* CONVOLUTION CODING:-

In block codes, a block of 'n' digits generated by the encoder in a particular time unit depends only on one block of 'k' input message digits within that time unit.

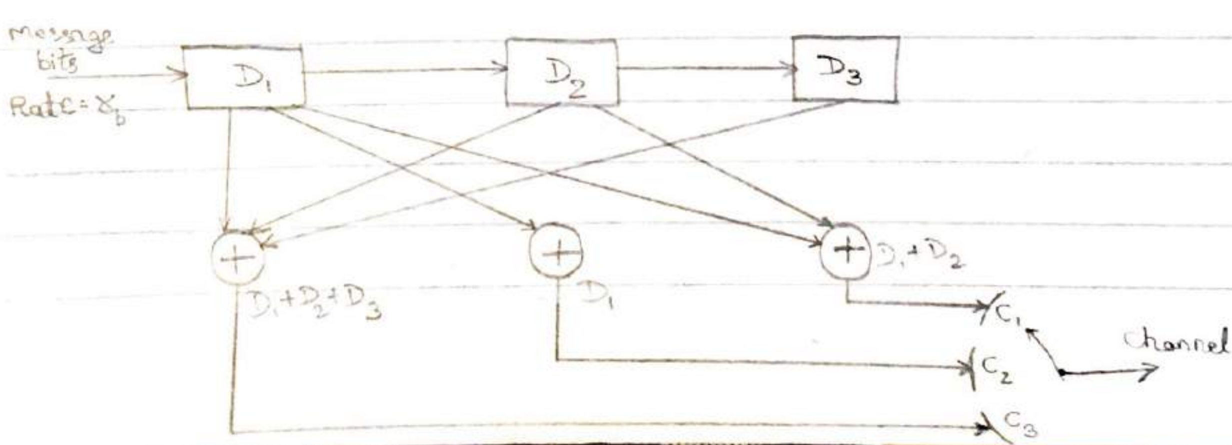
A convolution encoder takes a sequence of message digits & generates a sequence of code digits. In any time unit, a message block consisting of 'k' digits is fed into the encoder & the encoder generates a code block consisting of 'n' code digits.

The 'n' digit code word depends not only on 'k' digit message block of the same time unit but also on the previous (m-1) message block.

The code generated by the above encoder is called (n, k, m) convolution code of constrained length "nm" digits & rate efficiency "k/n" where n = no of outputs = no of modulo 2 adders,
k = no of i/p bits entering at any time
m = no of stages of the flip-flop.

The block codes are better suited for error detection & convolution codes for error correction.

Ex:- Consider an encoder for (n, k, m) = (3, 1, 3) to generate a convolution code as shown below:



Let $d_K = \underset{d_1 d_2 d_3 d_4 d_5}{10110}$

	0 – T_b	T_b – $2T_b$	$2T_b$ – $3T_b$	$3T_b$ – $4T_b$	$4T_b$ – $5T_b$	$5T_b$ – $6T_b$	$6T_b$ – $7T_b$
	d_1	d_2	d_3	d_4	d_5		
Input →	1	0	1	1	0	0	0
Contents of SR →	100	010	101	110	011	001	000
output →	111	101	011	010	001	100	000

In convolution encoder, the message ~~tree~~ stream continuously runs through the encoder whereas in block coding schemes, the message stream is first divided into long blocks & then encoded.

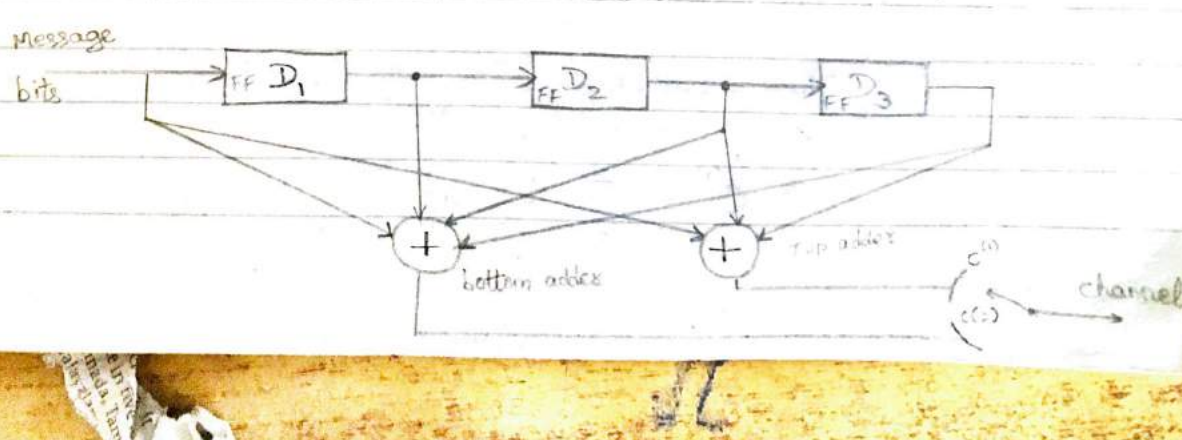
In general, there are 2 methods of generating convolution codes.

i) Time domain approach

ii) Transfer domain approach

⇒ Encoding of convolution codes using time domain approach :-

P) Consider a $(n, K, m) = (2, 1, 3)$ convolution encoder as shown in fig. Determine the codes using time domain approach & transfer domain approach.



The time domain behaviour of a binary convolution encoder may be defined in terms of a set of 'n' impulse responses. Let the sequence $[g_1^{(j)}\ g_2^{(j)}\ g_3^{(j)} \ldots\ g_{m+1}^{(j)}]$ denote the impulse responses, also called GENERATOR sequences of the input/output path of 'n' no of modulo-2 adders.

In the encoder, there are 2 modulo-2 adders labelled top adder & bottom adder. Hence, there will be 2 generator sequences.

Let $d_1 d_2 d_3 \ldots d_L$ represent the input message sequence that enters into the encoder one bit at a time starting with $\underline{d_1}$.

Then, the encoder generates 2 o/p sequences $C^{(1)}$ & $C^{(2)}$ defined by the discrete convolution sum given by

$$C^{(1)} = [d] * g^{(1)}$$
$$C^{(2)} = [d] * g^{(2)}$$

We have, $g^{(1)} = [1011]$; $g^{(2)} = [1111]$

From the definition of discrete convolution,

$$C_l^{j} = \sum_{i=0}^{m} d_{l-i}\, g_{i+1}^{j}$$

L = no of message sequence bits

where i varies from 0 to m = (0 to 3)

l varies from 1 to $(L+m)$ = (1 to 8)

$d_{l-i} = 0$, for $l \leq i$

Let the message sequence be $\underset{d_1 d_2 d_3 d_4 d_5}{10111}$

The o/p sequence is calculated as follows:

For $j = 1$,

$$C_l^{(1)} = \sum_{i=0}^{3} d_{l-i}\, g_{i+1}^{(1)}$$

Codeword = L + m

L = no. of message bits
m = no. of FFs.

$$c_l^{(1)} = d_l g_1^{(1)} + d_{l-1} g_2^{(1)} + d_{l-2} g_3^{(1)} + d_{l-3} g_4^{(1)}$$

$l=1$ $\quad c_1^{(1)} = d_1 g_1^{(1)} + 0 + 0 + 0 = (1)(1) = 1$

$l=2$ $\quad c_2^{(1)} = d_2 g_1^{(1)} + d_1 g_2^{(1)} + 0 + 0 = (0)(1) + 1(0) = 0$

$l=3$ $\quad c_3^{(1)} = d_3 g_1^{(1)} + d_2 g_2^{(1)} + d_1 g_3^{(1)} + 0 = (1)(1) + (0)(0) + (1)(1) = 0$

$l=4$ $\quad c_4^{(1)} = d_4 g_1^{(1)} + d_3 g_2^{(1)} + d_2 g_3^{(1)} + d_1 g_4^{(1)} = (1)(1) + (1)(0) + (0)(1) + (1)(1) = 0$

$c_5^{(1)} = d_5 g_1^{(1)} + d_4 g_2^{(1)} + d_3 g_3^{(1)} + d_2 g_4^{(1)} = (1)(1) + (0)(1) + (1)(1) + (1)(0) = 0$

$c_6^{(1)} = d_6 g_1^{(1)} + d_5 g_2^{(1)} + d_4 g_3^{(1)} + d_3 g_4^{(1)} = (0)(1) + (1)(0) + (1)(1) + (1)(1) = 0$

$c_7^{(1)} = d_7 g_1^{(1)} + d_6 g_2^{(1)} + d_5 g_3^{(1)} + d_4 g_4^{(1)} = (0) + 0 + (1)(1) + (1)(1) = 0$

