77)^2 + (0.771)^2}\right]$$

formula

$$= 1.54\left[\frac{1 - e^{-0.77} \sin(0.771) z^{-1}}{1 - 2e^{-0.77}\cos(0.771) z^{-1} + e^{-2(0.77)} z^{-2}}\right]$$

$$= 1.54\left[\frac{1 - 0.3227 z^{-1}}{1 - 0.664 z^{-1} + 0.214 z^{-2}}\right]$$

$$= \frac{1.54 - 0.497 z^{-1}}{1 - 0.664 z^{-1} + 0.214 z^{-2}}$$

Verification of designed analog filter:-

$$H_a(s) = \frac{1.188}{s^2 + 1.541s + 1.188}$$

Substitute $s = j\omega$.

$$H_a(j\Omega) = \frac{1.188}{\underbrace{(j\Omega)^2}_{-\Omega^2} + j(1.5418\,\Omega) + 1.188}$$

At $\Omega = \Omega_p = 0.35\pi$ r/s.

$$20\log|H_a(j\Omega)| = A_p = -3.01\,dB$$

$$|H_a(j\Omega)|_{\Omega=\Omega_p=0.35\pi} = \frac{1.18\pi}{-(0.35\pi)^2 + j1.5418 \times 0.35\pi + 1.188}$$

$$= \frac{1.18\pi}{-1.209 + j(1.694) + 1.188}$$

$$|H_a(j\Omega)| = \frac{1.188}{-0.021 + j(1.694)}$$

$$|H_a(j\Omega)| = \frac{1.188}{\sqrt{(0.021)^2 + (1.694)^2}}$$

$$= 0.702$$

$$A_p = 20\log|H_a(j\Omega)| = 20\log(0.70)$$

$$= -3.09\,dB$$

∴ Verified. Koranch diff value nadd Kalagam. Ω_c formula

Module - 2

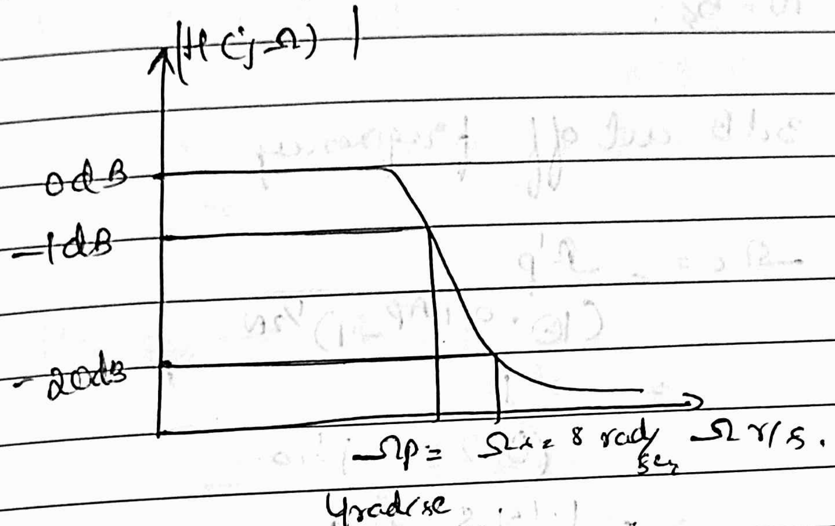
1. Butterworth LPF specification.

Passband gain = $A_p = -1dB$ at $\Omega_p = 4 rad/sec$.

Stop band attenuation greater than or equal to 20dB at $\Omega_s = 8 rad/sec$.

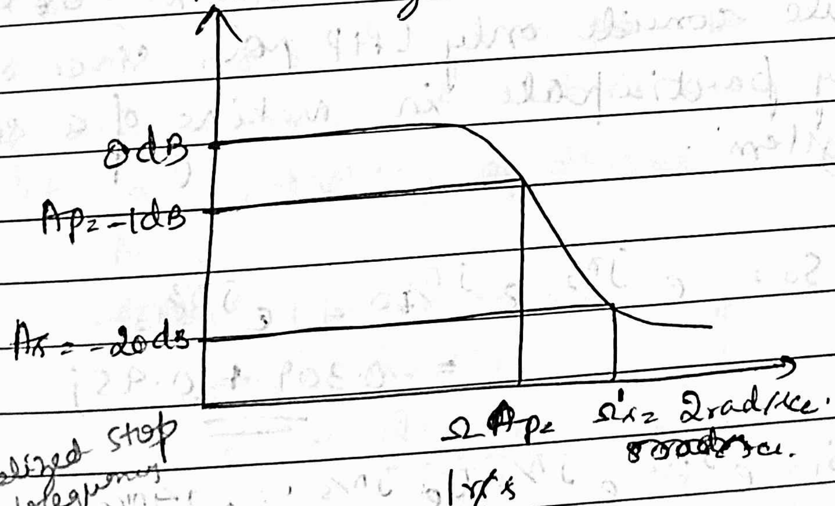
Determine T.F $H_a(s)$ of the lowest order Butterworth filter to.

Soln:-



The above ~~design~~ specifics can't be ~~designe~~ designed directly so convert them to normalized LPF.

Normalized LPF



normalized stop band frequency

Ω'_s for a LPF is $\Omega'_s = \frac{\Omega_s}{\Omega_p} = \frac{8}{4} = 2 rad/s$

① Calculation of order.

$$N \geq \frac{\log\{(10^{-0.1A_p} - 1) / (10^{-0.1A_s} - 1)\}}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}.$$

$$\geq \frac{\log\left(\left(10^{-0.1(-1)}-1\right)/\left(10^{-0.1(-20)}-1\right)\right)}{2\log\left(\frac{1}{2}\right)}$$

$$\geq \frac{\log(0.258/99)}{-0.6021}$$

$N \geq$ ~~...~~ 4.29

$N = 5$ ✓.

