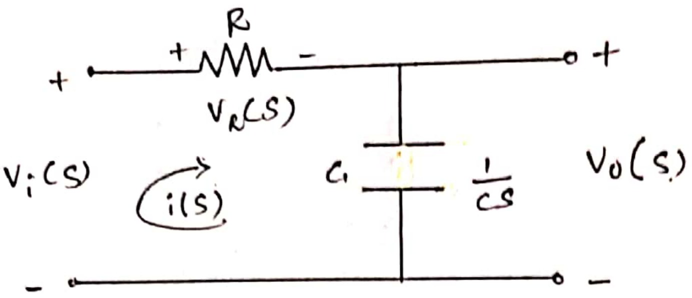
First order analog passive LPF



$$X_c = \frac{1}{2\pi f C_1}$$

$f \uparrow \; X_c \downarrow$ o/p get more & more short circuited

$\Rightarrow V_o(S) = 0$ for high frequencies.

Apply KVL

$$V_i(S) = V_R(S) + V_o(S)$$

$$V_i(S) = \frac{V_i(S)}{R + \frac{1}{CS}} R + V_o(S)$$

$$V_i(S)\left[1 - \frac{R}{R + \frac{1}{CS}}\right] = V_o(S)$$

$$V_i(S)\left[\frac{\cancel{R} + \frac{1}{CS} - \cancel{R}}{R + \frac{1}{CS}}\right] = V_o(S)$$

$$V_i(S)\left[\frac{1}{1 + SRC}\right] = V_o(S)$$

$$\frac{V_o(S)}{V_i(S)} = \frac{1}{1 + SRC} \longrightarrow (1)$$

But $\frac{V_o(S)}{V_i(S)} = H(S)$; the transfer function

$$f_c = \frac{1}{2\pi RC} \; ; \quad 2\pi f_c = \frac{1}{RC} \; ; \quad \Omega_c = \frac{1}{RC} \qquad RC = \frac{1}{\Omega_c}$$

$\therefore$ Eqn (1) $\Rightarrow$

$$H(S) = \frac{1}{1 + \frac{S}{\Omega_c}}$$

put $S = j\Omega$

$$H(j\Omega) = \frac{1}{1 + \frac{j\Omega}{\Omega_c}}$$

Taking magnitude.

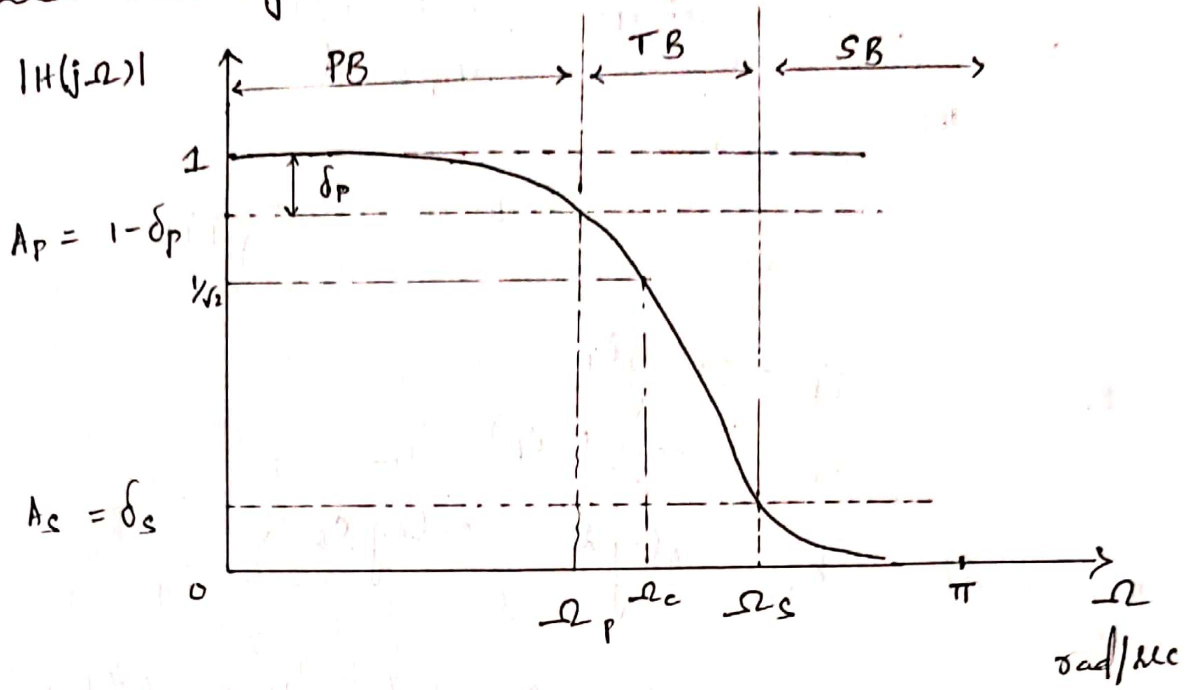
$$\boxed{|H(j\Omega)| = \frac{1}{\left[1 + \left(\frac{\Omega}{\Omega_c}\right)^2\right]^{1/2}}} \longrightarrow (2)$$

Eqn (2) is the magnitude frequency response of a first order LPF

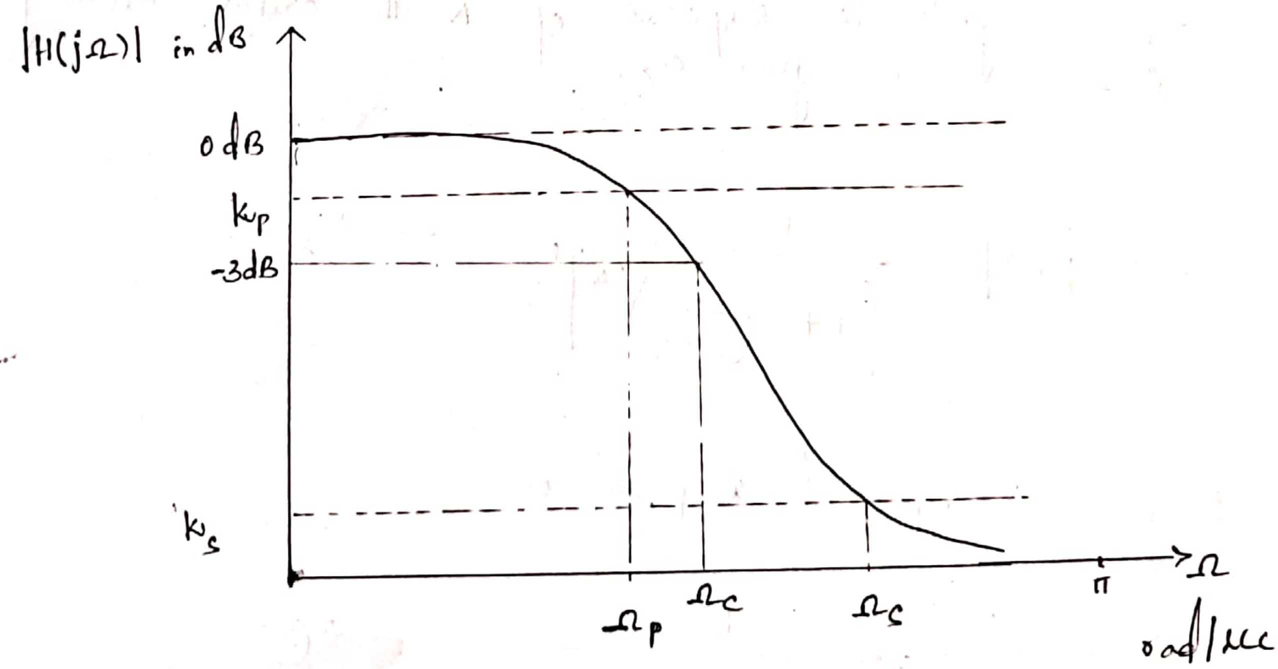
The magnitude frequency response of a N th order LPF is given as

$$|H(j\Omega)| = \frac{1}{\left[1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{1/2}} \qquad N = 1, 2, 3 \ldots$$

Magnitude response of analog Lowpass Butterworth filter with gain in decimal value.



magnitude response with gain in decibel (dB)



δ_p : Maximum tolerance in pass band (ripple)

δ_s : Maximum tolerance in stop band (ripple)

A_p : Minimum gain in pass band (PB)

A_s : maximum gain in stop band.

$$A_p = 1 - \delta_p = \frac{1}{\sqrt{1+\epsilon^2}} \qquad A_s = \delta_s = \frac{1}{\sqrt{1+\lambda^2}}$$

A_p in dB: $k_p = 20 \log A_p$

$k_p = 20 \log (1 - \delta_p)$

A_s in dB: $k_s = 20 \log A_s = 20 \log \delta_s$

To derive the expression for order of the filter Butterworth filter.

