Double error pattern	Syndrome
0 0 1 1 0 0 0 0	1 1 0 0
0 0 1 0 1 0 0 0	1 1 1 1
0 0 1 0 0 1 0 0	0 0 1 1
0 0 1 0 0 0 1 0	0 1 0 1
0 0 1 0 0 0 0 1	0 1 1 0
0 0 0 1 1 0 0 0	0 0 1 1
0 0 0 1 0 1 0 0	1 1 1 1
0 0 0 1 0 0 1 0	1 0 0 1
0 0 0 1 0 0 0 1	1 0 1 0
0 0 0 0 1 1 0 0	1 1 0 0
[redacted] 0 0 0 1 0 1 0	1 0 1 0
0 [redacted] 0 0 1 0 0 1	1 0 0 1
0 0 [redacted] 0 0 1 1 0	0 1 1 0
0 0 0 [redacted] 1 0 1	0 1 0 1
0 0 0 0 0 [redacted]	0 0 1 1

BINARY CYCLIC CODES :-

Binary Cyclic codes form the sub-class of linear block code.

ADVANTAGES :-

- Encoding & Syndrome calculation circuit can be easily implemented by simple shift register & feedback connection & by some basic gates.
- Cyclic codes have a mathematical structure that makes it possible to design the codes with useful error correcting property.

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* ALGEBRAIC STRUCTURE OF CYCLIC CODES :-

"A (n, k) linear block code is said to be a cyclic code if every cyclic shift of the code is also a code vector."

Ex:- If $C_1 = 0111110$

$C_2 = 0011111$

$C_3 = 1001111$

$C_4 = 1100111$

$\vdots \quad \vdots$

If $C_1, C_2, C_3 \ldots$ are also code vectors belonging to the same code, then the code is called CYCLIC CODE.

In general, let the n-bit vector be represented as

$$V = (V_0\ V_1\ V_2 \ldots\ldots V_{n-1})$$

$$V(1) = (V_{n-1}\ V_0\ V_1\ V_2 \ldots\ldots V_{n-2})$$

$$V(2) = (V_{n-2}\ V_{n-1}\ V_0\ V_1 \ldots\ldots V_{n-3})$$

$\vdots \quad \vdots$

$$V(i) = (V_{n-i}\ V_{n-i+1} \ldots\ldots V_0\ V_1\ V_2 \ldots\ldots V_{n-i-1})$$

These equations which are obtained by shifting the 'V' vector cyclically successively are also the code vectors 'C'. This property of cyclic codes also allows to treat the elements of each code vector as a the co-efficients of polynomial of degree ~~(n-1)~~. n

∴ The equation will be

$$V(x) = V_0 + V_1 x + V_2 x^2 + V_3 x^3 + \ldots\ldots + V_{n-1} x^{n-1}$$

$$V'(x) = V_{n-1} + V_0 x + V_1 x^2 + V_2 x^3 + \ldots\ldots + V_{n-2} x^{n-1}$$

$$V^2(x) = V_{n-2} + V_{n-1} x + V_0 x^2 + V_1 x^3 + \ldots\ldots + V_{n-3} x^{n-1}$$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$$V^i(x) = V_{n-i} + V_{n-i+1} x + V_{n-i+2} x^2 + V_{n-i+3} x^3 + \ldots\ldots + V_{n-i-1} x^{n-1}$$

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* MODULO-2 ALGEBRA :-

P1) Find the product of polynomials $f_1(x) = x+1$ & $f_2(x) = x^3+x+1$ using modulo-2 algebra.

$$f_1(x) \cdot f_2(x) = (x+1)(x^3+x+1)$$
$$= x^4 + x^2 + x + x^3 + x + 1$$
$$= x^4 + x^2 + x^3 + x(\cancel{1+1}) + 1$$
$$= \underline{\underline{x^4 + x^3 + x^2 + 1}}$$

P2) Multiply $f_1(x) = 1+x+x^3$ and $f_2(x) = 1+x+x^2+x^4$

$$f_1(x) \cdot f_2(x) = (1+x+x^3)(1+x+x^2+x^4)$$
$$= 1+x+x^2+x^4 + x+x^2+x^3+x^5 + x^3+x^4+x^5+x^7$$
$$= 1 + x(\cancel{1+1}) + x^2(\cancel{1+1}) + x^3(\cancel{1+1}) + x^4(\cancel{1+1}) + x^5(\cancel{1+1}) + x^7$$
$$= \underline{\underline{1+x^7}}$$