② 3dB cut off frequency

$$\Omega_c = \frac{\Omega'_p}{(10^{-0.1A_P}-1)^{1/2N}}$$

$$= \frac{1}{(0.258-1)^{1/10}}$$

$= 1.145$ r/s

③ To calculate poles

$$S_K = e^{j\pi/2}\, e^{j\pi/2N}\, e^{\frac{j\pi K}{N}} \quad ; K = 0,1,\ldots$$

We consider only LHP poles since only they participate in making of a stable system.

$$\therefore S_0 = e^{j\pi/2} \cdot e^{j\pi/10} = e^{j\pi 3/5}$$

$$= -0.309 + 0.95j$$

$$S_1 = e^{j\pi/2} \cdot e^{j\pi/10} \cdot e^{j\pi/5} = e^{j4\pi/5}$$

$$= -0.809 - 0.588$$

$$S_2 = e^{j\pi/2} \cdot e^{j\pi/10} \cdot e^{j2\pi/5} = e^{j\pi} = -1$$

$$S_3 = e^{j\pi/2}\, e^{j\pi/10} \cdot e^{j3\pi/5} = e^{j6\pi/5} =$$

$$-0.809 - 0.587j$$

$$S_4 = e^{j\pi/2} \cdot e^{j\pi/10}\, e^{j4\pi/5} = e^{j9\pi/5} =$$

$$-0.309 - 0.951j$$

(4) $H_n(s)$.

$$H_n(s) = \frac{1}{\prod (s - s_k)}$$

$$= \frac{1}{(s - s_0)(s - s_1)(s - s_2)(s - s_3)}$$

$$= \frac{1}{1-(-0.307+0.95j)(1-(-0.807-0.588)(s+1)(s-(0.809-0.587j))(s-(-0.309-0.950j))}$$

(5) $H_a(s) \rightarrow H_n(s)\Big|_{s = s/\Omega_c'}$

$$\Omega_c' = \Omega_p \times \Omega_c$$
$$= 4 \times 1.145 = 4.58.$$

2. Consider a 5th order LPF with pass band of 1KHz & a maximum pass band attenuation of 1dB. What is the actual attenuation in dB of LPF at a frequency of 2KHz?

$\omega = 2\pi f$.

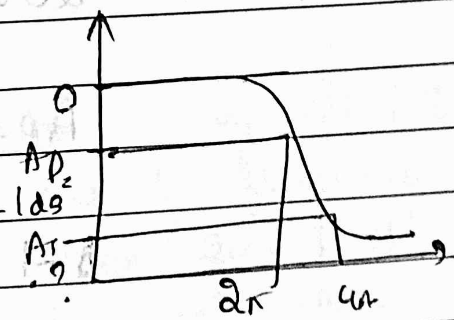
Soln:- $N = 5$.

$f_p = 1KHz$.

$\Omega_p = 2\pi \times f_p$

$= 2\pi K\, r/s$

$A_p = -1\, dB$



$\Omega_s = 2KHz.$

$\Omega_s = 2K \times 2\pi.$

$= 4\pi K\, r/s$

$$\Omega_1 = \frac{\Omega_s}{\Omega_p}$$

$$= \frac{4\pi}{2\pi} = 2.$$

$$N \geq \frac{\log (10^{-0.1Ap} - 1) / (10^{-0.1As} - 1)}{2 \log \left(\frac{\Omega p}{\Omega s} \right)}.$$

$$5 \geq \frac{\log (10^{-0.1Ap} - 1) / (10^{-0.1As} - 1))}{2 \log \left(\frac{1}{2} \right)}$$

$$-3.031 \geq \log (10^{-0.1} - 1) - \log (10^{-0.1As} - 1)$$

$$-3.031 - (-0.586) \geq \log (10^{-0.1As} - 1)$$

$$-2.44 \geq \log (10^{-0.1As} - 1)$$

$$\text{antilog} (-2.44) \leq 10^{-0.1As} - 1$$

$$0.0036 \leq 10^{-0.1As} - 1$$

$$1.0036 \leq 10^{-0.1As}$$

$$\log (1.0036) \leq \log (10^{-0.1As})$$

$$\leq -0.1 As \log (10)$$

$$0.0016 \leq -0.1 As$$

$$\frac{0.0016}{-0.1} \geq As$$

$$-0.016 \geq As (x).$$

Wkt, $20 \log |H(j\Omega)| = Ap.$

$$Ap = -10 \log \left(1 + \left(\frac{\Omega p}{\Omega c} \right)^{2N} \right).$$

$$-1 = -10 \log \left(1 + \frac{(2\pi \times 10^3)^{10}}{(\Omega c)^{2 \times 5}} \right).$$

$$\frac{-1}{10} = \log \left(1 + \frac{(2\pi \times 10^3)^{10}}{(\Omega c)^{2 \times 5}} \right)$$

$$1.25 - 1 = \frac{(2\pi \times 10^3)^{10}}{(\Omega c)^{10}}.$$

$$0.25 \times (\Omega_c)^{10} = (2\pi \times 10^3)^{10}$$

$$\Omega_c = \frac{2\pi \times 10^3}{(0.25)^{1/10}}$$

$$\Omega_c = 7213.82 \text{ rad/sec}$$

$$20 |H(j\Omega)| = A_s$$

$$A_s = -10 \log\left(1 + \left(\frac{\Omega_s}{\Omega_c}\right)^{2N}\right)$$

$$A_s = -10\log\left(1 + \left(\frac{4000\times\pi}{7213.8}\right)^{10}\right)$$

$$= -24.25 \text{dB}$$

$$20\log H(j\Omega) = A_p$$

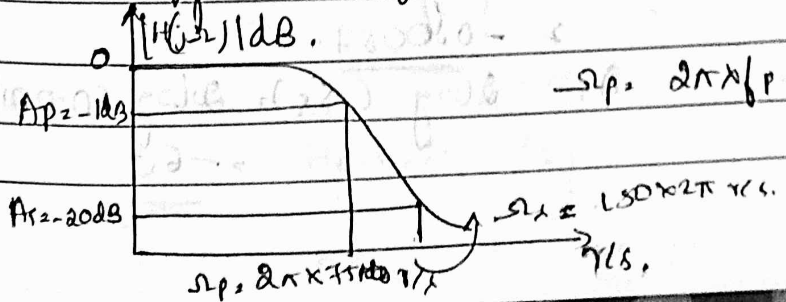
$$\boxed{\begin{aligned} A_p &= -10\log\left(1 + \left(\frac{\Omega_p}{\Omega_c}\right)^{2N}\right) \\ A_s &= -10\log\left(1 + \left(\frac{\Omega_s}{\Omega_c}\right)^{2N}\right) \end{aligned}}$$

LPF :- ① $\Omega'_s = \dfrac{\Omega_s}{\Omega_p}$ ② N ③ 3dB cut Ω_c ④ S_k ⑤ $H_n(s)$.

