# DIGITAL COMMUNICATION SYSTEM
# 19EC5DCDCS

# COURSE OBJECTIVES:

- To provide knowledge about source coding techniques.
- Analyze the channel capacity using Shannon's theorem.
- Introduce error control coding techniques.
- Get knowledge of theoretical aspects of detection of the transmitted data from the noisy received data.
- Analyze the generation and detection of basic digital modulation techniques and arrive at the respective probability of errors and PSD.
- Develop communication system for specific application by taking example of Spread spectrum communication

# COURSE OUTCOMES

- Apply the concept of Probability to measure information and capacity of channel.
- Apply various channel coding techniques for error detection and correction.
- Recognize the theoretical models, ideal receivers for the detection of the message from noisy received signal.
- Analyze transmitter and receiver blocks for binary and quaternary modulation techniques to arrive at probability of error and Power Spectral Density.
- Interpret Spread spectrum concepts, its types and applications.
- Work in a team to simulate digital communication systems for different applications.

# MODULE 1

***INFORMATION THEORY:***

- Digital Communication block diagram
- Information
- Entropy

***SOURCE ENCODING:***

- Shannon’s Encoding Algorithm
- Huffman coding

***CHANNEL***

- Discrete memoryless channels
- Binary Symmetric Channel
- Channel capacity
- Shannon Hartley Theorem and its implications.

# Prerequisite

- Knowledge of subjects like :

  **Basics of Probability Theory**

  **Signals and Systems**

  **Analog communication.**

***Self Steady Components:***

| | |
|---|---|
| **Module 1 :** | **Mutual Information and its Properties, extension of source, other discrete channels, Simulation** |
| **Module 2 :** | Types of Errors, Methods of Controlling Errors, Simulation. |
| **Module 3 :** | Matched filter for RF pulse, Simulation |
| **Module 4 :** | Simulation of digital modulation techniques: ASK, FSK, PSK, QPSK, DPSK |
| **Module 5 :** | Construction of state Diagrams, Counter Design and Maximum length sequence generation by simulation. |

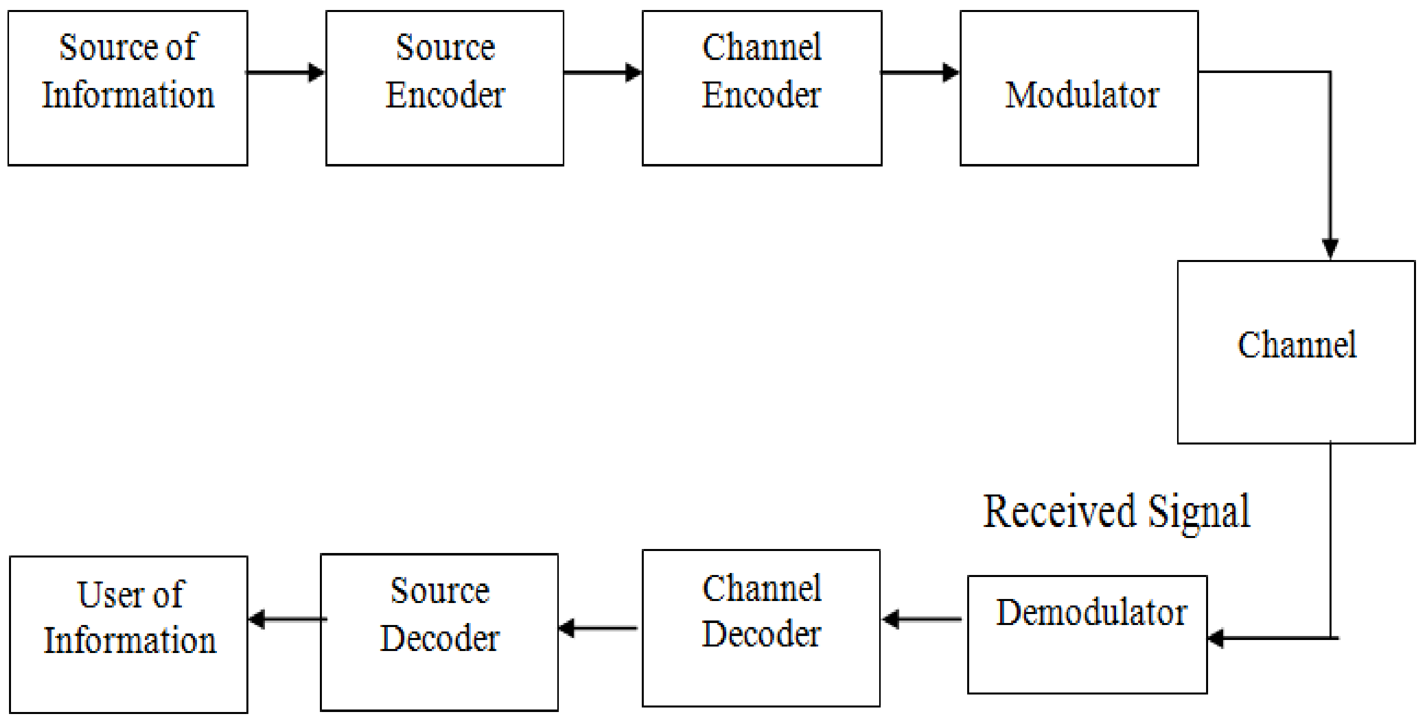
# TEXT BOOKS :

| | |
|---|---|
| | **Simon Haykin, "Digital communication", ISBN-9971-51-205-X, John Wiley & Sons (Asia), Pvt. Ltd, 2008** |
| | K. Sam Shanmugam, "Digital and analog communication systems", John Wiley India Pvt. Ltd, 1996. |
| | Shu Lin, Daniel J Costello Jr., "Error Control Coding", Pearson Education Asia, Second Edition, 2011. |
| | John. G.Proakis, "Communication Systems Engineering", 2nd Edition, Pearson. |
| | John. G. Proakis, Masoud Salehi, "Digital Communications", Mac Graw Hill, 2008 |

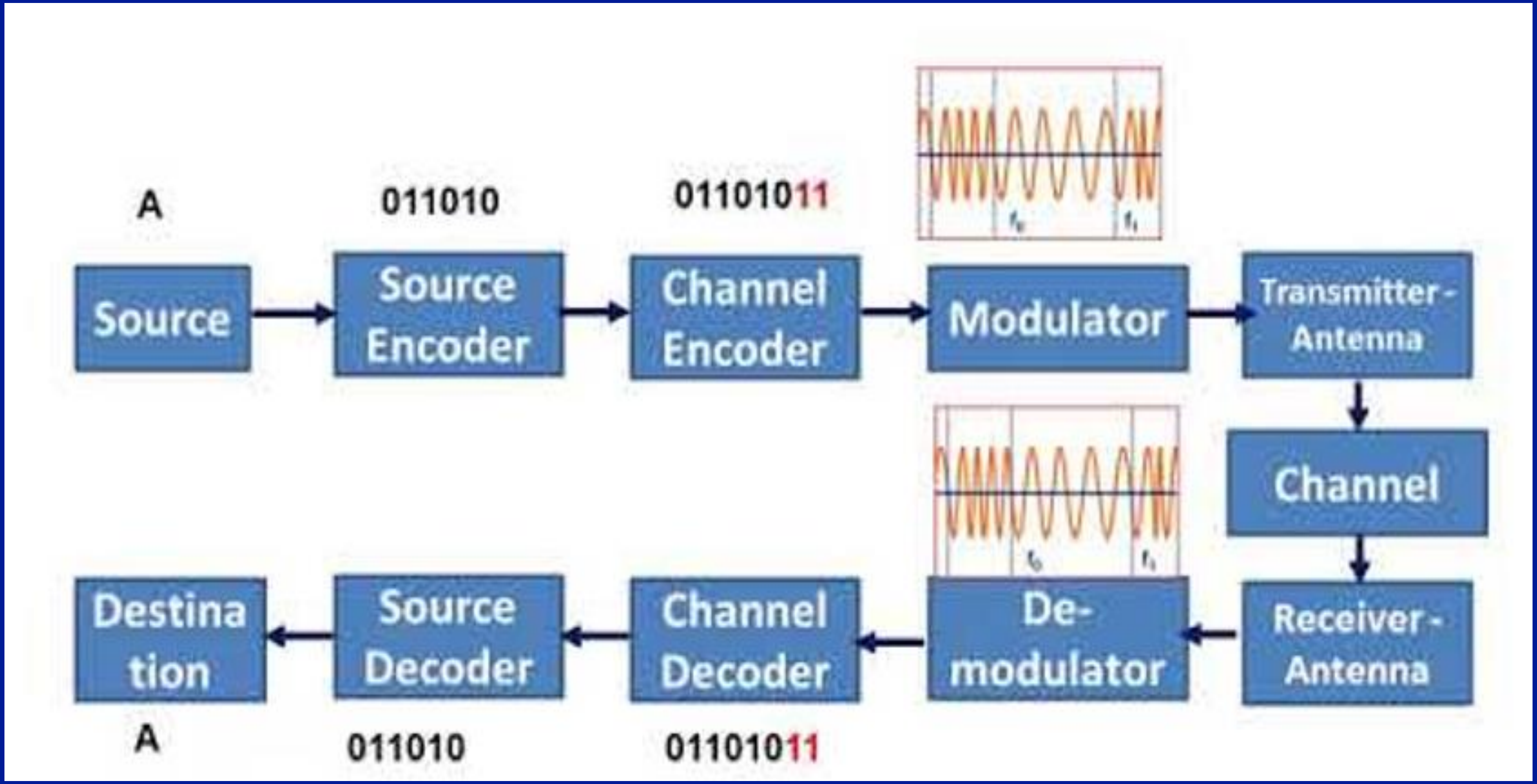
## REFERENCES BOOKS:

| | |
|---|---|
| | **Simon Haykin, "Digital and Analog Communication", John Wiley, India Pvt. Ltd., 2008.** |
| | Bernard Sklar, "Digital Communication", Pearson Education, 2007. |
| | John G. Proakis , Masoud Salehi, Gerhard Bauch, "Contemporary Communication Systems Using MATLAB", 3rd Edition. |
| | B.P. Lathi, "Modern digital and analog Communication systems", Oxford University Press, 4thedn., 2010. |
| | Ranjan Bose, " ITC and Cryptography", TMH, 2nd edition, 2007 |

# Digital communication system



# Digital communication system



## Information Source:

- The source of information can be analog or digital.
- Ex: Analog-Audio or video signal
- Video-Teletype signal.