P3) Divide $f_2(x) = x^6+x^5+x^2$ by $f_1(x) = x^3+x+1$

$$
\begin{array}{r|l}
 & x^3+x^2+x \longrightarrow \text{Quotient polynomial} \\
\hline
x^3+x+1 & \cancel{x^6}+x^5+x^2 \\
 & \cancel{x^6}+x^4+x^3 \\
\hline
 & \cancel{x^5}+x^4+\cancel{x^3}+\cancel{x^2} \\
 & \cancel{x^5}+\quad \cancel{x^3}+\cancel{x^2} \\
\hline
 & \cancel{x^4} \\
 & \cancel{x^4}+x^2+x \\
\hline
 & x^2+x \longrightarrow \text{Remainder polynomial}
\end{array}
$$

* PROPERTIES OF CYCLIC CODES :-

i) For a (n,k) cyclic code, there exists a generator polynomial of degree $(n-k)$ given by $g(x)$

$$g(x) = g_0 + g_1 x + g_2 x^2 + \cdots + g_{n-k-1}\, x^{n-k-1}$$

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ii) The generator polynomial $g(x)$ of a (n, k) cyclic code is a factor of x^n+1

i.e., $x^n + 1 = g(x)\, h(x)$

where, $h(x)$ is another polynomial of degree 'k' called PARITY-CHECK Polynomial.

iii) If $g(x)$ is a polynomial of degree $(n-k)$ & is a factor of x^n+1, then it generates the (n,k) cyclic code.

iv) The code vector polynomial can be found using

$$V(x) = D(x) \cdot g(x)$$

where $D(x) = d_0 + d_1 x + d_2 x^2 + \ldots\ldots + d_{k-1} x^{k-1}$ is the message vector polynomial of degree 'k'.

This method generates NON-SYSTEMATIC CYCLIC codes.

v) To generate a systematic cyclic code, the remainder polynomial $R(x)$ is obtained from the division of

$$\frac{x^{n-k} D(x)}{g(x)} = R(x)$$

The co-efficients of $R(x)$ are placed in beginning of code vector followed by co-efficients of message polynomial $D(x)$ to get the code vector.

n-bit code vector	
Co-efficients of $R(x)$	Co-efficients of $D(x)$

P) For (7,4) single error correcting cyclic code, $D(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3$ and $x^n + 1 = x^7 + 1 = (1+x+x^3)(1+x+x^2+x^4)$. Using the generator polynomial $g(x) = (1+x+x^3)$, find all 16 code vectors of cyclic code both in NON-SYSTEMATIC & SYSTEMATIC form.

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Non-systematic form:

$$V(x) = D(x)g(x)$$

Consider the message vector $D = [1011]$

The message vector polynomial $D(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3$

$$= 1 + 0.x + 1.x^2 + 1.x^3$$

$$= 1 + x^2 + x^3$$

$$V(x) = D(x).\ g(x)$$

$$V(x) = (1+x^2+x^3)(1+x+x^3)$$

$$= 1 + x + x^3 + x^2 + x^3 + x^5 + x^3 + x^4 + x^6$$

$$= 1 + x + x^2 + x^3(1+1+1) + x^4 + x^5 + x^6$$

$$= 1 + x + x^2 + x^3 + x^4 + x^5 + x^6$$

$\therefore V = [1111111]$ is the code vector

Let $D = 1001$

$$\therefore D(x) = 1 + 0.x + 0.x^2 + 1.x^3$$

$$= 1 + x^3$$

$$V(x) = (1+x^3)(1+x+x^3)$$

$$= 1 + x + x^3 + x^3 + x^4 + x^6$$

$$= 1 + x + x^4 + x^6$$

$\therefore V = [1100101]$

Systematic form:

$$\frac{x^{n-k} d(x)}{g(x)} = R(x)$$

Let $D = 1011 \Rightarrow D(x) = 1 + x^2 + x^3$

$$\therefore \frac{x^3(1+x^2+x^3)}{1+x+x^3} = \frac{x^3+x^5+x^6}{1+x+x^3}$$

$$x^3 + x^2 + x + 1$$

$$x^3+x+1 \,\big|\, \cancel{x^6} + x^5 + \cancel{x^3}$$

$$\cancel{x^6} + x^4 + \cancel{x^3}$$

$$\cancel{x^5} + x^4$$

$$\cancel{x^5} + x^3 + x^2$$

$$\cancel{x^4} + x^3 + \cancel{x^2}$$

$$\cancel{x^4} + \cancel{x^2} + x$$

$$\cancel{x^3} + \cancel{x}$$

$$\cancel{x^3} + \cancel{x} + 1$$

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$$\therefore \quad R(x) = 1 = R_0 + R_1 x^1 + R_2 x^2$$