⑥ $H_a(s) \rightarrow H_n(s)\big|_{s \rightarrow s/\Omega'_c}$ $\Omega'_c = \Omega_p \times \Omega_c$

Q. Design an analog LPF (maximally flat) filter that will have a -1dB cut off frequency at 75Hz & have a greater than 20 dB of attenuation for all frequencies greater than 150Hz. Verify the design.

Soln:-



$\Omega_p = 2\pi \times f_p = 2\pi \times 75\text{Hz}$

$$\Omega'_s = \frac{\Omega_s}{\Omega_p} = \frac{150 \times 2\pi}{75 \times 2\pi} = 2 \text{ rad/sec}$$

$$N \geq \frac{\log\{(10^{-0.1A_p} - 1)/(10^{-0.1A_s} - 1)\}}{2\log\left(\frac{\Omega'_p}{\Omega'_s}\right)}$$

$$N \geq \frac{\log\{(10^{-0.1(-1)} - 1)/(10^{-0.1(-20)} - 1)\}}{2\log\left(\frac{1}{2}\right)}$$

$$N \geq \frac{\log(0.2589/99)}{2\log(1/2)} = 4.28$$

$$N \geq 5$$

$$\Omega_c = \frac{\Omega'_p}{(10^{-0.1A_p} - 1)^{1/2N}}$$

$$H_a(s) = H_n(s)\Big|_{s \to s/\Omega_c}$$

$$\Omega_c = \Omega_p \times \Omega_c.$$

$$= 2 \times \pi \times 75 \times 539.41$$

8. ~~Design a Butterwor~~ Find the order of N of LPF Butterworth filter given $\delta_p = 0.001$ $\delta_s = 0.001$

$\Omega_p = 1$ rad/sec $\Omega_s = 2$ rad/sec.

Sol^n: $A_p = 20\log(1 - \delta_p)$.

$= 20\log(1 - 0.001)$

$= -0.0087$.

$A_s = 20\log(\delta_s) = 20\log(0.001)$

$= -60$

$$N \geq \frac{\log\{(10^{-0.1 A_P} - 1) / (10^{-0.1 A_S} - 1)\}}{2\log\left(\frac{\Omega'_p}{\Omega'_s}\right)}$$

$$\Omega'_s = \frac{\Omega_c}{\Omega_p} = \frac{2}{1} = 2.$$

$$N \geq \frac{\log\{\{10^{-0.1(-0.0087)}\}/\{10^{-0.1(-60)} - 1\}\}}{2\log\left(\frac{1}{2}\right)}$$

$$N \geq \frac{\log(0.0020/999999)}{2\log(1/2)}$$

$N \geq 14.4$

$N = 15$

4. Let $H(s) = \frac{1}{s^2+s+1}$ represent a TF of LPF (not Butterworth) with pass band of 1rad/sec. Use the frequency transformations to find the transfer functions of the following analog filter.

(1) A LPF with pass band of 10rad/sec.

Soln:- $H(s) = \frac{1}{s^2+s+1}$.

$\Omega_p = 1$ rad/sec.

LPF $H_a(s) = H_0(s)\Big|_{s \to s/\Omega = s/10}$.

$$= \frac{1}{(s/10)^2 + (s/10) + 1}.$$

(2) A HPF with cutoff frequency of 1rad/sec.

$H(s) = \frac{1}{s^2+s+1}$.

$\Omega'_c = \frac{\Omega_p}{\Omega_{pc}} = \frac{1}{1}$

$H_{a}(s) = H_s\Big|_{s \to \Omega'_c/s}$

$H_a(s) = H_s\Big|_{s \to 1/s}$.

c. A HPF with cutoff frequency of 10rad/sec.

$$H_a(s) \rightarrow \left. H(s) \right|_{s \rightarrow \frac{\Omega'_c}{s}}$$

$$\Omega'_c = \frac{\Omega_p}{\Omega_c} = \frac{1}{10}$$

$$H_a(s) = \left. \frac{1}{s^2+s+1} \right|_{s \rightarrow \frac{10}{s}}$$

d) A BPF with a passband of 10rad/sec & centre frequency of 100rad/sec.

e) A BSF filter with stop band of 2rad/sec & a center frequency of 10rad/sec.

3/12/2021

Bilinear transformation:-

IIV → disadvantage (not one-to-one mapping)

① → Also called as conformal mapping / One-to one mapping. (aliasing)

② → The $j\Omega$ axis is mapped on to the unit circle thus no aliasing.

③ In bilinear transformation the approx. of integration is done using trapezoidal formula.

Wkt,

Let $\frac{dy(t)}{dt} = x(t)$

Integrating on both sides,

$$\int_{(n-1)T}^{nT} \frac{dy(t)}{dt}\,dt = \int_{(n-1)T}^{nT} x(t)\cdot dt$$

The above integration can be estimated using the trapezoidal formula given by

$$\int_a^b f(x)\cdot dx = \frac{b-a}{2}\left[f(b)+f(a)\right]$$

$$y(t)\Big|_{(n-1)T}^{nT} = \frac{nT-(n-1)T}{2}\left[x(nT)+x((n-1)T)\right]$$

$$y(nT)-y((n-1)T) = \frac{T}{2}\left[x(nT)+x(n-1)T\right]$$

If the above is discrete time LTI system, it can be rewritten as

it is equivalent $\boxed{y(n)-y(n-1) = \frac{T}{2}\left[x(n)+x(n-1)\right]}$

$y(nT) = y(n)$ is $y(n)$ is $y(nT)$'s sampled version

Taking z transform on both sides,

$$Y(z) - Y(z)z^{-1} = \frac{T}{2}\left[X(z) + X(z)z^{-1}\right]$$

$$Y(z)\left[1-z^{-1}\right] = \frac{T}{2}X(z)\left[1+z^{-1}\right]$$

$$X(z) = \frac{2}{T}\left\{Y(z)\frac{[1-z^{-1}]}{[1+z^{-1}]}\right\} \longrightarrow (1)$$

On taking Laplace transform on the above eq^n.

$$X(s) = s \cdot Y(s) \longrightarrow (2)$$

Comparing eq (1) & (2) the relation

Relation b/w s & z.

$$\boxed{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

→ 1/pole
→ s↔ ... de
→ b/a

imp. IIR. trans.

