Realisation of filters :-

→ General form of LCCDE eqⁿ is given by

$$\sum_{k=0}^{N-1} a_k \cdot y(n-k) = \sum_{k=0}^{M-1} b_k x(n-k) \longrightarrow (1)$$

(or)

$$a_0 \cdot y(n) + a_1 y(n-1) + \cdots = b_0 x(n) + b_1 x(n-1) + \cdots$$

assume $a_0 = 1$

$$y(n) = \{b_0 x(n) + b_1 x(n-1) + \cdots\} - \{a_1 y(n-1) + a_2 y(n-2) + \cdots\} \longrightarrow (2)$$

↑ present o/p ↑ Present i/p & past i/p's ↑ Past o/p's

(or)

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k) - \sum_{k=1}^{N-1} a_k \cdot y(n-k)$$

Taking Z.T (1)

$$\Rightarrow \sum_{k=0}^{N-1} a_k \cdot z^{-k} y(z) = \sum_{k=0}^{M-1} b_k \cdot z^{-k} x(z)$$

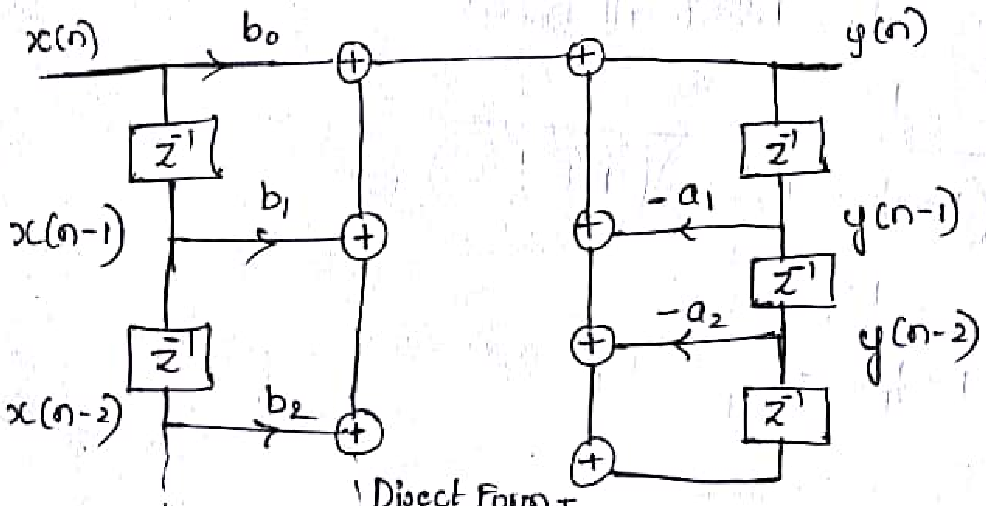
(or)

$$H(z) = \frac{y(z)}{x(z)} = \frac{\sum_{k=0}^{M-1} b_k \cdot z^{-k}}{\sum_{k=0}^{N-1} a_k \cdot z^{-k}}$$

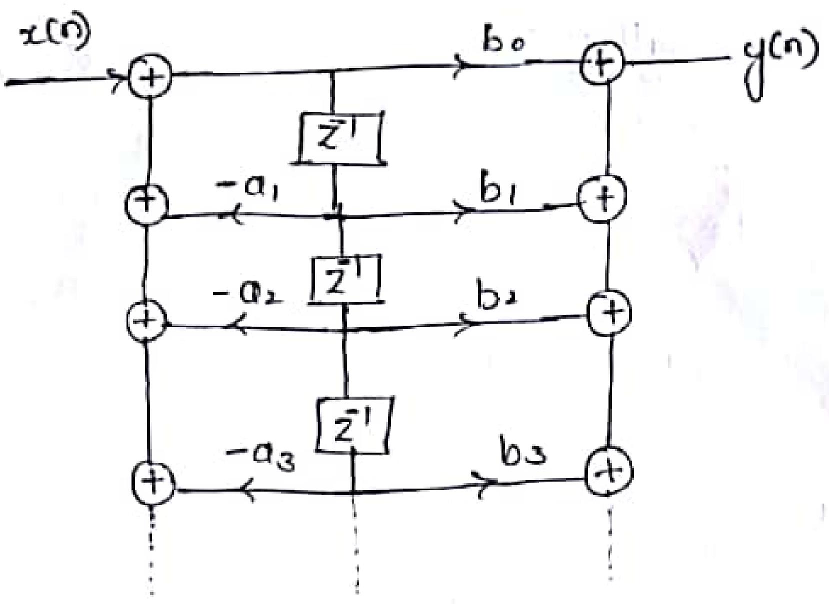
(or)

$$H(z) = \frac{y(z)}{x(z)} = \frac{\sum_{k=0}^{M-1} b_k \cdot z^{-k} \longrightarrow \text{Zero's}}{1 + \sum_{k=1}^{N-1} a_k z^{-k} \longrightarrow \text{poles}} \longrightarrow (3)$$

using eqⁿ (2) we can sketch D.F.T structure

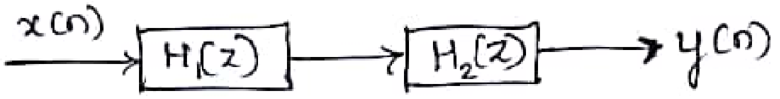


Direct Form-I

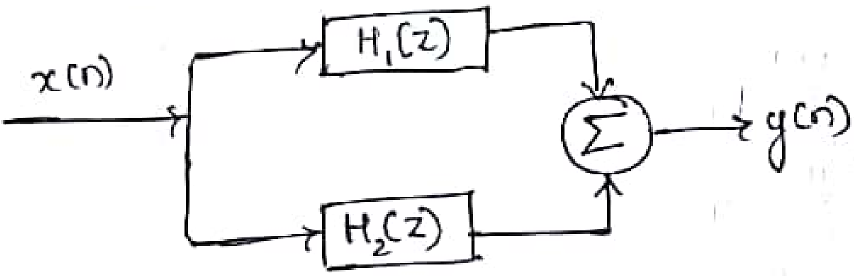


Direct Form - II

Cascade



Parallel



1) sketch DFT, DF-II, parallel and cascade realization of system described by

$$H(z) = \frac{1-\frac{1}{5}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}\right)\left(1+\frac{1}{4}z^{-1}\right)} \longrightarrow (1)$$

Soln :- H(z) can also be written as

$$H(z) = \frac{1-\frac{1}{5}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

$$H(z) = \frac{1-\frac{1}{5}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3}}$$

(265)

W.K.T

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{1}{5}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3}}$$

$$Y(z)\left[1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3}\right] = X(z)\left[1-\frac{1}{5}z^{-1}\right]$$

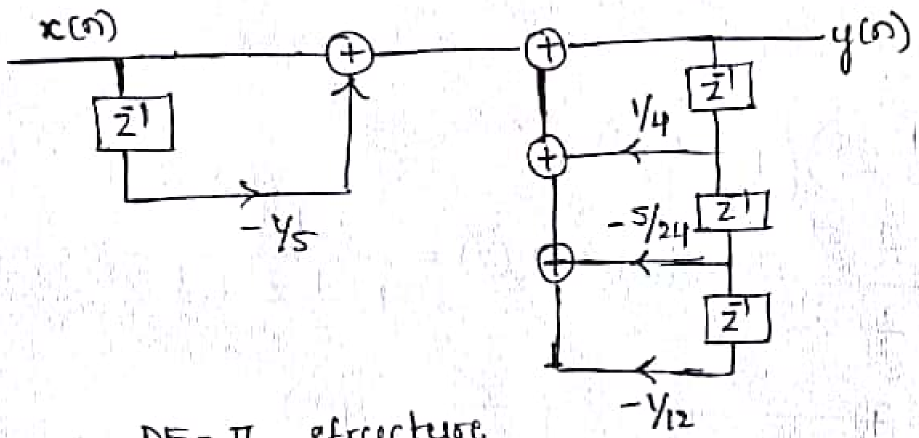
Taking I.Z.T

$$y(n) - \frac{1}{4}y(n-1) + \frac{5}{24}y(n-2) + \frac{1}{12}y(n-3) = x(n) - \frac{1}{5}x(n-1)$$

(or) $$y(n) = x(n) - \frac{1}{5}x(n-1) + \frac{1}{4}y(n-1) - \frac{5}{24}y(n-2) - \frac{1}{12}y(n-3) \longrightarrow (2)$$

using eq^n (2) we can sketch

DF-I structure



DF-II structure

Let $$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{V(z)}\cdot\frac{V(z)}{X(z)} = \frac{1-\frac{1}{5}z^{-1}}{\left[1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3}\right]}$$

& $$\frac{Y(z)}{V(z)} = 1-\frac{1}{5}z^{-1}$$

$$Y(z) = V(z)\left[1-\frac{1}{5}z^{-1}\right]$$

Taking I.Z.T. $y(n) = v(n) - \frac{1}{5}v(n-1) \longrightarrow (a)$

$$\frac{V(z)}{X(z)} = \frac{1}{1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3}}$$

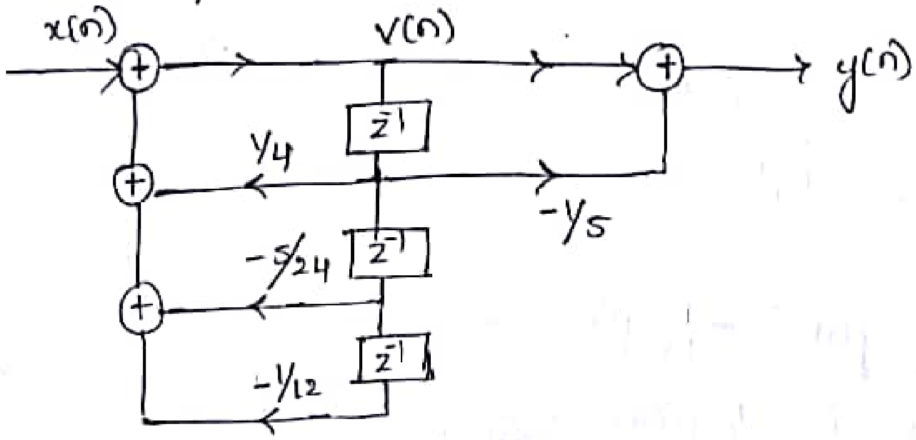
$$V(z)\left[1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3}\right] = X(z)$$

Taking I.Z.T

$$v(n) - \frac{1}{4}v(n-1) + \frac{5}{24}v(n-2) + \frac{1}{12}v(n-3) = x(n)$$

(or) $$v(n) = x(n) + \frac{1}{4}v(n-1) - \frac{5}{24}v(n-2) - \frac{1}{12}v(n-3) \longrightarrow (b)$$

using eq^n (a) and (b) we can sketch DF - II structure.



DF - II structure

cascade form

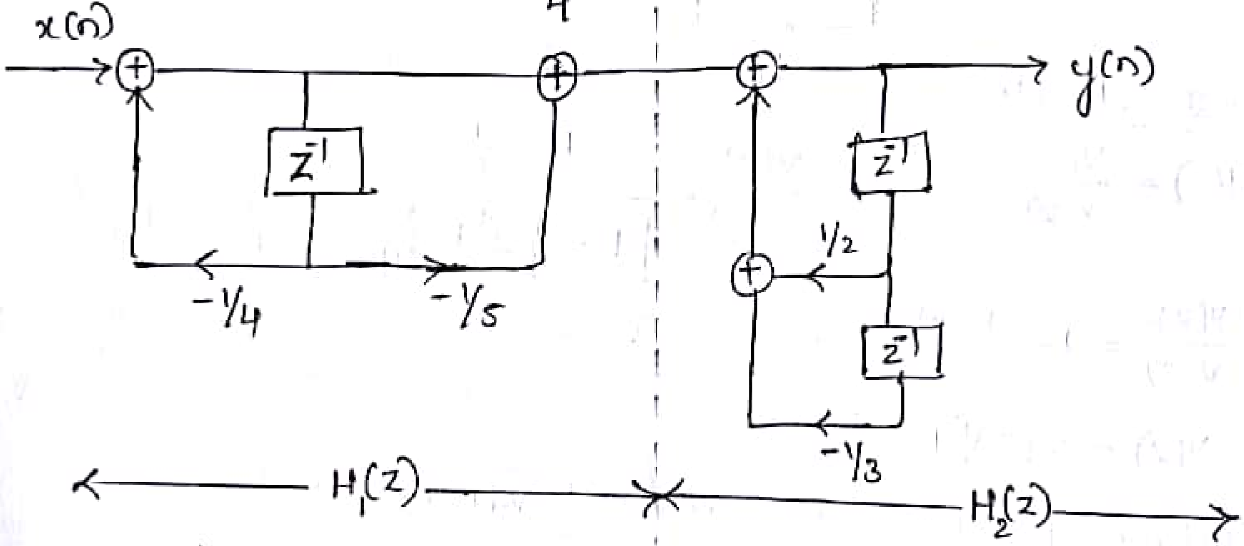
Let $H(z) = H_1(z) \cdot H_2(z)$

given

$$H(z) = \frac{1 - \frac{1}{5} z^{-1}}{\left(1 - \frac{1}{2} z^{-1} + \frac{1}{3} z^{-2}\right)\left(1 + \frac{1}{4} z^{-1}\right)}$$

Let

$$H_1(z) = \frac{1 - \frac{1}{5} z^{-1}}{1 + \frac{1}{4} z^{-1}} \quad \text{and} \quad H_2(z) = \frac{1}{1 - \frac{1}{2} z^{-1} + \frac{1}{3} z^{-2}}$$



Parallel form :-

let $H(z) = H_1(z) + H_2(z) + \ldots$

given

$$H(z) = \frac{1 - 1/5\, z^{-1}}{\left(1 - \frac{1}{2} z^{-1} + \frac{1}{3} z^{-2}\right)\left(1 + \frac{1}{4} z^{-1}\right)}$$

put $z^{-1} = p$

$$\frac{1 - \frac{1}{5} p}{\left(1 - \frac{1}{2} p + \frac{1}{3} p^2\right)\left(1 + \frac{1}{4} p\right)} = \frac{A}{\left(1 + \frac{1}{4} p\right)} + \frac{Bp + C}{\left(1 - \frac{1}{2} p + \frac{1}{3} p^2\right)}$$

solving

$$A = \left.\frac{1-\frac{1}{5}P}{1-\frac{1}{2}P+\frac{1}{3}P^2}\right|_{P=-4} = \frac{27}{125}$$

using co-efficient comparision method we find B & C

$$\frac{1-\frac{1}{5}P}{(1-\frac{1}{2}P+\frac{1}{3}P^2)(1+\frac{1}{4}P)} = \frac{A(1-\frac{1}{2}P+\frac{1}{3}P^2)+(BP+C)(1+\frac{1}{4}P)}{(1+\frac{1}{4}P)(1-\frac{1}{2}P+\frac{1}{3}P^2)}$$

comparing constant term

$1 = A + C$ (or) $C = 1 - A$

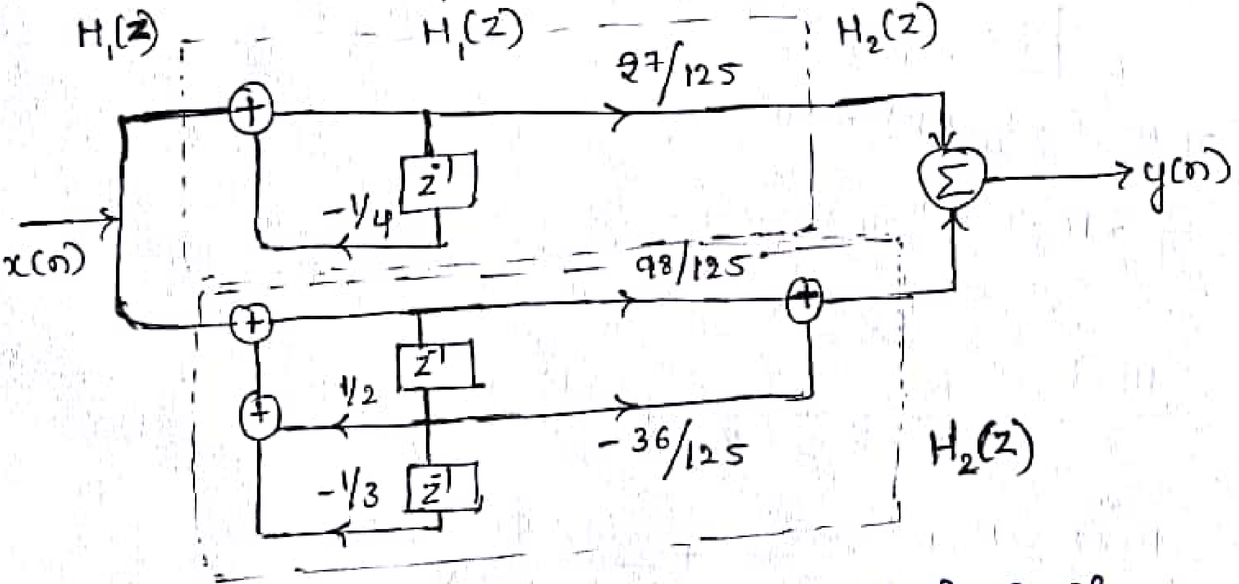
$$\boxed{C = \frac{98}{125}}$$

comparing co-efficients of P

$$-\frac{1}{5} = -\frac{1}{2}A + B + \frac{1}{4}C$$

solving $\boxed{B = \frac{-36}{125}}$

$$\therefore \quad H(z) = \underbrace{\frac{27/125}{1+\frac{1}{4}z^{-1}}}_{H_1(z)} + \underbrace{\frac{-\frac{36}{125}z^{-1}+\frac{98}{125}}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}}_{H_2(z)}$$