$$\therefore \quad R = [1\,0\,0]$$

$$D = [1\,0\,1\,1]$$

$$C = [R \,\vdots\, D] = \underline{[1\,0\,0\,1\,0\,1\,1]}$$

Let $D = 1001$

$$D(x) = 1 + x^3$$

$$\therefore \quad \frac{x^3(1+x^3)}{1+x+x^3} = \frac{x^3+x^6}{1+x+x^3}$$

Quotient: $x^3 + x$

Divisor: x^3+x+1

$$\cancel{x^6} + \cancel{x^3}$$

$$\cancel{x^6} + x^4 + \cancel{x^3}$$

$$\cancel{x^4}$$

$$\cancel{x^4} + x^2 + x$$

$$x^2 + x$$

$$\therefore \quad R(x) = x^2 + x = R_0 + R_1 x^1 + R_2 x^2$$

$$R = [0\,1\,1]$$

$$\therefore \quad C = \underline{[0\,1\,1\,1\,0\,0\,1]}$$

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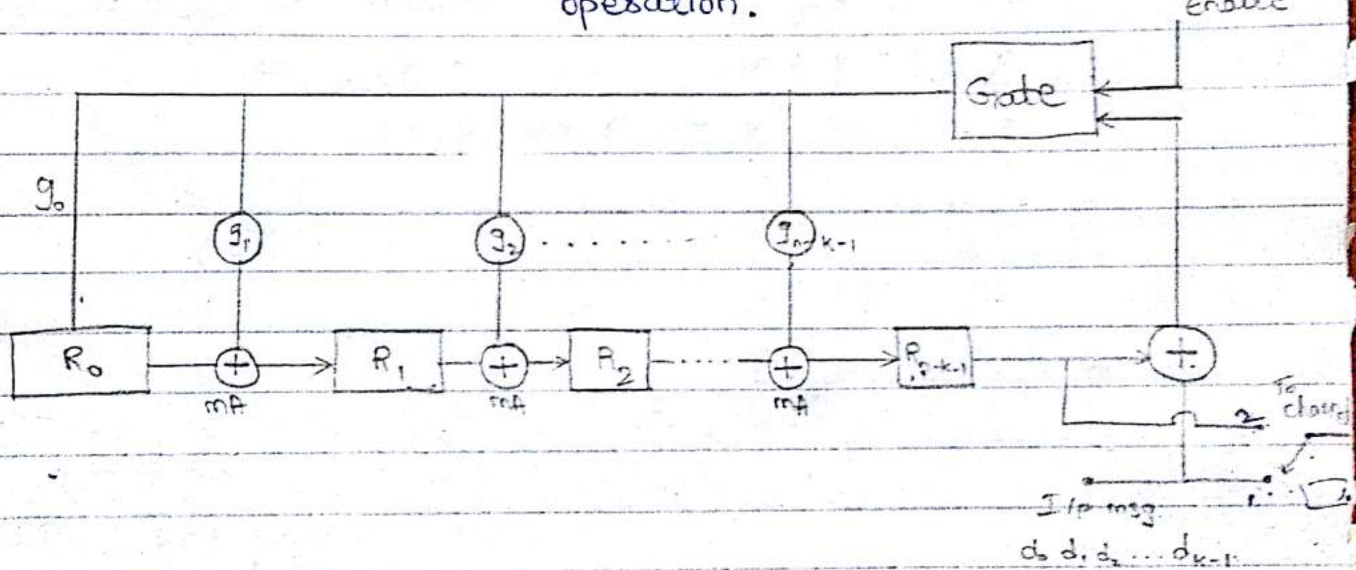
* ENCODING USING (n-k) BIT SHIFT REGISTER :-

In order to obtain the remainder polynomial $R(x)$, the division of $x^{n-k} D(x)$ by generator polynomial $g(x)$ is done to calculate the parity check polynomial $V(x)$. The hardware required to implement the encoding system consists of
i) $(n-k)$ bit shift register
ii) $(n-k)$ modulo-2 adder
iii) AND gate
iv) Counter to keep track of shifting operation.



It is assumed that at the occurence of the clock pulse, the inputs are shifted into the register and appear at the output at the end of clock pulse.

Step 1: With the gate turned on with the switch in position 1, with the information bits or digits $d_0 d_1 d_2 \ldots d_{k-1}$ are shifted into the register (with d_{k-1} first) & simultaneously into the communication channel. As soon as the k information digits have been shifted into the register, the register contains parity check bits $r_0, r_1, r_2 \ldots r_{n-k-1}$.