Relation b/w analog & digital poles

Wkt, the transformation b/w s & z domain is given by

$$\boxed{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$\frac{T}{2}s = \frac{z-1}{z+1}$$ multiply N^r & D^r by z

$$\frac{T}{2}s(z+1) = z-1$$

$$\boxed{z = \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s}}$$

WKt, $s_i = \sigma_i + j\Omega_i$
Substituting on the above eqn

$$z_i = \frac{1 + T/2\,[\sigma_i + j\Omega_i]}{1 - [T/2(\sigma_i + j\Omega_i)]}$$

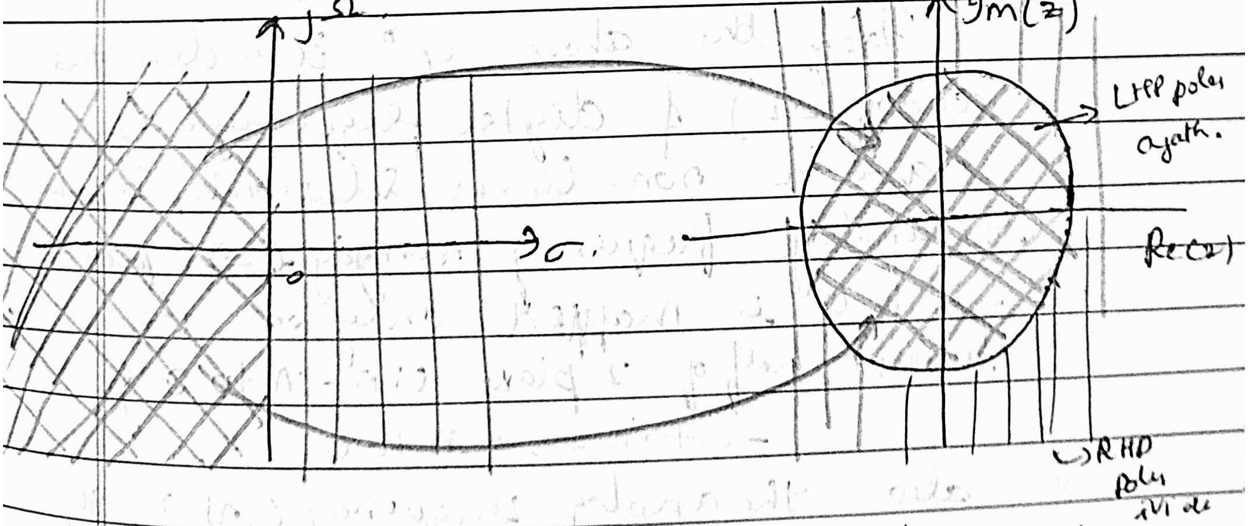
$$z_i = \frac{1 + T/2\,\sigma_i + j(T/2\,\Omega_i)}{1 - T/2\,\sigma_i - j(T/2\,\Omega_i)}$$

$$|z_i| = \sqrt{\frac{(1 + T/2\,\sigma_i)^2 + (T/2\,\Omega_i)^2}{(1 - T/2\,\sigma_i)^2 + (T/2\,\Omega_i)^2}}$$

① $\sigma_i < 0$; poles will lie on LHP in s-plane
$|z| < 1$

② $\sigma_i = 0$; poles on $j\Omega_i$ axis in s plane,
$|z| = 1$.

③ $\sigma_i > 0$; Poles will lie on RHP of s-plane,
$|z| > 1$



Relations b/w analog & digital frequency:-

WKt,

In BLT,

$$S = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

Substitute $s = j\Omega$ & $z = e^{j\omega}$

$z = re^{j\omega}$, $r = 1$ (because 1HP poles mostly)

$$j\Omega = \frac{2}{T}\left(\frac{1-e^{-j\omega}}{1+e^{-j\omega}}\right)$$

$\because 1 = e^{j\omega/2 - j\omega/2}$.

$$j\Omega = \frac{2}{T}\left[\frac{e^{j\omega/2}\cdot e^{-j\omega/2} - e^{-j\omega}}{e^{j\omega/2}\cdot e^{-j\omega/2} + e^{-j\omega}}\right]$$

$$j\Omega = \frac{2}{T}\left[\frac{e^{-j\omega/2}\left[e^{j\omega/2} - e^{-j\omega/2}\right]}{e^{-j\omega/2}\left(e^{j\omega/2} + e^{-j\omega/2}\right)}\right]$$

$$j\Omega = \frac{2}{T}\left[\frac{2j\,\sin(\omega/2)}{2\cos(\omega/2)}\right]$$

$$\boxed{\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)}$$

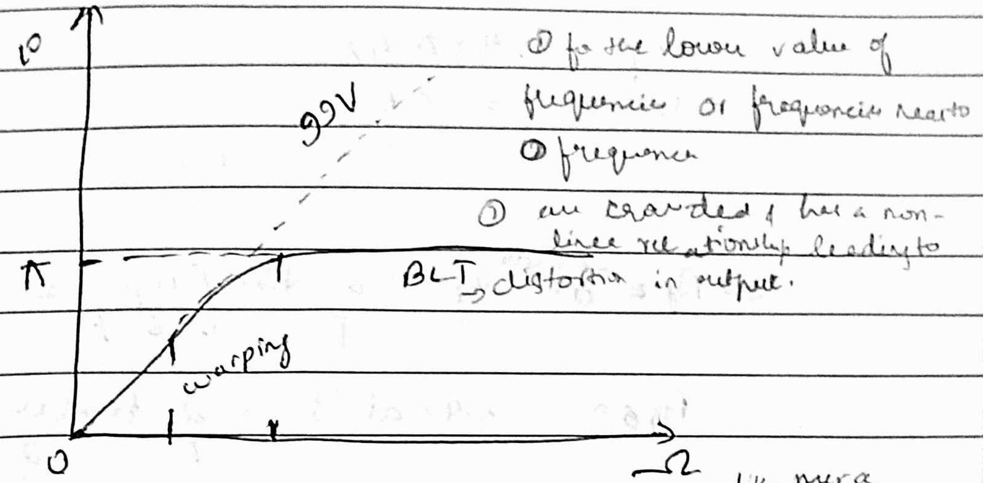
From the above eq^n it is clear that analog (Ω) & digital frequency (ω) have a non-linear relationship because the analog frequency ranging (Ω) from $-\infty$ to 0 is mapped only on the lower half of z plane (i.e) $-\pi$ to 0 &