② Draw DF I, DF II, parallel & cascade form of

$$H(z) = \frac{8z^3-4z^2+11z-2}{(z-\frac{1}{4})(z^2-z+\frac{1}{2})}$$

Parallel form :-

To perform partial fractions, power of Numerator must be less than power of denominator. H(z) can also be written as

$$\frac{H(z)}{z} = \frac{8z^3 - 4z^2 + 11z - 2}{z(z - 1/4)(z^2 - z + 1/2)}$$

$$= \frac{A}{z} + \frac{B}{z - 1/4} + \frac{Cz + D}{z^2 - z + 1/2}$$

$$A = \frac{H(z)}{z} \times z \Big|_{z=0} = \frac{-2}{-\frac{1}{4} \times \frac{1}{2}} = \underline{\underline{16}}$$

$$B = \frac{H(z)}{z} \times (z - 1/4) \Big|_{z = 1/4}$$

$$= \frac{8(1/4)^3 - 4(1/4)^2 + 11 \times \frac{1}{4} - 2}{\frac{1}{4} \times \left[\left(\frac{1}{4}\right)^2 - \frac{1}{4} + \frac{1}{2}\right]} = \underline{\underline{8}}$$

$$8z^3 - 4z^2 + 11z - 2 = A(z - 1/4)(z^2 - z + 1/2) + B \cdot z(z^2 - z + 1/2) + (Cz + D)z(z - 1/4).$$

By co-efficient comparision method

$$A + B + C = 8$$

$$16 + 8 + C = 8$$

$$\boxed{C = -16}$$

$$\left(-\frac{5}{4}A - B - \frac{C}{4} + D\right) = -11$$

$$-\frac{5}{4} \times 16 - 8 + \frac{16}{4} + D = -4$$

$$\underline{\underline{D = 20}}$$

$$\therefore \frac{H(z)}{z} = \frac{16}{z} + \frac{8}{z - 1/4} + \frac{-16z + 20}{z^2 - z + 1/2}$$

(or)

$$H(z) = 16 + 8 \cdot \frac{z}{z - 1/4} + \frac{-16z^2 + 20z}{z^2 - z + 1/2}$$

By simplifying

$$H(z) = 16 + 8 \frac{1}{1 - 1/4 z^{-1}} + \frac{-16 + 20z^{-1}}{1 - z^{-1} + 1/2 z^{-2}}$$

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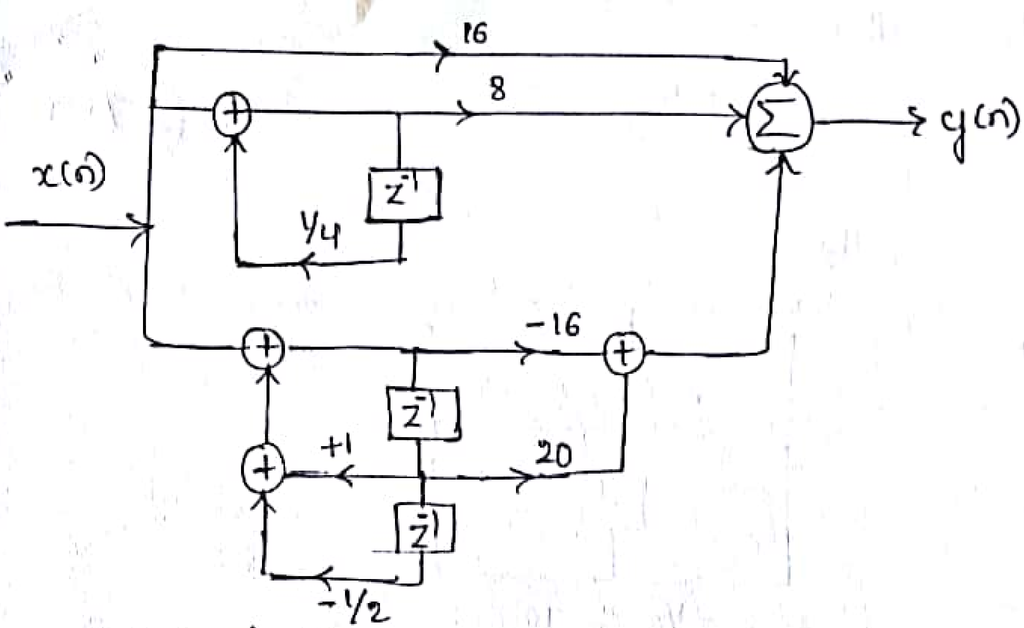


Cascade form

$$H(z) = \frac{8z^3 - 4z^2 + 11z - 2}{(z - 1/4)\,(z^2 - z + 1/2)}$$

numerator can be represented in factorial form as

$$8z^3 - 4z^2 + 11z - 2 = 0$$

at $z = 0$, $= -2$

$z = 1$, $8 - 4 + 11 - 2 = 13$

$z = 0.5$, $= 8.5$

$z = 0.25$, $= 0.625$

$z = 0.2$, $= 0.104$

$z = 0.15$, $= -0.413$

$\left.\right\}$ $\therefore$ b/w 0.2 & 0.15

$z = 0.18$, $= -0.1029$

$z = 0.19$, $= 0.000472 \simeq 0$

$\therefore$ $\underline{z = 0.19}$ or $\underline{z = -0.19}$

0.19	8	-4	11	-2
	0	1.52	-0.4712	2
	8	-2.48	10.5288	0

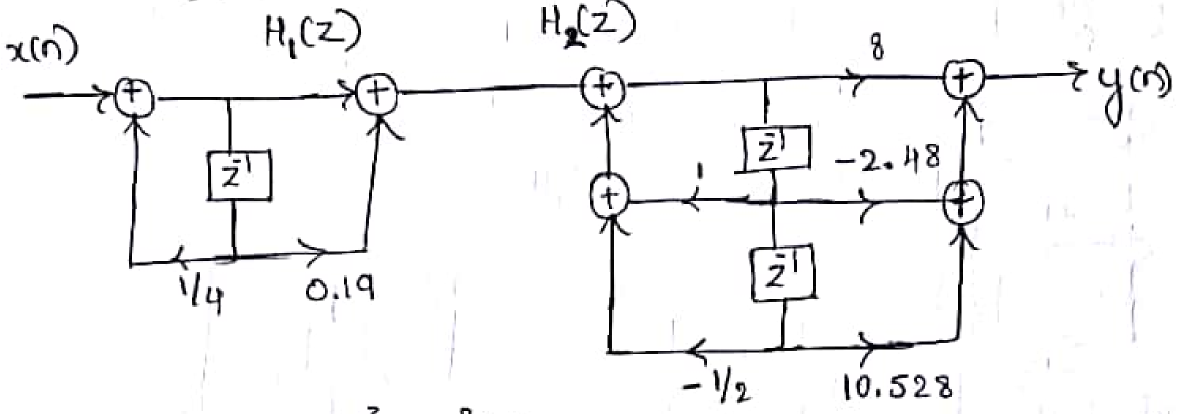
$\therefore$ Above polynomial can be written as

$$(z - 0.19)\,(8z^2 - 2.48z + 10.5288)$$

$$\therefore \quad H(z) = \frac{(z - 0.19)\,(8z^2 - 2.48z + 10.5288)}{(z - 1/4)\,(z^2 - z + 1/2)}$$

above H(z) can be written as

$$H(z) = \frac{(1-0.19z^{-1})}{(1-\tfrac{1}{4}z^{-1})}\cdot\frac{(8-2.48z^{-1}+10.528z^{-2})}{(1-z^{-1}+\tfrac{1}{2}z^{-2})}$$



$$H(z) = \frac{8z^3-4z^2+11z-2}{(z-\tfrac{1}{4})(z^2-z+\tfrac{1}{2})}$$

consider $(z-\tfrac{1}{4})(z^2-z+\tfrac{1}{2})$

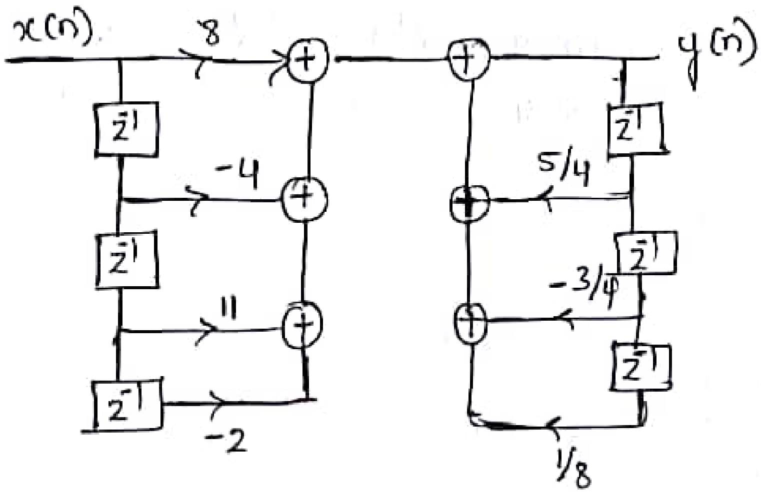
$$z^3-z^2+\frac{z}{2}-\frac{1}{4}z^2+\frac{z}{4}-\frac{1}{8}$$

$$\left(z^3-\frac{5}{4}z^2+\frac{3}{4}z-\frac{1}{8}\right)$$

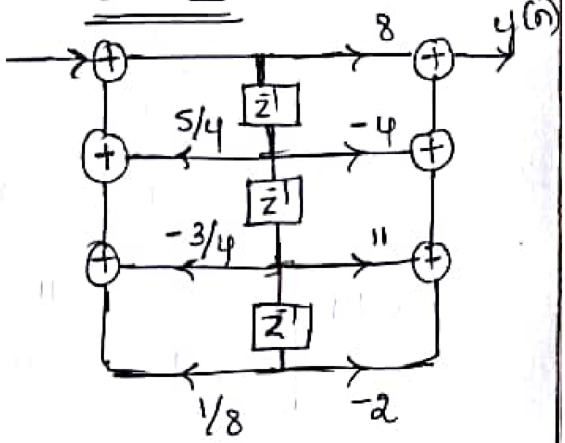
$$\therefore\quad H(z) = \frac{8z^3-4z^2+11z-2}{z^3-\frac{5}{4}z^2+\frac{3}{4}z-\frac{1}{8}}$$

$$H(z) = \frac{\cancel{z^3}\left[8-4z^{-1}+11z^{-2}-2z^{-3}\right]}{\cancel{z^3}\left[1-\frac{5}{4}z^{-1}+\frac{3}{4}z^{-2}-\frac{1}{8}z^{-3}\right]}$$

DF - I



DF - II



3) Draw DF - I , DF - II , parallel and cascade

$$H(z) = \frac{z(z-1)(z-2)(z+1)}{\left[z-\left(\frac{1}{2}+j\frac{1}{2}\right)\right]\left[z-\left(\frac{1}{2}-j\frac{1}{2}\right)\right](z-j/4)(z+j/4)}$$

$$\left[(z-\tfrac{1}{2})+j\tfrac{1}{2}\right]\cdot\left[(z-\tfrac{1}{2})-j\tfrac{1}{2}\right] = (z-\tfrac{1}{2})^2+(\tfrac{1}{2})^2$$

$$= z^2+(\tfrac{1}{2})^2-2\times z\times\tfrac{1}{2}+(\tfrac{1}{2})^2$$

$$= z^2-z+\tfrac{1}{2}$$

$$\text{\&}\ (z-j\tfrac{1}{4})(z+j\tfrac{1}{4}) = z^2+(\tfrac{1}{4})^2$$

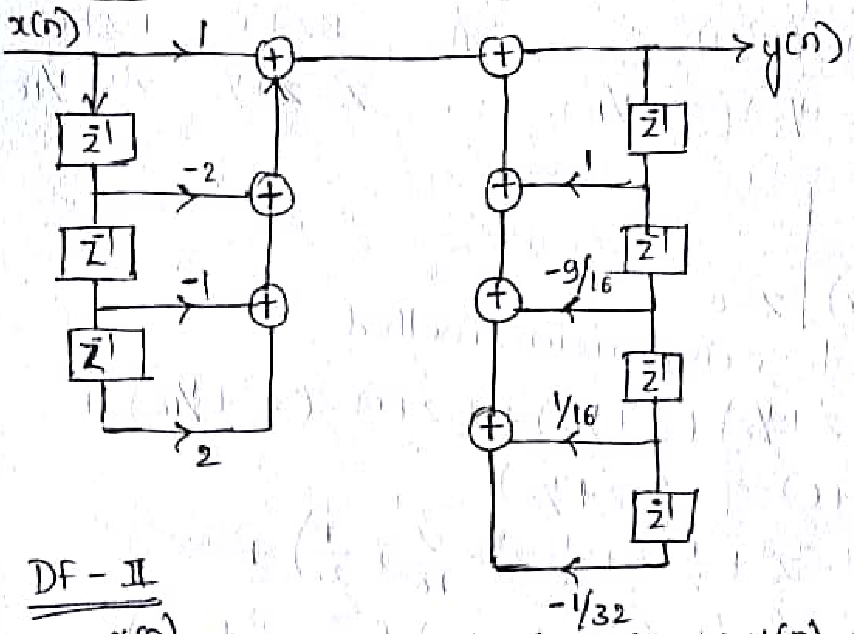
$$= z^2+\tfrac{1}{16}$$

$$H(z) = \frac{(z^2-2z)(z^2-1)}{(z^2-z+\tfrac{1}{2})(z^2+\tfrac{1}{16})}$$

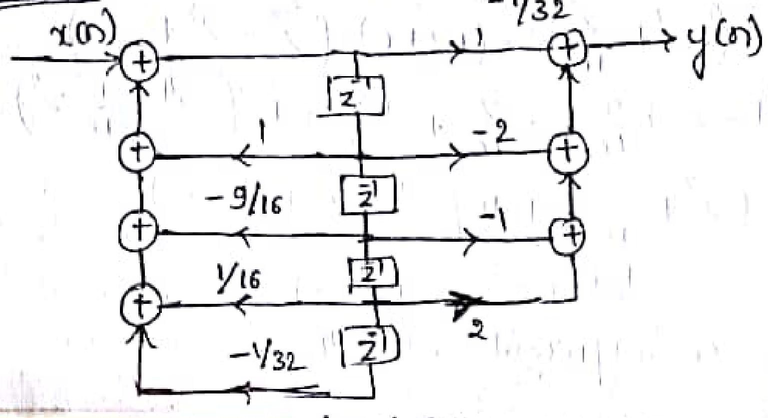
or $$H(z) = \frac{z^4-2z^3-z^2+2z}{z^4-z^3+\frac{9}{16}z^2-\frac{1}{6}z+\frac{1}{32}}$$

or $$H(z) = \frac{1-2z^{-1}-z^{-2}+2z^{-3}}{1-z^{-1}+\frac{9}{16}z^{-2}-\frac{1}{16}z^{-3}+\frac{1}{32}z^{-4}}$$

