# Module-5
# Spread Spectrum Modulation

- Pseudo noise sequences-PN Sequence
- Notion of spread spectrum
- Direct sequence spread spectrum
- Frequency Hop Spread Spectrum
- Applications

# Spread Spectrum Modulation

- **Bandwidth** and **Power** are the two primary communication resources that they need to be used with care in the design of most communication systems.

- There are situations where it is necessary to sacrifice the efficient utilization of these two resources in order to meet certain other design objectives.

- For example, the system may be required to provide a form of secure

  communication in a hostile environment such that the transmitted signal is not easily detected or recognized by unwanted listeners. This requirement is catered to by a class of signalling techniques known collectively as spread-spectrum modulation.

- The primary advantage of a spread-spectrum communication system is its ability to reject interference whether it be the unintentional interference of another user simultaneously attempting to transmit through the channel, or the intentional interference of a hostile transmitter attempting to jam the transmission.

# Spread Spectrum Modulation

The definition of spread spectrum may be stated in two parts:

1. Spread spectrum is a means of transmission in which the data of interest occupies a bandwidth in excess of the minimum bandwidth necessary to send the data.
2. The spectrum spreading is accomplished before transmission through the use of a code that is independent of the data sequence. The same code is used in the receiver (operating in synchronism with the transmitter) to despread the received signal so that the original data may be recovered.

Although standard modulation techniques such as frequency modulation and pulse-code modulation do satisfy Part 1 of this definition, they are not spread spectrum techniques because they do not satisfy Part 2 of the definition.

# Cont'd..

- Spread-spectrum modulation was originally developed for **military applications** where resistance to jamming (interference) is of major concern.
- However, there are civilian applications that also benefit from the unique characteristics of spread-spectrum modulation.
  - For example, it can be used to provide multipath rejection in a ground-based mobile radio environment.
  - Another application is in multiple-access communication in which a number of independent users are required to share a common channel without an external synchronizing mechanism; example, we may mention a ground-based mobile radio environment involving mobile vehicles that must communicate with a central station.

# Pseudo Noise sequences

- A pseudo-noise (PN) sequence is defined as a coded sequence of 1's and 0's with certain autocorrelation properties.
- The class of sequences used in spread-spectrum communications is usually periodic in that a sequence of 1's and 0's repeats itself exactly with a known period.
- The maximum-length sequence, a type of cyclic code, represents a commonly used periodic PN sequence.
- Such sequences have long periods and require simple instrumentation in the form of a linear feedback shift register. Indeed, they possess the longest possible period for this method of generation.

# Maximum length sequence generator

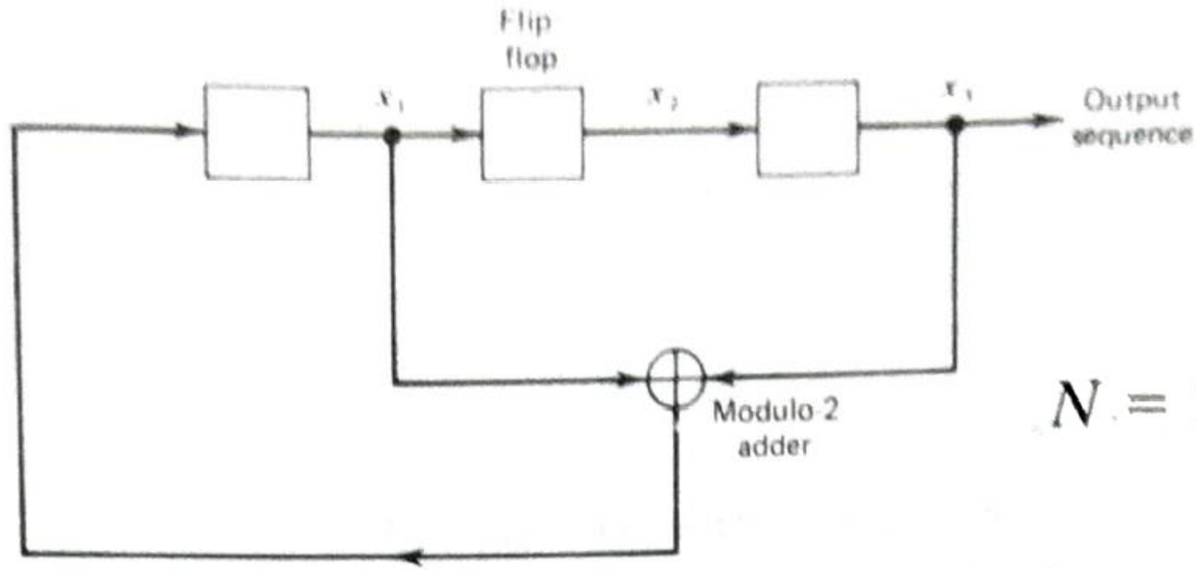



**Figure 9.1** Maximum-length sequence generator.

$$N = 2^m - 1$$

Consider the three-stage feedback shift register shown in Fig. 9.1. It is assumed that the initial state of the shift register is 100 (reading the contents of the three flip-flops from left to right). Then, the succession of states will be as follows:

100, 110, 111, 011, 101, 010, 001, 100, . . . .

The output sequence (the last position of each state of the shift register) is therefore

0011101

| x1 | x2 | x3 | output=x3 |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

**PN Sequence**

**0011101**

**N=7**

**Balance Property**

**Num of 0's= 3**

**Num of 1's=4**

**Num of 1's is one(count) greater than Num of 0's**

**Run Property**

**00 111 0 1**

Total runs=4

**length 1=2 .. (0 1)**

**50% of Total runs(4/2)**

**length 2= 1.. (00)**

**25% of Total runs(4/4)**

**length 3= 1..(111)**

**12.5% of Total runs(4/8)**

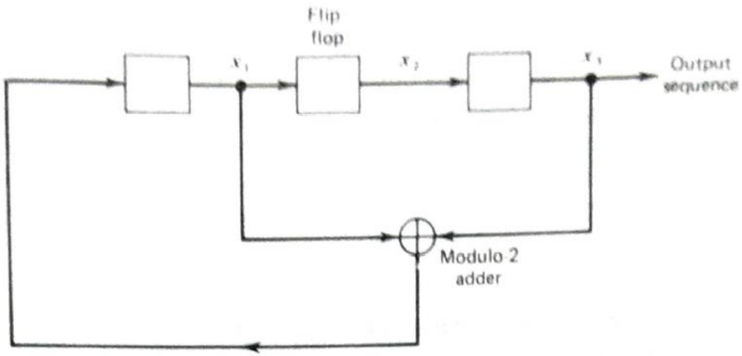


**Figure 9.1** Maximum-length sequence generator.

| x1 | x2 | x3 | output=x3 |
|---|---|---|---|
| **1** | **1** | **0** | **0** |
| **1** | **1** | **1** | **1** |
| **0** | **1** | **1** | **1** |
| **1** | **0** | **1** | **1** |
| **0** | **1** | **0** | **0** |
| **0** | **0** | **1** | **1** |
| **1** | **0** | **0** | **0** |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| | | | |
| | | | |
| | | | |

**Table 9.1** Range of PN Sequence Lengths

| Length of Shift Register, $m$ | PN Sequence Length, $N$ |
|---|---|
| 7 | 127 |
| 8 | 255 |
| 9 | 511 |
| 10 | 1023 |
| 11 | 2047 |
| 12 | 4095 |
| 13 | 8191 |
| 17 | 131071 |
| 19 | 524287 |

## (1) Properties of Maximum-length Sequences

Maximum-length sequences* have many of the properties possessed by a truly *random binary sequence*. A random binary sequence is a sequence in which the presence of a binary symbol 1 or 0 is equally probable. Some properties of maximum-length sequences are listed below:

### PROPERTY 1

*In each period of a maximum-length sequence, the number of 1s is always one more than the number of 0s*. This property is called the *balance property*.

### PROPERTY 2

*Among the runs of 1s and of 0s in each period of a maximum-length sequence, one-half the runs of each kind are of length one, one-fourth are of length two, one-eighth are of length three, and so on as long as these fractions represent meaningful numbers of runs*. This property is called the *run property*.

By a "run" we mean a subsequence of identical symbols (1s or 0s) within one period of the sequence. The length of this subsequence is the length of the run. For a maximum-length sequence generated by a feedback shift register of length $m$, the total number of runs is $(m + 1)/2$.

**PROPERTY 3**

*The autocorrelation function of a maximum-length sequence is periodic and binary-valued.* This property is called the *correlation property*.

Let binary symbols 0 and 1 be represented by $-1$ volt and $+1$ volt, respectively. By definition, the *autocorrelation sequence* of a binary sequence $\{c_n\}$, so represented, equals

$$R_c(k) = \frac{1}{N} \sum_{n=1}^{N} c_n c_{n-k}$$

where $N$ is the *length* or *period* of the sequence and $k$ is the *lag* of the autocorrelation sequence. For a maximum-length sequence of length $N$, the autocorrelation sequence is periodic with period $N$ and two-valued, as shown by

$$R_c(k) = \begin{cases} 1 & k = lN \\ -\dfrac{1}{N} & k \neq lN \end{cases}$$

where $l$ is any integer. When the length $N$ is infinitely large, the autocorrelation sequence $R_c(k)$ approaches that of a completely random binary sequence.

**EXAMPLE 2**

Consider again the maximum-length sequence generated by the feedback shift register of Fig. 9.1. The output sequence (represented in terms of binary symbols 0 and 1) is

$$\{c_n\} = \underbrace{0011101}_{N = 7} \ldots \tag{9.4}$$

In terms of the levels $-1$ and $+1$, the output sequence is

$$\{c_n\} = \underbrace{-1, -1, +1, +1, +1, -1, +1}_{N = 7}, \ldots \tag{9.5}$$

We see that there are three 0s (or $-1$'s) and four 1s (or $+1$'s) in one period of the sequence, which satisfies Property 1.

With $N = 7$, there are a total of four runs in one period of the sequence. Reading them from left to right in Eq. 9.4, the four runs are 00, 111, 0, and 1. Two of the runs (a half of the total) are of length one, and one run (a quarter of the total) is of length two, which satisfies Property 2.

Figure 9.2*a* shows two full periods of the maximum-length sequence. Figure 9.2*b* shows the corresponding autocorrelation function $R_c(\tau)$ plotted as a function of the time lag $\tau$. In this figure, the parameter $T_c$ denotes the duration of binary symbol 1 or 0 in the sequence, and $N$ is the length of one period of the sequence. The periodic and two-valued correlation property of the sequence is clearly seen in Fig. 9.2*b*.

