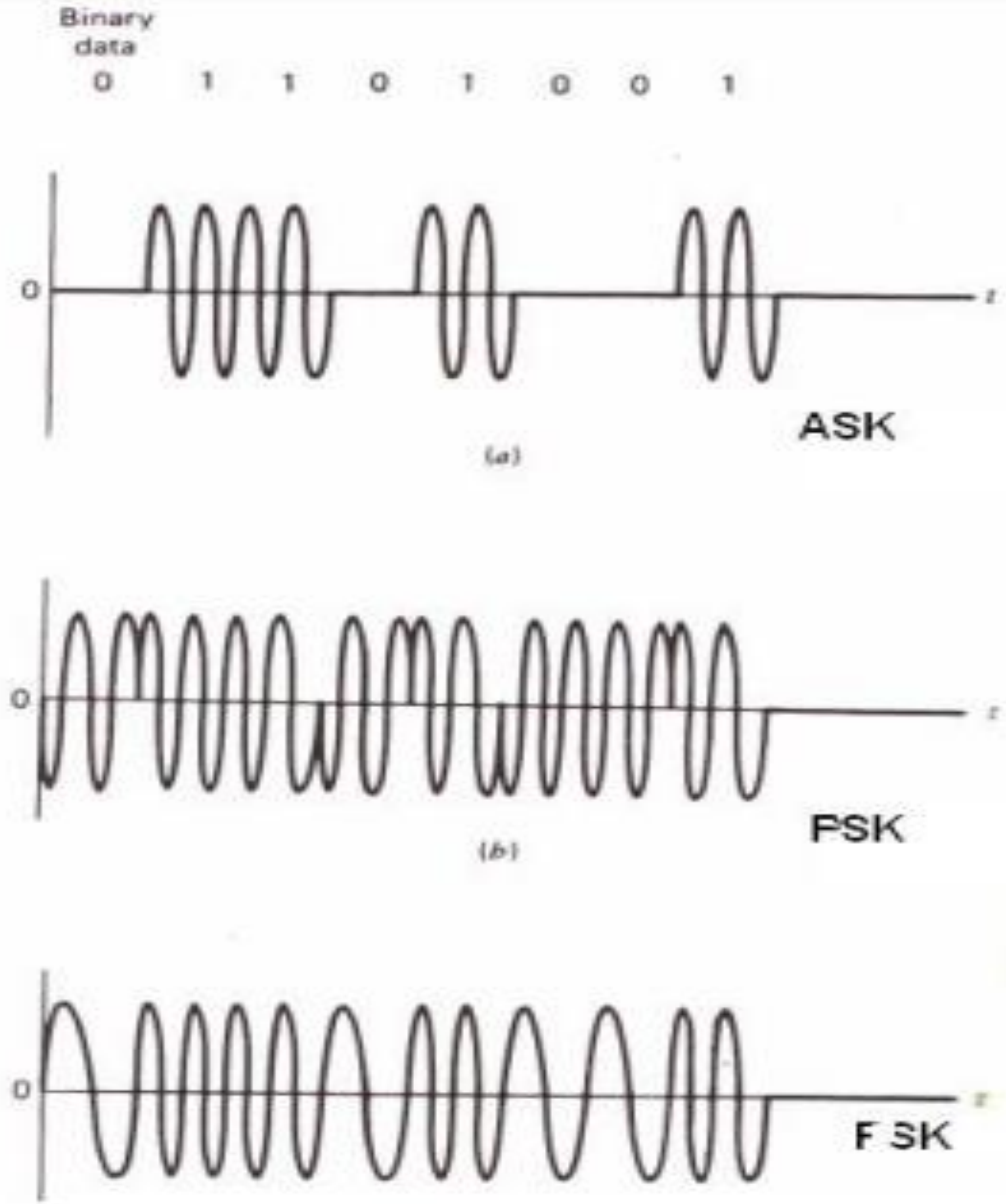
Module-4

Digital Modulation Techniques: Digital Modulation formats, Coherent binary modulation techniques, Probability of error derivation of PSK and FSK, M-ary modulations-QPSK, QAM, PSD for different digital modulation techniques, Non-coherent binary modulation techniques -DPSK

Digital Modulation formats

- Modulation is defined as the process by which some characteristics of a carrier is varied in accordance with a modulating wave.
- In digital communications, the modulating wave consists of binary data or an M-ary encoded version of it.
- In M-ary signalling, the modulator produces one of an available set of $M=2^m$ distinct signals in response to m bits of source data at a time.
- Binary modulation is a special case of M-ary modulation with M=2.
- For modulation, it is customary to use a sinusoidal wave.
- There are 3 basic modulation techniques for the transmission of digital data.
  - Amplitude Shift Keying
  - Frequency Shift Keying
  - Phase Shift Keying


Binary data
0 1 1 0 1 0 0 1
0
t
ASK
(a)
0
t
PSK
(b)
0
t
F SK

Coherent vs. Non-Coherent Detection

- Coherent detection requires a copy of the carrier to be recovered from the received signal for use in the detection process. It is more efficient because it uses all phase information, but requires added complexity
- Non-coherent detection using an envelope detector is much easier to implement, but less efficient because it uses only the envelope information and not the phase information.

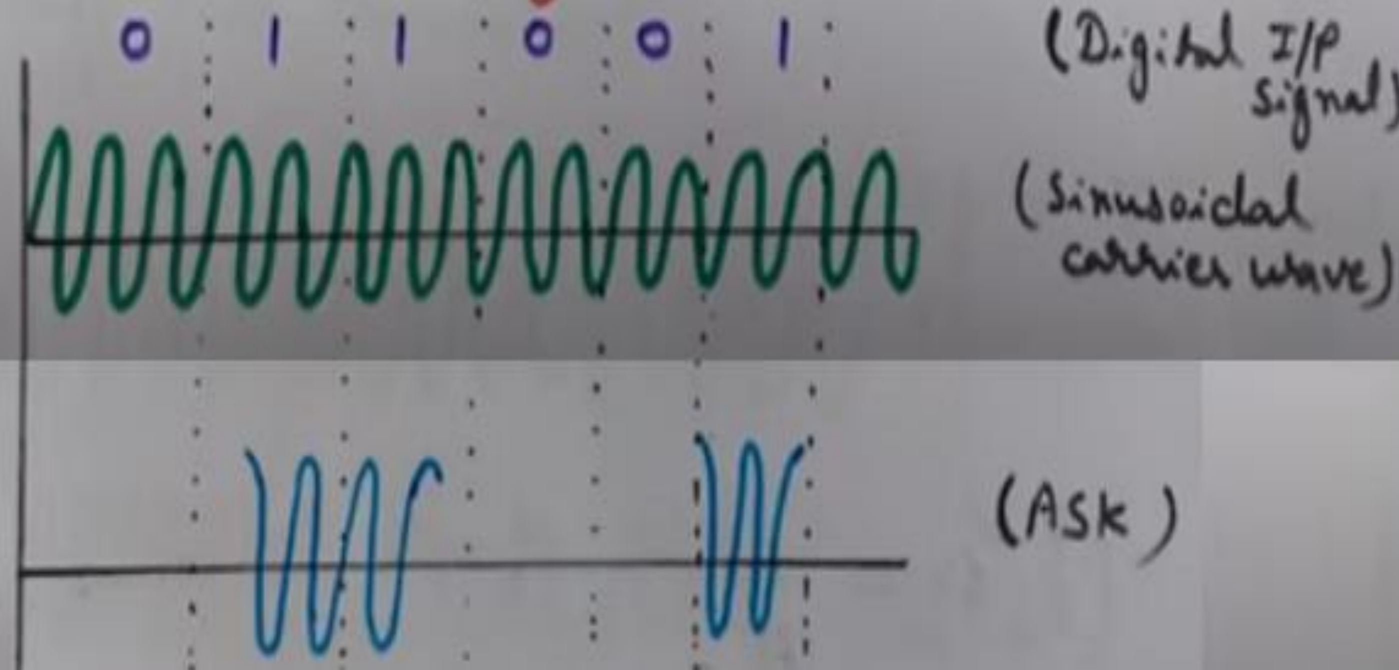
Coherent and Non-Coherent Detection

Coherent Detection - A synchronous detection in which the digital receiver is phase-locked to the carrier signal of the incoming digitally modulated signal.

Non-Coherent Detection - A non-synchronous detection in which the digital receiver does not require locally-generated receiver carrier signal to be phase-locked with transmitter carrier signal.

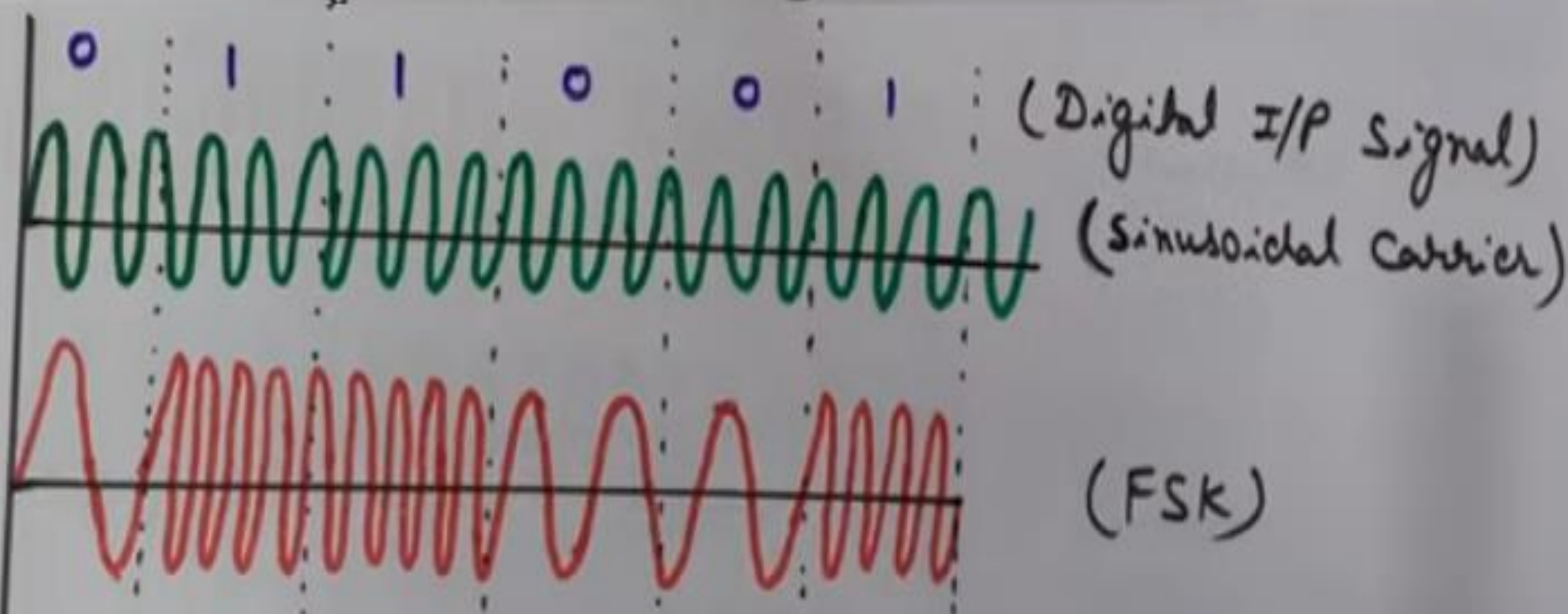
Amplitude Shift keying (ASK)

In ASK, the amplitude of the carrier wave is changed (switched) according to the digital input signal (modulating signal). Therefore, ASK is analogous to AM (analog modulation)



