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Roadmap of DSP | VLSI :-

Science
↓
Physics
↓
E&C    E&E    CS    — — — — —

(engg.)

E&C
→ commu'n
→ electronics
} → DSP

1) science : All Innovations

physics : Branch of science
deal with study of current through copper wire
(or)
study of flow of current through 220 V device

electronics :- Branch of physics, sub-branch of science
Deals with flow of current through semiconductor device only
Ge | si (5V)

Communication :- Branch of physics
Sub-branch of science
deals with exchange of Information from

study of flow of current through 5V device.

Advantage of E&C :-

current application reduces usage of power from 220 V to 5V only.

→ Tdy's world almost for all applications usage is dependent on electronic (devices).

→ This Branch is till yesterday, today and next generations also giving more usage

DSP :-

Today's world is digital almost for all applications regarding communication Digital signal processing has been used

DSP / VLSI processor works with in 5V.

Fourier Transform and DFT :-

Many applications demand signal processing in frequency domain

for example,

frequency content, periodicity, energy, Power spectrum can be better analised in freq. domain than in time domain

Hence signals of are transformed from time domain to frequency domain such transformations can be done from tools like discrete fourier transform, FT.

Once the required analysis and processing is done in freq. domain the signals are then transformed back to time domain by using IDFT.

$$\begin{tikzcd} x(n) \arrow[r, "DFT"] & X(K) \arrow[l, bend left, "IDFT"] \end{tikzcd}$$

Time      freq

SSP      DSP

Analog signals like speech, music etc are converted to digital form using by ADC

$$x(t) \xrightarrow{ADC} x(n) \xrightarrow{DFT} \boxed{\begin{matrix} X(K) \\ DSP \end{matrix}} \xrightarrow{IDFT} x(n) \xrightarrow{DAC} x(t)$$

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where $x(t)$ is original signal that exists in real world i.e speech, music

$x(n) \rightarrow$ descrete signals in time domain

throughout S & S we study & analyse signals in time domain only

$X(K) \rightarrow$ signals in frequency domain

$x(t) \rightarrow$ continuous signals in time

throughout DSP, we study & analyse signals in frequency domain

☆ DFT :-

It is a tool used to convert signals from

frequency domain

## IDFT :-

It is a tool used to convert frequency domain signals to time signals

(or)

to get back the original signals from freq. domain signals

$$x(n) \underset{IDFT}{\overset{DFT}{\rightleftarrows}} X(K)$$

using expressions :-

### DFT :-

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\left(\frac{2\pi}{N}\cdot K\right)n}$$

$K = 0, 1, 2 \ldots N-1$

### IDFT :-

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j\left(\frac{2\pi}{N}\cdot K\right)\cdot n}$$

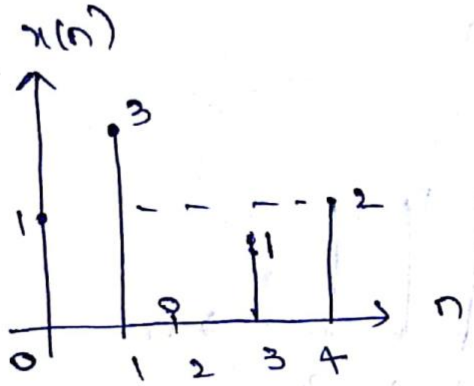
where $n = 0, 1, 2 \ldots N-1$

## Discrete time signals & analog signals :-

Analog signals are continuous signals exist at all points

Descrete signals are signals where values exists at different intervals of time and it is denoted by

eg: $x(n) = \{1, 3, 0, 1, 2\}$
$\uparrow$



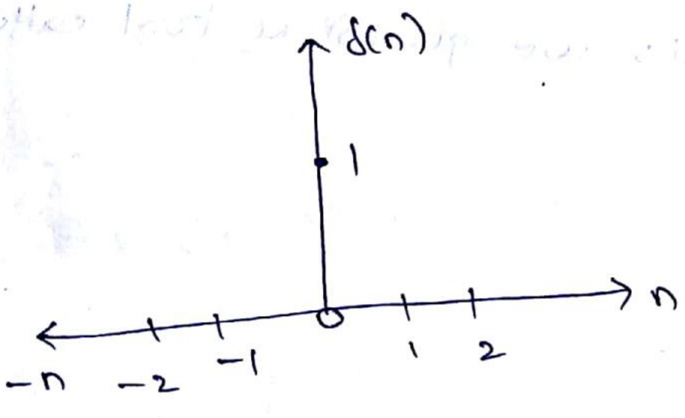

Most of the communication we are doing it now is descrete only

Some of the common descrete type signals are

i, Impulse functions
ii, step functions
iii, ramp functions

i, **Impulse function** :- It is one where output exist at one point i.e @ n=0 at all other points its value is zero

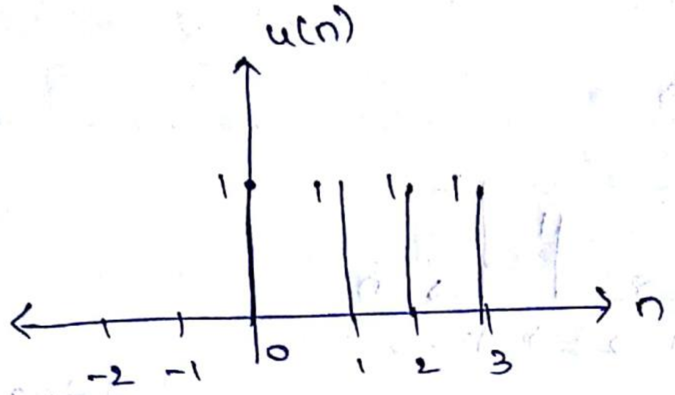
$$\delta(n) = \begin{cases} 1 & ; n=0 \\ 0 & \text{otherwise} \end{cases}$$




eg:- $\delta(n) = \{ \dots 0, 0, 1, 0, 0 \dots \}$

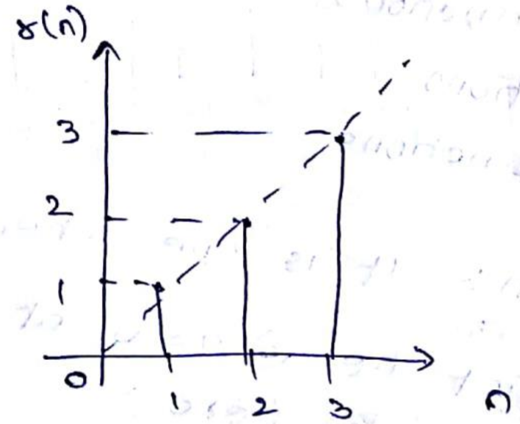
ii, **unit step function** :-

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$




iii, <u>Ramp function</u> :-

$$r(n) = \begin{cases} n & \text{for } n > 0 \\ 0 & \text{otherwise} \end{cases}$$




Descrete time signals in time domain gives less Information to analyse signals

To do better analysis we go for a tool called FT & DFT.

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# Module : 1

## DFT

### Fourier Transform :-

1) what is the main disadvantage of FT of a signal?

