IIR FILTER [Analog]

-> Also called as all pole filter

-> designed from analog filter, because ~~once~~ in digital domain, processing all the infinite length h(n) is not possible

An analog filter can be described as

① $H_a(s) = \frac{B(s)}{A(s)} = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}$

② Through its impulse response

$H_a(s) = \int_{-\infty}^{+\infty} h(t).e^{-st} dt$

③ $\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \cdot \frac{d^k x(t)}{dt^k}$

The properties need to ~~be~~ satisfied for proper mapping of analog domain into digital domain.

① The $j\Omega$ axis in the s-plane must map onto the unit circle in the z-plane.

② The LHP of s-plane must map into inside of unit circle in the z-plane

-> Many times it's necessary to approximate the characteristics of an analog filter to implement a causal filter, otherwise the ~~exact~~ implementation of ideal characteristics will lead to a non causal filter.

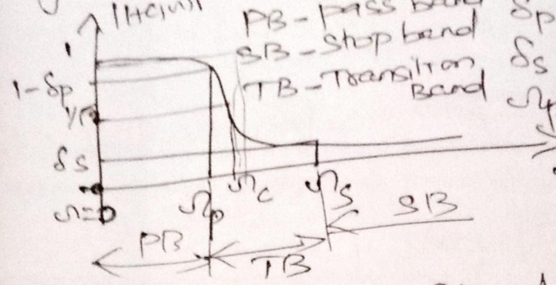
-> There are 3 different types of approximation

① Butterworth filter approximation

② Chebyshev filter approximation

③ Elliptic filter approximation.

mag. characteristics of BW filter approximation



δ_p - pass band ripple

δ_s - stop band ripple

Ω_p - PB edge frequency

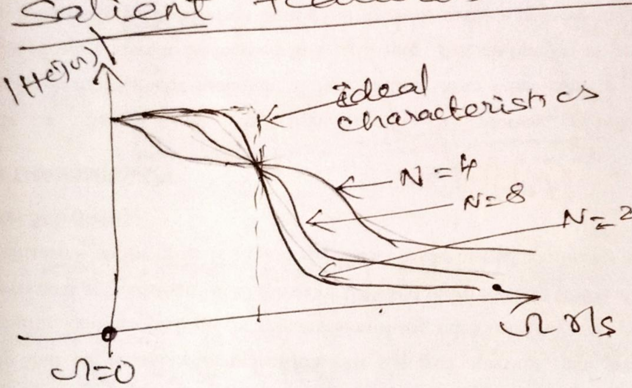
Ω_s - stop band edge freq

Ω_c - 3dB cut off freq

$20 \log(1-\delta_p) = A_p$ - passband gain in dB

$20 \log(\delta_s) = A_s$ - stopband attenuation in dB

Salient features of low pass Butterworth filter



From the characteristics it's clear that

(1) As N ↑ses, magnitude characteristics approaches ideal

(2) At $\Omega=0$, $|H(j\Omega)| = 1$

(3) Magnitude response is nearly constant (equal to 1) at lower frequencies ie PB is maximally flat.

(4) There are no ripples in the PB and SB.

(5) The magnitude response is monotonically decreasing

(6) As Ω tends to ∞, the magnitude approaches to zero.

(7) At $\Omega = \Omega_p$, $|H(j\Omega)| = A_p$ in dB or $(1-\delta_p)$, At $\Omega = \Omega_s$, $|H(j\Omega)| = A_s$ or δ_s

→ Designing of digital filter is an indirect procedure means, it's derived from an equivalent analog filter.

→ The analog normalised or prototype LPF is designed using the below designed steps & converted into a filter of any type and any frequency using analog to analog transformation.

