ERROR CONTROL CODING:

CHANNEL CODING:-

The main task required in digital communication is to construct cost effective system for transmitting information from end of system at a rate of and level of reliability that are acceptable to the user [at the other end].

The coding techniques discussed previously deals with minimizing the average wordlength of the codes with an objective of achieving higher efficiency.

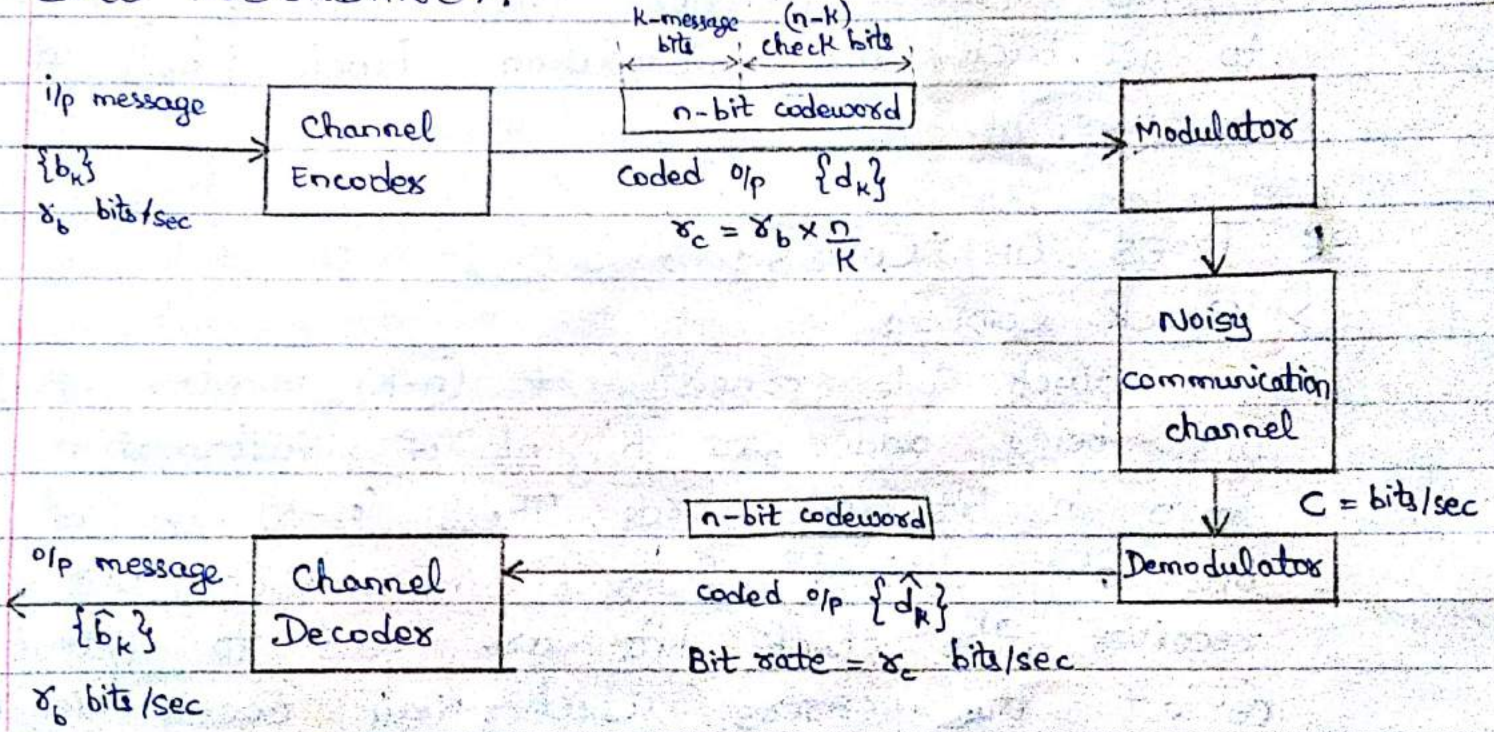
The main disadvantage with this type of coding is that they are variable length codes. Due to this, a single error which occurs due to noise present in the channel affects more than one block code word. Another disadvantage of variable length codes is that the time will fluctuate widely.

When fixed length codes are used, a single error will affect only that block which can be easily detected & corrected.

To detect & correct the errors, error control coding techniques are used that rely on the systematic addition of redundant symbols.

Error control coding is nothing but the calculated use of redundancy. The functional blocks that are used are channel encoder at the transmitter and channel decoder at the receiver.

The channel encoder at the transmitter systematically ~~adds~~ adds the digits to the transmitted message bits. These additional digits carry no information but makes it possible for the channel decoder to detect & correct the error in information bearing digits. This reduces the overall probability of error, thereby achieving the desired goal. These additional digits which carry no information are called REDUNDANT digits & the process of adding these digits is called REDUNDANCY.