Step 2: With gate turned off & swith in position 2,

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contents of shift register are shifted into the channel.

Thus, the code vector $R_0 R_1 \ldots R_{n-k-1} D_0 D_1 D_2 \ldots D_{K-1}$ is generated & sent over the channel.

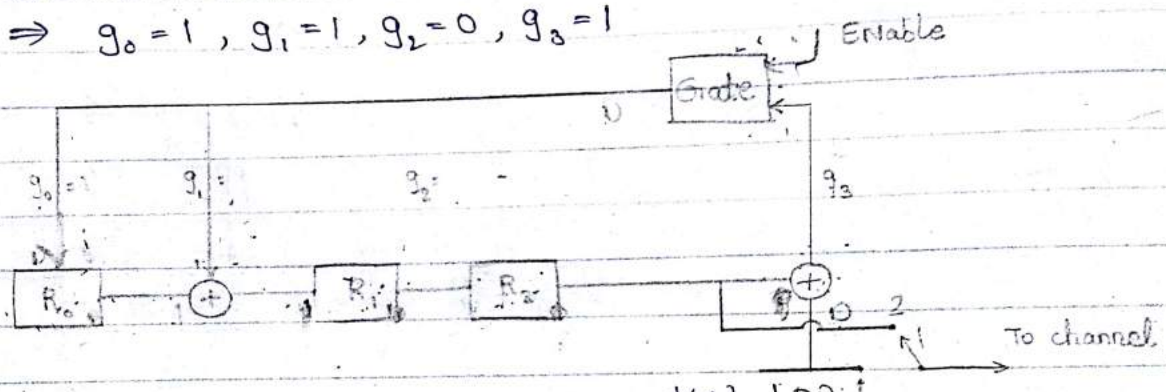
P) Design an encoder for (7,4) binary cyclic code generated by $g(x) = 1 + x + x^3$ and verify its operation using message vector (1001) and (1011)

$n - K = 7 - 4 = 3$ bit shift register

In general, the generator polynomial is given by

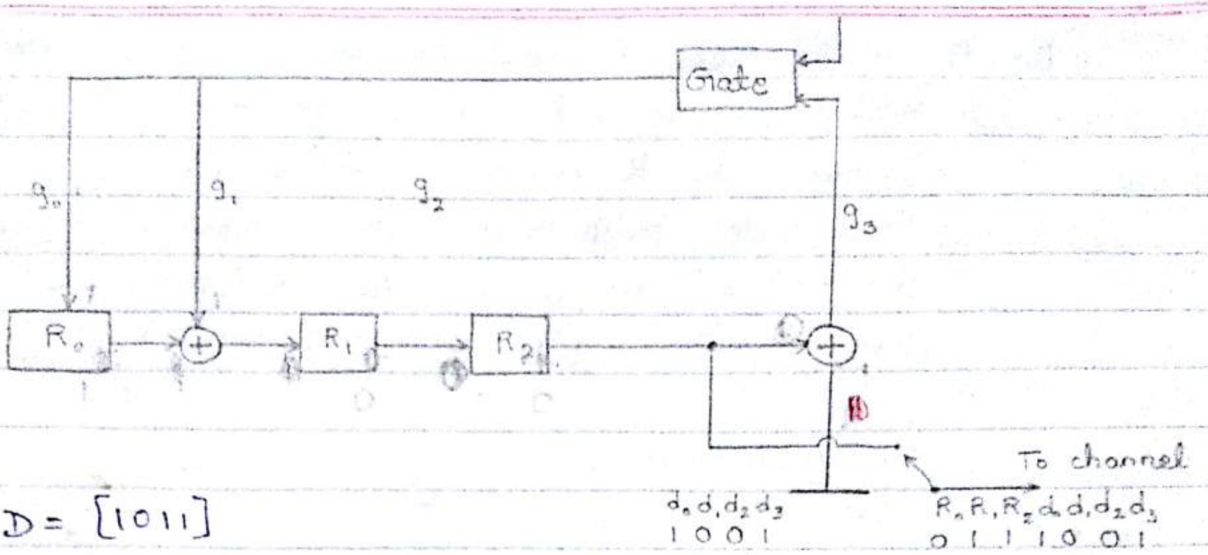
$g_0 + g_1 x + g_2 x^2 + \ldots + g_{n-K-1} x^{n-K-1}$