## Source Encoder :

- Converting the output of whether analog or digital source into a sequence of binary digits.
- The source decoder converts the binary output of the channel decoder into a symbol sequence.
- Aim : remove the redundancy in the transmitting information, so that bandwidth required for transmission is minimized.

## Channel encoder:

- Introduce some redundancy in the binary information sequence.
- Ex: Let k be the information sequence and map that k bits to unique n bit sequence called code word.
- These extra bits do not convey any information but helps the receiver to detect and / or correct some of the errors in the information bearing bits.
- The Channel decoder recovers the information bearing bits from the coded binary stream
- Error detection and possible correction is also performed by the channel decoder.

# Modulator Demodulator

- Efficient transmission of the signal.
- It operates by keying shifts in the amplitude, frequency or phase of a sinusoidal carrier wave to the channel encoder output.
- The digital modulation techniques are referred to as amplitude- shift keying, frequency- shift keying or phase-shift keying respectively.
- converts the input bit stream into an electrical waveform suitable for transmission over the communication channel.

- The detector performs demodulation, thereby producing a signal the follows the time variations in the channel encoder output.

# Information

- **Information theory, originally known as mathematical theory of communication, deals with mathematical modelling and analysis of a communication system rather than with physical sources and physical channels.**

- **The output of a discrete information source is a message that consists of a sequence of symbols.**

- **Information of an event depends only on its probability of occurrence and is not dependent on its content.**

- **The randomness of happening of an event and the probability of its prediction as a news is known as information.**

# INTRODUCTION

- The discrete information source consist of a discrete set of letters or symbol. A message emitted by a source consist of sequence of symbols.

- Every message coming out of the source contains some information.

- However the information contained by the message varies from one another. Some message convey more information than others.

- In order to identify the information content of the message and the average information content of symbols in messages 'measure of information' is necessary.

# BASICS OF PROBABILITY:

* The probability of an event A is a non negative number between 0 and 1.

$$0 \leq P(A) \leq 1$$

* Probability of a sure event is equal to unity.

$P(S) = 1$ , $S \rightarrow$ sample space

* Probability of a null event is equal to zero.

$$P(\phi) = 0.$$

* For any two events $A$ and $B$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A + B) = P(A) + P(B) - P(AB)$$

* If $A$ and $B$ are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

* $P(A) + P(\bar{A}) = 1$

* $\sum_i P_i = P_1 + P_2 + \dots + P_n = 1$

* <u>Conditional Probability:</u>

The probability of some event $B$ occuring given that some other event $A$ has already occured is given by

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P\left(\frac{B}{A}\right) P(A) = P\left(\frac{A}{B}\right) \cdot P(B)$$

# Measure of Information

Let $S = \{S_1, S_2, S_3 \ldots S_q\}$ be the symbols with probabilities $P_1, P_2, P_3 \ldots P_q$ such that $P_1 + P_2 + P_3 + \ldots + P_q = 1$

i.e., $\sum_{i=1}^{q} P_i = 1$

Let $S_K$ be the symbol chosen for transmission with a probability $P_K$. Suppose that a receiver correctly identifies the messages, then amount of information or self information is given by

$$I_K = \log_2 \left( \frac{1}{P_K} \right)$$

If the base of the log is 2, then unit is 'bits'. If the base is 10, unit is 'Hartleys' / 'Decits'. If base is 'e', unit is 'Nats'. ( Bits → BInary uniTS)

# Reason for using log

- The self information cannot be negative
- The lowest value for self information is zero, since the prob of the certain event is equal to one.
- More information is carried by the message which is less likely one or rare. The message associated with an event least likely to occur contains more information.
- The total amount of self information conveyed should be sum of individual information

# BASIC LOGARITHMIC FORMULAE:

1. If 'n' and 'a' are positive real numbers, and $a \neq 1$, then if $a^x = n$, then $\log_a n = x$.
   (definition of logarithm)
2. log of 1 to any base is zero.
   i.e., $\log_b 1 = 0$
3. log of any number to the base as itself is 1.
   i.e., $\log_a a = 1$.
4. $\log_a (pq) = \log_a p + \log_a q$
5. $\log_a (p/q) = \log_a p - \log_a q$
6. $\log_a p^n = n \log_a p$
7. $a^{(\log_a p)} = p$
8. $\log_a n = \log_b n \times \log_a b$
9. $\log_a b = \dfrac{\log_n b}{\log_n a}$
10. $\dfrac{1}{\log_b a} = \log_a b$

# Problem -1

The binary symbols 0's and 1's are transmitted with probabilities 1/4 and 3/4 respectively. Calculate the amount of information.

The binary symbols 0's and 1's are transmitted with probabilities 1/4 and 3/4 respectively. Calculate the amount of information.

$$P(0) = P_0 = 1/4 \quad ; \quad P(1) = P_1 = 3/4$$

$$I_0 = \log_2\left(\frac{1}{P_0}\right) = \log_2\left(1/\tfrac{1}{4}\right) = \log_2 4 = 2$$

$\therefore \underline{\underline{I_0 = 2 \text{ bits}}}$

$$I_1 = \log_2\left(\frac{1}{P_1}\right) = \log_2\left(\frac{1}{3/4}\right) = \log_2\left(\frac{4}{3}\right) = 0.415$$

$\therefore I_1 = \underline{\underline{0.415 \text{ bits}}}$

# Relation between Bits, Hartleys and Nats

Consider 3 equations given by

$$I = \log_{10}(1/p) \quad \text{Hartley} \quad \rightsquigarrow (1)$$

$$I = \log_{e}(1/p) \quad \text{Nats} \quad \rightsquigarrow (2)$$

$$I = \log_{2}(1/p) \quad \text{Bits} \quad \rightsquigarrow (3)$$

From eq$^n$ (1) and (2),

$$\log_{10}(1/p) \cdot \text{Hartley} = \log_e(1/p) \cdot \text{Nats}$$

$$\therefore 1 \text{ Hartley} = \frac{\log_e(1/p)}{\log_{10}(1/p)} \cdot \text{Nats} = \frac{-\log_e p}{-\log_{10} p} = \frac{\log_p 10}{\log_p e}$$

$$1 \text{ Hartley} = \log_e 10 \text{ Nats} \quad \Big| \because \frac{1}{\log_b a} = \log_a b$$

$$\boxed{\begin{array}{l} 1 \text{ Hartley} = 2.303 \text{ Nats} \\ 1 \text{ Nat} = \frac{1}{2.303} \text{ Hartley} = 0.434 \text{ Hartley} \end{array}}$$

From (1) and (3),

$$\log_{10}(1/p) \text{ Hartley} = \log_2(1/p) \text{ Bits}$$

$$\therefore \text{Hartley} = \frac{\log_2(1/p)}{\log_{10}(1/p)} \text{ Bits}$$

$$= \log_2 10$$

$$= 3.32 \text{ bits}$$

$$\boxed{\begin{array}{ll} 1 \text{ Hartley} & = 3.32 \text{ bits} \\ 1 \text{ bit} & = \frac{1}{3.32} \text{ Hartley} \\ 1 \text{ Bit} & = 0.301 \text{ Hartley} \end{array}}$$

From (2) and (3),

$$\log_e (1/p) \text{ Nats} = \log_2 (1/p) \text{ Bits}$$

$$1 \text{ Nat} = \frac{\log_2 (1/p)}{\log_e (1/p)} \text{ Bits}$$

$$\boxed{\begin{aligned} 1 \text{ Nat} &= \log_2 e = 1.443 \text{ bits} \\ 1 \text{ bit} &= 1/1.443 = 0.693 \text{ Nats} \end{aligned}}$$

# Average Information content(Entropy) of symbols in long independent sequences

Consider message Length N

- $p_1N$ number of messages of type $s_1$ contains $p_1 N \log_2\left(\frac{1}{p_1}\right)$bits
- $p_2N$ number of messages of type $s_2$ contains $p_2 N \log_2\left(\frac{1}{p_2}\right)$bits.
- .
- .
- $I_{total} = p_1 N \log_2\left(\frac{1}{p_1}\right) + p_2 N \log_2\left(\frac{1}{p_2}\right) + \cdots \ldots + p_q N \log_2\left(\frac{1}{p_q}\right)$

$$\boldsymbol{I_{total} = N \sum_{i=1}^{q} p_i \, log_2\left(\frac{1}{p_i}\right)} \textbf{ bits}$$

- The average information *per* symbol is obtained by dividing the total information content of the message by the number of symbols in the message

$$Entropy = H(s) = \frac{I_{total}}{N} = \sum_{i=1}^{q} p_i \log_2\left(\frac{1}{p_i}\right) \text{ bits/symbol}$$

The average information content per symbol is called the source entropy

## Average information rate:

If the symbols are emitted by source at a fixed time rate $r_{s}$, then the average information rate $R_s$ is given by

$$\boldsymbol{R_s} = \boldsymbol{r_s} * \boldsymbol{H}(\mathrm{s}) \text{ bits/sec}$$

bits/sec Symbol/sec bits/symbol

# Problem 2

Consider a source S = {S1, S2, S3} with probabilities {0.5, 0.25, 0.25}. Find

i) Self information of each message
ii) Entropy

# Problem 2

Sol$^n$: i) $I_1 = \log_2\left(\frac{1}{P_1}\right) = \log_2\left(\frac{1}{0.5}\right) = \underline{\underline{1 \text{ bit}}}$

$I_2 = \log_2\left(\frac{1}{P_2}\right) = \log_2\left(\frac{1}{0.25}\right) = \underline{\underline{2 \text{ bits}}}$

$I_3 = \log_2\left(\frac{1}{P_3}\right) = \log_2\left(\frac{1}{0.25}\right) = \underline{\underline{2 \text{ bits}}}$

ii) Entropy $= H(s) = \sum_{i=1}^{q} P_i \log_2\left(\frac{1}{P_i}\right)$

$= 0.5 \log_2\left(\frac{1}{0.5}\right) + 0.25 \log_2\left(\frac{1}{0.25}\right) + 0.25 \log_2(4)$

$= 0.5 \times 1 + 0.25 \times 2 + 0.25 \times 2$

$H(s) = \underline{\underline{1.5}}$ bits / symbol

# Problem 3

Q> The collector voltage of a circuit is to lie between -5 and -12. The voltage can take on only these values -5, -6, -7, -9, -10, -11, -12 volts with respective probabilities $\{ 1/6, 2/6, 1/12, 1/12, 1/12, 1/6, 1/12 \}$. This voltage is recorded with a pen recorder. Determine the average self info associated with the record in terms of bits/level.