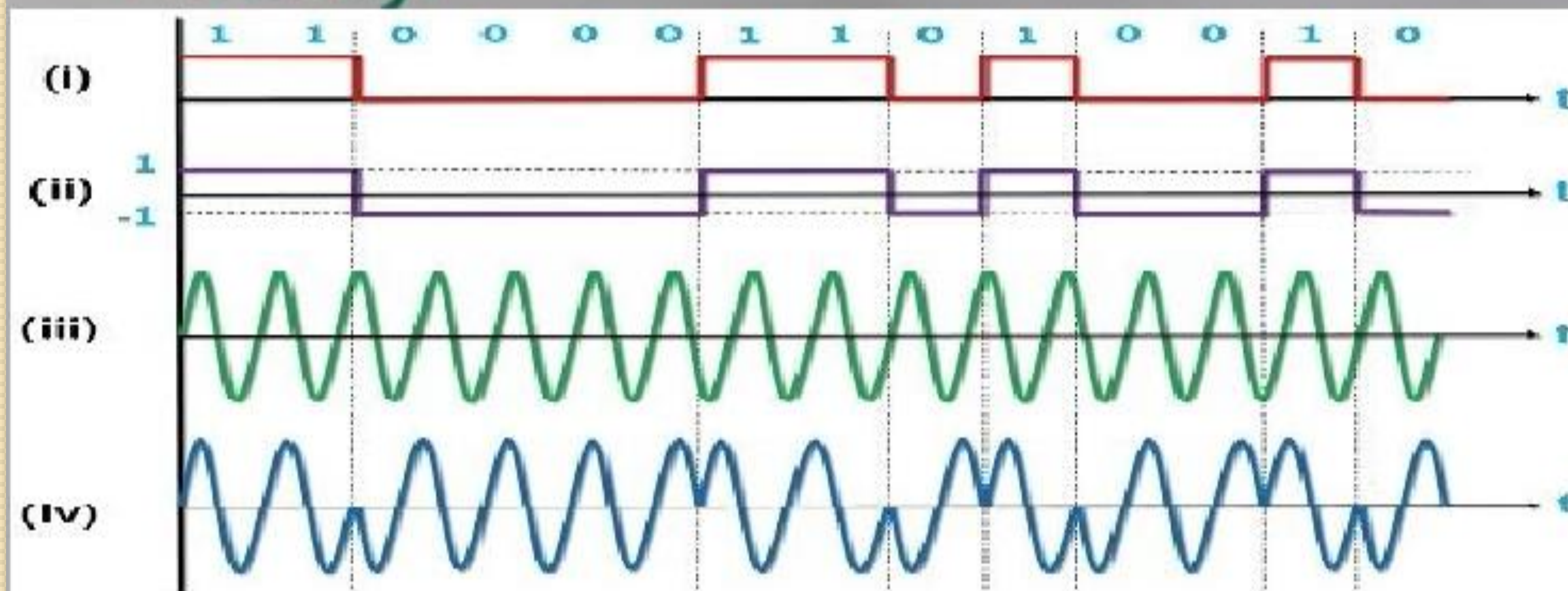
Frequency Shift keying (FSK)

If the frequency of sinusoidal carrier wave is varied (switched) depending on the digital input signal, then it is known as the Frequency shift keying (FSK). It is analogous to FM (Analog freq Modulation)



Phase Shift Keying (PSK)

In phase shift keying, phase of the carrier wave (analog) is switched as per the input digital signal. This is analogous to PM (analog phase modulation).



(i) Binary data sequence
(ii) Bipolar NRZ sequence
(iii) Carrier wave
(iv) BPSK waveform

Electronics Coach

Coherent vs. Non-Coherent Detection

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Coherent and Non-Coherent Detection

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Coherent binary modulation techniques:

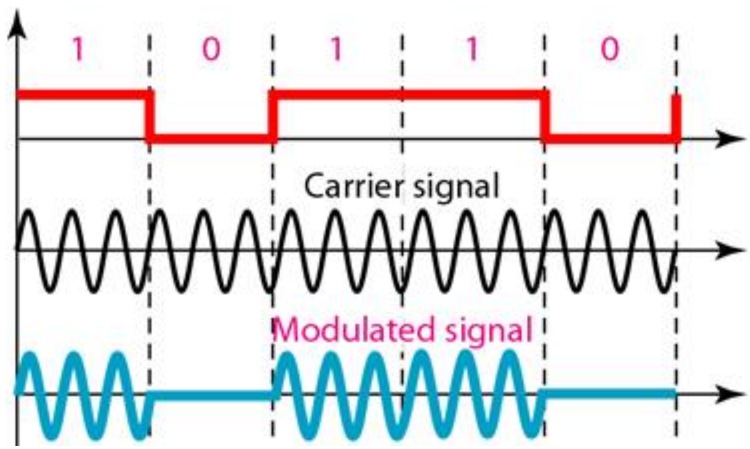
1. Coherent Binary ASK
2. Coherent Binary FSK
3. Coherent Binary PSK

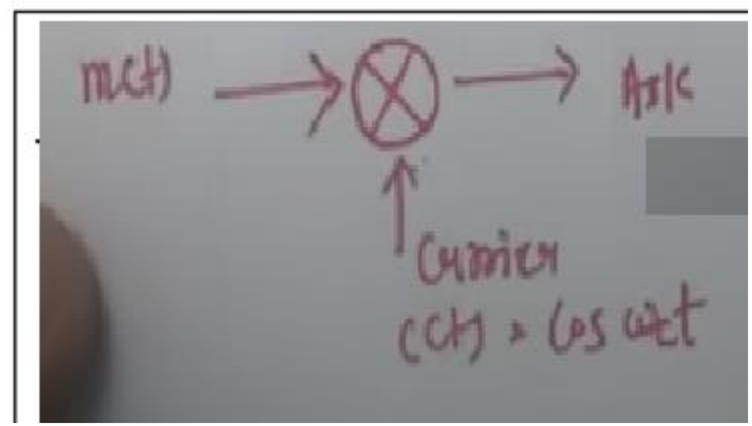
Coherent Binary ASK

A binary ASK wave can be defined as

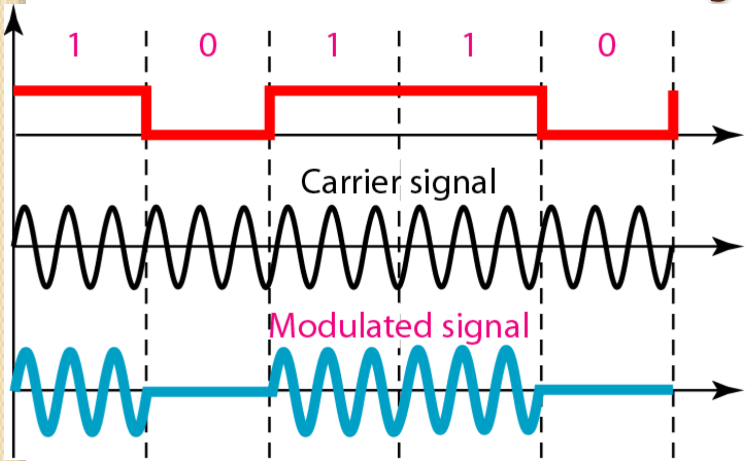
$S_t = A_c\ m(t)\ \cos 2\pi f_c t\ ,\ 0 \le t \le T_b$

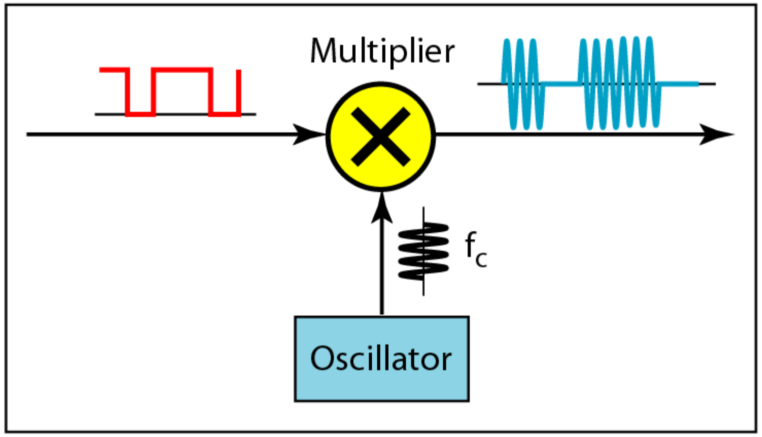
where A_c is amplitude of carrier, m(t) is digital information signal.
f_c is carrier frequency, T_b is bit duration.





Coherent Binary ASK





In binary ASK system, symbol 1 & 0 are represented as

$$s(t)= \begin{cases} s_1(t)= \sqrt{\dfrac{2E_b}{T_b}}\cos 2\pi f_c t & \text{for } 0\le t\le T_b \quad \text{for symbol 1} \\ s_2(t)= 0 & \text{for } 0\le t\le T_b \quad \text{for symbol 0} \end{cases}$$

Basis function $\Phi_1(t) = \sqrt{\dfrac{2}{Tb}}\cos 2\pi f_c t$

Binary ASK can be written as

$$s(t)= \begin{cases} s_1(t)= \sqrt{E_b}\,\Phi_1(t) & \text{for } 0\le t\le T_b \quad \text{for symbol 1} \\ s_2(t)= 0 & \text{for } 0\le t\le T_b \quad \text{for symbol 0} \end{cases}$$

Wher E_b is transmitted signal energy/bit

Demodulation of ASK

Demodulation of ASK

- Synchronous (Coherent)
- Non Synchronous (Non Coherent).

Synchronous (Coherent)

$$m(t)\cos\omega_c t \rightarrow \otimes \xrightarrow{m(t)\cos^2\omega_c t} \boxed{\text{LPF}} \rightarrow m(t).$$

$\uparrow$ $\cos\omega_c t$

Adv - It is efficient

Disadv - It is Costly

Non Synchronous (Non Coherent)

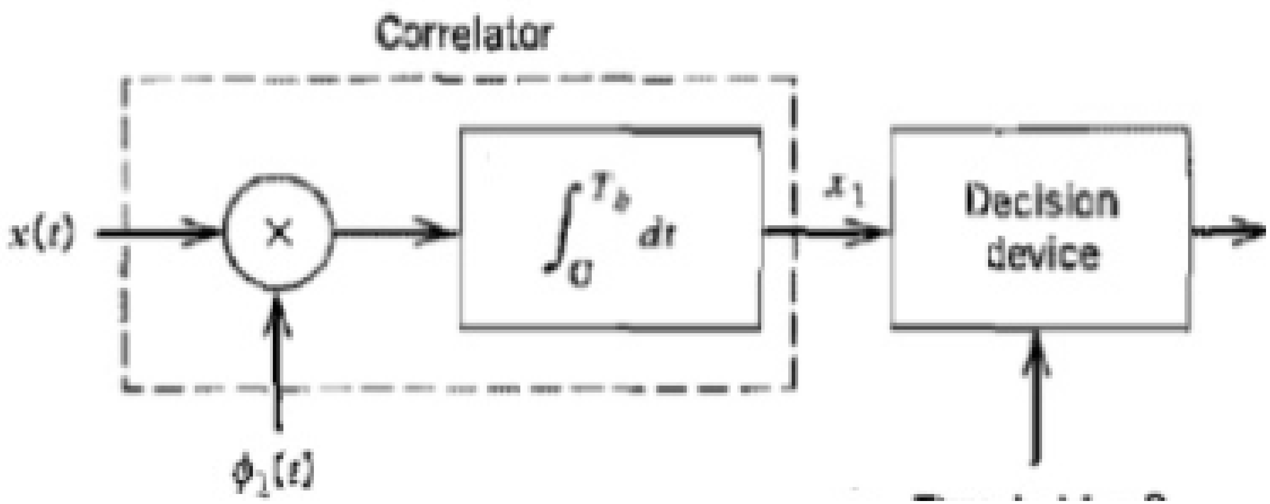
$$\xrightarrow{m(t)\cos\omega_c t} \boxed{\text{Envelope}} \rightarrow m(t).$$

Adv - Cost is low.

Disadv - Performance is Poor with less SNR received signal.