$$-\infty \text{ to } 0 \Rightarrow -\pi \text{ to } 0$$

also the analog frequency (Ω) ranging from 0 to $+\infty$ is mapped onto the z plane b/w 0 & $+\pi$.

$$0 \text{ to } \infty \Rightarrow 0 \text{ to } \pi$$

∴ for the lower value of frequencies or near to zero the frequencies are crowded or having a non-linear relationship leading to distortion in output called frequency warping (Imp)



To avoid the effect of warping while designing the filter, it need to be prewarped using the relation,

$$\boxed{\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)}$$

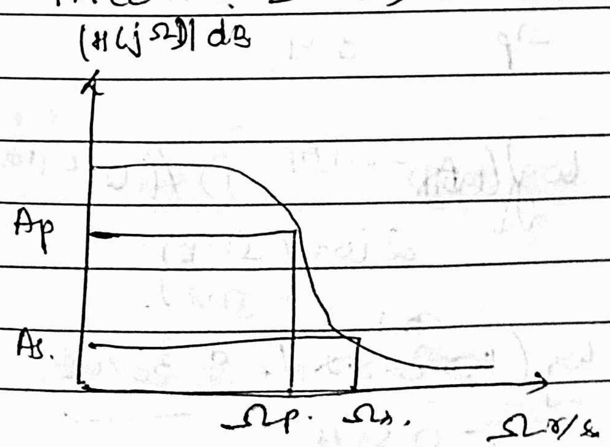
6/12/21 Qth B.T.

Design a butterworth digital IIR low pass filter by taking $t = 0.5$ seconds to satisfy the following specifications.

$$0.707 \le |H(e^{j\omega})| \le 1.0, \quad 0 \le \omega \le 0.45\pi.$$

$$|H(e^{j\omega})| \le 0.2; \quad 0.65\pi \le \omega \le \pi.$$

Sol:-



IIV → sampling frequency (Ωp)
where GT, 7K if not given

$1-\delta_p = 0.707$

$\delta_s = 0.2$

$A_p = 20\log(1-\delta_p) = -3.01\,dB$

$A_s = 20\log \delta_s = -13.97\,dB$

$\omega_p = 0.45\pi$ r/s.

$\omega_s = 0.65\pi$ r/s

warping ondagum (note)

non-linear relationship

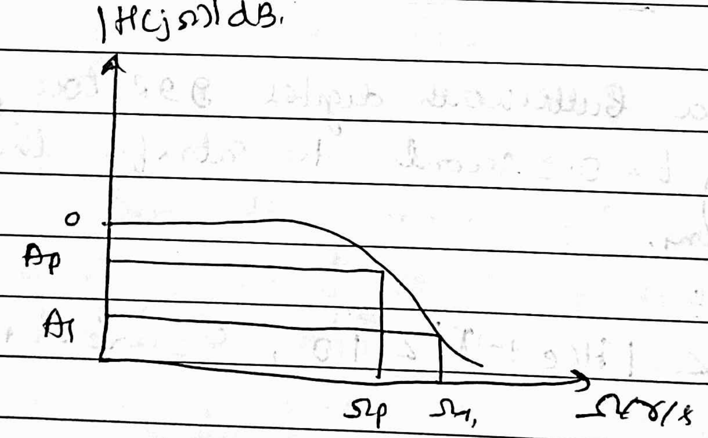
$A_p = -3.01\,dB$ at $\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = 3.4$ r/s rad.

$A_s = -13.97$ at $\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = 6.5$ r/s

$\Omega_p = 3.4$ r/s at $A_p = -3.01\,dB$,

$\Omega_s = 6.5$ r/s at $A_s = -13.97\,dB$

Convert to normalised



$\Omega'_s = \frac{\Omega_s}{\Omega_p} = \frac{6.5}{3.4} = 1.9$ r/s,

$$N \geq \frac{\log\left((10^{-0.1A_p}-1)/(10^{-0.1A_s}-1)\right)}{2\log\left(\frac{\Omega'_p}{\Omega'_s}\right)}$$

$$N \geq \frac{\log\left(\cancel{\ldots}1.009 / \cancel{\ldots}(10^{-0.1A_s}-1)\right)}{-0.5575}$$

$N \geq 2.45$.

$\underline{N = 3}$

$$\Omega_c = \frac{1}{(10^{-0.1 A_P} - 1)^{1/2N}}$$

$$= \frac{1}{(1.009)^{1/6}}$$

$$= 0.9985 \, r/s = \underline{1 \, r/s}$$

(5) $N = 3$.

$$H_N(s) = \frac{1}{\prod (s - s_k)} \text{ or } \frac{1}{B_N(s)}$$

$$H_N(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$H_N(s) = \frac{1}{s_n^3 + 2s_n^2 + 2s_n + 1}$$

$$H_a(s) = |H_n(s)|$$

$$s \to s/\Omega'_c$$

$$\Omega'_c = \Omega_p \times \Omega_c$$

$$= 3.4 \times 1$$

$$\underline{\Omega'_c = 3.4 \, r/s}$$

$$H_a(s) = \frac{1}{\left(\frac{s}{3.4}\right)^3 + 2\left(\frac{s}{3.4}\right)^2 + 2\left(\frac{s}{3.4}\right) + 1}$$

$$\boxed{H_a(s) = \frac{39.30}{s_n^3 + 6.8 s^2 + 23.12 s + 39.30}}$$

(1) $H(z) = H_a(s)\Big|_{s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

& $s \to \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)_{T=0.5}$

$s \to 4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$

$$H(z) = \frac{39.30}{\left(4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^3 + 6.8\left[4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 23.25\left[4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right] + 39.30}$$

$$= \frac{39.30}{64\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^3 + 108.8\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + 93\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 39.30}$$

$$H(z) = \frac{39.30\,(1+z^{-1})^3}{64(1-3z^{-1}+3z^{-2}-z^{-3}) + 108.8(1+z^{-2}-2z^{-1})(1+z^{-1}) + 93(1-z^{-1})(1+z^{-1}) + 39.30}$$

$$H(z) = \frac{39.30(1+z^{-1})^3}{64(1-3z^{-1}+3z^{-2}-z^{-3}) + 108.8(1+z^{-2}-2z^{-1}+z^{-1}+z^{-3}-2z^{-2}) + 93(1-z^{-1})(1+z^{-2}+2z^{-1}) + 39.30}$$