$c_8^{(1)} = d_8 g_1^{(1)} + d_7 g_2^{(1)} + d_6 g_3^{(1)} + d_5 g_4^{(1)} = 0 + 0 + 0 + 1 = 1$

$$\therefore \boxed{c^{(1)} = 10000001}$$

For $j = 2$

$$c_l^{(2)} = \sum_{i=0}^{3} d_{l-i}\, g_{i+1}^{(2)}$$

$$= d_l g_1^{(2)} + d_{l-1} g_2^{(2)} + d_{l-2} g_3^{(2)} + d_{l-3} g_4^{(2)}$$

But $g^{(2)} = [1111]$

$$\therefore \quad c_l^{(2)} = d_l + d_{l-1} + d_{l-2} + d_{l-3}$$

$c_1^{(2)} = d_1 + 0 + 0 + 0 = 1$

$c_2^{(2)} = d_2 + d_1 + 0 + 0 = 1$

$$C_3^{(2)} = d_3 + d_2 + d_1 + 0 = 0$$
$$C_4^{(2)} = d_4 + d_3 + d_2 + d_1 = 1$$
$$C_5^{(2)} = d_5 + d_4 + d_3 + d_2 = 1$$
$$C_6^{(2)} = 0 + d_5 + d_4 + d_3 = 1$$
$$C_7^{(2)} = 0 + 0 + d_5 + d_4 = 0$$
$$C_8^{(2)} = 0 + 0 + 0 + d_5 = 1$$

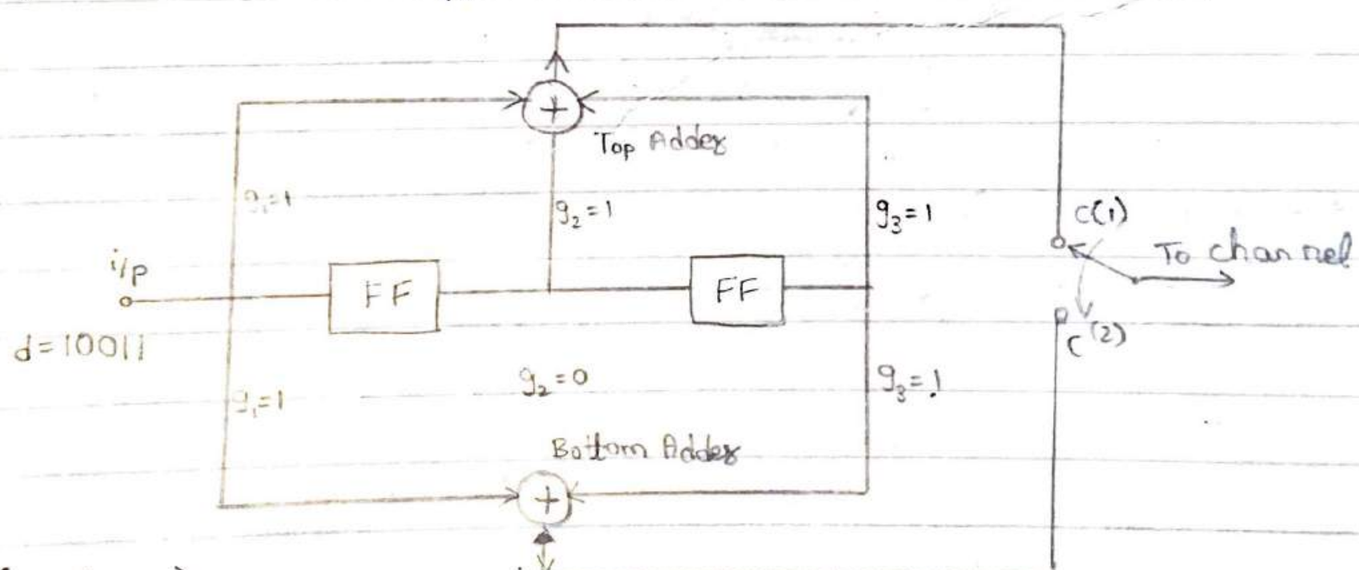
$$\therefore \boxed{C^{(2)} = [1\,1\,0\,1\,1\,1\,0\,1]}$$

After encoding, the 2 o/p sequences are multiplexing into a single sequence called Code word for transmission over channel.
The code word at o/p of convolution encoder is given in general as

$$[C_1^{(1)}\ C_1^{(2)}\ C_2^{(1)}\ C_2^{(2)}\ C_3^{(1)}\ C_3^{(2)}\ \ldots\ldots\ C_l^{(1)}\ C_l^{(2)}]$$

$\therefore$ Code word = 1101000101010011

P> For the convolution encoder shown, d = 10011 ($d_1 d_2 d_3 d_4 d_5$). Find the output sequence using the following 2 approaches.
i) Time domain approach ii) Transfer domain approach.



(n, k, m)
= (2, 1, 2)

$g^{(1)} = g_1^{(1)} g_2^{(1)} g_3^{(1)} g_4^{(1)} \ldots\ldots g_{m+1}^{(1)}$

$g^{(2)} = g_1^{(2)} g_2^{(2)} g_3^{(2)} g_4^{(2)} \ldots\ldots g_{m+1}^{(2)}$

$\therefore \quad g^{(1)} = (111) \quad ; \quad g^{(2)} = (101)$

$L = 5 \quad ; \quad m = 2 \quad ; \quad l = L + m = 7$

$$C_l^{(j)} = \sum_{i=0}^{m} d_{l-i}\, g_{i+1}^{j}$$

10011
$d_1 d_2 d_3 d_4 d_5$

$j=1$ $\quad C_l^{(1)} = \sum_{i=0}^{2} d_{l-i}\, g_{i+1}^{(1)}$

$C_l^{(1)} = d_l g_1^{(1)} + d_{l-1} g_2^{(1)} + d_{l-2} g_3^{(1)}$

$= d_l(1) + d_{l-1}(1) + d_{l-2}(1)$

$C_l^{(1)} = d_l + d_{l-1} + d_{l-2}$

$l = 1 \Rightarrow C_1^{(1)} = d_1 + 0 + 0 = 1$
$l = 2 \Rightarrow C_2^{(1)} = d_2 + d_1 + 0 = 1$
$l = 3 \Rightarrow C_3^{(1)} = d_3 + d_2 + d_1 = 1$
$l = 4 \Rightarrow C_4^{(1)} = d_4 + d_3 + d_2 = 1$
$l = 5 \Rightarrow C_5^{(1)} = d_5 + d_4 + d_3 = 0$
$l = 6 \Rightarrow C_6^{(1)} = 0 + d_5 + d_4 = 0$
$l = 7 \Rightarrow C_7^{(1)} = 0 + 0 + d_5 = 1$

$\therefore \quad \boxed{C^{(1)} = 1110001}$

$j=2$ $\quad C_l^{(2)} = \sum_{i=0}^{2} d_{l-i}\, g_{i+1}^{(2)}$

$= d_l g_1^{(2)} + d_{l-1} g_2^{(2)} + d_{l-2} g_3^{(2)}$

$$= d_l(1) + d_{l-1}(0) + d_{l-2}(1)$$

1 0 0 1 1
$d_1 d_2 d_3 d_4 d_5$

$$c_l^{(2)} = d_l + d_{l-2}$$

$l=1 \Rightarrow c_1^{(2)} = d_1 + 0 = 1$
$l=2 \Rightarrow c_2^{(2)} = d_2 + 0 = 0$
$l=3 \Rightarrow c_3^{(2)} = d_3 + d_1 = 1$
$l=4 \Rightarrow c_4^{(2)} = d_4 + d_2 = 1$
$l=5 \Rightarrow c_5^{(2)} = d_5 + d_3 = 1$
$l=6 \Rightarrow c_6^{(2)} = d_6 + d_4 = 0 + d_4 = 1$
$l=7 \Rightarrow c_7^{(2)} = d_7 + d_5 = 0 + d_5 = 1$

$\therefore \boxed{c^{(2)} = 1011111}$

$\therefore$ $c^{(1)} = 1111001$
$c^{(2)} = 1011111$

$\therefore$ Code word $= [11,10,11,11,01,01,11]$

* MATRIX METHOD :-

The generator sequence
$g_1^{(1)} g_2^{(1)} g_3^{(1)} \ldots g_{m+1}^{(1)}$
for the top adder and
$g_1^{(2)} g_2^{(2)} g_3^{(2)} \ldots g_{m+1}^{(2)}$
for the bottom adder can be interlaced & arranged in a matrix form with the no of rows equal to no of digits in the message sequence i.e, L rows & no of columns equal to $n(L+m)$. Such matrix of the order $\{L \times n(L+m)\}$ is called GENERATOR MATRIX of the

convolution encoder. In general, for 2 modulo-2 adders convolution encoder, the generator matrix is given by

$$G = \begin{bmatrix} g_1^{(1)} g_1^{(2)} & g_2^{(1)} g_2^{(2)} & g_3^{(1)} g_3^{(2)} & \cdots & g_{m+1}^{(1)} g_{m+1}^{(2)} & 0 0 \cdots & 0 0 \\ 0 \; 0 & g_1^{(1)} g_1^{(2)} & g_2^{(1)} g_2^{(2)} & \cdots & g_m^{(1)} \; g_m^{(2)} & g_{m+1}^{(1)} g_{m+1}^{(2)} & \cdots 0 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 \; 0 & 0 \; 0 & \cdots & & \cdots & & g_{m+1}^{(1)} g_{m+1}^{(2)} \end{bmatrix}$$

P) Previous same problem:

$d = 10111$

$g^{(1)} = 1011$

$g^{(2)} = 1111$

$L = 5$; $C = n(L+m) = 2(5+3) = 16$

$\therefore$ A matrix of (5×16)

$$G = \begin{bmatrix} 11 & 01 & 11 & 11 & 00 & 00 & 00 & 00 \\ 00 & 11 & 01 & 11 & 11 & 00 & 00 & 00 \\ 00 & 00 & 11 & 01 & 11 & 11 & 00 & 00 \\ 00 & 00 & 00 & 11 & 01 & 11 & 11 & 00 \\ 00 & 00 & 00 & 00 & 11 & 01 & 11 & 11 \end{bmatrix}$$

$C = [d][G]$

$= [10111][G]$

$$\boxed{C = 11, 01, 00, 01, 01, 01, 00, 11}$$

P) Same previous problem:

$d = 10011$

$g^{(1)} = 111$

$g^{(2)} = 101$

$R = L = 5$ rows

$C = n(L+m) = 2(5+2) = 14$ columns

$\therefore$ Matrix is of order (5×14)

$$G = \begin{bmatrix} 11 & 10 & 11 & 00 & 00 & 00 & 00 \\ 00 & 11 & 10 & 11 & 00 & 00 & 00 \\ 00 & 00 & 11 & 10 & 11 & 00 & 00 \\ 00 & 00 & 00 & 11 & 10 & 11 & 00 \\ 00 & 00 & 00 & 00 & 11 & 10 & 11 \end{bmatrix}$$

$C = [d][G]$

$= [10011][G]$

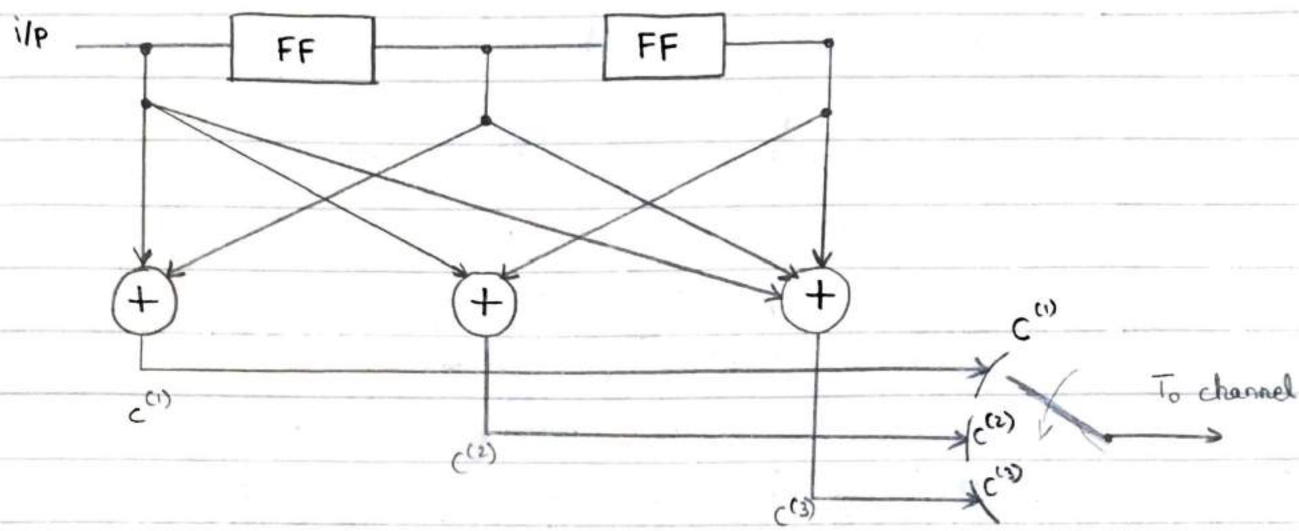
$$\boxed{C = [11, 10, 11, 11, 01, 01, 11]}$$

P) Consider a $(3,1,2)$ convolution code with $g^{(1)} = 110$, $g^{(2)} = 101$ and $g^{(3)} = 111$.

a) Draw the encoder block diagram

b) Find the generator matrix

c) Find the code word corresponding to $d = 11110$ using time domain approach.



Matrix will have $L=6$ rows

& $n(L+m) = 3(6+2) = 24$ columns

$g^{(1)} = 110$
$g^{(2)} = 101$
$g^{(3)} = 111$

$$G = \begin{bmatrix} 111 & 101 & 011 & 000 & 000 & 000 & 000 & 000 \\ 000 & 111 & 101 & 011 & 000 & 000 & 000 & 000 \\ 000 & 000 & 111 & 101 & 011 & 000 & 000 & 000 \\ 000 & 000 & 000 & 111 & 101 & 011 & 000 & 000 \\ 000 & 000 & 000 & 000 & 111 & 101 & 011 & 000 \\ 000 & 000 & 000 & 000 & 000 & 111 & 101 & 011 \end{bmatrix}$$

$d = [111101]$

$C = [d][G]$

$= [111; 010; 001; 001; 110; 100; 101; 011]$

* TRANSFORM DOMAIN METHOD :-

For j no of modulo-2 adders (where j varies from 1 to n), the Generator Polynomial is

$$g^j(x) = g_1^j + x g_2^j + x^2 g_3^j + x^3 g_4^j + \ldots\ldots + x^m g_{m+1}^j$$

where $j \to 1$ to n

The corresponding o/p of each of the adder is given by $C^j(x) = d(x)\, g^j(x)$

where $d(x)$ is message vector polynomial.

After getting the polynomials at the o/p of each of the adder, the final encoder o/p polynomial is obtained in the form

$$C(x) = c^{(1)}(x^n) + x\, c^{(2)}(x^n) + x^2 c^{(3)}(x^n) + \ldots + x^{n-1} c^{(n)}(x^n)$$

p>

$g^{(1)} = 1011$

$g^{(2)} = 1111$

$$g^{(1)}(x) = 1 + 0.x + 1.x^2 + 1.x^3 = 1 + x^2 + x^3$$

$$g^{(2)}(x) = 1 + 1.x + 1.x^2 + 1.x^3 = 1 + x + x^2 + x^3$$

$d = 10111$

$$d(x) = 1 + 0\cdot x + 1.x^2 + 1.x^3 + 1.x^4 = 1 + x^2 + x^3 + x^4$$

$$c^{(1)}(x) = d(x)\, g^{(1)}(x)$$

$$= (1 + x^2 + x^3 + x^4)(1 + x^2 + x^3)$$

$$= 1 + \cancel{x^2} + \cancel{x^3} + \cancel{x^2} + \cancel{x^4} + \cancel{x^5} + \cancel{x^3} + \cancel{x^5} + \cancel{x^6} + \cancel{x^4} + \cancel{x^6} + x^7$$

$$= 1 + x^7$$

$$c^{(1)}(x) = 1 + x^7$$

$$c^{(2)}(x) = d(x)\, g^{(2)}(x)$$
$$= (1+x^2+x^3+x^4)(1+x+x^2+x^3)$$
$$= 1+x+\cancel{x^2}+\cancel{x^3}+\cancel{x^2}+\cancel{x^3}+\cancel{x^4}+\cancel{x^5}+x^3+\cancel{x^4}+\cancel{x^5}+\cancel{x^6}$$
$$+x^4+x^5+\cancel{x^6}+x^7$$
$$= 1+x+x^3+x^4+x^5+x^7$$

$$C(x) = c^{(1)}x^n + x\, c^{(2)}x^n$$
$$= c^{(1)}(x)^2 + x\, c^{(2)}(x)^2$$
$$\cancel{= (1+x^7)x^2 + x(1+x+x^3+x^4+x^5+x^7)x^2}$$
$$\cancel{= x^2+x^9+}$$
$$= (1+x^7)^2 + x\,(1+x+x^3+x^4+x^5+x^7)^2$$
$$= 1+x^{14} + x(1+x^2+x^6+x^8+x^{10}+x^{14})$$
$$= 1+x^{14}+x+x^3+x^7+x^9+x^{11}+x^{15}$$
$$= 1+x+x^3+x^7+x^9+x^{11}+x^{14}+x^{15}$$
$$\therefore\ C = [11,01,00,01,01,01,00,11]$$

$\left[(1+x^7)^2 = 1+x^7+x^7+x^{14}\right.$
$= 1+x^{14}$
$\left.\text{III}^{ly} \text{ for } 2^{nd} \text{ term also}\right]$