# Solution 3

$$H(S) = \sum_{i=1}^{q} P_i \log_2\left(\frac{1}{P_i}\right) = \sum_{i=1}^{7} P_i \log_2\left(\frac{1}{P_i}\right)$$

$$H(S) = \frac{1}{6}\log_2(6) + \frac{2}{6}\log_2\left(\frac{6}{2}\right) + \frac{1}{12}\log_2(12) + \frac{1}{12}\log_2(12) + \frac{1}{12}\log_2(12) + \frac{1}{6}\log_2(6) + \frac{1}{12}\log_2(12)$$

$$= \frac{2}{6}\times 2.585 + \frac{4}{12}\times 3.585 + \frac{2}{6}\times 1.585$$

$$= 2.585$$

$$H(S) = \underline{2.585} \text{ bits/level}$$

# Problem 4

Q5> The output of an information source consists of 128 bits, 16 of which occur with probability 1/32 and remaining 112 occur with probability 1/224. The source emits 1000 symbols/sec. Assuming that symbols are chosen independently, find the average information rate of this source.

# Solution 4

Sol$^n$ :

$$H(S) = \sum_{i=1}^{128} P_i \log\left(\frac{1}{P_i}\right)$$

$$= 16 \times \frac{1}{32} \log_2(32) + 112 \times \frac{1}{224} \log_2(224)$$

$$= \frac{1}{2} \times 5 + \frac{1}{2} \times 7.807$$

$$H(S) = \underline{6.4035} \text{ bits/symbol}$$

Given $r_s = 1000$ symbols/sec

$$\therefore R_s = 1000 \times 6.4035$$

$$R_s = \underline{6403.5} \text{ bps}$$

**A card is drawn from a deck of playing cards.**

**i) You are informed that the card you draw is spade. How much information did you receive in bits?**

**ii) How much information did you receive if you are told that the card you drew is an ace?**

**iii) How much information did you receive if you are told that the card you drew is an ace of spades?**

**iv) Is the information content of the message “ace of spades” the sum of the information contents of the messages ”spade” and “ace”?**

## Basic concept on drawing a card:

In a pack or deck of 52 playing cards, they are divided into 4 suits of 13 cards each i.e. spades ♠ hearts ♥, diamonds ♦, clubs ♣.

Cards of Spades and clubs are black cards.

Cards of hearts and diamonds are red cards.

The card in each suit, are ace, king, queen, jack or knaves, 10, 9, 8, 7, 6, 5, 4, 3 and 2.

King, Queen and Jack (or Knaves) are face cards. So, there are 12 face cards in the deck of 52 playing cards.

Sol$^n$: In a deck, there are 13 cards of spade and 4 aces.

In total, there are 52 cards.

$\therefore$ Probability of getting a spade $= \frac{13}{52} = \frac{1}{4}$ (Let it be event A)

Probability of getting an ace $= \frac{4}{52} = \frac{1}{13}$ (Let it be event B)

Probability of getting an ace of spade $= \frac{1}{52}$ (event c)

Here event 'c' has the least probability of occurance followed by B and A. Let us find the information content of each event.

a) $I_A = \log_2 \left(\frac{1}{P_A}\right) = \log_2 (4) = \underline{\underline{2 \text{ bits}}} \longrightarrow (1)$

b) $I_B = \log_2 \left(\frac{1}{P_B}\right) = \log_2 (13) = \underline{\underline{3.7 \text{ bits}}} \longrightarrow (2)$

c) $I_c = \log_2 \left(\frac{1}{P_c}\right) = \log_2 (52) = \underline{\underline{5.7 \text{ bits}}} \longrightarrow (3)$

From results (1), (2) and (3), we can infer that the message with least probability of occurance contains more information.

Also, d) $I_c = I_A + I_B$

$\therefore \quad I_{\text{Ace of spade}} = I_{\text{spade}} + I_{\text{ace}}$

# Problem 6

Q5 The international Morse code uses a sequence of dot & dash to transmit the letters of english alphabet. The dash is represented by a current pulse that has a duration of 3 units and the dot has a duration of 1 unit. The probability of occurance of a dash is 1/3 of the probability of occurance of a dot.

a) Calculate information content of a dot and dash.

b) Calculate average information in dot-dash code.

c) Assume that the dot last for 10 msec which is the same time interval as the pause between the symbols. Find

**the average rate of information of transmission.**

average rate of information in transmission

<u>Sol$^n$</u>: Wkt $P_{dot} + P_{dash} = 1 \longrightarrow (1)$

Given $\frac{1}{3} P_{dot} = P_{dash} \implies P_{dot} = 3 P_{dash}$

$$\therefore P_{dot} + \frac{1}{3} P_{dot} = 1$$

$$P_{dot} = \frac{3}{4} = 0.75 \quad \text{and} \quad P_{dash} = 1 - 0.75 = 0.25$$

a> $I_{dot} = \log_2\left(\frac{1}{P_{dot}}\right) = \log_2\left(\frac{4}{3}\right) = \underline{\underline{0.415 \text{ bits}}}$

$I_{dash} = \log_2\left(\frac{1}{P_{dash}}\right) = \log_2(4) = \underline{\underline{2 \text{ bits}}}$

b> $H(S) = \sum_{i=1}^{2} P_i \log_2\left(\frac{1}{P_i}\right)$

$$= \frac{3}{4} \log_2\left(\frac{4}{3}\right) + \frac{1}{4} \log_2(4)$$

$$= \underline{\underline{0.81125}} \text{ bits/symbol}$$

c> In every 4 symbols, 3 will be dots ($P_{dot} = 3/4$) and 1 will be dash ($P_{dash} = 1/4$)

1m · 1m 1m · 1m 1m · 1m 3m 1m } 10m

$\therefore$ Totally 4 symbols will occur in 10 m sec

$$r_s = \frac{4}{10m} = 400 \text{ symbols/sec}$$

$$R_s = r_s H(s) = 400 \times 0.81125$$

$$R_s = \underline{\underline{324.5}} \text{ bits/sec}$$

## PROPERTIES OF ENTROPY:

1. Entropy is average self information. It is given by

$$H(S) = \sum_{i=1}^{q} P_i \log (1/P_i)$$

$$H(S) = \sum_{i=1}^{q} P_i I_i \quad \text{bits/symbol}$$

where $q$ represents number of symbols and also $\sum_{i=1}^{q} P_i = 1$.

2. For null event and sure event, the entropy vanishes.
3. The entropy is a symmetrical function of its arguments. The value of $H(S)$ remains the same irrespective of location of probabilities.

4. Extremal property: When all the source symbols become equiprobable, then the entropy attains the maximum value.

$$H(s)_{max} = \log_2 q.$$

5. When source symbols are not equiprobable, then entropy is less than maximum value.

6. The source efficiency, $\eta_s$ is given by

$$\eta_s = \frac{H(S)}{H(S)_{max}}$$

7. The source redundancy $R_{\eta_s}$ is given by

$$R_{\eta_s} = 1 - \eta_s$$

Usually efficiency and redundancy are represented in percentage.

Q> A binary source is emitting an independent sequence of 0's and 1's with probabilities P and (1-P) respectively. Plot the entropy of the source versus P.

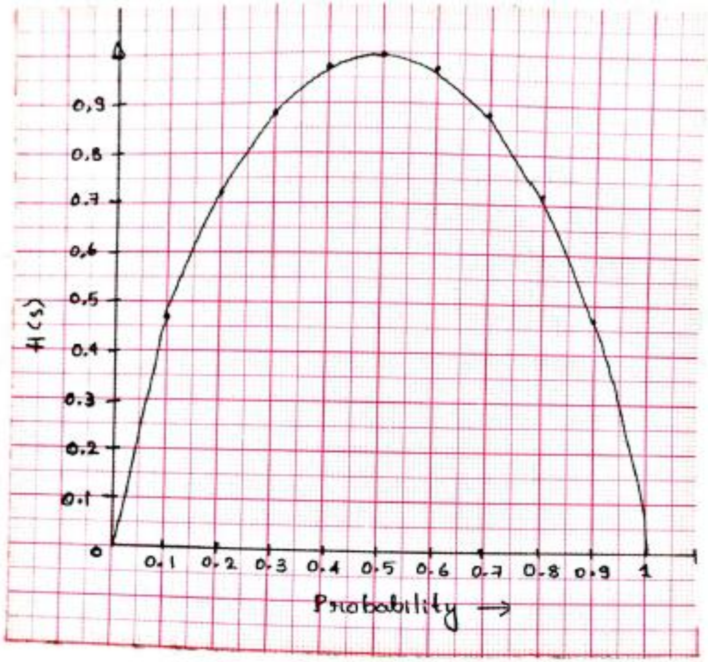
Sol$^n$: The entropy of the binary source is given by

$$H(S) = \sum_{i=1}^{2} P_i \log_2 \left(\frac{1}{P_i}\right)$$

$$H(S) = P \log_2 \left(\frac{1}{P}\right) + (1-P) \log_2 \left(\frac{1}{1-P}\right) \rightarrow (1)$$

Type eq$^n$ (1) in calculator and 'calculate' the values of H(s) for assumed values of P. P should vary from 0.1 to 1

| P | H(s) |
|---|---|
| 0.1 | 0.469 |
| 0.2 | 0.722 |
| 0.3 | 0.881 |
| 0.4 | 0.971 |
| 0.5 | 1 |
| 0.6 | 0.971 |
| 0.7 | 0.881 |
| 0.8 | 0.722 |
| 0.9 | 0.469 |
| 1.0 | — |



A discrete message source 's' emits two independent symbols x and y with probability 0.55 and 0.45 respectively. Calculate the efficiency of the source and its redundancy.