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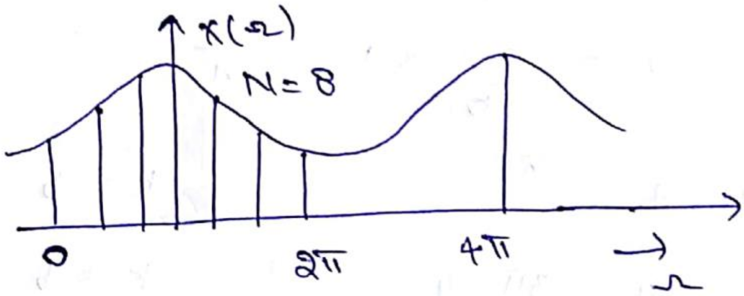
FT is a tool using this we can convert descrete signals in time domain to frequency domain and it is given by

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\Omega n} \rightarrow (1)$$




where $x(n) \rightarrow$ descrete time sequence

$\Omega \rightarrow$ frequency varies from $0-2\pi$

$X(\Omega) \rightarrow$ Its spectrum is continuous

The disadvantage here is eventhough the $x(n)$ is descrete in nature, it spectrum is continuous . such continuous spectrum cannot

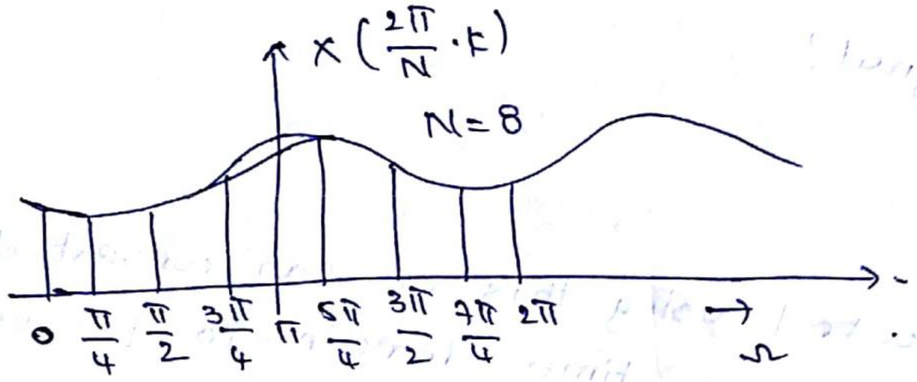
So to avoid this problem, we go for sampling

substitute $\Omega = \left(\frac{2\pi}{N} \cdot k\right)$

where $k \rightarrow$ Index for samples

$N \rightarrow$ no. of sample b/w 0 to $2\pi$

then we get sampled function




$k = 0, 1, 2, 3 \ldots k = 7$

where,

$k = 0 \quad \Omega = \left(\frac{2\pi}{8} \times 0\right) = 0$

$k = 1 \quad \Omega = \frac{2\pi}{8} \times 1 = \frac{\pi}{4}$

$k = 2, \quad \Omega = \frac{\pi}{2}$ $\qquad k = 5, \quad \Omega = \frac{5\pi}{4}$

$k = 3, \quad \Omega = 3\pi/4$ $\qquad k = 6, \quad \Omega = \frac{3\pi}{2}$

$k = 4, \quad \Omega = \pi$ $\qquad k = 7, \quad \Omega = \frac{7\pi}{4}$

for $N = 8$, we get 8 descrete samples at

$k = 0, 1 \ldots 7$

These samples can be processed easily using digital processor which is actually DFT

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10M

# ★ Frequency domain sampling and reconstructions of ~~peripheral~~ Original signals $x(n)$ :

impo

WKT,

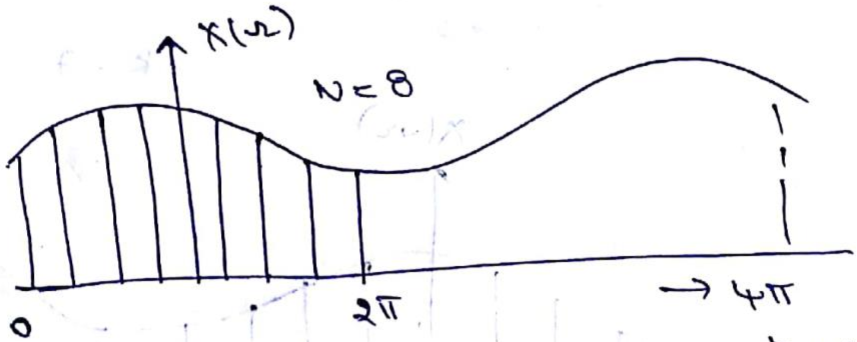
fourier transform of discrete time signals $x(n)$ is given by

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n} \rightarrow (1)$$

$x(n) \rightarrow$ discrete time signals

$\Omega \rightarrow$ frequency ,

It is continuous fun'n of 0 to $2\pi$




Even though $x(n)$ is descrete its spectrum $X(\Omega)$ is continuous , such continuous fun'ns can not be vaild by digital processors. To avoid this ~~sp~~ sampling problem we go for freq. domain

Part 2 :-

## Const'n ~~of~~ freq. domain samples

* Sample the spectrum uniformly by substitutin

$\Omega = \frac{2\pi}{N}.k$ where , $N = 8$

$k \rightarrow$ Index for samples

$$X\left(\frac{2\pi}{N}\cdot k\right) = \sum_{n=-\infty}^{\infty} x(n)\cdot e^{-j\left(\frac{2\pi}{N}k\right)n} \rightarrow (2)$$

where $k = 0,1,2 \ldots N$.

when

$k=0,\ N=8 \quad \Omega = \frac{2\pi}{8}(0) = 0$

$k=1, \quad \Omega = \frac{2\pi}{8} = \frac{\pi}{4}$

$k=2, \quad \Omega = \frac{2\pi}{8}(2) = \frac{\pi}{2}$

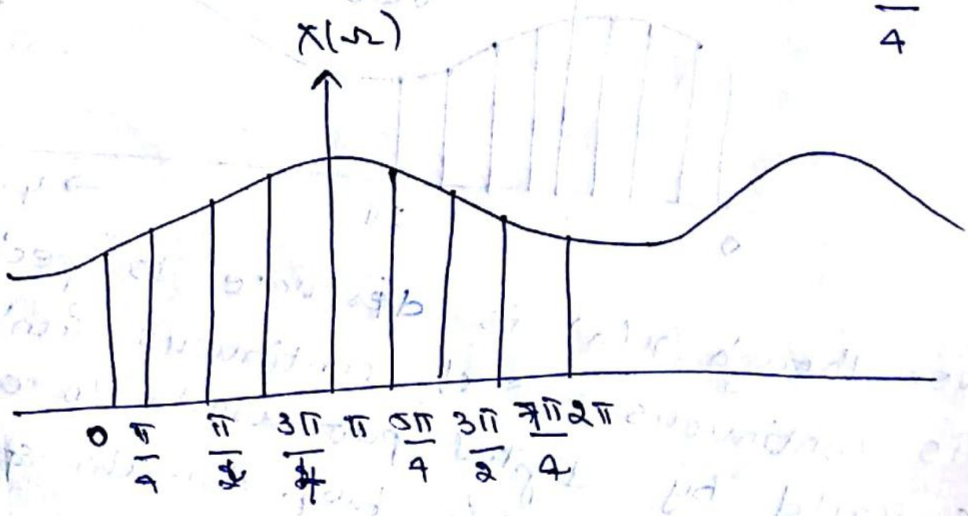
$k=3, \quad \Omega = \frac{3\pi}{4}$

$k=4 = \pi$

$k=5 \quad \Omega = \frac{5\pi}{4}$

$k=6 \quad \Omega = \frac{3\pi}{2}$

$k=7 \quad \Omega = \frac{7\pi}{4}$




Part 3:-

Determination of minimum value of 'N'

* Divide the summations of equ (2) into individual summation containing only N samples of $x(n)$

$$X\left(\frac{2\pi}{N}k\right) = \ldots\ldots + \sum_{n=-N}^{-1} x(n)\, e^{-j\left(\frac{2\pi}{N}k\right)n} + \sum_{n=0}^{N-1} \ldots e^{-j\left(\frac{2\pi k}{N}\right)n} \ldots \sum^{2N-1} x(n)\, e^{-j\left(\frac{2\pi}{N}k\right)\ldots}$$

The above individual summation can be represented as

$$X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n)\, e^{-j\left(\frac{2\pi k}{N}\right)n}$$

change inner index $n \to n+lN$

$n = 0$ to $N-1$

$$X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n+lN)\, e^{-j\left(\frac{2\pi k}{N}\right)(n+lN)}$$

| | |
|---|---|
| when $n = lN$ | $l = \infty$ |
| $n + lN = lN$ | $n = lN + ln - 1$ |
| $n = 0$ | $n + l\cancel{N} = l\cancel{N} + n - 1$ |
| | $n = N-1$ |

$$X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n+lN)\, e^{-j\frac{2\pi k}{N}n}.$$

$$\rightarrow e^{-j\left(2\pi k/\cancel{N}\right)l\cancel{N}} = 1$$

$$\rightarrow = \left[ e^{-j2\pi(kl)} = 1 \right]\left[\begin{matrix}0 \\ 1\end{matrix}\right]$$

$$\cos 2\pi(kl) - j\sin 2\pi(kl)$$

$$\boxed{X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n+lN)\, e^{-j\left(\frac{2\pi}{N}\right)kn}}$$

change of order of summation

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} \sum_{l=-\infty}^{\infty} x(n+lN)\, e^{-j\left(\frac{2\pi}{N}k\right)n}$$

$$\boxed{X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\left(\frac{2\pi k}{N}\right)n} \quad k=0,1,2\ldots}$$