Above figure shows block diagram of digital comm^n system employing error control coding. The main functional blocks are channel encoder, channel decoder, modulator, demodulator & noisy communication channel.

The source generates a message block {b_k} at a rate of r_b bits/sec & feeds it to the channel encoder. The channel encoder then adds (n-k) no of redundant bits to these k-bit messages to form n-bit code words. These (n-k) no of additional bits are also called as CHECK BITS which donot carry any information but helps the channel decoder to detect & correct the errors. The bit rate of coded output block $\{d_k\}$ will be r_c bits/sec. This is the rate at which the MODEM operates to produce a message block $\{\hat{d}_k\}$ at the receiver.

$$r_c = r_b \times \frac{n}{k}$$

The channel decoder then decodes this message to get back the information block $\{\hat{b}_k\}$ at the receiver.

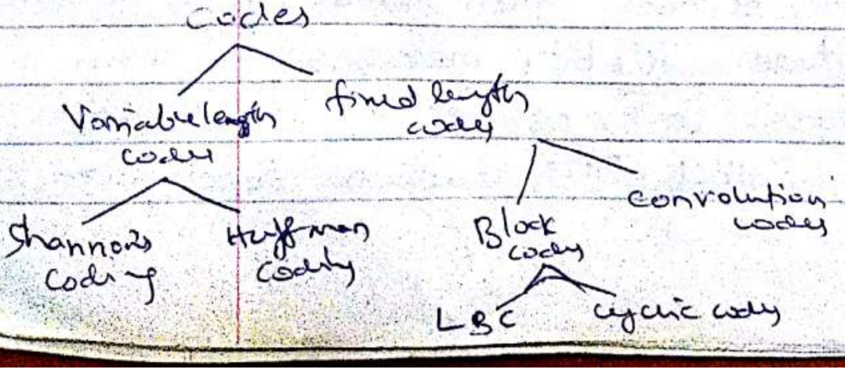
* TYPES OF CODES:

> BLOCK CODES:

Block Code consists of (n-k) number of check bits being added to 'k' no of information bits to form 'n' bit code words. These (n-k) no of check bits are derived from 'k' information bits. At the receiver, the check bits are used to detect & correct the errors which may occur in the entire n-bit code word.

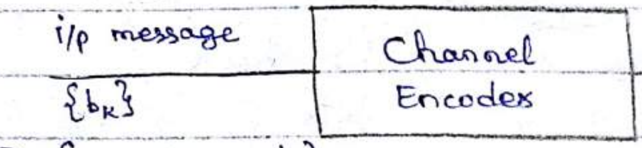
> CONVOLUTION CODES:

In convolution codes, the check bits are continuously interleaved with information bits. These check bits verify the information bits not only in the block immediately preceeding but in other blocks also.



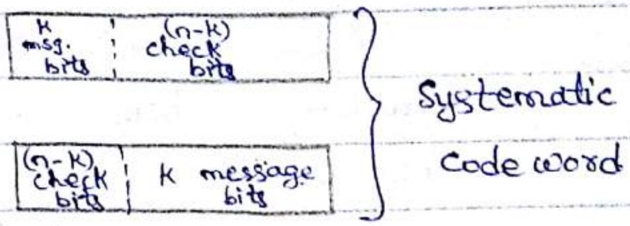
* LINEAR BLOCK CODES :-

$$\overbrace{c_1 c_2 \ldots c_k}^{\text{msg. bits}}\ \overbrace{c_{k+1} \ldots c_n}^{\text{check bits}}$$



$$C = c_1 c_2 c_3 \ldots \ldots c_n$$

$D = \{d_1, d_2, d_3 \ldots d_k\}$



A (n, k) block code is said to be (n, k) linear block code if it satisfies the condition given below.
Let C_a & C_b be any 2 code words [n-bits] belonging to a set of (n, k) block code. Then, if $C_a \oplus C_b$ [modulo-2 arithmetic] is also a n-bit code-word belonging to the same set of (n, k) block code, then, such a block code is called (n, k) LINEAR BLOCK CODE.

* MATRIX DESCRIPTION OF LINEAR BLOCK CODES:

Let the message block of k-bits be represented as a row vector called message vector given by

$$D = \{d_1 d_2 d_3 \ldots d_k\} \quad \text{— (i)}$$

where each message bit can be '0' or '1'. Thus, there are 2^k distinct message vectors.