Coherent detection of ASK signal

- In demodulator, the received signal x(t) is cross correlated with local reference signal $\Phi_1(t)$.
- The output of correlator is applied to decision device.
- The correlator output x is compared with threshold λ.
- If $x > \lambda$ the receiver decides in favour of symbol 1.
- If $x < \lambda$ the receiver decides in favour of symbol 0.
- . In coherent detection the output of local oscillator is in perfect synchronization with the carrier used in the transmitter

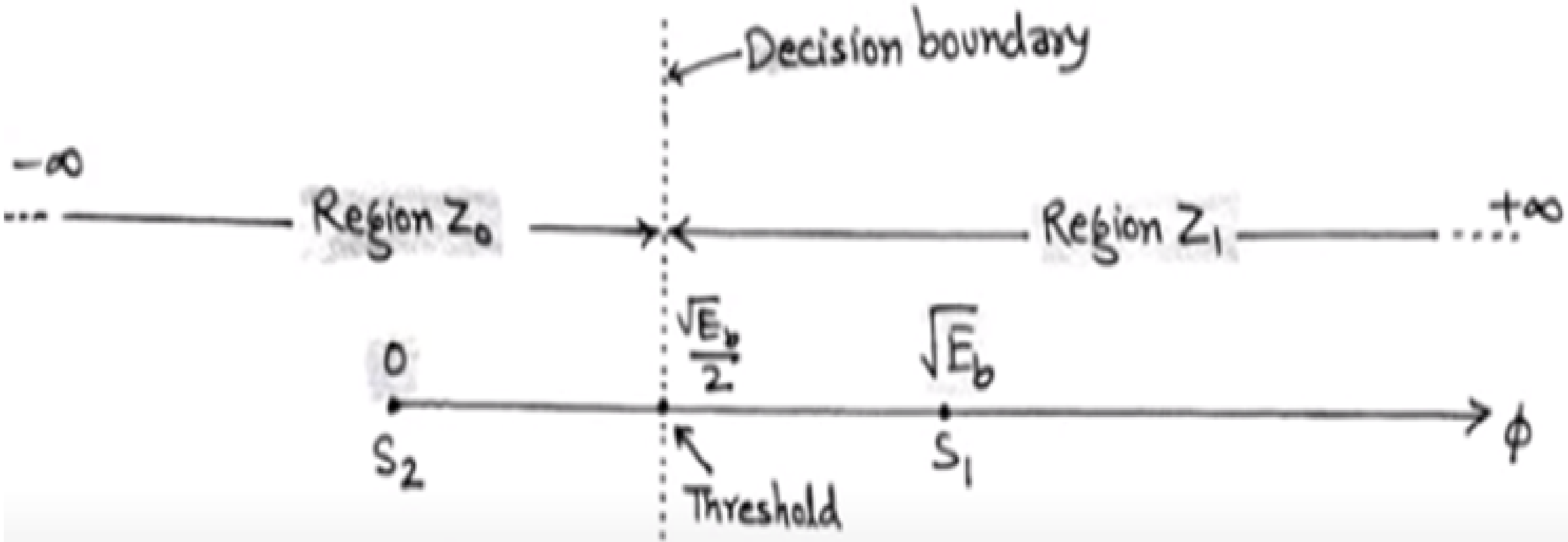


Constellation diagram for ASK

Binary ASK can be written as

$$s(t) = \begin{cases} s_1(t) = \sqrt{E_b}\,\Phi_1(t) & \text{for } 0 \le t \le T_b \quad \text{for symbol 1} \\ s_2(t) = 0 & \text{for } 0 \le t \le T_b \quad \text{for symbol 0} \end{cases}$$

Wher E_b is transmitted signal energy/bit



Signal-space diagram / signal constellation for ASK

Performance Analysis of Digital Communication System

Probability of Error of ASK

$$P_e = \frac{1}{2}\operatorname{erfc}\left\{\frac{\gamma_{max}^2}{8}\right\}^{1/2}$$

$$\gamma_{max}^2 = \frac{2}{N_0}\int_0^T p^2(t)\,dt$$

$$p(t) = x_{01}(t) - x_{02}(t)$$

$$1 \rightarrow A\cos\omega_c t = x_{01}(t)$$

$$0 \rightarrow 0 \quad = x_{02}(t)$$

$$p(t) = A\cos\omega_c t - 0 = A\cos\omega_c t$$

$$\gamma_{max}^2 = \frac{2}{N_0}\int_0^T A^2\cos^2\omega_c t\,dt$$

$$= \frac{2A^2}{N_0}\int_0^T \cos^2\omega_c t\,dt$$

$$\cos^2\omega_c t = \frac{1+\cos 2\omega_c t}{2}$$

$$\gamma_{max}^2 = \frac{2A^2}{N_0}\int_0^T \frac{1+\cos 2\omega_c t}{2}\,dt$$

$$\gamma_{max}^2 = \frac{\cancel{2}A^2}{\cancel{2}N_0}\int_0^T (1+\cos 2\omega_c t)\,dt$$

$[-1,1]$

$$\gamma_{max}^2 = \frac{A^2}{N_0}\left\{t + \cancel{\frac{\sin 2\omega_c t}{2\omega_c}}\right\}_0^T$$

$\omega_c \uparrow\uparrow$

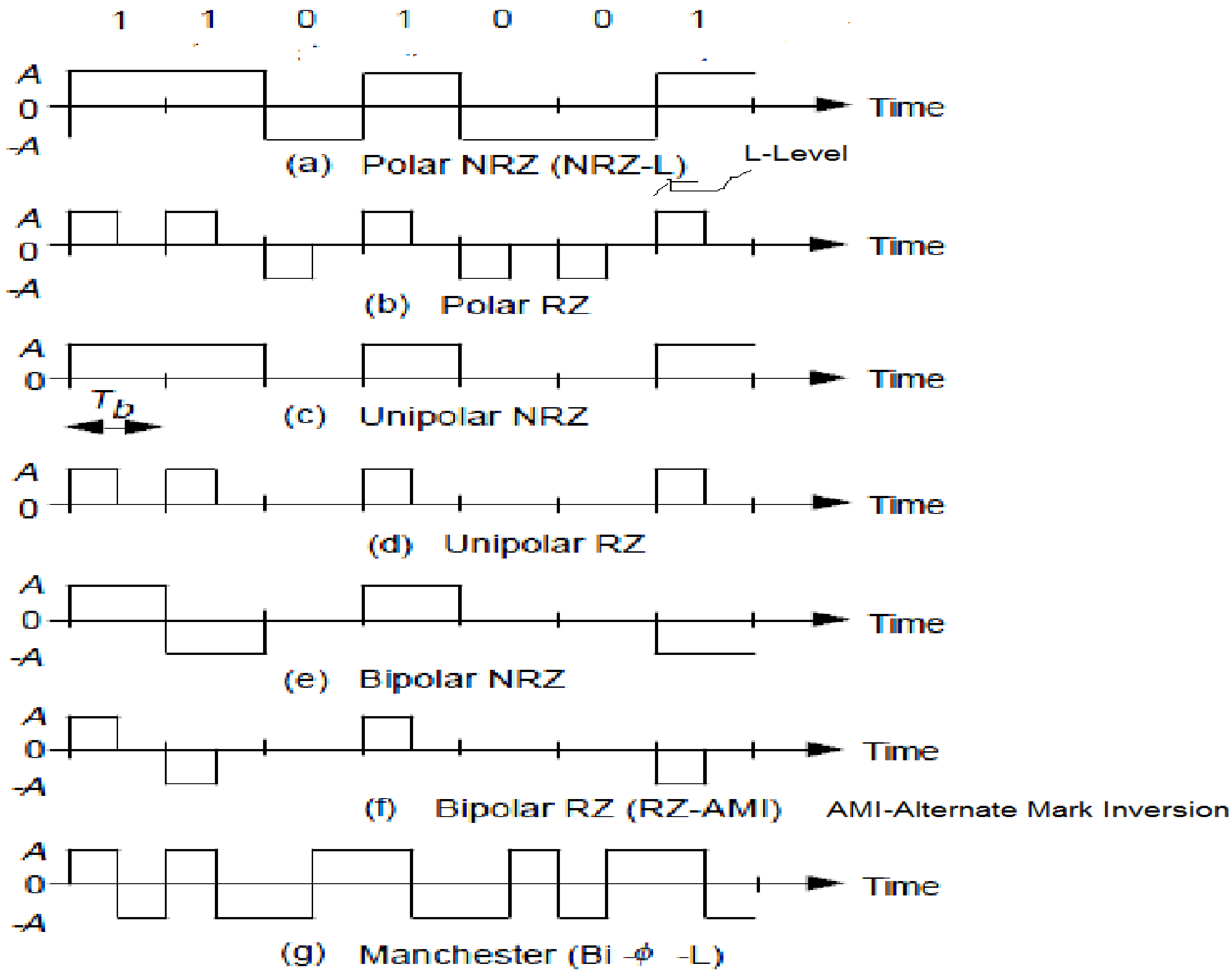
$$\gamma_{max}^2 = \frac{A^2 \times T}{N_0}$$

$$P_e = \frac{1}{2}\operatorname{erfc}\left\{\frac{\frac{A^2}{N_0}\times T}{8}\right\}^{1/2}$$

$$\Rightarrow \frac{A^2}{2} = E$$

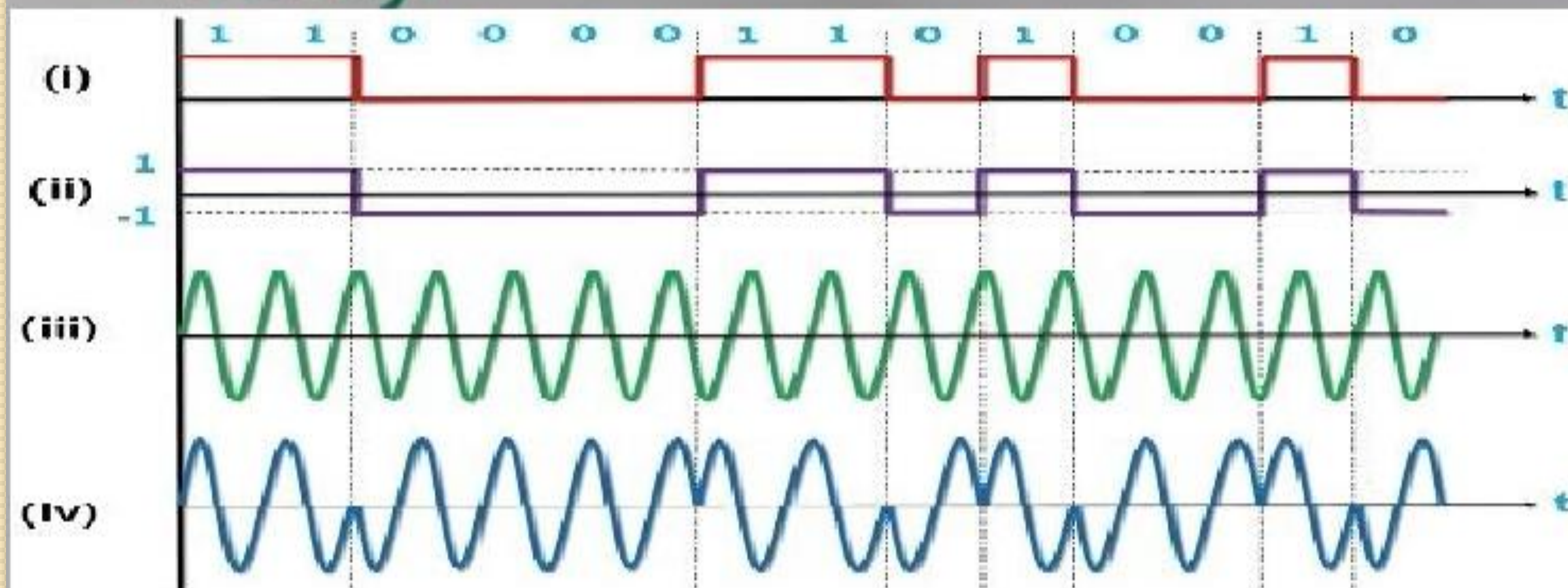
$$P_e = \frac{1}{2}\operatorname{erfc}\left\{\frac{E}{4N_0}\right\}$$

Line coding techniques



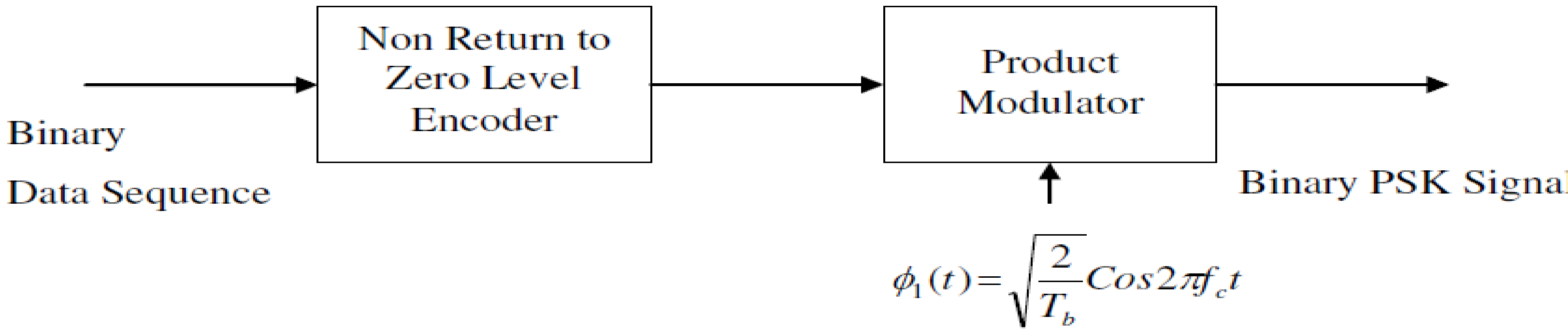
Phase Shift Keying (PSK)

In phase shift keying, phase of the carrier wave (analog) is switched as per the input digital signal. This is analogous to PM (analog phase modulation).