DF - I



DF - II

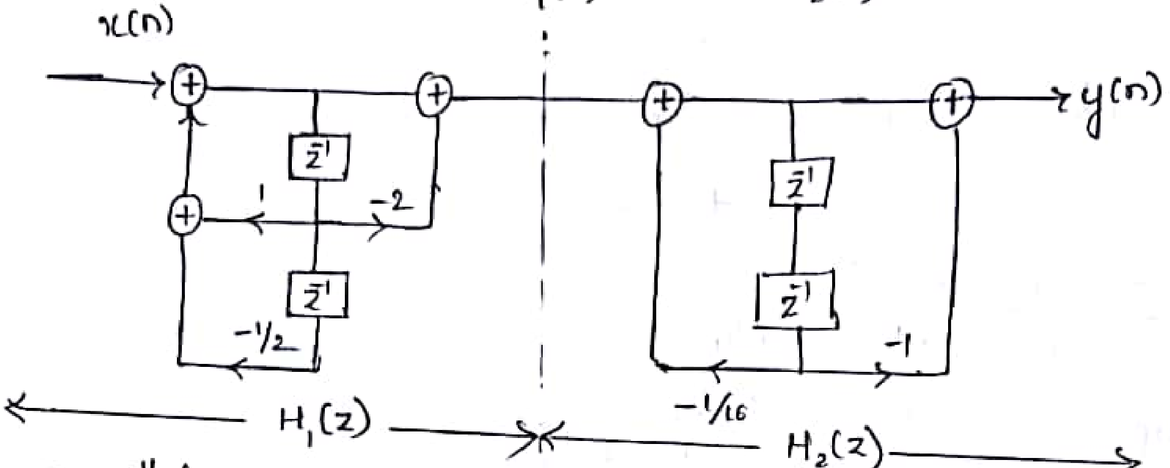


cascade

consider $H(z) = \dfrac{(z^2-2z)(z^2-1)}{(z^2-z+1/2)(z^2+1/16)}$

or

$$H(z) = \underbrace{\frac{(1-2z^{-1})}{(1-z^{-1}+\frac{1}{2}z^{-2})}}_{H_1(z)}\underbrace{\frac{(1-z^{-2})}{(1+\frac{1}{16}z^{-2})}}_{H_2(z)}$$



parallel

consider $H(z) = \dfrac{(z^2-2z)(z^2-1)}{(z^2-z+1/2)(z^2+1/16)}$

using partial fraction method

$$\frac{H(z)}{z} = \frac{(z^2-2z)(z^2-1)}{z(z^2-z+1/2)(z^2+1/16)} = \frac{A}{z} + \frac{Bz+C}{z^2-z+1/2} + \frac{Dz+E}{z^2+1/16}$$

$$A = \left.\frac{(z^2-2z)(z^2-1)}{(z^2-z+1/2)(z^2+1/16)}\right|_{z=0} = 0$$

using co-efficient comparision method

$$(z^2-2z)(z^2-1) = A(z^2-z+1/2)(z^2+1/16) + (Bz+C)z(z^2+1/16) + (Dz+E)z(z^2-z+1/2)$$

$$z^4-2z^3-z^2+2z = A\left(z^4-z^3+\frac{1}{2}z^2+\frac{1}{16}z^2-\frac{z}{16}+\frac{1}{32}\right) + (Bz+C)\left(z^3+\frac{z}{16}\right) + (Dz+E)\left(z^3-z^2+\frac{z}{2}\right)$$

$$= A\left(z^4-z^3+\frac{9}{16}z^2-\frac{z}{16}+\frac{1}{32}\right) + B\left(z^4+\frac{1}{16}z^2\right) + C\left(z^3+\frac{z}{16}\right) + D\left(z^4-z^3+\frac{1}{2}z^2\right) + E\left(z^3-z^2+\frac{1}{2}z\right)$$

comparing co-efficient of z^4

$1 = A + B + D$

since $A = 0$. $\therefore$ $\boxed{B = 1 - D}$

comp. co-effts of z^3

$-2 = -A + C - D + E$

$\boxed{-2 = C - D + E} \longrightarrow$ ⓐ

comp. co-efft of z^2

$-1 = -\frac{9}{16}A + \frac{1}{16}B + \frac{1}{16}C + \frac{1}{2}D - E$

$-1 = \frac{1}{16}(1-D) + \frac{1}{16}C + \frac{1}{2}D - E$

$-1 = \frac{1}{16} - \frac{1}{16}D + \frac{1}{16}C + \frac{1}{2}D - E \Rightarrow \boxed{\frac{-17}{16} = \frac{1}{16}C + \frac{7}{16}D - E} \longrightarrow$ ⓑ

comp. co-efftn of z

$2 = -\frac{1}{16}A + \frac{1}{16}C + \frac{1}{2}E$

$\boxed{2 = \frac{1}{16}C + \frac{1}{2}E} \longrightarrow$ ⓒ

comp. co-efft of constant

$0 = \frac{1}{32}A$ $\boxed{A = 0}$

solving eqn. ⓐ.ⓑ.ⓒ we get

$\therefore$ $B = 1 - D$

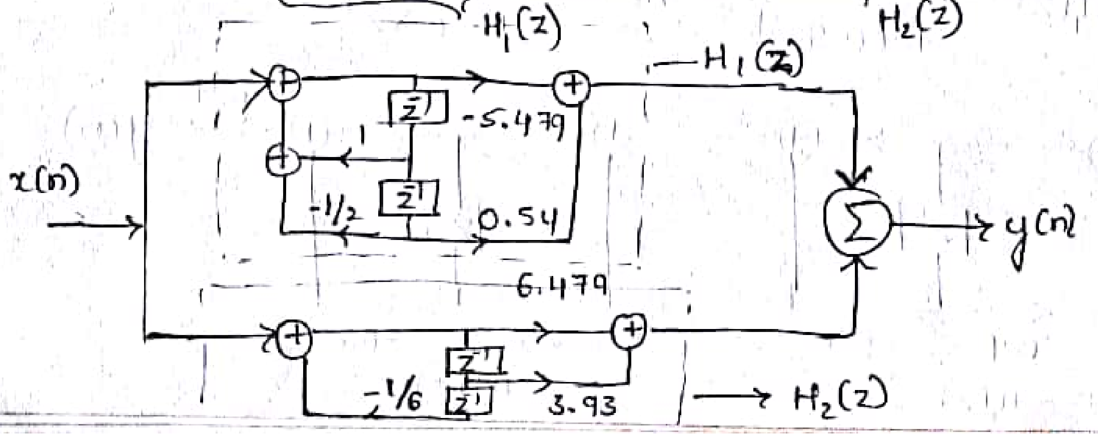
$B = -5.479$ $C = 0.5479$ $D = 6.479$ $E = 3.93$

$$\therefore\ H(z) = \frac{z(-5.479z + 0.54)}{(z^2 - z + 1/2)} + \frac{z(6.479z + 3.93)}{(z^2 + 1/16)}$$

$$H(z) = \frac{-5.479z^2 + 0.54z}{z^2 - z + 1/2} + \frac{6.479z^2 + 3.93z}{z^2 + 1/16}$$

or

$$H(z) = \underbrace{\frac{-5.479 + 0.54z^{-2}}{1 - z^{-1} + \frac{1}{2}z^{-2}}}_{H_1(z)} + \underbrace{\frac{6.479 + 3.93z^{-1}}{1 + \frac{1}{16}z^{-2}}}_{H_2(z)}$$



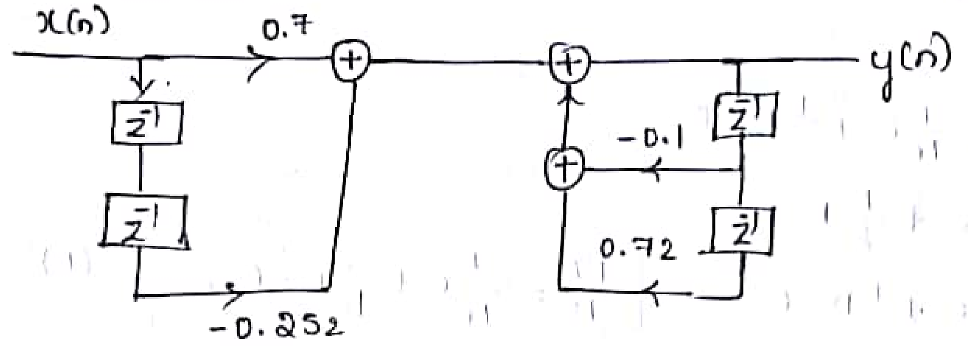
④ obtain direct form I, DF-II, cascade & parallel structure for the system described by

$$y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252\, x(n-2)$$

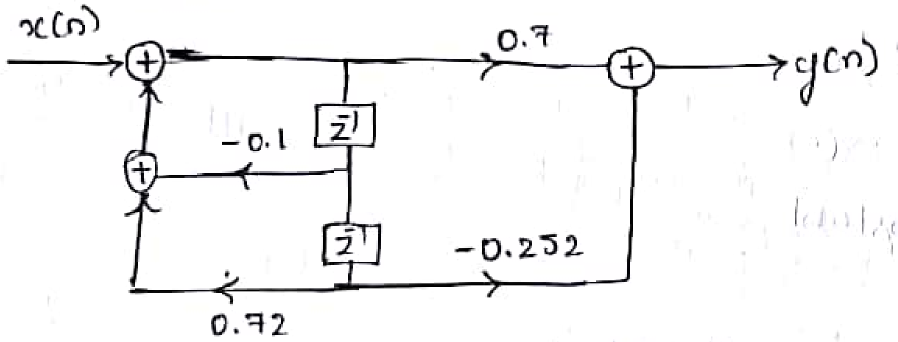
(274)

Soln :- June 2010 (16M).

DF - I



DF - II



Cascade :-

given, $y(n) = -0.1\, y(n-1) + 0.72\, y(n-2) + 0.7\, x(n) - 0.252\, x(n-2)$

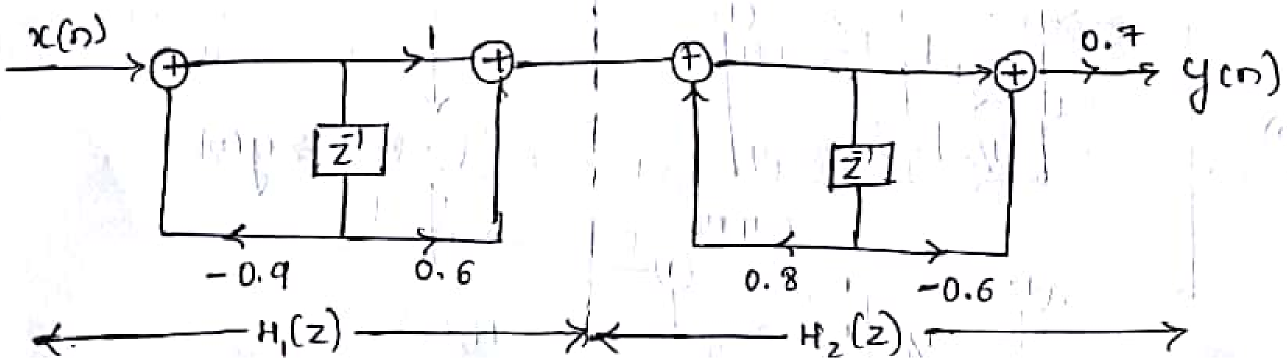
applying ZT

$$Y(z)\left[1 + 0.1 z^{-1} - 0.72 z^{-2}\right] = X(z)\left[0.7 - 0.252 z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.7 - 0.252 z^{-2}}{1 + 0.1 z^{-1} - 0.72 z^{-2}}$$

$$H(z) = \frac{0.7(1 - 0.6 z^{-1})(1 + 0.6 z^{-1})}{(1 + 0.9 z^{-1})(1 - 0.8 z^{-1})}$$

let $H_1(z) = \dfrac{1 + 0.6 z^{-1}}{1 + 0.9 z^{-1}}$ $\quad H_2(z) = \dfrac{1 - 0.6 z^{-1}}{1 - 0.8 z^{-1}}$



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Parallel :-

consider $H(z) = \dfrac{0.7 - 0.252\,z^{-2}}{1 + 0.1\,z^{-1} - 0.72\,z^{-2}}$

Let $z^{-1} = P$ & using long division method

$$\frac{0.7 - 0.252P^2}{1 + 0.1P - 0.72P^2}$$

$$\begin{array}{r|l} & 0.35 \\ -0.72P^2 + 0.1P + 1 & -0.252\,P^2 + 0.7 \\ & -0.252\,P^2 + 0.035P + 0.35 \\ & (+) \qquad (-) \qquad (-) \\ \hline & -0.035P + 0.35 \end{array}$$

$$\therefore \; H(z) = 0.35 + \frac{(-0.035P + 0.35)}{1 + 0.1P - 0.72P^2}$$

(or)

$$H(z) = 0.35 + \frac{(-0.035\,z^{-1} + 0.35)}{1 + 0.1\,z^{-1} - 0.72\,z^{-2}}$$

$$= 0.35 + \boxed{\frac{0.35 - 0.035\,z^{-1}}{(1 + 0.9\,z^{-1})(1 - 0.8\,z^{-1})}}$$

let $H(z) = \dfrac{0.35 - 0.035\,z^{-1}}{(1 + 0.9\,z^{-1})(1 - 0.8\,z^{-1})}$

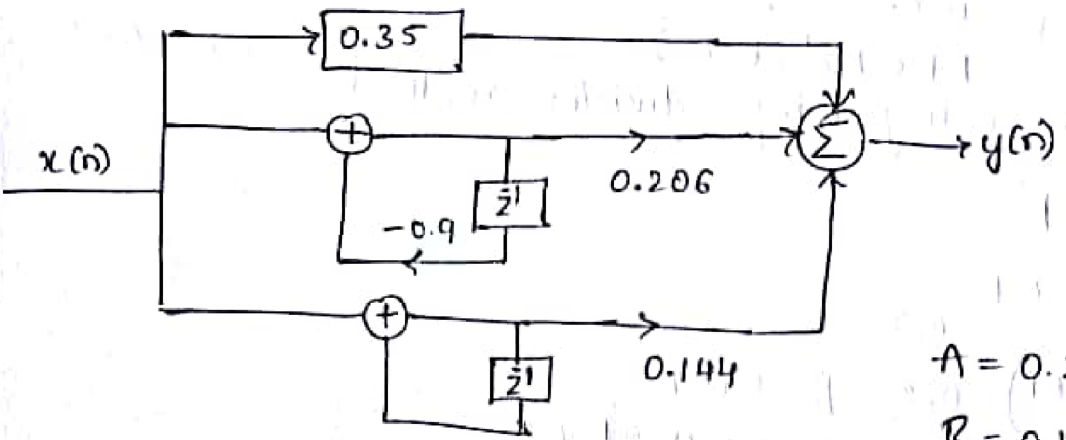
using partial fraction expansion method

$$\frac{0.35 - 0.035\,z^{-1}}{(1 + 0.9\,z^{-1})(1 - 0.8\,z^{-1})} = \frac{A}{1 + 0.9\,z^{-1}} + \frac{B}{1 - 0.8\,z^{-1}}$$

$$A = \left.\frac{0.35 - 0.035\,z^{-1}}{(1 - 0.8\,z^{-1})}\right|_{z^{-1} = \frac{-1}{0.9}} \qquad \boxed{A = 0.206}$$

$$B = \left.\frac{0.35 - 0.035\,z^{-1}}{1 + 0.9\,z^{-1}}\right|_{z^{-1} = \frac{1}{0.8}} \qquad \underline{\underline{B = 0.144}}$$

$$\therefore \; H(z) = \underbrace{0.35}_{H_1(z)} + \underbrace{\frac{0.206}{1 + 0.9\,z^{-1}}}_{H_2(z)} + \underbrace{\frac{0.144}{1 - 0.8\,z^{-1}}}_{H_3(z)}$$



A = 0.206
B = 0.144

5) Obtain DF-I and cascade realization for the IIR filter having ~~baxi~~ transfer function.