Each message block is transformed to a codeword of length n-bits & there are 2^k code vectors.

$$C = \{c_1 c_2 c_3 \ldots c_n\} \quad \text{— (ii)}$$

In a systematic linear block code, the message bits appear at beginning of code vector &

remaining $(n-k)$ bits are check bits.

$$C = \{\underbrace{c_1\, c_2\, c_2 \, ---- \, c_k}_{k \text{ message bits}}\, \underbrace{c_{k+1}\, c_{k+2} \, ---- \, c_n}_{(n-k) \text{ check bits}}\}$$

These $(n-k)$ check bits are derived from the k message bits using a predefined rule as given below.

$$\left.\begin{aligned}
C_{k+1} &= P_{11}d_1 + P_{21}d_2 + P_{31}d_3 + P_{41}d_4 + \dots + P_{k1}d_k \\
C_{k+2} &= P_{12}d_1 + P_{22}d_2 + P_{32}d_3 + P_{42}d_4 + \dots + P_{k2}d_k \\
C_{k+3} &= P_{13}d_1 + P_{23}d_2 + P_{33}d_3 + P_{43}d_4 + \dots + P_{k3}d_k \\
&\vdots \qquad\qquad \vdots \\
C_n &= P_{1(n-k)}d_1 + P_{2(n-k)}d_2 + P_{3(n-k)}d_3 + \dots + P_{k(n-k)}d_k
\end{aligned}\right\} \text{(iii)}$$

where $P_{11}, P_{21}, P_{12} \dots$ are either '0' or '1' & the addition operation is modulo-2 arithmetic.

Combining above 3 equations, the equation result can be expressed in matrix form as.

$$[c_1\, c_2\, c_3 \dots\dots c_n] = [d_1\, d_2\, d_3 \dots d_k] \left[\begin{array}{c|cccc}
1\,0\,0\,0 \dots 0 & P_{11} & P_{12} & \dots & P_{1(n-k)} \\
0\,1\,0\,0 \dots 0 & P_{21} & P_{22} & \dots & P_{2(n-k)} \\
0\,0\,1\,0 \dots 0 & P_{31} & P_{32} & \dots & P_{3(n-k)} \\
\vdots & \vdots & \vdots & & \vdots \\
0\,0\,0\,0 \dots 1 & P_{k1} & P_{k2} & \dots & P_{k(n-k)}
\end{array}\right]$$

$$(1\times n) \quad = \quad (1\times k) \quad \underbrace{(k\times k) \qquad (k\times (n-k))}_{k\times n}$$

$$[C] = [D]\,[G]$$

where $[G] \rightarrow$ generator matrix which consists of an identity matrix of order k & a parity matrix.

$$[G] = [I \,\vdots\, P_{k}]$$

$[G]$ is called Generator matrix of order $(k \times n)$ given by

$$[G] = [I_k \,\vdots\, P]_{k\times n}$$

where I_k is unit matrix of order k
P is an arbitrary matrix called parity matrix of the order $k \times (n-k)$

The Generator matrix can also be expressed as

$$[G] = [P \,\vdots\, I_k]_{k\times n}$$

In this case, the message bits will be present at the end & the check bits at the beginning of code vector.

P) The Generator matrix for a (6,3) block code is given below. Find all the code vectors.

$$G = \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 & 1 & 1 \\ 0 & 1 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 0 \end{bmatrix}$$

The message block size for this code is 3 & the length of code vector 'n' is 6. Since $k=3$, there are $2^k = 2^3 = 8$ message vectors present.
The code vectors are found as follows.

$$[C] = [D]\,[G]$$

$$= [d_1 \; d_2 \; d_3] \begin{bmatrix} 1 & 0 & 0 & \vdots & 0 & 1 & 1 \\ 0 & 1 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 0 \end{bmatrix}$$

$$[C] = [d_1 \;\; d_2 \;\; d_3 \,\vdots\, (d_2+d_3) \;\; (d_1+d_3) \;\; (d_1+d_2)]$$

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Message vector $d_1\ d_2\ d_3$	Code Vector $C_1\ C_2\ C_3\ C_4\ C_5\ C_6$
0 0 0	0 0 0 0 0 0
0 0 1	0 0 1 1 1 0
0 1 0	0 1 0 1 0 1
0 1 1	0 1 1 0 1 1
1 0 0	1 0 0 0 1 1
1 0 1	1 0 1 1 0 1
1 1 0	1 1 0 1 1 0
1 1 1	1 1 1 0 0 0

It can be verified that addition of any 2 code vector is a code vector belonging to the same (6,3) code.

Ex: Consider, $C_4 \oplus C_5$

$$C_4 = 100011$$
$$C_5 = 101101$$
$$\overline{001110} = C_1$$ is a code belonging to same (6,3) code.

$\therefore$ Above code is a linear block code.