(i) Binary data sequence
(ii) Bipolar NRZ sequence
(iii) Carrier wave
(iv) BPSK waveform

Coherent Binary PSK:



Fig(a) Block diagram of BPSK transmitter

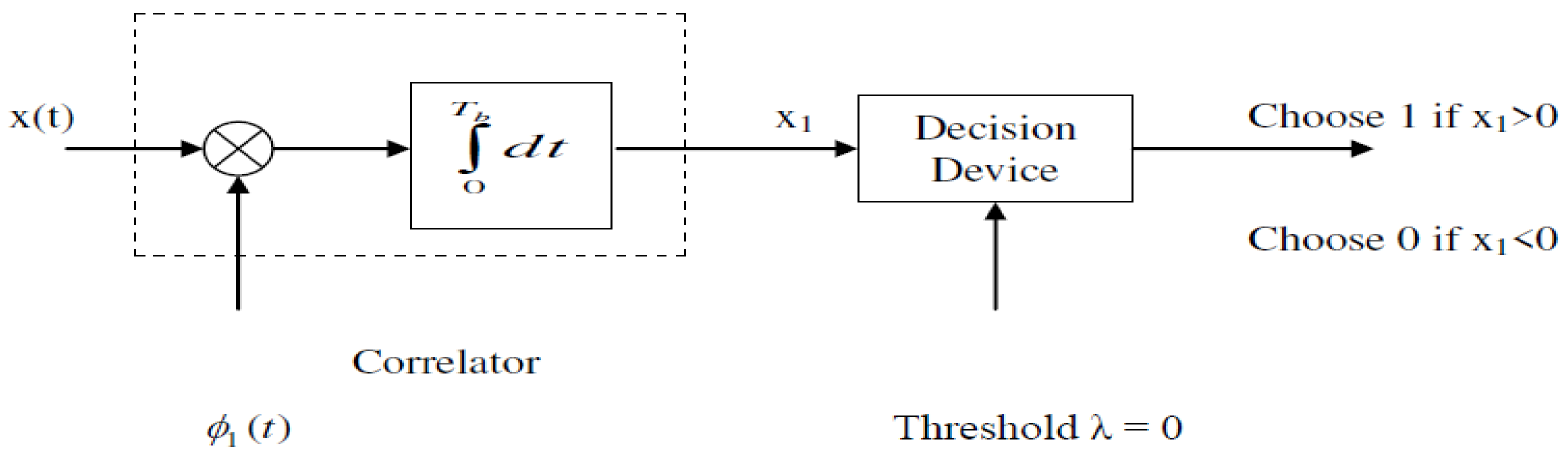


Fig (b) Coherent binary PSK receiver

Coherent Binary PSK

In binary PSK system, symbol 1 & 0 are represented as

$$s(t)=\begin{cases} s_1(t)=\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t & \text{for } 0\le t\le T_b \quad \text{for symbol 1} \\ s_2(t)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t+\pi) & \text{for } 0\le t\le T_b \quad \text{for symbol 0} \\ \quad = -\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t & \end{cases}$$

Basis function $\Phi_1(t) = \sqrt{\frac{2}{T_b}}\cos 2\pi f_c t$ for $0\le t\le T_b$

Binary PSK can be written as

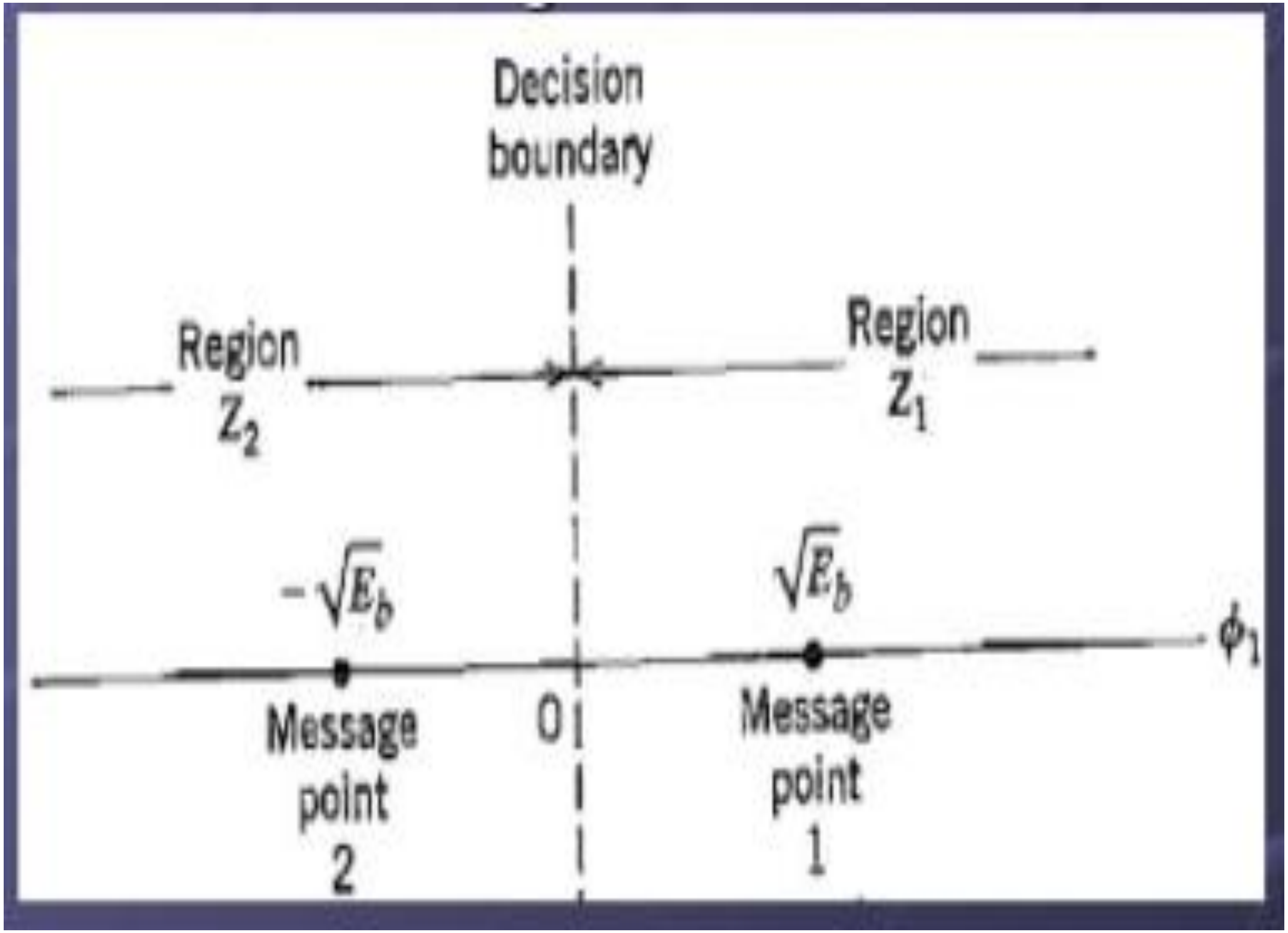
$$s(t)=\begin{cases} s_1(t)=\sqrt{E_b}\,\Phi_1(t) & \text{for } 0\le t\le T_b \quad \text{for symbol 1} \\ s_2(t)=-\sqrt{E_b}\,\Phi_1(t) & \text{for } 0\le t\le T_b \quad \text{for symbol 0} \end{cases}$$

A coherent binary PSK system is characterized by one-dimensional signal space. (N=1), and with two message points(M=2)

The coordinates o message points equal

$$s_{11}=\int_0^{T_b} s_1(t)\,\Phi_1(t)\,dt= +\sqrt{E_b} \qquad s_{21}=\int_0^{T_b} s_2(t)\,\Phi_1(t)\,dt= -\sqrt{E_b}$$

Signal space diagram for coherent binary PSK system



Performance Analysis of Digital Communication System

Probability of Error of PSK

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{\gamma_{max}^2}{8}\right\}^{1/2}$$

$$\gamma_{max}^2 = \frac{2}{N_0}\int_0^T P^2(t)\,dt$$

PSK :-

$1 \rightarrow A\cos\omega_c t\ ;\ x_{01}(t)$

$0 \rightarrow -A\cos\omega_c t\ ;\ x_{02}(t)$

$$P(t) = x_{01}(t) - x_{02}(t)$$

$$P(t) = A\cos\omega_c t - \{-A\cos\omega_c t\}$$

$$= 2A\cos\omega_c t$$

$$\gamma_{max}^2 = \frac{2}{N_0}\int_0^T (2A\cos\omega_c t)^2\,dt$$

$$= \frac{2}{N_0}\cdot 4A^2\int_0^T \cos^2\omega_c t\,dt$$

$$\cos^2\omega_c t = \frac{1+\cos 2\omega_c t}{2}$$

$$\gamma_{max}^2 = \frac{8A^2}{N_0}\int_0^T \frac{1+\cos 2\omega_c t}{2}\,dt$$

$$= \frac{4A^2}{N_0}\int_0^T (1+\cos 2\omega_c t)\,dt$$

$$= \frac{4A^2}{N_0}\left\{t + \cancel{\frac{\sin 2\omega_c t}{2\omega_c}}^{\,0}\right\}_0^t$$

$\omega_c \uparrow\uparrow$ (carrier frequency)