Design steps for Analog normalised (or) prototype LPF

① Determination of order N

$$N \geq \frac{\log\left[\left(10^{0.1A_p}-1\right)\Big/\left(10^{0.1A_s}-1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

Derivation

We know that

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\Omega/\Omega_c\right)^{2N}}$$

(or)

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\Omega/\Omega_c\right)^{2N}}}$$

Taking 20 log on both sides

$$20\log_{10}|H(j\Omega)| = 20\log_{10}\left[\frac{1}{\sqrt{1+\left(\Omega/\Omega_c\right)^{2N}}}\right]$$

$$20\log_{10}|H(j\Omega)| = -20\log_{10}\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]^{1/2}$$

$$\boxed{20\log_{10}|H(j\Omega)| = -10\log_{10}\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right]}$$

We know that at $\Omega = \Omega_p$, $|H(j\Omega)| = A_p$

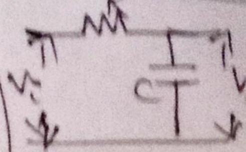
$$\Rightarrow A_p = -10\log_{10}\left[1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}\right]$$

$$\log_{10}\left[1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}\right] = \frac{-A_p}{10}$$

Taking antilog on both sides

$$1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = 10^{-0.1A_p}$$

$$\boxed{\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = 10^{-0.1A_p}-1} \longrightarrow ①$$



$V_i = I[R - jX_c]$

$V_o = I[jX_c]$

$\Rightarrow V_i = I\left[R+\frac{1}{j\Omega C}\right]$ & $V_o = \frac{I}{j\Omega C}$

$$H(j\Omega) = \frac{V_o}{V_{in}} = \frac{I/j\Omega C}{I\left[R+\frac{1}{j\Omega C}\right]} = \frac{I}{j\Omega C} \times \frac{j\Omega C}{I\,[Rj\Omega C+1]}$$

$$H(j\Omega) = \frac{1}{jRC\Omega+1} \qquad \boxed{\text{w.k.t } \Omega_c = \frac{1}{RC}}$$

$f_c = \frac{1}{2\pi RC} \Rightarrow \frac{1}{RC} = \Omega_c$

sub Ω_c

$$H(j\Omega) = \frac{1}{1+j\left(\frac{\Omega}{\Omega_c}\right)}$$

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\Omega/\Omega_c\right)^2}}$$

for order N

$$|H(j\Omega)| = \frac{1}{\sqrt{1+\left(\Omega/\Omega_c\right)^{2N}}}$$

Squared magnitude response

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\Omega/\Omega_c\right)^{2N}}$$

$V_o = \frac{I}{j\omega C}$

$V_{in} = I\left(R+\frac{1}{j\omega C}\right) = I\left(\frac{R\omega C j+1}{j\omega C}\right)$

$$\frac{V_o}{V_{in}} = \frac{1/j\omega C}{\frac{jR\omega C+1}{j\omega C}} = \frac{1}{1+jRC\omega} = \frac{1}{1+j\frac{\omega}{\omega_c}}$$

Also from figure magnitude characteristics

At $\Omega = \Omega_s$, $|H(j\Omega)| = A_s$ in dB or δ_s

$$20\log_{10}|H(j\Omega)| = 20\log_{10}\left(\frac{1}{\sqrt{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}}}\right)$$

$$\Rightarrow \boxed{\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = 10^{-0.1A_s} - 1} \longrightarrow (2)$$

(1) ÷ (2)

$$\Rightarrow \left(\frac{\Omega_p}{\Omega_s}\right)^{2N} = \frac{10^{-0.1A_p}-1}{10^{-0.1A_s}-1}$$

Taking log on both sides

$$2N\log\left(\frac{\Omega_p}{\Omega_s}\right) = \log\left[\frac{10^{-0.1A_p}-1}{10^{-0.1A_s}-1}\right]$$

solving for N

$$N \geq \frac{\log\left[\frac{10^{-0.1A_p}-1}{10^{-0.1A_s}-1}\right]}{2\log\left[\frac{\Omega_p}{\Omega_s}\right]}$$

The order of a filter always should be an integer. ∴ N is always rounded of to next highest integer

$$\Rightarrow \boxed{N \geq \frac{\log\left[(10^{-0.1A_p}-1)/(10^{-0.1A_s}-1)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}}$$

(2) <u>Determination of 3 dB cut off frequency Ω_c</u>

from eqn (1) $\Omega_{c1} = \dfrac{\Omega_p}{(10^{-0.1A_p}-1)^{1/2N}}$ or $\longrightarrow (3)$

from eqn (2) $\Omega_{c2} = \dfrac{\Omega_s}{(10^{-0.1A_s}-1)^{1/2N}}$ or $\longrightarrow (4)$

Ω_c can be calculated by taking average of above 2 eqns (3) & (4).