P) Repeat the above problem for (7,4) block code generated by

$$G = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$n=7$

$k=4$

$(n-k)=3$

$2^k = 2^4 = 16$ message vectors will be present.

$$[C] = [D][G]$$

$$= [d_1 \; d_2 \; d_3 \; d_4] \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$[C] = \left[d_1, d_2, d_3, d_4, (d_1+d_2+d_3), (d_1+d_2+d_4), (d_1+d_3+d_4)\right]$$

Message vector				Code vector						
d_1	d_2	d_3	d_4	c_1	c_2	c_3	c_4	c_5	c_6	c_7
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	0	1	1
0	0	1	0	0	0	1	0	1	0	1
0	0	1	1	0	0	1	1	1	1	0
0	1	0	0	0	1	0	0	1	1	0
0	1	0	1	0	1	0	1	1	0	1
0	1	1	0	0	1	1	0	0	1	1
0	1	1	1	0	1	1	1	0	0	0
1	0	0	0	1	0	0	0	1	1	1
1	0	0	1	1	0	0	1	1	0	0
1	0	1	0	1	0	1	0	0	1	0
1	0	1	1	1	0	1	1	0	0	1
1	1	0	0	1	1	0	0	0	0	1
1	1	0	1	1	1	0	1	0	1	0
1	1	1	0	1	1	1	0	1	0	0
1	1	1	1	1	1	1	1	1	1	1

P) For a systematic (6,3) linear block code, the parity matrix is given by

$$[P] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Find all the possible code vectors.

$n = 6$

$k = 3$

$2^3 = 8$ message vectors are present.

$$[G] = [I_k \,\vdots\, P]$$

$$= \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$[C] = [D]\,[G]$$

$$= [d_1\ d_2\ d_3] \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$= [d_1,\ d_2,\ d_3,\ (d_1+d_3),\ (d_2+d_3),\ (d_1+d_2)]$$

Message ~~Code~~ vectors	Code vectors
000	000000
001	001110
010	010011
011	011101
100	100101
101	101011
110	110110
111	111000

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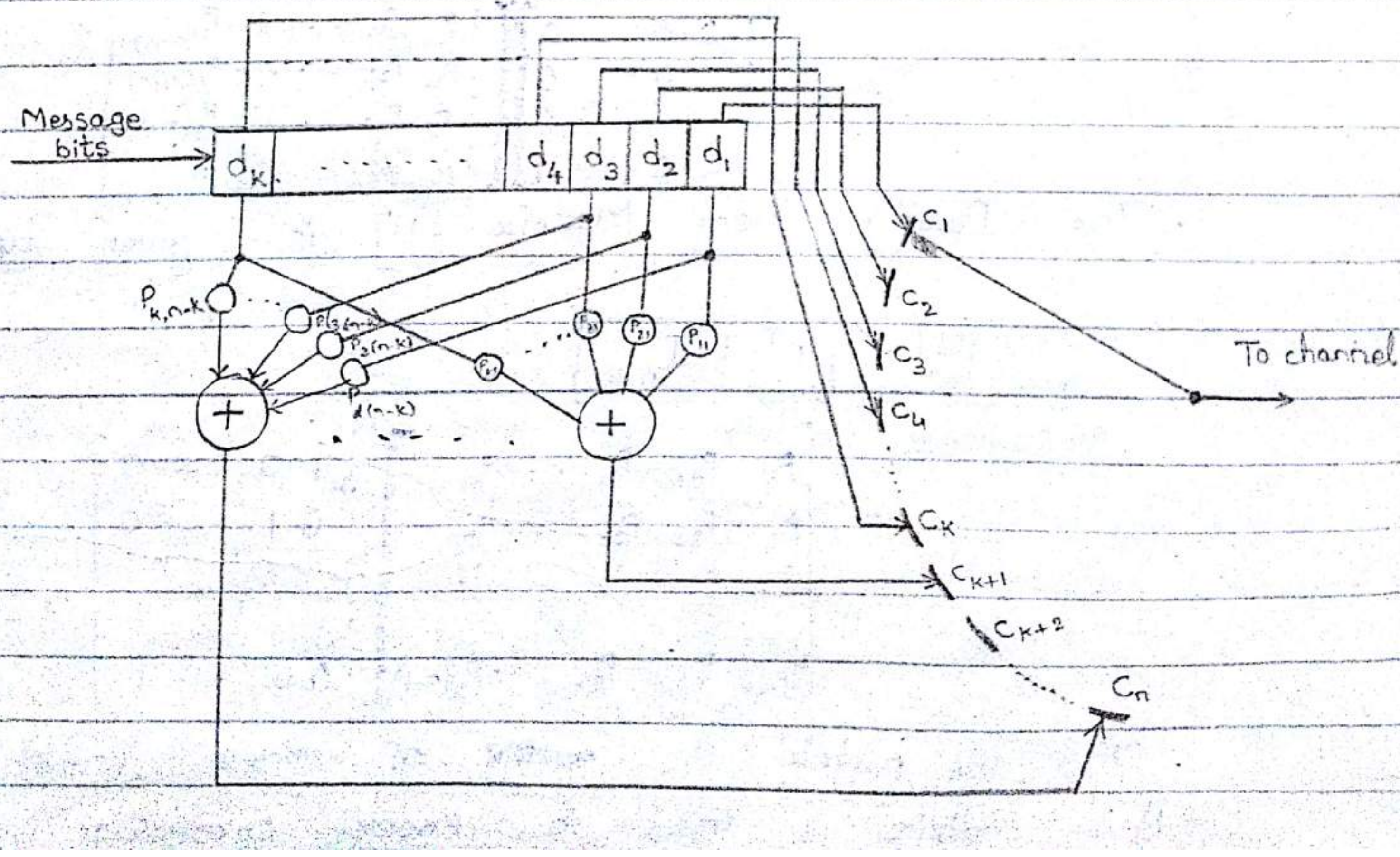
* ENCODING CIRCUIT FOR (n, K) LINEAR BLOCK CODE :