The magnitude response of N th order analog filter is given as.

$$|H(j\Omega)| = \frac{1}{\left[1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{1/2}} \longrightarrow (1).$$

Magnitude squared response is

$$|H(j\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}} \longrightarrow (2).$$

From figure (1) at $\Omega = \Omega_p$, $|H(j\Omega)| = A_p = 1 - \delta_p$

Using this condition in Eqn (2).

$$A_p^2 = \frac{1}{1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}}$$

$$1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = \frac{1}{A_p^2}$$

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = \frac{1}{A_p^2} - 1 \longrightarrow ③.$$

Now at $\Omega = \Omega_s$ $|H(j\Omega)| = A_s = \delta_s$

$$\therefore \quad A_s^2 = \delta_s^2 = \frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}}$$

$$1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = \frac{1}{A_s^2}$$

$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = \frac{1}{A_s^2} - 1 \longrightarrow ④.$$

Eqn ③ $\div$ ④.

$$\left(\frac{\Omega_p}{\Omega_s}\right)^{2N} = \frac{\frac{1}{A_p^2} - 1}{\frac{1}{A_s^2} - 1}$$

Taking $\log_{10}$ on bothside.

$$2N \log\left(\frac{\Omega_p}{\Omega_s}\right) = \log\left[\frac{\frac{1}{A_p^2} - 1}{\frac{1}{A_s^2} - 1}\right]$$

$$N = \frac{\log\left[\dfrac{\frac{1}{A_p^2}-1}{\frac{1}{A_s^2}-1}\right]}{2\log\left(\dfrac{\Omega_p}{\Omega_s}\right)} \longrightarrow (5)$$

From eqn (3)

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = \frac{1}{A_p^2} - 1$$

$$\frac{\Omega_p}{\Omega_c} = \left(\frac{1}{A_p^2} - 1\right)^{\frac{1}{2N}}$$

$$\boxed{\Omega_c = \frac{\Omega_p}{\left(\frac{1}{A_p^2} - 1\right)^{1/2N}}} \longrightarrow (6)$$

From eqn (4)

$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = \frac{1}{A_s^2} - 1$$

$$\frac{\Omega_s}{\Omega_c} = \left(\frac{1}{A_s^2} - 1\right)^{1/2N}$$

$$\boxed{\Omega_c = \frac{\Omega_s}{\left(\frac{1}{A_s^2} - 1\right)^{1/2N}}} \longrightarrow (7)$$

If A_p given in dB : $k_p = 20\log A_p$

$$\log A_p = k_p/20$$

$$A_p = 10^{k_p/20}$$

squaring Both side

$$A_p^2 = 10^{k_p/10}$$

$$\frac{1}{A_p^2} = 10^{-0.1 k_p} \longrightarrow (8)$$

Similarly $\frac{1}{A_s^2} = 10^{-0.1 k_s} \longrightarrow (9)$

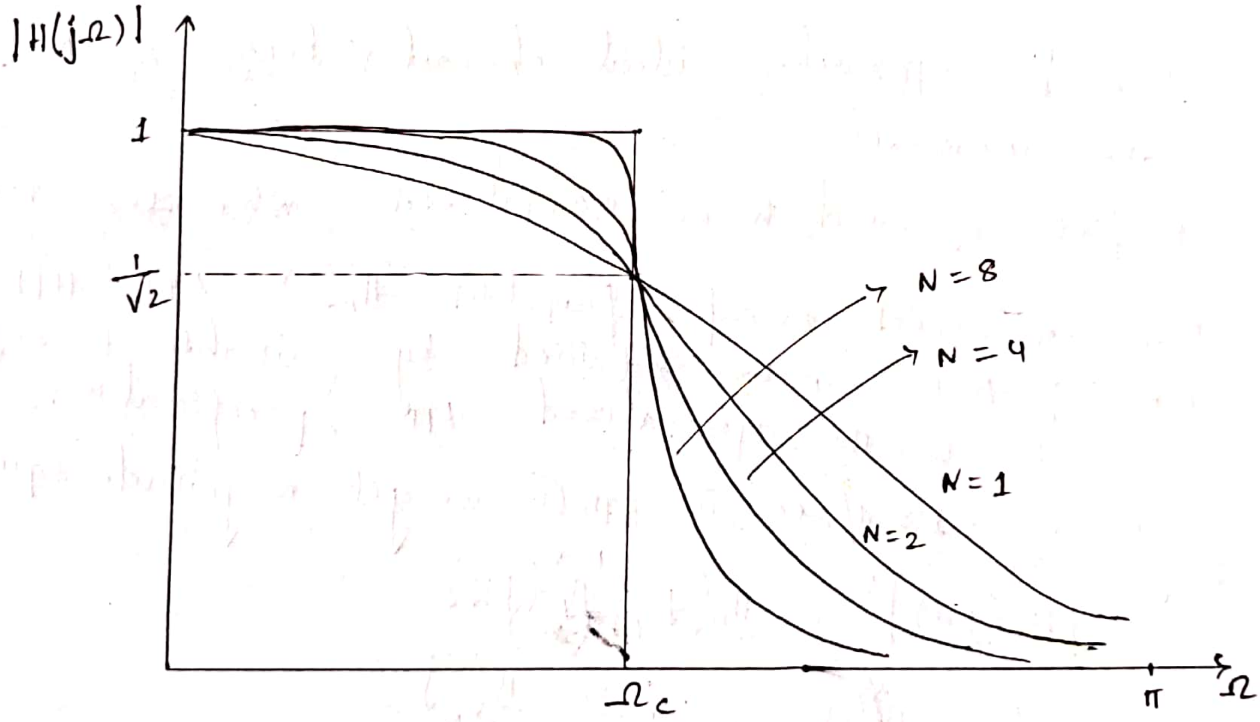
put eqn (8) & (9) in eqn (5).

$$N = \frac{\log\left[\dfrac{10^{-0.1k_p}-1}{10^{-0.1k_s}-1}\right]}{2\log\left(\dfrac{\Omega_p}{\Omega_s}\right)} \longrightarrow (10).$$

Eqn (10) is the expression for finding order of the filter when the specifications are given in dB.

To derive the Transfer function for analog Low pass

Normalized filter is one for which $\Omega_c = 1$ rad/sec with magnitude response denoted as $|H_n(j\Omega)|$



The Magnitude squared response of Butterworth LP filter is (unnormalized)

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\dfrac{\Omega}{\Omega_c}\right)^{2N}} \longrightarrow (1)$$

with $\Omega_c = 1$ rad/sec $\quad |H_n(j\Omega)|^2 = \dfrac{1}{1+\Omega^{2N}}$

Properties of Butterworth filter

1) $|H_n(j\Omega)|^2_{\Omega=0} = 1$ for all N.

2) $|H_n(j\Omega)|^2_{\Omega=\Omega_c} = 0.5$ for all finite N.