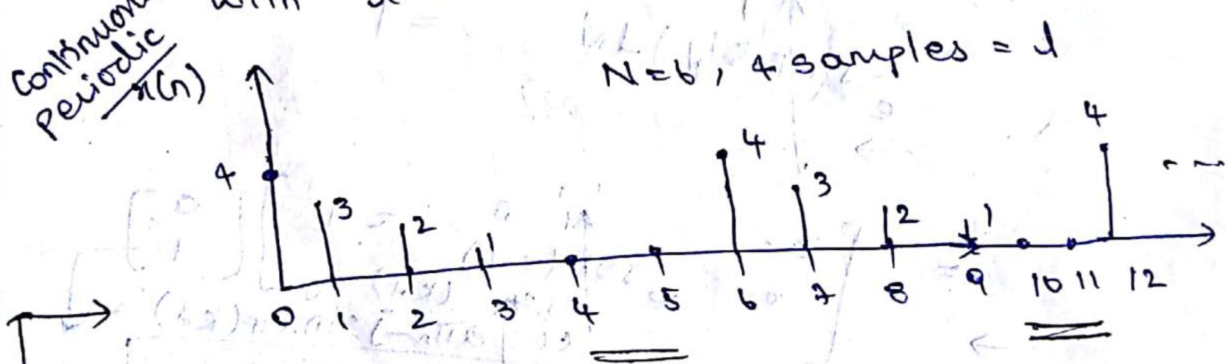
where $x_p(n) = \sum_{l=-\infty}^{\infty} x(n+lN)$

$$x_p(n) = \cdots + x(n-2N) + x(n-N) + x(n) + x(n+N) + x(n+2N) + \cdots$$

Imp
6M

* Explain aliasing and Non-aliasing with an example

Sol Consider non-periodic arbitrary signal $x(n)$ with $l = 4$ samples



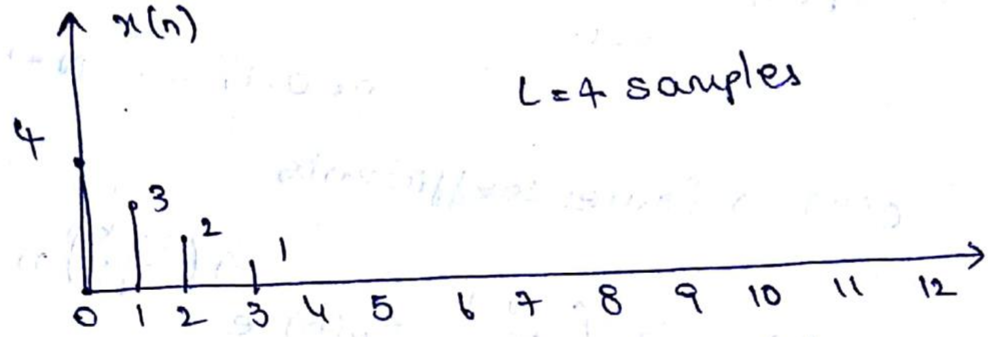

let us get $x_p(n)$ from $x(n)$ using 2 cases

case:1 $N > L$

$N = 6$; $L = 4$ samples

Non overlapping (or) Non aliasing

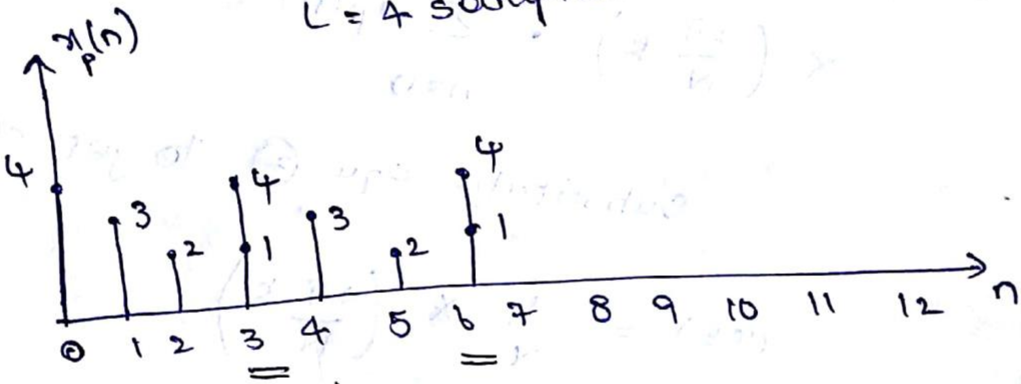
non continuous




Case :2

aliasing (or) overlapping

N=3 N<L
L=4 samples




① Sample value at 4, 5 points are zeros signal repeats at 6, 12 gives no overlapping conditions | non-aliasing cond's.

② Here two components are overlapped at 3, 6 This is called aliasing [It is always un derived condition] In order to avoid this select $\underline{N > L}$.

6M
★ Imp
Reconstructions of original sample x(n) from $X\left(\frac{2\pi k}{N}\right)$

Using DFS, periodic signal $x_p(n)$ with 'N' samples be written as,

$$x_p(n) = \sum_{k=0}^{N-1} c(k)\, e^{j\left(\frac{2\pi k}{N}\right)n} \rightarrow (1)$$

$n = 0, 1, \ldots, N-1$

$c(k) \rightarrow$ fourier coefficients

$$c(k) = \frac{1}{N}\left(\sum_{n=0}^{N-1} x_p(n)\, e^{-j\left(\frac{2\pi k}{N}\right)n}\right) \rightarrow (2)$$

Accordy freq. domain samples

WKT,

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\left(\frac{2\pi k}{N}\right)n}$$

substitute equ (2) to get $c(k)$

$$c(k) = \frac{1}{N} X\left(\frac{2\pi k}{N}\right)$$

Substitute $c(k)$ interms of samples in equ (1)

$$\boxed{x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}k\right) e^{j\left(\frac{2\pi}{N}k\right)n}}$$

DFT & IDFT using $W_N$ i.e Twiddle factor or phase factor

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\left(\frac{2\pi k}{N}\right)n} \rightarrow (1)$$

$k = 0, 1, \ldots, N-1$

IDFT

$$\sum^{N-1} X(k)\, e^{j\left(\frac{2\pi k}{N}\right)n} \rightarrow (2)$$

using $W_N$

$$W_N = e^{-j\frac{2\pi}{N}}$$

using $W_N$ DFT,

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{nK} \rightarrow (3)$$

$$W_n = \left(e^{-j\frac{2\pi}{N}}\right)^{nK} = e^{-j\left(\frac{2\pi K}{N}\right)n}$$

$K = 0, 1, \ldots N-1$

Similarly IDFT,

$$x(n) = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, W_N^{-nK} \longrightarrow (4)$$

$n = 0, 1 \ldots N-1$

8-10M Imp

★ Compute 8pt DFT of a sequence

$$x(n) = \{1\,1\,1\,1\,0\,0\,0\,0\}$$

Sol $N = 8$

$$X(K) = \sum_{n=0}^{7} x(n)\,(W_8^{nK}) \qquad K = 0, 1, \ldots 7$$

$$W_8^0 = \left(e^{-j\frac{2\pi}{8}}\right)^0 = 1$$

$$W_8^1 = \left(e^{-j2\pi/8}\right)^1 = e^{-j\pi/4}$$

$$= \cos\frac{\pi}{4} - j\sin\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$W_8^2 = -j$$

$$W_8^3 = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$W_8^4 = -1$$

$$W_8^5 = \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

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$$x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 0, 0, 0, 0\}$$

$$X(k) = \sum_{n=0}^{7} x(n) \cdot W_8^{nk} \qquad k = 0, 1, 2 \ldots 7$$

$$X(k) = 1 \times W_8^0 + 1 \times W_8^{1k} + 1 \times W_8^{2k} + 1 \times W_8^{3k}$$

$$X(k) = 1 + W_8^k + W_8^{2k} + W_8^{3k}$$

$$k = 0, 1, 2 \ldots 7$$

$k = 0$ $$X(0) = 1 + W_8^0 + W_8^0 + W_8^0$$
$$= 4$$

$k = 1$ $$X(1) = 1 + W_8^1 + W_8^2 + W_8^3$$
$$= 1 + \left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) - j + \left(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)$$
$$= 1 - j2.414$$