Sol$^n$: $P(X) = P_x = 0.55$ ; $P(y) = P_y = 0.45$

$$H(S) = \sum_{i=1}^{2} P_i \log 1/P_i$$

$$= 0.55 \log \left(\frac{1}{0.55}\right) + 0.45 \log \left(\frac{1}{0.45}\right)$$

$$H(S) = 0.9928 \text{ bits/message symbol}$$

$$H(S)_{max} = \log_2 q \qquad | \text{ Here } q = 2$$

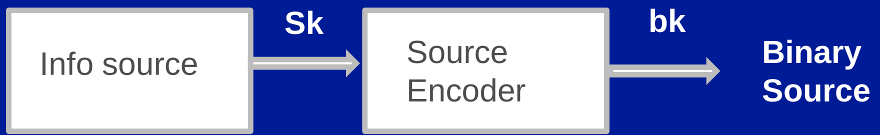
$$\therefore H(S)_{max} = \log_2 2 = 1$$

$$\eta_s = \frac{H(S)}{H(S)_{max}} = \frac{0.9928}{1} = \underline{\underline{0.9928}} \;/\; 99.28\%$$

$$R_{\eta_s} = 1 - \eta_s = \underline{\underline{0.0072}} \;/\; 0.72\%$$

# Source Encoding

- Output of an information source is converted in to an r-array sequence.




- Coding: transformation of each of the source symbol S=$\{s_{1,}s_{2,}s_3 \dots \dots \dots s_q\}$ where q-number of source
- using the symbols from the code alphabet
- X={$q_{1,}q_{2,}q_3 \dots \dots \dots q_r$) where r- number of coding symbols

- In binary X=(0,1).
- Ternary X=(0, 1 2)
- Qudranary X=(0, 1, 2 , 3)

- Transmission process is easier and Efficiency of the system can be increased.

# Shannon's Encoding Algorithm

- Let the source symbols in the order of decreasing probabilities

  **S={**$s_{1,}s_{2,}s_3 \dots \dots . . s_q$**)**

  **P={**$p_{1,}p_{2,}p_3 \dots \dots . . p_q$**)**

  $p_1 \geq p_2 \geq p_3 \dots \dots . \geq p_q$

- Compute the sequence

  $\alpha_1$**=0**

  $\alpha_2$**=**$p_1 = p_{1+}\alpha_1$

  $\alpha_3 = p_2 + p_1 = p_2 + \alpha_2$

  $\alpha_4 = p_3 + p_2 + p_1 = p_3 + \alpha_3$

  $\alpha_{q+1} = p_q + \alpha_q = 1$

- Determine the smallest integer for $l_i$ (length of code word) using the inequality

$$2^{l^i} \geq \frac{1}{p_i} \qquad \textbf{for all i=1 to q}$$

- Expand the decimal numbers $\alpha_i$ in binary form up to $l_i$ places neglecting the expansion beyond $l_i$ places.
- Remove the decimal point to get the desired code.

- **Code efficiency**
- The average length 'L' of any code is given by
- $\boldsymbol{L = \sum_{i=1}^{q} p_i l_i}$ binits/symbol
- where $l_i = l_1 l_2 l_3 l_4 \ldots \ldots . l_q$ is respective word length in expressed in binits
- Code efficiency, $\boldsymbol{\eta_c = \frac{H(S)}{L} * 100}$

- **Ex: 1**. Construct the Shannon’s binary code for the following message symbols S=($s_{1,}s_{2,}s_3, s_4$) with probabilities P=(0.4, 0.1,0.2,0.3).
- Solution:
- ***0.4 > 0.3 > 0.2 > 0.1***

| $S_i$ | $P_i$ | $\alpha_i$ | $l_i$ | binary | Code |
|---|---|---|---|---|---|
| $S_1$ | 0.4 | 0 | 2 | $(0.00000\cdots)_2$ | 00 |
| $S_2$ | 0.3 | 0.4 | 2 | $(0.01100\cdots)_2$ | 01 |
| $S_3$ | 0.2 | 0.7 | 3 | $(0.101100\cdots)_2$ | 101 |
| $S_4$ | 0.1 | 0.9 | 4. | $(0.11100\cdots)_2$ | 1110 |

$$2^{-l_1} \le 0.4 \rightarrow l_1 = 2$$
$$2^{-l_2} \le 0.3 \rightarrow l_2 = 2$$
$$2^{-l_3} \le 0.2 \rightarrow l_3 = 3$$
$$2^{-l_4} \le 0.1 \rightarrow l_4 = 4$$

$0.4\times2$
$0.8\times2 \rightarrow 0$
$1.6 \rightarrow 1$
$0.6\times2$
$1.2 \rightarrow 1$
$0.2\times2$
$0.4 \rightarrow 0$
$\Rightarrow 0.0110\cdots$

$0.7\times2$
$1.4 \rightarrow 1$
$0.4\times2$
$0.8\times2 \rightarrow 0$
$1.6 \rightarrow 1$
$0.6\times2 \rightarrow 1$
$1.2$
$0.2\times2$
$0.4\times2 \rightarrow 0$
$0.8 \rightarrow 0$
$\Rightarrow 0.101100\cdots$

$0.9\times2$
$1.8 \rightarrow 1$
$0.8\times2$
$1.6 \rightarrow 1$
$0.6\times2 \rightarrow 1$
$1.2$
$0.2\times2 \rightarrow 0$
$0.4\times2$
$0.8\times2 \rightarrow 0$
$\Rightarrow 0.11100\cdots$

The codes are ***S1***: ***00, s2***: ***01, s3***: ***101, s4***: ***1110***

- The average length of this code is $L = \sum_{i=1}^{q} P_i l_i$

$$L = 2 \times 0.4 + 2 \times 0.3 + 3 \times 0.2 + 3 \times 0.1 = 2.4 \text{ Binits / message}$$

$$H(S) = 0.4 \log \frac{1}{0.4} + 0.3 \log \frac{1}{0.3} + 0.2 \log \frac{1}{0.2} + 0.1 \log \frac{1}{0.1} = 1.84644 \text{ bits / message;}$$

- $\%\eta_c = \frac{H(S)}{L} * 100 = \frac{1.8464}{2.4} * 100 = 76.93\%$

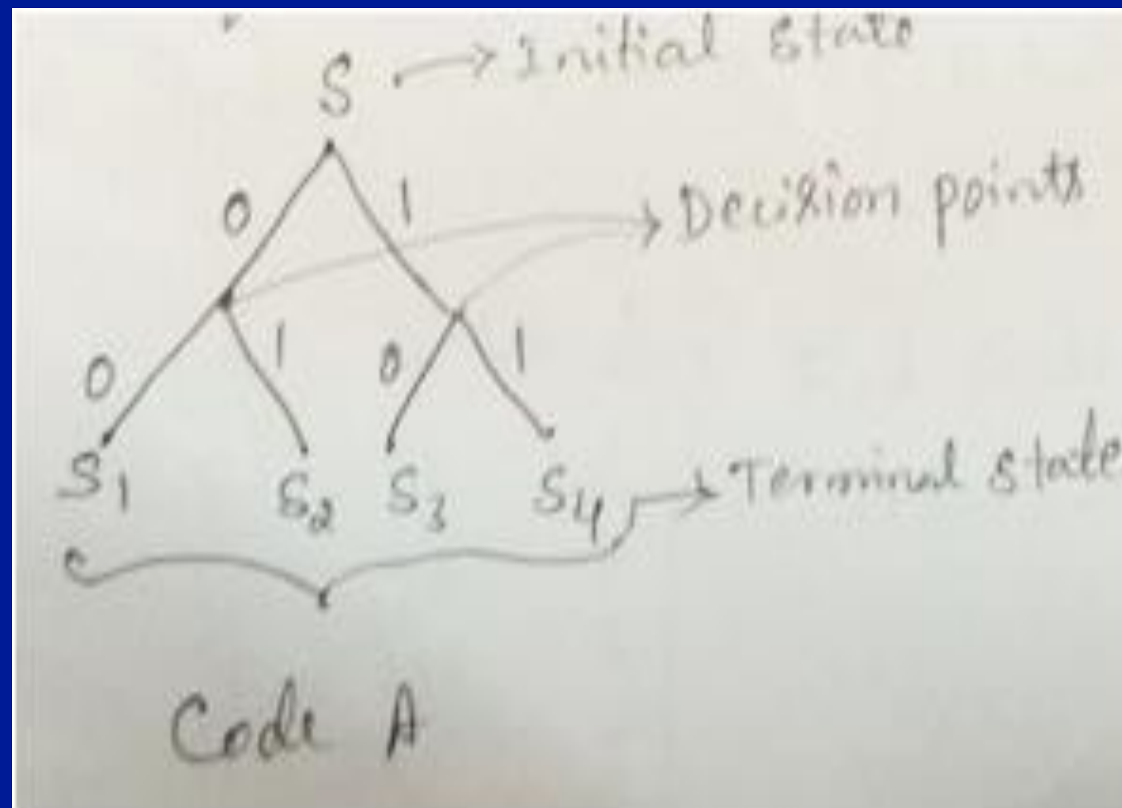
- Apply Shannon’s binary encoding procedure to the following set of messages and obtain code efficiency and redundancy. 1/8, 1/16, 3/16, 1/4, 3/8
- S={ S1, S2, S3, S4, S5}

| $S_i$ | $P_i$ | $\alpha_i$ | $l_i$ | binary | Code |
|---|---|---|---|---|---|
| $S_1$ | 3/8 | 0 | 2 | 0.00 | 00 |
| $S_2$ | 1/4 | 0.375 | 2 | 0.01 | 01 |
| $S_3$ | 3/16 | 0.625 | 3 | 0.101 | 101 |
| $S_4$ | 1/8 | 0.8125 | 3 | 0.110 | 110 |
| $S_5$ | 1/16 | 0.9375 | 4 | 0.1111 | 1111 |