3) $|H_n(j\Omega)|_{\Omega=\Omega_c} = \frac{1}{\sqrt{2}} = 0.707.$

i.e., $20\log|H_n(j\Omega)|_{\Omega=\Omega_c} = -3.01\,dB.$

$20\log 0.707 = -3.01\,dB$

$|H_n(j\Omega)|^2$ is a monotonically decreasing function of Ω.

$|H_n(j\Omega)|^2$ approaches ideal characteristics as N value increases.

A filter is said to be normalized when $\Omega_c = 1\,rad/sec$.

From normalized transfer function $H_n(S)$, LPF HPF BPF & BSF can be obtained by suitable transformation to the normalized LPF specification.

With $\Omega_c = 1\,rad/sec$ in Eqn (1) we get magnitude squared Response as

$$|H_n(j\Omega)|^2 = \frac{1}{1+(\Omega)^{2N}}$$

$$H_n(j\Omega)\,H_n^*(j\Omega) = \frac{1}{1+(\Omega)^{2N}}.$$

$$H_n(j\Omega)\,H_n(-j\Omega) = \frac{1}{1+(\Omega)^{2N}}$$

$$H_n(j\Omega)\, H_n(-j\Omega) = \frac{1}{1+(\Omega)^{2N}}$$

since $s = j\Omega$; $\Omega = \frac{s}{j}$

$$H_n(s)\, H_n(-s) = \frac{1}{1+\left(\frac{s}{j}\right)^{2N}}$$

The poles are found by equating denominator to zero.

$$1+\left(\frac{s}{j}\right)^{2N} = 0.$$

$$1+(-js)^{2N} = 0$$

$$1+(-j)^{2N} s^{2N} = 0$$

$$1+(-1)^{2N}(j^2)^N s^{2N} = 0$$

$$1+(-1)^N s^{2N} = 0 \longrightarrow (2).$$

If N is odd in Eqn (2).

$$1-s^{2N} = 0$$

$$s^{2N} = 1$$

$$s^{2N} = e^{j2\pi k} \qquad k = 1\ 2\ 3 \ldots 2N.$$

Rise power by $\frac{1}{2N}$.

$$S_k = 1\, e^{j\frac{\pi k}{N}} \qquad k = 1\ 2 \ldots 2N.$$

$$\boxed{S_k = 1 \angle \frac{\pi k}{N} \qquad k = 1\ 2 \ldots 2N.}$$ N odd.

If N is even in Eqn (2).

$$1 + (-1)^N s^{2N} = 0$$

$$1 + s^{2N} = 0$$

$$s^{2N} = -1$$

$$s^{2N} = e^{j(2k-1)\pi} \qquad k = 1\ 2 \dots 2N.$$

Raise power by $\frac{1}{2N}$.

$$S_k = e^{j\frac{(2k-1)\pi}{2N}} \qquad k = 1, 2, 3 \dots 2N$$

$$\boxed{S_k = 1 \angle \frac{\pi k}{N} - \frac{\pi}{2N} \qquad k = 1\ 2 \dots 2N.} \quad \text{N Even.}$$

To Locate (find) the poles and hence obtain the Expression for transfer function of Normalized Butterworth LPF

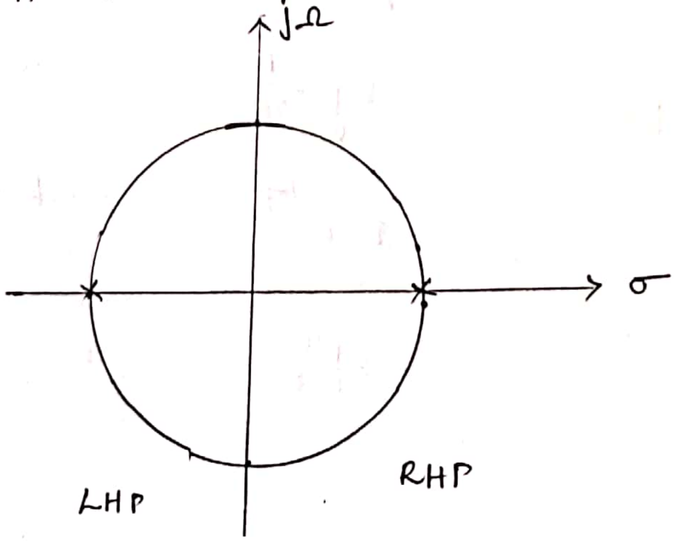
I First order $N = 1$ (N odd)

$$S_k = 1 \angle \frac{\pi k}{N} \qquad k = 1\ 2\ 3 \dots 2N$$

$$S_k = 1 \angle \pi k \qquad k = 1, 2$$

$k=1$ $\quad S_1 = 1 \angle \pi$

$k=2$ $\quad S_2 = 1 \angle 2\pi$



The transfer function is given as

$$H_n(S) = \frac{1}{\prod\limits_{LHP} (S - S_k)}$$

$$= \frac{1}{[S - (-1)]}$$

$$\boxed{H_n(S) = \frac{1}{S+1}}$$

N = 2 2nd order (N is even)

$$S_k = 1 \angle \frac{\pi k}{N} - \frac{\pi}{2N} \qquad k = 1\,2 \ldots 2N.$$

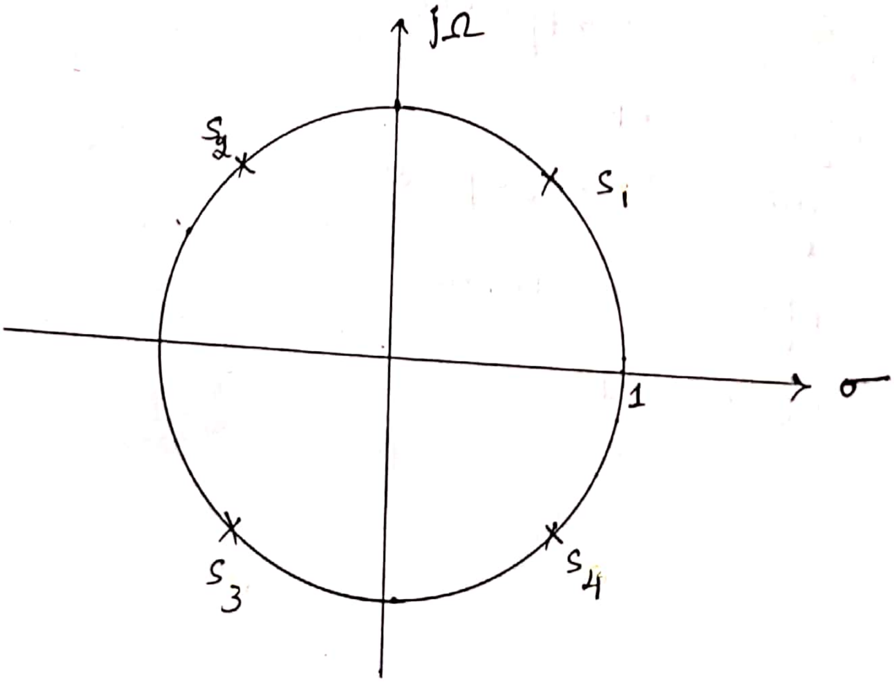
$$S_k = 1 \angle \frac{\pi k}{2} - \frac{\pi}{4} \qquad k = 1, 2, 3, 4$$

$$S_1 = 1 \angle \frac{\pi}{2} - \frac{\pi}{4} \; ; \; S_1 = 1 \angle \pi/4 \qquad S_1 = 0.707 + j0.707$$

$$S_2 = 1 \angle \frac{2\pi}{2} - \frac{\pi}{4} \qquad S_2 = 1 \angle \frac{3\pi}{4} \qquad S_2 = -0.707 + j0.707$$