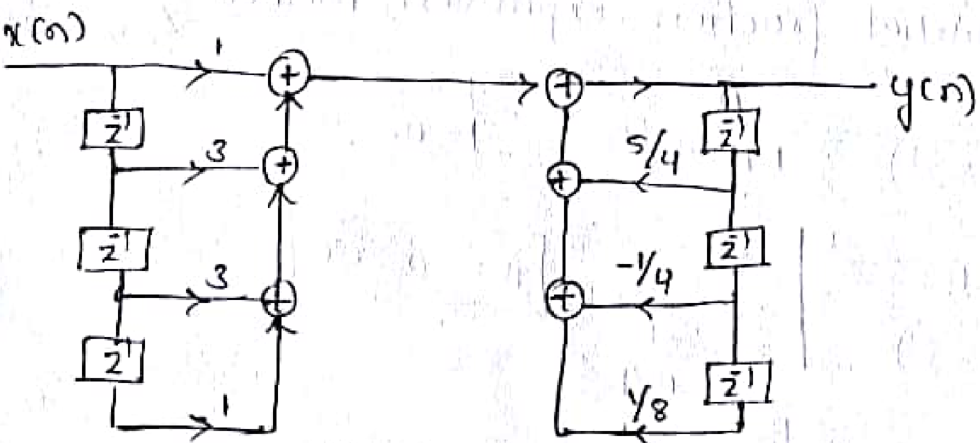
$$H(z) = \frac{(1+z^{-1})^3}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-z^{-1}+\frac{1}{2}z^{-2}\right)}$$

<u>Sol^n</u>:-

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+3z^{-1}+3z^{-2}+z^{-3}}{1-\frac{5}{4}z^{-1}+\frac{1}{4}z^{-2}-\frac{1}{8}z^{-3}}$$

Taking I.Z.T and solving

$$y(n) = \frac{5}{4}y(n-1) - \frac{1}{4}y(n-2) + \frac{1}{8}y(n-3) + x(n) + 3x(n-1) + 3x(n-2) + x(n-3).$$

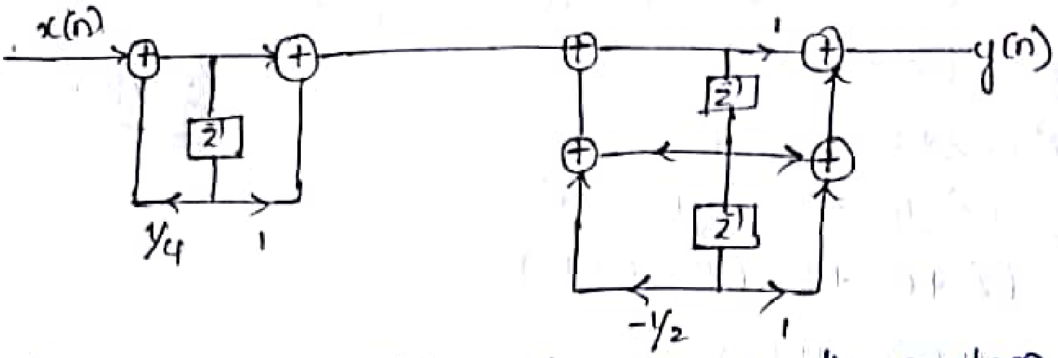


<u>DF - I</u>

<u>cascade</u>

$$H(z) = H_1(z) \cdot H_2(z)$$

$$H(z) = \left(\frac{1+z^{-1}}{1-\frac{1}{4}z^{-1}}\right)\left(\frac{1+2z^{-1}+z^{-2}}{1-z^{-1}+\frac{1}{2}z^{-2}}\right)$$



(2-77)

6) Obtain a parallel realization for the system described by

$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1+\frac{1}{8}z^{-1})}.$$

put $z^{-1} = p$. and using partial fraction method.

$$\frac{(1+p)(1+2p)}{(1+\frac{1}{2}p)(1-\frac{1}{4}p)(1+\frac{1}{8}p)} = \frac{A}{1+\frac{1}{2}p} + \frac{B}{1-\frac{1}{4}p} + \frac{C}{1+\frac{1}{8}p}$$

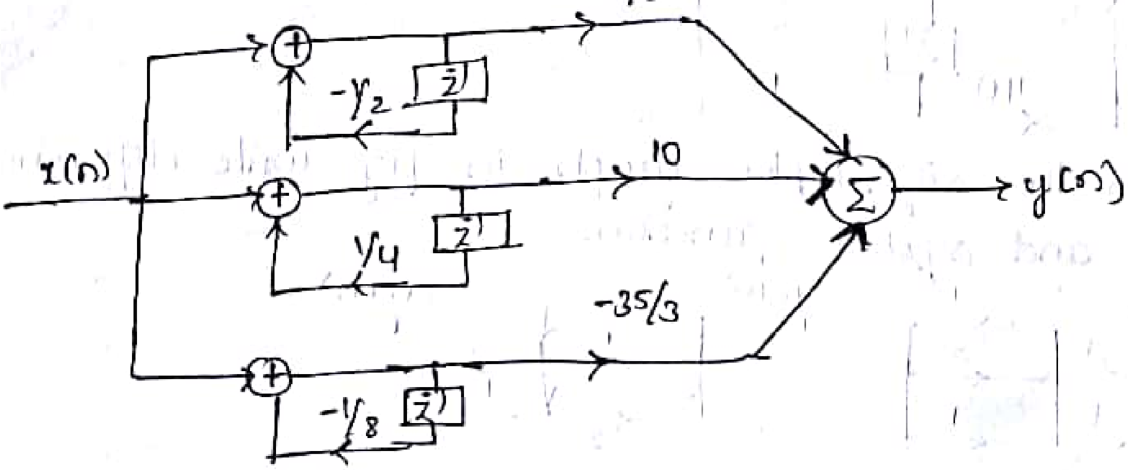
$$A = \left.\frac{(1+p)(1+2p)}{(1-\frac{1}{4}p)(1+\frac{1}{8}p)}\right|_{p=-2} = \frac{(1-2)(1-4)}{(1+\frac{2}{4})(1-\frac{2}{8})} = \frac{3}{\frac{3}{2}\times\frac{6}{8}} = \frac{8}{3}$$

$$B = \left.\frac{(1+p)(1+2p)}{(1+\frac{1}{2}p)(1+\frac{1}{8}p)}\right|_{p=4} = \frac{5\times 9}{3\times 3/2} = 10$$

$$C = \left.\frac{(1+p)(1+2p)}{(1+\frac{1}{2}p)(1-\frac{1}{4}p)}\right|_{p=-8} = \frac{-7\times -15}{-3\times 3} = \frac{-35}{3}$$

$$\therefore H(z) = \frac{8/3}{1+\frac{1}{2}z^{-1}} + \frac{10}{1-\frac{1}{4}z^{-1}} + \frac{(-35/3)}{1+\frac{1}{8}z^{-1}}$$

i.e $H(z) = H_1(z) + H_2(z) + H_3(z)$



⇒ Obtain DF - II and cascade realisation of

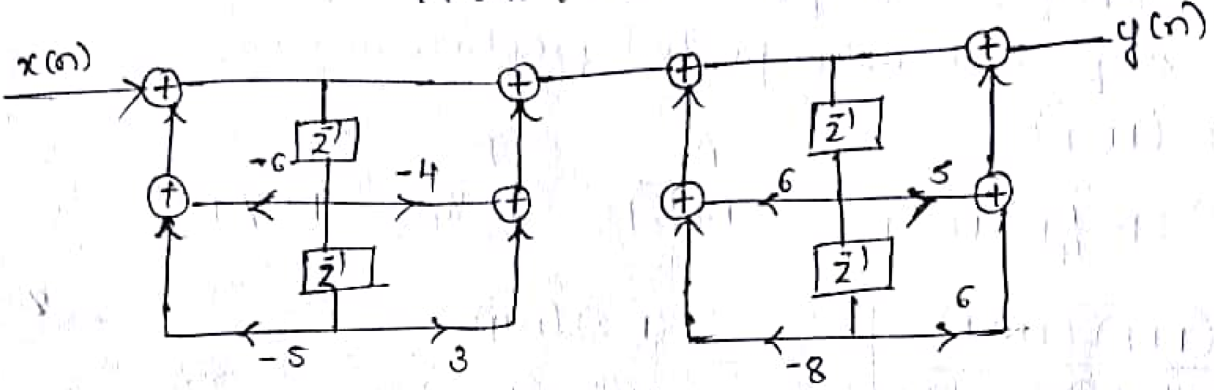
$$H(z) = \frac{(z-1)(z^2+5z+6)(z-3)}{(z^2+6z+5)(z^2-6z+8)}$$

Sol^n:-

$$H(z) = \frac{(z^2-4z+3)(z^2+5z+6)}{(z^2+6z+5)(z^2-6z+8)}$$

$$H(z) = H_1(z) \cdot H_2(z)$$

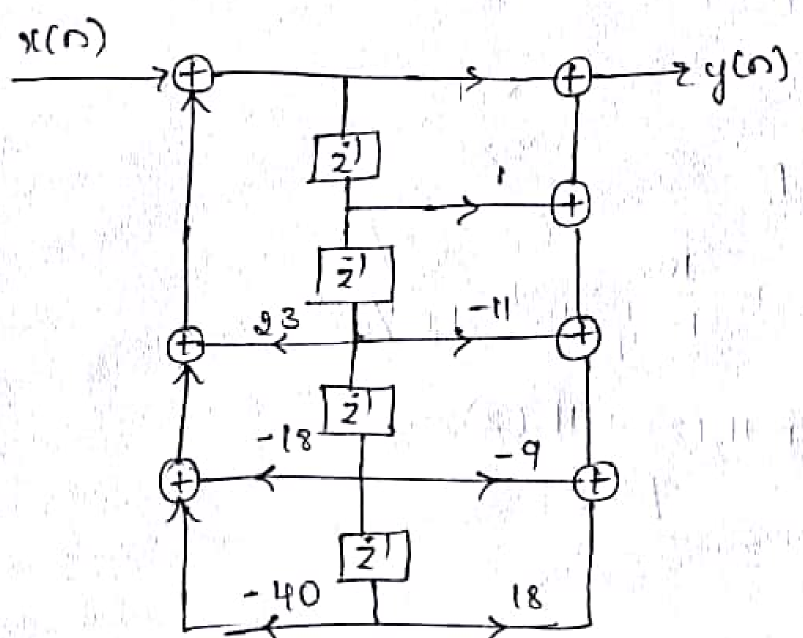
$$H(z) = \frac{1-4z^{-1}+3z^{-2}}{1+6z^{-1}+5z^{-2}} \cdot \frac{1+5z^{-1}+6z^{-2}}{1-6z^{-1}+8z^{-2}}$$



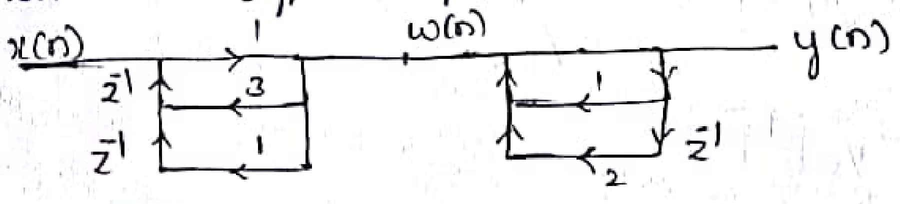
Cascade

DF - II

consider. $H(z) = \dfrac{1+z^{-1}-11z^{-2}-9z^{-3}+18z^{-4}}{1-23z^{-2}+18z^{-3}+40z^{-4}}$



8) consider the signal flow graph in fig. write difference equation and system function.



Sol^n:- $w(n) = x(n) + 3w(n-1) + w(n-2) \longrightarrow (a)$ (9)

$y(n) = w(n) + y(n-1) + 2y(n-2) \longrightarrow (b)$

(279)

Taking Z.T of eq^n (a) & (b)

$W(z)\left[1 - 3z^{-1} - z^{-2}\right] = X(z)$

$Y(z)\left[1 - z^{-1} - 2z^{-2}\right] = W(z)$

$$H(z) = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{1}{(1 - z^{-1} + 2z^{-2})} \cdot \frac{1}{(1 - 3z^{-1} - z^{-2})}$$

$$H(z) = \frac{1}{1 - z^{-1} - 2z^{-2} - 3z^{-1} + 3z^{-2} + 6z^{-3} - z^{-2} + z^{-3} + 2z^{-4}}$$

Transfer function

$$H(z) = \frac{1}{1 - 4z^{-1} + 7z^{-3} + 2z^{-4}}$$

Difference eq^n

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 4z^{-1} + 7z^{-3} + 2z^{-4}}$$

Taking I.Z.T

$y(n) - 4y(n-1) + 7y(n-3) + 2y(n-4) = x(n)$

(or) $y(n) = x(n) + 4y(n-1) - 7y(n-3) - 2y(n-4)$

(9) Obtain the D.F-II and cascade Realization of

Dec-12 10M

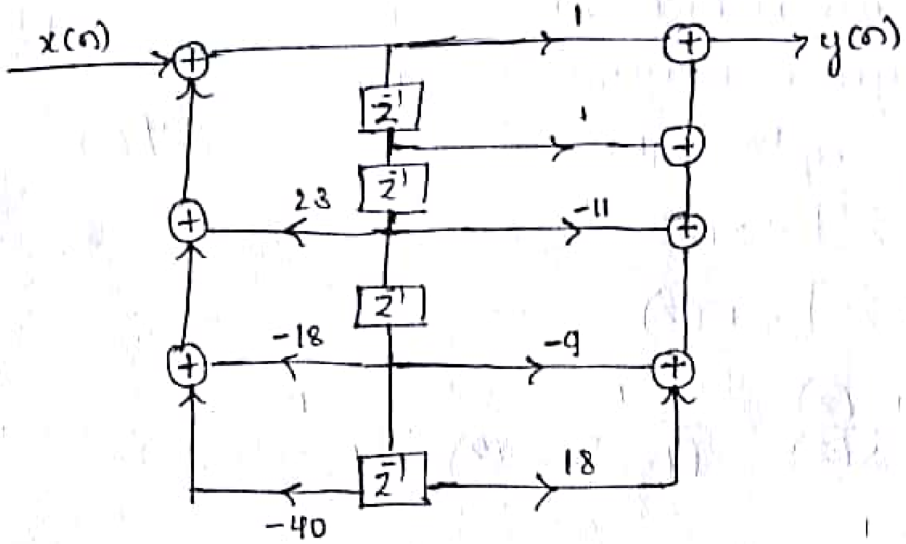
$$H(z) = \frac{(z-1)(z^2+5z+6)(z-3)}{(z^2+6z+5)(z^2-6z+8)}$$

The cascade should consists of two biquadratic sections.

Sol^n:- D.F-II

$$H(z) = \frac{(z^2 - 4z + 3)(z^2 + 5z + 6)}{(z^2 + 6z + 5)(z^2 - 6z + 8)}$$

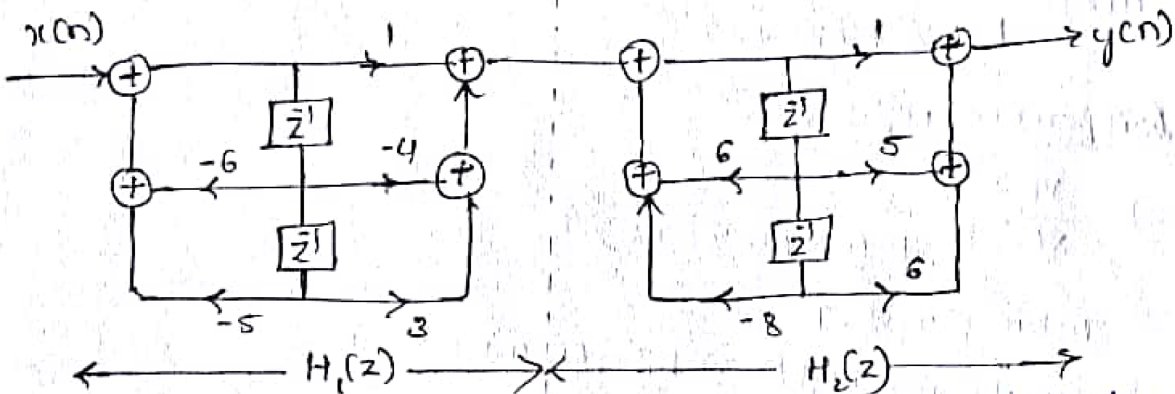
(or) $$H(z) = \frac{1 + z^{-1} - 11z^{-2} - 9z^{-3} + 18z^{-4}}{1 - 23z^{-2} + 18z^{-3} + 40z^{-4}}$$



cascade form

$$H(z) = H_1(z) \cdot H_2(z)$$

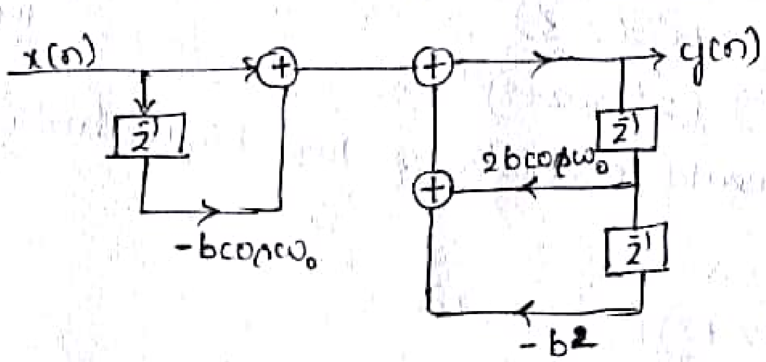
let $\quad H_1(z) = \dfrac{1-4z^{-1}+3z^{-2}}{1+6z^{-1}+5z^{-2}} \qquad H_2(z) = \dfrac{1+5z^{-1}+6z^{-2}}{1-6z^{-1}+8z^{-2}}$



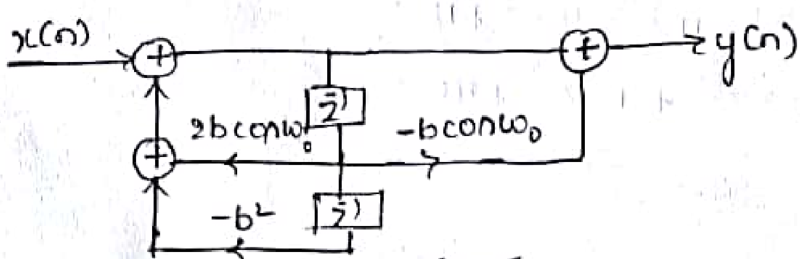
(10) Determine DF-I and DF-II for the filter given by

$$y(n) = 2b\cos\omega_0\, y(n-1) - b^2 y(n-2) + x(n) - b\cos\omega_0\, x(n-1)$$

Solⁿ :-



DF - I



DF - II

(11) Given $h(n) = e^{j\omega_0 n} . u(n)$ (10)

Feb 06 (281)

Taking Z.T

$$H(z) = \sum_{n=0}^{\infty} e^{j\omega_0 n} . z^{-n}$$

$$= \sum_{n=0}^{\infty} (e^{j\omega_0} . z^{-1})^n$$

$$H(z) = \frac{1}{1-e^{j\omega_0} . z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-e^{j\omega_0} z^{-1}}$$

$$Y(z)\left[1-e^{j\omega_0} . z^{-1}\right] = X(z)$$

Taking I.Z.T

$$y(n) - e^{j\omega_0} y(n-1) = x(n)$$

(or) $$y(n) = e^{j\omega_0} y(n-1) + x(n)$$

o/p $y(n) = y_R(n) + j y_I(n)$

$$\therefore y_R(n) + j y_I(n) = (\cos\omega_0 + j\sin\omega_0)(y_R(n-1) + j y_I(n-1)) + x(n)$$