$k = 2$ $$X(2) = 1 + W_8^2 + W_8^4 + W_8^6$$
$$= 1 - j - 1 + j$$
$$= 0$$

$k = 3$ $$X(3) = 1 + W_8^3 + W_8^6 + W_8^9$$
$$= 1 + \left(\cancel{\frac{-1}{\sqrt{2}}} \mp j\frac{1}{\sqrt{2}}\right) + j + \cancel{\frac{1}{\sqrt{2}}} - j\frac{1}{\sqrt{2}}$$
$$= \cancel{2 - \frac{1}{\sqrt{2}} - j\left(1 + \frac{1}{\sqrt{2}}\right)}$$
$$= 1 - j\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right)$$
$$= 1 - j(\sqrt{2} - 1)$$

$k=4 \Rightarrow X(4) = 1+W_8^4 + W_8^8 + W_8^{12}$

$$= 1+\left(\frac{-1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}\right) +1 + \left(\frac{-1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}\right)$$

$$= 0$$

$k=5 \qquad X(5) = 1+W_8^5 + W_8^{10} + W_8^{15}$

$$= 1 + \left(\frac{-1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right) + W_8^2 + W_8^7$$

$$= 1 \pm j\frac{1}{\sqrt{2}} + \cancel{\frac{1}{\sqrt{2}}} + j\frac{1}{\sqrt{2}} - \cancel{\frac{1}{\sqrt{2}}} - j$$

$$= 1+j(0.414)$$

$k=6$

$X(6) = 1+W_8^6 + W_8^{12} + W_8^{18}$

$$= 1+W_8^6 + W_8^4 + W_8^2$$

$$= \cancel{1} + \cancel{j} - \cancel{1} - \cancel{j}$$

$$= 0$$

$k=7 \qquad X(7) = 1+W_8^7 + W_8^{14} + W_8^{21}$

$$= 1+W_8^7 + W_8^6 + W_8^5$$

$$= 1+j2.414$$

$$X(k) = \{\underset{\uparrow}{4}, 1-j2.414, 0, 1-j0.414, 0, 1+0.414j, 0, 1+j2.414\}$$

* compute 8pt DFT $x(n) = \{10101010\}$

Sol $\quad X(k) = \sum_{n=0}^{7} x(n)\, W_8^{nk} \qquad k=0,1,2\ldots 7$

$N=8$

$$k=0 \quad x(k) = 1 + W_8^{2k} + W_8^{4k} + W_8^{6k}$$

$$= 1 + W_8^{0} + W_8^{0} + W_8^{0}$$

$$= 4$$

$$k=1 \quad x(k) = 1 + W_8^{2} + W_8^{4} + W_8^{6}$$

$$= 1 + \left(\frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) + \left(\frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

$$+ \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

$$= 1 - j - 1 + j = 0$$

$$k=2 \quad x(k) = 1 + W_8^{4} + W_8^{8} + W_8^{12}$$

$$= 1 + W_8^{4} + W_8^{0} + W_8^{4}$$

$$= 2 - 2$$

$$= 0$$

$$k=3 \quad x(k) = 1 + W_8^{6} + W_8^{12} + W_8^{18}$$

$$= 1 + W_8^{6} + W_8^{4} + W_8^{2}$$

$$= 1 + j + (-1) + (-j)$$

$$= 0$$

$$k=4 \quad x(k) = 1 + W_8^{8} + W_8^{16} + W_8^{24}$$

$$= 1 + 1 + 1 + 1 = 4$$

$$k=5 \quad x(k) = 1 + W_8^{10} + W_8^{20} + W_8^{30}$$

$$= 1 + W_8^{2} + W_8^{4} + W_8^{4}$$

$$= 0$$

$k=6 \quad x(k) = 1 + w_8^{12} + w_8^{24} + w_8^{36}$

$= 0$

$k=7 \quad x(k) = 1 + w_8^{14} + w_8^{28} + w_8^{42}$

$= 0$

$$x(k) = \{\underset{\uparrow}{4}, 0, 0, 0, 4, 0, 0, 0\}$$

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Compute $x(n) = \{01010101\}$ 8pt DFT

Sol

$$X(k) = \sum_{n=0}^{7} x(n) w_8^{nk} \quad k = 0, 1, \dots 7$$

$N = 8$

~~$k=0$~~ $\quad x(k) = w_8^{k} + w_8^{3k} + w_8^{5k} + w_8^{7k}$

$k=0 \quad X(0) = w_8^0 + w_8^0 + w_8^0 + w_8^0$

$= 4$

$k=1 \quad X(1) = w_8^1 + w_8^3 + w_8^5 + w_8^7$

$$= \left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) + \left(\frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right) + \left(\frac{-1}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

$= 0$

$k=2 \quad X(2) = w_8^2 + w_8^6 + w_8^{10} + w_8^{14}$

$= -j + j + j - j$

$= 0$

$w_8 = -w_0$
$w_9 = -w_1$
$w_{10} = -w_2$
$w_{11} = -w_3$
$w_{12} = -w_4$
$w_{13} = -w_5$
$w_{14} = -w_6$
$w_{15} =$

$k=3 \quad X(3) = w_8^3 + w_8^9 + w_8^{15} + w_8^{21}$

$$= \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$-j\frac{1}{\quad} \quad -1 \quad +j\frac{1}{\quad} \quad = 0$

$X(4) = W_8^4 + W_8^{12} + W_8^{20} + W_8^{28}$

$= W_8^4 + W_8^4 + W_8^4 + W_8^4$

$= -4$

$X(5) = W_8^5 + W_8^{15} + W_8^{20} + W_8^{35}$

$= W_8^5 + W_8^7 + W_8^4 + W_8^3$

$= 0$

$X(6) = W_8^6 + W_8^{18} + W_8^{24} + W_8^{42}$

$= j - j + 1 - 1$

$= 0$

$X(7) = W_8^7 + W_8^{21} + W_8^{35} + W_8^{28}$

$= 0$

$$X(k) = \{\underset{\uparrow}{4}, 0, 0, 0, -4, 0, 0, 0\}$$

2) find 4 pt DFT of a sequence $x(n) = \cos\left(\frac{n\pi}{4}\right)$

Sol $N = 4$

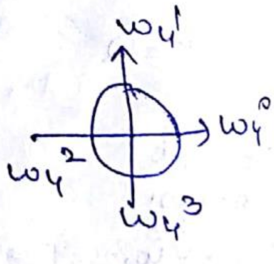
| $n$ | $x(n)$ |
|---|---|
| 0 | 1 |
| 1 | $1/\sqrt{2}$ |
| 2 | 0 |
| 3 | $-1/\sqrt{2}$ |

$W_4^0 = W_8^0 = 1$

$W_4^1 = W_8^2 = -j$

$W_4^2 = -1$

$W_4^3 = +j$




$$x(n) = \{\underset{\uparrow}{1}, 1/\sqrt{2}, 0, \frac{-1}{\sqrt{2}}\}$$

$X(k) = \sum^{3} x(n) W_4^{nk}$

$X(0) = 1 + \frac{1}{\sqrt{2}} w_4^0 - \frac{1}{\sqrt{2}} w_4^0$

$= 1$

$X(1) = 1 + \frac{1}{\sqrt{2}} w_4^1 - \frac{1}{\sqrt{2}} w_4^3$

$= 1 - 1.414j$

$X(2) = 1 + \frac{1}{\sqrt{2}} \cancel{w_4}^2 - \frac{1}{\sqrt{2}} \cancel{w_4}^6 = 1$

$X(3) = 1 + \frac{1}{\sqrt{2}} w_4^3 - \frac{1}{\sqrt{2}} w_4^9 = 1 + j\phi.414$

$X(k) = \{\underset{\uparrow}{1}, 1 - 1.414j, 1, 1 + \phi.414j\}$

ii) $x(n) = \sin\left(\frac{n\pi}{4}\right)$

$N = 4$

Sol

| $n$ | $x(n)$ |
|---|---|
| 0 | 0 |
| 1 | $1/\sqrt{2}$ |
| 2 | 1 |
| 3 | $+1/\sqrt{2}$ |