$1+z^{-2}-2z^{-1}+z^{-2}$

$1-z^{-2}+z^{-1}-z^{-3}$

$1+z^{-2}+2z^{-1}-z^{-1}-z^{-3}-2z^{-2}$

HPF

& BPF

$$= \frac{39.30(1+z^{-1})^3}{64-192z^{-1}+192z^{-2}-64z^{-3}+108.8 - 108.8z^{-2} - 108.8z^{-1}+z^{-3}+93-z^{-2}+93z^{-1}-93z^{-3}+39.30}$$

$$= \frac{39.30(1+3z^{-1}+3z^{-2}+z^{-3})}{305.1 - 207.8}$$

FIR filter - finite impulse response:

→ Length of $h(n)$ is finite
  ↳ windowing } technique.
  ↳ frequency sampling
→ Can be designed directly from IIR
→ Will have a linear phase & always a stable filter. an IIR system can be represented by

$$\sum_{k=0}^{M-1} b_k\, y(n-k) = \sum_{k=0}^{N} a_k\, x(n-k).$$

FIR	IIR
→ directly des	
→ linear phase	
stability	

Taking ZT,

$$\sum_{k=0}^{M-1} b_k\, Y(z)\, z^{-1} = \sum_{k=0}^{N-1} a_k X(z)\, z^{-1}$$

zth degree

$$H(z) = \frac{\sum_{k=0}^{M-1} a_k \cdot z^{-1}}{\sum_{k=0}^{M-1} b_k z^{-1}}$$

FIR-il phase depends
linear aayirikyum.

In case of $\boxed{\text{FIR- all zero filter}}$

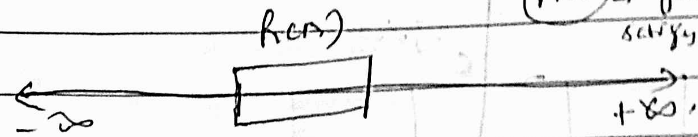
IIR-il phase constant alla.

$$H(z) = \frac{\sum_{k=0}^{N-1} a_k z^{-1}}{1}$$ ⇒ all poles, always stable.

FIR filter should have
$$h(n) = \pm h(N-1-n).$$
i.e

$$\boxed{\begin{aligned} h(n) &= h(N-1-n) \\ &= -h(N-1-n) \end{aligned}}$$

→ symmetry (ith ondayal) mathrmal linear phase ondagum.
↳ (IM) → filters should satisfy to have linear phase



Infinite length $h(n)$ a window of length N. → finite length $h(n)$ of N.

Windows:-

① Rectangular window.
② Bartlett window (triangular).
③ Hamming window

Rectangular window:-

$$W_R(n) = \begin{cases} 1; & 0 \le n \le N-1 \\ 0 & \text{otherwise.} \end{cases} \quad = \frac{8\pi}{N} = -21\,dB$$

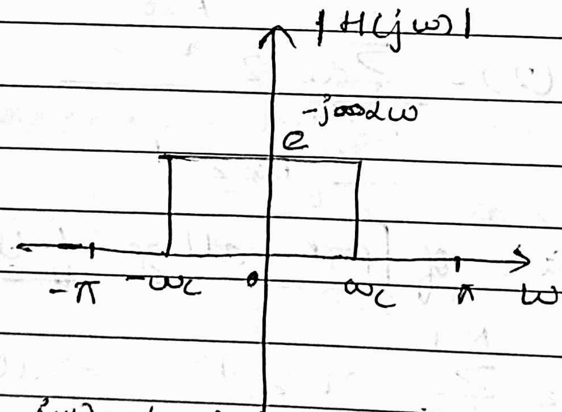
Bartlett window (triangular)

$$W_{BART} = W_{TRI}(n) = \begin{cases} 1 - \dfrac{2\left|n - \frac{N-1}{2}\right|}{N-1}; & 0 < n \le N-1 \\ 0 & \text{otherwise.} \end{cases}$$

Hamming window

$$W_{hamm}(n) = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{N-1}\right); & 0 < n \le N-1 \\ 0 & \text{otherwise.} \end{cases}$$

LPF:-



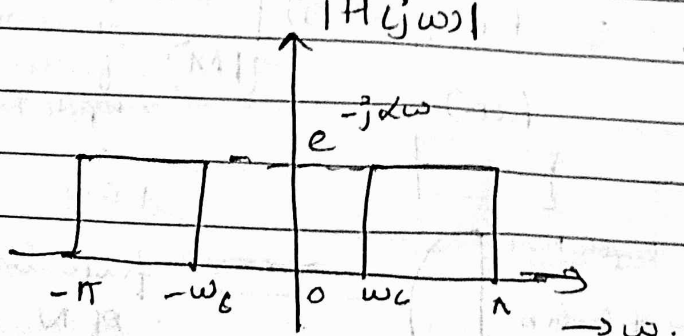
$$H(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega}; & -\omega_c \le \omega \le \omega_c \\ 0; & \text{otherwise} \end{cases}$$

or $-\omega_c \le \omega \le \omega_c$

$|\omega| \le \omega_c$

make out from characteristics and shape.