- $H(S) = \frac{1}{4}log_2 4 + \frac{3}{8}log_2 \frac{8}{3} + \frac{1}{8}log_2 8 + \frac{3}{16}log_2 \frac{16}{3} + \frac{1}{4}log_2 4 + \frac{1}{16}log_2 16$
- $H(S) = 2.1085\ bits/symbol$
- $L = \sum_{i=1}^{q} P_i l_i = \frac{1}{4}(2) + \frac{3}{8}(2) + \frac{1}{8}(3) + \frac{3}{16}(3) + \frac{1}{16}(4)$
- $L = 2.4375\ bits/symbol$
- $\%\eta = \frac{H(S)}{L} * 100 = 86.5\%$
- Redundancy=1- $\eta$=100-86.5=13.5%

# Code Tree

| Source Symbol | A |
|---|---|
| $S_1$ | 00 |
| $S_2$ | 01 |
| $S_3$ | 10 |
| $S_4$ | 11 |



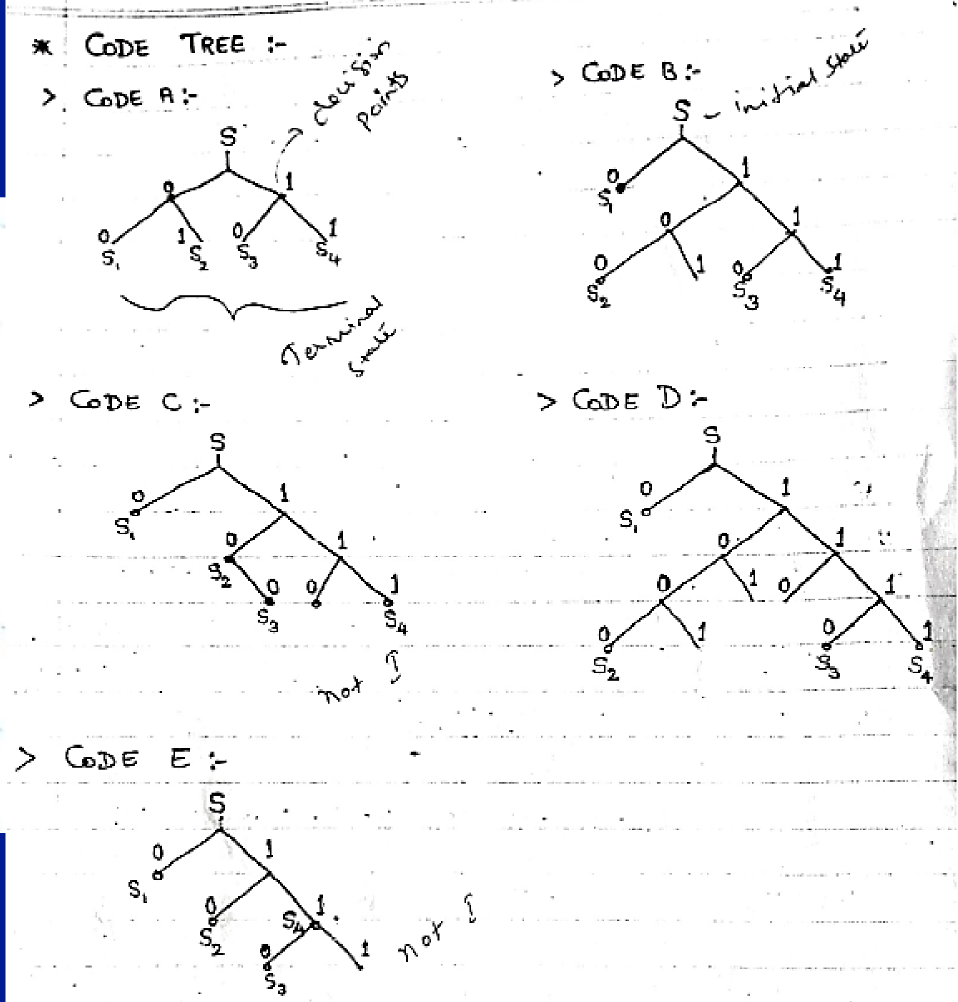
Consider a set of 5 codes as shown

| Source Symbol | A | B | C | D | E |
|---|---|---|---|---|---|
| $S_1$ | 00 | 0 | 0 | 0 | 0 |
| $S_2$ | 01 | 100 | 10 | 1000 | 10 |
| $S_3$ | 10 | 110 | 100 | 1110 | 110 |
| $S_4$ | 11 | 111 | 111 | 1111 | 11 |

# Code Tree

Consider a set of 5 codes as shown

| Source Symbol | A | B | C | D | E |
|---|---|---|---|---|---|
| $S_1$ | 00 | 0 | 0 | 0 | 0 |
| $S_2$ | 01 | 100 | 10 | 1000 | 10 |
| $S_3$ | 10 | 110 | 100 | 1110 | 110 |
| $S_4$ | 11 | 111 | 111 | 1111 | 11 |




NOTE :- If a decision point represents a code, then that code is not instantaneous.

# Huffman Coding

- The source symbols are listed in the decreasing order of probabilities.
- Check

  if $q = r + a(r-1)$ is satisfied and

  find the integer 'a',

  q is number of source symbols and

  r is number of symbols used in code alphabets.
- If 'a' is not integer, add dummy symbols of zero probability of occurrence.
- Combine the last 'r' symbols into a single composite symbol by adding their probability to get a reduced source.
- Repeat the above three steps, until in the final step exactly ***r***- symbols are left.

- The last source with ‘r’ symbols are encoded with ‘r’ different codes 0,1,2,3,….r-1
- In binary coding the last source are encoded with 0 and 1
- As we pass from source to source working backward, decomposition of one code word each time is done in order to form 2 new code words.
- This procedure is repeated till we assign the code words to all the source symbols of alphabet of source ‘s’ discarding the dummy symbols.

- Construct a Huffman Code for symbols having probabilities $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right\}$. Also find efficiency and redundancy.
- Solution:

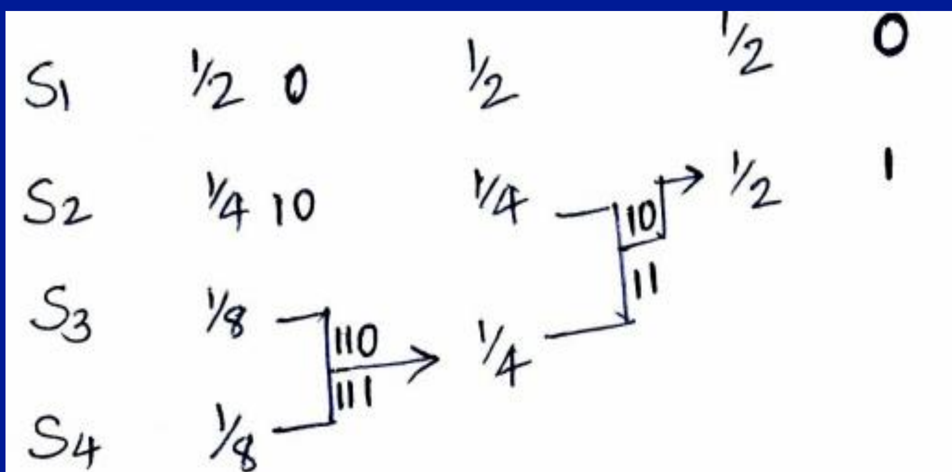
$$q = r + \boldsymbol{a}(r-1)$$

**4=2+a(1) => a=2**

- Construct a Huffman Code for symbols having probabilities $\left\{\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{8}\right\}$. Also find efficiency and redundancy.
- Solution:

$$q = r + \alpha(r-1)$$

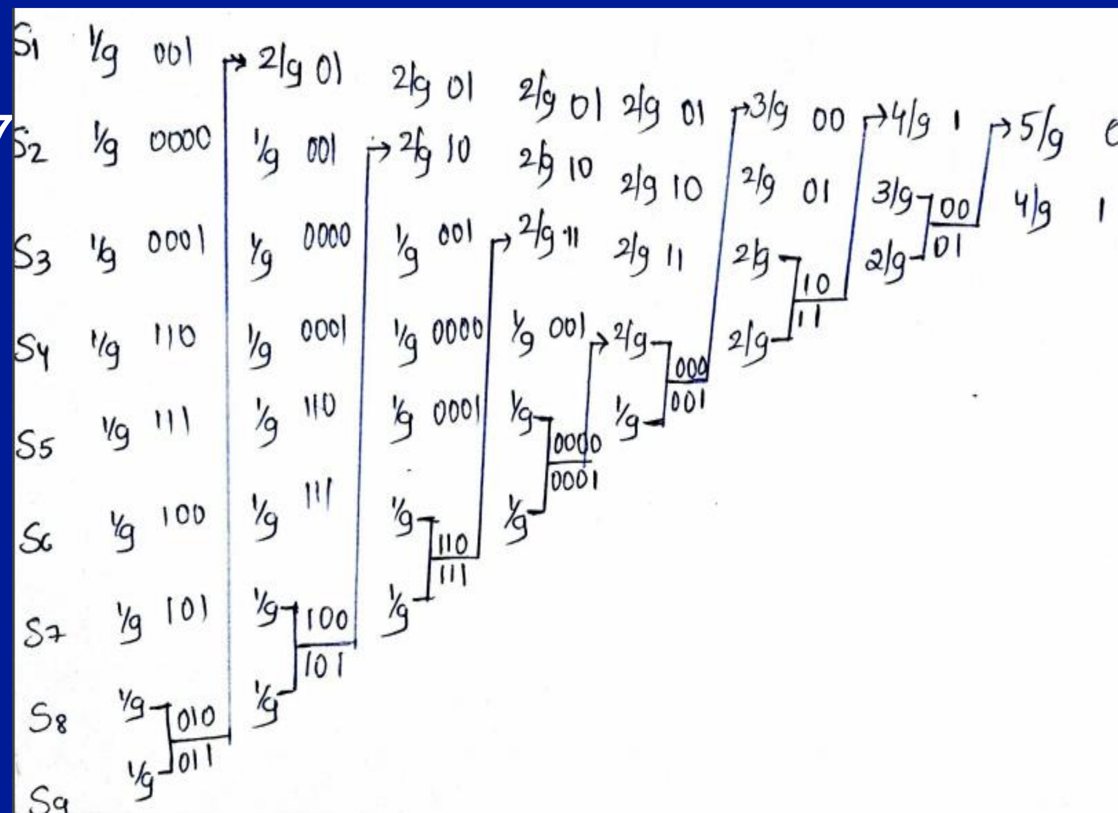
**4=2+α(1) => α=2**




| Symbols | Codes | Probabilities | Length |
|---|---|---|---|
| $S_1$ | 0 | ½ | 1 |
| $S_2$ | 10 | ¼ | 2 |
| $S_3$ | 110 | 1/8 | 3 |
| $S_4$ | 111 | 1/8 | 3 |