(3) Determination of poles

we know that $|H(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$

put $s = j\Omega$ $\Rightarrow \Omega = s/j$, $\Omega_c = 1 r/s$

$$|H(s)| \cdot |H(-s)| = \frac{1}{1+(s/j)^{2N}}$$

In the above TF, there are no finite zeros.
The poles of the above transfer function are determined by equaling the denominator term to zero.

$$\Rightarrow 1+\left(\frac{s}{j}\right)^{2N} = 0$$

$$\left(\frac{s}{j}\right)^{2N} = -1$$

$$\left(\frac{s}{j}\right) = (-1)^{1/2N}$$

$$s = j(-1)^{1/2N}$$

(-1) can be replaced with $e^{+j\pi(2k+1)}$
and j can be replaced with $j = e^{j\pi/2}$
$k = 0$ to $2N-1$

$$s = e^{j\pi/2} \cdot e^{\frac{j\pi(2k+1)}{2N}} ; \quad 0 \le k \le 2N-1$$

$$\boxed{s_k = e^{j\left(\pi/2 + \frac{\pi k}{N} + \frac{\pi}{2N}\right)} ; \quad 0 \le k \le 2N-1}$$

The poles of $H(s)$ and $H(-s)$ falls on the circle of unit radius and hence relatively phase difference of $\frac{\pi}{N}$ radians.

when N odd

$$s_k = e^{\frac{j\pi k}{N}} ; \quad k = 1,2,3 \ldots 2N$$

N even

$$s_k = e^{j\left(\frac{(2k-1)\pi}{2N}\right)} ; \quad k = 1,2,3, \ldots 2N$$

(4) Transfer function of a normalised LPF

$$H_{Nor}(s) \text{ or } H_{pro}(s) = \frac{1}{\prod_{k=0}^{N-1}(s-s_k)} \text{ or } \frac{1}{B_N(s)}$$

where s_k - poles, $B_N(s)$ - Butterworth polynomial

For N=1

WKT
$$s_k = e^{j(\pi/2 + \frac{\pi k}{N} + \frac{\pi}{2N})} \; ; \; 0 \le k \le 2N-1$$

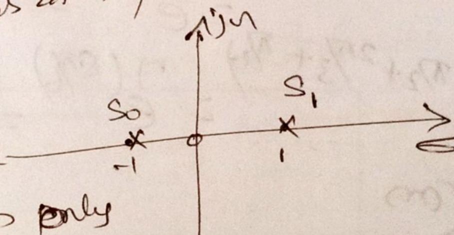
$\therefore$ k will take up values 0 & 1

k=0
$$s_0 = e^{j(\pi/2 + 0 + \frac{\pi}{2})} = e^{j\pi} = \cos\pi + j\cancel{\sin\pi}^{0} = -1$$

$$\boxed{s_0 = -1}$$

k=1
$$s_1 = e^{j(\pi/2 + \frac{\pi}{1} + \frac{\pi}{2})} = e^{j2\pi} = \cos 2\pi - j\cancel{\sin 2\pi}^{0} = 1$$

$$\boxed{s_1 = 1}$$



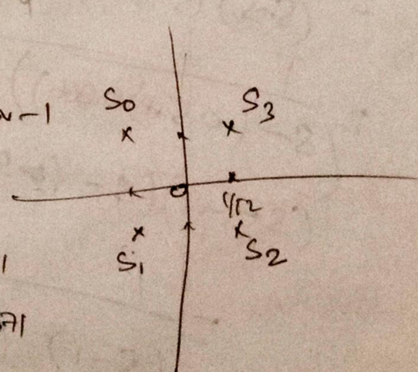
To implement a stable s/m, poles lying on LHP only need to be considered. $\therefore$ only s_0

$$H_{Nor}(s) = \frac{1}{s-s_0} = \frac{1}{s-(-1)} = \frac{1}{s+1}$$

$$\boxed{H_1(s) = \frac{1}{s+1}}$$

$\frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{4}$

N=2
$$s_k = e^{j(\pi/2 + \frac{\pi k}{N} + \frac{\pi}{2N})} \; ; \; 0 \le k \le 2N-1$$