$\Rightarrow g_0 = 1, g_1 = 1, g_2 = 0, g_3 = 1$



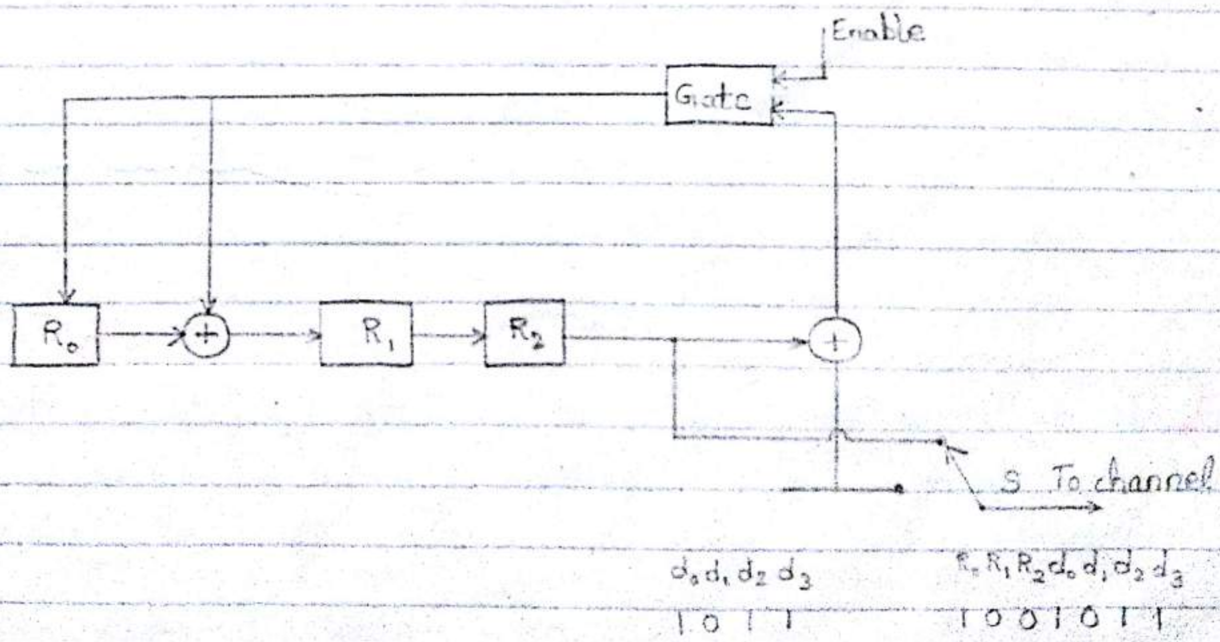
i) For the message vector (1001) the shift register contents are

	No of Shifts	I/p	Shift register contents R_0	R_1	R_2	Remainder
Switch S in position 1 & gate ON	1	1	0	0	0	—
	2	0	1	1	0	—
	3	0	0	1	1	—
	4	1	1	1	1	—
Switch S in position 2 & gate OFF	5	x	0	1	1	1
	6	x	0	0	1	1
	7	x	0	0	0	0
		x	0	0	0	



ii) $D = [1011]$

	No of Shifts	I/P	SR contents R_0	R_1	R_2	Remainder
Switch S in position 1 & gate ON	1	1	0	0	0	—
	2	1	1	1	0	—
	3	0	1	0	0	—
	4	1	1	0	0	—
Switch S in position 2 & gate OFF	5	X	0	1	0	0
	6	X	0	0	1	0
	7	X	0	0	0	1



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A (15, 5) Algebraic code (cyclic) is generated using generator polynomial $g(x) = 1+x+x^2+x^4+x^5+x^8+x^{10}$

i) Draw block diagram of encoder

ii) Find code polynomial for message polynomial $d = 1+x^2+x^4$ using encoder diagram

$d = (10101)$ $g(x) = (11101100101)$

Switch : position 1

Gate : Turned on

No of Shifts	I/p	Shift register contents
		1 1 1 0 1 1 0 0 1 0
1	1	
2	0	
3	1	
4	0	
5	1	

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* SYNDROME CALCULATION CIRCUIT :-