It is known that $[C] = [D][G]$.

Expanding above equation & equating the corresponding elements on both sides,

$$c_1 = d_1$$
$$c_2 = d_2$$
$$\vdots \quad \vdots$$
$$c_K = d_K$$
$$c_{K+1} = P_{11}d_1 + P_{21}d_2 + P_{31}d_3 + \cdots + P_{K1}d_K$$
$$c_{K+2} = P_{12}d_1 + P_{22}d_2 + P_{32}d_3 + \cdots + P_{K2}d_K$$
$$\vdots \qquad \vdots$$
$$c_n = P_{1(n-K)}d_1 + P_{2(n-K)}d_2 + P_{3(n-K)}d_3 + \cdots + P_{K(n-K)}d_K$$

The implimentation of above equation in a circuit results in the encoder for (n, K) linear block code. The realization of encoder circuit shown below consists of a k-bit shift register, n-segment commutator & (n-k) no of modulo-2 adders.



The entire data $d_1 d_2 d_3 \ldots d_K$ is shifted into the k-bit shift register. The small circles $P_{11}, P_{21}, P_{31} \ldots P_{K1}$ are either open circuit or short circuit depending either 0 or 1.

If $P_{11} = 0$, then there is no connection from d_1 to modulo-2 adder & if $P_{11} = 1$, then there is a connection.

When the message is shifted into the shift register, the modulo-2 adders generate the check bits which are fed into the commutator segment along with the message bits as shown in figure. When the commutator brush rotates & makes contact with the segment successively, the code vector bits will be transmitted through the channel.

* PARITY CHECK MATRIX:

The generator matrix is given by

$$[G] = [I_K \,\vdots\, P]$$

$$= \left[\begin{array}{ccccc|cccc} 1 & 0 & 0 & \cdots & 0 & P_{11} & P_{12} & \cdots & P_{1(n-K)} \\ 0 & 1 & 0 & \cdots & 0 & P_{21} & P_{22} & \cdots & P_{2(n-K)} \\ \vdots & \vdots & \vdots & \cdots & 0 & P_{31} & P_{32} & \cdots & P_{3(n-K)} \\ 0 & 0 & 0 & \cdots & 1 & P_{K1} & P_{K2} & \cdots & P_{K(n-K)} \end{array}\right]$$

The Parity Check Matrix $[H]$ is given by

$$[H] = [P^T \,\vdots\, I_{n-K}]$$

$$= \left[\begin{array}{ccccc|cccc} P_{11} & P_{21} & P_{31} & \cdots & P_{K1} & 1 & 0 & \cdots & 0 \\ P_{12} & P_{22} & P_{32} & \cdots & P_{K2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{1(n-K)} & P_{2(n-K)} & P_{3(n-K)} & \cdots & P_{K(n-K)} & 0 & 0 & \cdots & 1 \end{array}\right]$$

The $[H]$ matrix is of order $(n-K) \times n$ and this matrix is used in error correction.

* To PROVE $CH^T = 0$:-

Consider the Generator [G] & [H] matrix as given above.

The i^{th} row of [G] matrix is given by

$$g_i = 0\,0\,0 \ldots \underset{\substack{\uparrow \\ j^{th} \\ \text{element}}}{1} \ldots 0 \; P_{i1} \; P_{i2} \ldots \underset{\substack{\uparrow \\ (k+j)^{th} \\ \text{element}}}{P_{ij}} \ldots P_{i(n-k)}$$

$|||^{ly}$, consider j^{th} row of [H] matrix.

$$h_j = P_{1j} \; P_{2j} \ldots \underset{\substack{\uparrow \\ i^{th} \\ \text{element}}}{P_{ij}} \ldots P_{kj} \; 0\,0\,0 \ldots \underset{\substack{\uparrow \\ (k+j)^{th} \\ \text{element}}}{1} \ldots 0$$