$\sin 2\omega_c t \rightarrow [-1, 1]$

$$\gamma_{max}^2 = \frac{4A^2\cdot T}{N_0}$$

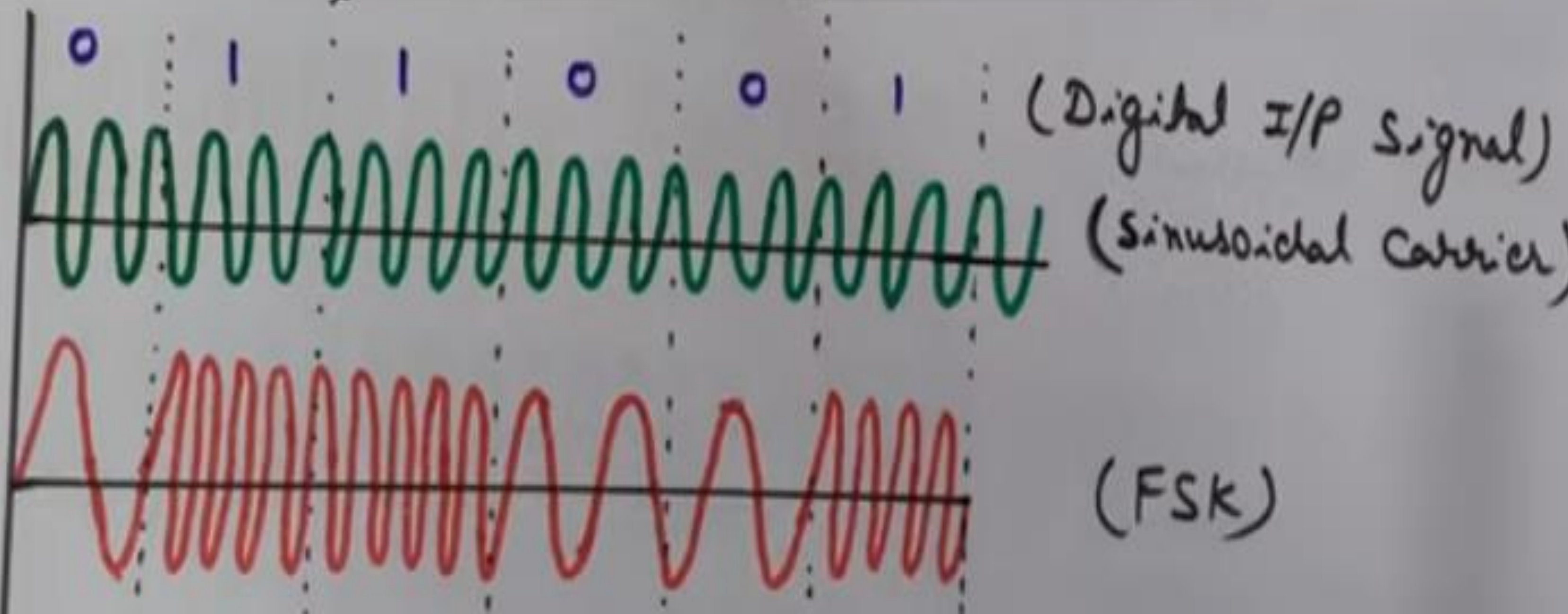
$$P_e = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{\cancel{4}^{\,1}A^2 T}{N_0\cdot \cancel{8}_{\,2}}\right\}^{1/2}$$

$$= \frac{1}{2}\,\mathrm{erfc}\left\{\frac{A^2 T}{2N_0}\right\}$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{E}{N_0}\right\}$$

Frequency Shift keying (FSK)

If the frequency of sinusoidal carrier wave is varied (switched) depending on the digital input signal, then it is known as the Frequency shift keying (FSK). It is analogous to FM (Analog freq Modulation)



Coherent Binary FSK

In binary FSK system, symbol 1 & 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount.

$$s_i(t)= \begin{cases} \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t & \text{for } 0\le t\le T_b \\ 0 & \text{elsewhere} \end{cases} \qquad \text{where } i=1,2.$$

Transmitted frequency equals $f_i = \dfrac{n_c + i}{T_b}$ for some fixed integer n_c and i=1,2

$$\Phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_b}}\cos 2\pi f_c t & \text{for } 0\le t\le T_b \\ 0 & \text{elsewhere} \end{cases}$$

$$S_{ij} = \int_0^{T_b} s_i(t)\,\Phi_j(t)\,dt = \begin{cases} \sqrt{E_b} & i=j \\ 0 & i\neq j \end{cases}$$

FSK in terms of orthonormal basis functions is

$$s(t)= \begin{cases} s_1(t)= \sqrt{E_b}\ \Phi_1(t) & \text{for } 0\leq t\leq T_b \quad \text{for symbol 1} \\ s_2(t)= \sqrt{E_b}\ \Phi_2(t) & \text{for } 0\leq t\leq T_b \quad \text{for symbol 0} \end{cases}$$

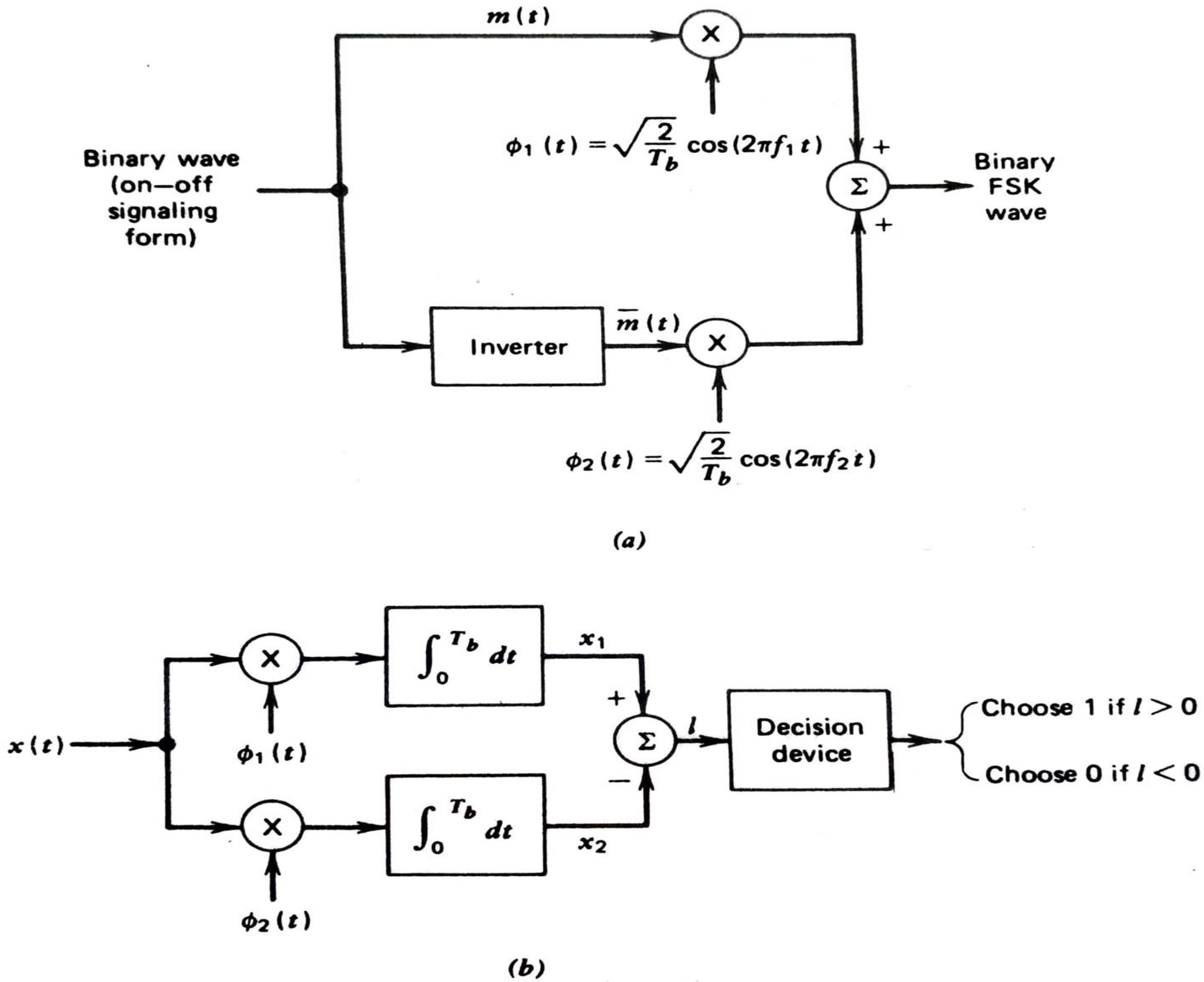
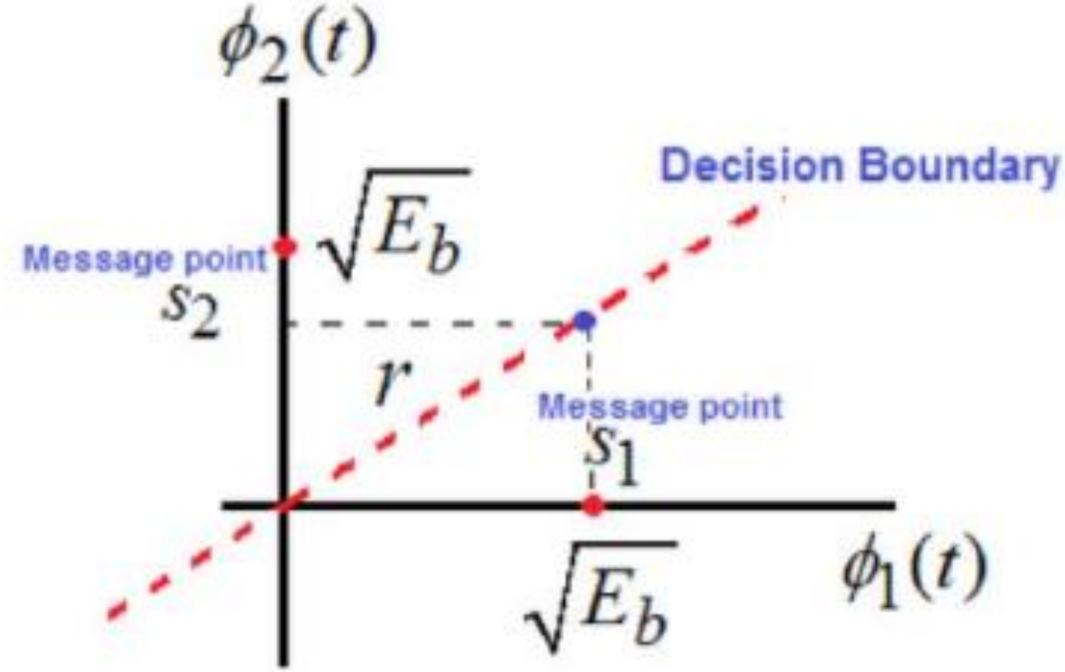


Figure 7.5 Block diagrams for (*a*) binary FSK transmitter, and (*b*) coherent binary FSK receiver.

Signal space diagram for coherent binary FSK system



Performance Analysis of Digital Communication System

Probability of Error of FSK

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{\gamma_{max}^2}{8}\right\}^{1/2}$$

$$\gamma_{max}^2 = \frac{2}{N_0}\int_0^T p^2(t)\,dt$$

$$p(t) = x_{01}(t) - x_{02}(t);$$

$1 \to A\cos(\omega+\tau)t \to x_{01}(t)$

$0 \to A\cos(\omega-\tau)t \to x_{02}(t)$

$$\gamma_{max}^2 = \frac{2}{N_0}\int_0^T \left(A\cos(\omega+\tau)t - A\cos(\omega-\tau)t\right)^2 dt$$

$$= \frac{2A^2}{N_0}\int_0^T \underbrace{\cos^2(\omega+\tau)t}_{\frac{1+\cos 2(\omega+\tau)t}{2}} + \underbrace{\cos^2(\omega-\tau)t}_{\frac{1+\cos 2(\omega-\tau)t}{2}} - 2\cos(\omega+\tau)t\cdot\cos(\omega-\tau)t\,dt$$

$$\int_0^T 2\cos A\cos B \to \frac{\sin(A+B)T}{2(A+B)} + \frac{\sin(A-B)T}{2(A-B)}$$

$$= \frac{2A^2}{N_0}\left\{\frac{T}{2} + \cancelto{0}{\frac{\sin(2(\omega+\tau))T}{4(\omega+\tau)}} + \frac{T}{2} + \cancelto{0}{\frac{\sin(2(\omega-\tau))T}{4(\omega-\tau)}} - \cancelto{0}{\frac{\sin 2\omega T}{4\omega}} - \frac{\sin 2\tau T}{4\tau}\right\}$$

$\omega \to$ carrier freq $\to$ high

$$\gamma_{max}^2 = \frac{2A^2}{N_0}\left\{T - \underbrace{\frac{\sin 2\tau T}{4\tau}}_{\text{Sinc } f^n}\right\}$$

max at $T = 3\pi/2$

γ_{max}^2 @ $3\pi/2$

$$= \frac{2A^2}{N_0}T\cdot\{2.42\}$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{\frac{2A^2T}{N_0}\cdot 2.42}{8}\right\}$$

$$= \frac{1}{2}\,\mathrm{erfc}\left\{0.6\,\frac{A^2T}{N_0}\right\}$$

$$= \frac{1}{2}\,\mathrm{erfc}\left\{0.3E/N_0\right\}$$

- Quadrature phase shift keying (QPSK) is a form of PSK (Phase Shift keying), in which two bits are modulated at once.