$$S_3 = 1 \angle \frac{3\pi}{2} - \frac{\pi}{4} \qquad S_3 = 1 \angle \frac{5\pi}{4} \qquad S_3 = -0.707 - j0.707$$

$$S_4 = 1 \angle \frac{4\pi}{2} - \frac{\pi}{4} \qquad S_4 = 1 \angle 7\pi/4 \qquad S_4 = 0.707 - j0.707$$



$$H_n(s) = \frac{1}{\prod_{LHP} (s - s_k)}$$

$$H_n(s) = \frac{1}{(s-s_2)(s-s_3)}$$

$$H_n(s) = \frac{1}{[s-(-0.707+j0.707)][s-(-0.707-j0.707)]}$$

$$H_n(s) = \frac{1}{[(s+0.707)-j0.707][(s+0.707)+j0.707]}$$

$$H_n(s) = \frac{1}{(s+0.707)^2-(j0.707)^2}$$

$$H_n(s) = \frac{1}{s^2+1.414s+1}$$

$\underline{N=3}$ $\underline{\text{order } 3}$ $\underline{N \text{ odd}}$

$S_k = 1 \angle \frac{\pi k}{N}$ $\quad k = 1, 2 \ldots 2N.$

$S_k = \angle \frac{\pi k}{3}$ $\quad k = 1\ 2\ 3\ 4\ 5\ 6.$

$s_1 = 1 \angle \pi/3 = 0.5 + j0.866.$

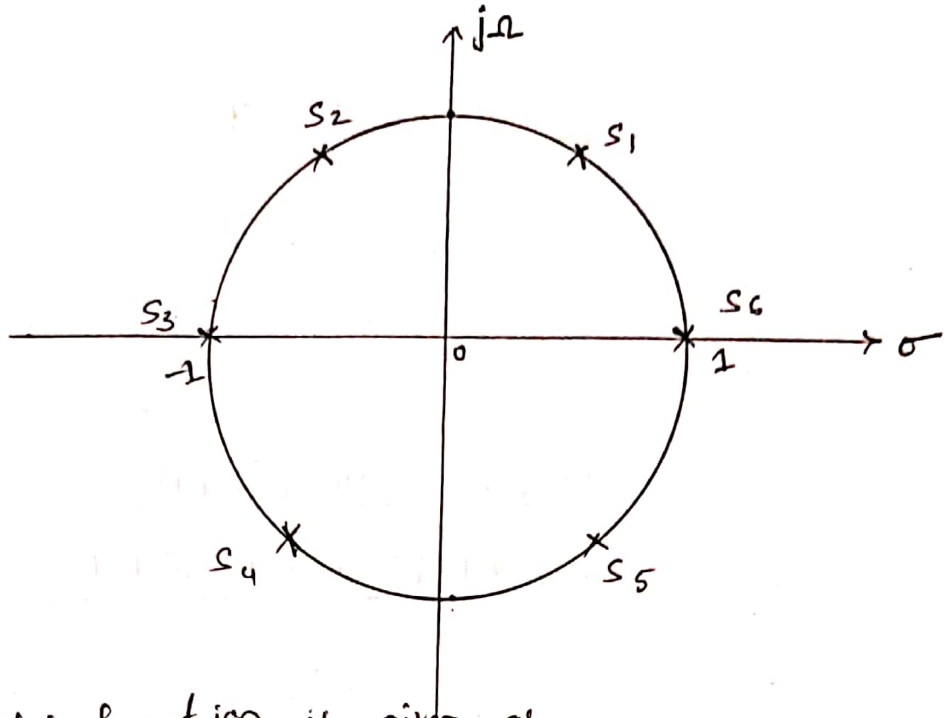
$s_2 = 1 \angle 2\pi/3 = -0.5 + j0.866$

$s_3 = 1 \angle \pi = -1$

$s_4 = 1 \angle 4\pi/3 = -0.5 - j0.866$

$s_5 = 1 \angle 5\pi/3 = 0.5 - j0.866$

$s_6 = 1 \angle 2\pi = 1$



The transfer function is given as

$$H_n(S) = \frac{1}{\prod_{LHP} (S - S_k)}$$

$$H_n(S) = \frac{1}{(S-S_2)(S-S_3)(S-S_4)}$$

$$= \frac{1}{(S-(-0.5+j0.866))(S-(-1))(S-(-0.5-j0.866))}$$

$$= \frac{1}{(S+1)((S+0.5)-j0.866)((S+0.5)+j0.866)}$$

$$H_n(S) = \frac{1}{(S+1)(S^2+S+1)}$$

Assignment: obtain Expression for transfer function $H_n(S)$ of Normalized LPF for $N=4$, $N=5$.

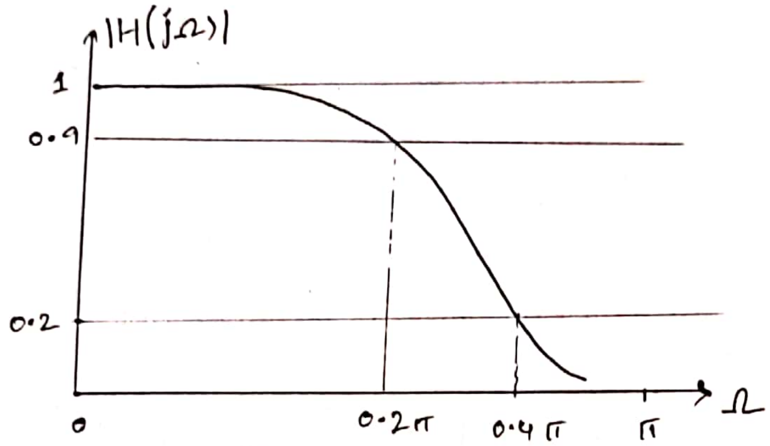
Order	Butterworth Polynomial
1	$(s+1)$
2	$(s^2+\sqrt{2}s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76536s+1)(s^2+1.847765s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1$

① Design an analog Butterworth LPF to meet the following specification.

$$0.9 \le |H(j\Omega)| \le 1 \quad \text{for} \quad 0 \le \Omega \le 0.2\pi$$

$$|H(j\Omega)| \le 0.2 \quad \text{for} \quad 0.4\pi \le \Omega \le \pi$$

Soln:



Step 1: Given

$A_p = 0.9 \qquad k_p = 20\log 0.9 = -0.915\ dB$

$A_s = 0.2 \qquad k_s = 20\log 0.2 = -13.98\ dB$

$\Omega_p = 0.2\pi$

$\Omega_s = 0.4\pi$

Step 2:
$$N = \frac{\log\left[\frac{10^{-0.1k_p}-1}{10^{-0.1k_s}-1}\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$N = \frac{\log\left[\frac{10^{+0.1\times0.915}-1}{10^{0.1\times13.98}-1}\right]}{2\log\left(\frac{0.2\pi}{0.4\pi}\right)}$$

$N = 3.33$

Step 3: Round off N $\boxed{N=4}$

Step 4:

$$\Omega_c = \frac{\Omega_p}{(10^{-0.1k_p}-1)^{1/2N}} = \frac{0.2\pi}{(10^{0.1\times 0.915}-1)^{\frac{1}{8}}}$$

$\Omega_c = 0.7532$ rad/sec.

