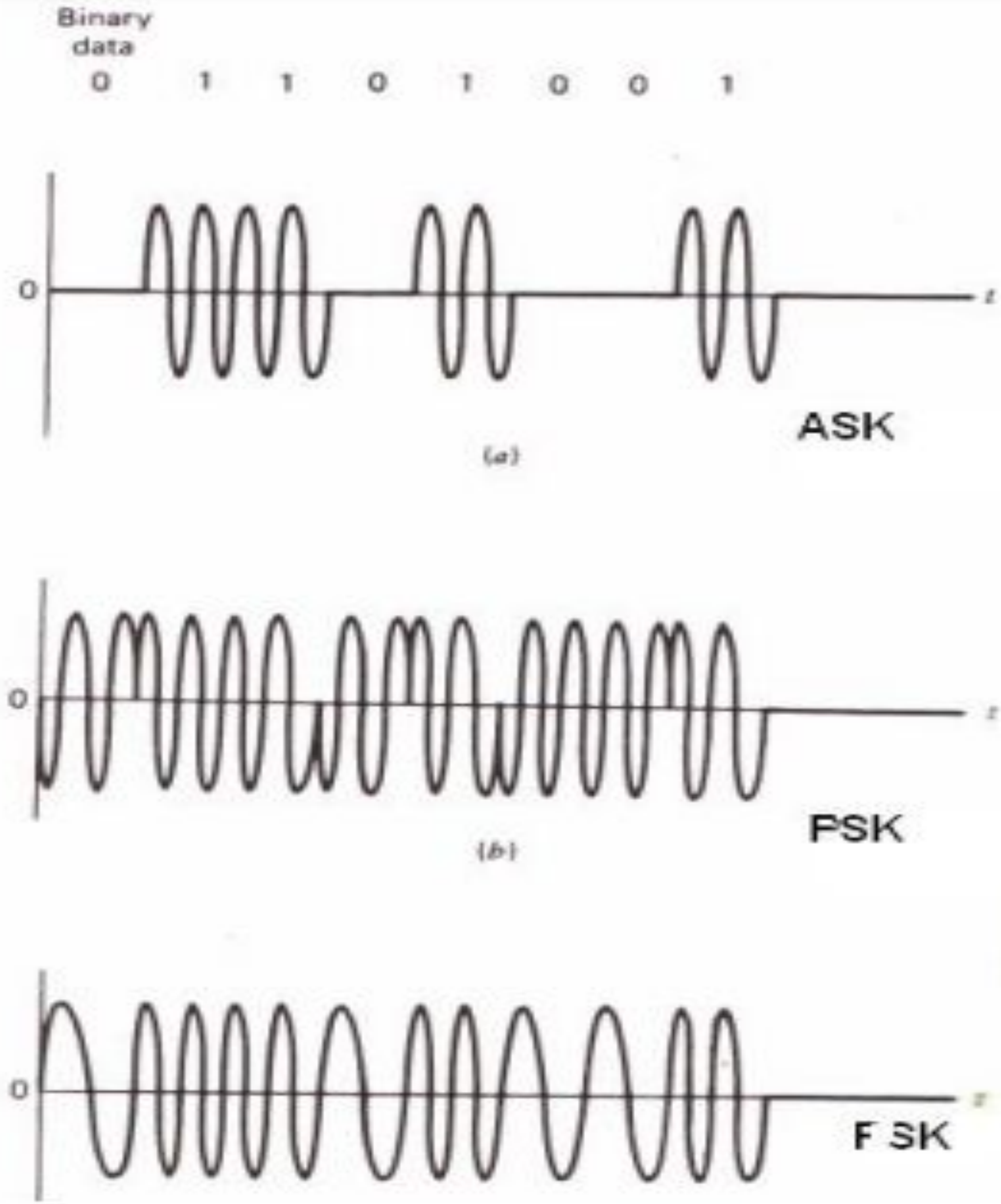
Module-4

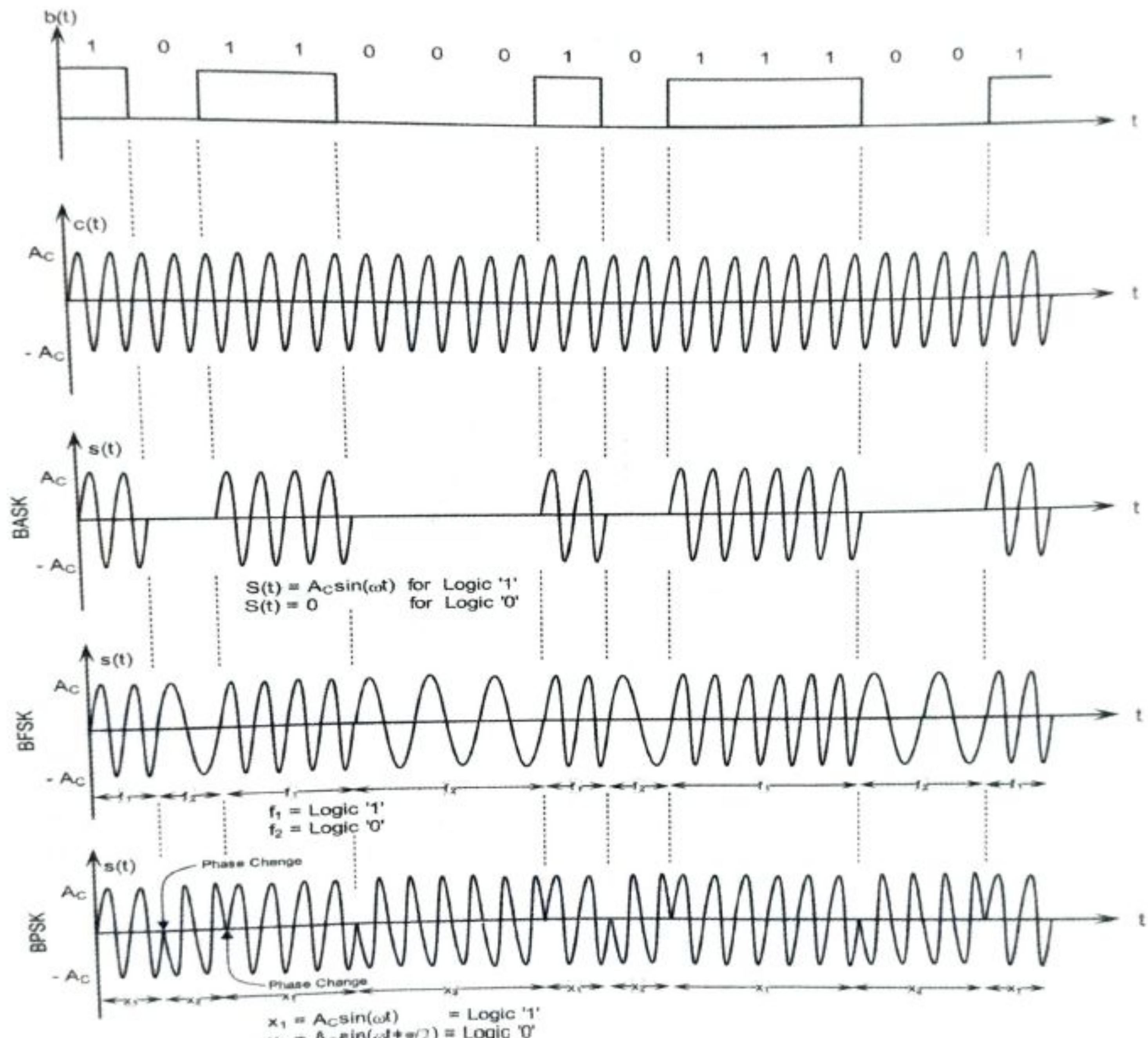
Digital Modulation Techniques:

- Digital Modulation formats
- Coherent binary modulation techniques
- Probability of error derivation of PSK and FSK
- M-ary modulations-QPSK
- QAM
- PSD for different digital modulation techniques
- Non-coherent binary modulation techniques -DPSK

Digital Modulation formats

- Modulation is defined as the process by which some characteristics of a carrier is varied in accordance with a modulating wave.
- In digital communications, the modulating wave consists of binary data or an M-ary encoded version of it. For carrier a sinusoidal wave is used.
- In M-ary signalling, the modulator produces one of an available set of $M=2^m$ distinct signals in response to m bits of source data at a time.
- Binary modulation is a special case of M-ary modulation with M=2.
- For modulation, it is customary to use a sinusoidal wave.
- The modulation process involves switching or keying the amplitude, frequency or phase of the carrier in accordance with the incoming data
- There are 3 basic modulation techniques for the transmission of digital data.
  - Amplitude Shift Keying
  - Frequency Shift Keying
  - Phase Shift Keying


Binary data
0 1 1 0 1 0 0 1
0
t
ASK
(a)
0
t
PSK
(b)
0
t
F SK



b(t)
1 0 1 1 0 0 0 1 0 1 1 1 0 0 1
t
c(t)
Ac
- Ac
BASK
s(t)
S(t) = Acsin(ωt) for Logic '1'
S(t) = 0 for Logic '0'
BFSK
f1 = Logic '1'
f2 = Logic '0'
BPSK
Phase Change
x1 = Acsin(ωt) = Logic '1'
x2 = Acsin(ωt+π/2) = Logic '0'

- PSK and FSK have constant envelope. Therefore they are impervious to amplitude nonlinearities.
- Used in Microwave radio links and satellite channels.
- In practice, PSK and FSK signals are much more widely used than ASK signals.
- Note: Sometimes, a hybrid form of modulation is used.

For example, changes in both amplitude and phase of the carrier are combined to produce amplitude-phase keying (APK).

Demodulation at the receiver:

Demodulation can be either

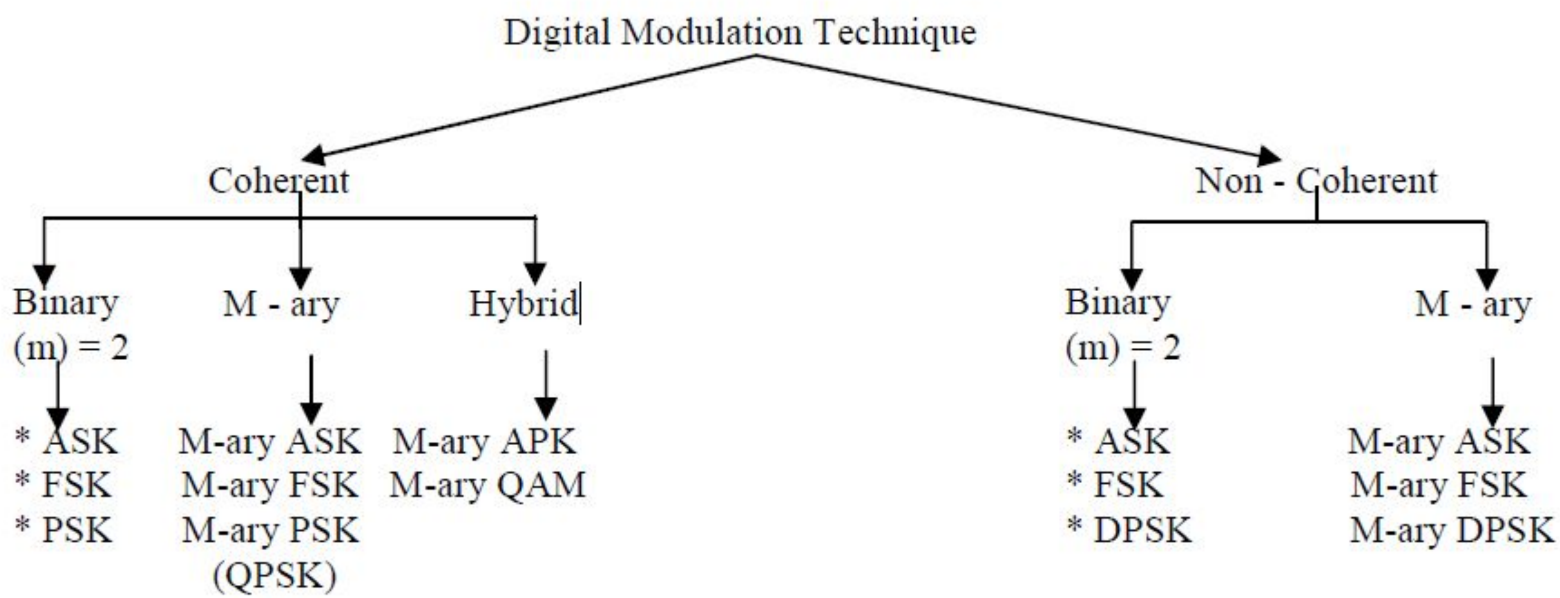
- Coherent
- Non-coherent detection.