P)
$$g^{(1)} = 111$$
$$g^{(2)} = 101$$
$$d = 10011$$
$$g^{(1)}(x) = 1+x+x^2$$
$$g^{(2)}(x) = 1+x^2 \quad ; \quad d(x) = 1+x^3+x^4$$
$$c^{(1)}(x) = d(x)\, g^{(1)}(x)$$
$$= (1+x^3+x^4)(1+x+x^2)$$
$$= 1+x+x^2+x^3+\cancel{x^4}+\cancel{x^5}+\cancel{x^4}+\cancel{x^5}+x^6$$
$$c^{(1)}(x) = 1+x+x^2+x^3+x^6$$
$$c^{(2)}(x) = d(x)\, g^{(2)}(x) = (1+x^3+x^4)(1+x^2)$$
$$= 1+x^2+x^3+x^5+x^4+x^6$$
$$= 1+x^2+x^3+x^4+x^5+x^6$$

$$C(x) = c^{(1)}(x)^{n} + x.c^{(2)}(x)^{n}$$
$$= c^{(1)}(x)^2 + x\cdot c^{(2)}(x)^2$$
$$= \{1+x+x^2+x^3+x^6\}^2 + x\{1+x^2+x^3+x^4+x^5+x^6\}^2$$
$$= 1+x^2+x^4+x^6+x^{12}+x+x^5+x^7+x^9+x^{11}+x^{13}$$
$$= 1+x+x^2+x^4+x^5+x^6+x^7+x^9+x^{11}+x^{12}+x^{13}$$
$$C = \boxed{[11101111010111]}$$

$$g^{(1)} = 110$$
$$g^{(2)} = 101$$
$$g^{(3)} = 111$$
$$d = 111101$$
$$g^{(1)}(x) = 1+x$$
$$g^{(2)}(x) = 1+x^2$$
$$g^{(3)}(x) = 1+x+x^2$$
$$d(x) = 1+x+x^2+x^3+x^5$$

$$c^{(1)}(x) = d(x)\cdot g^{(1)}(x)$$
$$= (1+x+x^2+x^3+x^5)(1+x)$$
$$= 1+\cancel{x}+\cancel{x^2}+\cancel{x^3}+x^5+\cancel{x}+\cancel{x^2}+\cancel{x^3}+x^4+x^6$$
$$= 1+x^4+x^5+x^6$$

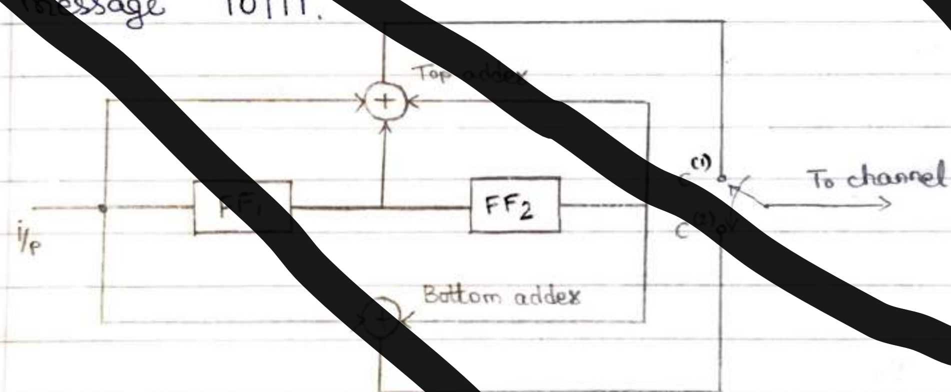
$$c^{(2)}(x) = d(x)\, g^{(2)}(x)$$
$$= (1+x+x^2+x^3+x^5)(1+x^2)$$
$$= 1+x+\cancel{x^2}+\cancel{x^3}+\cancel{x^5}+\cancel{x^2}+\cancel{x^3}+x^4+\cancel{x^5}+x^7$$
$$= 1+x+x^4+x^7$$

$$c^{(3)}(x) = d(x)\, g^{(3)}(x)$$
$$= (1+x+x^2+x^3+x^5)(1+x+x^2)$$
$$= 1+\cancel{x}+x^2+x^3+\cancel{x^5}+\cancel{x}+\cancel{x^2}+\cancel{x^3}+\cancel{x^4}+x^6$$
$$+\cancel{x^2}+\cancel{x^3}+\cancel{x^4}+\cancel{x^5}+x^7$$
$$= 1+x^2+x^3+x^6+x^7$$

$$c(x) = c^{(1)}(x)^n + x\,c^{(2)}(x)^n + x^2 c^{(3)}(x)^n$$
$$= c^{(1)}(x^3) + x\,c^{(2)}(x^3) + x^2 c^{(3)}(x^3)$$
$$= 1 + x^{12} + x^{15} + x^{18} + x\{1 + x^3 + x^{12} + x^{21}\}$$
$$+ x^2\{1 + x^6 + x^9 + x^{18} + x^{21}\}$$
$$= 1 + x^{12} + x^{15} + x^{18} + x + x^4 + x^{13} + x^{22} + x^2 + x^8 + x^{11} + x^{20} + x^{23}$$
$$= 1 + x + x^2 + x^4 + x^8 + x^{11} + x^{12} + x^{13} + x^{15} + x^{18} + x^{20} + x^{22} + x^{23}$$
$$C = [111, 010, 001, 001, 110, 100, 101, 011]$$

* [redacted]TATE DIAGRAM & CO[redacted]E TREE:-

P) [redacted]nsider the binary con[redacted]tion encoder shown in the [redacted]e. Draw the state ta[redacted], state transition table, [redacted] diagram & correspondi[redacted] code tree. Using the code t[redacted] find the encoded seque[redacted] for the [redacted]ssage 10111.



[redacted]tate table :-

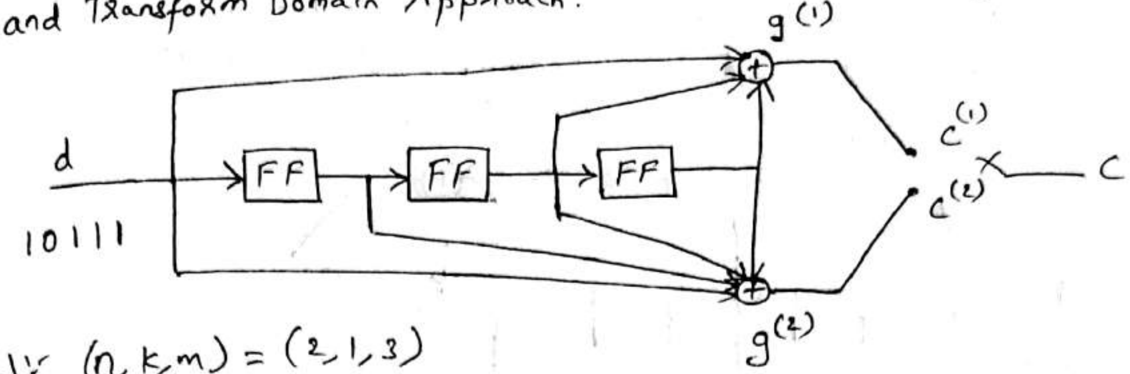
$S_0 \leftarrow 00$

[redacted] $\leftarrow 01$

S_2 [redacted] 10

S_3 [redacted]

Obtain the output of the Convolutional encoder using Time Domain and Transform Domain Approach.



Sol: $(n, k, m) = (2, 1, 3)$

i) Time Domain Approach

$g^{(1)} = 1011 \qquad g^{(2)} = 1111$

Order of $[G] = L \times n(L+m) = 5 \times 16$

$$G = \begin{bmatrix} 11 & 01 & 11 & 11 & 00 & 00 & 00 & 00 \\ 00 & 11 & 01 & 11 & 11 & 00 & 00 & 00 \\ 00 & 00 & 11 & 01 & 11 & 11 & 00 & 00 \\ 00 & 00 & 00 & 11 & 01 & 11 & 11 & 00 \\ 00 & 00 & 00 & 00 & 11 & 01 & 11 & 11 \end{bmatrix}$$

$$C = [D][G]$$

$$= [10111]\begin{bmatrix} 11 & 01 & 11 & 11 & 00 & 00 & 00 & 00 \\ 00 & 11 & 01 & 11 & 11 & 00 & 00 & 00 \\ 00 & 00 & 11 & 01 & 11 & 11 & 00 & 00 \\ 00 & 00 & 00 & 11 & 01 & 11 & 11 & 00 \\ 00 & 00 & 00 & 00 & 11 & 01 & 11 & 11 \end{bmatrix}$$

$$C = [11 \ 01 \ 00 \ 01 \ 01 \ 01 \ 00 \ 11]$$

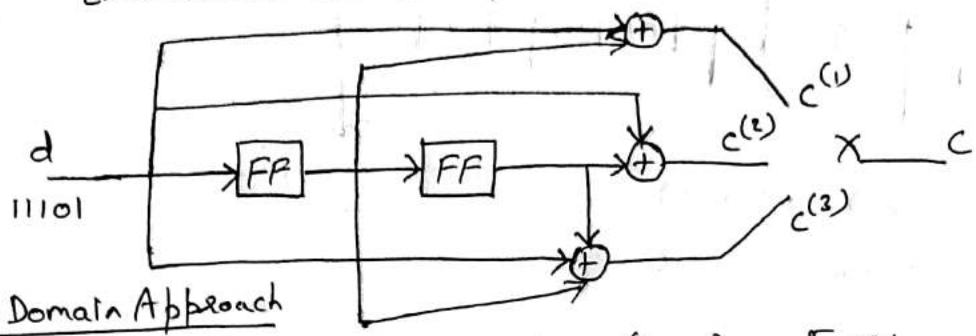
ii) Transform Domain Approach

$$c^{(1)}(x) = d(x)\,g^{(1)}(x) = (1+x^2+x^3+x^4)(1+x^2+x^3)$$
$$= 1+\cancel{x^2}+\cancel{x^3}+\cancel{x^4}+\cancel{x^2}+\cancel{x^4}+\cancel{x^5}+\cancel{x^6}+\cancel{x^3}+\cancel{x^5}+\cancel{x^6}+x^7$$
$$= 1+x^7$$
$$c^{(1)}(x^n) = c^{(1)}(x^2) = 1+x^{14}$$
$$c^{(2)}(x) = d(x)\,g^{(2)}(x) = (1+x^2+x^3+x^4)(1+x+x^2+x^3)$$
$$= 1+\cancel{x^2}+\cancel{x^3}+\cancel{x^4}+x+\cancel{x^3}+\cancel{x^4}+\cancel{x^5}+\cancel{x^2}+x^4+\cancel{x^5}+\cancel{x^6}+x^3+x^5+\cancel{x^6}+x^7$$
$$= 1+x+x^3+x^4+x^5+x^7$$
$$x \cdot c^{(2)}(x^n) = x \cdot c^{(2)}(x^2) = x(1+x^2+x^6+x^8+x^{10}+x^{14})$$
$$= x+x^3+x^7+x^9+x^{11}+x^{15}$$
$$\therefore C(x) = 1+x+x^3+x^7+x^9+x^{11}+x^{14}+x^{15}$$
$$\Rightarrow C = [11 \ 01 \ 00 \ 01 \ 01 \ 01 \ 00 \ 11]$$