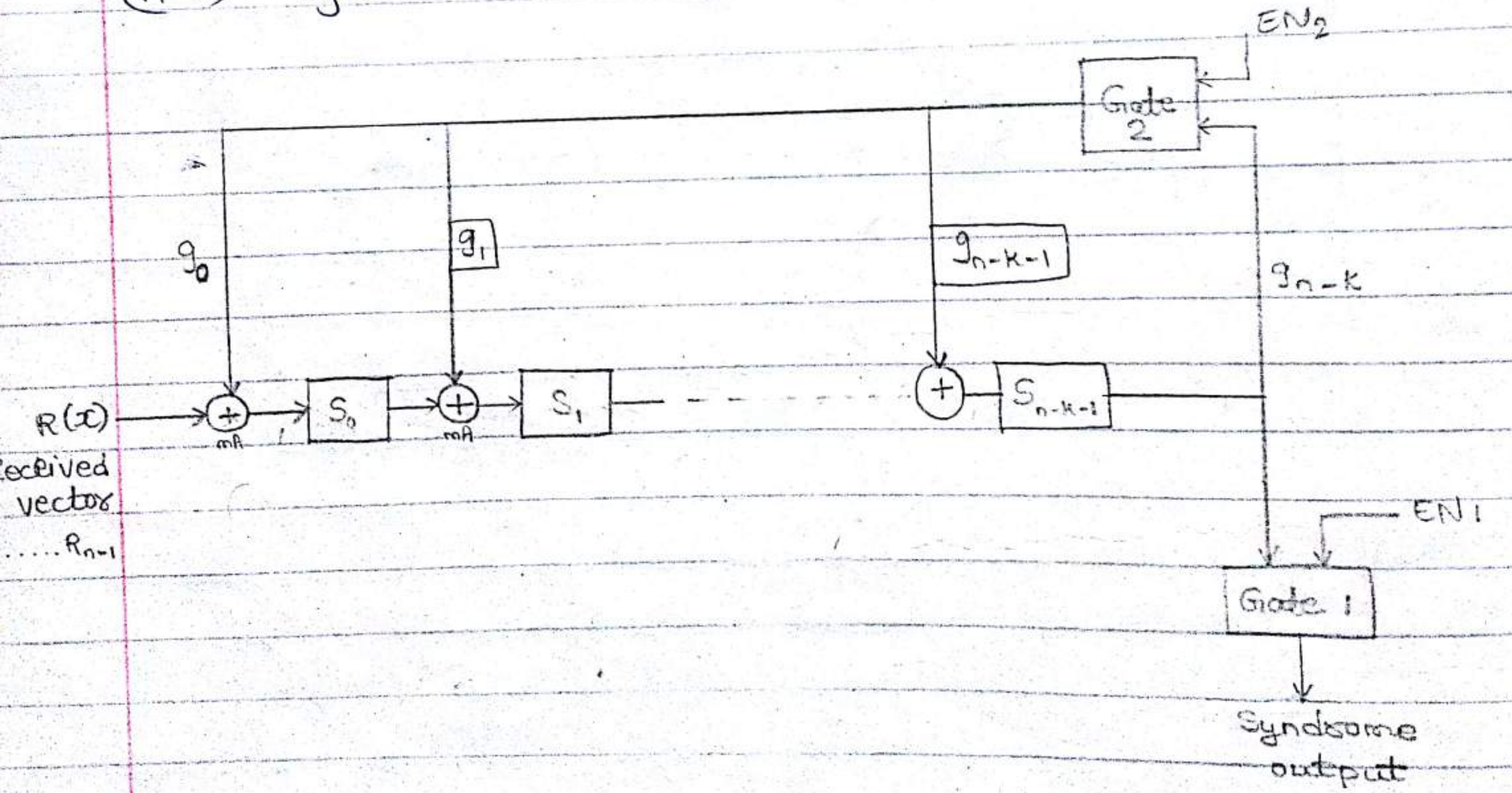
If $V(x)$ is the transmitted code vector & $R(x)$ is received code vector & if $V(x) = R(x)$, then the Syndrome polynomial $S(x) = 0$

If $V(x) \neq R(x)$, then $S(x) \neq 0$

To calculate syndrome polynomial, the received code vector is divided by generator polynomial. If remainder of division is '0', then there is no error in received code vector. The remainder of division gives the error syndrome.

$\frac{R(x)}{g(x)}$

The error polynomial depends upon syndrome polynomial. To Determine the co-efficients of syndrome polynomial, the dividing circuit for a (n-k) cyclic code is shown below.