- Coherent:
  - Exact replicas of possible arriving signals are available at the receiver.
  - Receiver is phase-locked to the transmitter.
  - It is performed by cross-correlating the received signal with each one of the replicas, and then making a decision based on comparisons with preselected thresholds.

- Non-coherent:
  - Knowledge of the carrier wave's phase is not required.
  - Complexity of the receiver is reduced.
  - It exhibits an inferior error performance, compared to a coherent system.

- The choice among different scheme is made based on attaining as many of the following design goals.
  - ✔ Maximum data rate.
  - ✔ Minimum probability of error.
  - ✔ Minimum transmitted power.
  - ✔ Minimum channel bandwidth
  - ✔ Maximum resistance to interfering signals
  - ✔ Minimum circuit complexity

Hierarchy of digital modulation technique



Important formulae

1. Mean

$$\mu = E[x] = \int_{-\infty}^{\infty} x f_X(x)dx$$

2. Mean Square Value

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_X(x)dx$$

3. Variance

$$\sigma_x^2 = \int_{\infty}^{\infty} (x-\mu)^2 f_X(x)dx = E(x^2)$$

4. Guassian Random Variable

$$P_{Df} = f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]}$$

$$P(x) = \int_{x_1}^{x_2} f_X(x)dx$$

$$\boxed{P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{x_1}^{x_2} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx}$$

5. Error function

$$\boxed{\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{u^2} du}$$

6. Complementary error function

$$\boxed{\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{+x}^{\infty} e^{-u^2} du}$$

Relation between Amplitude and bit Energy

$$E = \text{Power} \times \text{Time}$$

$$x(t) = A_c \cos(2\pi f_c t)$$

$$P = \left(\frac{A_c}{\sqrt{2}}\right)^2 = \frac{A_c^2}{2}$$

$$E_b = \frac{A_c^2}{2} \times T_b$$

$$A_c^2 = \frac{2E_b}{T_b}$$

$$\boxed{A_c = \sqrt{\frac{2E_b}{T_b}}}$$

$$\boxed{\therefore x(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)}$$

$A_c \rightarrow$ Amplitude

$T_b \rightarrow$ Bit Duration

$E_b \rightarrow$ Bit Energy

Orthogonal Basis Function

It represents the carrier signal with a unit bit energy for ‘n’ number of basis functions. They are represented suing Φ1(t) Φ2(t),…. While all of them are orthogonal to each other. They will never get mixed up during transmission

$$x(t) = A_c \cos(2\pi f_c t)$$

$$A_c = \sqrt{\frac{2E_b}{T_b}}$$

$$\phi(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

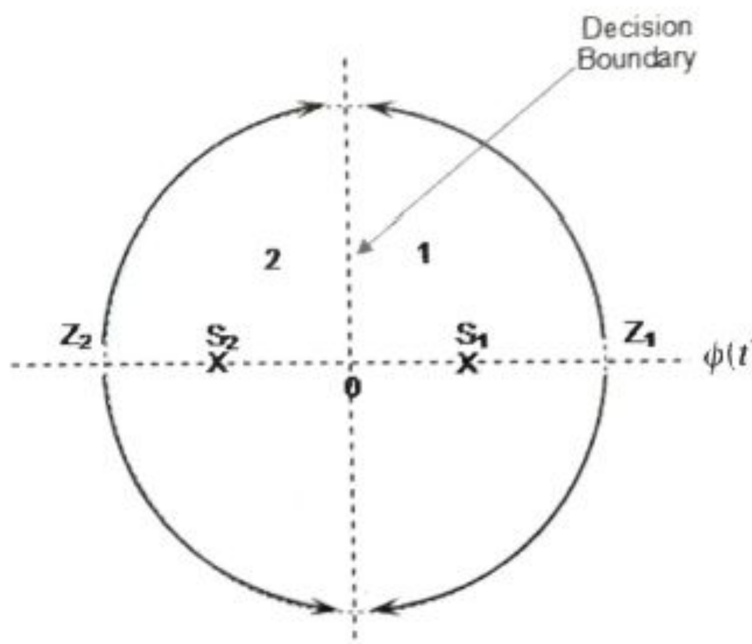
$$x(t) = \sqrt{E_b}\sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

$$\boxed{\therefore x(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)}$$

$$\boxed{x(t) = \sqrt{E_b}\,\phi(t)}$$

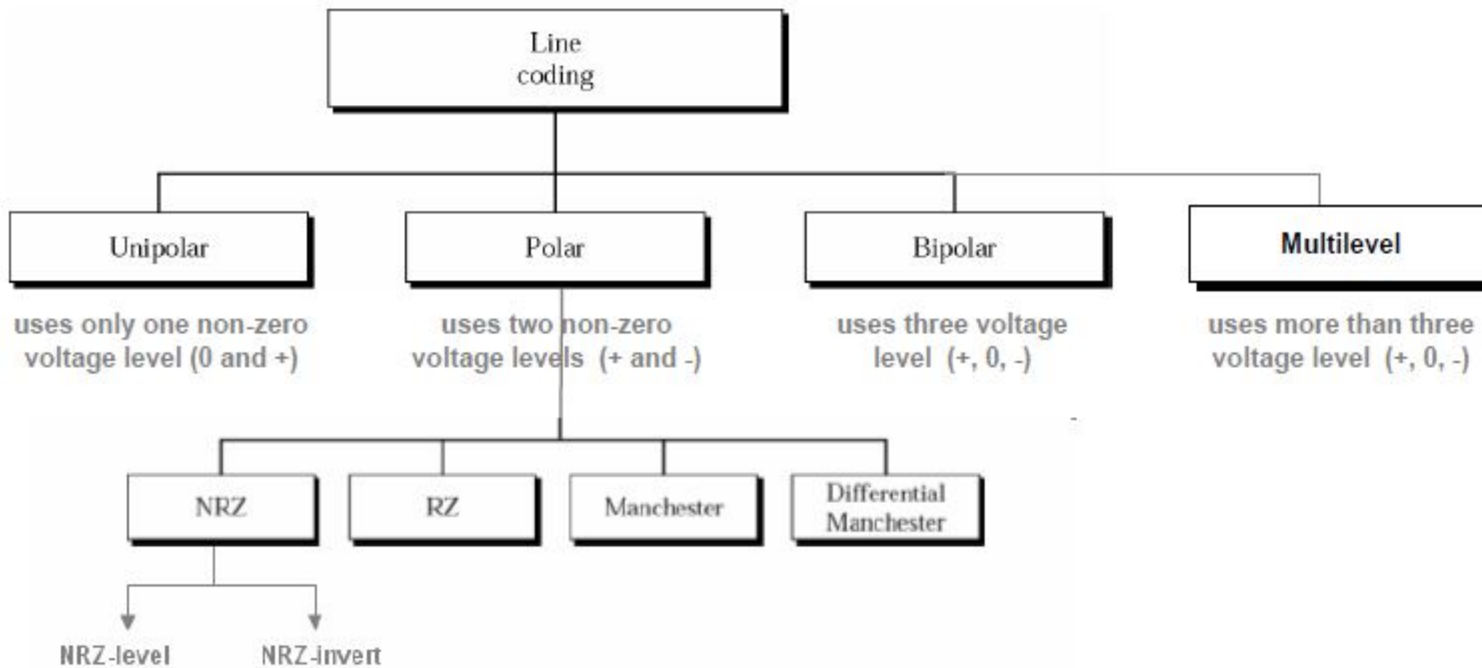
Signal Space

It indicates symbol space with respect to basis function. Here decision boundary is used to take appropriate decision in favour of expected signals. It's a sample space to find probability of error or probability of correctness for the given observation point. In the given space diagram, if observation point in region Z1 them the decision is taken in favour of symbol S1. Similarly if the observation point is in area Z2. Then the decision is taken in favour of symbol s2. If the observation point lies exactly on decision boundary. Then the decision is arbitrary.



Decision Boundary

Line Coding Schemes – can be divided into four broad categories



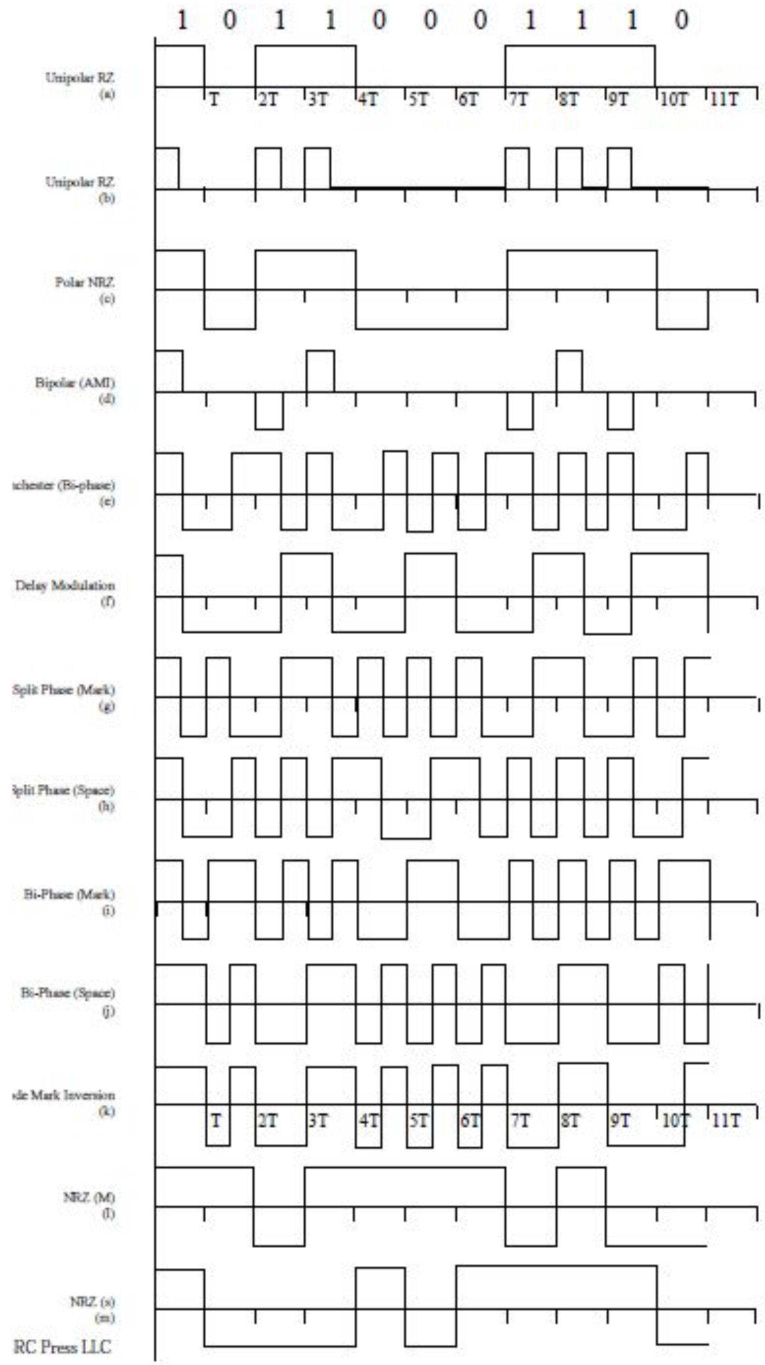


FIGURE 6.1: Waveforms for different line codes.