\# Consider (3,1,2) Convolutional Code with $g^{(1)} = 110$, $g^{(2)} = 101$ and $g^{(3)} = 111$. Draw the encoder block diagram. Find G. Find the code word corresponding to $d = 11101$ using the time domain and transform domain approach.

Jan-14 20M
July-13 20M
Jan-16 20M

Sol:-



Time Domain Approach

Order of generator matrix $= L \times n(L+m) = 5 \times 21$

$g^{(1)} = 110$, $g^{(2)} = 101$, $g^{(3)} = 111$

$$G = \begin{bmatrix} 111 & 101 & 011 & 000 & 000 & 000 & 000 \\ 000 & 111 & 101 & 011 & 000 & 000 & 000 \\ 000 & 000 & 111 & 101 & 011 & 000 & 000 \\ 000 & 000 & 000 & 111 & 101 & 011 & 000 \\ 000 & 000 & 000 & 000 & 111 & 101 & 011 \end{bmatrix}$$

$C = [D][G]$

$$= [11101] \cdot \begin{bmatrix} 111 & 101 & 011 & 000 & 000 & 000 & 000 \\ 000 & 111 & 101 & 011 & 000 & 000 & 000 \\ 000 & 000 & 111 & 101 & 011 & 000 & 000 \\ 000 & 000 & 000 & 111 & 101 & 011 & 000 \\ 000 & 000 & 000 & 000 & 111 & 101 & 011 \end{bmatrix}$$

$C = [111\ 010\ 001\ 110\ 100\ 101\ 011]$

Transform Domain Approach:-

$c^{(1)}(x) = d(x) * g^{(1)}(x) = (1+x+x^2+x^4)(1+x)$

$= 1+\cancel{x}+\cancel{x^2}+x^4+\cancel{x}+\cancel{x^2}+x^3+x^5$

$= 1+x^3+x^4+x^5$

$c^{(1)}(x^n) = c^{(1)}(x^3) = 1+x^9+x^{12}+x^{15}$

$c^{(2)}(x) = d(x) * g^{(2)}(x) = (1+x+x^2+x^4)(1+x^2)$

$= 1+x+\cancel{x^2}+\cancel{x^4}+\cancel{x^2}+x^3+\cancel{x^4}+x^6$

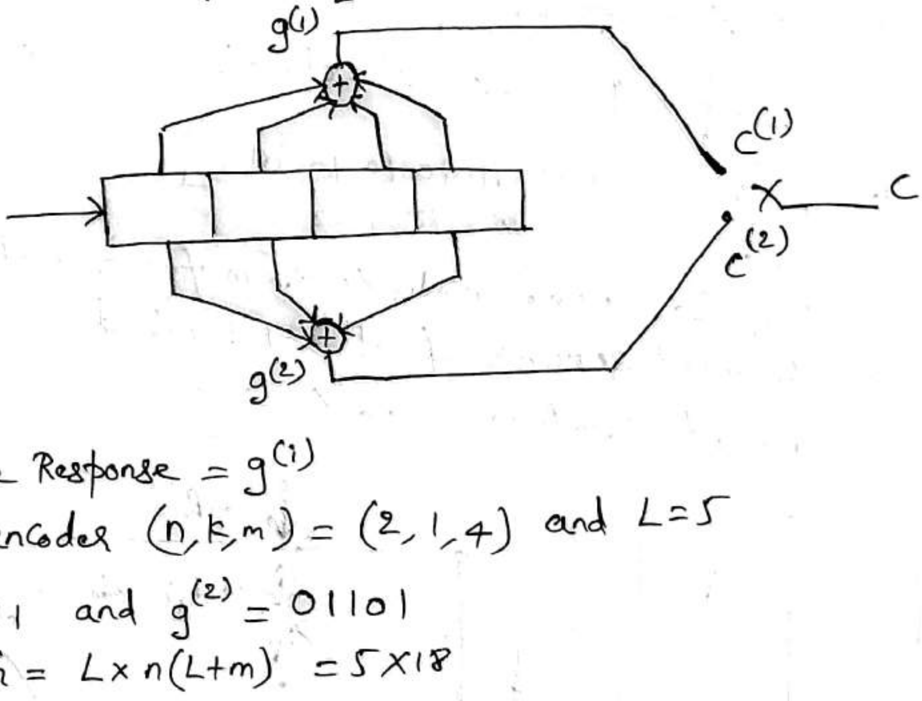
$= 1+x+x^3+x^6$

$x\,c^{(2)}(x^n) = x\,c^{(2)}(x^3) = x(1+x^3+x^9+x^{18})$

$= x+x^4+x^{10}+x^{19}$

$$c^{(3)}(x) = d(x) * g^{(3)}(x) = (1+x+x^2+x^4)(1+x+x^2)$$
$$= 1+\cancel{x}+\cancel{x^2}+\cancel{x^4}+\cancel{x}+\cancel{x^2}+\cancel{x^3}+x^5+x^2+\cancel{x^3}+\cancel{x^4}+x^6$$
$$= 1+x^2+x^5+x^6$$
$$x^2 c^{(3)}(x^n) = x^2 c^{(3)}(x^3) = x^2(1+x^6+x^{15}+x^{18})$$
$$= x^2+x^8+x^{17}+x^{20}$$
$$\therefore c(x) = c^{(1)}(x^3) + x\,c^{(2)}(x^3) + x^2 c^{(3)}(x^3)$$
$$= 1+x^9+x^{12}+x^{15}+x+x^4+x^{10}+x^{19}+x^2+x^8+x^{17}+x^{20}$$
$$= 1+x+x^2+x^4+x^8+x^9+x^{10}+x^{12}+x^{15}+x^{17}+x^{19}+x^{20}$$
$$\Rightarrow C = [111\ 010\ 001\ 110\ 100\ 101\ 011]$$

\# Dec-12 8M For the Convolutional encoder shown in figure, find the impulse response and hence calculate the output produced by the message sequence 10111. Write the generator polynomials of encoder and find the output sequence.



Sol:- Impulse Response $= g^{(1)}$

From the encoder $(n,k,m) = (2,1,4)$ and $L=5$

$g^{(1)} = 01111$ and $g^{(2)} = 01101$

order of $G = L \times n(L+m) = 5 \times 18$

$$G = \begin{bmatrix} 00 & 11 & 11 & 10 & 11 & 00 & 00 & 00 & 00 \\ 00 & 00 & 11 & 11 & 10 & 11 & 00 & 00 & 00 \\ 00 & 00 & 00 & 11 & 11 & 10 & 11 & 00 & 00 \\ 00 & 00 & 00 & 00 & 11 & 11 & 10 & 11 & 00 \\ 00 & 00 & 00 & 00 & 00 & 11 & 11 & 10 & 11 \end{bmatrix}$$

$C = [D][G]$

$$= [10111] \begin{bmatrix} 00 & 11 & 11 & 10 & 11 & 00 & 00 & 00 & 00 \\ 00 & 00 & 11 & 11 & 10 & 11 & 00 & 00 & 00 \\ 00 & 00 & 00 & 11 & 11 & 10 & 11 & 00 & 00 \\ 00 & 00 & 00 & 00 & 11 & 11 & 10 & 11 & 00 \\ 00 & 00 & 00 & 00 & 00 & 11 & 11 & 10 & 11 \end{bmatrix}$$

$C = [00\ 11\ 11\ 01\ 11\ 10\ 10\ 01\ 11]$

$d(x) = 1+x^2+x^3+x^4$

$g^{(1)}(x) = x+x^2+x^3+x^4$, $g^{(2)}(x) = x+x^2+x^4$

$c^{(1)}(x) = d(x) * g^{(1)}(x) = (1+x^2+x^3+x^4)(x+x^2+x^3+x^4)$

$= x+\cancel{x^3}+\cancel{x^4}+\cancel{x^5}+x^2+\cancel{x^4}+\cancel{x^5}+\cancel{x^6}+x^4+x^6+\cancel{x^7}+x^8$