Consider

$$g_i h_j^T = \begin{bmatrix} 0\,0\,0 \ldots 1 \ldots 0 \; P_{i1} \; P_{i2} \ldots P_{ij} \ldots P_{i(n-k)} \end{bmatrix} \begin{bmatrix} P_{ij} \\ P_{2j} \\ \vdots \\ P_{ij} \\ \vdots \\ P_{kj} \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

$$= 0 + 0 + \ldots + P_{ij} + \ldots 0 + P_{ij} + 0 \ldots + 0$$

$$= P_{ij}\,[1+1] = 0$$

$[\because 1 \oplus_2 1 = 0 \text{ modulo 2 addition}]$

$\therefore g_i h_j^T = 0$

This equation is true for every value of i & j & hence in the matrix form,

$$[G][H^T] = 0$$

(X)^{by} both sides by [D] message vector

$$\therefore [D][G][H^T] = [D]0 = 0$$

But $[D][G] = [C]$

$$\therefore \boxed{[C][H^T] = 0}$$

* ERROR CORRECTION AND SYNDROME:-

Let $[C] = [c_1\, c_2 \ldots c_n]$ be a valid code vector transmitted over a noisy communication channel belonging to a (n,K) linear block code.

Let $[R] = [r_1\, r_2 \ldots r_n]$ be a received vector. Due to the noise in the channel, $r_1, r_2 \ldots r_n$ may be different from $c_1, c_2, \ldots c_n$.

The error vector or error pattern E is defined as the difference b/w R & C.

$$\therefore \quad E = R - C$$

Therefore, the error vector can be represented by

$$E = [e_1\, e_2 \ldots e_n]$$

From above equation, it is clear that E is also a vector where $e_i = 1$ if $R \neq C$
& $e_i = 0$ if $R = C$

The 1's present in error vector E represent the error's caused by the noise in the channel.

In the equation $E = R - C$, the receiver knows only R & it doesn't know C & E. In order to find E & then C, the receiver does the decoding operation by determining a (n-k) vector S defined as

$$S = RH^T = [S_1\, S_2\, S_3 \ldots S_{n-k}]$$

This (n-k) vector is called ERROR SYNDROME of R.

Consider $S = RH^T$

$$= [C+E][H^T]$$

$$= \cancel{CH^T}^{\,0} + EH^T = EH^T$$

$$\therefore \quad \boxed{S = EH^T}$$

The receiver finds E from the above equation as S & H^T are known. Then, from the equation $R = C + E$, the transmitted code vector C can be found out.

Note that the syndrome S of the received vector will be zero if R is a valid code vector. When $R \neq C$, then $S \neq 0$. The receiver then detects & corrects the error.

P> For a systematic (6,3) code, find all the transmitted code vector, draw the encoding circuit, if received vector $[R] = [110010]$, detect & correct the ~~single~~ error that has occured due to noise.

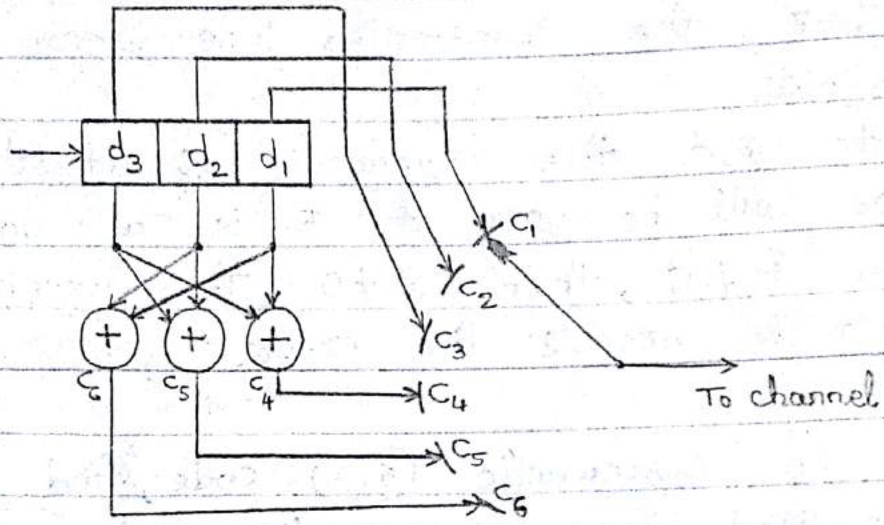
$$[P] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad G = \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 1 \\ 0 & 1 & 0 & \vdots & 0 & 1 & 1 \\ 0 & 0 & 1 & \vdots & 1 & 1 & 0 \end{bmatrix}$$

$$[C] = [D][G]$$
$$= [d_1 \; d_2 \; d_3] [\;]$$

$$[C] = [d_1 \; d_2 \; d_3 \; (d_1 + d_3) \; (d_2 + d_3) \; (d_1 + d_2)]$$

Message vectors $d_1 \; d_2 \; d_3$	Code vectors
000	000 000
001	001110
010	010011
011	011101
100	100101
101	101011
110	110110
111	111000