Step 5: $$H_n(s) = \frac{1}{(s^2+0.765365s+1)(s^2+1.847765s+1)}$$

Step 6: $$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_c} = \frac{s}{0.7532}}$$

$$H_a(s) = \frac{1}{\left[\frac{s^2}{0.7532^2}+0.76536\frac{s}{0.7532}+1\right]\left[\frac{s^2}{0.7532^2}+\frac{1.84776s}{0.7532}+1\right]}$$

$$\boxed{H_a(s) = \frac{0.323}{(s^2+0.576s+0.567)(s^2+1.39s+0.567)}}$$

verification: At $\Omega=\Omega_p = 0.2\pi$ $\quad A_p = 0.9 = |H(j\Omega_p)|$

put $s=j\Omega_p$ in $H_a(s)$

$$H_a(j\Omega_p) = \frac{0.323}{((j\Omega_p)^2+0.576j\Omega_p+0.567)((j\Omega_p)^2+1.39j\Omega_p+0.567)}$$

$$H_a(j\Omega_p) = \frac{0.323}{[j^2(0.2\pi)^2+0.576(j0.2\pi)+0.567][j^2(0.2\pi)^2+1.39j0.2\pi+0.567]}$$

$$= \frac{0.323}{(-0.3947 + j\,0.362 + 0.567)(-0.3947 + 0.8733j + 0.567)}$$

$$|H(j\Omega_p)| = \frac{0.323}{(0.1723 + j\,0.362)(0.1723 + 0.8733j)}$$

Taking magnitude.

$$|H(j\Omega_p)| = 0.904$$

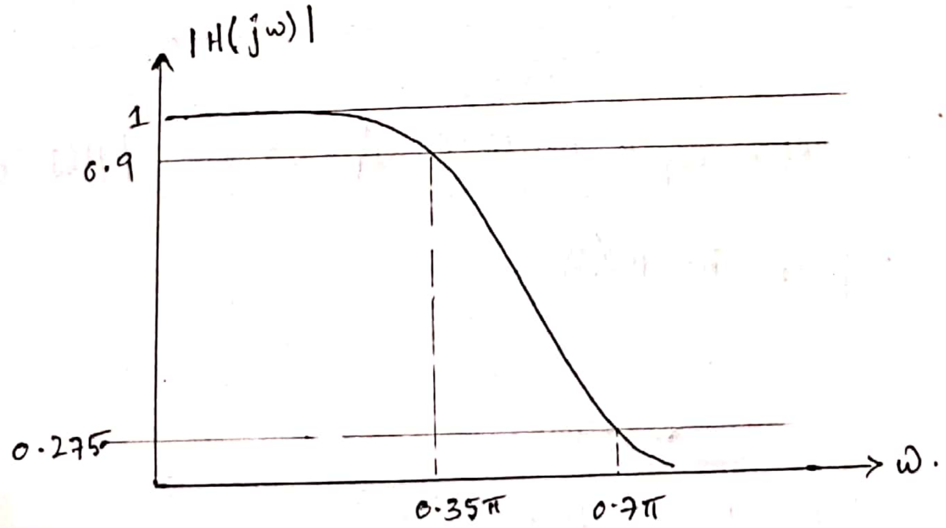
$$\simeq A_p = \underline{\underline{0.9}}$$

Qn ②

① Design an analog Butterworth LPF to meet the following specifications. Assume $T = 1$ sec.

$0.9 \le |H(j\omega)| \le 1 \qquad 0 \le \omega \le 0.35\pi$

$|H(j\omega)| \le 0.275 \qquad 0.7\pi \le \omega \le \pi.$



Step 1: $A_p = 0.9 \qquad k_p = 20 \log A_p = 20 \log 0.9 = -0.9151$

$A_s = 0.275 \qquad k_s = 20 \log A_s = 20 \log 0.275 = -11.2133$

Step 1:

$A_p = 0.9 \quad k_p = 20 \log A_p = 20 \log 0.9$

$A_s = 0.275 \quad k_s = 20 \log A_s = 20 \log(0.275)$

$\omega_s = 0.7\pi \quad \Omega_s = \frac{\omega_s}{T} = \frac{0.7\pi}{1} \quad \Omega_s = 0.7\pi$

$\omega_p = 0.35\pi \quad \Omega_p = \frac{\omega_p}{T} = \frac{0.35\pi}{1} \quad \Omega_p = 0.35\pi$

$k_p = -0.9151$

$k_s = -11.2133$

$\Omega_p = 0.35\pi$ rad/sec.

$\Omega_s = 0.7\pi$ rad/sec.

Step 2:

order of the filter.

$$N = \frac{\log\left[\frac{10^{-0.1k_p} - 1}{10^{-0.1k_s} - 1}\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)} \qquad N = \frac{\log\left[\frac{10^{0.1\times0.951} - 1}{10^{0.1\times11.2103} - 1}\right]}{2\log\left(\frac{0.35\pi}{0.7\pi}\right)}$$

$$N = \frac{-1.717}{-0.6020} \qquad N = 2.852$$

Step 3: Round off N. $\underline{N = 3}$

Step 4: T.F of normalized LPF (prototype) with N=3

$$H_n(s) = \frac{1}{(s+1)(s^2+s+1)}$$

Step 5 Find cutoff frequency Ω_c

$$\Omega_c = \frac{\Omega_p}{\left(10^{-0.1k_p} - 1\right)^{1/2N}}$$

$$\Omega_c = \frac{0.35\pi}{(10^{0.1\times 0.9151}-1)^{\frac{1}{6}}} \qquad \Omega_c = 1.4 \text{ rad/sec.}$$

step 6: Transfer function of required unnormalized filter

$$H_a(s) = H_n(s)\Big|_{s\to\frac{s}{\Omega_c}} = \frac{s}{1.4}$$

$$H_n(s) = \frac{1}{(s+1)(s^2+s+1)} = \frac{1}{s^3+2s^2+2s+1}$$

$$H_a(s) = \frac{1}{\left(\frac{s}{1.4}\right)^3 + 2\left(\frac{s}{1.4}\right)^2 + 2\frac{s}{1.4} + 1} \times \frac{(1.4)^3}{(1.4)^3}$$

$$H_a(s) = \frac{2.744}{s^3+2.8s^2+3.92s+2.744}$$

verification

put $s = j\Omega$ $\quad \Omega = \Omega_p = 0.35\pi$

$$H_a(s) = \frac{2.744}{(j\Omega)^3 + 2.8(j\Omega)^2 + 3.92(j\Omega) + 2.744}$$

$$= \frac{2.744}{-j\Omega^3 + 2.8(-1)\Omega^2 + 3.92(j\Omega) + 2.744}$$

$$H_a(s) = \frac{2.744}{j(3.92\Omega - \Omega^3) - 2.8\Omega^2 + 2.744}$$

$$H_a(s) = \frac{2.744}{j(3.92\times 0.35\pi - (0.35\pi)^2) - 2.8(0.35\pi)^2 + 2.744}$$

$$H_a(s) = \frac{2.744}{j3.101 - 3.38 + 2.744}$$

$$H_a(s) = \frac{2.744}{j3.101 - 0.636}$$

$$H_a(j\Omega_p) = 0.8668$$
$$\simeq 0.87 \qquad \text{Given } H_a(j\Omega_p) = 0.9$$

Steps to Design Analog High pass filter

Step 1: Determine the stopband frequency of the normalized filter as $\Omega_s = \frac{\Omega_p}{\Omega_s'}$ $\Omega_p = \frac{\Omega_p'}{\Omega_p'}$

where Ω_p' & Ω_s' are the given passband & stopband frequency.