$+\cancel{x^3}+x^5+\cancel{x^6}+\cancel{x^7}$

$= x+x^2+x^4+x^5+x^6+x^8$

$c^{(1)}(x^n) = c^{(1)}(x^2) \quad = x^2+x^4+x^8+x^{10}+x^{12}+x^{16}$

$c^{(2)}(x) = d(x) * g^{(2)}(x) = (1+x^2+x^3+x^4)(x+x^2+x^4)$

$= x+x^3+\cancel{x^4}+\cancel{x^5}+x^2+\cancel{x^4}+\cancel{x^5}+\cancel{x^6}+x^4+\cancel{x^6}+x^7+x^8$

$= x+x^2+x^3+x^4+x^7+x^8$

$x\,c^{(2)}(x^n) = x\,c^{(2)}(x^2) \quad = x(x^2+x^4+x^6+x^8+x^{14}+x^{16})$

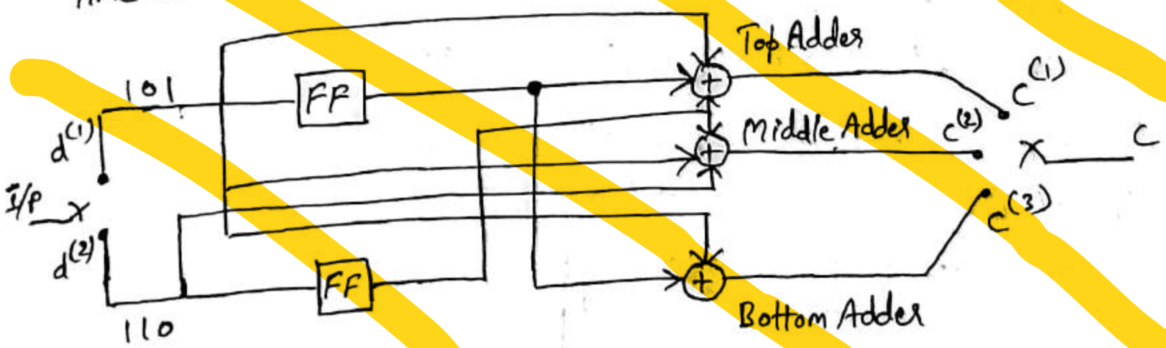
$= x^3+x^5+x^7+x^9+x^{15}+x^{17}$

$\therefore c(x) = x^2+x^4+x^8+x^{10}+x^{12}+x^{16}+x^3+x^5+x^7+x^9+x^{15}+x^{17}$

$= x^2+x^3+x^4+x^5+x^7+x^8+x^9+x^{10}+x^{12}+x^{15}+x^{16}+x^{17}$

$\Rightarrow c = [00\ 11\ 11\ 01\ 11\ 10\ 10\ 10\ 01\ 11]$

\# For (3,2,1) Convolutional encoder shown in figure find the Codeword 'c' for the input sequence of $d^{(1)} = 101$ and $d^{(2)} = 110$ using Time Domain and Transform Domain Approach.



Sol:- Given (3,2,1) Convolutional encoder with $L=3$

Order of Generator matrix $= 2L \times n(L+m) = 6 \times 12$

For $d^{(1)}$	For $d^{(2)}$
$g_1^{(1)} = 11$	$g_2^{(1)} = 01$
$g_1^{(2)} = 10$	$g_2^{(2)} = 11$
$g_1^{(3)} = 11$	$g_2^{(3)} = 00$

$d^{(1)} = 1\,0\,1$

$d^{(2)} = 1\,1\,0$

$d = [11\ 01\ 10]$

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Sol^n From the encoder diagram,

$c^{(1)} = D_{11} \oplus D_{12} \oplus D_{22}$

$c^{(2)} = D_{11} \oplus D_{21} \oplus D_{22}$

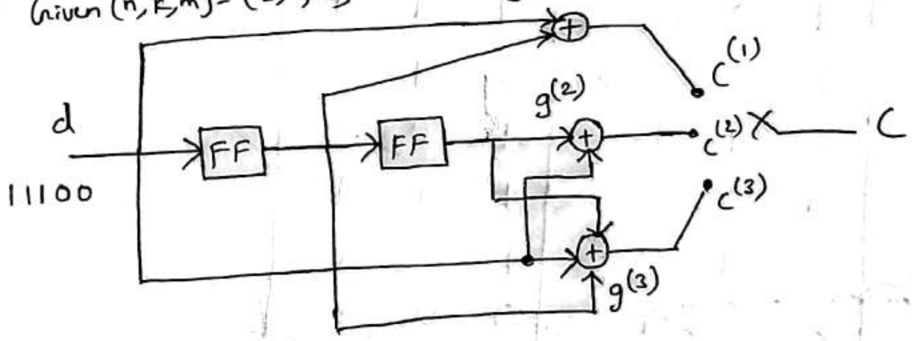
$c^{(3)} = D_{11} \oplus D_{21}$

Input	Shift Registers D_{11}	D_{12}	D_{21}	D_{22}	Code Vectors $c^{(1)}$	$c^{(2)}$	$c^{(3)}$	
	0	0	0	0	0	0	0	→ Initial State
01	0	1	0	0	1	0	0	
01	0	1	0	1	0	1	0	
11	1	1	0	1	1	0	1	
	0	0	1	1	1	0	1	→ Final state

Two bits at a time

Dec-12 12M Consider (3,1,2) Convolutional encoder with $g^{(1)} = 110$, $g^{(2)} = 101$ and $g^{(3)} = 111$. Draw the encoder block diagram. Write the state transition table. Draw the state diagram. Write the code tree. Find the encoder output for the message sequence 11100. Write Trellis diagram.

Sol:- Given $(n,k,m) = (3,1,2)$



Number of States $= 2^m = 2^2 = 4$

Let S_0, S_1, S_2 and S_3 be the 4 states with each state represented by 2 binary digits in bit reversed form.

In normal form, each state binary digits are represented as $S_0 = 00$, $S_1 = 01$, $S_2 = 10$ and $S_3 = 11$

But to represent in bit reversed form, write each binary digits in the reverse order i.e from right to left.

$\Rightarrow S_0 = 00$, $S_1 = 10$, $S_2 = 01$ and $S_3 = 11$

∴ The binary description for the 4 states in bit reversed form is as shown in the table.

(State Table)

States	Binary Description
S_0	00
S_1	10
S_2	01
S_3	11

State Transition Table :-

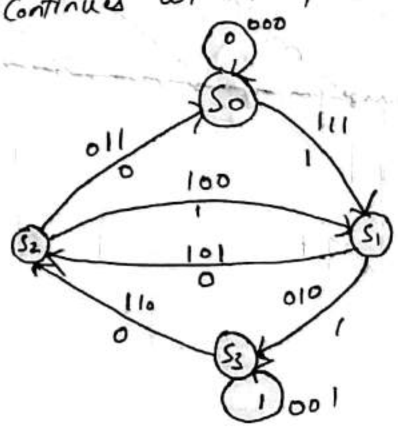
* Using present state and giving input '0' and '1', we will find the next state and shift register contents and corresponding code vectors.

Present State	Binary Description	Input	Next State	Binary Description	Shift registers d_1 d_2 d_3	Code Vector $c^{(1)}$ $c^{(2)}$ $c^{(3)}$
S_0	0 0	0	S_0	0 0	0 0 0	0 0 0
		1	S_1	1 0	1 0 0	1 1 1
S_1	1 0	0	S_2	0 1	0 1 0	1 0 1
		1	S_3	1 1	1 1 0	0 1 0
S_2	0 1	0	S_0	0 0	0 0 1	0 1 1
		1	S_1	1 0	1 0 1	1 0 0
S_3	1 1	0	S_2	0 1	0 1 1	1 1 0
		1	S_3	1 1	1 1 1	0 0 1

For the present state binary description, if the input either '0' or '1' is given d_1 and d_2 contents are shifted to d_2 and d_3 respectively and input will be put into d_1.
Now d_1 and d_2 contents will be the binary description of next state.
Based on d_1, d_2, d_3 which corresponds to $g^{(1)}, g^{(2)}, g^{(3)}$, the code vectors $c^{(1)}, c^{(2)}, c^{(3)}$ are found.
In this case $c^{(1)} = d_1 \oplus d_2$, $c^{(2)} = d_1 \oplus d_3$, $c^{(3)} = d_1 \oplus d_2 \oplus d_3$

State Diagram :-

* Based on State Transition Table, State diagram can be drawn.
For present state S_0, if input '0' is given, it will go to next state S_0 with code vector 000.
For present state S_0, if input '1' is given, it will go to next state S_1 with code vector 111.
Similarly the process continues with respect to different (other three) present states.