Coherent binary modulation techniques

1. Coherent Binary ASK
2. Coherent Binary FSK
3. Coherent Binary PSK

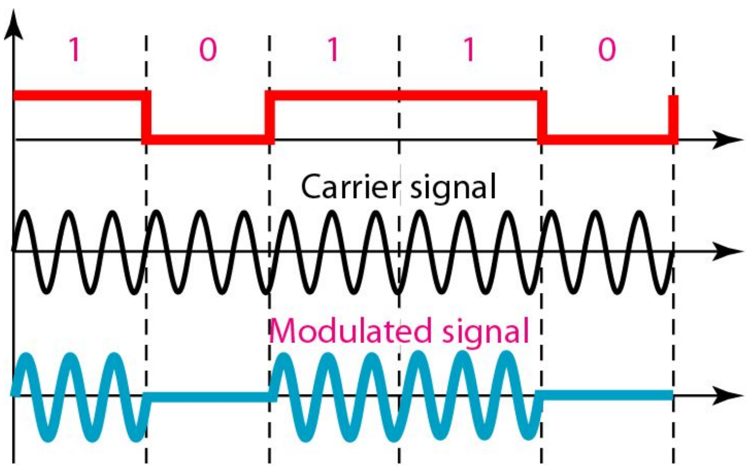
Refer PDF Document for waveform and operational explonations

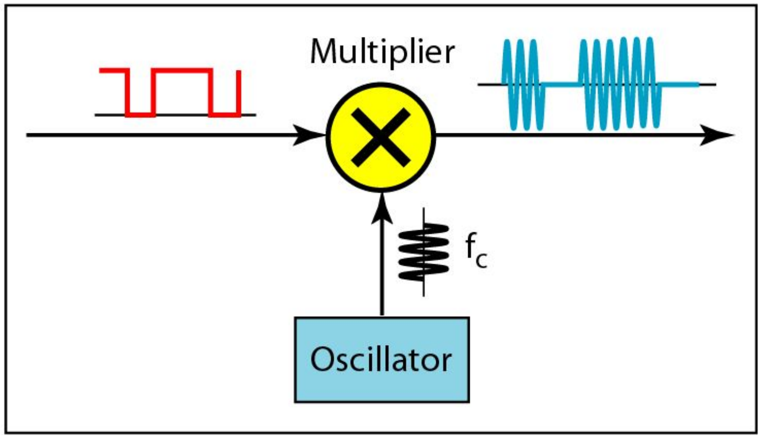
Coherent Binary ASK

A binary ASK wave can be defined as

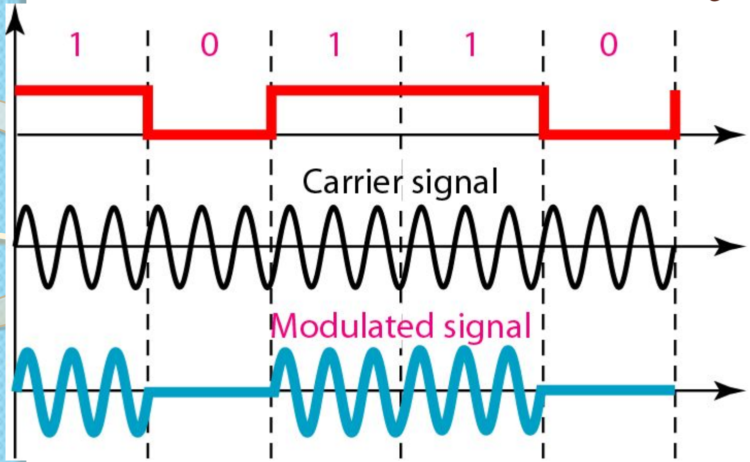
$S_t = A_c\ m(t)\ \cos 2\pi f_c t\ ,\ 0 \le t \le T_b$

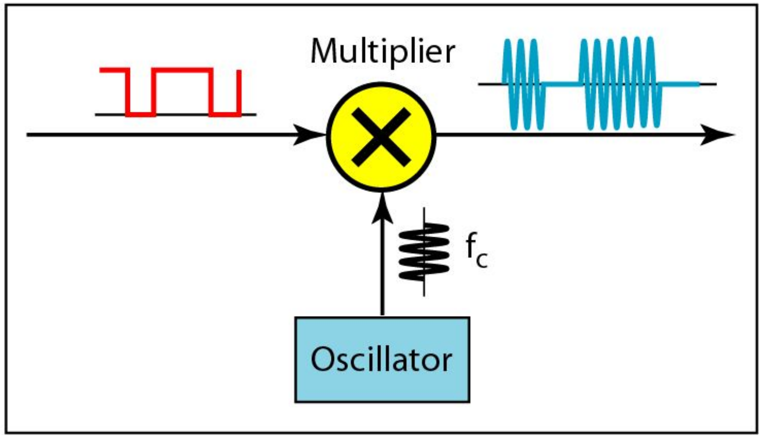
where A_c is amplitude of carrier, m(t) is digital information signal.
f_c is carrier frequency, T_b is bit duration.





Coherent Binary ASK





In binary ASK system, symbol 1 & 0 are represented as

$$s(t) = \begin{cases} s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t & \text{for } 0 \le t \le T_b \quad \text{for symbol 1} \\ s_2(t) = 0 & \text{for } 0 \le t \le T_b \quad \text{for symbol 0} \end{cases}$$

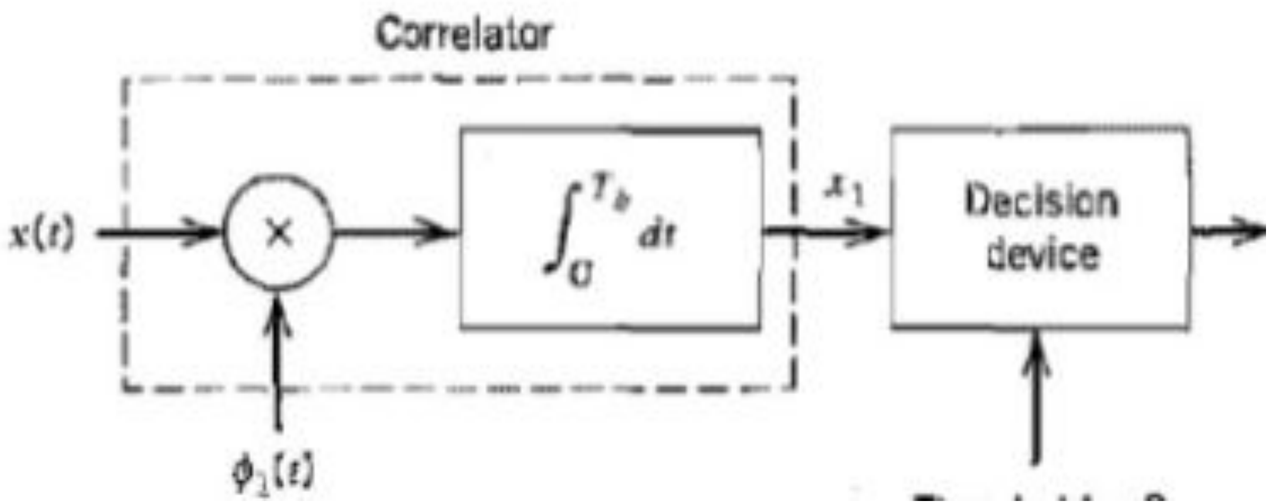
Basis function $\Phi_1(t) = \sqrt{\frac{2}{Tb}} \cos 2\pi f_c t$

Binary ASK can be written as

$$s(t) = \begin{cases} s_1(t) = \sqrt{E_b}\, \Phi_1(t) & \text{for } 0 \le t \le T_b \quad \text{for symbol 1} \\ s_2(t) = 0 & \text{for } 0 \le t \le T_b \quad \text{for symbol 0} \end{cases}$$

Coherent detection of ASK signal

- In demodulator, the received signal x(t) is cross correlated with local reference signal Φ_1 (t) .
- The output of correlator is applied to decision device.
- The correlator output x is compared with threshold λ.
- If $x > \lambda$ the receiver decides in favour of symbol 1.
- If $x < \lambda$ the receiver decides in favour of symbol 0.
- . In coherent detection the output of local oscillator is in perfect synchronisation with the carrier used in the transmitter



Coherent Binary PSK

In binary PSK system, symbol 1 & 0 are represented as

$$s(t)=\begin{cases} s_1(t)=\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t & \text{for } 0\le t\le T_b \quad \text{for symbol 1} \\ s_2(t)=\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t+\pi) & \text{for } 0\le t\le T_b \quad \text{for symbol 0} \\ \quad = -\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t & \end{cases}$$

Basis function $\Phi_1(t) = \sqrt{\frac{2}{Tb}}\cos 2\pi f_c t$ for $0\le t\le T_b$

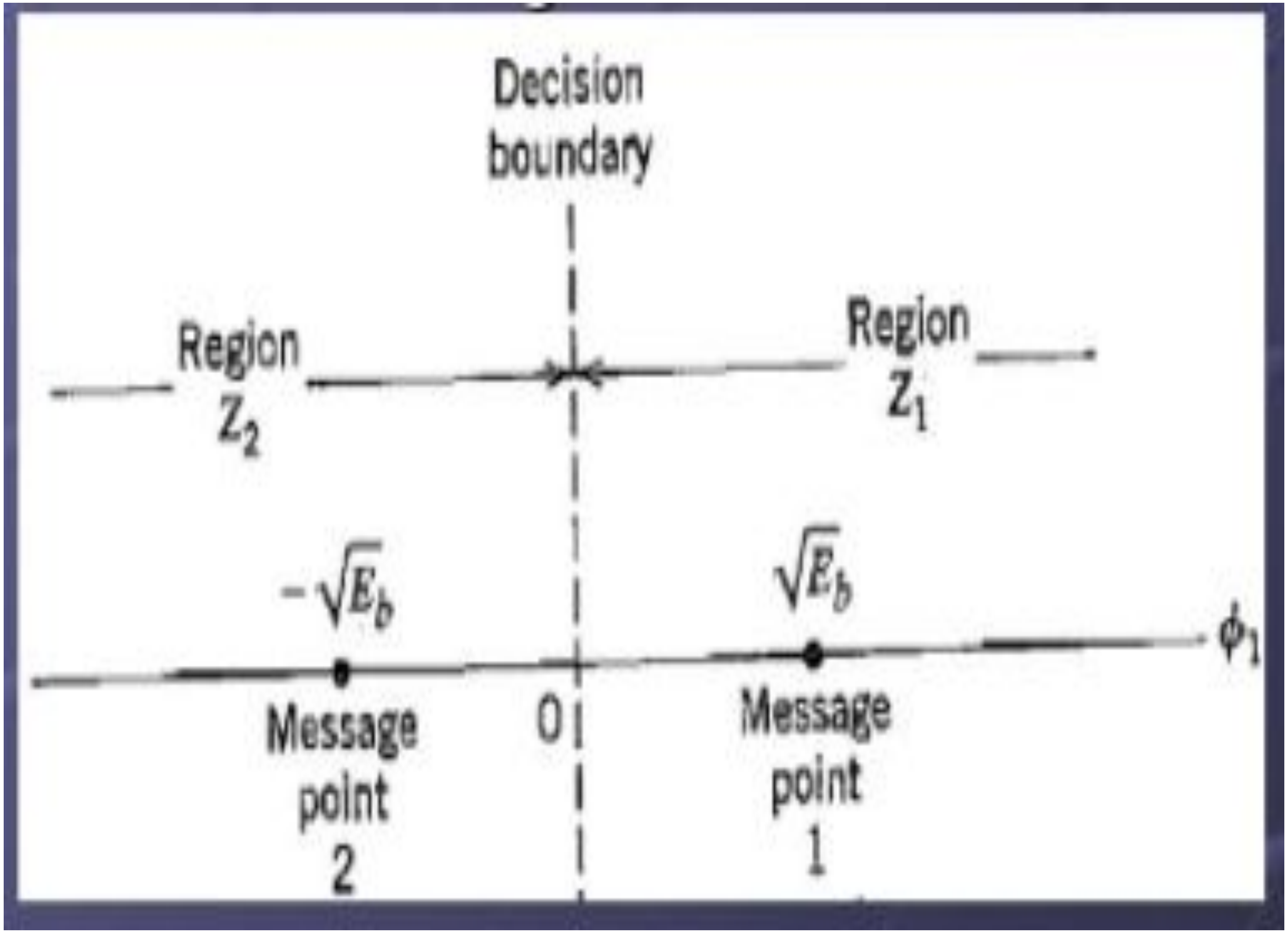
Binary PSK can be written as

$$s(t)=\begin{cases} s_1(t)=\sqrt{E_b}\,\Phi_1(t) & \text{for } 0\le t\le T_b \quad \text{for symbol 1} \\ s_2(t)=-\sqrt{E_b}\,\Phi_1(t) & \text{for } 0\le t\le T_b \quad \text{for symbol 0} \end{cases}$$