k = 0, 1, 2, 3

$s_0 = e^{j(\pi/2 + 0 + \frac{\pi}{4})} = e^{j(3\pi/4)} = -0.7071 + j0.7071$

$s_1 = e^{j(\pi/2 + \pi/2 + \pi/4)} = e^{j(5\pi/4)} = -0.7071 - j0.7071$

$s_2 = e^{j(\pi/2 + \pi + \pi/4)} = e^{j(7\pi/4)} = 0.707 - j0.707$

$s_3 = e^{j(\pi/2 + 3\pi/2 + \pi/4)} = e^{j(9\pi/4)} = 0.707 + j0.707$

only S_0 & S_1 will lead to a stable s/m.

$$\therefore H_{nor}(s) = \frac{1}{(s-s_0)(s-s_1)} = \frac{1}{\left(s-\left(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}\right)\right)\left(s-\left(-\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)\right)}$$

for $N=2$

$$\boxed{H_{nor}(s) = \frac{1}{s^2+\sqrt{2}s+1}}$$

$$\left(s+\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}\right)\left(s+\frac{1}{\sqrt{2}}+j\left(\frac{1}{\sqrt{2}}\right)\right)$$

$$= (a-jb)(a+jb) = a^2+b^2$$

$$\Rightarrow \left(s+\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= s^2+1.41421s+1.$$

similarly for $N=3$

$$S_k = e^{j\left[\pi/2+\frac{\pi k}{N}+\pi/2N\right]}$$

k will take up the values 0 to 2N-1. Among the '2N' poles only 'N' poles lying on the LHP are considered to implement a stable s/m. Therefore in the above S_k eqn k ranges from 0 to N-1 can be considered to extract out only LHP poles.

Note: S_k can also be written as

$$S_k = e^{j\left(\pi/2+\frac{\pi k}{N}+\pi/2N\right)}; 0 \le k \le N-1$$ to extract out only LHP poles.

$$H_3(s) = \frac{1}{s^3+2s^2+2s+1}$$

$N=2$

$k=1,2$

$k=1$ $\quad e^{j(\pi/2+\pi/2+\pi/4)} = e^{j(\pi+\pi/4)} = e^{j5\pi/4}$

$k=2$ $\quad e^{j(\pi/2+\pi+\pi/4)} = e^{j(\pi+\pi/4)}$

$e^{j5\pi/4} = -0.707-j0.707$

$e^{j7\pi/4} = 0.707-j0.707.$

(5) Transforming normalized LPF into a filter of any type and required frequency

Normalized LPF →

LPF

$H_{Nor}(s)\Big|_{s \to s/\Omega_c'}$; where $\Omega_c' = \Omega_p, \Omega_c$ (given)

HPF

$H_{Nor}(s)\Big|_{s \to \frac{\Omega_c'}{s}}$; where $\Omega_c' = \frac{\Omega_p}{\Omega_c}$ (given)

BPF

$H_{Nor}(s)\Big|_{s \to \frac{s^2+\Omega_0^2}{B_0 s}}$

BSF

$H_{Nor}(s)\Big|_{s \to \frac{B_0 s}{s^2+\Omega_0^2}}$

$\Omega_0^2 = \Omega_L \Omega_U$

$B_0 = \Omega_U - \Omega_L$

(6) once Analog filter of any type and required frequency is obtained, by applying either a

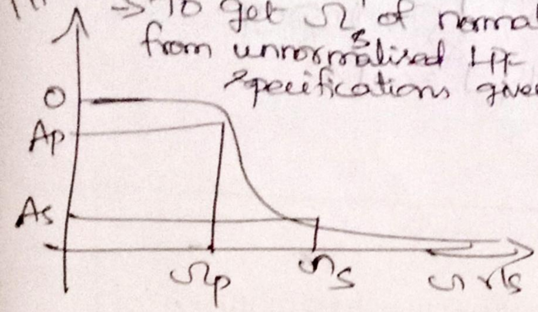
(1) Bilinear transformation (or)

(2) Impulse invariant transformation, a digital filter $H(z)$ can be obtained.