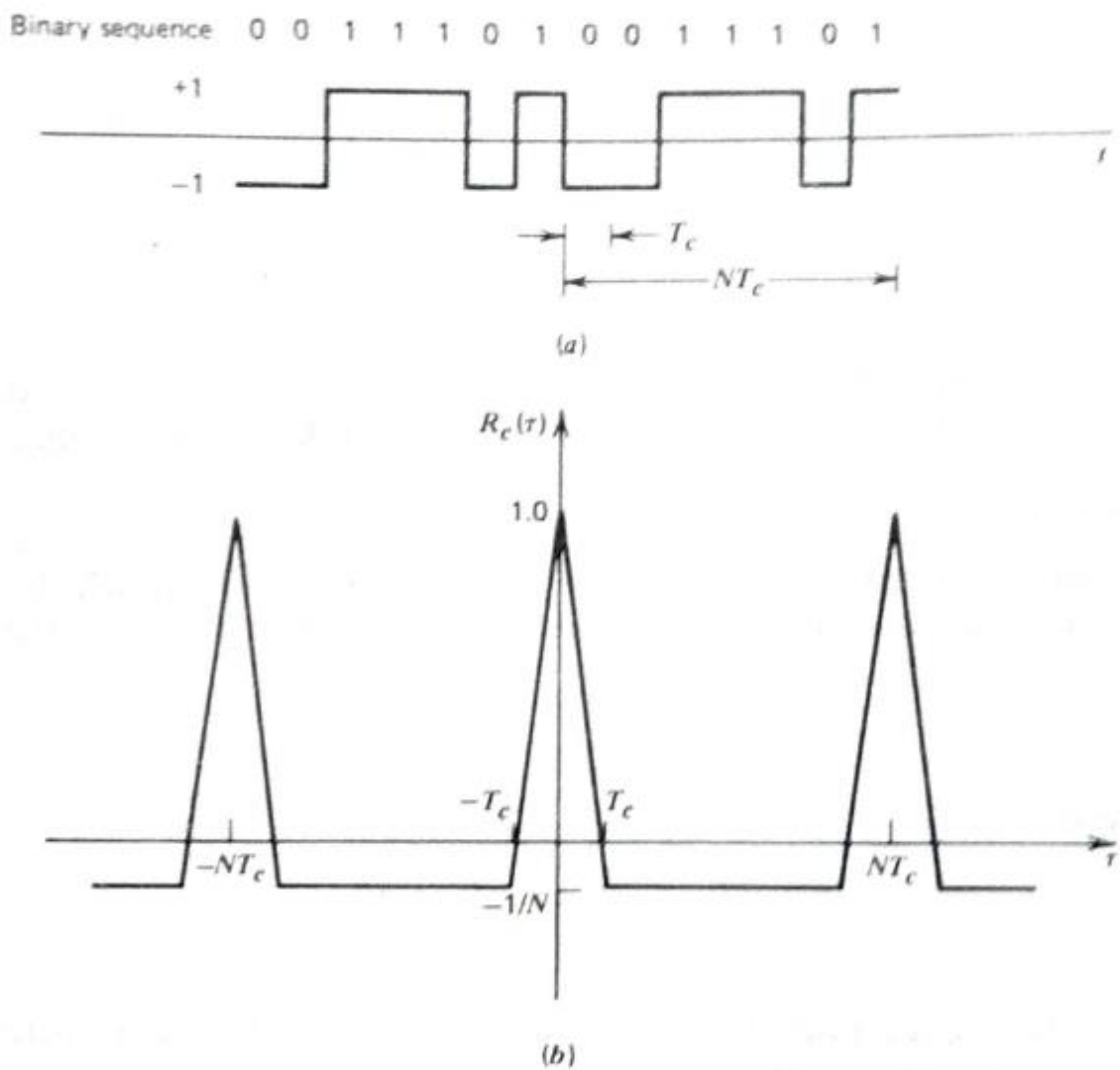
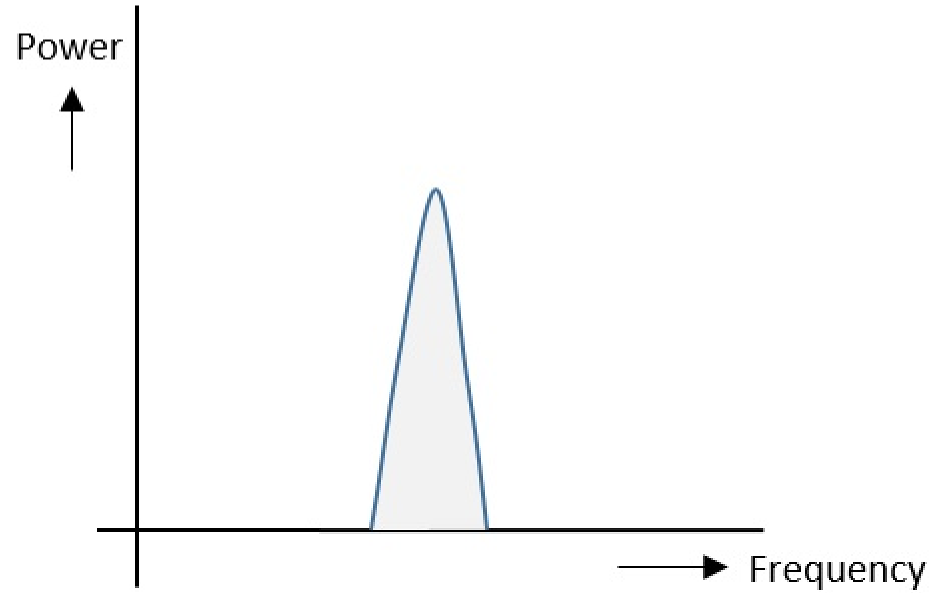



**Figure 9.2** (*a*) Waveform of maximum-length sequence. (*b*) Autocorrelation of maximum-length sequence. Both parts refer to the output of the feedback shift register of Fig. 9.1.

## Narrow-band Signals

The Narrow-band signals have the signal strength concentrated as shown in the following frequency spectrum figure.




Narrow-band Signal

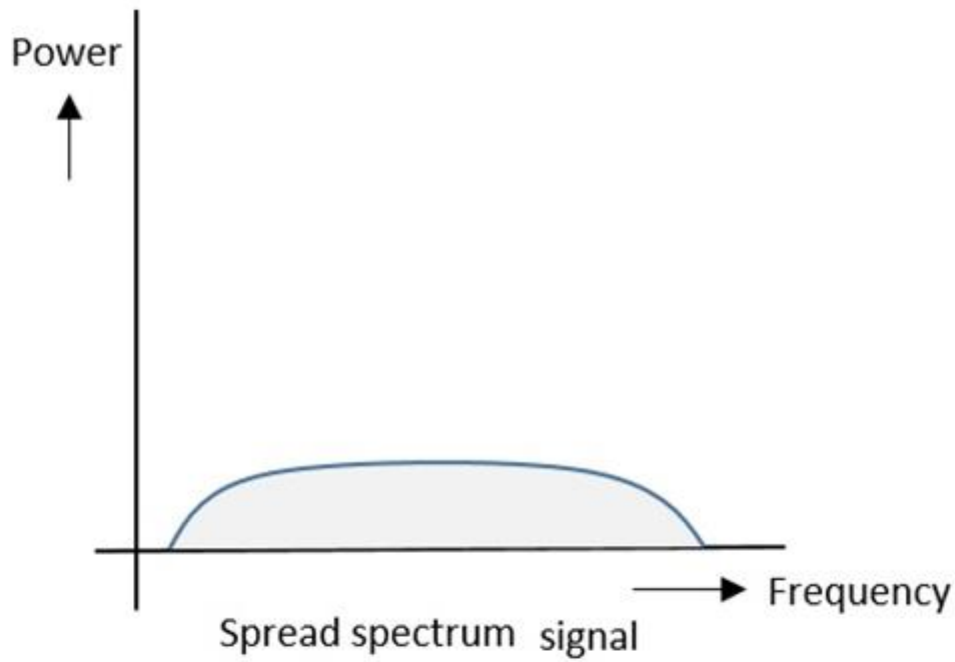
Following are some of its features −

- Band of signals occupy a narrow range of frequencies.
- Power density is high.
- Spread of energy is low and concentrated.

Though the features are good, these signals are prone to interference.

## Spread Spectrum Signals

The spread spectrum signals have the signal strength distributed as shown in the following frequency spectrum figure.




Following are some of its features −

- Band of signals occupy a wide range of frequencies.
- Power density is very low.
- Energy is wide spread.

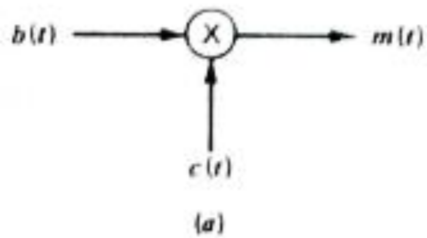
With these features, the spread spectrum signals are highly resistant to interference or jamming. Since multiple users can share the same spread spectrum bandwidth without interfering with one another, these can be called as **multiple access techniques**.

# Advantages of Spread Spectrum

Following are the advantages of spread spectrum −

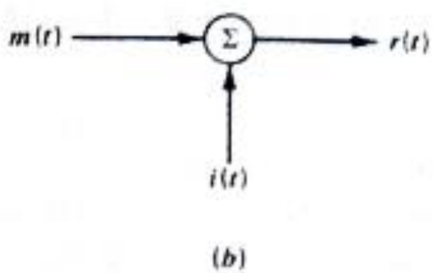
- Cross-talk elimination
- Better output with data integrity
- Reduced effect of multipath fading
- Better security
- Reduction in noise
- Co-existence with other systems
- Longer operative distances
- Hard to detect
- Not easy to demodulate/decode
- Difficult to jam the signals