Encoding circuit:-



$[H] = [P^T \vdots I_{n-k}]$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$C = R - E \equiv R \oplus E$

$1+0+0+0+0+0$

$[S] = [R][H^T]$

$$= [1\,1\,0\,0\,1\,0] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [1\ 0\ 0]$$

$E = [0\,0\,0\,1\,0\,0]$

$C = R - E = [1\,1\,0\,0\,1\,0] - [0\,0\,0\,1\,0\,0]$

$\therefore C = (1\,1\,0\,1\,1\,0)$

Comparing the $[S] = [100]$ with the rows of $[H^T]$ matrix

The syndrome vector $[S] = [100]$ is present in the 4^{th} row of $[H^T]$ matrix, & hence the 4^{th} bit in the received vector counting from left is in error.

$\therefore$ The corrected code vector is $[110110]$ which is a valid transmitted code vector.

P) For a systematic (6,3) linear block code, the parity matrix is given by $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

a) Find all possible code vectors
b) Draw the encoding circuit
c) If the received code vector $[R] = [110011]$, find the syndrome, detect and correct the error.

$$[G] = [I_k \vdots P]$$

$$= \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

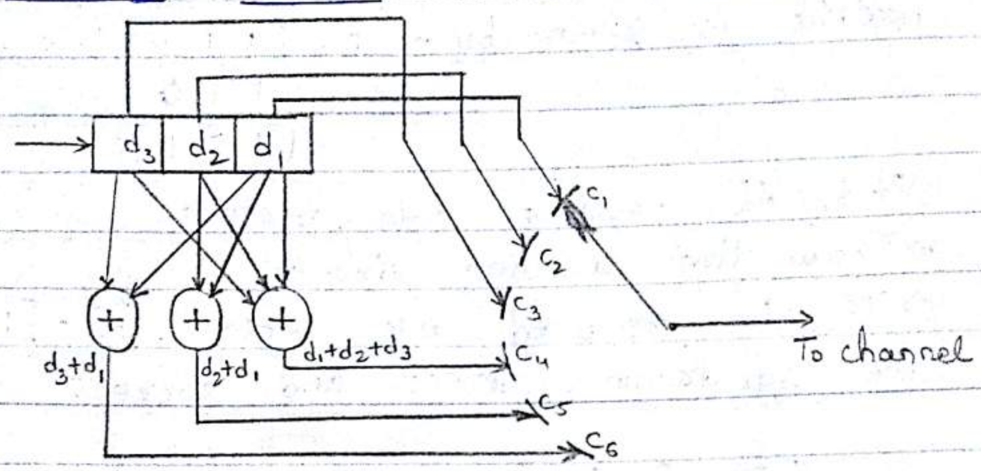
$$[C] = [D][G]$$

$$= [d_1 \ d_2 \ d_3] \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$[C] = [d_1 \ d_2 \ d_3 \ (d_1+d_2+d_3) \ (d_1+d_2) \ (d_1+d_3)]$$

Message vectors	Code vectors
000	000000
001	001101
010	010110
011	011011
100	100111
101	101010
110	110001
111	111100

Encoding circuit



$$[H] = [P^T \vdots I_{n-k}]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$[S] = [R][H^T]$$

$$= [110011] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$E = E - R$
$= [000010] - [110011]$
$= [110001]$

$$\boxed{[S] = [0\ 1\ 0]} \quad \therefore [E] = [0\,0\,0\,0\,1\,0]$$

The syndrome vector $[S] = [010]$ is present in 5^{th} row of $[H^T]$ matrix & hence 5^{th} bit in received vector from left is in error.

$\therefore$ The corrected code vector is $[110001]$ which is a valid transmitted code vector.

SYNDROME CALCULATION CIRCUIT :-

Let the received vector $R = [r_1\ r_2\ r_3 \dots r_n]$. The syndrome vector $[S]$ is given by the equation

$$[S] = [S_1\ S_2\ S_3 \dots S_{n-K}] = RH^T$$

Substituting each value,

$$[S_1\ S_2\ S_3 \dots S_{n-K}] = [r_1\ r_2\ r_3 \dots r_K\ r_{K+1} \dots r_n] \begin{bmatrix} P_{11} & P_{12} & P_{13} & \dots & P_{1(n-K)} \\ P_{21} & P_{22} & P_{23} & \dots & P_{2(n-K)} \\ P_{31} & P_{32} & P_{33} & \dots & P_{3(n-K)} \\ \vdots & \vdots & \vdots & & \vdots \\ P_{K1} & P_{K2} & P_{K3} & \dots & P_{K(n-K)} \\ \hline 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & 0 \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

multiplying both the equation using modulo 2 arithmetic, syndrome bits are given by

$$S_1 = P_{11}r_1 + P_{21}r_2 + P_{31}r_3 + \dots + P_{K1}r_K + r_{K+1}$$

$$S_2 = P_{12}r_1 + P_{22}r_2 + P_{32}r_3 + \dots + P_{K2}r_K + r_{K+2}$$

$$S_3 = P_{13}r_1 + P_{23}r_2 + P_{33}r_3 + \dots + P_{K3}r_K + r_{K+3}$$

$$\vdots$$

$$S_{n-K} = P_{1(n-K)}r_1 + P_{2(n-K)}r_2 + P_{3(n-K)}r_3 + \dots + P_{K(n-K)}r_K + r_n$$