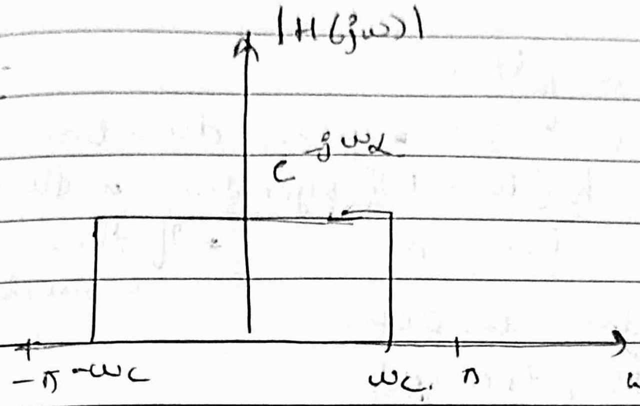
HPF:-



$$H(\omega) = \begin{cases} e^{-j\omega\alpha} \; ; -\omega_c \le \omega \le \pi \\ \quad \omega_c \le |\omega| \le \pi \\ e^{-j\omega\alpha\omega} \; ; -\pi \le \omega \le -\omega_c \\ 0 \; ; \text{otherwise} \end{cases}$$

6/12/21

LPF:-



$\alpha = \frac{N-1}{2}$

-25dB

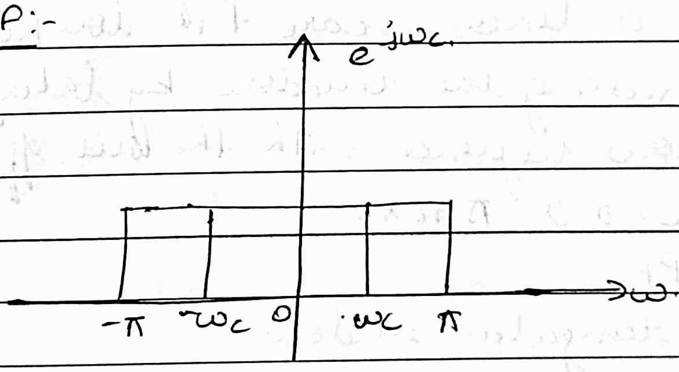
$= \frac{4\pi}{N}$

-25

$= \frac{8\pi}{N}$, -53dB

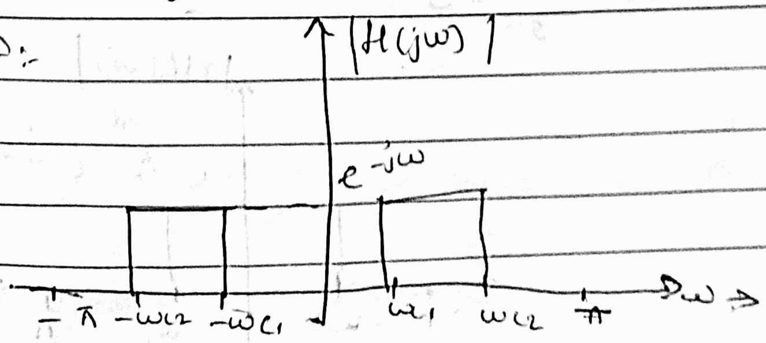
$$h_d(n) = \begin{cases} \frac{\sin \omega_c(n-\alpha)}{\pi(n-\alpha)} \; \forall \, n \ne \alpha \\ \frac{\omega_c}{\pi} \; , n = \alpha. \end{cases}$$

HPF:-



$$h_d(n) = \begin{cases} \sin\pi(n-\alpha) - \sin\omega_c(n-\alpha) \; \forall n \ne \alpha \\ 1 - \left(\omega_c/\pi\right) \; ; n = \alpha. \end{cases}$$

BPF:-



$$h_d(n) = \begin{cases} \dfrac{\sin \omega_{c2}(n-\alpha) - \sin \omega_{c1}(n-\alpha)}{\pi(n-\alpha)} & \forall n \neq \alpha \\ \dfrac{\omega_{c2} - \omega_{c1}}{\pi}, & n = \alpha. \end{cases}$$

FIR:-

→ All zero filter.

→ Linear ① N = finite duration sequence.

→ $h(n) = h(N-1-n)$ → symmetric. is always stable

$-h(N-1-n)$

* If there is zero, then it is possible to have a conjugate pair.

① Rectangular window:-

② Bartlett / triangular.

③ Hamming window.

Minimum SB attenuation

of

Transmission width.

1. Design a linear case FIR lowpass filter using rectangular window by taking 7 samples of window sequence with the cut-off frequency of $\omega_c = 0.2\pi$ r/s.

Soln:-

LPF

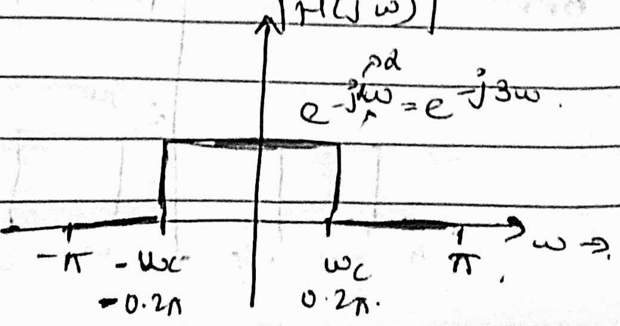
Rectangular window.

$\omega_c = 0.2\pi$ r/s.

$N = 7$.