- It selects one of four possible carrier phase shifts [$0^\circ, 90^\circ, 180^\circ, 270^\circ$]

- QPSK allows the signal to carry twice as much information as ordinary PSK using the same BW.

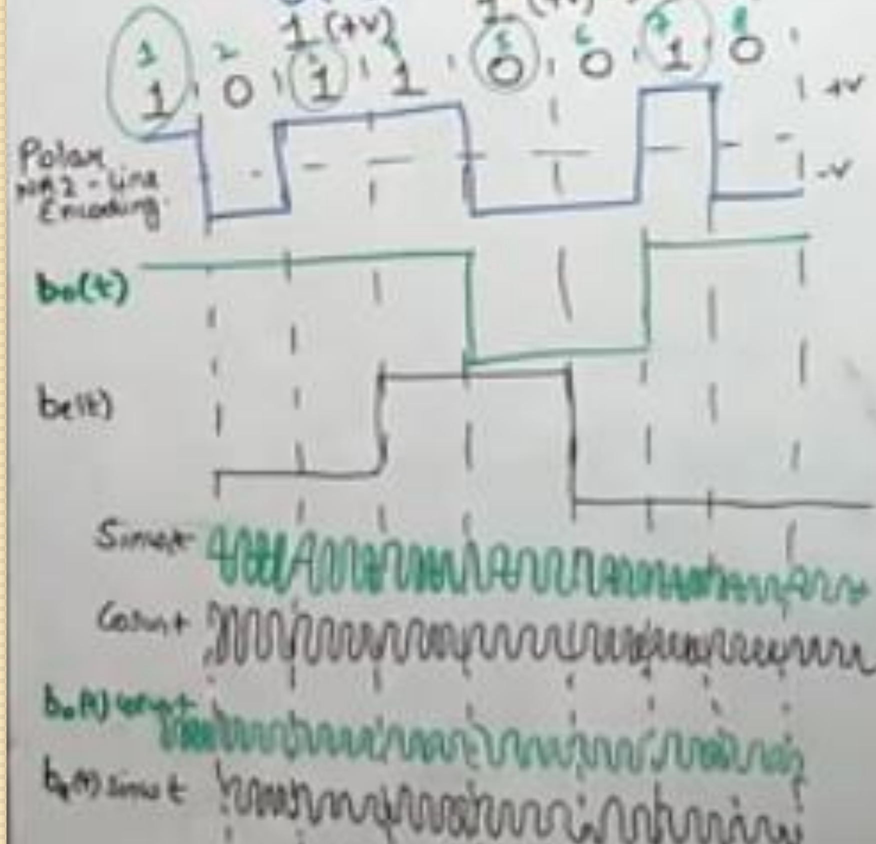
- QPSK is used for satellite transmission of MPEG2, cable modem, cellular phone system etc.

Digital Communications (QPSK)

QPSK ⇒ bit change corresponds to 45° phase change

⇒ 2 bit combined ⇒ 90° Phase change

1(+v)	0(-v)	→ $\pi/4$
0(-v)	0(-v)	→ $3\pi/4$
0(-v)	1(+v)	→ $5\pi/4$
1(+v)	1(+v)	→ $7\pi/4$



$$s(t) = b_o(t)\cos\omega_c t + b_e(t)\sin\omega_c t$$

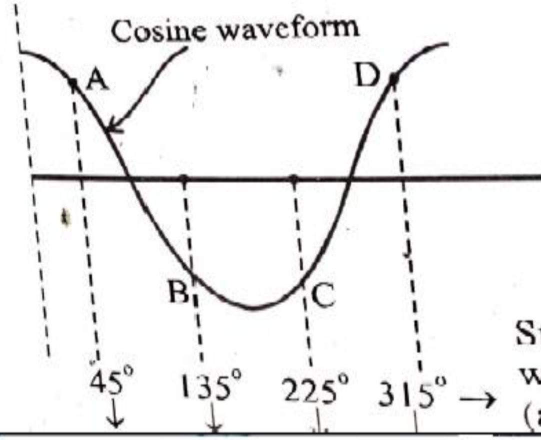
$$= \sqrt{2P_s}\cos\omega_c t + \sqrt{2P_s}\sin\omega_c t$$

$$= \sqrt{P_s T_b}\sqrt{\tfrac{2}{T_b}}\cos\omega_c t + \sqrt{P_s T_b}\sqrt{2/T_b}\sin\omega_c t$$

$\rightarrow \phi_1$ $\qquad \phi_2$

Example 7.1 : Draw the QPSK waveform for a binary sequence of 1001111000

Solution : The given binary sequence can be divided into five sets of dibits as 10, 01, 11, 10 and 00. Referring to table 7.1, The starting phase for the dibit 10 is 45°, for 01 it is 225°, for 11 → 315°, for 10 → 45° and for 00 → 135°. With respect to a cosine waveform shown in fig. 7.15, these phases can be marked as shown. Fig. 7.16 : QPSK waveform



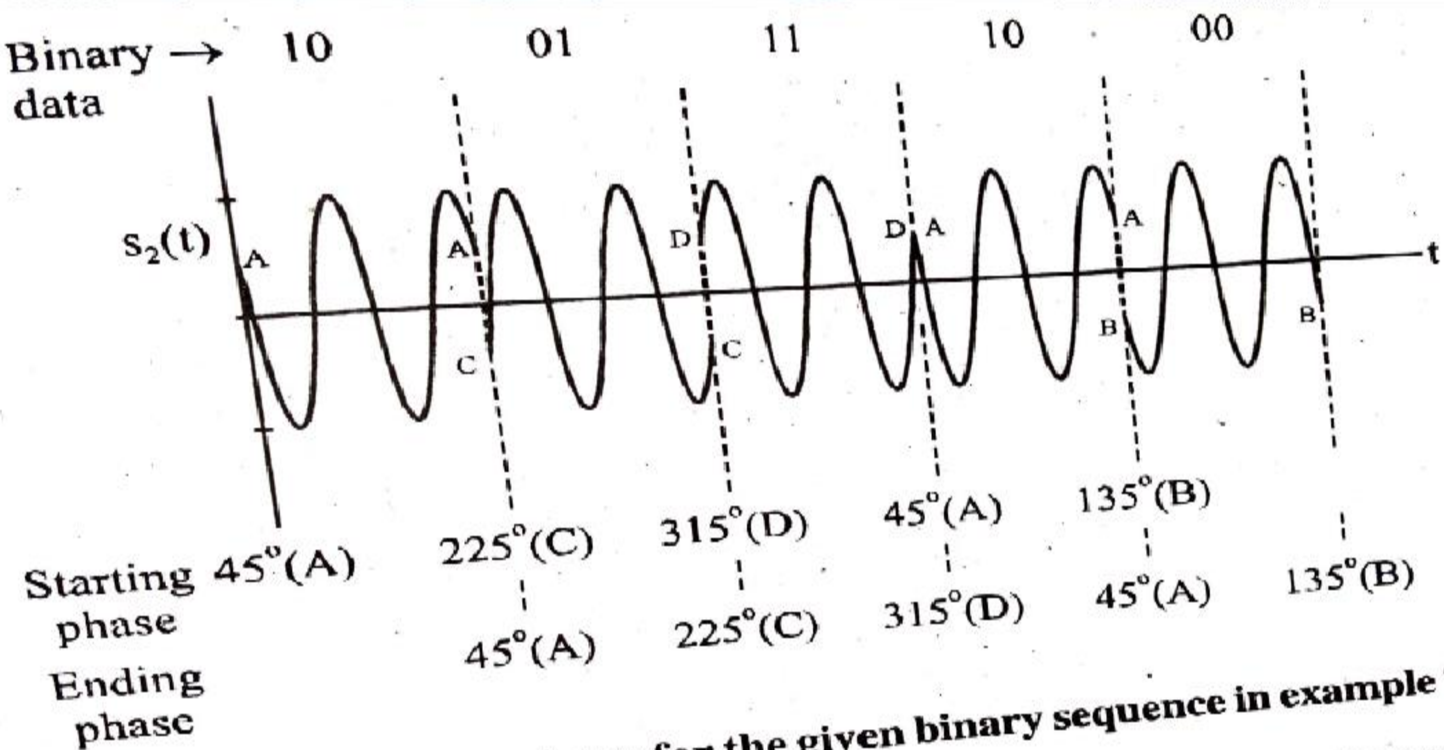
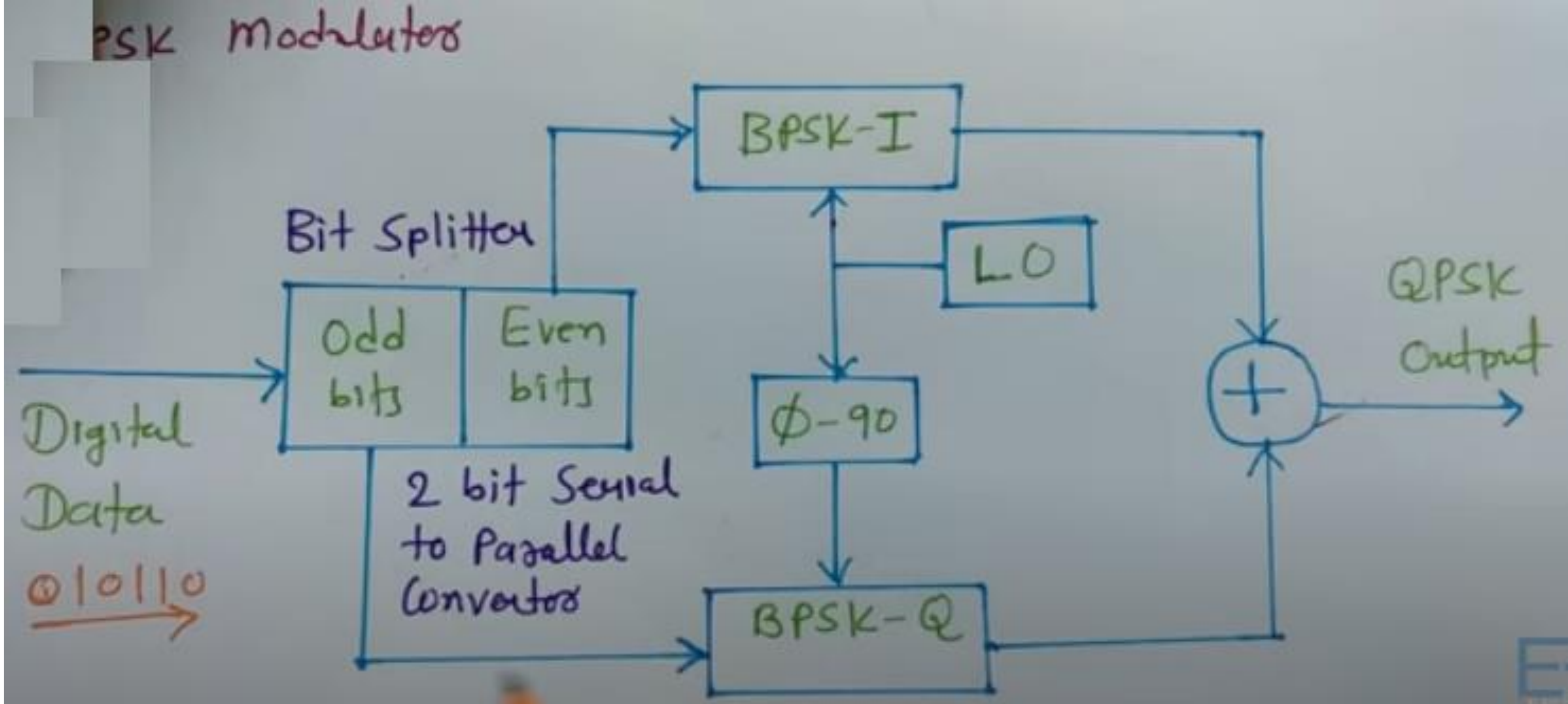
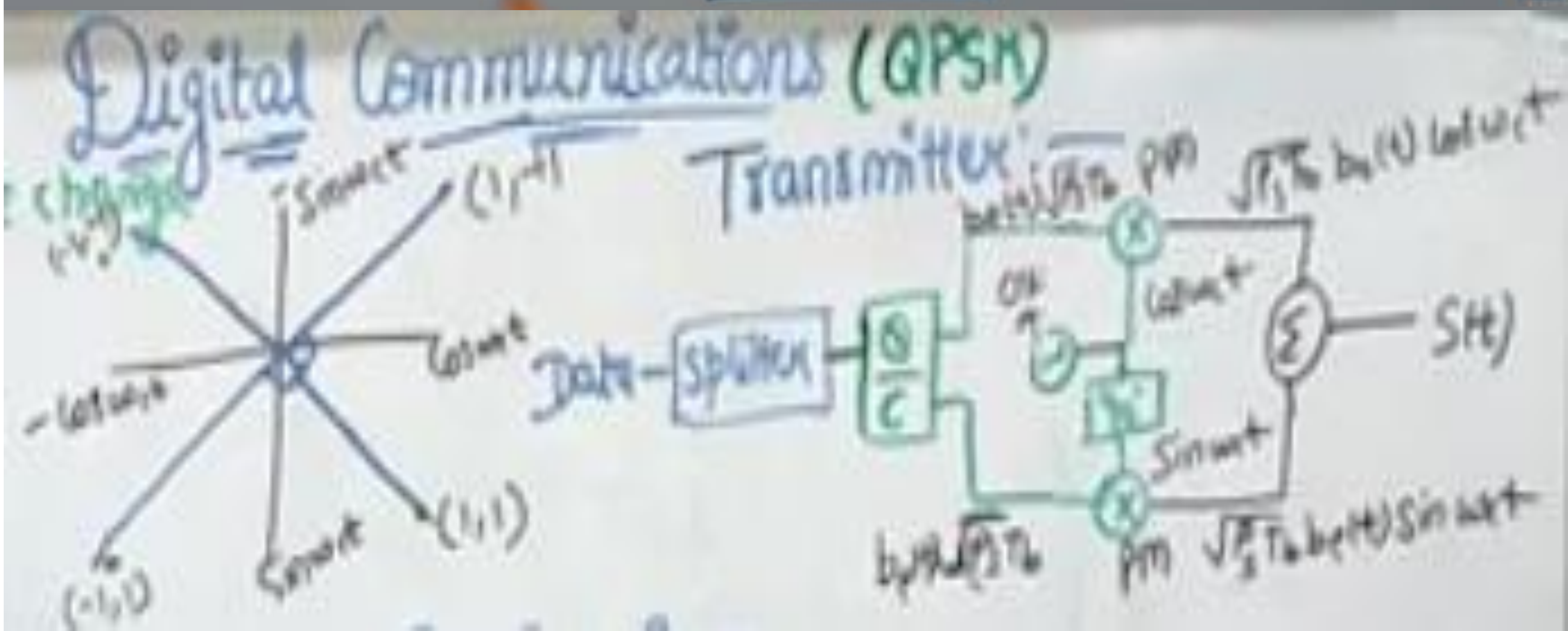