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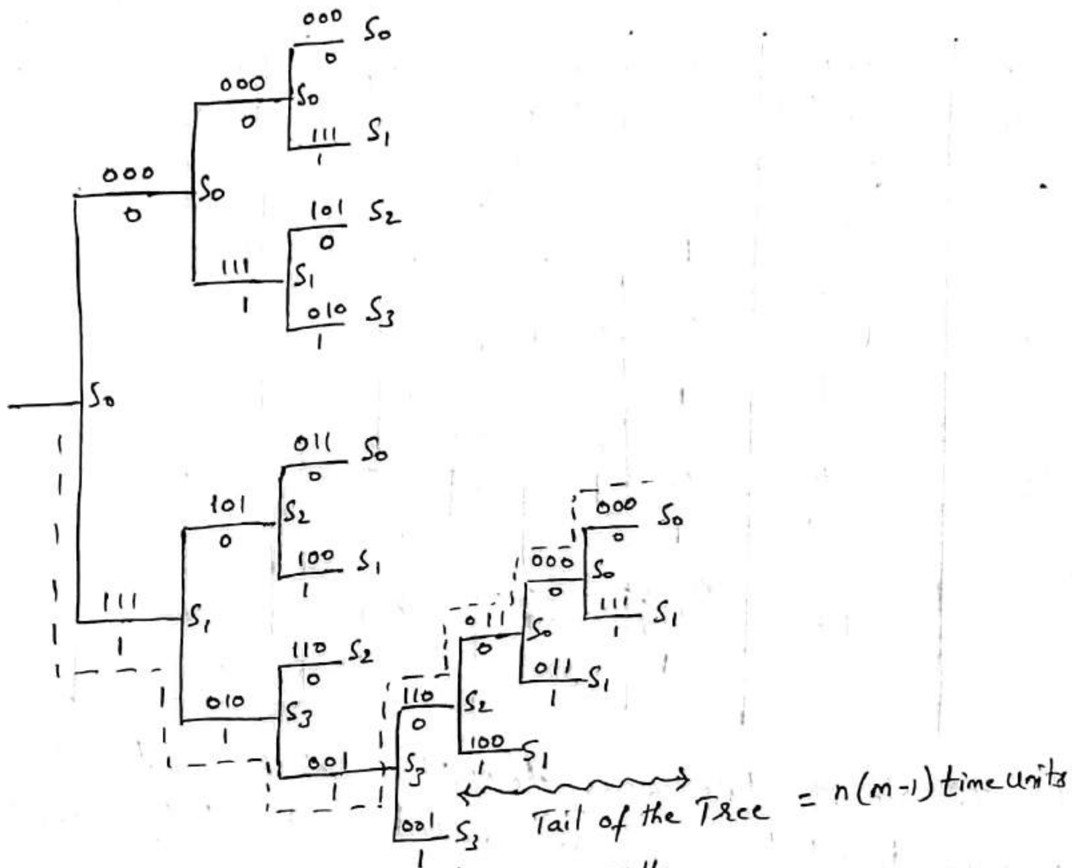
Code Tree :-



The codes which comes in the path are written.

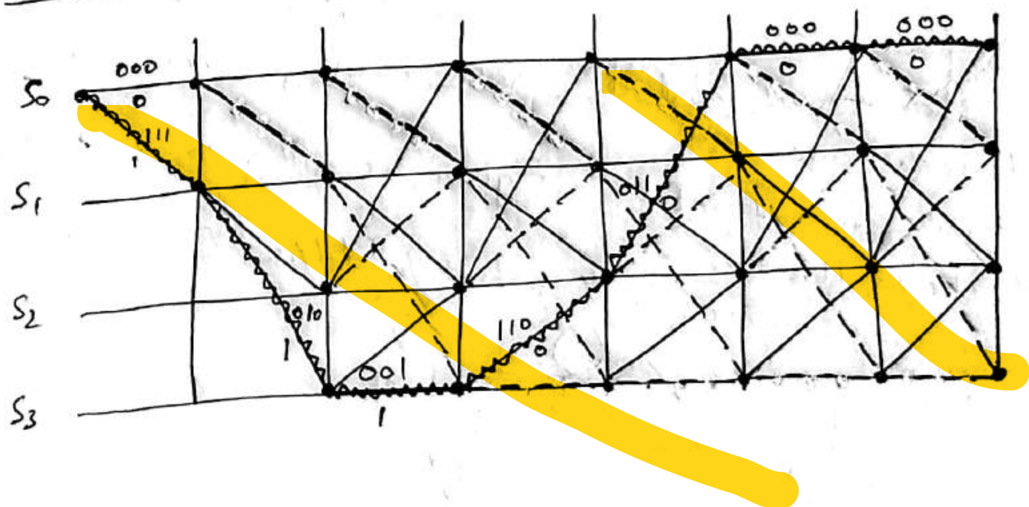
∴ Code vector C = [111 010 001 110 011 000 000]

Note :- To find output from code tree diagram, consider (L+m) number of bits as input

In the above case L+m = 5+2 = 7 bits

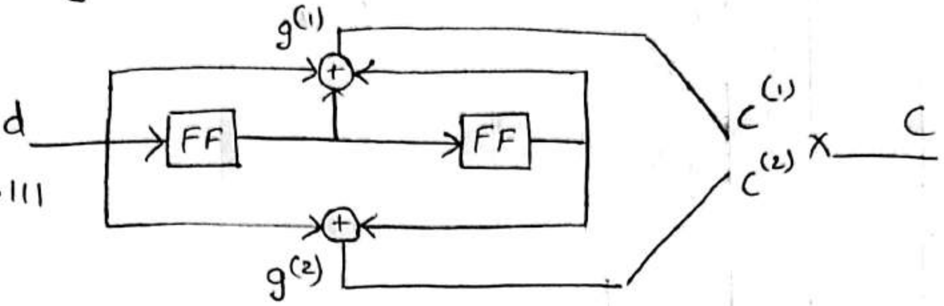
Trellis Diagram :-

Message sequence = 1110000



Dec-10 14M

Consider the binary Convolutional encoder shown in the figure. Draw the state table, state transition table, state diagram and the Corresponding code tree. Find the encoded sequence for the message 10111. Also find Constraint length and rate of efficiency.



Sol:- $(n, k, m) = (2, 1, 2)$

Since $m = 2$, number of States $= 2^m = 2^2 = 4$

State Table :-

$c^{(1)} = d_1 \oplus d_2 \oplus d_3$

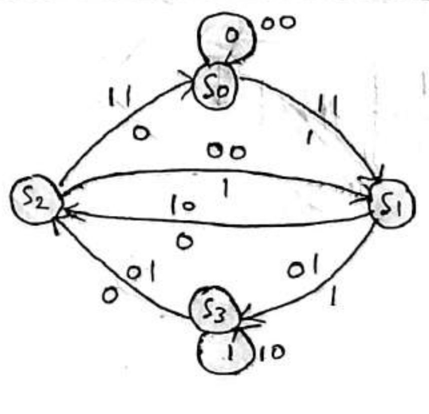
$c^{(2)} = d_1 \oplus d_3$

States	Binary Description
S_0	00
S_1	10
S_2	01
S_3	11

State Transition Table :-

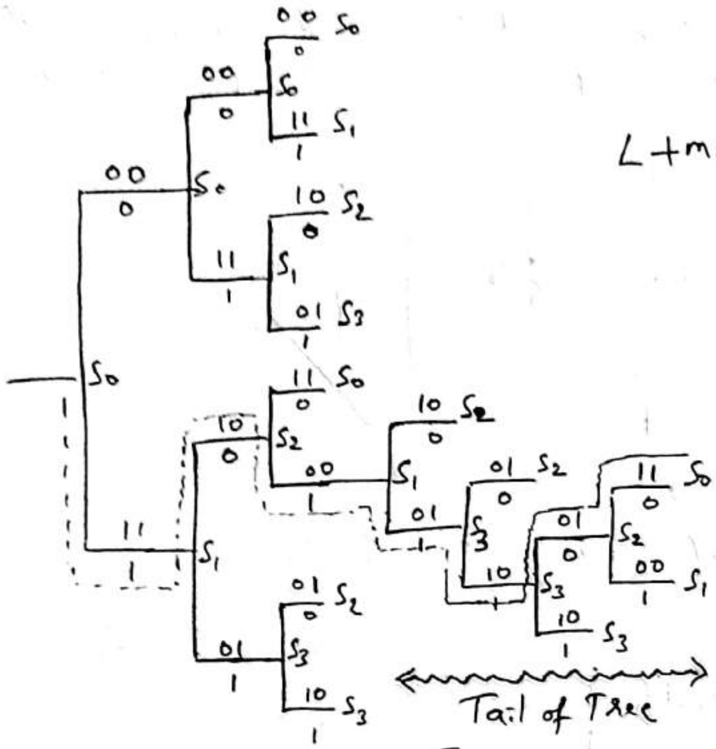
Present State	Binary Description	Input	Next State	Binary Description	Shift Registers d_1 d_2 d_3	Code Vectors $c^{(1)}$ $c^{(2)}$
S_0	00	0	S_0	00	0 0 0	0 0
		1	S_1	10	1 0 0	1 1
S_1	10	0	S_2	01	0 1 0	1 0
		1	S_3	11	1 1 0	0 1
S_2	01	0	S_0	00	0 0 1	1 1
		1	S_1	10	1 0 1	0 0
S_3	11	0	S_2	01	0 1 1	0 1
		1	S_3	11	1 1 1	1 0