① $\boxed{\alpha = \dfrac{N-1}{2}}$

$\alpha = \dfrac{7-1}{2} = 3$.



$$H(\omega) = \begin{cases} e^{-j3\omega} & ; -0.2\pi \le \omega \le 0.2\pi \text{ or } |\omega| \le 0.2\pi \\ 0, & \text{elsewhere.} \end{cases}$$

For the LPF, the impulse response is obtained by applying inverse Fourier transform & it is given by

$$h_d(n) = \begin{cases} \frac{\sin \omega_c (n-\alpha)}{\pi(n-\alpha)} & \forall n \ne \alpha \\ \frac{\omega_c}{\pi} & ; n = \alpha. \end{cases}$$

$n = 0.$

$$h_d(0) = \frac{\sin 0.2\pi(0-3)}{\pi(-3)}$$

$$= 0.1009$$

$n = 1$

$$h_d(n) = \frac{\sin 0.2\pi(0-2)}{\pi(-2)}.$$

$$= 0.1513$$

$n = 2$

$$h_d(n) = \frac{\sin 0.2\pi(-1)}{\pi(-1)}$$

$$= 0.1871.$$

$n = 3,$ $\quad h_d(n) = \frac{\omega_c}{\pi}$

$$= \frac{0.2\pi}{\pi} = 0.20$$

$n = 4,$ $\quad h_d = \frac{\sin 0.2\pi(1)}{\pi(1)} = 0.1871$

$$hd(5) = \frac{\sin 0.2\pi(2)}{\pi(2)}$$

$$= 0.1513$$

$$hd(6) = 0.1009.$$

$$h(n) = h(N-1-n).$$

$$h(4) = h(7-1-4).$$

$$= \underline{h(2)}$$

$$hd(n) = \{.1009, .153, 0.1871, 0.2, 0.187, 0.153, 0.1009\}.$$

$$\boxed{\text{To get the finite impulse response } h(n) = \hat{h}_d(n) \times W_R(n)}$$

$$W_R(n) = \{1, 1, 1, 1, 1, 1, 1\} \rightarrow \text{window}$$

$$h(n) = \{0.1009, 0.1513, 0.1871, 0.2, 0.1871, 0.1513, 0.1009\}$$

To get N point filter,

$$Z\{h(n)\} = H(Z).$$

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$H(z) = h(0)z^{-0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$$

$$H(z) = 0.1009 + 0.1513z^{-1} + 0.1871z^{-2} + 0.2z^{-3} + 0.1871z^{-4} + 0.1513z^{-5} + 0.1009z^{-6}$$

$$\boxed{H(z) = 0.1009(1+z^{-6}) + 0.1513(z^{-1}+z^{-5}) + 0.1871(z^{-2}+z^{-4}) + 0.2z^{-3}}$$

2. Design a linear phase FIR LPF using hamming window by taking 7 samples of window sequence & with the cut off frequency of 0.2π r/s.

Soln: $h_d(n) = \{0.1009, 0.1514, 0.1871, 0.2, 0.1871, 0.1514, 0.1009\}$.

$$W_{ham}(n) = \left\{0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right); 0 \le n \le 6\right\}.$$

$$W_{ham}(0) = 0.54 - 0.46\cos\left(\frac{2\pi 0}{N-1}\right).$$

$$= 0.54 \overset{-0.46}{\cancel{-0.0846}} = 0.08$$

$$\omega_{ham}(1) = 0.54 - 0.46\cos\left(\frac{2\pi}{6}\right).$$

$$= 0.31$$

$$\omega_{ham}(2) = 0.54 - 0.46\left(\frac{2\pi(2)}{6}\right) = 0.77.$$

$$\omega_{ham}(n) = \{0.08, 0.31, 0.77, 1, 0.77, 0.31, 0.08\}$$

$h(n) = h_d(n) \times W_n$ ham.

$h(n) = \{0.00807, 0.0477, 0.144, 0.2, 0.144, 0.0477, 0.00807\}$

$$H(z) = Z\{h(n)\} = \sum_{n=0}^{N-1} h(n) z^{-n}.$$

$$H(z) = \sum_{n=0}^{6} h(n) \cdot z^{-n}.$$

$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}$

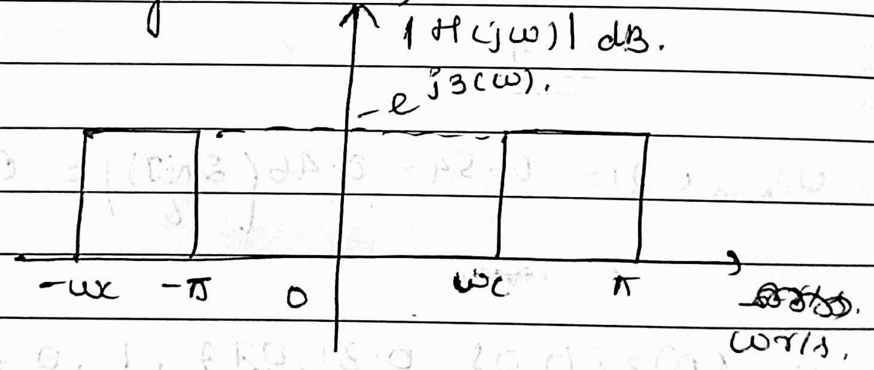
$= 0.00807 + 0.0477z^{-1} + 0.144z^{-2} + 0.2z^{-3} + 0.144z^{-4} + 0.0477z^{-5} + 0.00807z^{-6}$

$$\boxed{= 0.00807(1+z^{-6}) + 0.0477(z^{-1}+z^{-5}) + 0.144(z^{-2}+z^{-4}) + 0.2z^{-3}}$$

3. Design a linear phase FIR HPF using ~~rectangular~~ hamming window with the cutoff freq. $\omega_c = 0.8\pi$ ~~r/s~~, $N = 7$.

Sol:
$\omega_c = 0.8\pi$.
$N = 7$.

hamming window, for N=7 same as previous.



① $\alpha = \frac{N-1}{2} = 3$.

wkt, for HPF,

$$h_d(n) = \begin{cases} \frac{\sin\pi(n-\alpha) - \sin\omega_c(n-\alpha)}{\pi(n-\alpha)} & \forall n \neq \alpha \\ 1 - \left(\frac{\omega_c}{\pi}\right), & n = \alpha \end{cases}$$

$h_d(0) = \cancel{\sin\pi(0-3)} - \sin 0.8\pi(0-3)$

$= \cancel{0.9511}$

$= \frac{\sin(0-3) - \sin 0.8\pi(0-3)}{\pi(-3)}$

$h_d(1)$

$= 0.1513$ ✓

$$h_d(2) = \frac{\sin\pi(2-1) - \sin 0.8\pi(2-1)}{\pi(2-1)}$$

$= \underline{\underline{-0.187}}$

$hd(n) = \{-0.1009, 0.1513, -0.187, 0.2, -0.187, 0.1513, -0.1009\}$.

$w_{han}(n) = \{0.008, 0.31, 0.77, 1, 0.77, 0.31, 0.08\}$.