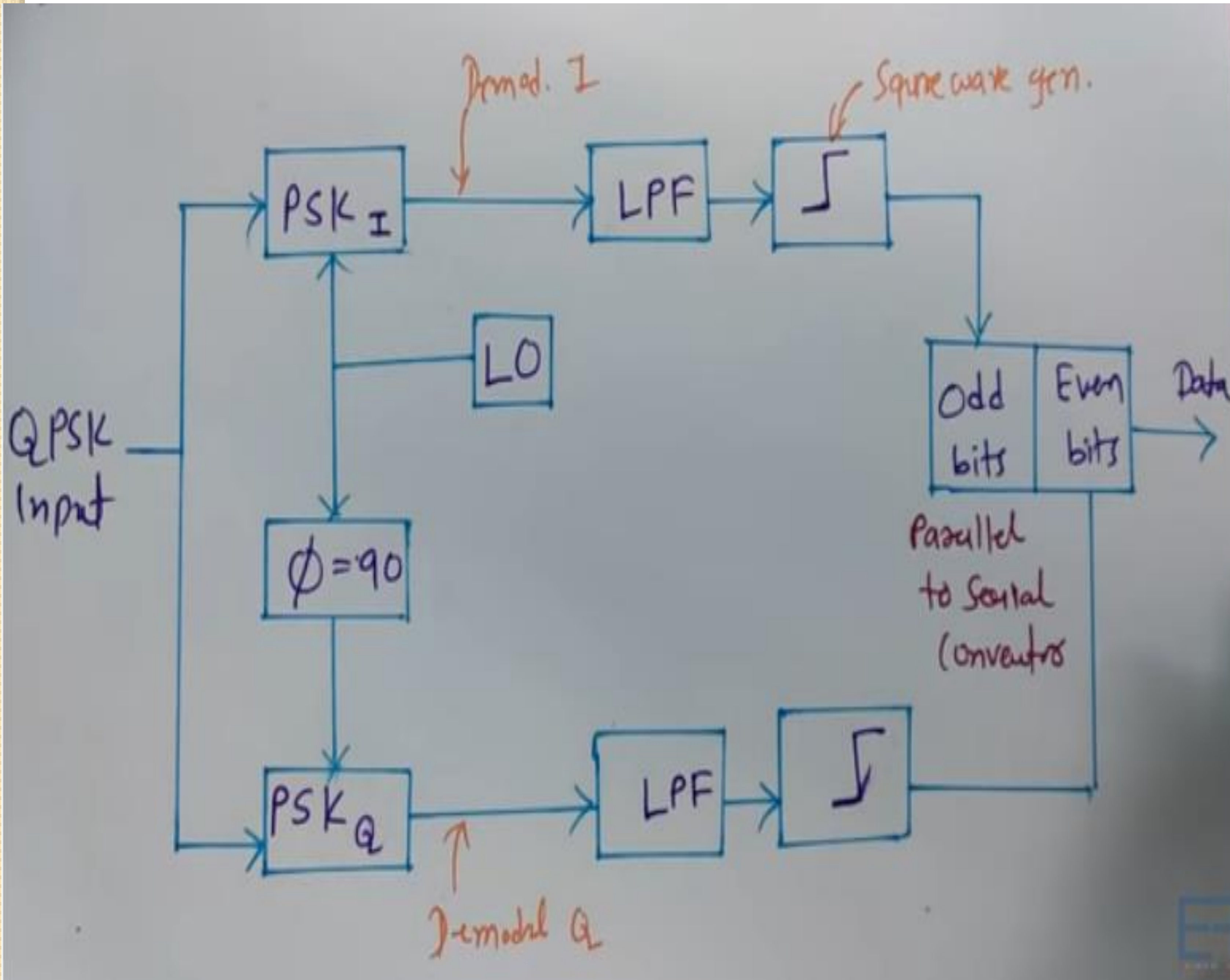
Fig. 7.16 : QPSK waveform for the given binary sequence in example 7.1



Modulator
BPSK-I
Bit Splitter
LO
QPSK Output
Odd bits
Even bits
ϕ-90
Digital Data
010110
2 bit Serial to Parallel Convertor
BPSK-Q



Digital Communications (QPSK)
Transmitter
Data
Splitter
S(t)


Demod. I
Squarewave gen.
PSK I
LPF
LO
QPSK Input
Odd bits
Even bits
Data
Parallel to Serial Convertor
ϕ=90
PSK Q
LPF
Demodul Q

QPSK (Quadriphase-shift keying)

- This modulation scheme is characterised by the fact that the information carried by the transmitted wave is contained in the phase.
- The phase of the carrier takes on one of four equally spaced values, such as п/4, 3п/4, 5п/4, 7п/4.
- Each possible value of the phase corresponds to a unique pair of bits called a dibit. (ex; set of phase values may represent the gray encoded set of dibits: 10,00,01 and 11)

$$s_i(t) = \sqrt{\frac{2E}{T}} Cos[2\pi f_c t + (2i-1)\pi/4] \qquad 0 \le t \le \mathrm{T} \qquad i = 1 \; to \; 4$$

$$s_i(t) = \sqrt{\frac{2E}{T}}\; Cos[(2i-1)\pi/4]\cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin[(2i-1)\pi/4]\sin(2\pi f_c t) \qquad 0 \le t \le T \qquad i = 1 \; to \; 4$$

- There are only two orthonormal basis functions $\Phi_1(t)$ and $\Phi_2(t)$, in the expansion $s_i(t)$.
- $\Phi_1(t)= \sqrt{\frac{2}{T}}\ \cos(2\pi f_c t) \qquad 0\le t\le T$
- $\Phi_2(t)= \sqrt{\frac{2}{T}}\ \sin(2\pi f_c t) \qquad 0\le t\le T$
- There are 4 message points, and the associated signal vectors are defined by

$$s_i = \begin{bmatrix} \sqrt{E}\ \cos((2i-1)\,\pi/4) \\ \sqrt{E}\ \sin((2i-1)\,\pi/4) \end{bmatrix} \qquad i= 1,2,3,4$$

Input dibit $0 \leq t \leq T$	Phase of QPSK signal (*radians*)	Coordinates of message points s_{i1}	Coordinates of message points s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