- $H(S) = \frac{1}{2} log_2 2 + \frac{2}{8} log_2 8 + \frac{1}{4} log_2 4$
- $H(S) = 1.75\ bits/symbol$
- $L = \sum_{i=1}^{4} P_i L_i = \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{8}(3)$
- $L = 1.75\ bits/symbol$
- $\%\eta = \frac{H(S)}{L} * 100 = 100\%$
- Redundancy=0%

- Ex.2: A source has 9 symbols and each occur with a probability of 1/9. Construct a binary Huffman code. Find efficiency and redundancy of coding.
- Solution:
- $q = r + \alpha(r-1)$
- $9=2+\alpha(1) \quad => \quad \alpha=7$



| Symbols | Codes | Probabilities | Length |
| --- | --- | --- | --- |
| $S_1$ | 001 | 1/9 | 3 |
| $S_2$ | 0000 | 1/9 | 4 |
| $S_3$ | 0001 | 1/9 | 4 |
| $S_4$ | 110 | 1/9 | 3 |
| $S_5$ | 111 | 1/9 | 3 |
| $S_6$ | 100 | 1/9 | 3 |
| $S_7$ | 101 | 1/9 | 3 |
| $S_8$ | 010 | 1/9 | 3 |
| $S_9$ | 011 | 1/9 | 3 |

- $H(S) = \frac{9}{9} log_2 9$
- $H(S) = 3.17\ bits/symbol$
- $L = \sum_{i=1}^{9} P_i L_i = \frac{1}{9}(3+4+4+3+3+3+3+3+3)$
- $L = 3.22\ bits/symbol$
- $\%\eta = \frac{H(S)}{L} * 100 = 98.45\%$
- Redundancy=100-% $\eta$=1.55%

- Ternary $\%\eta = \frac{H3(S)}{L} * 100$
- $H3(S)$= H(s)/ log3

- Ex. 3:

Given the messages $x_1, x_2, x_3, x_4, x_5$ &$x_6$ with probabilities 0.4, 0.2, 0.2, 0.1, 0.07, 0.03. Construct binary and trinary code by applying Huffman encoding procedure. Also find efficiency and redundancy.

- Solution:
- Binary

$$q = r + \alpha(r - 1)$$

$$6 = 2 + \alpha(1) \Rightarrow \alpha = 4$$

- Ex. 3: Given the messages $x_1, x_2, x_3, x_4, x_5$ & $x_6$ with probabilities 0.4, 0.2, 0.2, 0.1, 0.07, 0.03. Construct binary and trinary code by applying Huffman encoding procedure. Also find efficiency and redundancy.
- Solution:

$$q = r + \alpha(r-1)$$

$$6 = 2 + \alpha(1) \Rightarrow \alpha = 4 \in \mathbb{Z}$$

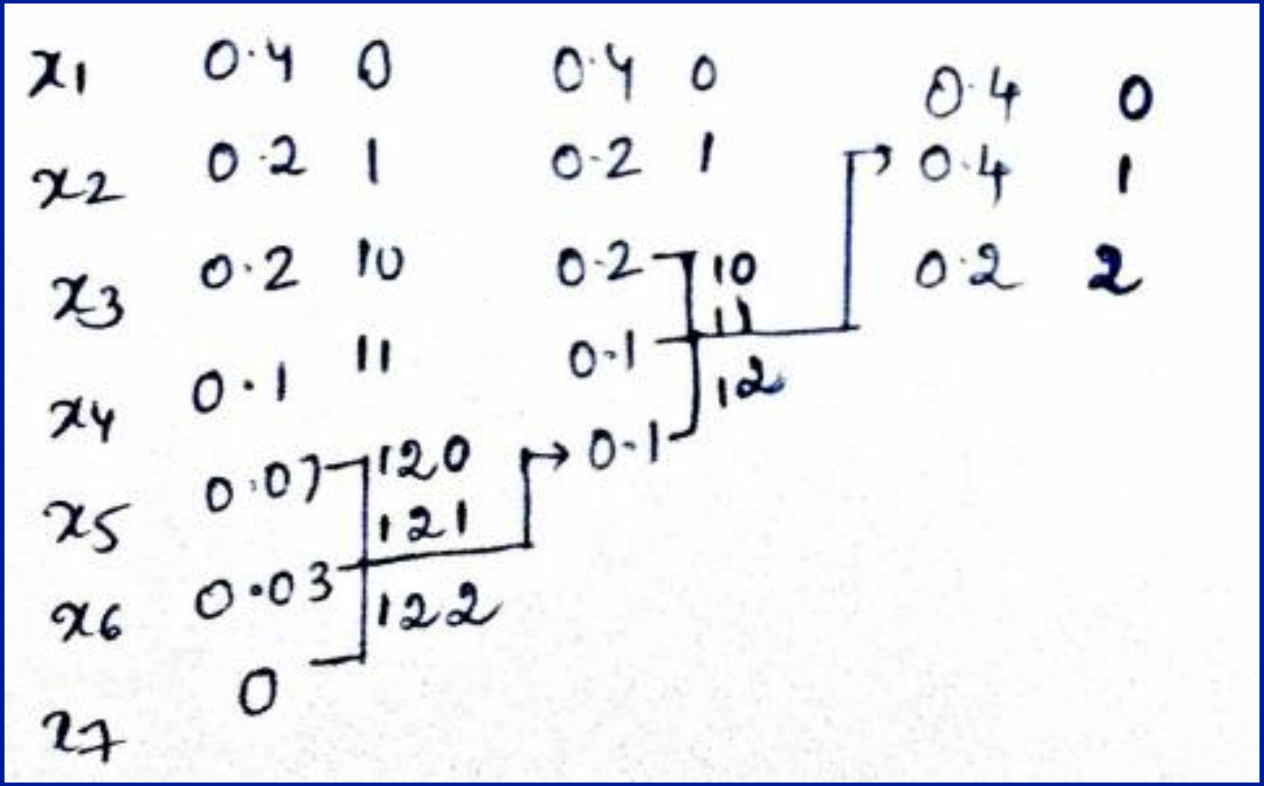
| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 0.4 | 1 | 0.4 | 1 | 0.4 | 1 | 0.4 | 1 | 0.6 | 0 |
| $x_2$ | 0.2 | 01 | 0.2 | 01 | 0.2 | 01 | 0.4 | 00 | 0.4 | 1 |
| $x_3$ | 0.2 | 000 | 0.2 | 000 | 0.2 | 000 | 0.2 | 01 | | |
| $x_4$ | 0.1 | 0010 | 0.1 | 0010 | 0.2 | 001 | | | | |
| $x_5$ | 0.07 | 00110 | 0.1 | 0011 | | | | | | |
| $x_6$ | 0.03 | 00111 | | | | | | | | |

Trinary: $q = r + \alpha(r-1)$

6=3+α(2) => α=3/2 ;

Let α=2, If α=2 => $q = 3 + 2(2)$= 7

Hence add a symbol $x_7$ with probability ‘0’.




| Symbols | Codes | Probabilities | Len |
|---|---|---|---|
| $x_1$ | 0 | 0.4 | 1 |
| $x_2$ | 1 | 0.2 | 1 |
| $x_3$ | 10 | 0.2 | 2 |
| $x_4$ | 11 | 0.1 | 2 |
| $x_5$ | 120 | 0.07 | 3 |
| $x_6$ | 121 | 0.03 | 3 |

# Problem

Consider a Zero memory source with S=[S1,S2,S3,S4,S5,S6,S7] and Probabilities P=[0.4,0.2,0.1,0.1,0.1,0.05,0.05]

i. Construct a binary Huffman code by placing the composite symbol as low as possible.

ii. Repeat (i) by moving a composite symbol as high as possible.

iii. In each of the cases (i) and (ii) above,

- Compute the variances of the word lengths and comment on the result.
- Find Efficiency and Redundancy.

iv. Considering Case(ii) table,

- Write the code tree and decode the message 01110110011000100….
- Determine probabilities of 0’s and 1’s.

***Tips:*** Variance= $\sum_{i=1}^{i=q} P_i \, (l_i - L)^2$

Probability of 0’s : P(0)= $\frac{1}{L}\sum_{i=1}^{i=q}(No. \, of \, 0's \text{ in } the \;\; code \; for \; Si) \;\; P_i$

As low as possible.

| | | | | | | |
|---|---|---|---|---|---|---|
| $S_1$ | 0.4 | 1 | 0.4 | as low as possible | $S_1$ | 1 |
| $S_2$ | 0.2 | 01 | 0.2 | 1 | $S_2$ | 01 |
| $S_3$ | 0.1 | 0010 | 0.1 | 0 | $S_3$ | 0010 |
| $S_4$ | 0.1 | 0011 | 0.1 | 0 | $S_4$ | 0011 |
| $S_5$ | 0.1 | 0000 | 0.1 | 1 | $S_5$ | 0000 |
| $S_6$ | 0.05 | 00010 | 0.1 | 0001 | $S_6$ | 00010 |
| $S_7$ | 0.05 | 00011 | | | $S_7$ | 00011 |

$H(S) = 2.421$

$L = 2.5$

Variance = ~~2.45~~ 2.25

Efficiency $\eta = 96.84\%$

Redundancy $R_\eta = 3.16\%$

As high.