# A Notion of Spread Spectrum



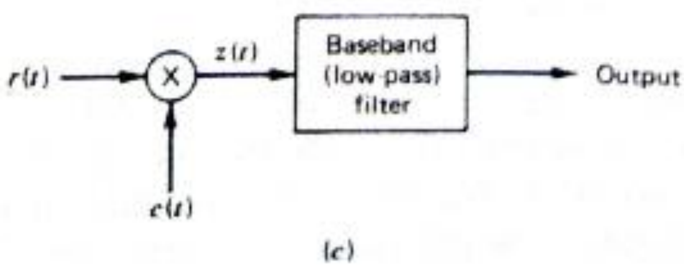

$$m(t) = c(t)b(t)$$

$$r(t) = m(t) + i(t)$$
$$= c(t)b(t) + i(t)$$




$$z(t) = c(t)r(t)$$
$$= c^2(t)b(t) + c(t)i(t)$$

$$c^2(t) = 1 \qquad \text{for all } t$$




$$z(t) = b(t) + c(t)i(t)$$

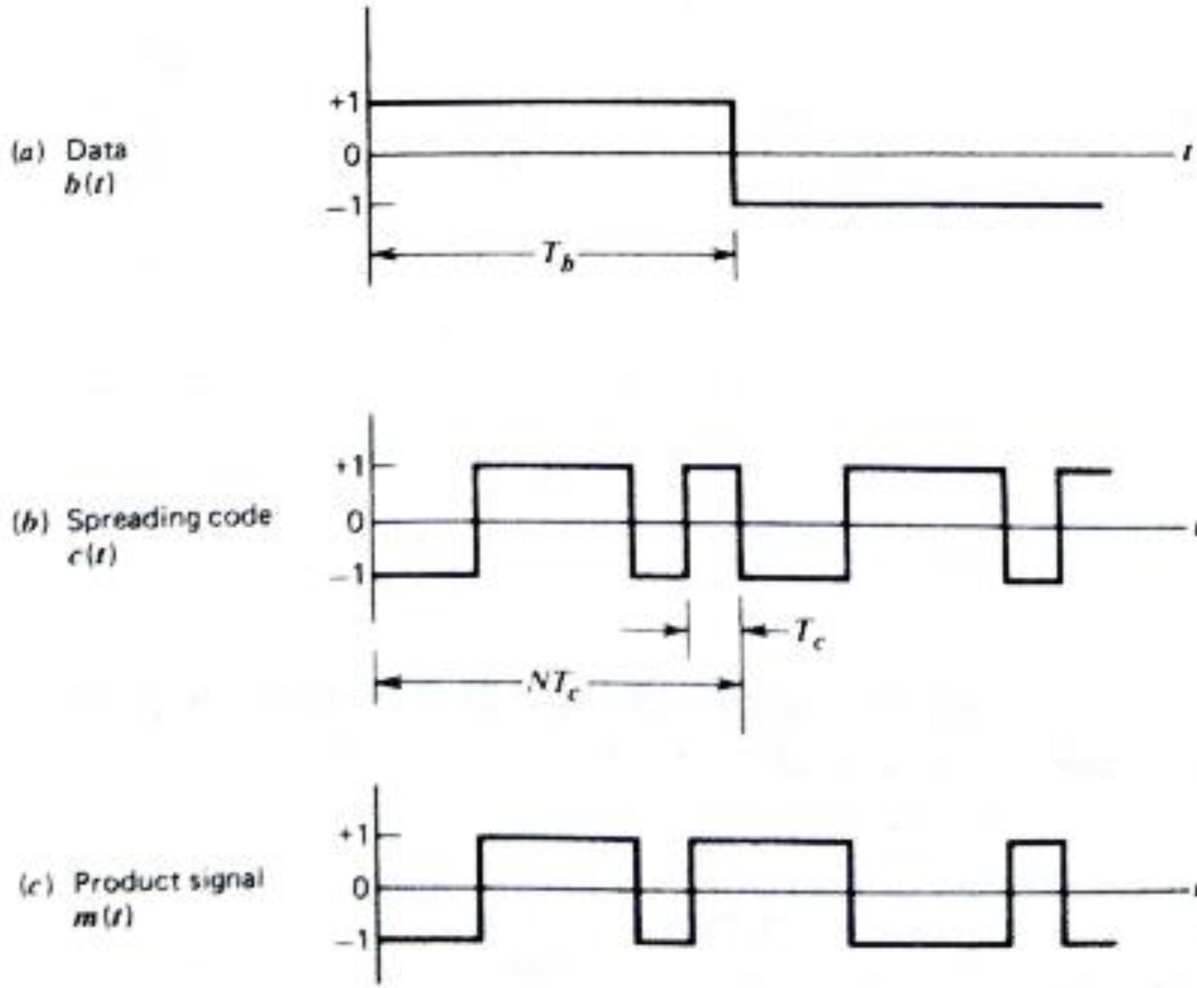
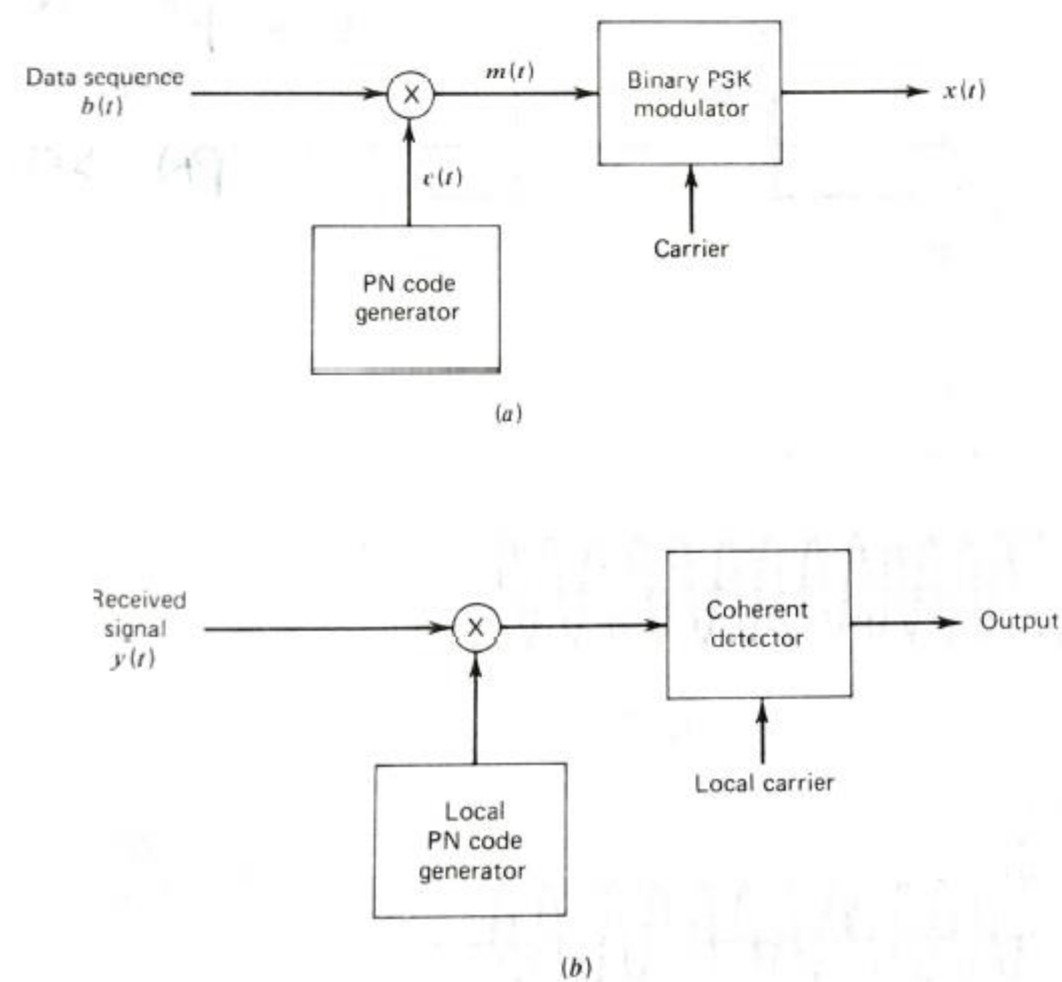
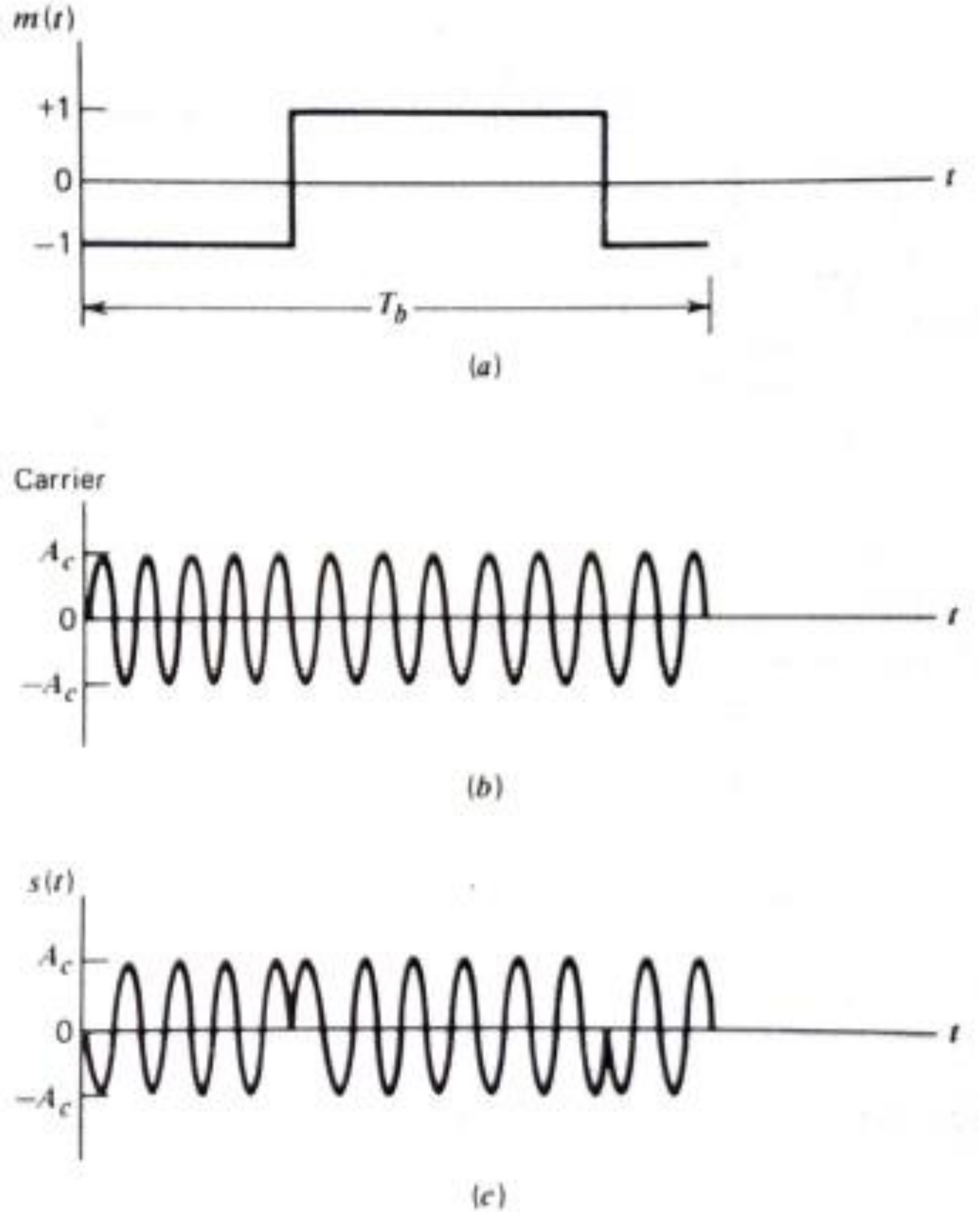