A coherent binary PSK system is characterized by one-dimensional signal space. (N=1), and with two message points(M=2)
The coordinates o message points equal

$$s_{11}=\int_0^{T_b} s_1(t)\,\Phi_1(t)\,dt = +\sqrt{E_b} \qquad s_{21}=\int_0^{T_b} s_2(t)\,\Phi_1(t)\,dt = -\sqrt{E_b}$$

Signal space diagram for coherent binary PSK system



Coherent Binary FSK

In binary FSK system, symbol 1 & 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by a fixed amount.

$$s_i(t)=\begin{cases}\sqrt{\dfrac{2E_b}{T_b}}\cos 2\pi f_c t & \text{for } 0\le t\le T_b \\ 0 & \text{elsewhere}\end{cases} \quad \text{where } i=1,2.$$

Transmitted frequency equals $f_i = \dfrac{n_c + i}{T_b}$ for some fixed integer n_c and i=1,2

$$\Phi_i(t) = \begin{cases}\sqrt{\dfrac{2}{T_b}}\cos 2\pi f_c t & \text{for } 0\le t\le T_b \\ 0 & \text{elsewhere}\end{cases}$$

$$S_{ij} = \int_0^{T_b} s_i(t)\,\Phi_j(t)\,dt = \begin{cases}\sqrt{E_b} & i=j \\ 0 & i\neq j\end{cases}$$

Probability of error derivation of PSK and FSK

- Refer Class notes

M-ary modulations- QPSK, QAM

- QPSK is an extension of BPSK
- In binary data transmission, we can transmit only one of two possible signals during each bit interval Tb. On the other hand, in M-ary data transmission, it is possible to send ny one of M possible signals during each signally interval T where T=nTb
- QPSK is an example of M-ary data transmission with M=4,
- $M=2^{m}$, if m=2, then M=4
- In QPSK, the phase of the carrier takes on one of four equally space values such as 45, 135, 225, 315

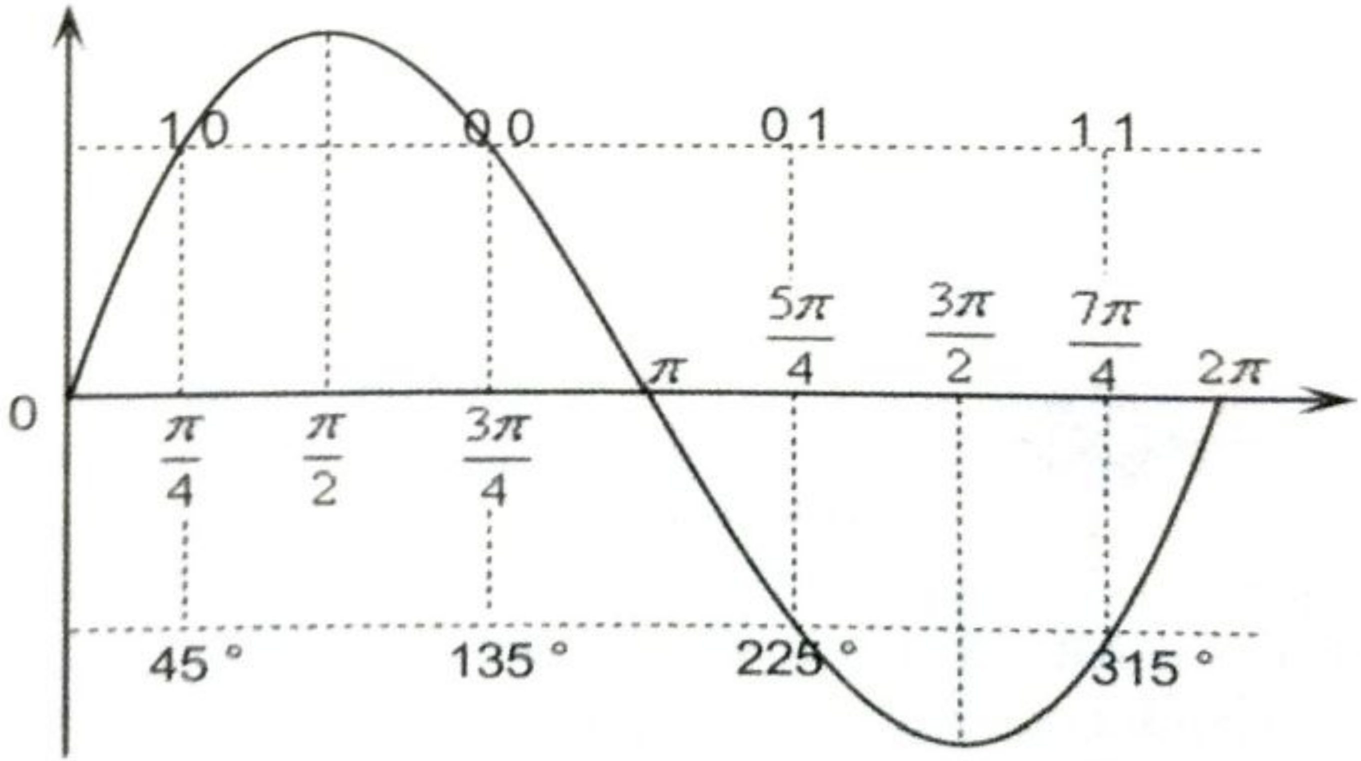
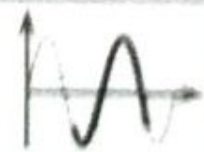
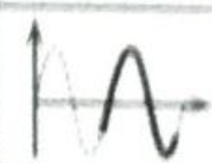


Figure 5.19: Waveform

QPSK Signal

Figure 5.20: Table Showing Co-ordinates of Message and Phase of QPSK Signal

Messages	Phase Shift	Dibits	S_{i1}	S_{i2}	Waveform
S_1	$\frac{\pi}{4}$	1 0	$\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$	
S_2	$\frac{3\pi}{4}$	0 0	$-\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$	
S_3	$\frac{5\pi}{4}$	0 1	$-\sqrt{\frac{E}{2}}$	$\sqrt{\frac{E}{2}}$	
S_4	$\frac{7\pi}{4}$	1 1	$\sqrt{\frac{E}{2}}$	$\sqrt{\frac{E}{2}}$	

There are four message points, and the associated signal vectors are defined by

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E}\cos\left((2i-1)\frac{\pi}{4}\right) \\ -\sqrt{E}\sin\left((2i-1)\frac{\pi}{4}\right) \end{bmatrix} \qquad i = 1, 2, 3, 4 \qquad (7.36)$$

Error function Table :

Table Error Function

u	erf(u)	u	erf(u)
0.00	0.00000	1.10	0.88021
0.05	0.05637	1.15	0.89612
0.10	0.11246	1.20	0.91031
0.15	0.16800	1.25	0.92290
0.20	0.22270	1.30	0.93401
0.25	0.27633	1.35	0.94376
0.30	0.32863	1.40	0.95229
0.35	0.37938	1.45	0.95970
0.40	0.42839	1.50	0.96611
0.45	0.47548	1.55	0.97162
0.50	0.52050	1.60	0.97635
0.55	0.56332	1.65	0.98038
0.60	0.60386	1.70	0.98379
0.65	0.64203	1.75	0.98667
0.70	0.67780	1.80	0.98909
0.75	0.71116	1.85	0.99111
0.80	0.74210	1.90	0.99279
0.85	0.77067	1.95	0.99418
0.90	0.79691	2.00	0.99532
0.95	0.82089	2.50	0.99959
1.00	0.84270	3.00	0.99998
1.05	0.86244	3.30	0.999998

A binary FSK System transmits data at a rate of 2Mbps over an AWGN Channel. The noise power Spectral density $\frac{N_0}{2} = 10^{-20}$ Watts/Hz. Determine the probability error 'P_e' for Coherent detection of FSK. Assume amplitude of the received Signal = 1 μV

$$R_b = 2\,\text{Mbps}, \quad \frac{N_0}{2} = 10^{-20}\ \text{W/Hz}, \quad A_c = 1 \times 10^{-6}\ \text{V}$$

$$T_b = \frac{1}{R_b} = \frac{1}{2 \times 10^{6}}, \quad N_0 = 2 \times 10^{-20}\ \text{W/Hz}$$

* $E_b = PT_b$

* $P = \frac{A_c^2}{2} = \frac{(1\times10^{-6})^2}{2} = 5\times10^{-13}$ W

* $E_b = 5\times10^{-13} \times \frac{1}{2\times10^{6}} = 2.083\times10^{-19}$ Joules

* WKT 'P_e' for FSK System is

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{2.083\times10^{-19}}{2\times2\times10^{-20}}}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{5.2075}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(2.2819\right)$$

$$= \frac{1}{2}\left[1 - \mathrm{erf}(2.2819)\right]$$

$$= \frac{1}{2}\left[1 - 0.99959\right]$$

$$= \frac{1}{2}\left(4.1\times10^{-4}\right)$$

$$\boxed{P_e = 2.05\times10^{-4}}$$

$\because$ $\boxed{\mathrm{erfc}(u) = 1 - \mathrm{erf}(u)}$

A binary FSK System transmits data at a rate of 2 Mbps over an AWGN channel. The noise is zero mean with PSD, $\frac{N_0}{2} = 10^{-20}$ W/Hz. The amplitude of received Signal in the absence of noise is 1 μv. Determine the average probability of error for Coherent detection of FSK. Take $\operatorname{erfc}\sqrt{6.25} = 0.00041$.

Given :-

$R_b = 2\,\text{Mbps}$, $\frac{N_0}{2} = 10^{-20}$ W/Hz

$A_c = 1 \times 10^{-6}$ V , $\operatorname{erfc}\sqrt{6.25} = 0.00041$

$$T_b = \frac{1}{R_b} = \frac{1}{2\times10^6}$$

$N_0 = 2\times10^{-20}$ W/Hz.

WKT 'P_e' for FSK System

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

* $E_b = PT_b$

* $P = \frac{A_c^2}{2} = \frac{(1\times10^{-6})^2}{2} = 0.5\times10^{-12}\ W$

* $E_b = 0.5\times10^{-12}\times\frac{1}{2\times10^{6}} = \underline{0.25\times10^{-18}}$ Joules

* $P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{0.25\times10^{-18}}{2\times2\times10^{-20}}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{6.25}\right) = \frac{1}{2}\left[0.00041\right]$

$\because\ \mathrm{erfc}\left(\sqrt{6.25}\right) = 0.00041$

$$\boxed{P_e = 2.05\times10^{-4}}$$

Binary data is transmitted over AWGN channel using BPSK at a rate of 1 Mbps. It is desired to have average probability of error $P_e \leq 10^{-4}$. Noise PSD is $\frac{N_0}{2} = 10^{-12}$ W/Hz. Determine the average carrier power required at receiver I/p if the detector is of coherent type. [Assume $erfc(3.5) = 0.00025$].

$$R_b = 1\text{Mbps}, \quad T_b = \frac{1}{R_b} = \frac{1}{1\times10^6}$$

$$\frac{N_0}{2} = 10^{-12}\ \text{W/Hz}, \quad N_0 = 2\times10^{-12}\ \text{W/Hz}$$

$$P_e \leq 10^{-4}, \quad erfc(3.5) = 0.00025.$$

Sol:-

WKT for BPSK System

$$P_e = \frac{1}{2} \, erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$10^{-4} = \frac{1}{2} \, erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$erfc\left(\sqrt{\frac{E_b}{N_0}}\right) = 2\times10^{-4}$$

$$erf(u) = 1 - 2\times10^{-4}$$

$$erf(u) = 1 - erfc(u)$$

$$erf(u) = 0.9998$$

From erf function table

$$\boxed{u \approx 2.8 \text{ to } 2.9}$$

$$\therefore \sqrt{\frac{E_b}{N_0}} = 2.8$$

$$\frac{E_b}{N_0} = (2.8)^2$$

$$\frac{E_b}{N_0} = 7.84$$

$$E_b = N_0 \times 7.84$$

$$E_b = 2\times10^{-12} \times 7.8$$

$$PT_b = 1.568\times10^{-11}$$

$$P = \frac{1.568\times10^{-11}}{T_b} = \frac{1.568\times10^{-11}}{1\times10^{-6}}$$

$$\boxed{P = 1.568\times10^{-5} \text{ W}}$$

Binary data are transmitted at a rate of 10^6 bits per Second over a microwave link. Assuming Channel noise is AWGN with Zero mean & power Spectral density at the receiver I/P is 10^{-10} W/Hz, find the average carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for Coherent binary PSK. Determine the minimum Channel bandwidth required.