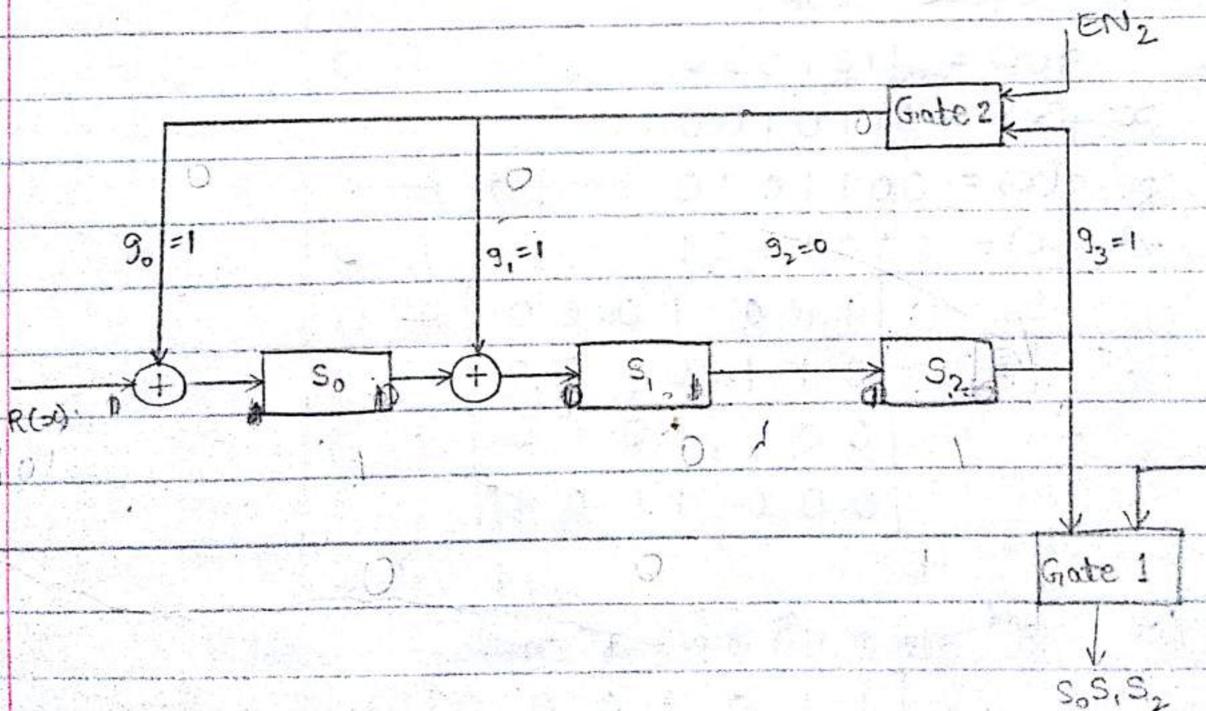
with Gate 1 turned OFF & Gate 2 turned ON, the received code vector is loaded into the shift register with (R_{n-1}) as first digit. At the end of 'n' clock pulses, the flip-flops will have the co-efficients of syndrome polynomial. After the message is loaded into the shift register, gate 2 is turned OFF & gate 1 is turned ON and the information present in syndrome calculating circuit is shifted to an error detection & correction circuit.

P) For a (7,4) cyclic code, the received vector is 1110101 and the generator polynomial $g(x) = 1 + x + x^3$. Draw the syndrome calculation circuit & correct the single error in the received vector.

$n-k = 7-4 = 3$ bit shift register

$$g(x) = g_0 + g_1 x + g_2 x^2 + g_3 x^3$$

$$g_0 = 1 \; ; \; g_1 = 1 \; ; \; g_2 = 0 \; ; \; g_3 = 1$$



Initially
Gate-1 is OFF
Gate-2 is ON

No of shifts	I/p	Shift registers S_0	S_1	S_2	Remainder
	LSB	0	0	0	
1	1	1	0	0	
2	0	0	1	0	
3	1	1	0	1	
4	0	1	0	0	
5	1	1	1	0	
6	1	1	1	1	
7	1	0	0	1	

Indicates error

Consider $g(x) = 1 + x + x^3$

It is known that $g(x)$, $x\,g(x)$, $x^2 g(x)$ & $x^3 g(x)$ also represent the code vector polynomial of the same cyclic code.

$g(x) = 1101000$

$x.g(x) = 0110100$

$x^2.g(x) = 0011010$

$x^3.g(x) = 0001101$

$$\therefore [G] = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

Let 3^{rd} row $= 1^{st}$ row $+ 3^{rd}$ row

$$[G] = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

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Let 4^{th} row $= 1^{st}$ row $+ 2^{nd}$ row $+ 4^{th}$ row

$$\therefore [G] = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ \bullet & \bullet & 1 & 0 & 0 & 1 & 0 \\ \bullet 1 & 0 & 1\bullet & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\therefore [G] = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Generally
$G = [I_K : P]$
$H = [P^T : I_{n-k}]$