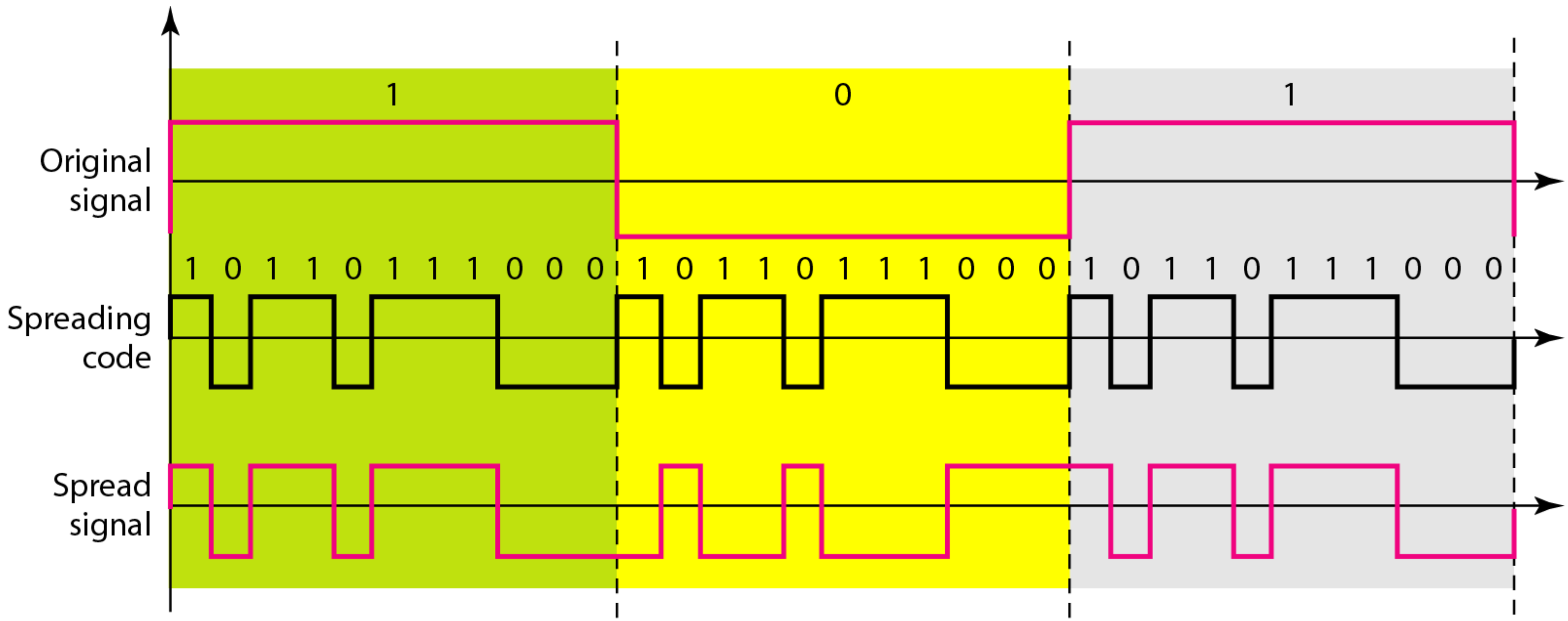

**Figure 9.4** Illustrating the waveforms in the transmitter of Fig. 9.3*a*.

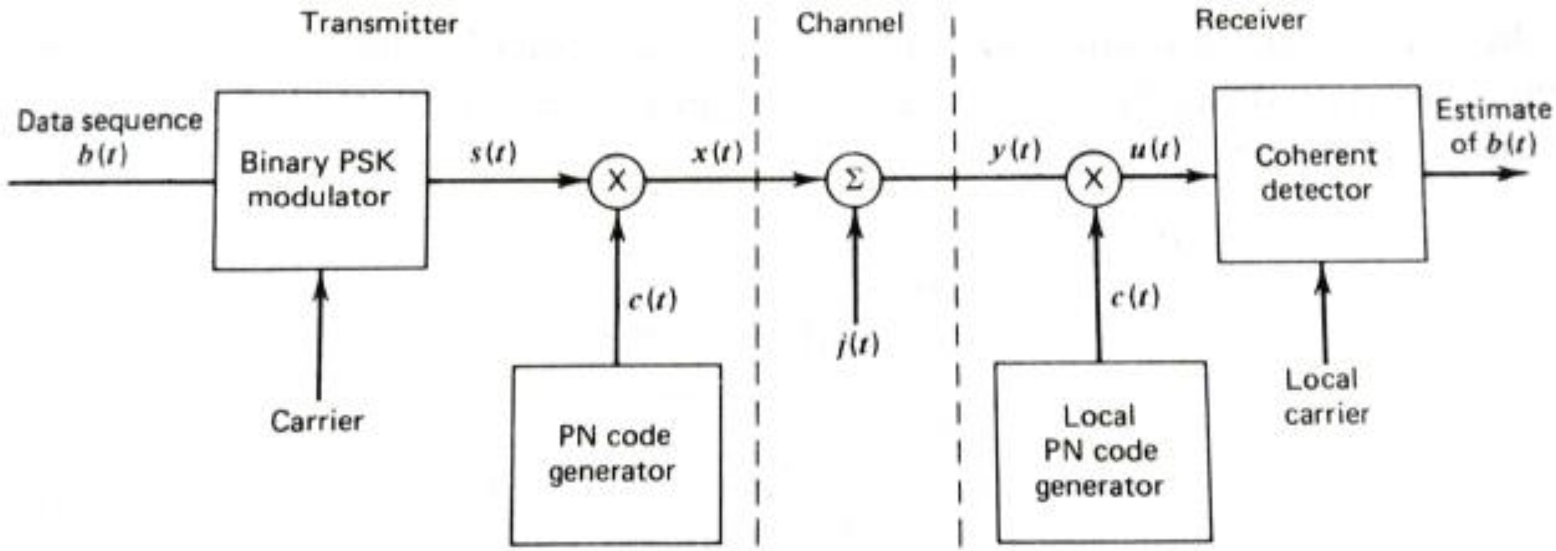
# Direct sequence spread spectrum




m(t)
+1
0
−1
t
T_b
(a)
Carrier
A_c
0
−A_c
t
(b)
s(t)
A_c
0
−A_c
t
(c)

# Example


1
0
1
Original
signal
1 0 1 1 0 1 1 1 0 0 0 1 0 1 1 0 1 1 1 0 0 0 1 0 1 1 0 1 1 1 0 0 0
Spreading
code
Spread
signal




Model of direct-sequence spread binary PSK system.

$$\begin{aligned} y(t) &= x(t) + j(t) \\ &= c(t)s(t) + j(t) \end{aligned}$$

$$\begin{aligned} u(t) &= c(t)\, y(t) \\ &= c^2(t)\, s(t) + c(t)\, j(t) \\ &= s(t) + c(t)\, j(t), \end{aligned}$$

$$c^2(t) = 1 \qquad \text{for all } t$$

# Signal space dimensionality and processing gain

Signal space representation of the transmission and interfering signal

Orthogonal basis functions

$$\phi_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}} \cos(2\pi f_c t) & kT_c \leq t \leq (k+1)T_c \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{\phi}_k(t) = \begin{cases} \sqrt{\dfrac{2}{T_c}} \sin(2\pi f_c t) & kT_c \leq t \leq (k+1)T_c \\ 0 & \text{otherwise} \end{cases}$$

$$k = 0, 1, \ldots, N-1$$

Tc= Chip duration, N=Number of chips per bit

The transmitted signal X(t) is

$$\begin{aligned} x(t) &= c(t)s(t) \\ &= \pm \sqrt{\frac{2E_b}{T_b}}\, c(t) \cos(2\pi f_c t) \\ &= \pm \sqrt{\frac{E_b}{N}} \sum_{k=0}^{N-1} c_k \phi_k(t) \qquad 0 \leq t \leq T_b \end{aligned}$$

$$(\mathrm{SNR})_I = \frac{E_b/T_b}{J}$$

$$(\mathrm{SNR})_O = \frac{2T_b}{T_c}(\mathrm{SNR})_I \qquad (9.36)$$

It is customary practice to express signal-to-noise ratios in decibels. We may thus write Eq. 9.36 in the equivalent form

$$10\log_{10}(\mathrm{SNR})_O = 10\log_{10}(\mathrm{SNR})_I + 3 + 10\log_{10}(PG),\ \mathrm{dB} \qquad (9.37)$$

where

$$PG = \frac{T_b}{T_c} \qquad (9.38)$$

$$spread\ factor\ N = T_b/T_c$$

Average Interference power of j(t) $J = \dfrac{1}{T_b}\displaystyle\int_0^{T_b} j^2(t)\,dt$

$$(\text{SNR})_I = \frac{E_b/T_b}{J} \qquad (\text{SNR})_O = \frac{2T_b}{T_c}(\text{SNR})_I \tag{9.36}$$

It is customary practice to express signal-to-noise ratios in decibels. We may thus write Eq. 9.36 in the equivalent form

$$10\log_{10}(\text{SNR})_O = 10\log_{10}(\text{SNR})_I + 3 + 10\log_{10}(PG), \text{ dB} \tag{9.37}$$

where

$$PG = \frac{T_b}{T_c} \tag{9.38}$$

$$\textit{spread factor } N = T_b/T_c$$

Average Interference power of j(t) $$J = \frac{1}{T_b}\int_0^{T_b} j^2(t)\,dt$$

The 3-dB term on the right side of Eq. 9.37 accounts for the gain in SNR that is obtained through the use of coherent detection (which presumes exact knowledge of the signal phase by the receiver). This gain in SNR has nothing to do with the use of spread spectrum. Rather, it is the last term, $10\log_{10}(PG)$, that accounts for the *gain in SNR obtained by the use of spread spectrum*. The ratio $PG$, defined in Eq. 9.38, is therefore referred to as the *processing gain*. Specifically, it represents the gain achieved by processing a spread-spectrum signal over an unspread signal. Note that both the processing gain $PG$ and the spread factor $N$ (i.e., PN sequence length) equal the ratio $T_b/T_c$. Thus the longer we make the PN sequence (or, correspondingly, the smaller the chip time $T_c$ is), the larger will the processing gain be.

We may define the processing gain in another way by making two observations:

1. The *bit rate* of the binary data entering the transmitter input is given by

$$R_b = \frac{1}{T_b}$$

2. The bandwidth of the PN sequence $c(t)$, defined in terms of the main lobe of its spectrum, is given by

$$W_c = \frac{1}{T_c}$$

Note that both $R_b$ and $W_c$ are baseband parameters. Hence, we may reformulate the processing gain of Eq. 9.38 as

$$PG = \frac{W_c}{R_b}$$

# Probability of Error

BPSK

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right)$$

# Probability of Error

BPSK

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{JT_c}}\right)$$

## Antijam characteristics

$$\frac{N_0}{2} = \frac{JT_c}{2}$$

$$E_b = PT_b$$

$$\frac{E_b}{N_0} = \left(\frac{T_b}{T_c}\right)\left(\frac{P}{J}\right)$$

$$\frac{J}{P} = \frac{PG}{E_b/N_0}$$

$$(\text{Jamming margin})_{\text{dB}} = (\text{Processing gain})_{\text{dB}} - 10\log_{10}\left(\frac{E_b}{N_0}\right)_{\min}$$

## EXAMPLE 3

A spread-spectrum communication system has the following parameters:

$$\text{Information bit duration, } T_b = 4.095 \text{ ms}$$

$$\text{PN chip duration, } T_c = 1\ \mu\text{s}$$

Hence, using Eq. 9.38, we find that the processing gain is

$$PG = 4095$$

Correspondingly, the required PN sequence is $N = 4095$, and the feedback shift length is $m = 12$.

For a satisfactory reception, we may assume that the average probability of error is not to exceed $10^{-5}$. From the formula for a coherent binary PSK receiver, we find that $E_b/N_0 = 10$ yields an average probability of error equal to $0.387 \times 10^{-5}$. Hence, using this value for $E_b/N_0$, and the value calculated for the processing gain, we find from Eq. 9.49 that the jamming margin is

$$\begin{aligned}(\text{Jamming margin})_{\text{dB}} &= 10 \log_{10} 4095 - 10 \log_{10}(10) \\ &= 36.1 - 10 \\ &= 26.1 \text{ dB}\end{aligned}$$