Given : $R_b = 1\times10^6$, $T_b = \frac{1}{R_b} = \frac{1}{1\times10^6}$

$\frac{N_0}{2} = 10^{-10}$ W/Hz, $N_0 = 2\times10^{-10}$ W/Hz , $P_e \leq 10^{-4}$

Sol :- WKT for PSK System: $P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$

$$10^{-4} = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$\boxed{\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = 2\times10^{-4}} \rightarrow ①$$

WKT $\operatorname{erf}(u) = 1 - \operatorname{erfc}(u)$

$$= 1 - 2\times10^{-4}$$

$$\boxed{\operatorname{erf}(u) = 0.9998}$$

From error function table $u \approx 2.8$.

From eq ① $\sqrt{\frac{E_b}{N_0}} = 2.8$

$$\frac{E_b}{N_0} = (2.8)^2 = 7.84$$

$$E_b = 7.84 \times 2\times10^{-10}$$

$$\boxed{E_b = 1.568\times10^{-9} \text{ Joules}}$$

* WKT $E_b = PT_b$

$$P = \frac{E}{T_b} = \frac{1.568\times10^{-9}}{\frac{1}{1\times10^{6}}}$$

∴ Average carrier power

$$\boxed{P = 1.568\times10^{-3} \text{ W}}$$

* Channel bandwidth $B_T = \frac{1}{T_b} = R_b$

$$\boxed{B_T = 1\text{MHz}}$$

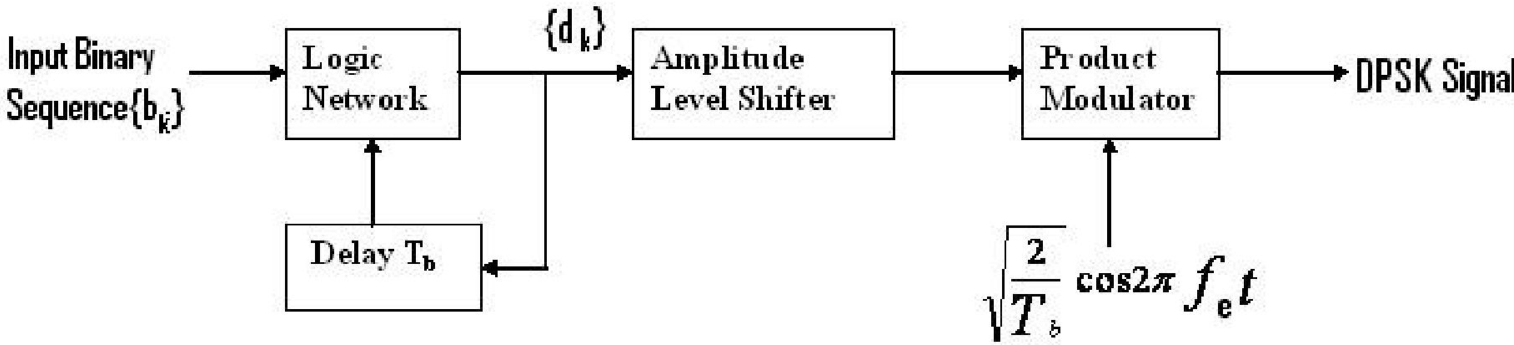
Non-coherent binary modulation techniques -DPSK

- A DPSK system may be viewed as the non coherent version of the PSK. It eliminates the need for coherent reference signal at the receiver by combining two basic operations at the transmitter

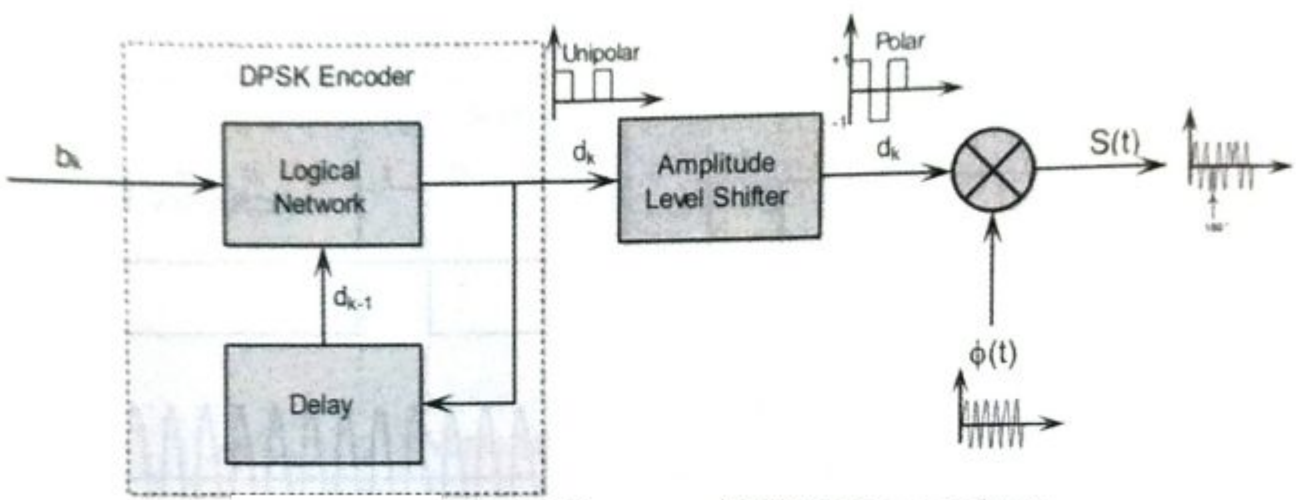
(1) Differential encoding of the input binary wave and

(2) Phase shift keying

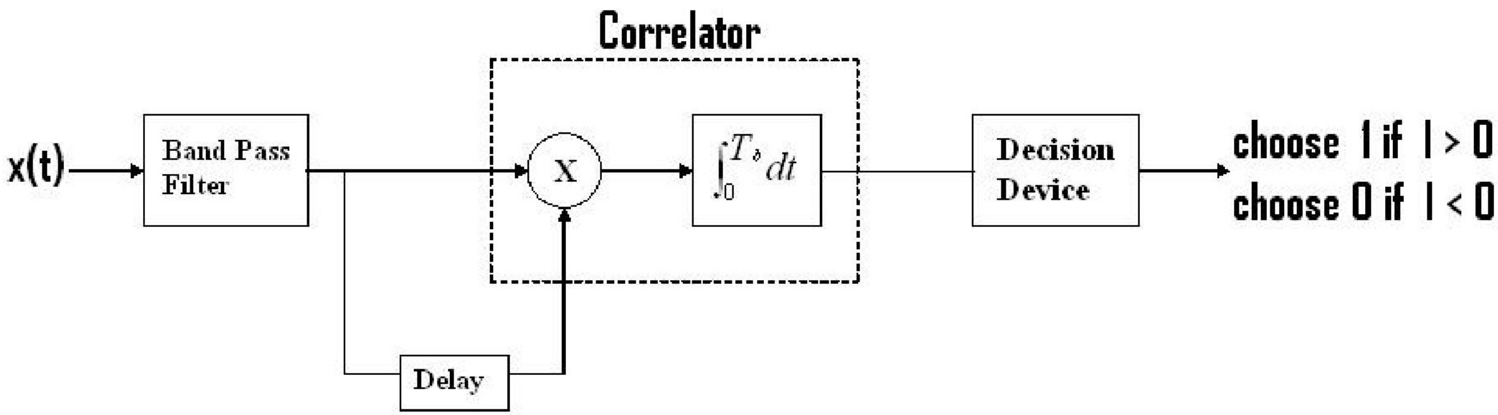
Non-coherent binary modulation techniques -DPSK