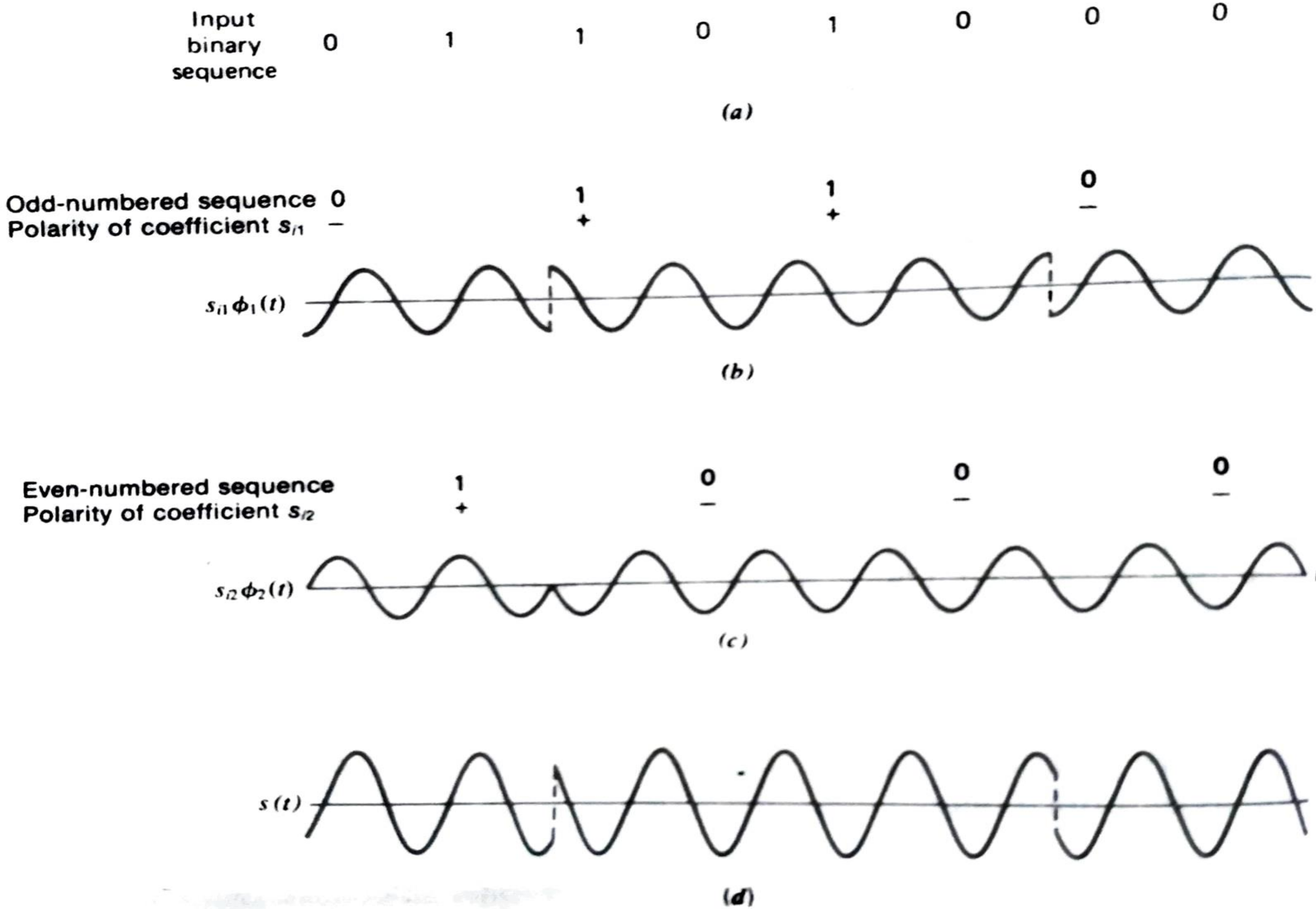
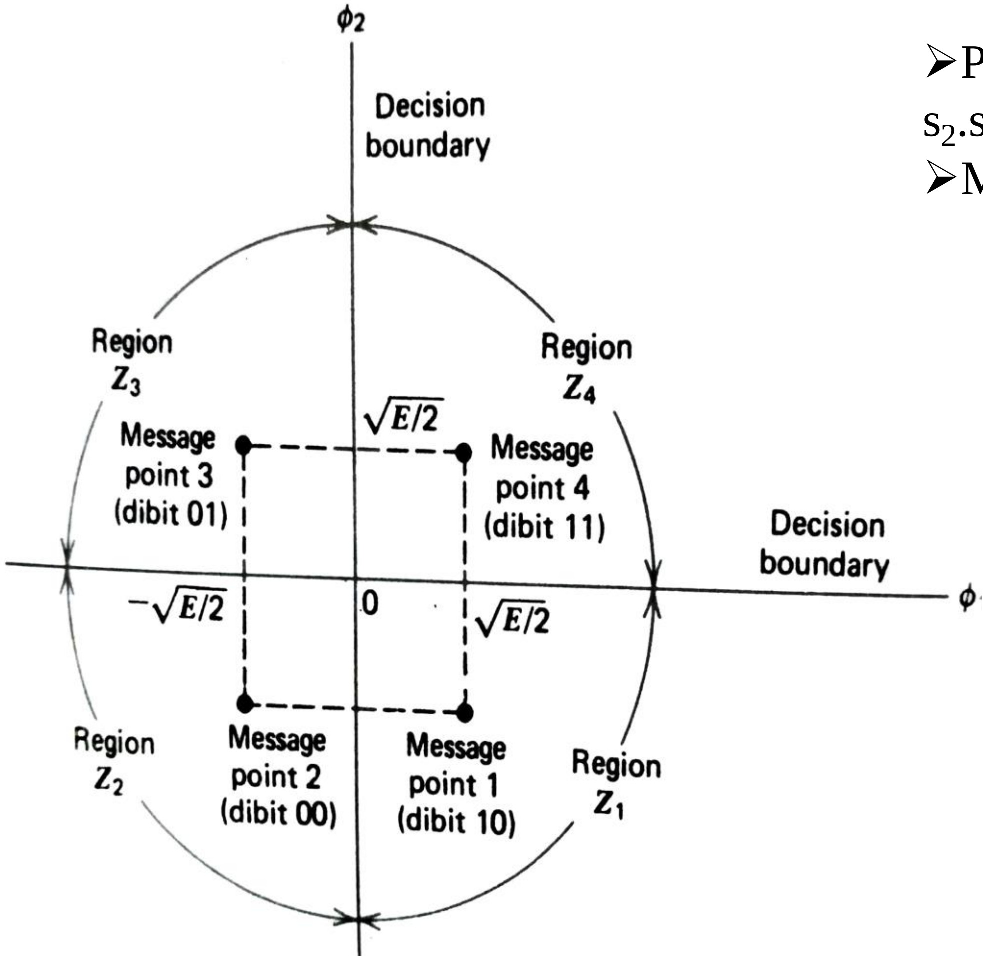


Figure 7.7 (*a*) Input binary sequence. (*b*) Odd-numbered bits of input sequence and associated binary PSK wave. (*c*) Even-numbered bits of input sequence and associated binary PSK wave. (*d*) QPSK waveform.

Signal space diagram for coherent QPSK system



- Partition the signal space into four regions: s_1, s_2.s_3.s_4.
- Mark the regions Z_1, Z_2, Z_3, Z_4.

DPSK(Differential Phase–Shift Keying)

- Its non coherent version of PSK.
- It eliminates the need for a coherent reference signal at the receiver by combining two basic operations at the transmitter.

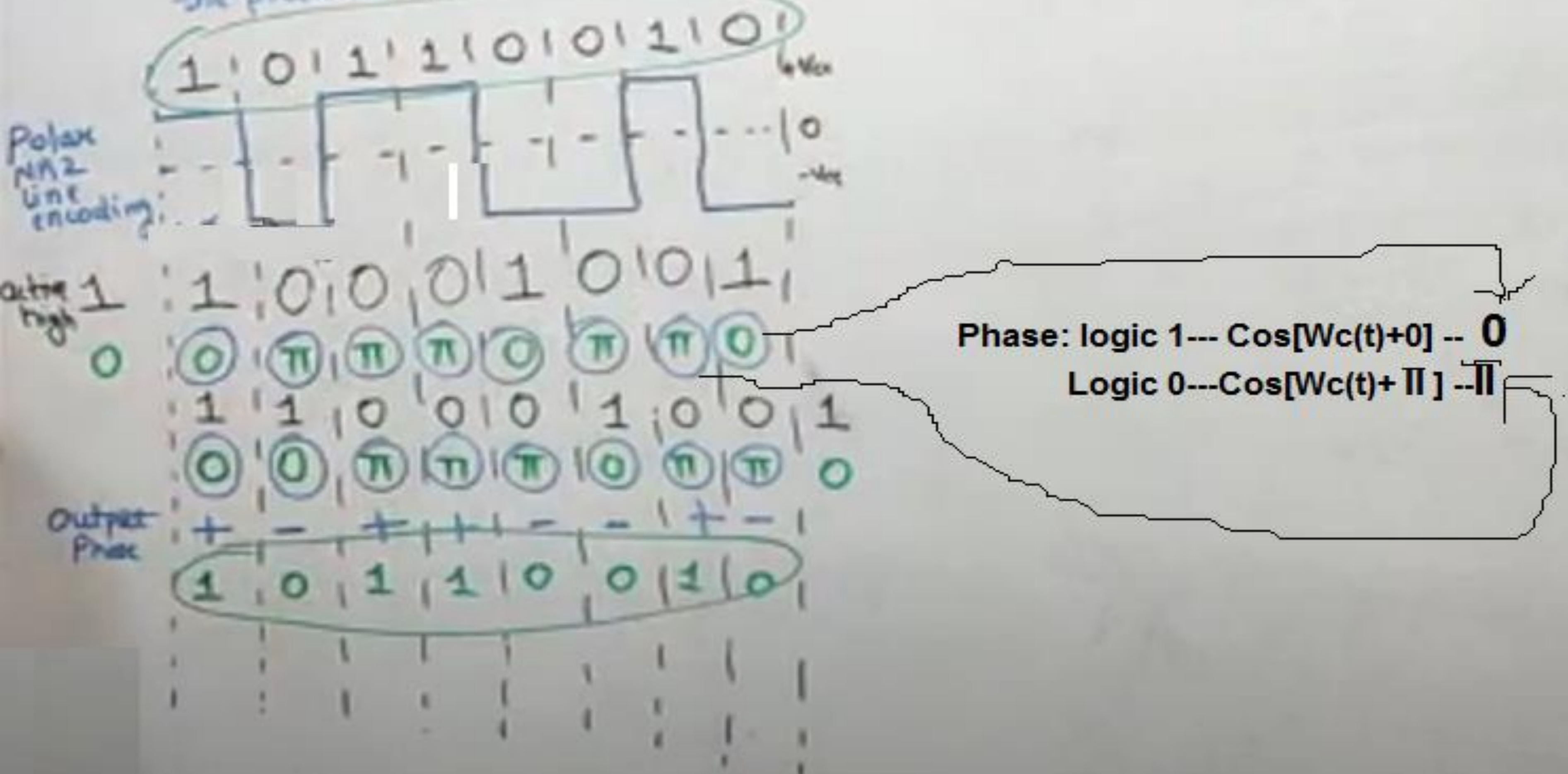
1)Differential-encoding of the input binary wave.

2)Phase-shift keying

Digital Communications

DPSK (Differential Phase Shift Key)

DPSK: Non coherent mode of BPSK, where the output encoded bit depends upon the previous bit.



0 → transition

1 → No transition

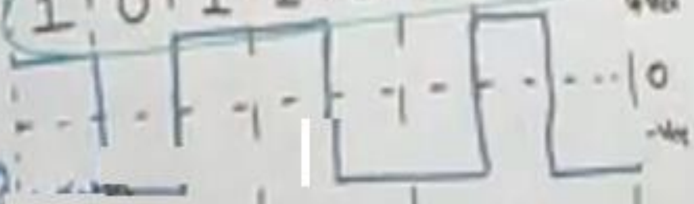
Digital Communications

DPSK (Differential Phase Shift Key)

DPSK: Non coherent mode of BPSK, where the output encoded bit depends upon the previous bit.

1 0 1 1 0 0 1 0

Polar NRZ line encoding:



Initial high 1	1	0	0	0	1	0	0	1	
0	0	π	π	π	0	π	π	0	
	1	1	0	0	0	1	0	0	1
	0	0	π	π	π	0	π	π	0
Output Phase	+	-	+	+	-	-	+	-	
	1	0	1	1	0	0	1	0	

0 → transition

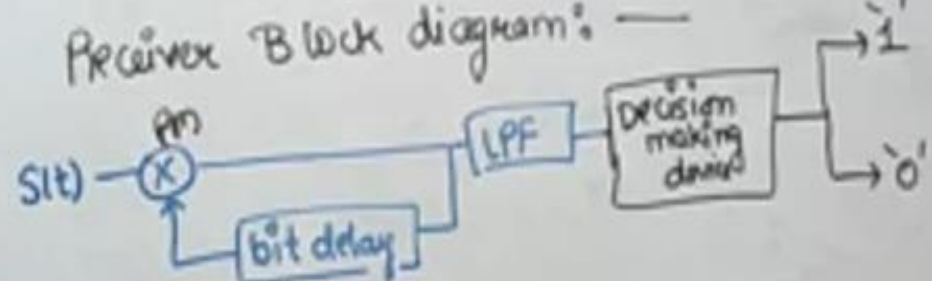
1 → No transition

Transmitter block diagram:—

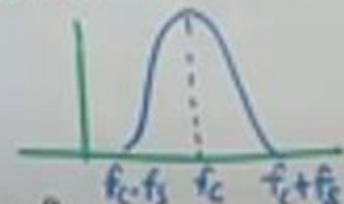
0 → transition

1 → No transition

Receiver Block diagram:—



Bandwidth:—



$$BW = f_c + f_s - (f_c - f_s)$$
$$= \cancel{f_c} + f_s - \cancel{f_c} + f_s$$
$$= 2f_s$$
$$= \frac{2}{T_s} = \frac{\cancel{2}}{\cancel{2}T_b}$$
$$\boxed{BW = f_b}$$

Advantages:—

→ Bandwidth is reduced

→ Receiver → price efficient.

Disadvantages:—

→ High probability of error.