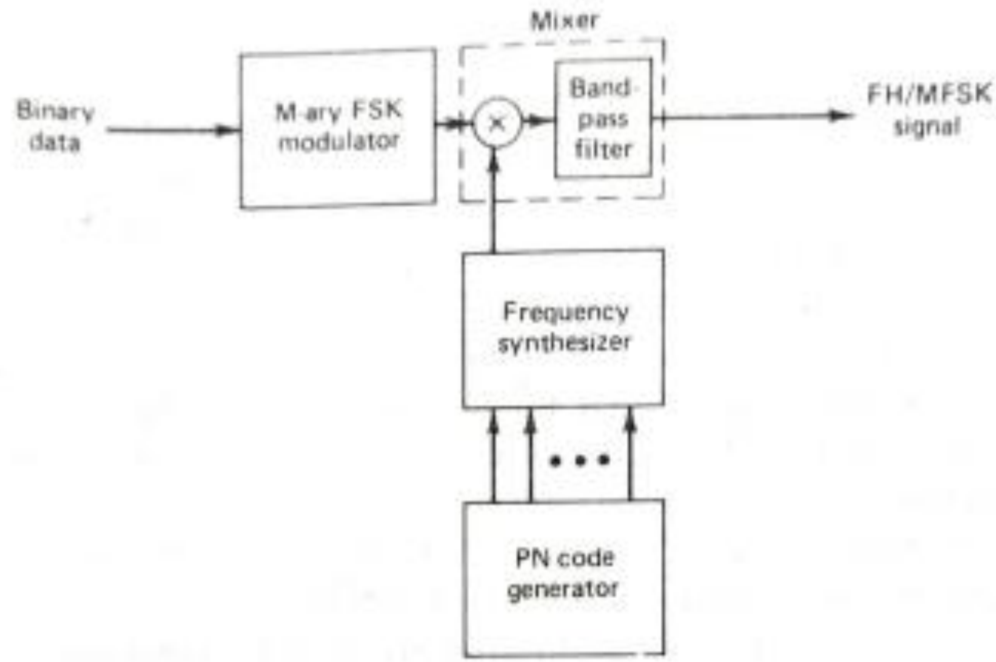
# Frequency-Hop Spread Spectrum

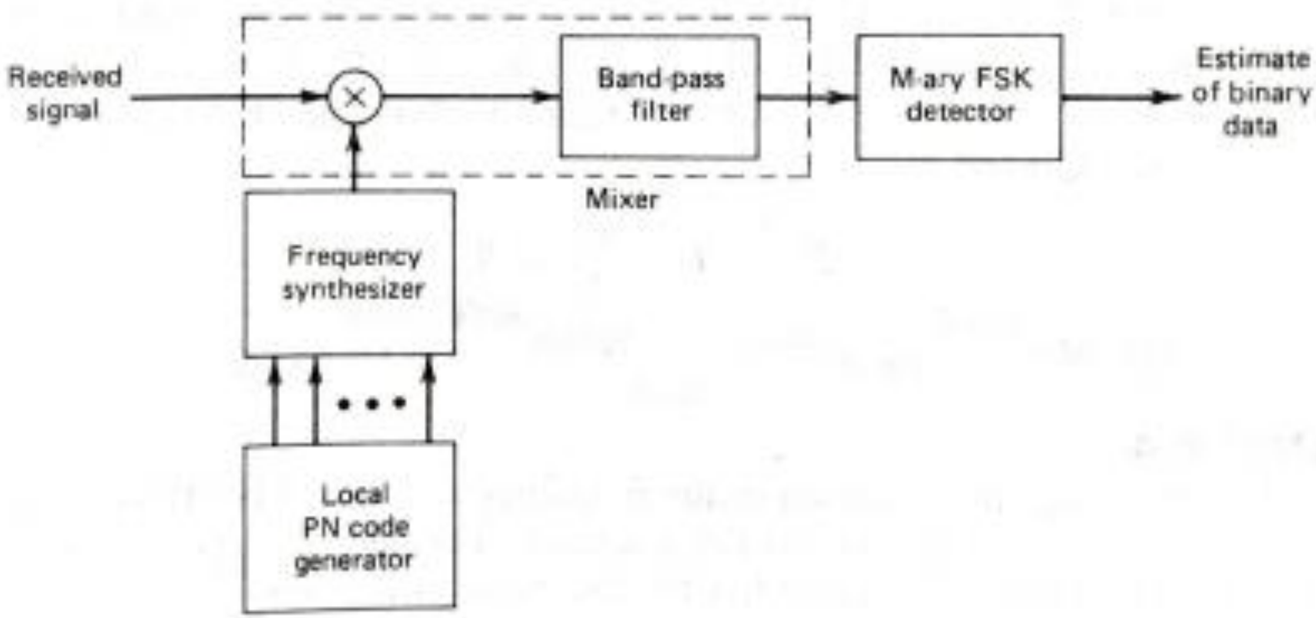
- The type of spread spectrum in which the carrier hops randomly from one frequency to another is called frequency- hop(FH) spread spectrum.
- A common modulation format for FH systems is that of M-ary frequency shift keying(MFSK). The combination is referred as FH/MFSK

Since frequency hopping does not cover the entire spread spectrum instantaneously, we are led to consider the rate at which the hops occur. In this context, we may identify two basic (technology-independent) characterizations of frequency hopping:

1. *Slow-frequency hopping*, in which the *symbol rate* $R_s$ of the MFSK signal is an integer multiple of the *hop rate* $R_h$. That is, several symbols are transmitted on each frequency hop.
2. *Fast-frequency hopping*, in which the hop rate $R_h$ is an integer multiple of the MFSK symbol rate $R_s$. That is, the carrier frequency will change or hop several times during the transmission of one symbol.

# Frequency hop spread M-ary frequency shift keying Transmitter and Receiver



# Slow Frequency Hopping

**EXAMPLE 4**

Figure 9.9*a* illustrates the variation of the frequency of a slow FH/MFSK signal with time for one complete period of the PN sequence. The period of the PN sequence is $2^4 - 1 = 15$. The FH/MFSK signal has the following parameters:

| | |
|---|---|
| Number of bits per MFSK symbol | $K = 2$ |
| Number of MFSK tones | $M = 2^K = 4$ |
| Length of PN segment per hop | $k = 3$ |
| Total number of frequency hops | $2^k = 8$ |

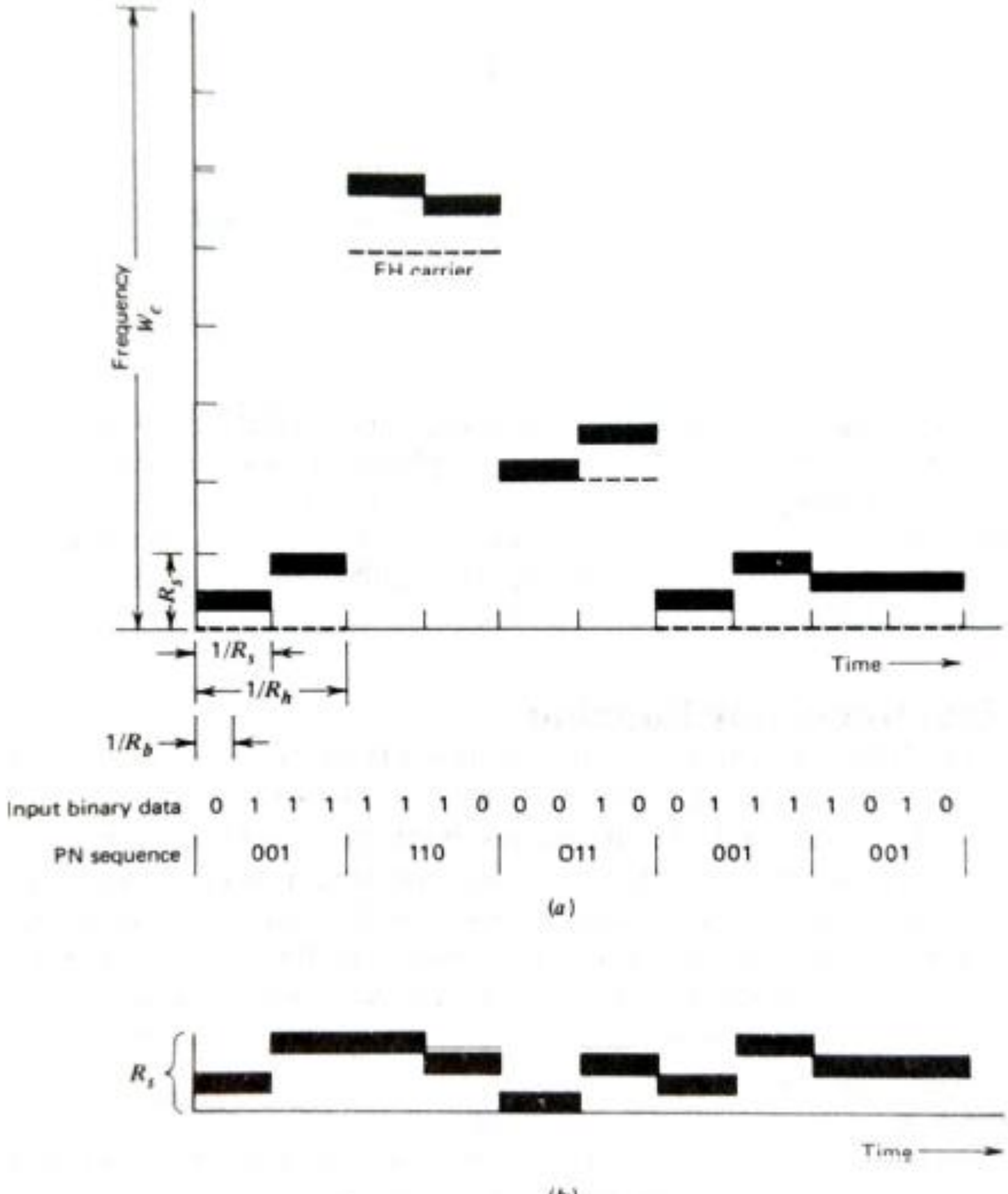
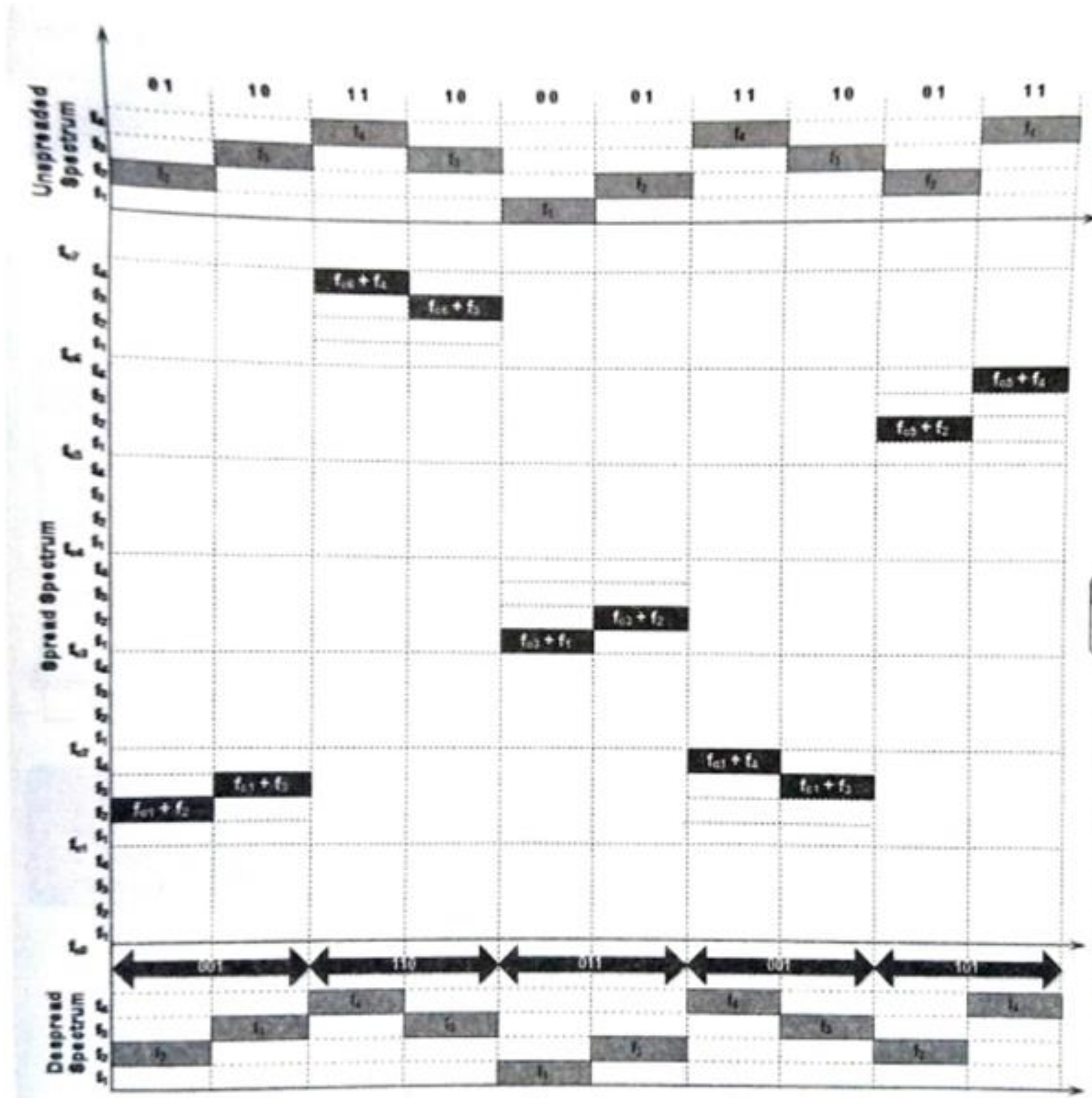