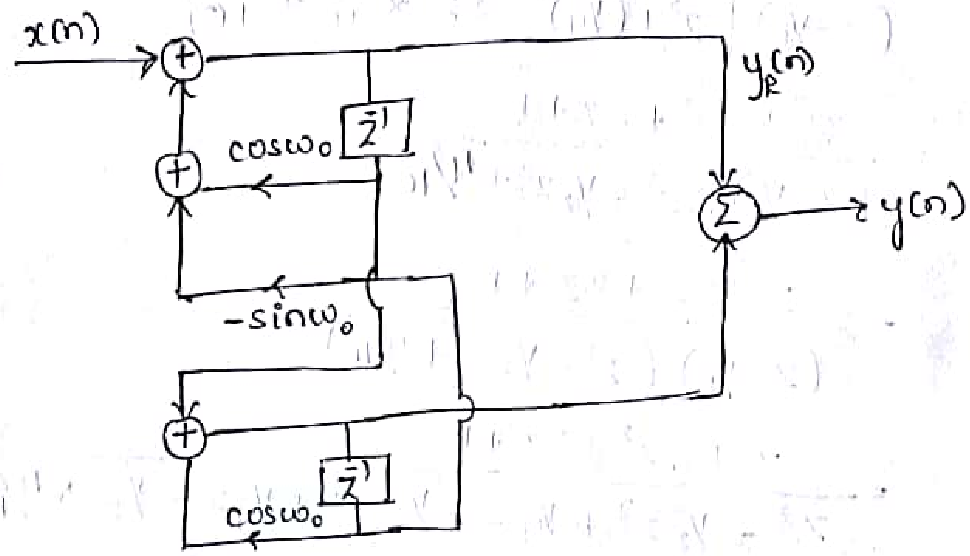
$$= x(n) + y_R(n-1)\cos\omega_0 + j y_I(n-1)\cos\omega_0$$

$$+ j y_R(n-1)\sin\omega_0 - y_I(n-1)\sin\omega_0 .$$

Equating real and imaginary parts

$$y_R(n) = x(n) + y_R(n-1)\cos\omega_0 - y_I(n-1)\sin\omega_0$$

$$y_I(n) = y_I(n-1)\cos\omega_0 + y_R(n-1)\sin\omega_0 .$$

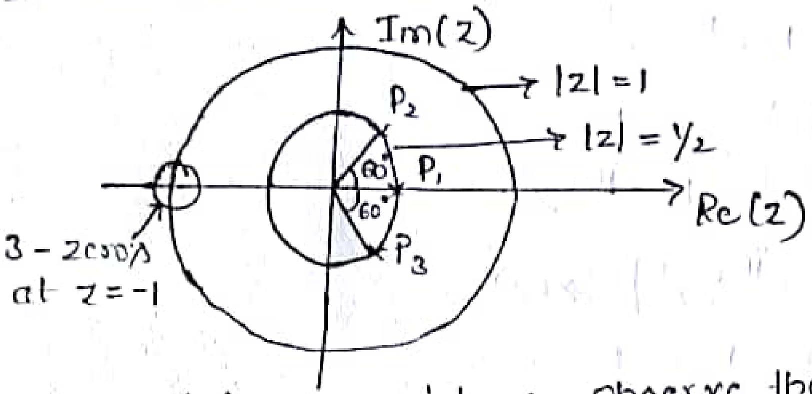


12) A z-plane pole - zero plot for a certain digital filter shown in fig. Determine the system function in July 13 (282)

$$H(z) = \frac{(1+a_1 z^{-1})(1+b_1 z^{-1}+b_2 z^{-2})}{(1+c_1 z^{-1})(1+d_1 z^{-1}+d_2 z^{-2})}$$

giving numerical values for parameters $a_1, b_1, c_1, b_2, d_1, d_2$.

Sketch DF - II and cascade realization of the system.



from above pole - zero plot we observe that

i) 3-zeros at $z=-1$. $\therefore (z+1)^3$

ii) 3-poles a) at $z = 1/2$ $\therefore (z-1/2)$

b) at $z = 1/2\, e^{j60}$, $z = \frac{1}{4} + j\frac{\sqrt{3}}{4}$

c) at $z = \frac{1}{2} e^{-j60}$, $z = \frac{1}{4} - j\frac{\sqrt{3}}{4}$

$$\therefore H(z) = \frac{(z+1)^3}{(z-1/2)(z-1/4+j\sqrt{3}/4)\left[z-\left(\frac{1}{4}-j\frac{\sqrt{3}}{4}\right)\right]}$$

$$= \frac{(z+1)(z^2+1+2z)}{(z-1/2)\left[(z-1/4)^2+\left(\frac{\sqrt{3}}{4}\right)^2\right]}$$

$$= \frac{z^3+z+2z^2+z^2+1+2z}{(z-1/2)\left[z^2+(1/4)^2-2z\times\frac{1}{4}+\frac{3}{16}\right]}$$

$$= \frac{z^3+3z^2+3z+1}{(z-1/2)(z^2-1/2z+4/16)}$$

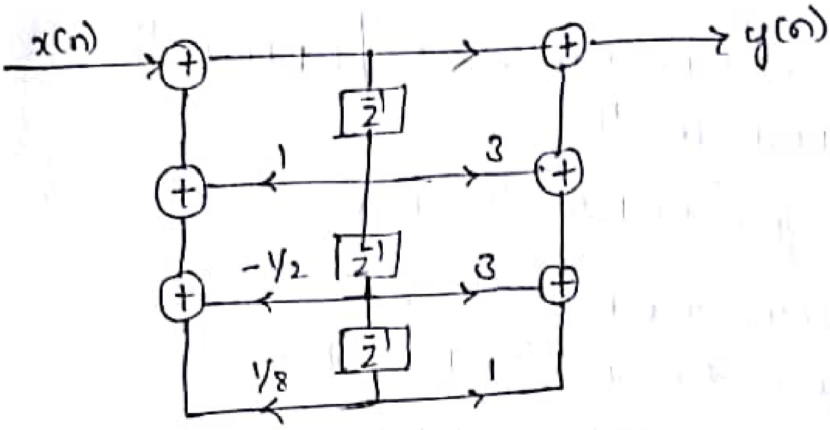
$$= \frac{z^3+3z^2+3z+1}{(z-1/2)(z^2-1/2z+4/16)}$$

$$= \frac{z^3+3z^2+3z+1}{z^3-1/2z^2+1/4z-1/2z^2+1/4z-1/2\times 4/16}$$

(11) (283)

$$= \frac{z^3+3z^2+3z+1}{z^3-z^2+\frac{1}{2}z-\frac{1}{8}}$$

$$H(z) = \frac{1+3z^{-1}+3z^{-2}+z^{-3}}{1-z^{-1}+\frac{1}{2}z^{-2}-\frac{1}{8}z^{-3}}$$

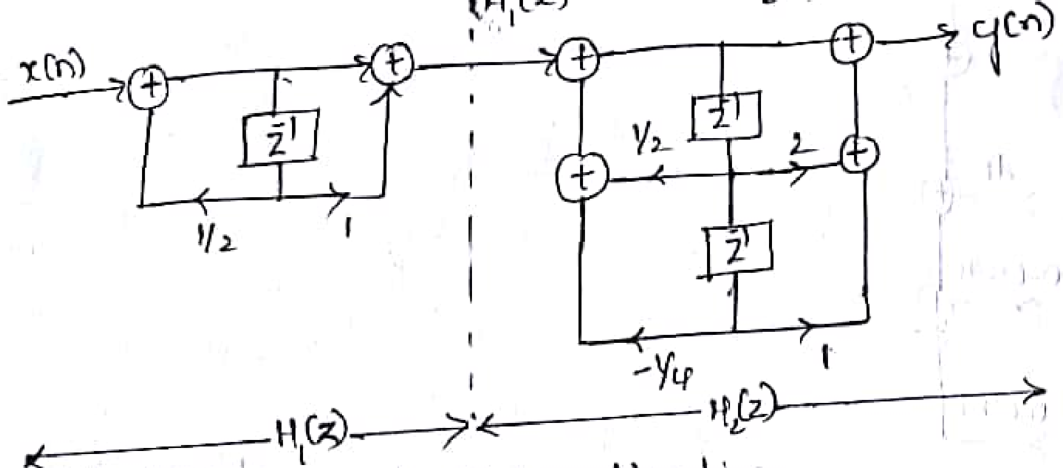


DF - II

cascade

consider $$H(z) = \frac{(z+1)\ (z^2+2z+1)}{(z-\frac{1}{2})\ (z^2-\frac{1}{2}z+\frac{1}{4})}$$

(or) $$H(z) = \frac{(1+z^{-1})\ (1+2z^{-1}+z^{-2})}{\underbrace{(1-\frac{1}{2}z^{-1})}_{H_1(z)}\ \underbrace{(1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2})}_{H_2(z)}}$$



cascade Realization.

(13) Given $H(z) = (1+0.6z^{-1})^5$

i) Realize in DF

ii) Realize as a cascade of first Order sections only.

iii) Realize as a cascade of Ist and IInd order sections.

1 Soln: given $H(z) = (1+0.6z^{-1})^5$ (284)

$$H(z) = (1+0.6z^{-1})(1+0.6z^{-1})(1+0.6z^{-1})(1+0.6z^{-1})(1+0.6z^{-1})$$

$$H(z) = (1+0.6z^{-1}+0.6z^{-1}+0.36z^{-2})(1+0.6z^{-1}+0.6z^{-1}+0.36z^{-2})(1+0.6z^{-1})$$

$$H(z) = (1+1.2z^{-1}+0.36z^{-2})(1+1.2z^{-1}+0.36z^{-2})(1+0.6z^{-1})$$

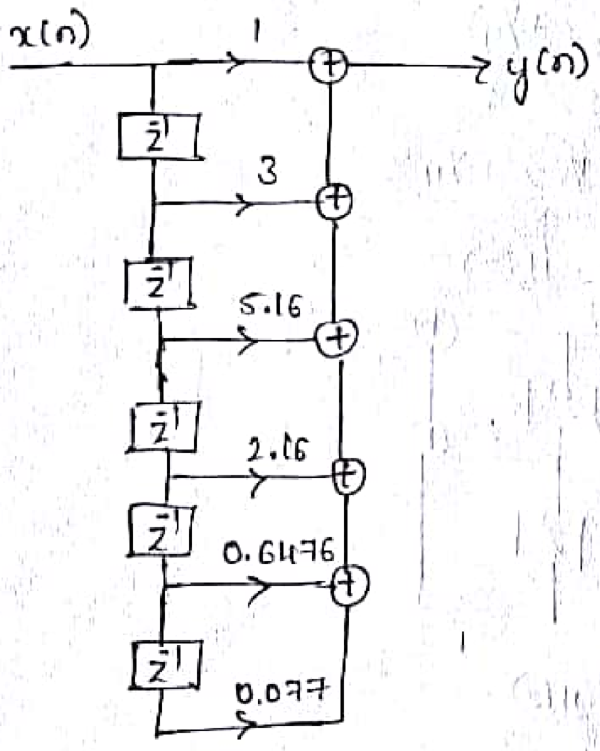
$$H(z) = \left[\begin{array}{l}1+1.2z^{-1}+0.36z^{-2}+1.2z^{-1}+1.44z^{-2}+0.432z^{-3} \\ +0.36z^{-2}+0.432z^{-3}+0.1296z^{-4}\end{array}\right](1+0.6z^{-1})$$

$$H(z) = (1+2.4z^{-1}+2.16z^{-2}+0.864z^{-3}+0.1296z^{-4})(1+0.6z^{-1})$$

$$H(z) = 1+2.4z^{-1}+2.16z^{-2}+0.864z^{-3}+0.1296z^{-4} + 0.6z^{-1}+3z^{-2}+1.296z^{-3}+0.518z^{-4}+0.0777z^{-5}$$

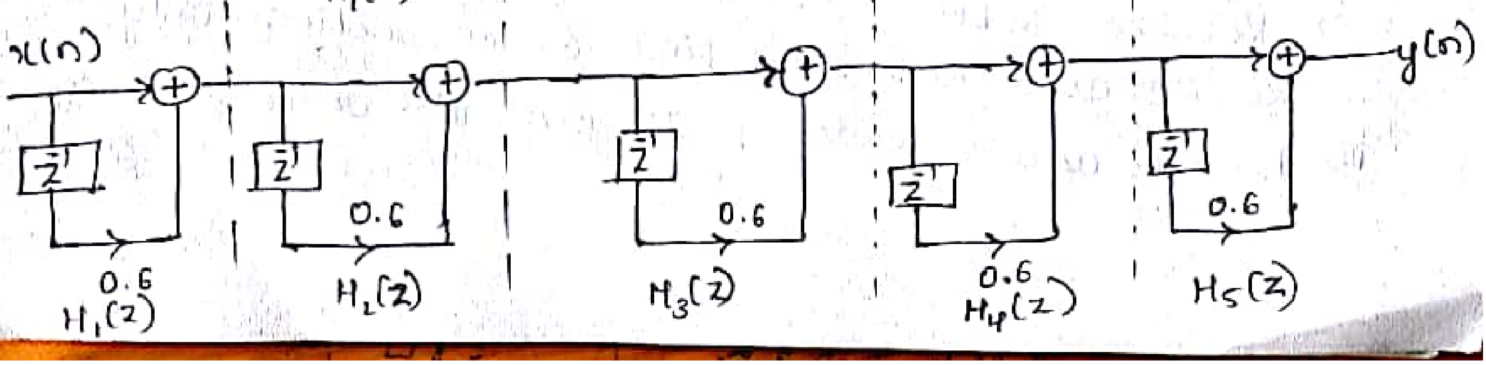
$$\boxed{H(z) = 1+3z^{-1}+5.16z^{-2}+2.16z^{-3}+0.6476z^{-4}+0.0777z^{-5}}$$

using above equation we can sketch DFI structure.



To realize as a cascade of first order sections only

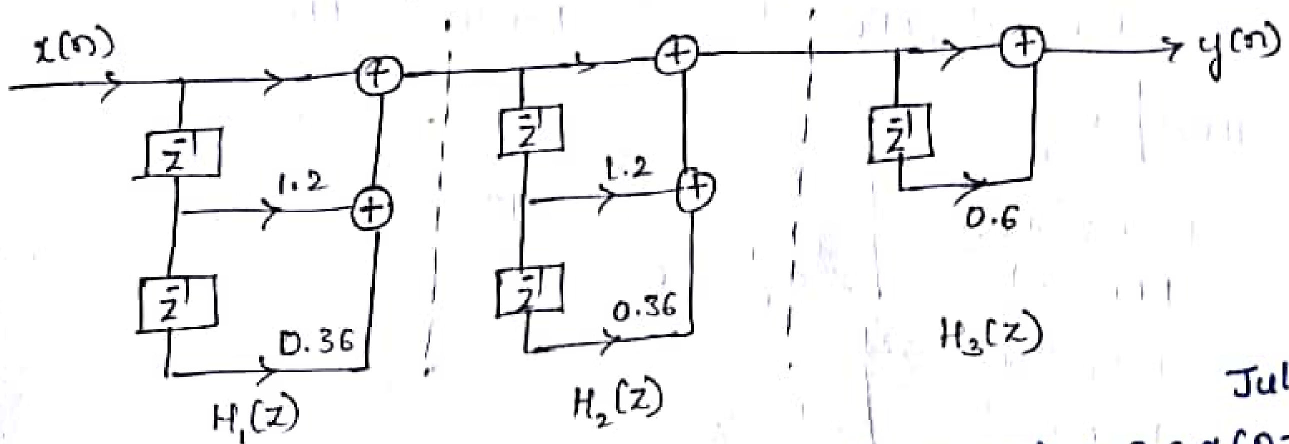
$$H(z) = \underbrace{(1+0.6z^{-1})}_{H_1(z)}\underbrace{(1+0.6z^{-1})}_{H_2(z)}\underbrace{(1+0.6z^{-1})}_{H_3(z)}\underbrace{(1+0.6z^{-1})}_{H_4(z)}\underbrace{(1+0.6z^{-1})}_{H_5(z)}$$