State Diagram :-



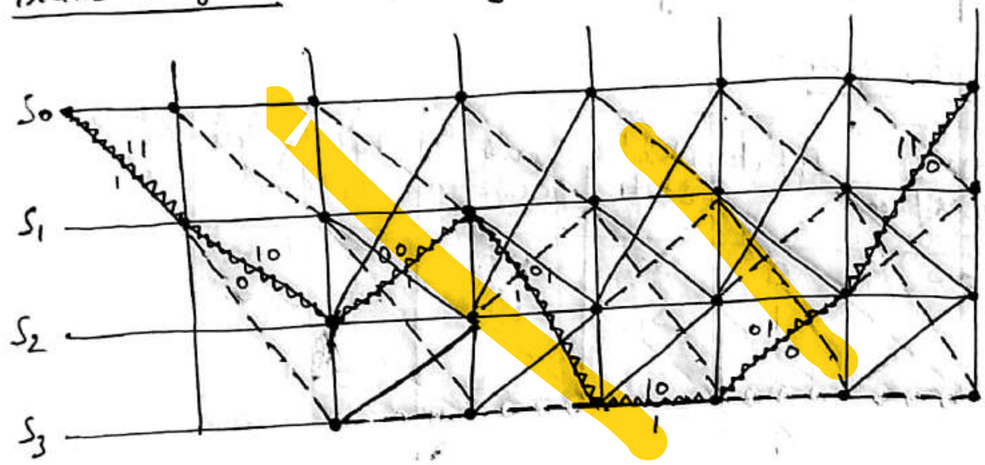
Code Tree :-

$L + m = 5 + 2 = 7$ bits



$C = [11\ 10\ 00\ 01\ 10\ 01\ 11]$

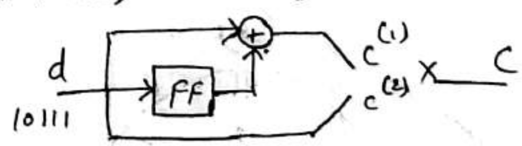
Trellis Diagram :- Message sequence = [1011100]



Constraint Length $= m * n = 2 * 2 = 4$

Rate of Efficiency $= k/n = 1/2$

\# Dec-10 6M For the Convolutional encoder shown in the figure, draw the state transition table, state diagram, Code tree and Trellis diagram.



Sol': $(n, k, m) = (2, 1, 1)$

Number of States $= 2^m = 2^1 = 2$

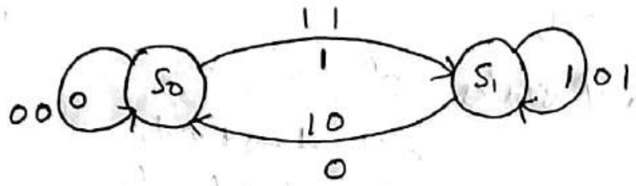
$c^{(1)} = d_1 \oplus d_2$

$c^{(2)} = d_1$

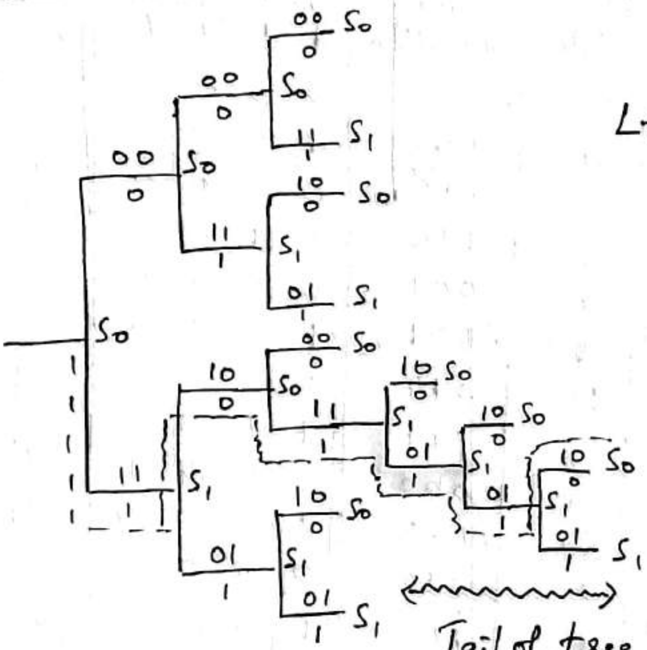
State Transition Table :-

Present State	Binary Description	Input	Next State	Binary Description	Shift Registers d_1 d_2	Code Vectors $c^{(1)}$ $c^{(2)}$
S_0	0	0	S_0	0	0 0	0 0
		1	S_1	1	1 0	1 1
S_1	1	0	S_0	0	0 1	1 0
		1	S_1	1	1 1	0 1

State Diagram :-



Code Tree :-

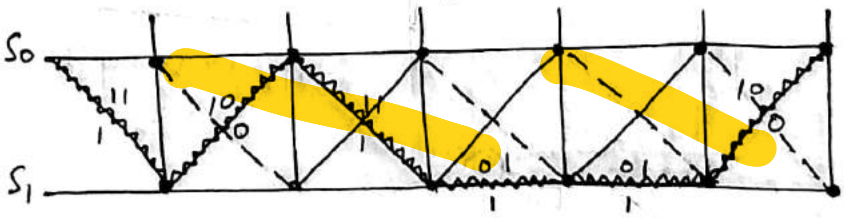


$L + m = 5 + 1 = 6$ bits

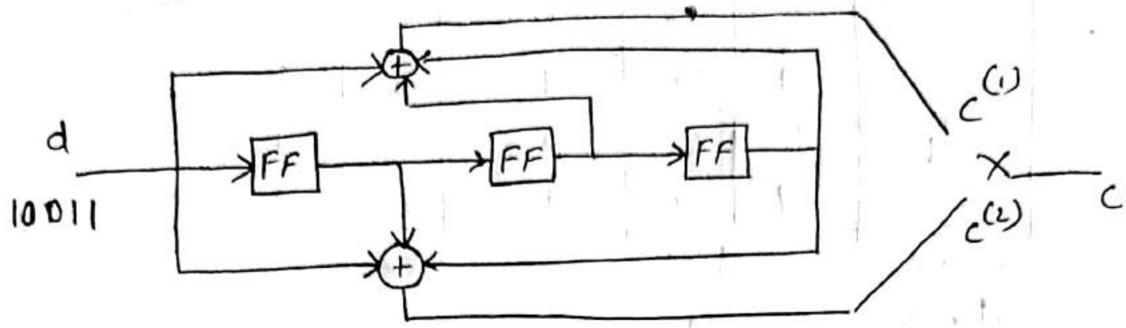
$C = [11\ 10\ 11\ 01\ 01\ 10]$

Trellis Diagram :-

Message sequence = [1 0 1 1 1 0]



Consider (2,1,3) Convolutional encoder shown in the figure. Draw State transition table, State diagram. Find the encoder output for the message sequence 10011 by traversing through the state diagram.



Sol:- Given $(n, k, m) = (2, 1, 3)$

Since $m = 3$, number of states $= 2^m = 2^3 = 8$

$c^{(1)} = d_1 \oplus d_3 \oplus d_4$

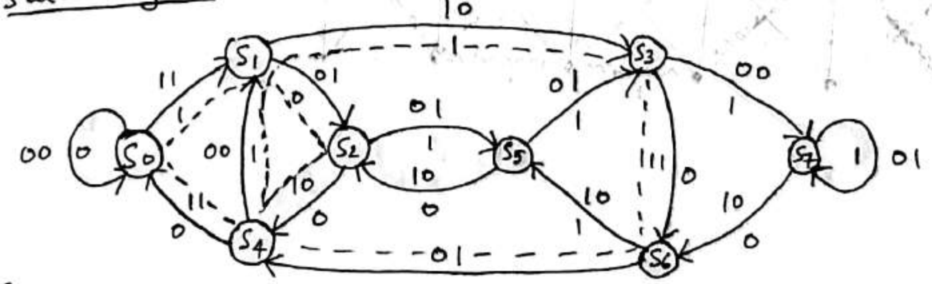
$c^{(2)} = d_1 \oplus d_2 \oplus d_4$

State Transition Table :-

Present state	Binary Description	Input	Next state	Binary Description	Shift Registers d_1 d_2 d_3 d_4	Code Vectors $c^{(1)}$ $c^{(2)}$
S_0	000	0	S_0	000	0 0 0 0	0 0
		1	S_1	100	1 0 0 0	1 1
S_1	100	0	S_2	010	0 1 0 0	1 1
		1	S_3	110	1 1 0 0	0 0
S_2	010	0	S_4	001	0 0 1 0	0 1
		1	S_5	101	1 0 1 0	1 0
S_3	110	0	S_6	011	0 1 1 0	1 0
		1	S_7	111	1 1 1 0	0 1
S_4	001	0	S_0	000	0 0 0 1	1 1
		1	S_1	100	1 0 0 1	0 0
S_5	101	0	S_2	010	0 1 0 1	0 0
		1	S_3	110	1 1 0 1	1 1
S_6	011	0	S_4	001	0 0 1 1	1 0
		1	S_5	101	1 0 1 1	0 1
S_7	111	0	S_6	011	0 1 1 1	0 1
		1	S_7	111	1 1 1 1	1 0

State Diagram :-

$L + m = 5 + 3 = 8$ bits



For message 10011000

$C = [11\ 01\ 10\ 00\ 10\ 11\ 01\ 11]$