**Figure 9.9** Illustrating slow-frequency hopping.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 111 | 11 | | | | | | | | | | |
| | 10 | | | | | | | | | | |
| | 01 | | | | | | | | | | |
| | 00 | | | | | | | | | | |
| 110 | 11 | | | | | | | | | | |
| | 10 | | | | | | | | | | |
| | 01 | | | | | | | | | | |
| | 00 | | | | | | | | | | |
| 101 | 11 | | | | | | | | | | |
| | 10 | | | | | | | | | | |
| | 01 | | | | | | | | | | |
| | 00 | | | | | | | | | | |
| 100 | 11 | | | | | | | | | | |
| | 10 | | | | | | | | | | |
| | 01 | | | | | | | | | | |
| | 00 | | | | | | | | | | |
| 011 | 11 | | | | | | | | | | |
| | 10 | | | | | | | | | | |
| | 01 | | | | | | | | | | |
| | 00 | | | | | | | | | | |
| 010 | 11 | | | | | | | | | | |
| | 10 | | | | | | | | | | |
| | 01 | | | | | | | | | | |
| | 00 | | | | | | | | | | |
| 001 | 11 | | | | | | | | | | |
| | 10 | | | | | | | | | | |
| | 01 | | | | | | | | | | |
| | 00 | | | | | | | | | | |
| Message | | 01 | 11 | 11 | 10 | 00 | 10 | 01 | 11 | 10 | 10 |
| PN Sequence | | 001 | | 110 | | 011 | | 001 | | 001 | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 11 | | | | | | | | | | |
| | 10 | | | | | | | | | | |
| | 01 | | | | | | | | | | |
| | 00 | | | | | | | | | | |




Slow FHSS

Frequency Hops for full Symbol duration

| PNS | Freq |
|---|---|
| 000 | $f_{c0}$ |
| 001 | $f_{c1}$ |
| 010 | $f_{c2}$ |
| 011 | $f_{c3}$ |
| 100 | $f_{c4}$ |
| 101 | $f_{c5}$ |
| 110 | $f_{c6}$ |
| 111 | $f_{c7}$ |

| Bits | Freq |
|---|---|
| 00 | $f_1$ |
| 01 | $f_2$ |
| 10 | $f_3$ |
| 11 | $f_4$ |

Slow Frequency Hopping for k=3

# Fast-Frequency Hopping

**EXAMPLE 5**

Figure 9.10*a* illustrates the variation of the transmitted frequency of a fast FH/MFSK signal with time. The signal has the following parameters:

| | |
|---|---|
| Number of bits per MFSK symbol | $K = 2$ |
| Number of MFSK tones | $M = 2^K = 4$ |
| Length of PN segment per hop | $k = 3$ |
| Total number of frequency hops | $2^k = 8$ |

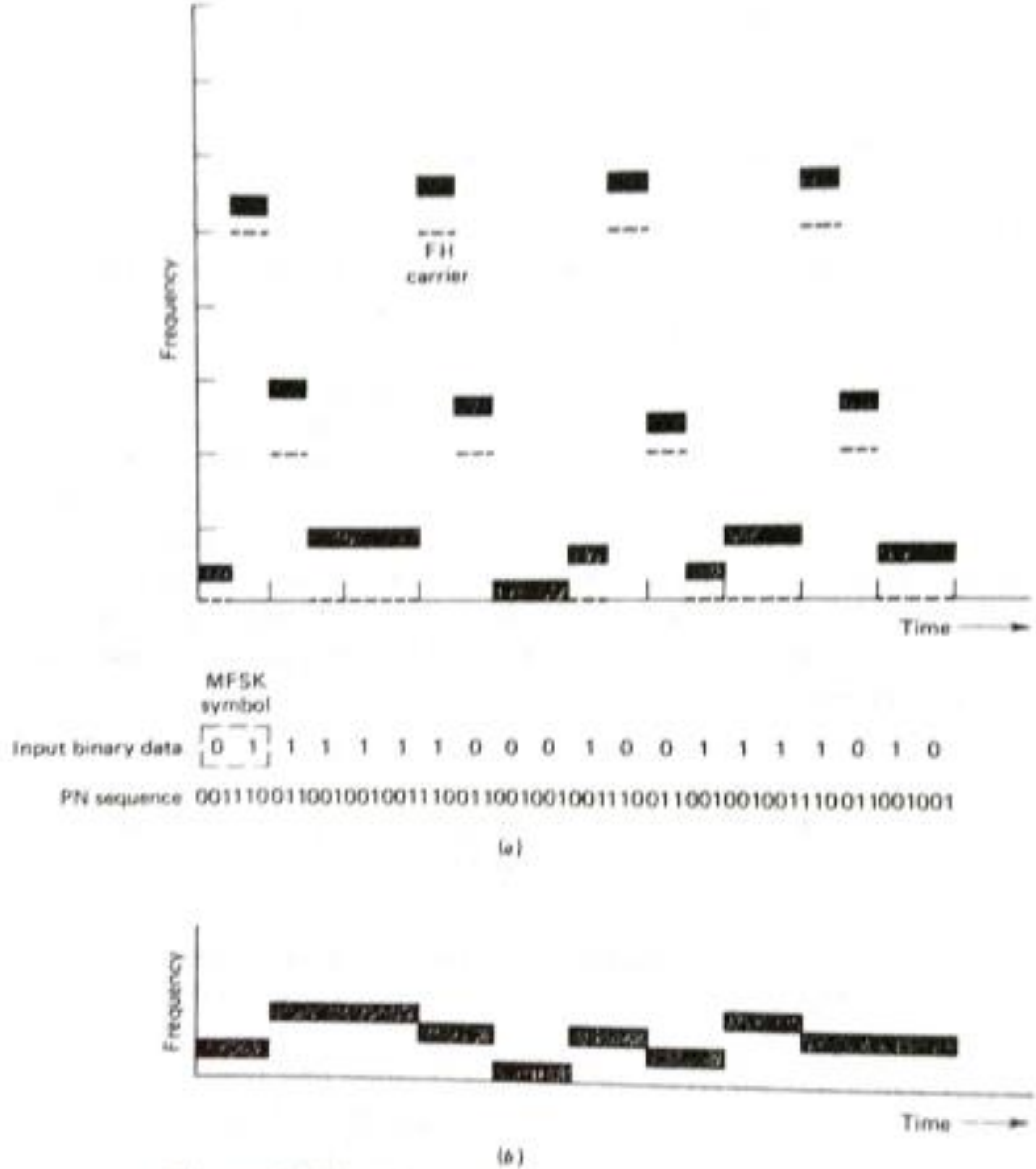
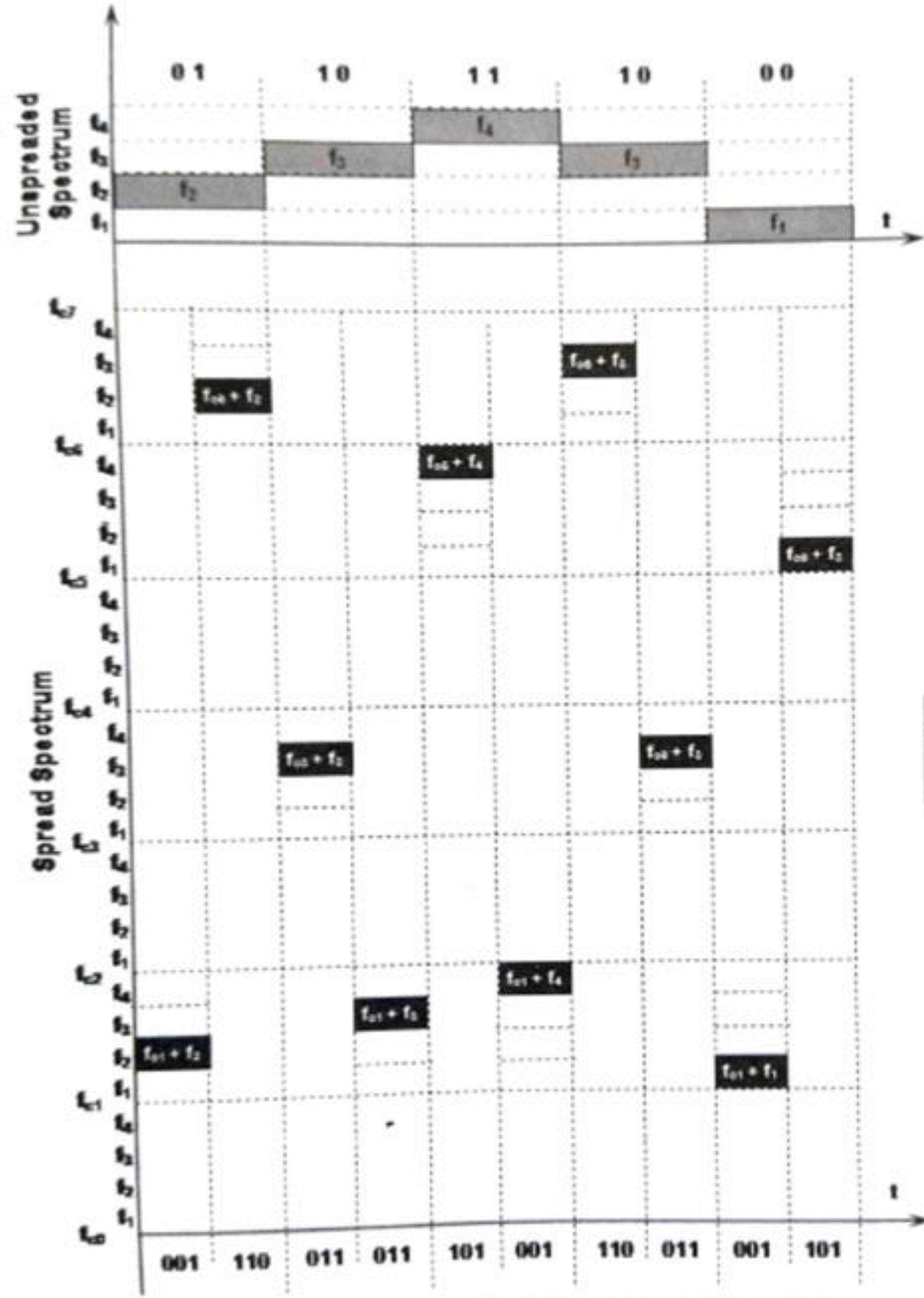



Figure 9.10 Illustrating fast-frequency hopping.




| Bits | Freq |
|---|---|
| 00 | $f_1$ |
| 01 | $f_2$ |
| 10 | $f_3$ |
| 11 | $f_4$ |

| PNS | Freq |
|---|---|
| 000 | $f_{c0}$ |
| 001 | $f_{c1}$ |
| 010 | $f_{c2}$ |
| 011 | $f_{c3}$ |
| 100 | $f_{c4}$ |
| 101 | $f_{c5}$ |
| 110 | $f_{c6}$ |
| 111 | $f_{c7}$ |