The above equations can be realized using the circuit shown next which is called SYNDROME CALCULATION CIRCUIT:

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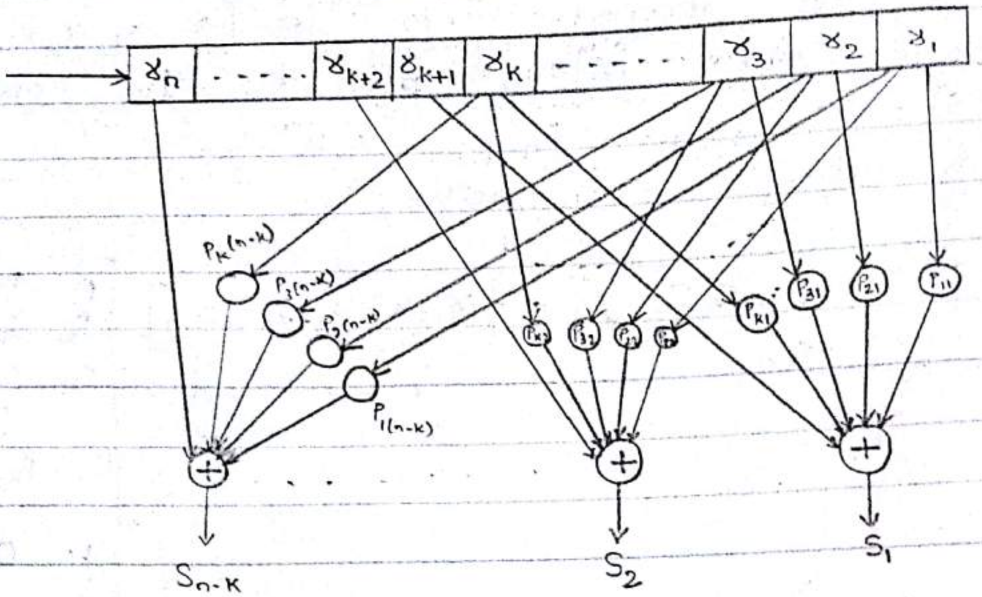


P) For a systematic (6,3) code, the received vector $R = [r_1\ r_2\ r_3\ r_4\ r_5\ r_6]$, construct the corresponding syndrome calculation circuit for $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

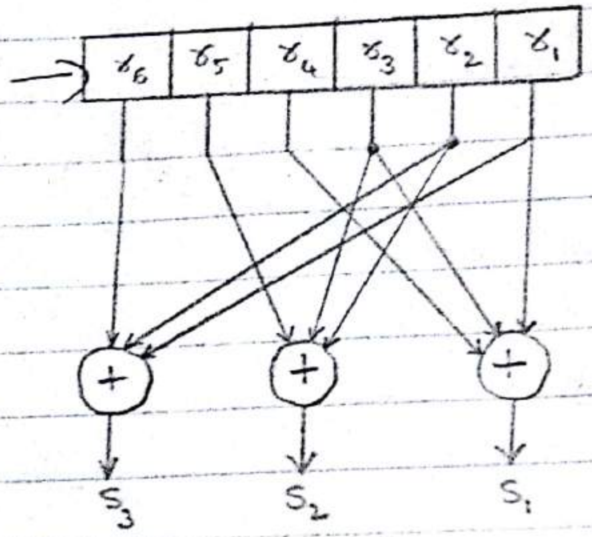
$$[H] = [P^T \,\vdots\, I_{n-k}]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$[S] = [R][H^T]$$

$$= [r_1\ r_2\ r_3\ r_4\ r_5\ r_6] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= [(r_1+r_3+r_4)\quad (r_2+r_3+r_5)\quad (r_1+r_2+r_6)]$$



) For a systematic (7,4) linear block code, the parity matrix is given by

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

a) Find all possible valid code vectors
b) Draw the corresponding encoding circuit
c) A single error has occurred in each of the received vectors. Detect & correct those errors

$R_A = [0111110]$

$R_B = [1011100]$

$R_C = [1010000]$

d) Draw the syndrome calculation circuit.

$$S = R\,H^T$$

$$H^T = \begin{bmatrix} P \\ - \\ I \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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