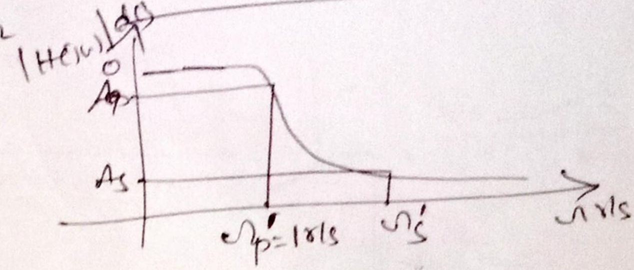
order N	Butterworth polynomial $B_N(s)$
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76536s+1)(s^2+1.84775s+1)$
5	$(s+1)(s^2+0.61805s+1)(s^2+1.61805s+1)$

Backward transformation

→ To get Ω_s' of normalised LPF from unnormalised LPF specifications given

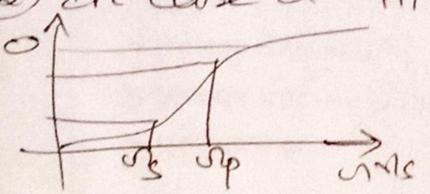


Normalised LPF



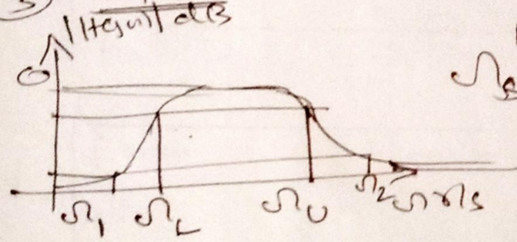
① To determine $\Omega_s' = \dfrac{\Omega_s}{\Omega_p}$

② In case of HPF



$$\Omega_s' = \frac{\Omega_p}{\Omega_s}$$

③ BPF

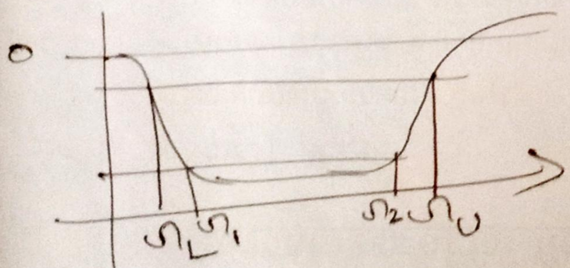


$$\Omega_s' = \min\{|A|, |B|\}$$

$$A = \frac{-\Omega_1^2 + \Omega_0^2}{B_0 \Omega_1} \; ; \; B = \frac{\Omega_2^2 - \Omega_0^2}{B_0 \Omega_2}$$

$$B_0 = \Omega_U - \Omega_L \; ; \; \Omega_0^2 = \Omega_L \Omega_U$$

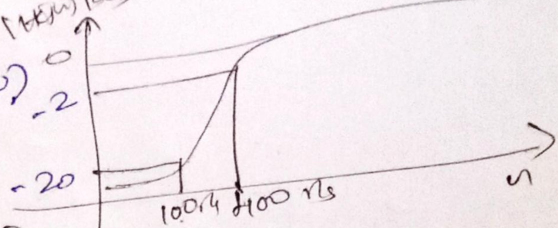
④ BSF



$$\Omega_s' = \min\{|A|, |B|\}$$

$$A = \frac{-\Omega_1^2 + \Omega_0^2}{B_0 \Omega_1} \; ; \; B = \frac{\Omega_2^2 - \Omega_0^2}{B_0 \Omega_2}$$

Design a High pass filter to satisfy foll. specifications

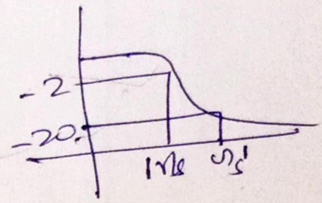


① Monotonic response (BW)

② $A_p = -2$dB at 400 r/s

③ $A_s = -20$dB at 100 r/s

To design a normalised LPF



① $\Omega_s' = \frac{\Omega_p}{\Omega_s} = \frac{400}{100} = 4$ r/s

② $N \geq \dfrac{\log\left[\left(10^{0.1A_p}-1\right)/\left(10^{0.1A_s}-1\right)\right]}{2\log\left(\frac{1}{4}\right)} = 2$