To realize as cascade of I and II order sections

$$H(z) = (1+1.2z^{-1}+0.36z^{-2})(1+1.2z^{-1}+0.36z^{-2})(1+0.6z^{-1})$$

$\uparrow H_1(z)$ $\uparrow H_2(z)$ $\uparrow H_3(z)$

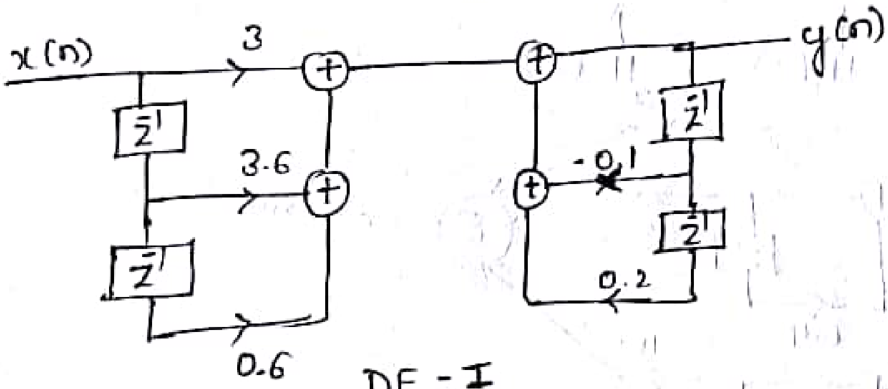


July-13, 10M

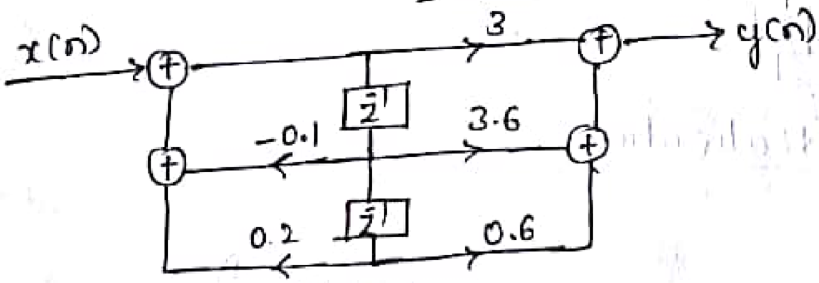
14) Given $y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2) \longrightarrow ①$

obtain DF-I and II, cascade form with single pole-zero system

Sol^n:-



DF-I



DF-II

parallel

applying Z.T to eq^n ①

$$y(z) = -0.1z^{-1}y(z) + 0.2z^{-2}y(z) + 3x(z) + 3.6z^{-1}x(z) + 0.6z^{-2}x(z)$$

$$y(z) + 0.1z^{-1}y(z) - 0.2z^{-2}y(z) = 3x(z) + 3.6z^{-1}x(z) + 0.6z^{-2}x(z)$$

$$y(z)[1+0.1z^{-1}-0.2z^{-2}] = x(z)[3+3.6z^{-1}+0.6z^{-2}]$$

$$H(z) = \frac{y(z)}{x(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

$\uparrow$ Transfer function

cascade

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{(1-0.4z^{-1})(1+0.5z^{-1})}$$

using partial fraction method

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{(1-0.4z^{-1})(1+0.5z^{-1})} = A + \frac{B}{1-0.4z^{-1}} + \frac{C}{1+0.5z^{-1}}$$

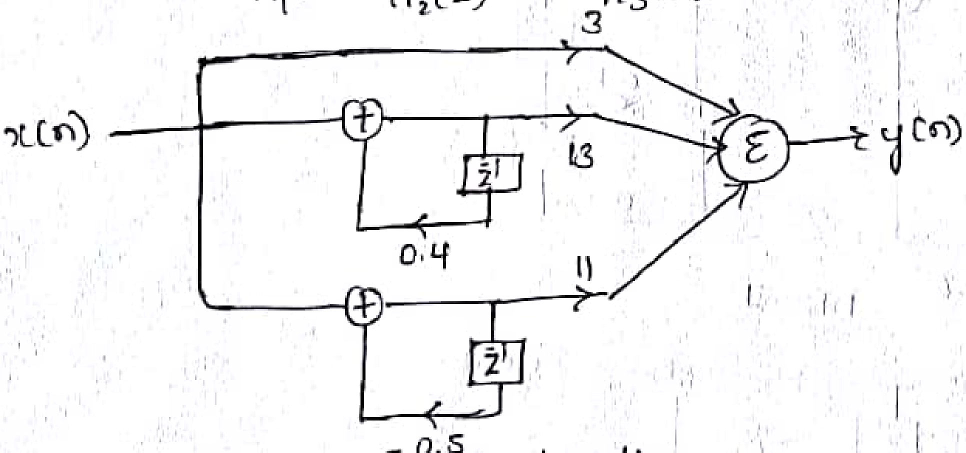
$$A = H(z)\Big|_{z^{-1}=0} = \underline{\underline{3}}$$

$$B = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.5z^{-1}}\Bigg|_{z^{-1}=\frac{1}{0.4}=2.5} = \underline{\underline{13}}$$

$$C = \frac{3+3.6z^{-1}+0.6z^{-2}}{1-0.4z^{-1}}\Bigg|_{z^{-1}=-2}$$

$\underline{\underline{C=11}}$

$$\therefore H(z) = \underset{\uparrow\, H_1(z)}{3} + \underset{\uparrow\, H_2(z)}{\frac{13}{1-0.4z^{-1}}} + \underset{\uparrow\, H_3(z)}{\frac{11}{1+0.5z^{-1}}}$$



parallel Realization

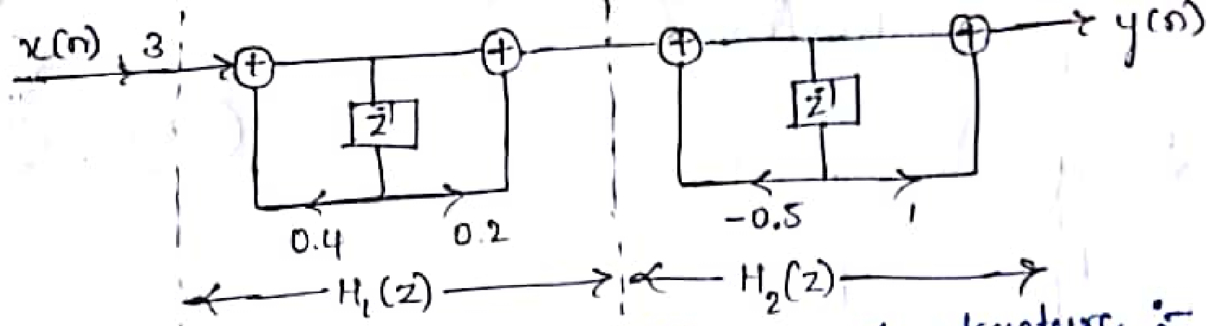
Cascade

consider

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}} = \frac{3(1+0.2z^{-1})(1+z^{-1})}{(1-0.4z^{-1})(1+0.5z^{-1})}$$

$$H(z) = \underset{\uparrow\, H_1(z)}{3\,\frac{(1+0.2z^{-1})}{(1-0.4z^{-1})}} \cdot \underset{\uparrow\, H_2(z)}{\frac{(1+z^{-1})}{(1+0.5z^{-1})}}$$

Scanned by CamScanner

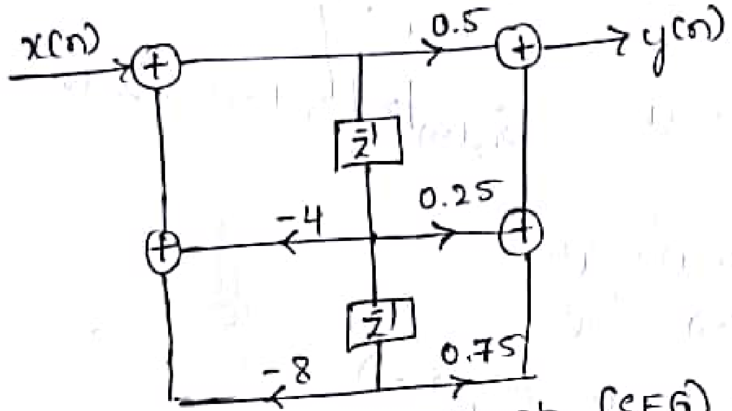


Signal flow graph and Transposed structure :-

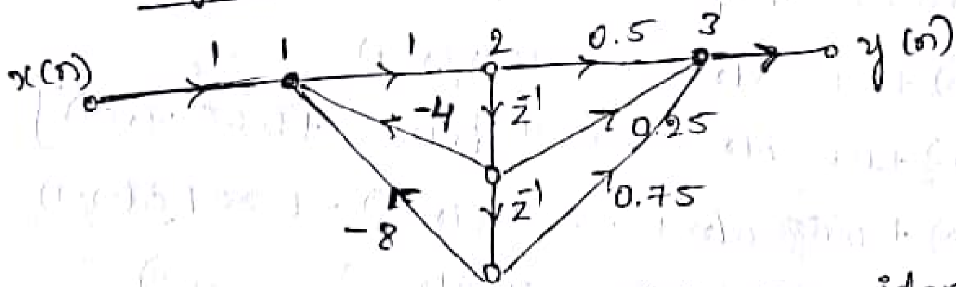
consider

$$H(z) = \frac{0.5 + 0.25z^{-1} + 0.75z^{-2}}{1 + 4z^{-1} + 8z^{-2}}$$

DF - II representation

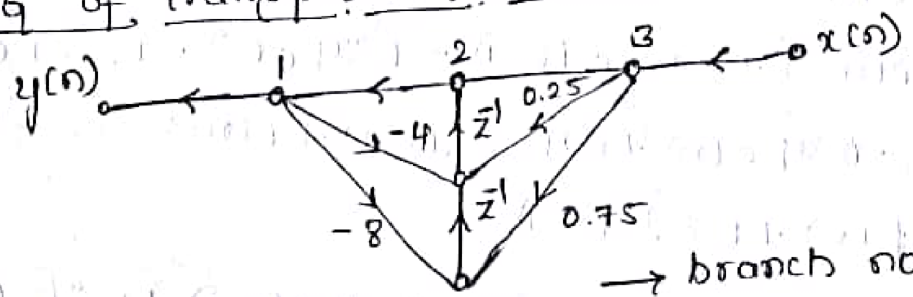


Signal flow graph (SFG)



Numbering is done for non-identical nodes and summing points.

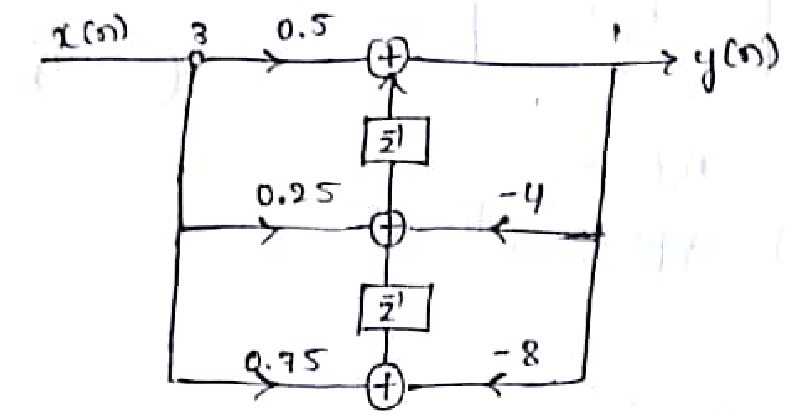
SFG of transposed structure



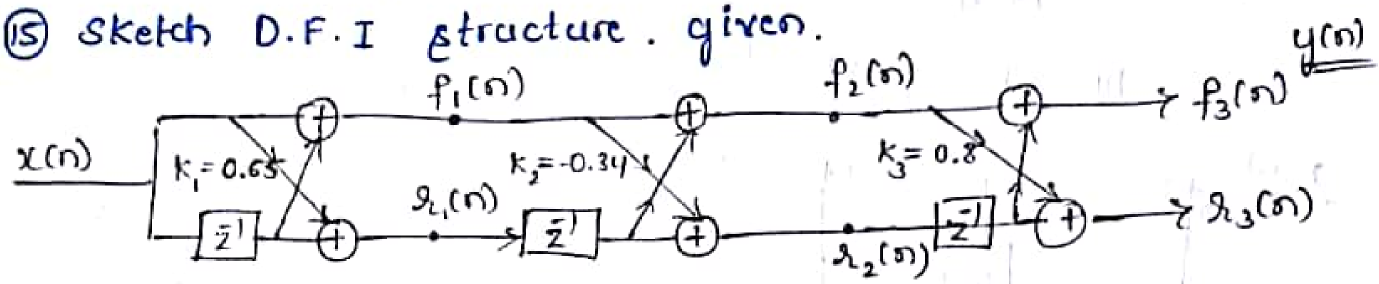
→ branch nodes becomes adders nodes and vice versa.

* Reverse the directions of all branch transmittance (gain b/w two nodes) and interchange i/p & o/p.

÷ Redrawn transposed structure :-



(15) Sketch D.F.I structure. given.



Let $x(n) = \delta(n)$

$f_1(n) = \delta(n) + 0.65\,\delta(n-1) \longrightarrow (1)$

$g_1(n)$ ~~$f_2(n)$~~ $= \delta(n-1) + 0.65\,\delta(n) \longrightarrow (2)$

$y(n) = f_2(n) + 0.8\,g_1(n-2) - 0.272 f_1(n-1)$

$f_2(n) = f_1(n) - 0.34\,g_1(n-1) \longrightarrow (3)$

$= \delta(n) + 0.65\,\delta(n-1) - 0.34\,g_1(n-1)$

$= \delta(n) + 0.65\,\delta(n-1) - 0.34\,[\delta(n-2) + 0.65\,\delta(n-1)]$

$= \delta(n) + 0.65\,\delta(n-1) - 0.34\,\delta(n-2) - 0.221\,\delta(n-1)$

$f_2(n) = \delta(n) + 0.429\,\delta(n-1) - 0.34\,\delta(n-2) \longrightarrow (3a)$

$g_2(n) = g_1(n-1) - 0.34 f_1(n) \longrightarrow (4)$

$= \delta(n-2) + 0.65\,\delta(n-1) - 0.34\,[\delta(n) + 0.65\,\delta(n-1)]$

$= \delta(n-2) + 0.65\,\delta(n-1) - 0.34\,\delta(n) - 0.221\,\delta(n-1)$

$g_2(n) = -0.34\,\delta(n) + 0.429\,\delta(n-1) + \delta(n-2) \longrightarrow (4a)$

$y(n) = f_2(n) + 0.8\,g_2(n-1)$

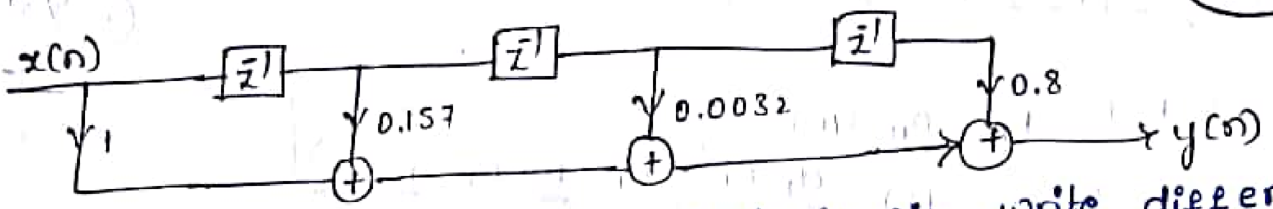
(01) $= \delta(n) + 0.429\,\delta(n-1) - 0.34\,\delta(n-2) + 0.8\,(-0.34\,\delta(n-1)$
$+ 0.429\,\delta(n-2) + \delta(n-3)]$

$h(n)$

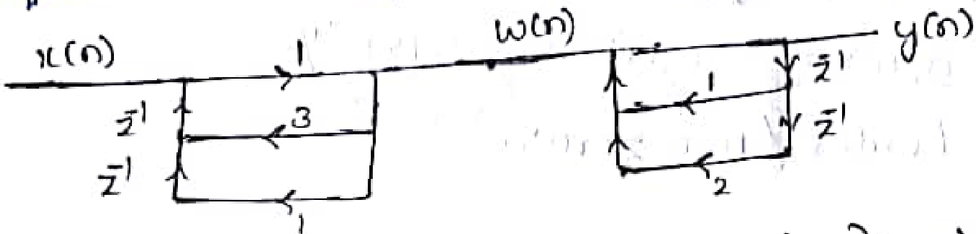
(08) $f_3(n) = \delta(n) + 0.157\,\delta(n-1) + 0.0032\,\delta(n-2) + \delta(n-3)$

Taking Z.T

$$\boxed{y(z) = 1 + 0.157\, z^{-1} + 0.0032\, z^{-2} + 0.8\, z^{-3}}$$



16) consider the signal flow graph in fig. write difference equation and system function.