(ii) as high as possible

| | |
|---|---|
| $S_1$ | 00 |
| $S_2$ | 11 |
| $S_3$ | 011 |
| $S_4$ | 100 |
| $S_5$ | 101 |
| $S_6$ | 0100 |
| $S_7$ | 0101 |

$H(S) = 2.421$

$L = 2.5$

Variance $= 0.45$

Probability of 0's $= 0.58$

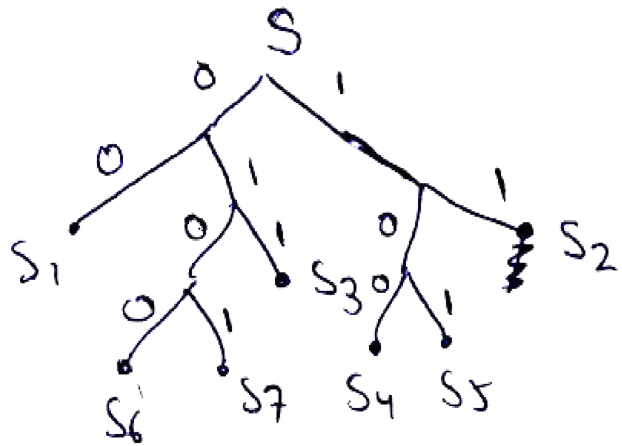
Probability of 1's $= 0.42$

Efficiency $\eta = 96.84\%$

Redundancy $R_\eta = 3.16\%$

0101

Code tree using code in case (ii)




Sequence $\Rightarrow$

$$\underbrace{011}_{S_3}\underbrace{101}_{S_5}\underbrace{100}_{S_4}\underbrace{11}_{S_2}\underbrace{00}_{S_1}\underbrace{0100}_{S_6}$$

Consider a source with 8 alphabet A to H with respective probabilities of 0.22, 0.20, 0.18, 0.15, 0.10, 0.08, 0.05, 0.02.

i] Construct a binary compact code and determine the code efficiency.

ii] Construct a ternary compact code and determine efficiency of the code

iii] Construct a quaternary compact code and determine the code efficiency.

Compare and comment on the result. Draw code trees for all three cases.

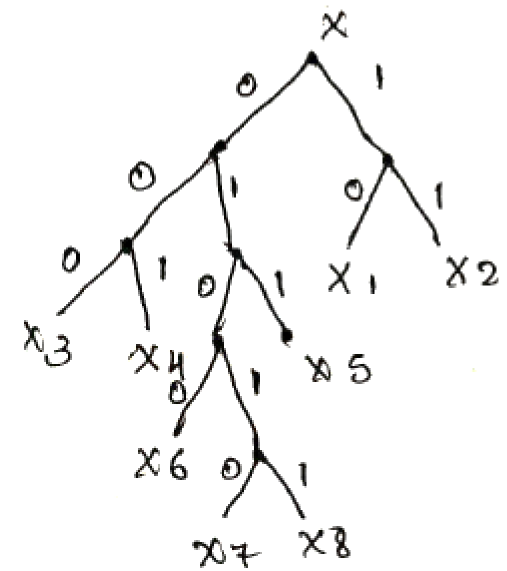
iv] Decode the messages using appropriate code trees

a) 0101001000001101011001…

b) 1211101102001200 2 ….

c) 031132020300100231 …..

| ymbol | Prob | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 0.22 | 10 | 0.22 | 10 | 0.22 | 10 | 0.25 | 01 | 0.33 | 00 | 0.42 | 1 | 0.58 | 0 |
| $x_2$ | 0.20 | 11 | 0.20 | 11 | 0.20 | 11 | 0.22 | 10 | 0.25 | 01 | 0.33 | 00 | 0.42 | 1 |
| $x_3$ | 0.18 | 000 | 0.18 | 000 | 0.18 | 000 | 0.20 | 11 | 0.22 | 10 | 0.25 | 01 | | |
| $x_4$ | 0.15 | 001 | 0.15 | 001 | 0.15 | 001 | 0.18 | 000 | 0.20 | 11 | | | | |
| $x_5$ | 0.10 | 011 | 0.10 | 011 | 0.15 | 010 | 0.15 | 001 | | | | | | |
| $x_6$ | 0.08 | 0100 | 0.08 | 0100 | 0.10 | 011 | | | | | | | | |
| $x_7$ | 0.05 | 01010 | 0.07 | 0101 | | | | | | | | | | |
| $x_8$ | 0.02 | 01011 | | | | | | | | | | | | |



$$H(S) = \sum_{i=1}^{q} P_i \log_2 \frac{1}{P_i}$$

$$H(S) = 2.75 \text{ bits / symbol}$$

$$L = \sum_{i=1}^{q} P_i l_i \qquad R\eta_c = 1.79\%$$

$$\eta_c = \frac{H(S)}{L}$$

$$L = 2.8$$

$$\eta_c = 98.25\%$$

iv a) 0101001001000110101114001

= $X_7 \; X_6 \; X_3 \; X_2 \; X_8 \; X_4$

ii) $q = 3 + \alpha(2)$

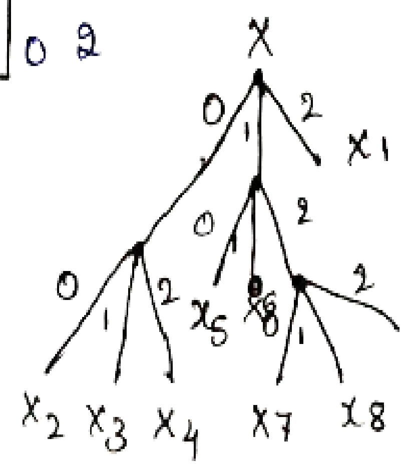
$\alpha 2 = 2.5$

$\alpha 2 = 3 \Rightarrow q = 9$

iv) 121 11 01 10 2 00 120 02

$x_8\ x_6\ x_3\ x_1\ x_2\ x_7\ x_4$

| Symbol | Prob | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x_1$ | 0.22 | 2 | 0.22 | 2 | 0.25 | 1 | 0.53 | 0 |
| $x_2$ | 0.20 | 00 | 0.20 | 00 | 0.22 | 2 | 0.25 | 1 |
| $x_3$ | 0.18 | 01 | 0.18 | 01 | 0.20 | 00 | 0.22 | 2 |
| $x_4$ | 0.15 | 02 | 0.15 | 02 | 0.18 | 01 | | |
| $x_5$ | 0.10 | 10 | 0.10 | 10 | 0.15 | 02 | | |
| $x_6$ | 0.08 | 11 | 0.08 | 11 | | | | |
| $x_7$ | 0.05 | 120 | 0.07 | 12 | | | | |
| $x_8$ | 0.02 | 121 | | | | | | |
| $x_9$ | 0 | 122 | | | | | | |




$H_r(S) = \frac{H(S)}{\log_2 r}$

$H_r(S) = \frac{H(S)}{\log_2(3)} = 1.735$

$L = \sum_{i=1}^{q} P_i l_i$

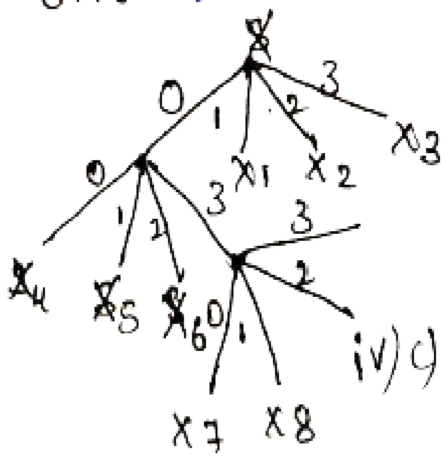
$L = 1.85$

$\eta_c = \frac{H(S)}{L} = 93.73\%$

$R_{\eta_c} = 1 - \eta_c = 6.21\%$

iii) $\alpha = 2 \quad q = 10$

| Symbol | Prob | Code | | Code | | Code |
|---|---|---|---|---|---|---|
| $x_1$ | 0.22 | 1 | 0.22 | 1 | 0.4 | 0 |
| $x_2$ | 0.20 | 2 | 0.20 | 2 | 0.22 | 1 |
| $x_3$ | 0.18 | 3 | 0.18 | 3 | 0.20 | 2 |
| $x_4$ | 0.15 | 00 | 0.15 | 00 | 0.18 | 3 |
| $x_5$ | 0.10 | 01 | 0.10 | 01 | | |
| $x_6$ | 0.08 | 02 | 0.08 | 02 | | |
| $x_7$ | 0.05 | 030 | 0.07 | 03 | | |
| $x_8$ | 0.02 | 031 | | | | |
| $x_9$ | 0 | 032 | | | | |
| $x_{10}$ | 0 | 033 | | | | |