Step 2: Determine the order of the filter using

$$N = \frac{\log\left[\frac{10^{-0.1k_p}-1}{10^{-0.1k_s}-1}\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

Step 3: Determine the cutoff frequency Ω_c as

$$\Omega_c = \frac{\Omega_p}{(10^{-0.1k_p}-1)^{1/2N}} \quad \text{(or)} \quad \Omega_c = \frac{\Omega_s}{(10^{-0.1k_s}-1)^{1/2N}}.$$

Step 4: Determine the transfer function of normalized Butterworth filter as

$$H_n(s) = \frac{1}{\prod_{LHP}(s - s_k)} = \frac{1}{B_n(s)}$$

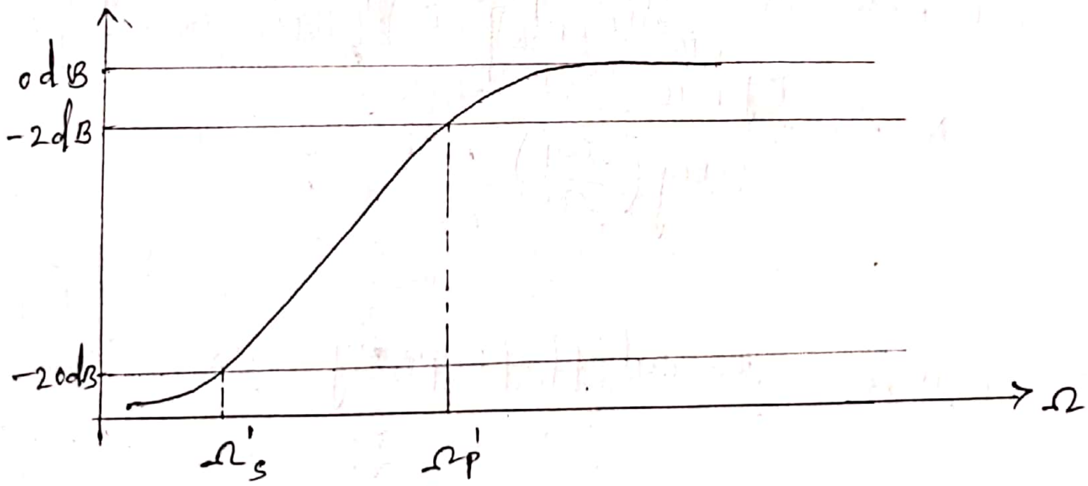
Step 5: Desired HPF T.F is obtained through frequency transformation as.

$$H_a(s) = H_n(s)\Big|_{s \rightarrow \frac{\Omega_p'}{\Omega_c s}}$$

(3)

Qn: Design a Butterworth analog highpass filter to meet the following specifications.

(i) Maximum passband attenuation of 2dB.
(ii) Passband edge frequency 200 rad/sec.
(iii) Minimum stopband attenuation = 20dB.
(iv) stopband edge frequency = 100 rad/sec.



Soln Given : $\Omega_s' = 100$ rad/sec.

$\Omega_p' = 200$ rad/sec.

Step 1 : Determine the passband & stopband edge frequency of normalized LPfilter.

$$\Omega_p = \frac{\Omega_p'}{\Omega_p'} \qquad \Omega_s = \frac{\Omega_p'}{\Omega_s'}$$

$$\Omega_p = \frac{200}{200} \qquad \Omega_s = \frac{200}{100}$$

$\Omega_p = 1$ rad/sec. $\qquad \Omega_s = 2$ rad/sec.

$k_p = -2dB \qquad k_s = -20dB$

Step 2:

$$N = \frac{\log\left[\dfrac{10^{-0.1k_p}-1}{10^{-0.1k_s}-1}\right]}{2\log\left(\dfrac{\Omega_p}{\Omega_s}\right)}$$

$$N = \frac{\log\left[\dfrac{10^{0.1\times2}-1}{10^{0.1\times20}-1}\right]}{2\log\left(1/2\right)}$$

$N = 3.7 \qquad N = 4$

Step 3: Cut-off frequency Ω_c

$$\Omega_c = \frac{\Omega_p}{\left[10^{-0.1k_p}-1\right]^{1/2N}}$$

$$= \frac{1}{\left[10^{+0.1\times2}-1\right]^{1/2\times4}}$$

$\Omega_c = 1.0693$ rad/sec.

Step 4: $$H_n(s) = \frac{1}{(s^2+0.7654s+1)(s^2+1.8478s+1)}$$

step 5:

$$H_a(S) = H_n(S)\Big|_{S \longrightarrow \frac{\Omega_p'}{\Omega_c S}}$$

$$S \longrightarrow \frac{200}{1.06935} = \frac{187.038}{S}$$

$$H_a(S) = \frac{1}{\left[\left(\frac{187.038}{S}\right)^2 + 0.7654 \times \frac{187.038}{S} + 1\right]\left[\left(\frac{187.038}{S}\right)^2 + 1.8478\left(\frac{187.038}{S}\right) + 1\right]}$$

$$H_a(S) = \frac{1}{(34983.21 S^{-2} + 143.158 S^{-1} + 1)(34983.21 S^{-2} + 345.61 S^{-1} + 1)}$$

$$H_a(S) = \frac{S^4}{(S^2 + 143.158S + 34983.21)(S^2 + 345.61S + 34983.21)}$$

verification

At $\Omega_p' = 200$ rad/sec. $\quad K_p = -2dB.$

$H_a(S)$ at $\Omega_p' = 200$ rad/sec.

$$H_a(j\Omega_p') = \frac{(j\Omega_p')^4}{((j\Omega_p')^2 + 143.158(j\Omega_p) + 34983.21)((j\Omega_p')^2 + 345.61(j\Omega_p') + 34983.21)}$$

$$H_a(j\Omega_p') = \frac{(200)^4}{\left(-200^2 + j143.158 \times 200 + 34983.21\right)\left(-200^2 + j345.61 \times 200 + 34983.21\right)}$$

$$= \frac{16 \times 10^8}{(-40000 + 28631.6\,j + 34983.21)(-40000 + 69122\,j + 34983.21)}$$

$$= \frac{16 \times 10^8}{(-5016.79 + 28631.6)(-5016.79 + 69122\,j)}$$

$$= 0.7942$$

$H_a(j\Omega_p') = -2.001\ dB$

Given

$H_a(j\Omega)\big|_{\Omega = \Omega_p' = 200} = -2\,dB$

Hence verified.

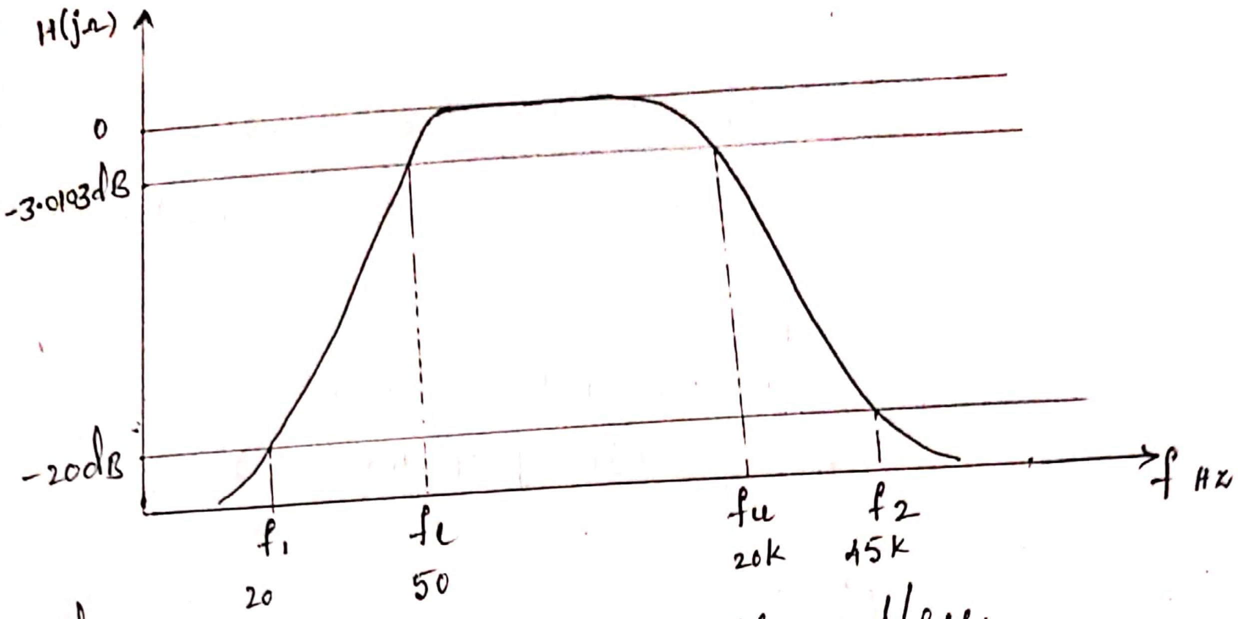
Qn: 4

Design a Butterworth analog bandpass filter that will meet the following specifications.