Soln:-

$$w(n) = x(n) + 3w(n-1) + w(n-2) \longrightarrow (a)$$

$$y(n) = w(n) + y(n-1) + 2y(n-2) \longrightarrow (b)$$

Taking Z.T of eqn (a) & (b)

$$W(z)\left[1 - 3z^{-1} - z^{-2}\right] = X(z)$$

$$Y(z)\left[1 - z^{-1} - 2z^{-2}\right] = W(z)$$

$$H(z) = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{1}{(1 - z^{-1} - 2z^{-2})} \times \frac{1}{(1 - 3z^{-1} - z^{-2})}$$

$$H(z) = \frac{1}{1 - z^{-1} - 2z^{-2} - 3z^{-1} + 3z^{-2} + 6z^{-3} - z^{-2} + z^{-3} + 2z^{-4}}$$

Transfer function

$$H(z) = \frac{1}{1 - 4z^{-1} + 7z^{-3} + 2z^{-4}}$$

Difference equation

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 4z^{-1} + 7z^{-3} + 2z^{-4}}$$

Taking I.Z.T

$$y(n) - 4y(n-1) + 7y(n-3) + 2y(n-4) = x(n)$$

(or) $y(n) = x(n) + 4y(n-1) - 7y(n-3) - 2y(n-4)$

→ Lattice structure of FIR filter :-

17) Draw the lattice structure of FIR filter with system function

$$H(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2} \longrightarrow ①$$

(290)

Sol^n :-

W.K.T $\quad k_m = a_m(m)$

& $\quad a_{m-1}(i) = \dfrac{a_m(i) - a_m(m)\,a_m(m-i)}{1 - k_m^2} \quad \boxed{1 \le i \le m-1}$

from eq^n ①, $H(z) = a_2(0) + a_2(1)z^{-1} + a_2(2)z^{-2}$

$a_2(0) = 1 \quad a_2(1) = 2 \quad a_2(2) = 1/3$

Second Order — $M = 2$ (order / two zero's)

case (i) :-

if $m = 2$,

$k_2 = a_2(2) = \frac{1}{3}$

at $m=2$, & $i=1$

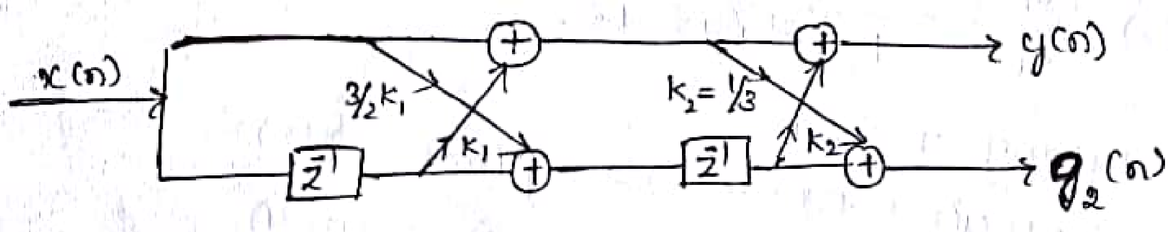
$$a_1(1) = \frac{a_2(1) - a_2(2)\,a_2(1)}{1 - k_2^2}$$

$$= \frac{a_2(1)\,[1 - a_2(2)]}{1 - a_2^2(2)}$$

$$= \frac{a_2(1)\,(1 - \cancel{a_2(2)})}{(1 - \cancel{a_2(2)})\,(1 + a_2(2))}$$

$$= \frac{a_2(1)}{1 + a_2(2)} = \frac{2}{1 + 1/3} = \frac{2}{4/3} = \frac{6}{4} = \frac{3}{2}$$

$\therefore \boxed{K_1 = a_1(1) = 3/2}$



18) A FIR filter is given by

$$y(n) = x(n) + \frac{2}{5} x(n-1) + \frac{3}{4} x(n-2) + \frac{1}{3} x(n-3).$$

Draw Lattice structure.

Soln:- Taking Z.T

$$Y(z) = X(z) + \frac{2}{5} z^{-1} X(z) + \frac{3}{4} z^{-2} X(z) + \frac{1}{3} z^{-3} X(z)$$

$a_3(0) = 1 \quad a_3(1) = 2/5 \quad a_3(2) = 3/4 \quad a_3(3) = 1/3$

W.K.T $\quad K_m = a_m(m)$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-i)}{1 - K_m^2}$$

M=3

case (i)

for $m=3$ $\quad \boxed{K_3 = a_3(3) = 1/3}$

$$a_2(i) = \frac{a_3(i) - a_3(3)\, a_3(3-i)}{1 - K_3^2}$$

at $i=1$

$$a_2(1) = \frac{a_3(1) - a_3(3)\, a_3(2)}{1 - a_3^2(3)}$$

$$= \frac{2/5 - 1/3 \times 3/4}{1 - (1/3)^2} = \frac{2/5 - 1/4}{1 - 1/9} = \frac{\frac{8-5}{20}}{8/9}$$

$$= \frac{3}{20} \times \frac{9}{8} = \frac{27}{160}$$

at $i=2$

$$a_2(2) = \frac{a_3(2) - a_3(3)\, a_3(1)}{1 - K_3^2}$$

$$= \frac{3/4 - 1/3 \times 2/5}{1 - (1/3)^2} = \frac{3/4 - 2/15}{8/9}$$

$$= \frac{\frac{45-8}{60}}{8/9} = \frac{37}{\cancel{60}_{20}} \times \frac{\cancel{9}^{3}}{8} = \frac{111}{160}$$

$$= 0.693$$

case (2)

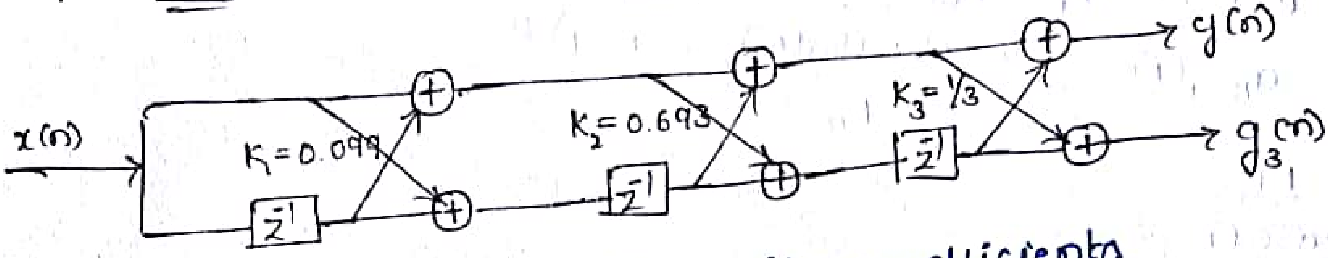
for $m=2$ $\boxed{K_2 = a_2(2) = 0.693}$

at $i=1$ $a_1(1) = \dfrac{a_2(1) - a_2(2)\,a_2(1)}{1-K_2^2}$

$$= \frac{\frac{27}{160} - 0.693 \times \frac{27}{160}}{1-(0.693)^2} = 0.099$$

case (3)

for $m=1$ $K_1 = a_1(1) = 0.099$



Feb-05

19) consider an FIR lattice filter with co-efficients $K_1 = 0.65$, $K_2 = -0.34$, $K_3 = 0.8$. Find its impulse response Draw the equivalent DF-I structure.

Soln:- I- Method

consider $a_m(0) = 1$ $a_m(m) = K_m$

$a_m(i) = a_{m-1}(i) + a_m(m) \cdot a_{m-1}(m-i)$

$1 \le i \le m-1$

Here $M=3$ i.e 3 stages in the FIR Lattice structure

for $m=1$. $a_1(0) = 1$

$a_1(1) = K_1 = 0.65$

for $m=2$.

$a_2(0) = 1$

$a_2(2) = K_2 = -0.34$

$a_2(1) = a_1(1) + a_2(2)\,a_1(2-1)$

$1 \le i \le 1$

i.e $a_2(1) = a_1(1) + a_2(2)\,a_1(1)$

$= 0.65 - 0.34 \times 0.65$

$a_2(1) = 0.429$

for $m=3$

$a_3(0) = 1$

$a_3(3) = K_3 = 0.8$

$a_3(i) = a_2(i) + a_3(3)\, a_2(3-i) \quad 1 \le i \le 2$

$i=1$

$a_3(1) = a_2(1) + a_3(3) \cdot a_2(2)$

$= 0.429 + 0.8\,(-0.34) = 0.157$

$i=2$

$a_3(2) = a_2(2) + a_3(3) \cdot a_2(1)$

$= -0.34 + 0.8\,(0.429) = 0.0032$

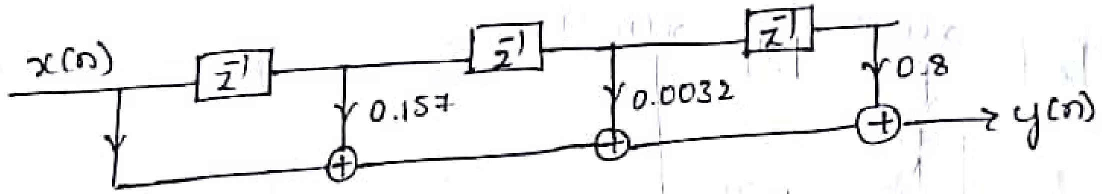
$H(z) = 1 + a_3(1)\, z^{-1} + a_3(2)\, z^{-2} + a_3(3)\, z^{-3}$

(or) $H(z) = 1 + 0.157\, z^{-1} + 0.0032\, z^{-2} + 0.8\, z^{-3}$

DF-I

Taking I.Z.T

$h(n) = \delta(n) + 0.157\,\delta(n-1) + 0.0032\,\delta(n-2) + 0.8\,\delta(n-3)$



Difference equation

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 0.157\, z^{-1} + 0.0032\, z^{-2} + 0.8\, z^{-3}$$

$$Y(z) = X(z)\left[1 + 0.157\, z^{-1} + 0.0032\, z^{-2} + 0.8\, z^{-3}\right]$$

I.Z.T

$$y(n) = x(n) + 0.157\, x(n-1) + 0.0032\, x(n-2) + 0.8\, x(n-3)$$

20) Obtain direct form Realization.

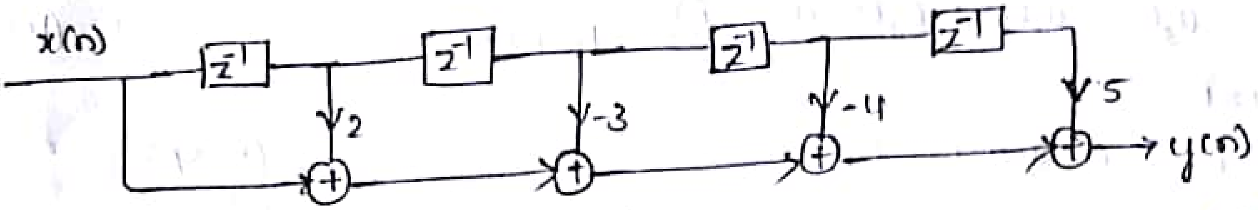
$H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

Sol^n:- wkt, $H(z) = \dfrac{Y(z)}{X(z)} = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

(or) $Y(z) = X(z) + 2z^{-1}X(z) - 3z^{-2}X(z) - 4z^{-3}X(z) + 5z^{-4}X(z)$

Taking I.Z.T

$$y(n) = x(n) + 2x(n-1) - 3x(n-2) - 4x(n-3) + 5x(n-4)$$



22) Obtain the direct form realization of linear phase FIR system given.

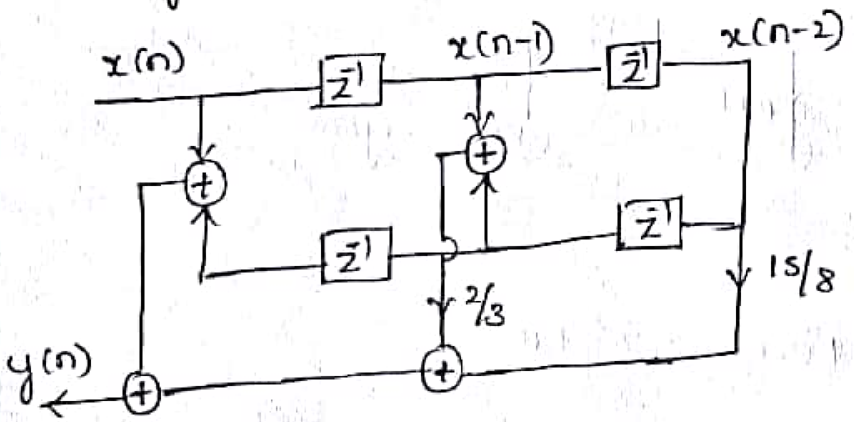
$$H(z) = 1+\frac{2}{3}z^{-1}+\frac{15}{8}z^{-2}+\frac{2}{3}z^{-3}+z^{-4}$$

$$H(z) = \frac{Y(z)}{X(z)} = 1+\frac{2}{3}z^{-1}+\frac{15}{8}z^{-2}+\frac{2}{3}z^{-3}+z^{-4}$$

$$Y(z) = \left[1+\frac{2}{3}z^{-1}+\frac{15}{8}z^{-2}+\frac{2}{3}z^{-3}+z^{-4}\right]X(z)$$

I.Z.T

$$y(n) = x(n)+\frac{2}{3}x(n-1)+\frac{15}{8}x(n-2)+\frac{2}{3}x(n-3)+x(n-4)$$



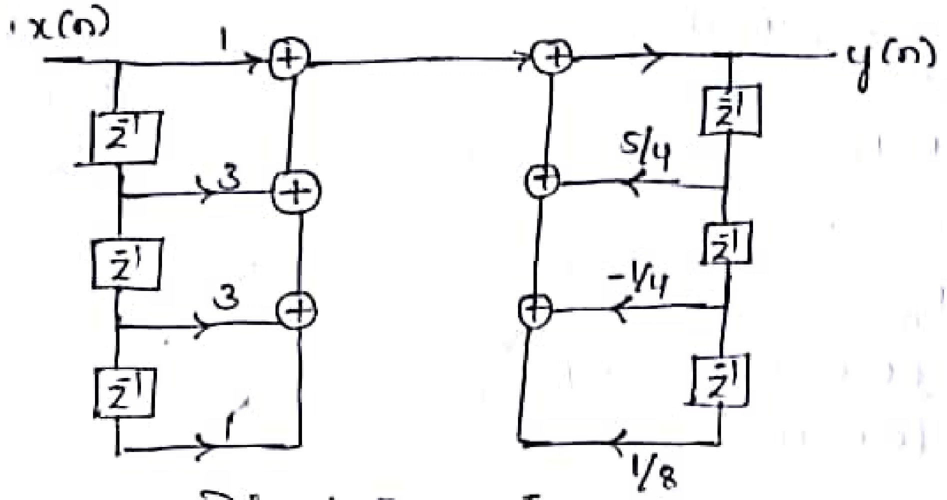
23) Obtain direct form I and cascade realization for the IIR filter having transfer function.

$$H(z) = \frac{(1+z^{-1})^3}{(1-\frac{1}{4}z^{-1})(1-z^{-1}+\frac{1}{2}z^{-2})}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+3z^{-1}+3z^{-2}+z^{-3}}{1-\frac{5}{4}z^{-1}+\frac{1}{4}z^{-2}-\frac{1}{8}z^{-3}}$$

Taking I.Z.T and solving

$$y(n) = \frac{5}{4}y(n-1)-\frac{1}{4}y(n-2)+\frac{1}{8}y(n-3)$$
$$+x(n)+3x(n-1)+3x(n-2)+x(n-3)$$

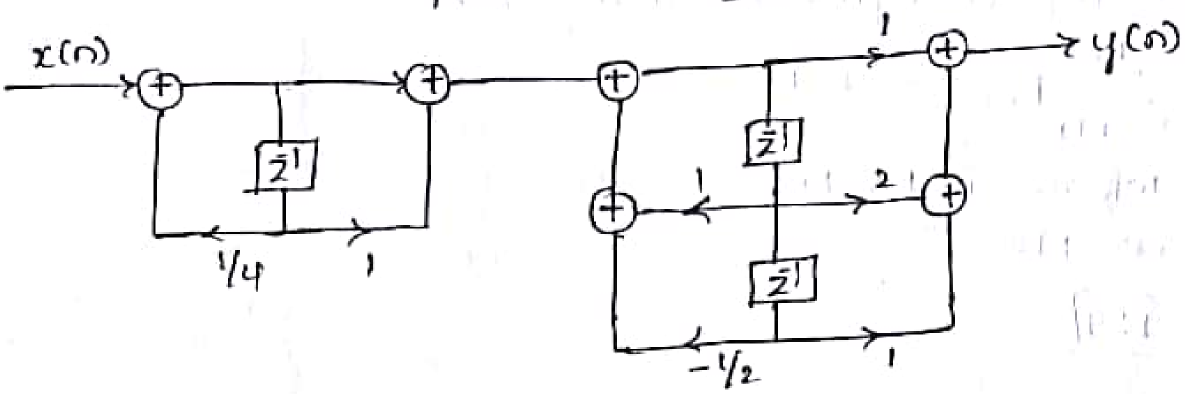


Direct Form - I

Cascade

$$H(z) = H_1(z) \cdot H_2(z)$$

$$H(z) = \left(\frac{1+z^{-1}}{1-\frac{1}{4}z^{-1}}\right)\left(\frac{1+2z^{-1}+z^{-2}}{1-z^{-1}+\frac{1}{2}z^{-2}}\right)$$



→ Lattice Ladder structure :-

24) Realize IIR filter using Lattice Ladder structure. given

$$H(z) = \frac{0.129 + 0.3867z^{-1} + 0.3869z^{-2} + 0.129z^{-3}}{1 - 0.2971z^{-1} + 0.3564z^{-2} - 0.0276z^{-3}}$$