$x(n) = \{\underset{\uparrow}{0}, \frac{1}{\sqrt{2}}, 1, \frac{+1}{\sqrt{2}}\}$

$X(k) = \sum_{n=0}^{3} w_4^{nk} x(n)$ $\quad k = 0, 1, 2, 3$

$X(0) = \frac{1}{\sqrt{2}} w_4^0 + w_4^0 + \frac{1}{\sqrt{2}} w_4^0$

$= 2.414$

$X(1) = \frac{1}{\sqrt{2}} w_4^1 + w_4^2 + \frac{1}{\sqrt{2}} w_4^3$

$= \frac{-1}{\sqrt{2}} j - 1 + \frac{1}{\sqrt{2}} j$

$= -1 \cancel{-1.414j}$

$X(3) = -1 \cancel{+j1.414}$

$X(2) = \frac{1}{\sqrt{2}} w_4^2 + w_4^4 + w_4^6 \frac{1}{\sqrt{2}}$

$= \frac{-1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}}$

$X(k) =$ $\{$ 2.414, −1, −0.414, −1 ~~1, −1−1.414j, 1, −1+j1.414~~ $\}$

(iii) find 5 pt DFT of sequence $x(n) = \{1, 1, 1\}$

$N = 5$

Sol $X(k) = \sum_{n=0}^{4} x(n) W_5^{nk}$ $k = 0, 1, 2, 3, 4$

$X(k) = W_5^0 + W_5^{1k} + W_5^{2k}$

Ans $X(k) = \{3, 0.5 - j1.538, 0.5 + j0.364, 0.5 - j0.364, 0.5 + j1.538\}$
$\uparrow$

$W_5^0 = \left(e^{-j\frac{2\pi}{5}}\right)^0 = 1$

$W_5^1 = \left(e^{-\frac{2\pi}{5}j}\right)^1 = \cos\frac{2\pi}{5} - j\sin\frac{2\pi}{5} = 0.309 - 0.951j$

$W_5^2 = \cos\frac{4\pi}{5} - j\sin\frac{4\pi}{5} = -0.809 - 0.587j$

$W_5^3 = \cos\frac{6\pi}{5} - j\sin\frac{6\pi}{5} = -0.809 + 0.587j$

$W_5^4 = 0.309 + 0.951j$

$X(0) = 1 + W_5^0 + W_5^0 = 3$

$X(1) = 1 + W_5^1 + W_5^2 = 1 + 0.309 - 0.951j - 0.809 - 0.587j$

$= 0.5 - j1.538$

$X(2) = 1 + (-0.809 - 0.587j) + 0.309 + 0.9$

$= 0.5 + j0.364$

$X(3) = 1 - 0.809 + 0.587j + 0.309 - 0.951j$

$= 0.5 - j0.364$

$X(4) = 1 + 0.309 + 0.951j - 0.809 + 0.587$

iv, $x(n) = \{1, 0, 1\}$

SPT

$N = 5$

$X(k) = w_5^0 + w_5^2 = 1 + w_5^{2k}$

$X(0) = 1 + w_5^0 = 2$

$X(1) = 1 + w_5^2 = 1 - 0.809 - 0.587j$

$= 0.191 - 0.587j$

$X(2) = 1 + w_5^4 = 1 + 0.309 + 0.951j$

$= 1.309 + 0.951j$

$X(3) = 1 + w_5^6 = 1 + 0.309 - 0.951j$

$= 1.309 - 0.951j$

$X(4)$

$= 1 + w_5^8 = 1 - 0.809 + 0.587j$

$= 0.191 + 0.587j$

$X(k) = \{\underset{\uparrow}{2}, 0.191 - 0.587j, 1.309 + 0.951j,$
$1.309 - 0.951j, 0.191 + 0.587j\}$

v, $x(n) = \{1, 1, 1\}$

$N = 7$

$w_7^0 = 1$

$w_7^1 = \cos\frac{2\pi}{7} - j\sin\frac{2\pi}{7} = 0.623 - 0.781j$

$w_7^2 = -0.222 - 0.974j$

$w_7^3 = -0.900 - 0.433j$

$w_7^4 = -0.900 + 0.433j$

$w_7^5 = -0.222 + 0.974j$

$0.623 + 0.781j$

$$X(k) = 1 + w_7^{k} + w_7^{2k}$$

$$X(0) = 1+1+1 = 3$$

$$X(1) = 1 + 0.623 - 0.781j + (-0.222 - 0.974j)$$

$$= 1.401 - 1.755j$$

$$X(2) = 1 + w_7^2 + w_7^4 = -0.122 - 0.541j$$

$$X(3) = 1 + w_7^3 + w_7^6 = 0.723 + 0.348j$$

$$X(4) = 1 + w_7^4 + w_7^8 = 0.723 - 0.348j$$

$$X(5) = 1 + w_7^5 + w_7^{10} = -0.122 + 0.541j$$

$$X(6) = 1 + w_7^6 + w_7^{12} = 1.401 + 1.755j$$

## IDFT

1) find IDFT seq, $X(k) = \{2, 1+j, 0, 1+j\}$

$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X(k) w_4^{-nk}$$

$$= \frac{1}{4}\left[(2) w_4^{-n(0)} + (1+j) w_4^{-n} + (1+j) w_4^{-3n}\right]$$

$$= \frac{1}{4}\left[2 + (1+j) w_4^{-n} + (1+j) w_4^{-3n}\right]$$

$$w_4^{-0} = w_4^{0} = 1$$

$$w_4^{-1} = (w_4^{1})^* = (-j)^* = j$$

$$w_4^{-2} = (w_4^{2})^* = (-1)^* = -1$$

$$w_4^{-3} = (w_4^{3})^* = (j)^* = -j$$

$$x(n) = \frac{1}{4}\left[2 + (1+j)\,w_4^{-n} + (1+j)w_4^{-3n}\right]$$

$$x(0) = \frac{1}{4}\left[2 + (1+j) + (1+j)\right]$$

$$x(0) = 1 + 0.5j$$

$$x(1) = \frac{1}{4}\left[2 + (1+j)(j) + (1+j)(-j)\right] = 0.5$$

$$x(2) = \frac{1}{4}\left[2 + (1+j)(-1) + (1+j)(-1)\right] = -0.5j$$

$$x(3) = \frac{1}{4}\left[2 + (1+j)(-j) + (1+j)(j)\right] = 0.5$$

$$x(n) = \{\underset{\uparrow}{1+0.5j},\ 0.5,\ -0.5j,\ 0.5\}$$

Imp ★

2) find IDFT of $X(K) = \{+4, -2j, 0, 2j\}$

sol
$$x(n) = \frac{1}{4}\sum_{K=0}^{3} X(K)\,w_4^{-nK}$$

$$x(n) = \frac{1}{4}\left[(4)w_4^{0} + (-2j)\,w_4^{-n} + (2j)\,w_4^{-3n}\right]$$

$$x(n) = \frac{1}{4}\left[4 - 2j\,w_4^{-n} + 2j\,w_4^{-3n}\right]$$

$$x(0) = \frac{1}{4}\left[4 - (2j) + 2j\right] = 1$$

$$x(1) = \frac{1}{4}\left[4 - (2j)(j) + (2j)(-j)\right] = 2$$

$$x(2) = \frac{1}{4}\left[4 - (2j)(-1) + (2j)(-1)\right] = 1$$

$$x(3) = \frac{1}{4}\left[4 - (2j)(-j) + (2j)(j)\right] = 0$$

3) five sample

$$X(k) = \{\overset{0}{5}, \overset{1}{0}, \overset{2}{1-j}, \overset{3}{0}, \overset{4}{1}, \overset{5}{0}, \overset{6}{1+j}, \overset{7}{0}\} \quad 8\text{pt}$$

Sol

$$x(n) = \frac{1}{8} \sum_{k=0}^{7} X(k)\, W_8^{-nk}$$

$$x(n) = \frac{1}{8}\left[5 W_8^{0} + (1-j) W_8^{-2n} + W_8^{-4n} + (1+j) W_8^{-6n}\right]$$