(i) Maximum attenuation of -3.0103 dB at lower and upper cut off frequency of 50 Hz & 20 kHz.

(ii) Minimum attenuation of 20 dB at 20Hz & 45 KHz.

(iii) Monotonic frequency response.



Soln

$\Omega_1 = 2\pi f_1 = 2\pi \times 20 = 125.663$ rad/sec.

$\Omega_2 = 2\pi f_2 = 2\pi \times 45 \times 10^3 = 2.827 \times 10^5$ rad/sec.

$\Omega_l = 2\pi f_l = 2\pi \times 50 = 314.159$ rad/sec.

$\Omega_u = 2\pi f_u = 2\pi \times 20 \times 10^3 = 1.257 \times 10^5$ rad/sec.

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)}$$

$$A = \frac{-(125.663)^2 + 314.159 \times 1.257 \times 10^5}{125.663\,(1.257 \times 10^5 - 314.159)}$$

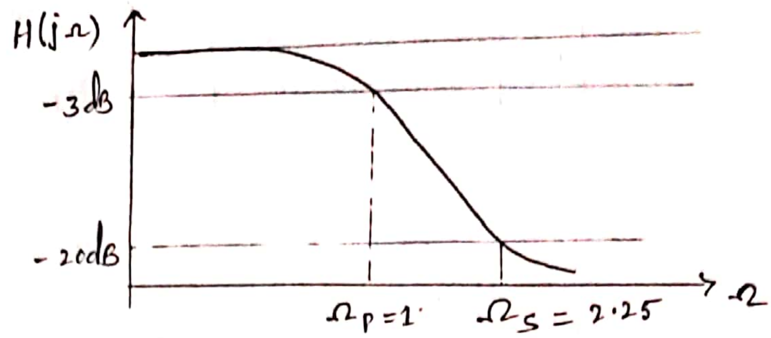
$$A = 2.51$$

$$B = \frac{-\Omega_2^2 + \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$$

$$= \frac{-(2.827 \times 10^5)^2 + 314.159 \times 1.257 \times 10^5}{2.827 \times 10^5\,(1.257 \times 10^5 - 314.159)}$$

$$B = -2.25$$

$$\Omega_s = \min(|A|, |B|) = 2.25$$



$$N = \frac{\log\left[\dfrac{10^{0.1 k_p} - 1}{10^{0.1 k_s} - 1}\right]}{2 \log\left(\dfrac{\Omega_p}{\Omega_s}\right)}$$

$$= \frac{\log\left[\dfrac{10^{0.1 \times 3} - 1}{10^{0.1 \times 20} - 1}\right]}{2 \log\left(\dfrac{1}{2.25}\right)}$$

$N = 2.83$ $\qquad$ $\underline{\underline{N = 3}}$

$$H_n(s) = \frac{1}{(s+1)(s^2 + s + 1)}$$

$$H_n(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}}$$

$$= H_n(s)\Big|_{s \to \frac{s^2 + 3.949 \times 10^7}{s(1.2538 \times 10^5)}}$$

$$H_a(S) = \frac{1}{\left[\frac{s^2+3.949\times10^7}{s(1.2538\times10^5)}\right]+2\left[\frac{s^2+3.949\times10^7}{s(1.2538\times10^5)}\right]+2\left[\frac{s^2+3.949\times10^7}{s(1.2538\times10^5)}\right]+1}$$

$$H_a(S) = \frac{1.9695\times10^{15}\, s^3}{(s^6+2.506\times10^5 s^5+3.154\times10^{10} s^4+1.989\times10^{15} s^3 + 1.2453\times10^{18} s^2+3.9073\times10^{20} s+6.1529\times10^{22})}$$

CHEBYSHEV FILTERS

The magnitude frequency response (squared) of a Chebyshev filter is

$$|H(j\Omega)|^2 = \frac{1}{\left[1+\epsilon^2 T_n^2 \frac{\Omega}{\Omega_p}\right]}$$

Properties of chebyshev filter

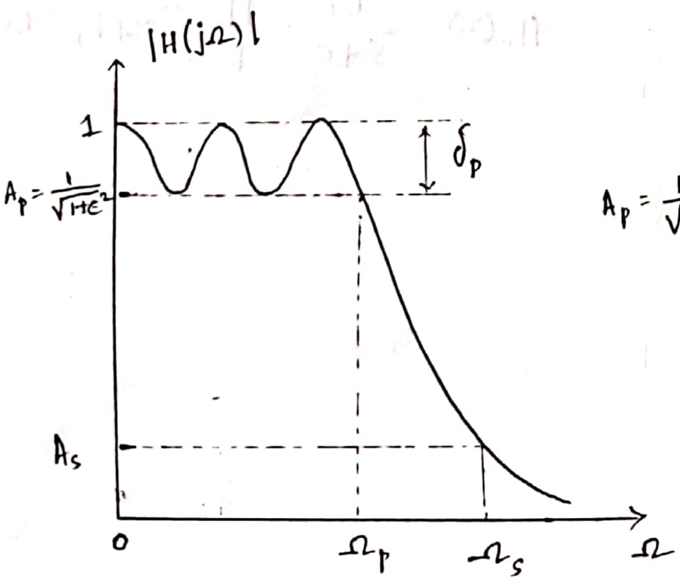
1) If $\Omega_p = 1$ rad/sec then it is called as type-1 normalized Chebyshev filter.

2) $|H_n(j\Omega)|^2_{\Omega=0} = 1$ for all N.

3) $|H(j0)| = 1$ for ODD N. and $|H(j0)| = \frac{1}{\sqrt{1+\epsilon^2}}$ for EVEN N.

4) The type I filter has uniform ripples in the passband and is monotonic outside the passband.

(i) N odd (ii) N Even



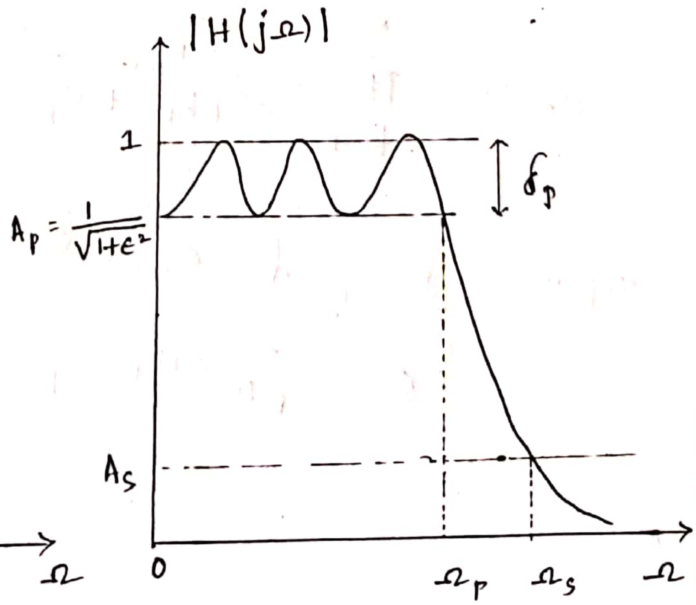


Fig: Magnitude Response of Chebyshev LPF.

The sum of the number of maxima and minima in the passband equals the order of the filter.

Steps to design a Chebyshev filter.

Step 1: To find order of the filter.

$$N = \frac{\cosh^{-1}\left[\sqrt{\dfrac{10^{-0.1k_s}-1}{10^{-0.1k_p}-1}}\right]}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

k_p : Passband attenuation in dB

k_s : Stopband attenuation in dB.