Now
$G = [P_{n-k} : I_K]$
$H = [I_{n-k} : P^T]$

$$H^T = \left[\begin{array}{c} I_{n-k} \\ \hline P \end{array}\right]$$

$[H] = [I_{n-k} : P^T]$

$$= \left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

$$[H^T] = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{array}\right] \leftarrow \text{Syndrome present in } 3^{rd} \text{ row}$$

$\therefore$ Error pattern $= [0010000]$ $\qquad R + E = C$

$\therefore$ Corrected vector $= [1100101]$

$||^{ly}$ do for $R(x) = \underline{0111011}$

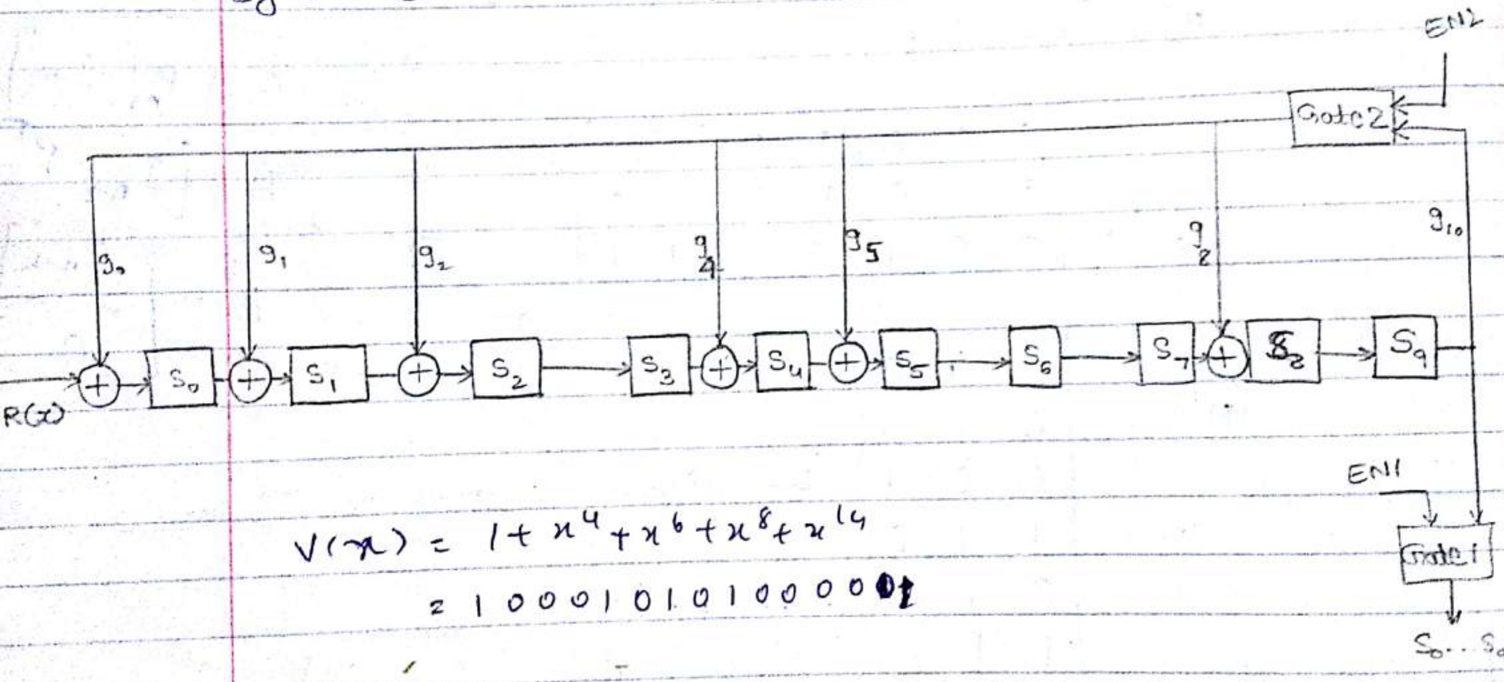
Syndrome $= 111$

P) Consider a (15,11) cyclic code generated by $g(x) = 1+x+x^4$.

a) Device a feedback register encoder for this code

b) Illustrate the encoding procedure with the message vector 11001101011 by listing the states of the registers.

P) A (15,5) linear cyclic code has a generator polynomial $g(x) = 1+x+x^2+x^4+x^5+x^8+x^{10}$. Draw the syndrome calculation circuit. Is $v(x) = 1+x^4+x^6+x^8+x^{14}$ a code polynomial? If not, find the syndrome for $v(x)$.



$V(x) = 1+x^4+x^6+x^8+x^{14}$

$= 100010101000001$

If $V(x)$ is a code polynomial it should be perfectly divisible by $g(x)$ with remainder zero.