~~x(0)=~~ $W_8^{0} = 1$

$W_8^{-1} = (W_8^{1})^{*} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$W_8^{-2} = \cancel{\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}}\ j$

$W_8^{-3} = \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$W_8^{-4} = -1$

$W_8^{-5} = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$W_8^{-6} = -j$

$W_8^{-7} = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$$x(0) = \frac{1}{8}\left[5 + (1-\cancel{j}) + 1 + (1+\cancel{j})\right] = 1$$

$$x(1) = \frac{1}{8}\left[5 + (1-j)j + (-1) - (1+j)j\right]$$

$$= \frac{1}{8}\left[5 + \cancel{j} - \cancel{j^2} - \cancel{1} - \cancel{j} \mp j^2\right] = \frac{3}{2} = 1.5$$

$$x(2) = \frac{1}{8}\left[5 + (1-j)(-1) + 1 + (1+j)(-1)\right]$$

$$= \frac{1}{8}\left[5 - \cancel{1} + \cancel{1} - 1\right] = \frac{1}{2} = 0.5$$

$$x(3) = \frac{1}{8}\left[5 + (1-j)(-j) - 1 + (1+j)j\right]$$

$= \ldots = 0.25$

$x(4) = \frac{1}{8}\left[5 + (1-j) + 1 + 1+j\right] = 1$

$x(5) = \frac{1}{8}\left[5 + (1-j)j - 1 - (1+j)j\right] = 1.5$

$x(6) = 0.5$

$x(7) = 0.25$

$$x(n) = \{\underset{\uparrow}{1}, 1.5, 0.5, 0.25, 1, 1.5, 0.5, 0.25\}$$

4) $X(K) = \{4, \cancel{0}, 1-j2.414, 0, 1-j.414, 0, 1+j0.414, 0, 1+j2.414\}$

sol

$$x(n) = \frac{1}{8}\sum_{0=K}^{7} X(K)\, W_8^{-nK}$$

$$x(n) = \frac{1}{8}\Big[4W_8^{0} + (1-j2.414)W_8^{-1n} + (1-j0.414)W_8^{-3n} + (1+j0.414)W_8^{-5n} + (1+j2.414)W_8^{-7n}\Big]$$

$$x(0) = \frac{1}{8}\Big[4 + (1-j2.\cancel{4}14)W_8^{0} + (1-j0.\cancel{4}14)W_8^{0} + (1+j0.\cancel{4}14)W_8^{0} + (1+j2.\cancel{4}14)W_8^{0}\Big]$$

$$= 8 \times \frac{1}{8} = 1$$

$$x(1) = \frac{1}{8}\Big[4 + (1-j2.414)\left(\cancel{0.414}\,\tfrac{1}{\sqrt{2}} + j\tfrac{1}{\sqrt{2}}\right) + (1-j0.414)\left(-\tfrac{1}{\sqrt{2}} + j\tfrac{1}{\sqrt{2}}\right) + (1+j0.414)\left(\tfrac{-1}{\sqrt{2}} - j\tfrac{1}{\sqrt{2}}\right) + (1+j2.414)\left(\tfrac{1}{\sqrt{2}} - j\tfrac{1}{\sqrt{2}}\right)\Big]$$

$$= \phi$$

$$x(2) = \frac{1}{8}\Big[4 + (1-j2.41\cancel{4})j + (1-j0.4\cancel{1}4)(-j) + (1+j0.\cancel{4}14)j + (1-j2\cancel{.}414)(-j)\Big]$$

$$x(3) = \frac{1}{8}\Big[4 + (1-j2.414)\left(\frac{-1}{\sqrt2}+j\frac{1}{\sqrt2}\right) + (1-j0.414)\left(\frac{1}{\sqrt2}+j\frac{1}{\sqrt2}\right) + (1+j0.414)\left(\frac{1}{\sqrt2}+j\frac{1}{\sqrt2}\right) + (1+j2.414)\left(\frac{-1}{\sqrt2}+j\frac{1}{\sqrt2}\right)\Big]$$

$$= \frac{1}{8}\times 8 = 1$$

$$x(4) = \frac{1}{8}\Big[4 + (1-j2.414)(-j) + (1-j0.414)j + (1+j0.414)(-j) + (1+j2.414)j\Big]$$

$$= \frac{1}{8}(0)$$

$$= 0$$

$x(5) = 0$

$x(6) = 0$

$x(7) = 0$

$$x(n) = \{\underset{\uparrow}{1},1,1,1,0,0,0,0\}$$

1/9/17

1) $X(k) = \{4,0,0,0,4,0,0,0\}$

Sol

$$x(n) = \frac{1}{8}\sum_{k=0}^{7} X(k)\, W_8^{-nk}$$

$$x(n) = \frac{1}{8}\left[4W_8^{0} + 4W_8^{-4n}\right]$$

$$x(0) = \frac{1}{8}[4+4] = 1$$

$$x(1) = \frac{1}{8}[4+(-4)] = 0$$

$$x(2) = \frac{1}{8}[4+4] = 1$$

$x(3) = \frac{1}{8}[4-4] = 0$

$x(4) = \frac{1}{8}[4+4] = 1$

$x(5) = \frac{1}{8}[4-4] = 0$

$x(6) = \frac{1}{8}[4+4] = 1$

$x(7) = \frac{1}{8}[4-4] = 0$

$x(n) = \{\underset{\uparrow}{1}0101010\}$

2) $X(K) = \{4, 0, 0, 0, -4, 0, 0, 0\}$

sol $x(n) = \frac{1}{8}\sum_{k=0}^{7} X(K)\, W_8^{-nK}$

$x(n) = \frac{1}{8}\left[4W_8^{0} - 4W_8^{-4n}\right]$

$x(0) = \frac{1}{8}[4-4] = 0$

$x(1) = \frac{1}{8}[4+4] = 1$

$x(2) = \frac{1}{8}[4-4] = 0$

$x(3) = \frac{1}{8}[4+4] = 1$

$x(4) = 0$

$x(5) = 1$

$x(6) = 0$

$x(7) = 1$

$x(n) = \{\underset{\uparrow}{0}, 1, 0, 1, 0, 1, 0, 1\}$

3) find DFT of sequence $x(n) = \{1, 1, 1, 1, 1, 1\}$ 7pt

sol $N = 7$

$X(K) = \sum_{n=0}^{\cancel{8}\,6} x(n)\, W_7^{nK}$ $\quad K = 0, 1, 2, 3 \dots 6$

$X(k) = w_7^0 + w_7^{1k} + w_7^{2k} + w_7^{3k} + w_7^{4k}$

$x(0) = 5$

$x(1) = 1 + 0.623 - 0.781j - 0.222 - 0.974j$
$\quad - 0.900 + 0.433j - 0.900 - 0.433j$

$= -0.399 - 1.755j$

$x(2) = 1 + (-0.222 - 0.974j) - 0.900 +$ $0.433j + 0.623 + 0.781j + 0.623$
$\quad - 0.781j$

$= 1.124 - 0.541j$

$x(3) = 1 + (-0.900 - 0.433j) + 0.623 +$ $0.781j - 0.222 - 0.974j - 0.222$
$\quad + 0.974j$

$= 0.279 + 0.348j$

$x(4)$

$= 1 - 0.900 + 0.433j - 0.\cancel{222}^{623} - 0.\cancel{974}^{781}j - 0.2$
$\quad + 0.974j - 0.222 - 0.974j$

$= -0.\cancel{279} - 0.\cancel{541}j$ 348

$x(5) = 1.124 + 0.541j$

$x(6) = -0.399 + 1.755j$

$X(k) = \{5, -0.399 - 1.755j, +1.124 - 0.541j,$
$\uparrow \; 0.279 + 0.348j, \; 0.279 - 0.348i,$
$1.124 + 0.541j, \; -0.399 + 1.755i\}$

* find DFT $x(n) = \{1, 2, 3, 4\}$ 4 pt's

$X_N = [W_N] x_N$

for $N = 4$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{array}{c|cccc} & n=0 & 1 & 2 & 3 \\ \hline k=0 & 1 & 1 & 1 & 1 \\ 1 & 1 & W_4^1 & W_4^2 & W_4^3 \\ 2 & 1 & W_4^2 & W_4^4 & W_4^6 \\ 3 & 1 & W_4^3 & W_4^6 & W_4^9 \end{array} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$W_4^{nk}$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