Ω_p : passband edge frequency in rad/sec.

Ω_s : stopband edge frequency in rad/sec.

Step 2: Round off N value to higher integer.

Step 3: Write the Transfer function of Normalized filter.

i) N even

$$H_n(S) = \prod_{k=1}^{N/2} \frac{B_k}{S^2+b_kS+C_k}$$

(ii) N odd

$$H_n(S) = \frac{B_0}{S+C_0}\prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{S^2+b_kS+C_k}$$

Step 4: To find ϵ value.

$$\epsilon = \sqrt{10^{-0.1k_p}-1}$$

Step 5:

To find denominator coefficients c_0, b_k & c_k of $H_n(s)$

$$y_N = \frac{1}{2}\left[\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}\right]$$

$$\mu = \frac{1}{\epsilon} + \sqrt{1 + \frac{1}{\epsilon^2}}$$

$$c_0 = y_N$$

$$b_k = 2 y_N \sin\left[\frac{(2k-1)\pi}{2N}\right]$$

$$c_k = y_N^2 + \cos^2\left[\frac{(2k-1)\pi}{2N}\right]$$

$k = 0, 1, \ldots \frac{N}{2}$ N even

OR

$k = 0, 1, \ldots \frac{N-1}{2}$ N odd.

Step 6: To find Numerator constants B_0, B_k of $H_n(s)$

i) N even

$$H_n(s)\Big|_{s=0} = \frac{1}{\sqrt{1+\epsilon^2}}$$

(ii) N odd

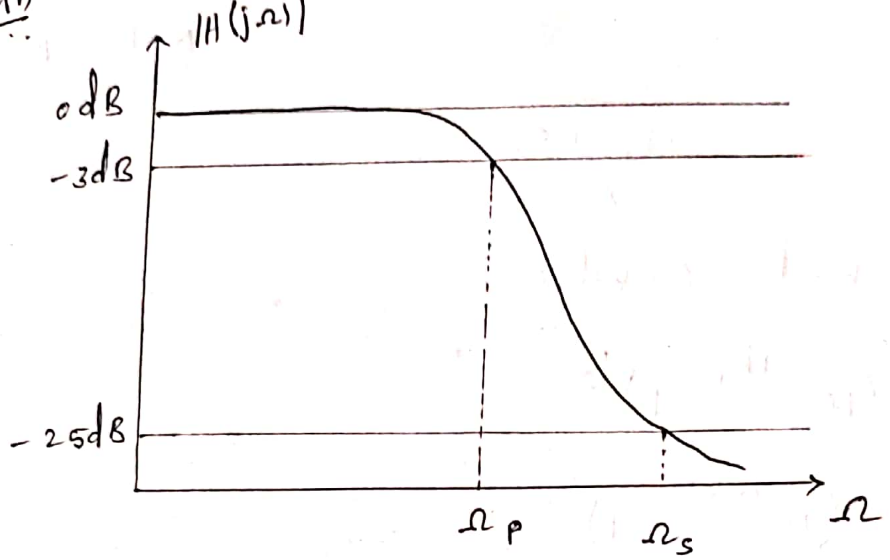
$$H_n(s)\Big|_{s=0} = 1$$

Step 7: obtain transfer function of unnormalized LP filter as

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_p}}$$

Design a Chebyshev IIR analog LPF that has -3dB frequency 100 rad/sec and stopband attenuation 25dB or greater or greater for all radian frequencies past 250 rad/sec.

soln



given:

$k_p = -3dB$ $\quad\quad$ $\Omega_p = 100$ rad/sec.

$k_s = -25dB$ $\quad\quad$ $\Omega_s = 250$ rad/sec.

step1:

$$N = \frac{\cosh^{-1}\left[\sqrt{\frac{10^{-0.1k_s}-1}{10^{-0.1k_p}-1}}\right]}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N = \frac{\cosh^{-1}\left[\sqrt{\frac{10^{0.1\times 25}-1}{10^{0.1\times 3}-1}}\right]}{\cosh^{-1}\left(\frac{250}{100}\right)}$$

$N = 2.279$

$N = 3$

Step: 3

Since N is odd

Transfer function of normalized filter is given as

$$H_n(S) = \frac{B_0}{S+C_0} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k}{S^2+b_k S + C_k}$$

$$\frac{N-1}{2} = \frac{3-1}{2}$$

$$\frac{N-1}{2} = 1$$

$$H_n(S) = \frac{B_0}{S+C_0} \quad \frac{B_1}{S^2+b_1 S + C_1}$$

Step 4: To find ϵ value.

$$\epsilon = \left(10^{-0.1k_p} - 1\right)^{1/2}$$

$$\epsilon = \left(10^{+0.1\times 3} - 1\right)^{1/2}$$

$$\epsilon = 0.9976$$

$$\boxed{\epsilon \simeq 1}$$

Step 5: To find denominator coefficients of $H_n(S)$

$$y_N = \frac{1}{2}\left[\mu^{1/N} - \mu^{-\frac{1}{N}}\right]$$

$$\mu = \sqrt{1+\frac{1}{\epsilon^2}} + \frac{1}{\epsilon}$$

$$\mu = \sqrt{1+\frac{1}{1^2}} + \frac{1}{1}$$

$$\mu = \sqrt{2} + 1$$

$$\mu = 2.414$$

$$y_N = \frac{1}{2}\left[(2.414)^{1/3} - (2.414)^{-1/3}\right]$$

$$y_N = 0.298$$

$c_0 = y_N = 0.298$, $k = 1$

$$b_k = 2 \times y_N \sin\left[\frac{(2k-1)}{2N}\pi\right] \qquad k = 1, 2 \ldots \frac{N-1}{2}$$

$k = 1$

$$b_1 = 2 \times 0.298 \sin\left(\frac{\pi}{6}\right)$$

$$= \cancel{2} \times 0.298 \times \frac{1}{\cancel{2}}$$

$$\underline{b_1 = 0.298}$$

$$c_k = y_N^2 + \cos^2\left[\frac{(2k-1)}{2N}\pi\right] \qquad k = 1$$

$$c_1 = (0.298)^2 + \left(\cos\frac{\pi}{6}\right)^2$$

$$= 0.088 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$c_1 = \underline{\underline{0.838}}$$

$$H_n(s) = \frac{B_0}{s + 0.298} \cdot \frac{B_1}{s^2 + 0.298s + 0.828}$$

~~Since~~

step 6: To find the numerator coefficients of $H_n(s)$

Since N is odd

$$H_n(s)\Big|_{s=0} = 1$$

$$\frac{B_0}{0 + 0.298} \cdot \frac{B_1}{0 + 0 + 0.838} = 1$$

$$B_0 B_1 = 0.249$$

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$B_0 B_1 = 0.25$

$B_0 = B_1$

$\boxed{B_0^2 = 0.25}$

$B_0 = \sqrt{0.25}$ $\boxed{B_0 = 0.5}$

$$H_n(s) = \frac{0.25}{(s+0.298)(s^2+0.296s+0.838)}$$

$$= \frac{0.25}{s^3+0.298s^2+0.838s+0.298s^2+0.0888s+0.249}$$

$$H_n(s) = \frac{0.25}{s^3+0.596s^2+0.9268s+0.25}$$

<u>step 7:</u>

The unnormalized (required analog LPF) is obtained as.

$$H_a(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_p} = \frac{s}{100}}$$

$$H_a(s) = \frac{0.25}{\frac{1}{100^3}s^3 + 0.596 s^2 \frac{1}{100^2} + \frac{1}{100}0.9268 s + 0.25} \times \frac{100^3}{100^3}$$

$$H_a(s) = \frac{0.25 \times 100^3}{s^3 + 59.6 s^2 + 9268 s + 0.25 \times 100^3}$$