$H(S) = 1.375$

$L = \sum_{i=1}^{q} p_i l_i$

$L = 1.47$

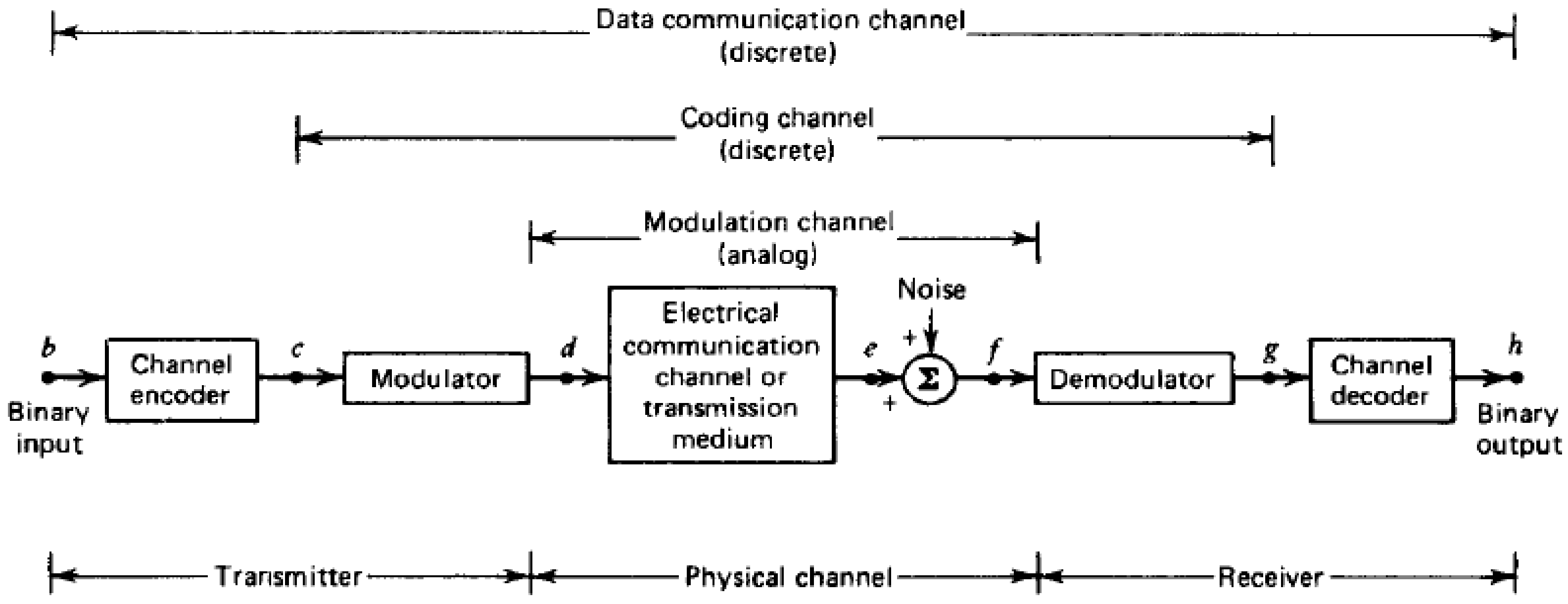
$\eta_c = \frac{H(S)}{L} = 93.53\%$

$R_{\eta_c} = 1 - \eta_c = 6.47\%$

iv) c) 031 1 3 20 203 001 002 3 1

$x_8 \, x_1 \, x_3 \, x_2 \, x_6 \, x_7 \, x_5 \, x_4 \, x_2 \, x_3 \, x_1$

# Communication Channel




Characterization of a binary communication channel.

# Discrete memoryless Channel:

- A channel is defined as the medium through which the coded signals which are generated by an information source are transmitted.
- In general, the input to the channel is a symbol belonging to an alphabet ‘A’ with ‘r’ symbols
- The output of a channel is a symbol belonging to an alphabet ‘B’ with ‘s’ symbols
- Due to errors in the channel, the output symbols may differ from input symbols.

- **Representation of a channel:**
- A=$(a_{1,}a_{2,}a_3 \dots \dots \dots a_r)$
- B=$(b_{1,}b_{2,}b_3 \dots \dots \dots b_s)$
- set of conditional probability P$({}^{b_i}/_{a_i})$ with i=1,2,….r and j=1,2,…….s

$$A \quad \left.\begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_r \end{matrix}\right\} \rightarrow \mathrm{P}({}^{b_i}/_{a_i}) \rightarrow \left\{\begin{matrix} b_1 \\ b_2 \\ \vdots \\ b_s \end{matrix}\right. \quad B$$

- The conditional probabilities come in to the existence due to the presence of noise in the channel.

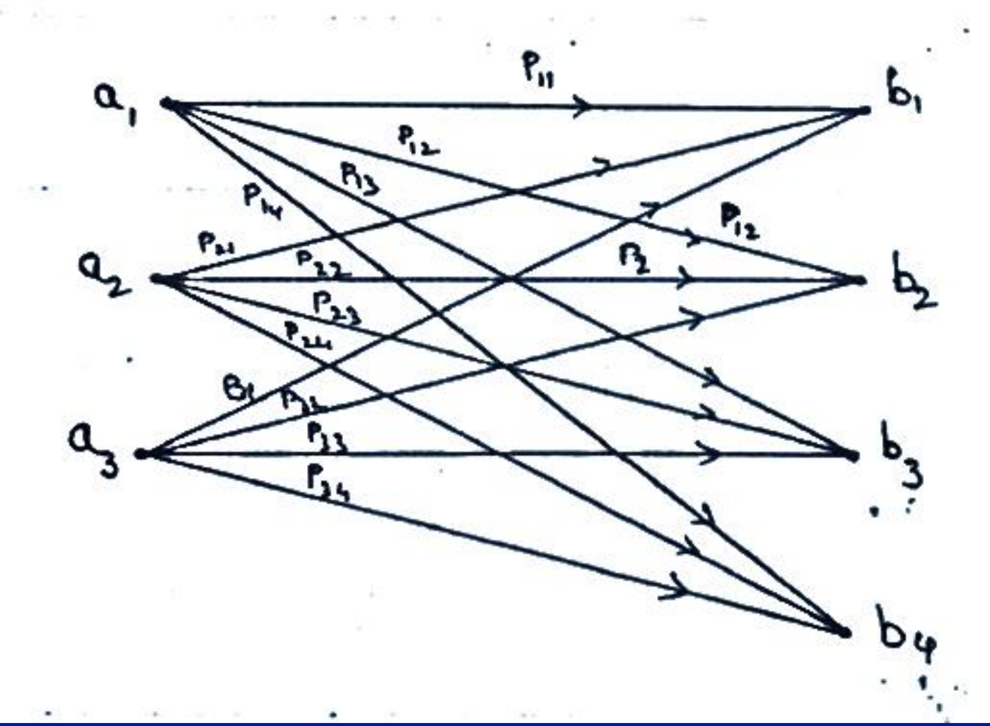
- There are ‘s’ number of symbols at the receiver from ‘r’ symbols at transmitter.

- Totally there are r * s conditional probabilities represented in a form of matrix which is called as Channel Matrix or Noise Matrix.

$$
P(b_j/a_i) = \begin{array}{c c}
 & \begin{array}{cccc} b_1 & b_2 & b_3 & \qquad b_s \end{array} \\
a_1 & \left[\begin{array}{cccc}
P(b_1/a_1) & P(b_2/a_1) & P(b_3/a_1) \cdots & P(b_s/a_1) \\
P(b_1/a_2) & P(b_2/a_2) & P(b_3/a_2) \cdots & P(b_s/a_2) \\
P(b_1/a_3) & P(b_2/a_3) & P(b_3/a_3) \cdots & P(b_s/a_3) \\
P(b_1/a_4) & P(b_2/a_4) & P(b_3/a_4) \cdots & P(b_s/a_4) \\
\vdots & & & \\
P(b_1/a_r) & P(b_2/a_r) & P(b_3/a_r) \cdots & P(b_s/a_r)
\end{array}\right]
\end{array}
$$

# Channel Diagram or Noise Diagram

**Channel matrix can also be represented as channel diagram. For Example, consider a source A={a1, a2, a3} and output alphabet B={b1, b2, b3, b4} which can be represented in a channel diagram as shown below.**



# Channel Continued

- When $a_1$ is transmitted, it can be received as any one of the output symbols $(b_{1,} b_{2,} b_3 \ldots \ldots \ldots b_s)$
- $P_{11} + P_{12} + P_{13} + \cdots \ldots P_S$=1
- P($^{b_1}/_{a_1}$)+ P($^{b_2}/_{a_1}$)+ P($^{b_3}/_{a_1}$)+……+ P($^{b_s}/_{a_1}$)=1
- $\sum_{j=1}^{s} \mathrm{P}(^{b_j}/_{a_i}) = 1$ for i= 1 to r
- Thus the sum of all the elements in any row of the channel matrix is equal to UNITY.

## Joint Probability:

- $\mathrm{P}(a_i \cap b_j) = \boldsymbol{P(a_i, b_j) = P(A, B) =}\ \mathbf{P}\left({}^{b_j}/_{a_i}\right)\boldsymbol{P(a_i)}$
- $\boldsymbol{P(a_i, b_j)} = \mathbf{P}\left({}^{a_i}/_{b_j}\right)\boldsymbol{P(b_j)}$

Consider,

$$P(a_i, b_j) = P(b_j/a_i)\, P(a_i)$$

Multiply all the elements of the first row of channel matrix by $P(a_1)$ & $2^{nd}$ row by $P(a_2)$ & $r^{th}$ row by $P(a_r)$

Then the matrix obtained is of the form :

$$P(b_j/a_i)\,P(a_i) = P(a_i, b_j) = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_r \end{array} \begin{array}{c} \begin{array}{ccccc} b_1 & b_2 & b_3 & \cdots & b_s \end{array} \\ \begin{bmatrix} P(b_1/a_1)P(a_1) & P(b_2/a_1)P(a_1) & P(b_3/a_1)P(a_1) & \cdots & P(b_s/a_1)P(a_1) \\ P(b_1/a_2)P(a_2) & P(b_2/a_2)P(a_2) & P(b_3/a_2)P(a_2) & \cdots & P(b_s/a_2)P(a_2) \\ P(b_1/a_3)P(a_3) & P(b_2/a_3)P(a_3) & P(b_3/a_3)P(a_3) & \cdots & P(b_s/a_3)P(a_3) \\ \vdots & \vdots & \vdots & & \vdots \\ P(b_1/a_r)P(a_r) & P(b_2/a_r)P(a_r) & P(b_3/a_r)P(a_r) & & P(b_s/a_r)P(a_r) \end{bmatrix} \end{array}$$

$$P(a_i, b_j) = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_r \end{array} \begin{array}{c} \begin{array}{ccccc} b_1 & b_2 & b_3 & \cdots & b_s \end{array} \\ \begin{bmatrix} P(a_1, b_1) & P(a_1, b_2) & P(a_1, b_3) & \cdots & P(a_1, b_s) \\ P(a_2, b_1) & P(a_2, b_2) & P(a_2, b_3) & \cdots & P(a_2, b_s) \\ P(a_3, b_1) & P(a_3, b_2) & P(a_3, b_3) & \cdots & P(a_3, b_s) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ P(a_r, b_1) & P(a_r, b_2) & P(a_r, b_3) & \cdots & P(a_r, b_s) \end{bmatrix} \end{array}$$

- The above matrix whose elements are various joint probabilities between input and output symbols is called joint probability matrix(JPM)

  The important properties of JPM are:
- The probabilities of input symbols can be obtained by adding the elements of JPM row wise
- The probabilities of output symbols can be obtained by adding the elements of JPM column wise
- The sum of all the elements of JPM is always equal to unity.