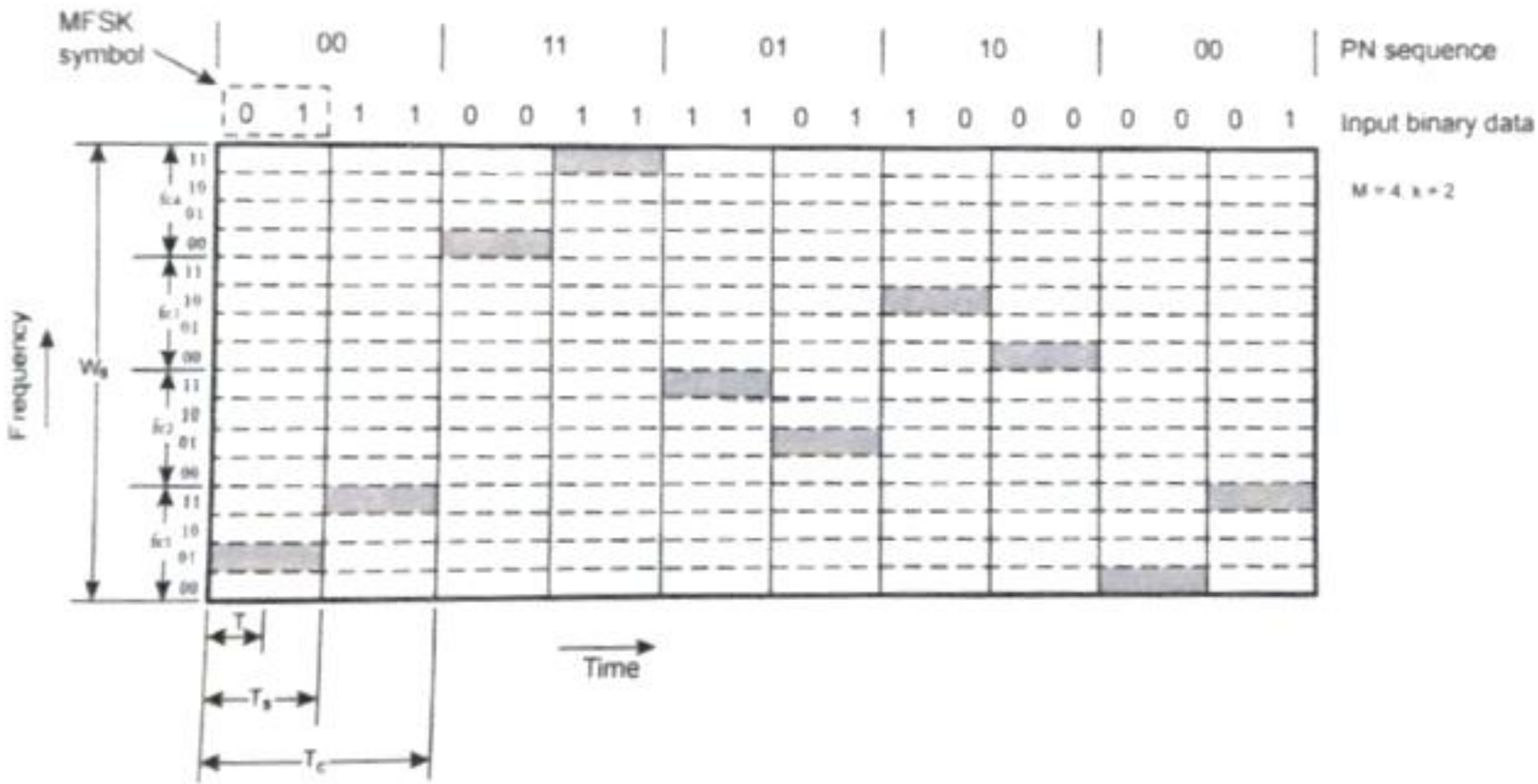
**Fast FHSS**

Frequency Hops for every **half** of symbol duration

Fast Frequency Hopping for k-3

# Example

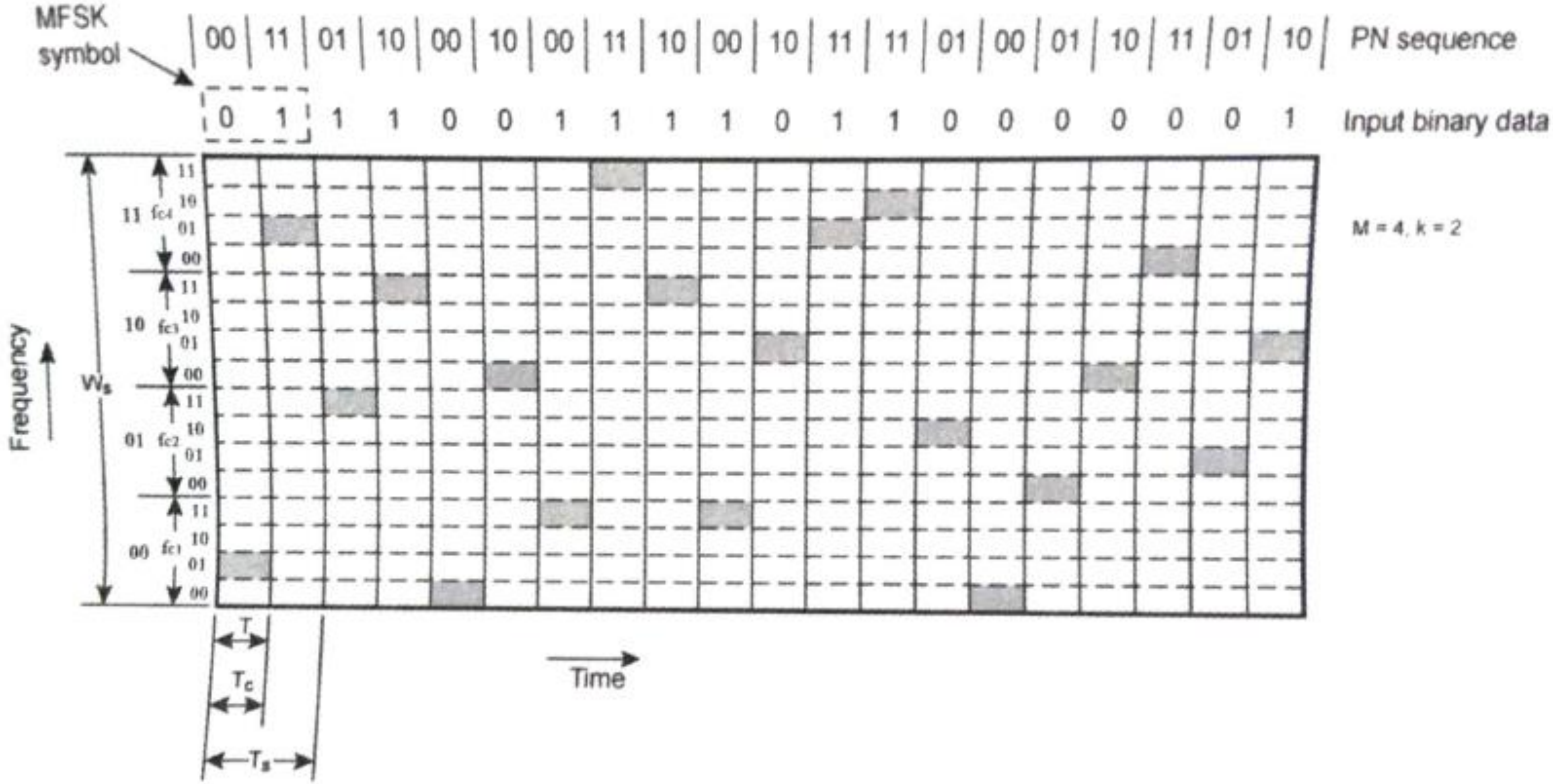
| | |
|---|---|
| Number of bits per MFSK symbol | $K = 2$ |
| Number of MFSK tones | $M = 2^K = 4$ |
| Length of PN segment per hop | $k = 2$ |
| Total number of frequency hops | $2^k = 4$ |




Slow Frequency Hop Spread Spectrum for k=2

# Example

| | |
|---|---|
| Number of bits per MFSK symbol | $K = 2$ |
| Number of MFSK tones | $M = 2^K = 4$ |
| Length of PN segment per hop | $k = 2$ |
| Total number of frequency hops | $2^k = 4$ |




Fast Frequency Hop Spread Spectrum for k=2

**DP9.9.** A PN sequence is generated using 4-stage linear feedback shift register as shown in Figure DP9.9(a), with initial conditon $(C_3C_2C_1C_0) = (1000)$. This sequence is used in a slow FH/MFSK system. The FH/MFSK signal has the following parameters.

Number of bits per MFSK symbol $K = 2$

Number of MFSK tones $M = 2^K = 4$

Length of PN segment per hop $k = 3$

Total number of frequency hops $2^k = 8$

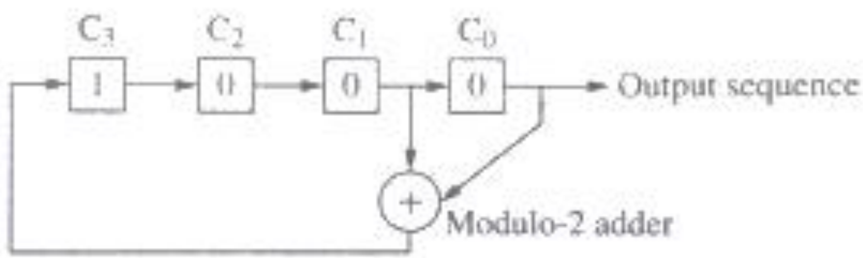



**Figure DP9.9(a)**

Determine the following:

a) Period of the PN sequence.
b) PN sequence for one periodic length.
c) Illustrate the variation of the frequency of FH/MFSK signal for one complete period of the PN sequence. Assume that the carrier hops to a new frequency after transmitting two MFSK symbols or four information bits. Assume binary data sequence to be 10001101000111111001.
d) Sketch the variation of dehopped frequency with time.

**Solution:**

a) The period of the PN sequence is $2^4 - 1 = 15$.

b) For the initial condition shown, the PN sequence is obtained by writing all the successive states of the shift register (SR), for one period. Table DP9.2 gives the successive states, the fed back bit and the output bit.

ii) The PN sequence of one periodic length is 000100110101111. The carrier is hopped to a new frequency after transmitting two MFSK symbols or four information bits. Number of bits per MFSK symbol $K = 2$. There are hence four MFSK frequencies corresponding to dibits 00, 01, 10 and 11. Length of PN segment per hop $k = 3$.

**Table DP9.2**

| States of SR | | | | Fed back bit | Output bit |
|---|---|---|---|---|---|
| $C_3$ | $C_2$ | $C_1$ | $C_0$ | $C_3 = C_1 \oplus C_0$ | $C_0$ |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | Repeats | Repeats |

Hence, there are $2^3 = 8$ hopping frequencies corresponding to each block of 3 PN sequence bits.

Let the hopping carrier frequencies corresponding to each block of 3 bits be selected as shown in Table DP9.3.