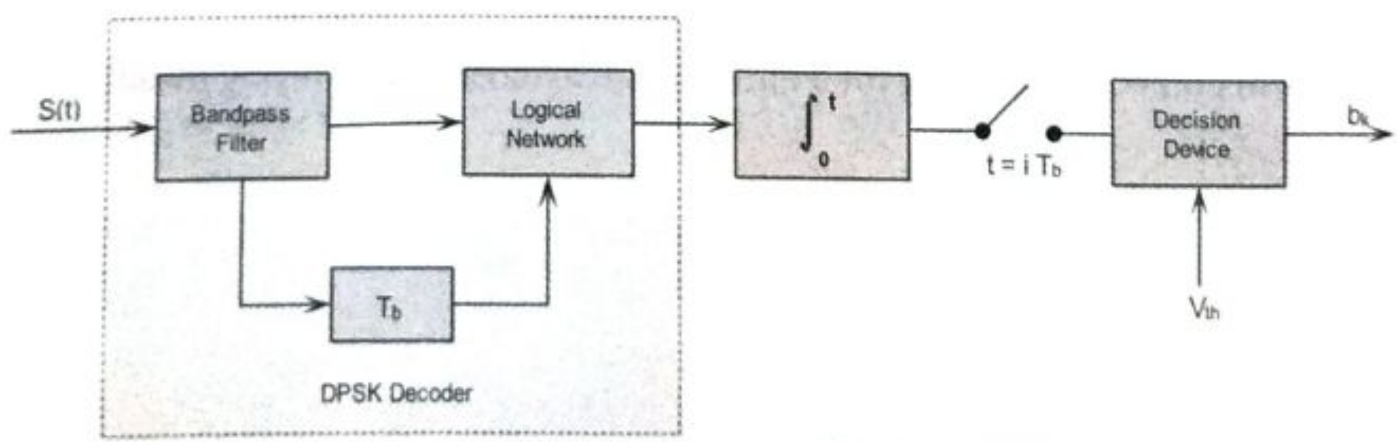
DPSK Transmitter



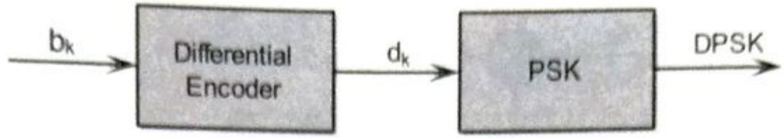
Block Diagram of DPSK Transmitter

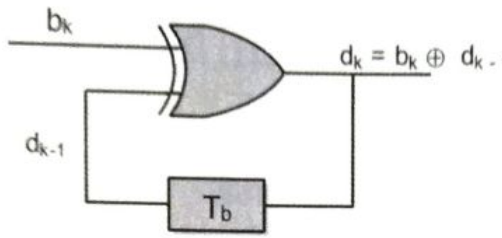


DPSK Receiver



Block Diagram of DPSK Receiver

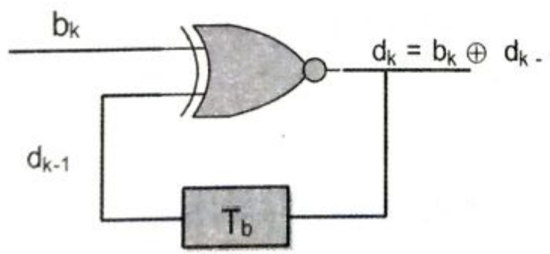




A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

b_k	1	0	1	1	1	0
d_{k-1}	0	1	1	0	1	0
dk	1	1	0	1	0	0

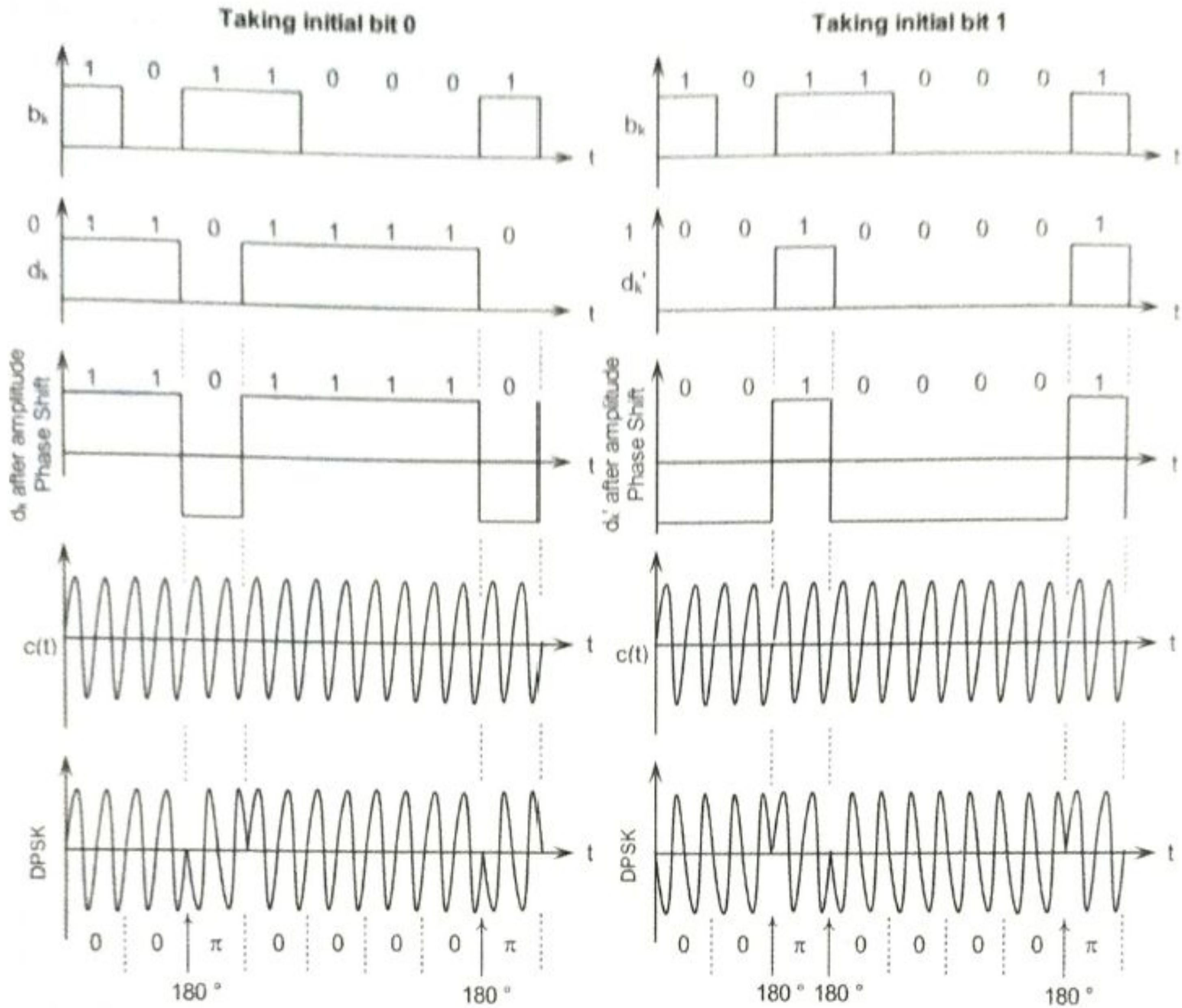
Differential encoding using XOR gate



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

b_k	1	0	1	1	1	0
d_{k-1}	0	0	1	1	1	1
dk	0	1	1	1	1	0

Differential encoding using XNOR gate



DPSK Waveforms for Initial bit 0 and Initial bit 1

Cont’d

- Hence the name differential phase shift keying [DPSK]. To send symbol ‘0’ we
- phase advance the current signal waveform by 1800 and to send symbol 1 we leave the
- phase of the current signal waveform unchanged.
- The differential encoding process at the transmitter input starts with an arbitrary
- first but, securing as reference and thereafter the differentially encoded sequence{dk} is
- generated by using the logical equation.

Cont'd

$$d_k = d_{k-1}\, b_k \oplus \overline{d_{k-1}}\; \overline{b_k}$$

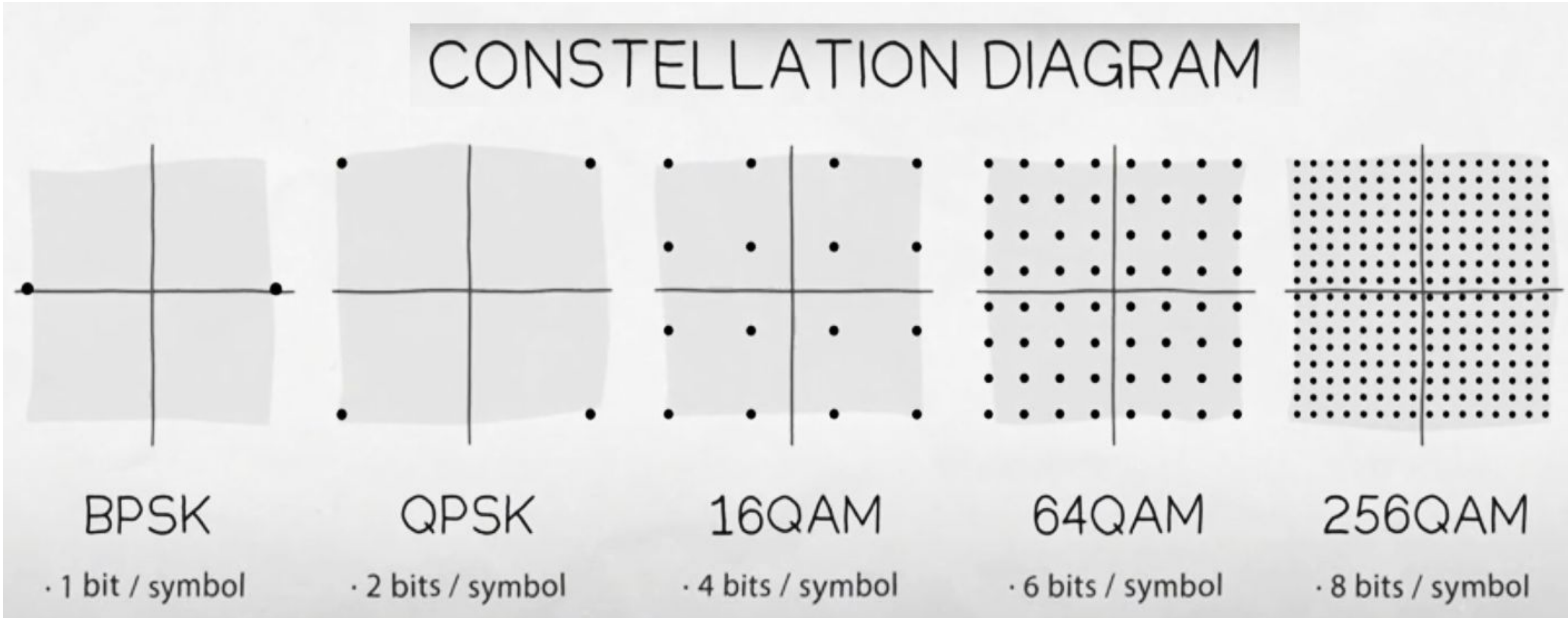
Where b_k is the input binary digit at time kT_b and d_{k-1} is the previous value of the differentially encoded digit. Table illustrate the logical operation involved in the generation of DPSK signal.

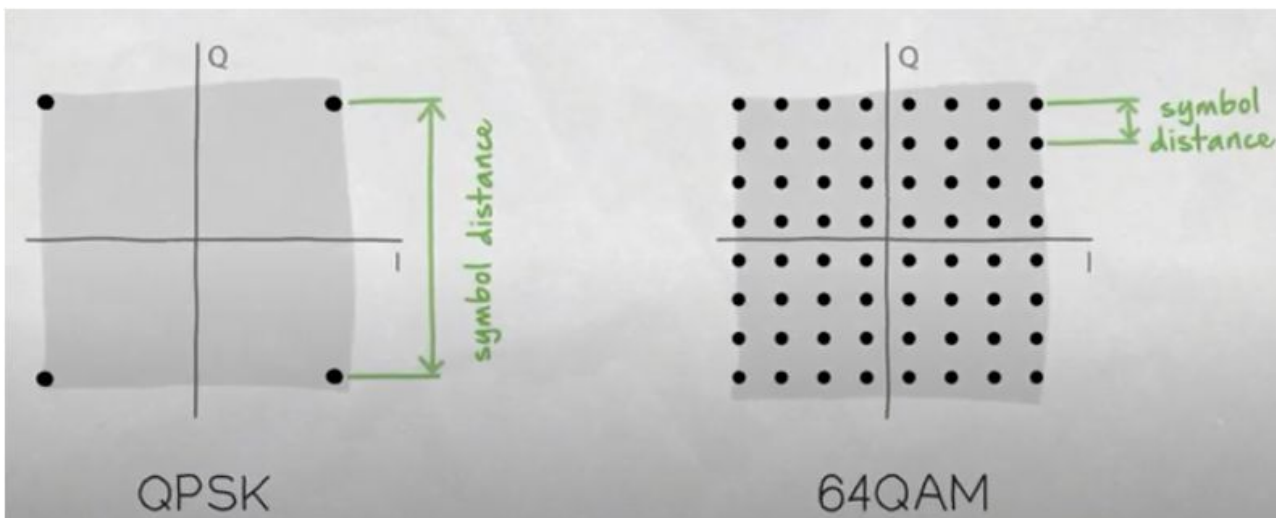
Input Binary Sequence $\{b_K\}$	1	0	0	1	0	0	1	1
Differentially Encoded sequence $\{d_K\}$ 1	1	0	1	1	0	1	1	1
Transmitted Phase 0	0	Π	0	0	Π	0	0	0
Received Sequence (Demodulated Sequence)	1	0	0	1	0	0	1	1

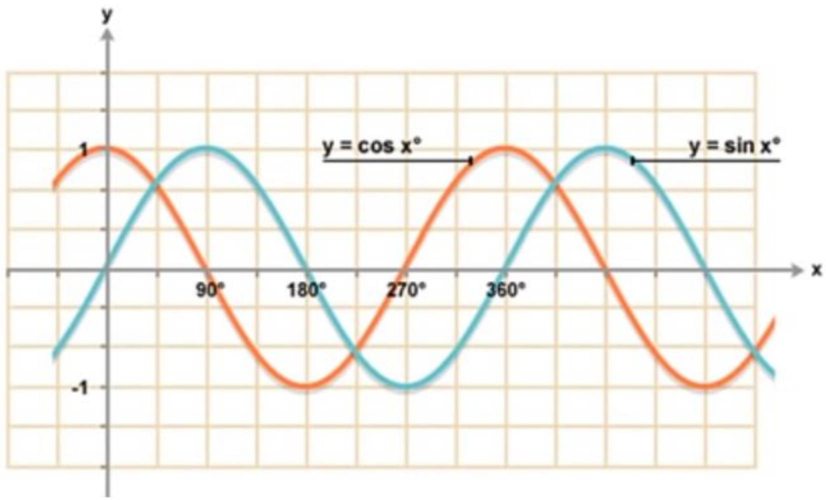
- A DPSK demodulator is as shown in fig(b). The received signal is first passed
- through a BPF centered at carrier frequency fc to limit noise power. The filter output and
- its delay version are applied to correlator the resulting output of correlator is proportional
- to the cosine of the difference between the carrier phase angles in the two correlator
- inputs. The correlator output is finally compared with threshold of ‘0’ volts .
- If correlator output is +ve -- A decision is made in favour of symbol ‘1’
- If correlator output is -ve --- A decision is made in favour of symbol ‘0’