Soln :- Let $H(z) = \frac{1}{A(z)} \cdot B(z)$

$\therefore A(z) = 1 - 0.2971z^{-1} + 0.3564z^{-2} - 0.0276z^{-3} \rightarrow$ poles

$B(z) = 0.129 + 0.3867z^{-1} + 0.3869z^{-2} + 0.129z^{-3} \rightarrow$ zeros

To find Lattice co-efficients of A(z)

$a_3(3) = -0.0276$

$a_3(2) = 0.3564$

$a_3(1) = -0.2971$

$K_m = a_m(m)$.

$$\boxed{K_3 = a_3(3) = -0.0276}$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m) \cdot a_m(m-i)}{1-k_m^2}$$

for $m=3$, & $i=1,2$

$$a_2(1) = \frac{a_3(1) - a_3(3) \cdot a_3(2)}{1-k_3^2}$$

$$= \frac{-0.2971 + 0.027 \times 0.3564}{1-(0.027)^2}$$

$$= -0.2875$$

Illy

$$a_2(2) = 0.3485 = k_2$$

for $m=2$, $i=1$

$$a_1(1) = k_1 = -0.2132$$

To find tap-co-efficients using $B(z)$

$$\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m \, a_m(m-i)$$

where $i = M, M-1, \ldots 0$

given $M=3$

$$\boxed{\beta_3 = 0.129}$$

$i=2$

$$\beta_2 = b_2 - \sum_{m=3}^{3} \beta_m \cdot a_m(m-2)$$

$$= 0.3869 - (0.129)(-0.2971)$$

$$\boxed{\beta_2 = 0.4252}$$

$$\beta_1 = b_1 - \sum_{m=2}^{3} \beta_m \cdot a_m(m-1)$$

$$= b_1 - \beta_2 \, a_2(1) - \beta_3 \, a_3(2)$$

$$= 0.3867 - (0.4252)(-0.2875) - (0.129)(0.3564)$$

$$\boxed{\beta_1 = 0.4630}$$

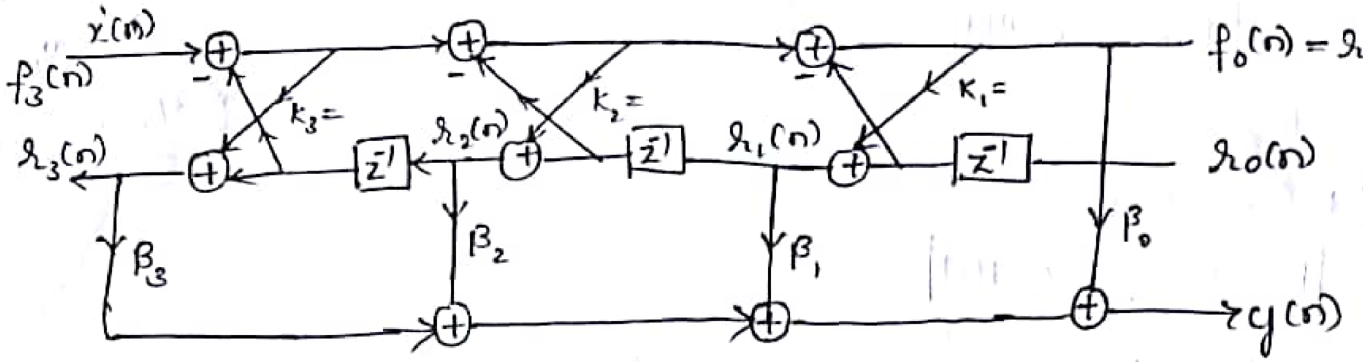
$$\beta_0 = b_0 - \sum_{m=1}^{3} \beta_m \cdot a_m(m-1)$$

$$= b_0 - \beta_1 a_1(1) - \beta_2 \, a_2(2) - \beta_3 \, a_3(3)$$

$$\boxed{\beta_0 = 0.0831}$$

18

(297)



25) Determine the parameters k_m of the lattice filter corresponding to the FIR filter described by

$$H(z) = 1 + 2.82 z^{-1} + 3.408 z^{-2} + 1.74 z^{-3}$$

Soln:- let $H(z) = a_3(0) + a_3(1) z^{-1} + a_3(2) z^{-2} + a_3(3) z^{-3}$

$\therefore$ $a_3(0) = 1$ $\quad$ $a_3(2) = 3.408$

$a_3(1) = 2.82$ $\quad$ $a_3(3) = 1.74$

w.k.t $\quad k_m = a_m(m) \longrightarrow ①$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m).a_m(m-i)}{1-k_m^2} \longrightarrow ② \qquad 1 \le i \le m-1$$

Order

$M=3$

Case(i)

of $m=3$

$k_3 = a_3(3) = 1.74$

$1 \le i \le 2$ $\quad$ i.e $\quad i = 1, 2$

with $m=3$ & $i=1$ eq ② becomes

$$a_2(1) = \frac{a_3(1) - a_3(3)\, a_3(2)}{1-k_3^2}$$

$$a_2(1) = \frac{2.82 - 1.74 \times 3.408}{1-(1.74)^2} = \frac{-3.1099}{-2.0276} = \underline{\underline{1.533}}$$

$a_2(1) = 1.533$

With $m=3$ & $i=2$ eq ② becomes

$$a_2(2) = \frac{a_3(2) - a_3(3).a_3(1)}{1-k_3^2}$$

$$= \frac{3.408 - 1.74 \times 2.82}{1-(1.74)^2} = \frac{-1.5028}{-2.0276} = 0.741$$

$$\boxed{a_2(2) = 0.741}$$

Case (2)

$m=2, \ i=1$

$$\boxed{k_2 = a_2(2) = 0.741}$$

eqⁿ (2) becomes

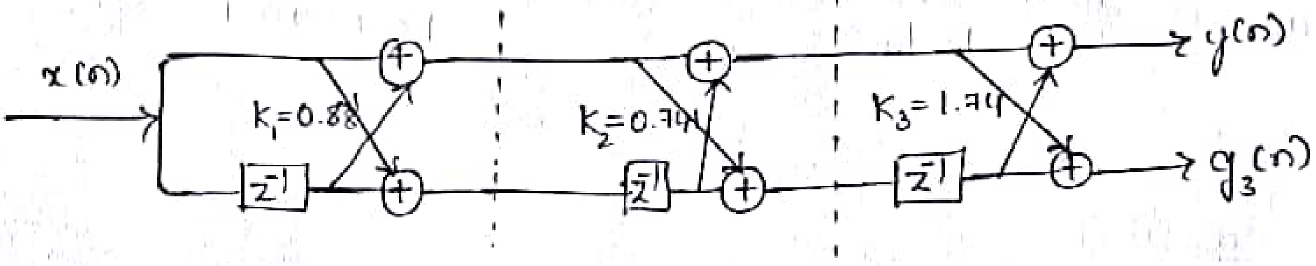
$$a_1(1) = \frac{a_2(1) - a_2(2) \cdot a_2(1)}{1-k_2^2}$$

$$= \frac{1.533 - 0.741 \times 1.533}{1-(0.741)^2} = \frac{0.397}{0.4509} = \underline{\underline{0.88}}$$

case (3)

$m=1$

$$\boxed{k_1 = a_1(1) = 0.88}$$



Lattice structure

July-13. 10M

26) Develop the lattice ladder structure for the filter with difference equation.

$$y(n) = \frac{3}{4} y(n-1) + \frac{1}{4} y(n-2) = x(n) + 2x(n-1) \longrightarrow (1)$$

Solⁿ:-

Apply Z.T to eqⁿ (1)

$$Y(z)\left[1 + \frac{3}{4} z^{-1} + \frac{1}{4} z^{-2}\right] = X(z)\left[1 + 2z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}} = \frac{B(z)}{A(z)}$$

i.e $B(z) = 1 + 2z^{-1}$

$A(z) = 1 + \frac{3}{4} z^{-1} + \frac{1}{4} z^{-2}$

To find Lattice co-efficients of A(z)

$A(z) = 1 + \frac{3}{4} z^{-1} + \frac{1}{4} z^{-2}$

$a_2(0) = 1$

$a_2(1) = 3/4$ $\quad$ $\underline{M=2}$

$\underline{\text{case(1)}}$

$a_2(2) = 1/4$ $\quad$ $m=2, \ \boxed{K_2 = a_2(2) = 1/4}$

$m=2$ & $i=1$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)\, a_m(m-i)}{1 - K_m^2}$$

$$a_1(1) = \frac{a_2(1) - a_2(2) \cdot a_2(1)}{1 - K_2^2}$$

$$a_1(1) = \frac{3/4 - 1/4 \times 3/4}{1 - (1/4)^2} = \frac{3/4 - 3/16}{1 - 1/16} = \frac{\frac{12-3}{16}}{\frac{16-1}{16}} = \frac{9}{15} = \frac{3}{5}$$

$$\boxed{a_1(1) = \frac{3}{5}}$$

$\underline{\text{case(2)}}$

$m=1$

$\therefore \boxed{K_1 = a_1(1) = 3/5}$

To find tap co-efficients of B(z)

given $B(z) = 1 + 2z^{-1}$

$\therefore b_0 = 1, \ b_1 = 2$

w.k.T, $\beta_i = b_i - \sum_{m=i}^{M} \beta_m \cdot a_m(m-i)$ $\quad$ $i = M, M-1, \ldots 0$

$\underline{M=1}$

$\underline{\text{case(1)}}$ $\quad m=1$ & $i=1$

$\beta_1 = b_1 - 0$

$\boxed{\beta_1 = 2}$

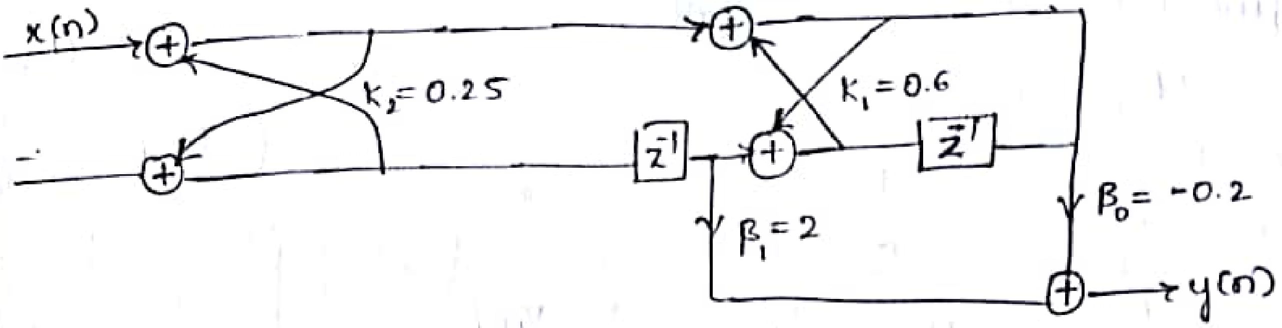
$\underline{\text{case(2)}}$ $\quad m=1$ & $i=0$

$\beta_0 = b_0 - \sum_{m=1}^{1} \beta_m a_m(m-0)$

$\beta_0 = b_0 - \beta_1 a_1(1)$

$= 1 - 2 \times 3/5$

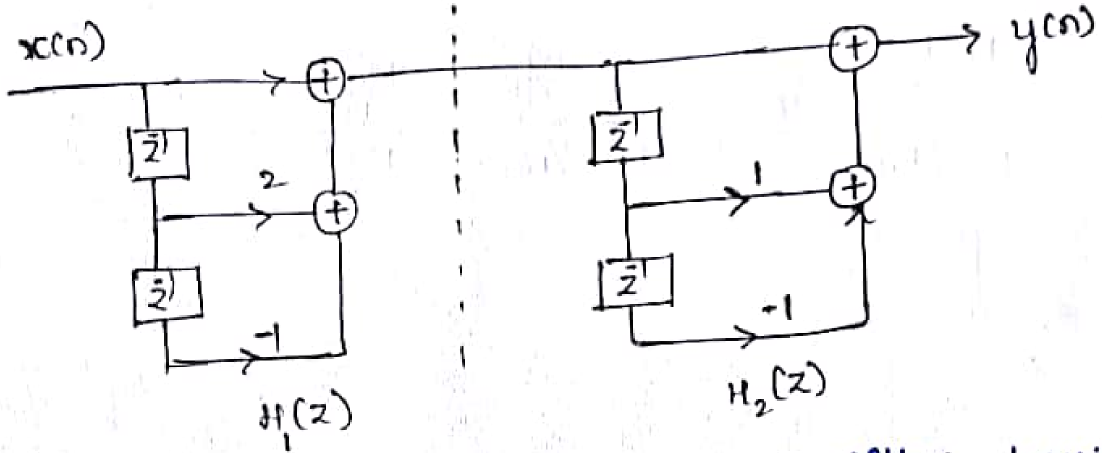
$\boxed{\beta_0 = -0.2}$



27) obtain cascade realization of

$$H(z) = (1+2z^{-1}-z^{-2})\,(1+z^{-1}-z^{-2})$$

Let $H(z) = H_1(z) \cdot H_2(z)$



28) Realize the linear phase FIR filter having the impulse Response.

$$h(n) = \{1, \tfrac{1}{4}, -\tfrac{1}{8}, \tfrac{1}{4}, 1\}$$

Soln :-

$$h(n) = \delta(n) + \tfrac{1}{4}\delta(n-1) - \tfrac{1}{8}\delta(n-2) + \tfrac{1}{4}\delta(n-3) + \delta(n-4)$$

taking Z.T

$$H(z) = 1 + \tfrac{1}{4}z^{-1} - \tfrac{1}{8}z^{-2} + \tfrac{1}{4}z^{-3} + z^{-4}$$

$$H(z) = \frac{y(z)}{x(z)} = 1 + \tfrac{1}{4}z^{-1} - \tfrac{1}{8}z^{-2} + \tfrac{1}{4}z^{-3} + z^{-4}$$

(or)

$$y(z) = x(z)\left[1 + \tfrac{1}{4}z^{-1} - \tfrac{1}{8}z^{-2} + \tfrac{1}{4}z^{-3} + z^{-4}\right]$$

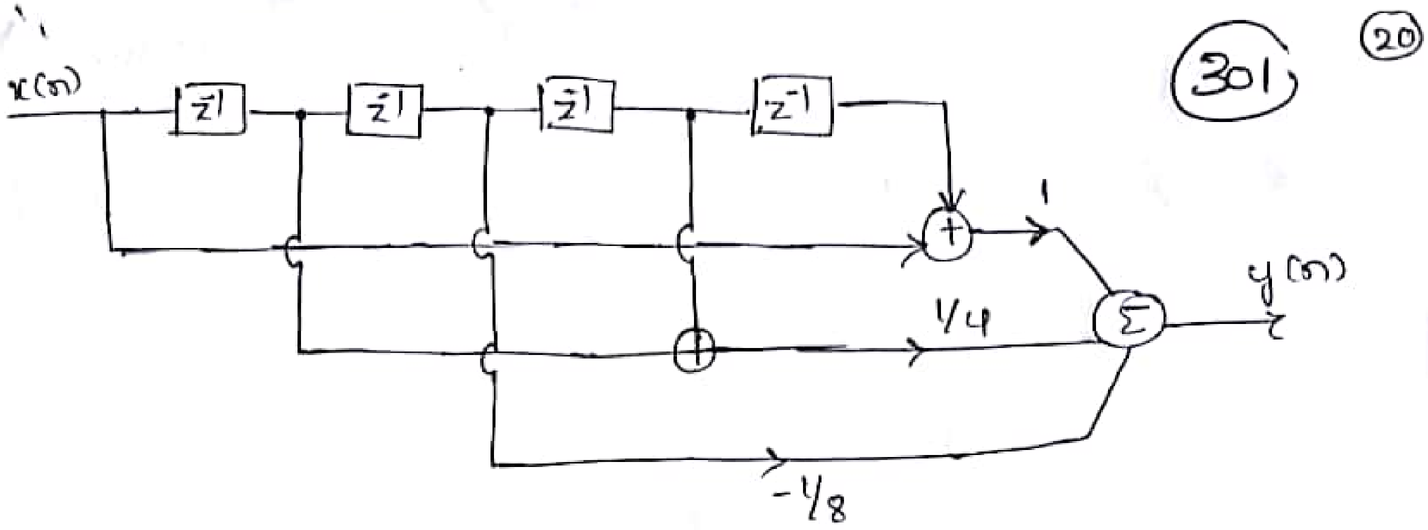
taking I.Z.T

$$y(n) = x(n) + \tfrac{1}{4}x(n-1) - \tfrac{1}{8}x(n-2) + \tfrac{1}{4}x(n-3) + x(n-4)$$

(or)

$$y(n) = \left[x(n) + x(n-4)\right] + \tfrac{1}{4}\left[x(n-1) + x(n-3)\right] - \tfrac{1}{8}x(n-2)$$

Scanned by CamScanner



x(n)
z^{-1}
z^{-1}
z^{-1}
z^{-1}
+
1
1/4
Σ
y(n)
-1/8