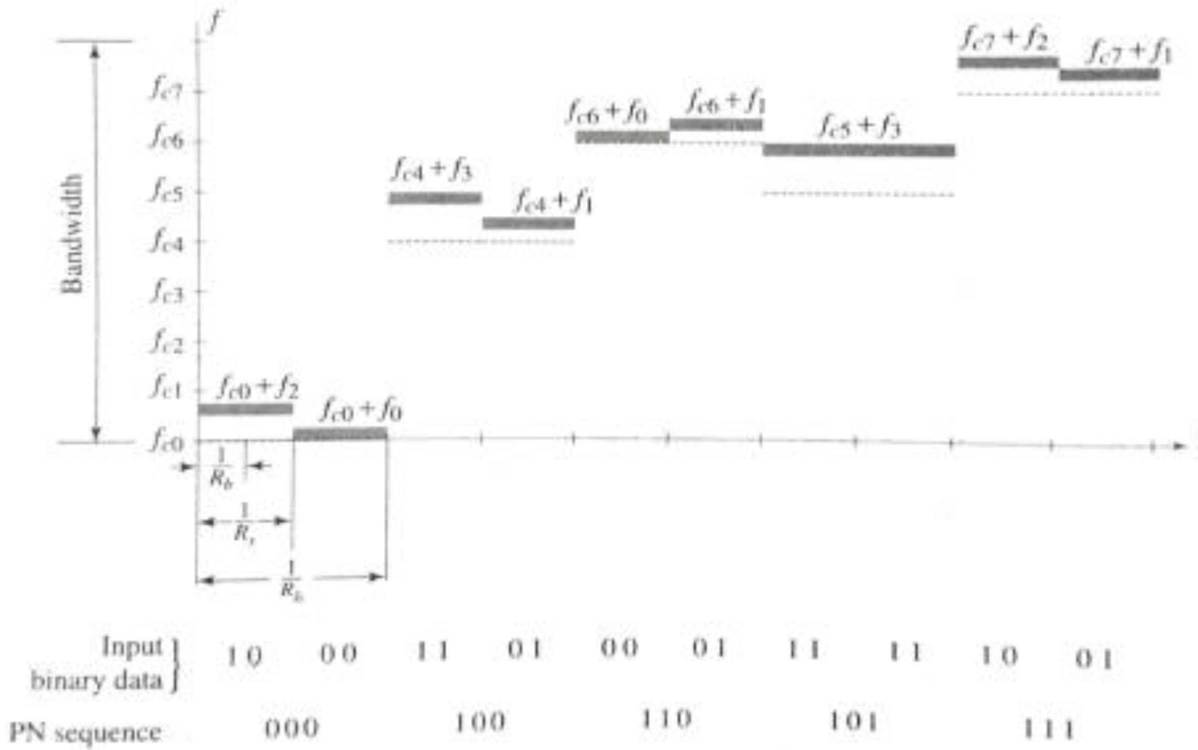
**Table DP9.3**

| PN sequence segment | Hopping carrier frequency in Hz |
|---|---|
| 000 | $f_{c0}$ |
| 001 | $f_{c1}$ |
| 010 | $f_{c2}$ |
| 011 | $f_{c3}$ |
| 100 | $f_{c4}$ |
| 101 | $f_{c5}$ |
| 110 | $f_{c6}$ |
| 111 | $f_{c7}$ |

The 4, MFSK tones be as shown in Table DP9.4.

**Table DP9.4**

| Bits of MFSK symbol | MFSK tone in Hz |
|---|---|
| 00 | $f_0$ |
| 01 | $f_1$ |
| 10 | $f_2$ |
| 11 | $f_3$ |




Transmitted frequencies versus time for the given binary data and PN sequence.

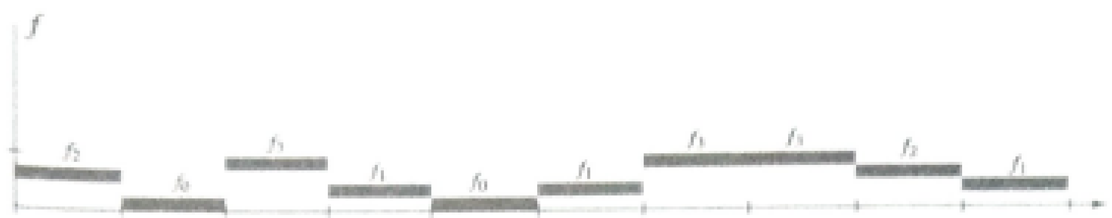



**Figure DP9.9(c)** Dehopped frequencies.

**DP9.10.** In a fast FH/MFSK system, the signal has the following parameters:

Number of bits per MFSK symbol: $K = 2$
Number of MFSK tones: $M = 2^K = 4$
Length of PN segment per hop: $k = 3$
Total number of frequency hops, $2^3 = 8$
Number of hops per MFSK symbol $= 2$
Period of PN sequence: $L = 15$.

i) Determine the relation between bit rate and chip rate.
ii) Sketch the variation of frequency of the transmitted signal with time.
Assume binary data sequence to be 01101100 and one period of PN sequence is 111100010011010.
iii) Sketch the dehopped MFSK signal.

**Solution:**

i) In a fast FH/MFSK, there are multiple hops per MFSK symbol. Hence in a fast FH/MFSK system, each hop is a chip. In this example there are 2 bits/MFSK symbol and 2 hops/MFSK symbol. Hence bit rate $R_b$ = hop rate $R_h$ = chip rate $R_c$.

ii) Let the MFSK tones be denoted by $f_0, f_1, f_2$ and $f_3$ corresponding to MFSK symbols 00, 01, 10, 11, respectively.

Let the hopping carrier frequencies be denoted by: $f_{c0}, f_{c1}, f_{c2}, f_{c3}, f_{c4}, f_{c5}, f_{c6}$ and $f_{c7}$ which correspond to the PN sequence segments 000, 001, 010, 011, 100, 101, 110, and 111, respectively.

During a hopping interval, if carrier frequency is $f_{cj}$ and MFSK tone is $f_i$, then the transmitted frequency is $f_{cj} + f_i$.

The transmitted frequency and dehopped MFSK signal are shown in Figure DP9.10(a) and (b) respectively.

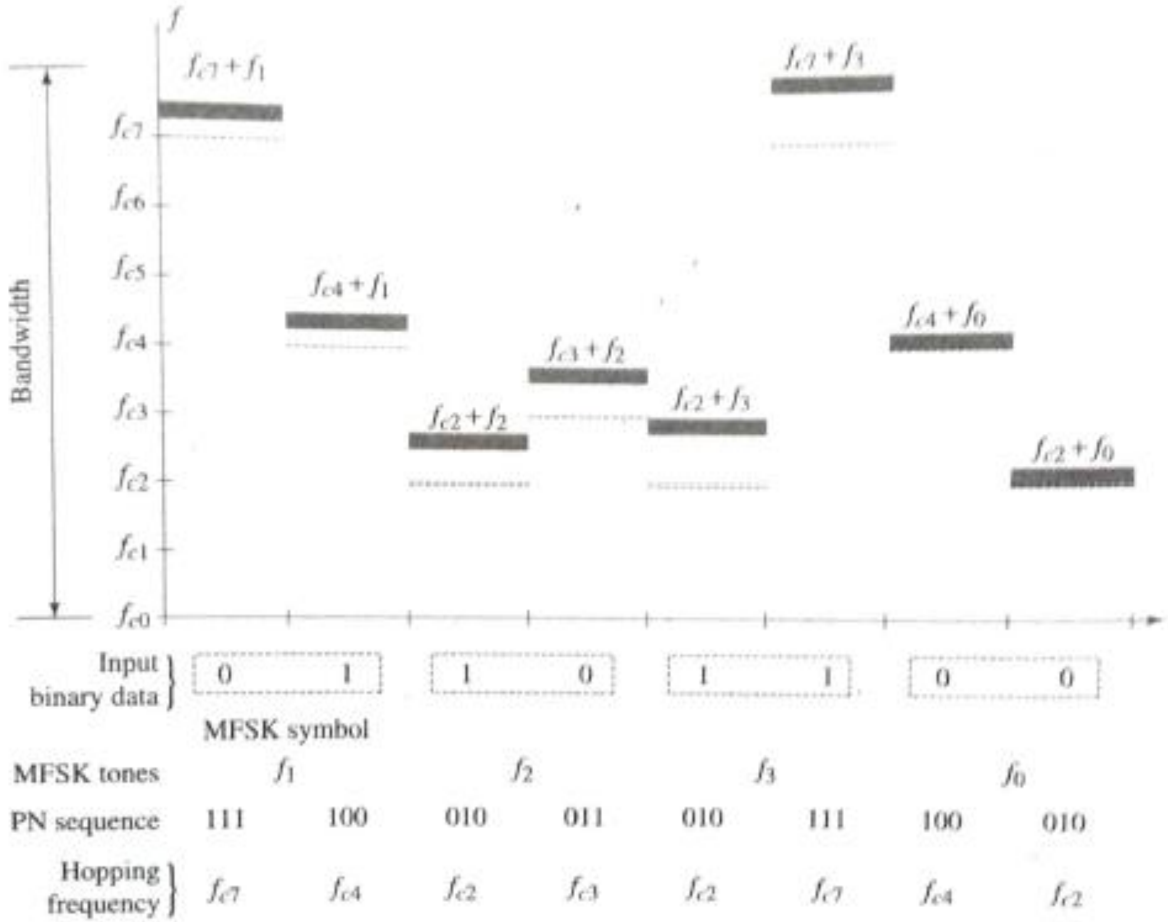



**Figure DP9.10(a)**


**Figure DP9.10(b)**

Comparsion Between Slow Frequency Hopping and Fast Frequency Hopping

| Sl. No | Parameter | Slow frequency Hopping | Fast frequency Hopping |
|---|---|---|---|
| 1 | *Definition* | Multiple Symbols are transmitted in one hop. | Multiple hops are taken to transmit one symbol |
| 2 | *Chip Rate* | Symbol rate = chip rate | Hop rate = Chip Rate |
| 3 | $R_h$ *and* $R_s$ | $R_h < R_s$ | $R_h > R_s$ |
| 4 | *Carrier Frequencies* | One or more symbols are transmitted over same carrier frequency | One symbol is transmitter over multiple carriers in different hops. |
| 5 | *Jammer Interference* | This signal can be detected by jammer if carrier frequency in one hop is known | This signal is difficult to detect since one symbol is transmitted on multiple carrier frequencies. |

| FHSS | DSSS / CDMA |
|---|---|
| Multiple frequencies are used | Single frequency is used |
| Hard to find the user's frequency at any instant of time | User frequency, once allotted is always the same |
| Frequency reuse is allowed | Frequency reuse is not allowed |
| Sender need not wait | Sender has to wait if the spectrum is busy |
| Power strength of the signal is high | Power strength of the signal is low |
| Stronger and penetrates through the obstacles | It is weaker compared to FHSS |
| It is never affected by interference | It can be affected by interference |
| It is cheaper | It is expensive |
| This is the commonly used technique | This technique is not frequently used |

## Comprasion Between Direct Sequence Spread Spectrum and Fast Hopping Spread Spectrum

| Sl. No | Parameter | DS - SS | FH -SS |
|---|---|---|---|
| 1 | *Definition* | PN Sequence of large Bandwidth is multiplied with Narrowband data signal | Data bits are transmitted in different frequency slots which are changed by PN Sequence |
| 2 | *Spectrum of Signal* | Data Sequence is spread over entire B.W of spread spectrum signal | Data Sequence is spread over small frequency slots of spread spectrum signal |
| 3 | *Chip rate* | $R_c = 1/T_c$ | $R_c = \max(R_h, R_s)$ |
| 4 | *modulator* | BPSK | M-ary FSK |
| 5 | *Effect Of Distance* | System is distance relative | Less distance effect |
| 6 | *Acquisition time* | Long | Short |

# Applications

- Multipath Suppression
- Code Division Multiple Access

To accomplish CDMA, spread spectrum is always used.* In particular, each user is assigned a code of its own, which performs the direct-sequence or frequency-hop spread-spectrum modulation. The design of the codes has to cater for two provisions:

1. Each code is approximately *orthogonal* (i.e., has low cross-correlation) with all the other codes.
2. The CDMA system operates *asynchronously*, which means that the transition times of a user's data symbols do not have to coincide with those of the other users.

The second requirement complicates the design of good codes for CDMA.†

The use of CDMA offers three attractive features over TDMA:

1. CDMA does not require an external synchronization network, which is an essential feature of TDMA.
2. CDMA offers a gradual degradation in performance as the number of users is increased. It is therefore relatively easy to add new users to the system.
3. CDMA offers an external interference rejection capability (e.g., multipath rejection or resistance to deliberate jamming).

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