③ $\Omega_c = \dfrac{\Omega_p'}{\left(10^{-0.1A_p}-1\right)^{1/2N}} = \dfrac{1}{(0.5848)^{0.25}} = \cancel{1.36 \text{ r/s}}$ 1.14 r/s

(or) $\Omega_c = \dfrac{\Omega_p}{\left(10^{-0.1A_p}-1\right)^{1/2N}}$

④ $B_N(s) = s^2 + \sqrt{2}s + 1$

⑤ $H_N(s) = \dfrac{1}{s^2+\sqrt{2}s+1}$

⑥ To get HPF of 400 r/s

If $\Omega_c = \frac{\Omega_p}{(\,)^{1/2N}}$ is used then $\boxed{\Omega_c' = \frac{\Omega_p}{\Omega_c}}$

If $\Omega_c = \frac{\Omega_p}{(\,)^{1/2N}}$ then $s_n \to \frac{s}{\Omega_c}$

$H_a(s) = H_N(s)\Big|_{s \to \frac{\Omega_c'}{s}}$; $\Omega_c' = \frac{400}{1.14} = 350.877$

$H_a(s) = \dfrac{1}{\left(\frac{350.8}{s}\right)^2 + \sqrt{2}\left(\frac{350.8}{s}\right) + 1} = \dfrac{s^2}{(350.8)^2 + 496.1s + s^2}$

$$\boxed{H_a(s) = \frac{s^2}{(350.8)^2 + 496.1s + s^2}}$$

Verification of the design

$s = j\Omega$ $\quad \Omega = \Omega_p = 400$

$H_a(j\Omega_p) = \dfrac{-\Omega_p^2}{(350.8)^2 + j496.1\Omega_p - \Omega_p^2} = \dfrac{-(400)^2}{\sqrt{(36939.36)^2 + (198440)^2}} = \dfrac{-(400)^2}{\ldots + 3.939 \times 10^{10}}$

$= \dfrac{-(400)^2}{202322.80} = 0.79$

$\boxed{20\log(0.79) = -2.047\text{dB}}$ verified.

Design a BW Analog LPF & HPF filters for the foll. spec:

① $A_p = -2\,dB$ at $\Omega_p = 100\,r/s$

② $A_s = -20\,dB$ at $\Omega_s = 400\,r/s$

① $N = \dfrac{\log\left[10^{0.2}-1 \,/\, (10^{2}-1)\right]}{2\log\left(\frac{100}{400}\right)} = \dfrac{\log(0.5848/99)}{} = \dfrac{-2.228}{1.204} = 1.85$

$\boxed{N=2}$

② $\Omega_c = \dfrac{100}{(0.5848)^{1/4}} = \dfrac{100}{0.8744} = 114.36\,r/s$ (or) $\Omega_c = \dfrac{1}{(0.5848)^{1/4}} = 1.14\,r/s$

③ $H_N(s) = \dfrac{\cancel{1}}{s^2+\sqrt{2}s+1}$ ④ $H_N(s) = \dfrac{1}{s^2+\sqrt{2}s+1}$

⑤ $H_a(s) = H_N(s)\Big|_{s\to \frac{s}{\Omega_c}} = \dfrac{1}{\left(\frac{s}{114.36}\right)^2+\sqrt{2}\left(\frac{s}{114.36}\right)+1} = \dfrac{(114.36)^2}{s^2+161.72s+(114.36)^2}$

Verification

$s=j\Omega$ at $\Omega=\Omega_p$ $\;20\log|H(j\Omega)| = -2\,dB$

$|H(j\Omega_p)| = \dfrac{(114.36)^2}{-\Omega_p^2+j\,161.72\,\Omega_p+(114.36)^2} = \dfrac{(114.36)^2}{\sqrt{(3078.20)^2+(16770)^2}}$

$= \dfrac{(114.36)^2}{\sqrt{9475315.24+281232900}} = \dfrac{(114.36)^2}{\sqrt{271008899.2}}$

$|H(j\Omega)| = \dfrac{(114.36)^2}{\cancel{17050.1676}\;16462.347} = 0.767$

$20\log|H(j\Omega)|_{\Omega=100} = 20\log(0.767) = -2.3\,dB$

$-2.047\,dB$

Another method:

$\Omega_{ch} = 100\times 1.14 = 114.36\,r/s$

$P_k = \pm\Omega_c\, e^{j(\pi/2 + \frac{2k\pi}{2N} + \pi/2N)}$; $0\le k\le N-1$

$k=0,1$

$P_0 = \pm 114.36\, e^{j\left(\frac{\pi}{2}+0+\frac{\pi}{4}\right)} = e^{j(3\pi/4)} = -0.707+j0.707$

$P_0 = -80.86+j80.86$

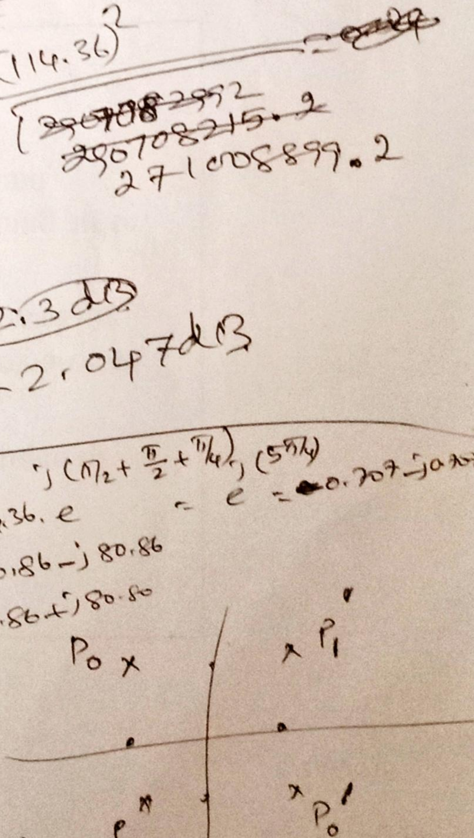
$P_0' = +80.86-j80.86$

$k=1$

$P_1 = \pm 114.36\, e^{j\left(\pi/2+\frac{\pi}{2}+\pi/4\right)} = e^{j(5\pi/4)} = -0.707-j0.707$

$= -80.86-j80.86$

$P_1' = +80.86+j80.86$



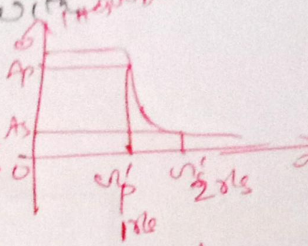
$(s+80.86-j80.86)$

$(s+80.86+j80.86)$

① Design an analog BW LPF with

① $A_p = -2dB$, $\Omega_p = 100 r/s$

② $A_s = -20dB$, $\Omega_s = 200 r/s$



$\Omega_s' = \frac{\Omega_s}{\Omega_p}$

$\Omega_s' = \frac{200}{100} = 2 r/s$

① $N = 4$

② $\Omega_c = \frac{1}{\left(10^{-0.1(-2)} - 1\right)^{1/8}} = 1.06906 \, r/s$

③ $B_N(s) = (s^2 + 0.7653s + 1)(s^2 + 1.8477s + 1)$

④ $H_N(s) = \frac{1}{s^4 + 1.8477s^3 + s^2 + 0.76533s^3 + 1.414s^2 + 0.7653s + s^2 + 1.8477s + 1}$

$H_N(s) = \frac{1}{s^4 + 1.8477s^3 + 4.1793s^2 + 2.613s + 1}$

$H_a(s) = H_N(s) \Big|_{s \to \frac{s}{\Omega_{ch}}}$

For LPF

$\boxed{\Omega_{ch} = \Omega_c \cdot \Omega_p} = 106.906 \, r/s$ which is same as $\Omega_c = \frac{\Omega_p}{\left(10^{0.2} - 1\right)^{1/8}}$

$= 106.93 \, r/s$

* In the case of ~~HPF~~ to get $\boxed{\Omega_{ch} = \frac{\Omega_p}{\Omega_c}}$

$H_a(s) = \frac{1}{\left(\frac{s}{106.9}\right)^4 + 1.8477\left(\frac{s}{106.9}\right)^3 + 4.179\left(\frac{s}{106.9}\right)^2 + 2.613\left(\frac{s}{106.9}\right) + 1}$