5/9/17

find IDFT of $X(k) = \{4, 1-j2.414, 0, 1-j0.414, 0, 1+j0.414, 0, 1+j2.414\}$

$N = 8$

$$x_N = \frac{1}{8}[W_N] X_N^*$$

$W_8^0 = 1 = W_8^8$ $\qquad$ $W_8^4 = -1 = W_8^{12}$

$W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} = W_8^9$ $\qquad$ $W_8^5 = \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} = W_8^{13}$

$W_8^2 = -j = W_8^{10}$ $\qquad$ $W_8^6 = +j = W_8^{14}$

$W_8^{3} = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$ $\qquad$ $W_8^7 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} = W_8^{15}$

$$
\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix}
= \frac{1}{8}
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\
1 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\
1 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\
1 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\
1 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\
1 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\
1 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49}
\end{bmatrix}
\begin{bmatrix} 4 \\ 1+j2.414 \\ 0 \\ 1+j0.414 \\ 0 \\ 1-j0.414 \\ 0 \\ 1-j2.414 \end{bmatrix}
$$

$$
\Rightarrow \frac{1}{8}
\begin{bmatrix}
4+1+j2.414+0+1+j0.414+0+1-j2.414+0+1-j0.414 \\ 8 \\ 8 \\ 8 \\ 0 \\ 0 \\ 0 \\ 0
\end{bmatrix}^T
$$

$$
\Rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T
$$

* find IDFT $X(k) = \{4, 0, 0, 0, 4, 0, 0, 0\}$ for $N = 8$pt

$\Rightarrow$

Sol

$$
x_N = \frac{1}{8} [W_N] X_N^*
$$

$$
\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \end{bmatrix}
=
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\
1 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\
1 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\
1 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\
1 & W_8^5 \\
1 & W_8^6
\end{bmatrix}
$$

$$\begin{bmatrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{+1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -j & \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -1 & \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & j & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -j & \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -j & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & j & \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & j & \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} & -1 & \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} & -j & \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 4+4 \\ 0 \\ 4+4 \\ 0 \\ 4+4 \\ 0 \\ 4+4 \\ 0 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$x(n) = \{1\,0\,1\,0\,1\,0\,1\,0\}$$

* find DFT of the sequence $x(n) = \{1\,1\,1\,1\,0\,0\,0\,0\}$ through matrix method

Sol

$$X_N = [W_N]\, x_N$$

$N = 8$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ 1 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ 1 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ 1 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ 1 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ 1 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ 1 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 1+1+1+1+0+0+0+0 \\ 1+\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} - j - \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} + 0+0+0+0 \\ 0 \\ \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} + j + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} - 1 + 0 + 0 + 0 + 0 \\ 0 \\ 1+j0.414 \\ 0 \\ 1+j2.414 \end{bmatrix}$$

$$X(k) = \{\underset{\uparrow}{4},\ 1-j2.414,\ 0,\ 1-j0.414,\ 0,\ 1+j0.414,\ 0,\ 1+j2.414\}$$

$$
\begin{bmatrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} \\
1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & j & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} \\
1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & j & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2}
\end{bmatrix}
\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

DFT $x(n) = \{1\,0\,1\,0\;1\,0\,1\,0\}$

DFT $x(n) = \{0\,1\,0\,1\,0\,1\,0\,1\}$

$\left.\right\}\; N = 8$

## Properties:-

### * Periodicity:-

For sequence $x(n)$ of $x(n+N) = x(n)$ then

PT, $X(K+N) = X(K)$

Proof:- WKT,

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{Kn}$$

$K = K+N$

$$X(K+N) = \sum_{n=0}^{N-1} x(n) W_N^{(K+N)n}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{Kn} \cdot \cancel{W_N^{Nn}}^{\,1}$$

$$\left[ W_N^{Nn} = e^{-j\frac{2\pi}{N}Nn} = e^{-j2\pi n} = 1 \right.$$

$$\Rightarrow X(K+N) = X(K)$$

### * Linearity:-

It states that $x_1(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X_1(K)$ &

$x_2(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X_2(K)$ then

PT, $a_1 x_1(n) + a_2 x_2(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} a_1 X_1(K) + a_2 X_2(K)$

Proof:-

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{Kn} \quad K = 0, 1, 2 \ldots N-1$$

let $x(n) = a_1 x_1(n) + a_2 x_2(n)$

$$X(K) = \sum^{N-1} (a_1 x_1(n) + a_2 x_2(n)) W_N^{Kn}$$

$$X(k) = a_1 X_1(k) + a_2 X_2(k)$$

6-8M

* find DFT of the following sequence, 4 pt

$$x(n) = \cos\frac{n\pi}{4} + \sin\frac{n\pi}{4}$$

(ii) $x(n) = \cos\frac{n\pi}{2} + \sin\frac{n\pi}{2}$

(iii) $x(n) = \cos\frac{n\pi}{3} + \sin\frac{n\pi}{3}$

Sol $N=4$

$x_1(n) = \cos\left(\frac{n\pi}{4}\right)$

| $n$ | $x_1(n) = \cos(n\pi/4)$ |
|---|---|
| 0 | 1 |
| 1 | $1/\sqrt{2}$ |
| 2 | 0 |
| 3 | $-1/\sqrt{2}$ |

$X_1(k)$

$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1 \\ 1-j1.414 \\ 1 \\ 1+j1.414 \end{bmatrix}$$

$X_1(k) = \{1, 1-j1.414, 1, 1+j1.414\}$

$x_2(n) = \sin\left(\frac{n\pi}{4}\right)$

| $n$ | $x_2(n) = \sin(n\pi/4)$ |
|---|---|
| 0 | 0 |
| 1 | $1/\sqrt{2}$ |
| 2 | 1 |
| 3 | $1/\sqrt{2}$ |

$X(k) = X_1(k) + X_2(k)$

$\Rightarrow \{3.414, -j0.414, 0.586, j1.414\}$

$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1 \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 2.414 \\ -1 \\ -0.414 \\ -1 \end{bmatrix}$$

(ii) HW

$x_1(n) = \cos \frac{n\pi}{2}$

| $n$ | $x_1(n)$ |
|---|---|
| 0 | 1 |
| 1 | 0 |
| 2 | −1 |
| 3 | 0 |

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$= \{0, 2, 0, 2\}$

$x_2(n) = \sin \frac{n\pi}{2}$

| $n$ | $x_2(n)$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 0 |
| 3 | −1 |

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$= \{0, -2j, 0, 2j\}$

$X(k) = \{0, 2-2j, 0, 2+2j\}$

(iii)

$x_1(n) = \cos \frac{n\pi}{3}$

| $n$ | $x_1(n)$ |
|---|---|
| 0 | 1 |
| 1 | ~~0 0.333~~ 0.5 |
| 2 | −0.5 |
| 3 | −1 |

$x_2(n) = \sin \frac{n\pi}{3}$

| $n$ | $x_2(n)$ |
|---|---|
| 0 | 0 |
| 1 | 0.8660 |
| 2 | 0.8660 |
| 3 | 0 |

$= \{0, 1-0.5j+0.5-j, 1, 1+0.5j-0.5+j\}$

$= \{1.732, -0.866-0.866j, 0, -0.866+0.866j\}$

$= \{0, 1.5-1.5j, 1, 0.5+1.5j\}$

H.W

* find DFT sequence for N=8 $x(n) = \{10101010\}$ and $x(n) = \{01010101\}$

Sol

$$X_N = [W_N]\, x_N$$

N=8

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ 1 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ 1 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ 1 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ 1 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ 1 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ 1 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ 1 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ 1 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ 1 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ 1 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ 1 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ 1 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