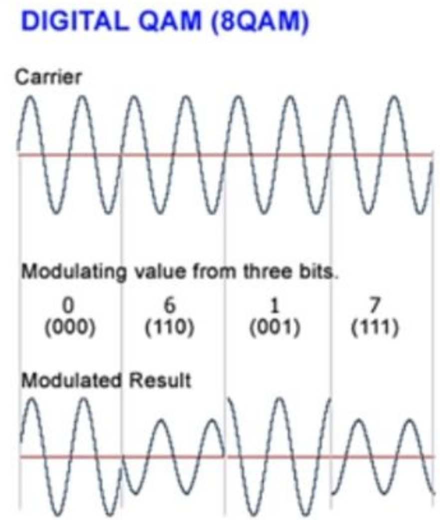
$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

QAM


CONSTELLATION DIAGRAM
BPSK
· 1 bit / symbol
QPSK
· 2 bits / symbol
16QAM
· 4 bits / symbol
64QAM
· 6 bits / symbol
256QAM
· 8 bits / symbol

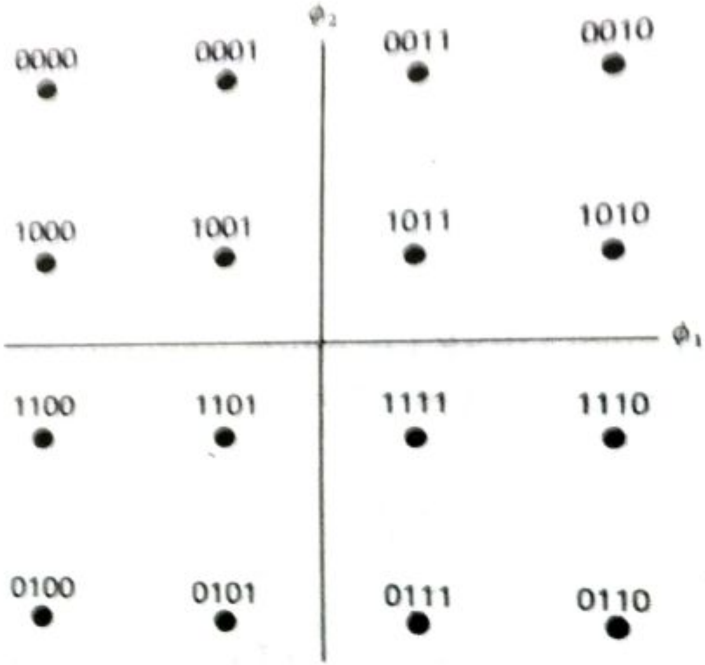

Q
I
symbol distance
QPSK
Q
I
symbol distance
64QAM

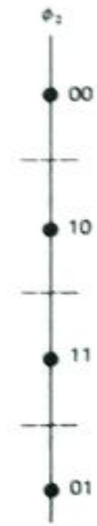

y
x
1
-1
90°
180°
270°
360°
y = cos x°
y = sin x°


DIGITAL QAM (8QAM)
Carrier
Modulating value from three bits.
0
(000)
6
(110)
1
(001)
7
(111)
Modulated Result

QAM

(a)





(b)

$$s_i(t) = \sqrt{\frac{2E_0}{T}}\, a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}}\, b_i \sin(2\pi f_c t) \qquad 0 \le t \le T$$

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \qquad 0 \le t \le T$$

The coordinates of the ith message point are $a_i\sqrt{E}$ and $b_i\sqrt{E_0}$, where (a_i, b_i) is an element of the L-by-L matrix:

$$\{a_i, b_i\} = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \dots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \dots & (L-1, L-3) \\ \vdots & \vdots & & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \dots & (L-1, -L+1) \end{bmatrix} \quad (7.116)$$

where

$$L = \sqrt{M} \quad (7.117)$$

For example, for the 16-QAM whose signal constellation is depicted in Fig. 7.24, where $L = 4$, we have the matrix

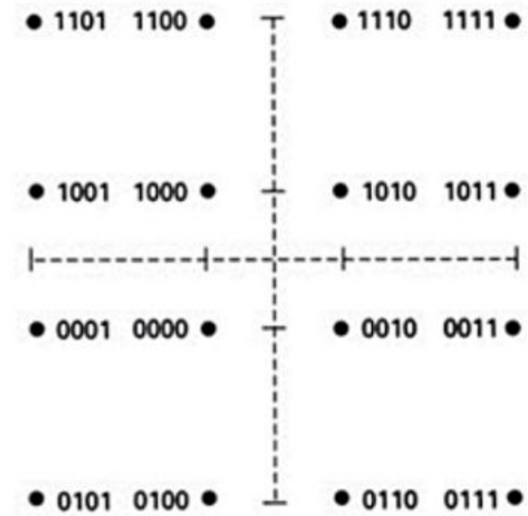
$$\{a_i, b_i\} = \begin{bmatrix} (-3,3) & (-1,3) & (1,3) & (3,3) \\ (-3,1) & (-1,1) & (1,1) & (3,1) \\ (-3,-1) & (-1,-1) & (1,-1) & (3,-1) \\ (-3,-3) & (-1,-3) & (1,-3) & (3,-3) \end{bmatrix} \quad (7.118)$$

16QAM Generation – Output Phases

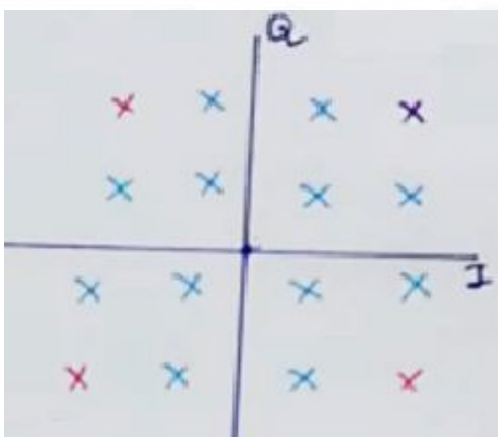
Binary input				16-QAM output	
Q	Q′	I	I′		
0	0	0	0	0.311 V	−135°
0	0	0	1	0.850 V	−165°
0	0	1	0	0.311 V	−45°
0	0	1	1	0.850 V	−15°
0	1	0	0	0.850 V	−105°
0	1	0	1	1.161 V	−135°
0	1	1	0	0.850 V	−75°
0	1	1	1	1.161 V	−45°
1	0	0	0	0.311 V	135°
1	0	0	1	0.850 V	165°
1	0	1	0	0.311 V	45°
1	0	1	1	0.850 V	15°
1	1	0	0	0.850 V	105°
1	1	0	1	1.161 V	135°
1	1	1	0	0.850 V	75°
1	1	1	1	1.161 V	45°

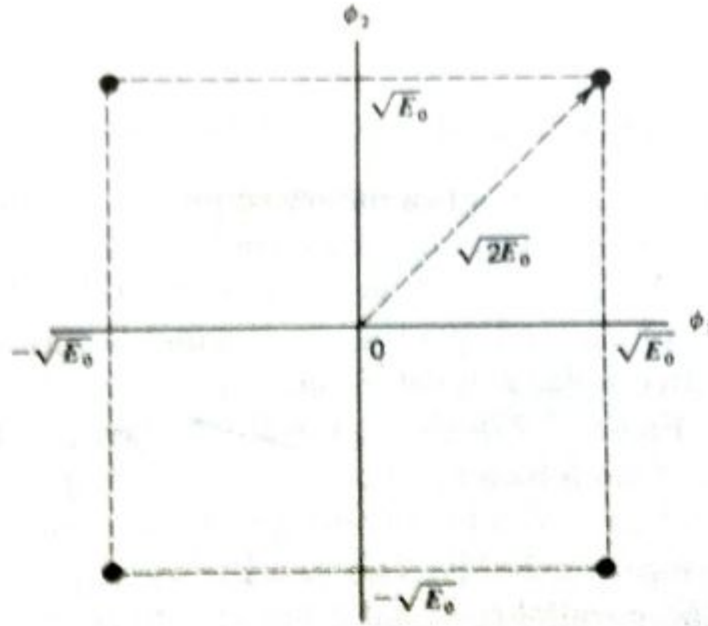
8 PHASES WITH SAME AMPLITUDE

4 PHASES WITH 2 AMPLITUDES EACH



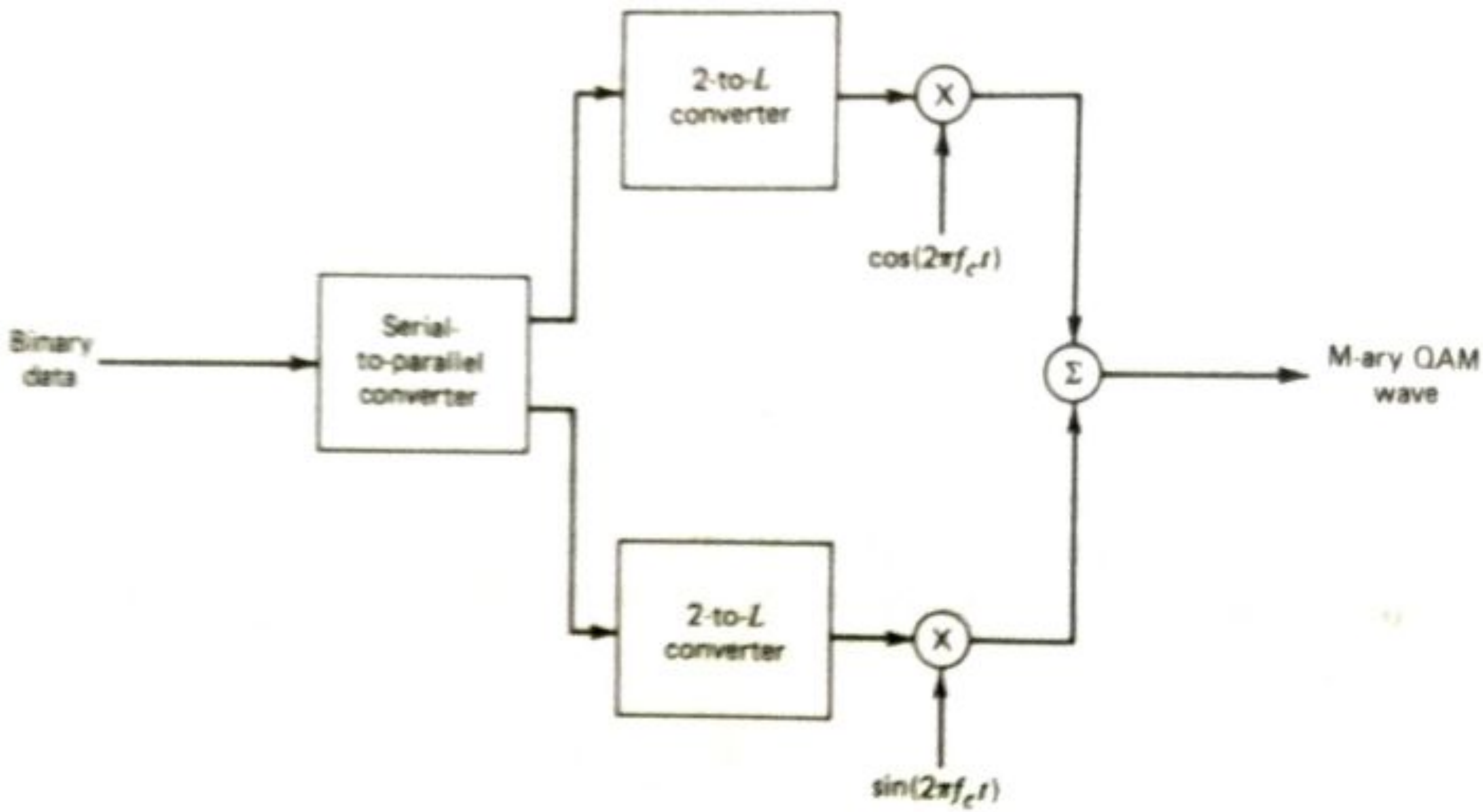
(c)