$H_a(s) = \frac{(106.9)^4}{s^4 + 197.5s^3 + 47755.9s^2 + 3192070.87s + 130590270.3}$

verification: $s = j\Omega$ at $\Omega = 100$ $|H(j\Omega)| = -2dB$

$= \frac{(106.9)^4}{(\Omega_p)^4 - j197.5\Omega_p^3 + 47755.9(\Omega_p)^2 + j3192070.87\Omega_p + 130590270.3}$

$|H(j\Omega_p)| = \frac{(106.9)^4}{\sqrt{\left(\cancel{[illegible]}(100)^4 + 47755.9(100)^2 + 130590270.3\right) + j\left(319207087 - 197500000\right)}}$

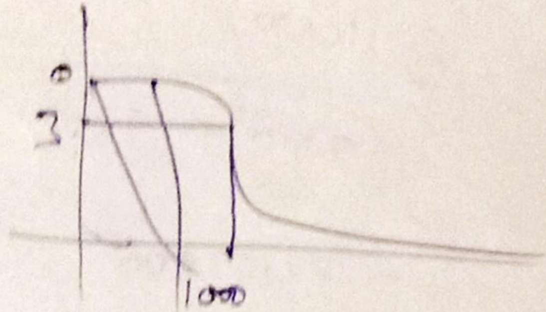
$= \frac{(106.9)^4}{\ldots + (121707087)^2} = \frac{(106.94)^4}{5.014\times10^{17} + 1.48126\times10^{16}}$

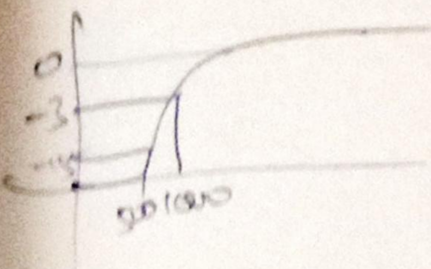
$= \frac{(106.94)^4}{\sqrt{5.1628\times10^{17}}} = \frac{(106.94)^4}{\cancel{7185328288}\ 2.27\times10^{17}} = \frac{\cancel{0.18}}{5.2\times10^{24}}$

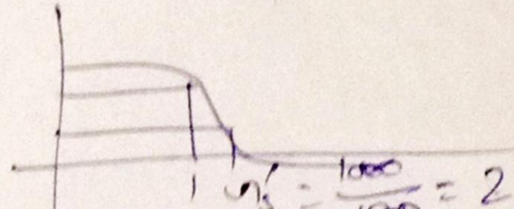
$20\log(0.18)$

$\alpha_p = 3dB$, $\Omega_p = 1000 r/s$

$\alpha_s = 15dB$ $\Omega_s = 500 r/s$







$\Omega_s' = \frac{1000}{500} = 2$

$$N \geq \frac{\log\left(\left(10^{0.3}-1\right)\left/\left(10^{1.5}-1\right)\right.\right)}{2\log(1/2)} = \frac{\log\left((2-1)/(31.62-1)\right)}{2(-0.30)} = \frac{-1.52}{-0.60} = 2.53$$

$N = 3$

$$H_n(s) = \frac{1}{(s_n+1)(s_n^2+s_n+1)} \quad ; \quad \Omega_c = \frac{1000}{(2-1)^{1/6}} = 1000 r/s$$

$$H_a(s) = \frac{1}{s_n^3 + 2s_n^2 + 2s_n + 1} = \frac{1}{\left(\frac{1000}{s}\right)^3 + 2\left(\frac{1000}{s}\right)^2 + 2\left(\frac{1000}{s}\right) + 1}$$

$$H_a(s) = \frac{s^3}{(1000)^3 + 2000000 s + 2000 s^2 + s^3}$$

$$H_a(j\Omega) = \frac{(j\Omega)^3}{(1000)^3 + 2000000 j\Omega + 2000 (j\Omega)^2 + (j\Omega)^3}$$

$$= \frac{-j\Omega^3}{(1000)^3 + j2000000\Omega - 2000\Omega^2 - j\Omega^3}$$

$$= \frac{-j(1000)^3}{\ldots - 2000000000 - j(1000)^3}$$