(i) $X(K) = \{4, 0, 0, 0, 4, 0, 0, 0\}$

(ii) $X(K) = \{4, 0, 0, 0, -4, 0, 0, 0\}$

$$\begin{bmatrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} \\ 1 & -j & -1 & j & 1 & j & 1 & -j \\ 1 & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & j & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} \\ 1 & j & -1 & -j & 1 & -j & 1 & +j \\ 1 & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & j & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+1+1+1 \\ 1-j-1+j \\ 1-1+1-j \\ 0 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} \\ 1 & -j & -1 & j & 1 & j & 1 & -j \\ 1 & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} & j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & j & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} \\ 1 & j & -1 & -j & 1 & -j & 1 & +j \\ 1 & \frac{-1}{\sqrt2}+j\frac{1}{\sqrt2} & -j & \frac{1}{\sqrt2}+j\frac{1}{\sqrt2} & -1 & \frac{1}{\sqrt2}-j\frac{1}{\sqrt2} & j & \frac{-1}{\sqrt2}-j\frac{1}{\sqrt2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+1+1+1 \\ 1-j-1+j \\ 1-1+1-1 \\ 0 \\ -1-1-1-1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ -4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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3) circular time shift property

Multiplication Exponential in freq. Domain

Statement :-

$$DFT\{x(n)\} \longrightarrow X(K) \text{ then}$$

$$DFT\{x(n-m)_N\} = W_N^{mk} X(K)$$

Proof :-

WKT,

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(K)\, W_N^{-nk}$$

sub $n = n-m$

$$x(n-m) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-(n-m)k}$$

Since circular ~~conv~~ time shifting

$$x(n-m)_N = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-nk}\, W_N^{mk}$$

$$= \frac{1}{N}\left[\sum_{k=0}^{N-1}\left(X(k)\, W_N^{mk}\right) W_N^{-nk}\right]$$

$$x(n-m)_N = IDFT\{X(K)\, W_N^{mk}\}$$

$$IDFT \text{ of } (X(k)) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-mk}$$

$$DFT\{x((n-m))_N\} = DFT\{IDFT\ X(k)\, W_N^{mk}\}$$

$$DFT\{x(n-m)_N\} = X(K)\, W_N^{mk}$$

Q8 6M

* find 4pt DFT of a sequence $x(n) = \{1,-1,1,-1\}$ also by using Time shift property find DFT of a sequence.

$$y(n) = x((n-2))_4$$

Sol

$$X(K) = \{\underset{\uparrow}{0},0,4,0\}$$

Given $y(n) = x((n-2))_4$

Take DFT Bothsides

$$Y(k) = DFT\{x(n-2)\}$$

Apply CTSP

$$Y(k) = w_4^{2k}\, X(k) \quad k=0,1,2,3$$

$Y(0) = 0$

$Y(1) = 0$

$Y(2) = 1\times 4 = 4$

$Y(3) = 0$

$$Y(k) = \{0,0,4,0\}$$

$X(k) = ?$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1-1+1-1 \\ 1+j-j-1 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

(ii)

$\{1,0,1,0\}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$$Y(k) = w_4^{2k}\, X(k)$$

$Y(0) = w_4^{0}\, X(0) = 2$

$Y(1) = w_4^{2}\, X(1) = 0$

$Y(2) = w_4^{4}\, X(2) = 2$

$Y(3) = w_4^{6}\, X(3) = 0$

$$Y(k) = \{2,0,2,0\}$$

4) circular frequency shift property

Multiplication exponential in time Domain

statement :- If DFT $\{x(n)\} = X(K)$ then

$$DFT\{x(n) \cdot \omega_N^{-ln}\} = X(K-l)_N$$

Proof:-

WKT, $X(K) = \sum_{n=0}^{N-1} x(n)\, \omega_N^{nK}$ $\quad K = 0, 1 \ldots N-1$

sub

$K = K-l$

$$X(K-l) = \sum_{n=0}^{N-1} x(n)\, \omega_N^{n(K-l)}$$

Since frequency shift circular (given)

$$X(K-l)_N = \sum_{n=0}^{N-1} x(n)\, \omega_N^{nK}\, \omega_N^{-ln}$$

$$= \sum_{n=0}^{N-1} \left(x(n) \cdot \omega_N^{-ln}\right) \omega_N^{nK}$$

$$X(K-l)_N = DFT\{x(n)\, \omega_N^{-ln}\}$$

$$DFT\{x(n) \cdot \omega_N^{-ln}\} = X(K-l)_N$$

6M

Compute 4 pt DFT of sequence $x(n) = \{1, 0, 1, 0\}$ also find $y(n)$, if $y(K) = X(K-2)_4$

$\rightarrow X(K) = \{2, 0, 2, 0\}$

Sol

$$Y(K) = X(K-2)_4$$

Using CISP

$$Y(K) = DFT\{x(n) \cdot \omega_4^{-2n}\}$$

Take IDFT on B.S

$$y(n) = IDFT\{DFT\{x(n)\, \omega_4^{-2n}\}\}$$

$$= x(n) \cdot \omega_4^{-2n}$$

$y(0) = x(0)\, \omega_4^{0} = 1$

$y(1) = x(1)\, \omega_4^{-2} = 0$

$y(2) = x(2)\, \omega_4^{-4} = 1$

$y(3) = 0$

$\Rightarrow \{1, 0, 1, 0\}$

8/9/17

$x(n) = \{1, -1, 1, -1\}$

$Y(K) = X(K-2)_4$

Using CISP

$Y(K) = DFT\{x(n) \cdot \omega_4^{-2n}\}$

Taking IDFT on BS

$x(n)\, \omega_4^{-2n}$

$n = 0, 1, 2, 3$

$y(0) = 1$

$y(1) = 1$

$y(2) = 1$

$y(3) = 1$

$\Rightarrow \{1, 1, 1, 1\}$

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* Imp
8M ★

## DFT with other Transform :-

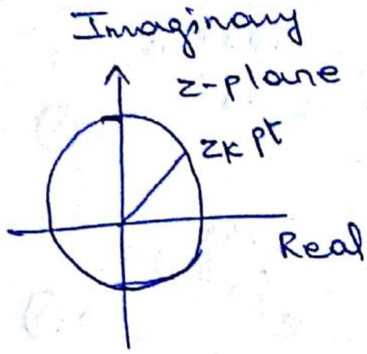
→ DFT with z Transform

→ DFT with DFS

→ DFT with DTFT

① WKT z-Transform of $x(n)$ is given by,

$X(z) = \sum^{\infty} x(n) z^{-n} \rightarrow ①$




Sample X(z) at N equally spaced pts. After the points and is written by equation

$$z_K = e^{\frac{j2\pi K}{N}}$$

$$K = 0, 1, 2 \ldots N-1$$

let us evaluate X(z) at the zk pts, then

$$X(z)\Big|_{z = e^{\frac{j2\pi K}{N}}} = \sum_{n=-\infty}^{\infty} x(n)\, e^{\left(\frac{j2\pi K}{N}\right)-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{\frac{-j2\pi K n}{N}}$$

For causal sequence x(n) & finite value of N

$$X(z)\Big|_{z = e^{\frac{j2\pi K}{N}}} = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi K n}{N}}$$

$$\boxed{X(z)\Big|_{z = e^{\frac{j2\pi K}{N}}} = X(K)}$$

2. DFS coefficients for the periodic sequence $x_p(n)$ over 'n' is given by

$$C(K) = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n)\, e^{\frac{-j2\pi K \cdot n}{N}} \rightarrow (1)$$

DFT pair wrt x(n) & $x_p(n)$ is given by

$$x(n) \xleftrightarrow[N]{DFT} X(K)$$

$$x_p(n) \xleftrightarrow[N]{DFT} X(K)$$

WKT,

$$X(k) = \sum_{n=0}^{N-1} x_p(n)\, e^{\frac{-j2\pi k}{N} n} \rightarrow (2)$$

$$= \sum_{n=0}^{N-1} x(n) \cdot e^{\frac{-j2\pi k}{N} n} \rightarrow (3)$$

Sub (2) in (1)

$$C(k) = \frac{1}{N} X(k)$$

$$\boxed{X(k) = N\, C(k)}$$

(3) DTFT of $x(n)$ is

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\Omega n} \rightarrow (1)$$

Substitute $\Omega = \frac{2\pi k}{N}$

$$X\left(\frac{2\pi k}{N}\right) = \sum_{n=0}^{N-1} x_p(n)\, e^{-j\left(\frac{2\pi k}{N}\right)n} \qquad k = 0, 1 \dots N-1$$

finally,

$$\boxed{X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi k n}{N}}} \qquad k = 0, 1 \dots N-1$$