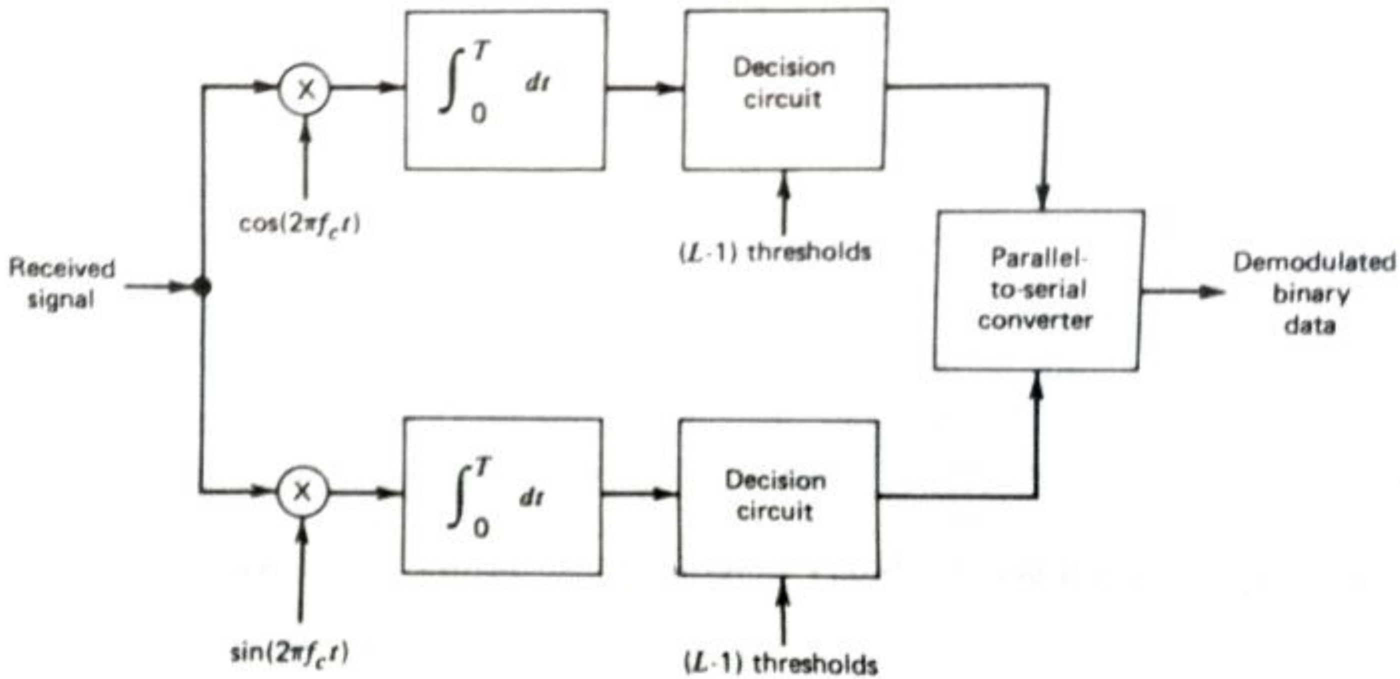


Signal constellation for the special case of M-ary QAM for $M = 4$.

QAM Transmitter



QAM Receiver



PSD

A Power Spectral Density (PSD) is the measure of signal's power content versus frequency.

A PSD is typically used to characterize broadband random signals. The amplitude of the PSD is normalized by the spectral resolution employed to digitize the signal.

PSD for different digital modulation techniques

PSD of BPSK

$$g(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases}$$

$$S_B(f) = \frac{2E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2} = 2E_b \operatorname{sinc}^2(T_b f)$$

Power spectrum falls off as the inverse square of frequency

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t \pm \frac{\pi t}{T_b}\right) \qquad 0 \leq t \leq T_b$$

$$\begin{aligned} s(t) &= \sqrt{\frac{2E_b}{T_b}} \cos\left(\pm \frac{\pi t}{T_b}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{T_b}} \sin\left(\pm \frac{\pi t}{T_b}\right) \sin(2\pi f_c t) \\ &= \sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi t}{T_b}\right) \cos(2\pi f_c t) \mp \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi t}{T_b}\right) \sin(2\pi f_c t) \qquad (7.138) \end{aligned}$$

$$g(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \sin\left(\dfrac{\pi t}{T_b}\right) & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases} \tag{7.139}$$

The energy spectral density of this symbol shaping function equals

$$\Psi_g(f) = \frac{8E_b T_b \cos^2(\pi T_b f)}{\pi^2(4T_b^2 f^2 - 1)^2} \tag{7.140}$$

PSD of BPSK and BFSK

BPSK

$$S_B(f) = \frac{2E_b \sin^2(\pi T_b f)}{(\pi T_b f)^2}$$
$$= 2E_b \operatorname{sinc}^2(T_b f)$$

BFSK

$$S_B(f) = \frac{E_b}{2T_b}\left[\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right] + \frac{8E_b \cos^2(\pi T_b f)}{\pi^2(4T_b^2 f^2 - 1)^2}$$

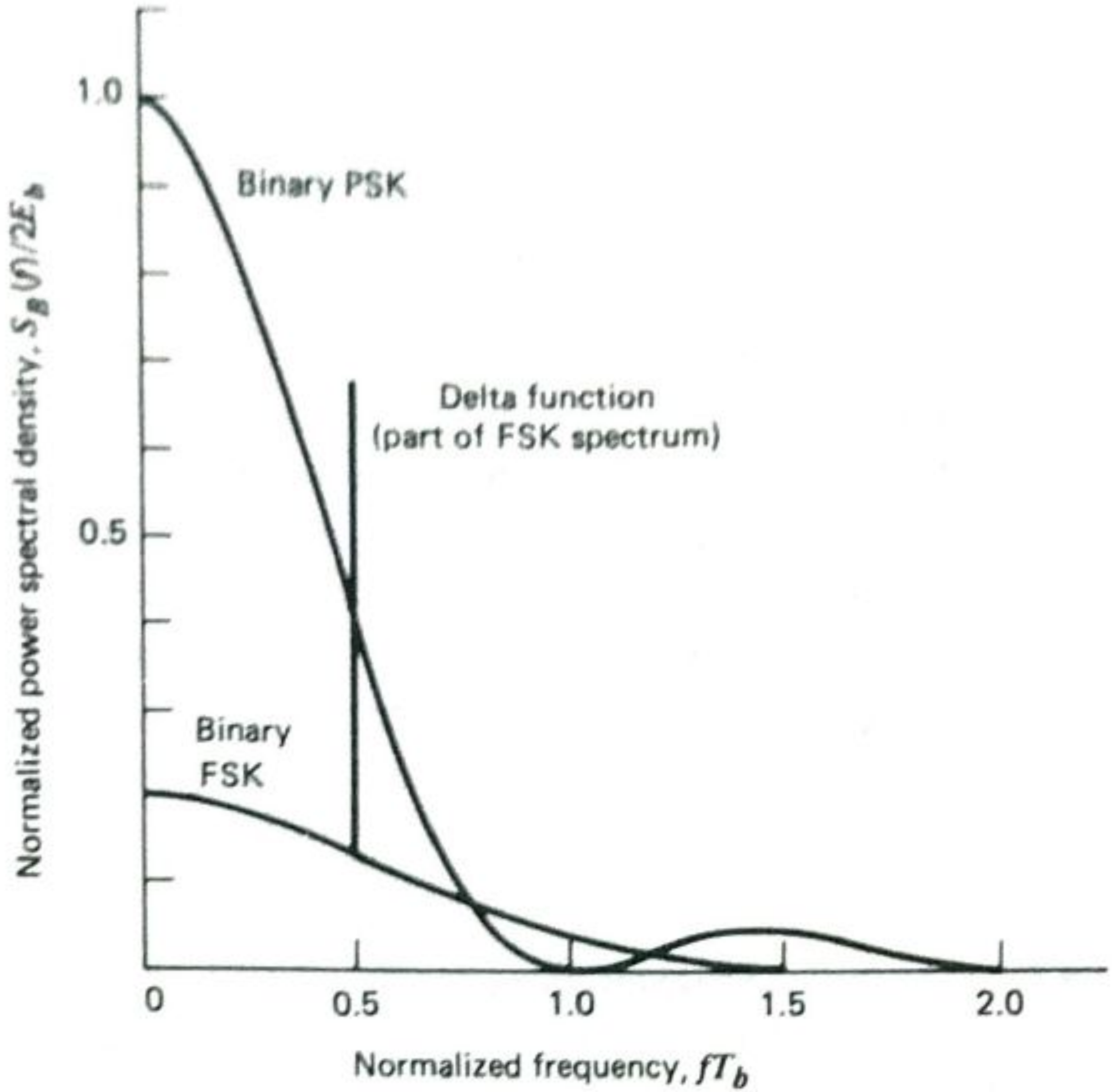


Figure 7.29 Power spectra of binary PSK and FSK signals.

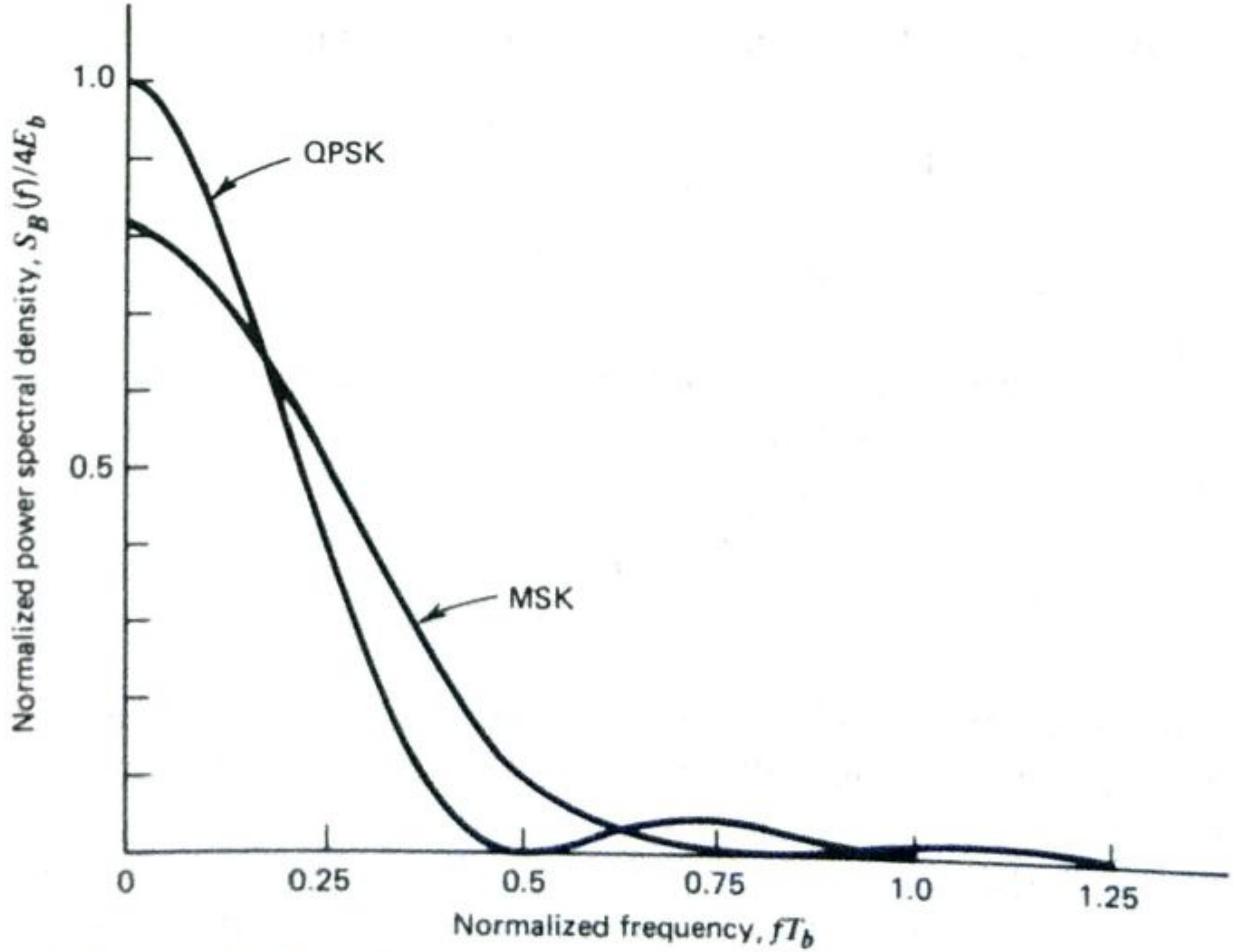


Figure 7.30 Power spectra of QPSK and MSK signals.