Floating Point Numbers

DIGITAL SYSTEM DESIGN USING VERILOG
Course Code: 19EC5DCDSV
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MODULE-4C

Faculty In-Charge: Dr. Dinesha P

drdinesh-ece@dayanandasagar.edu

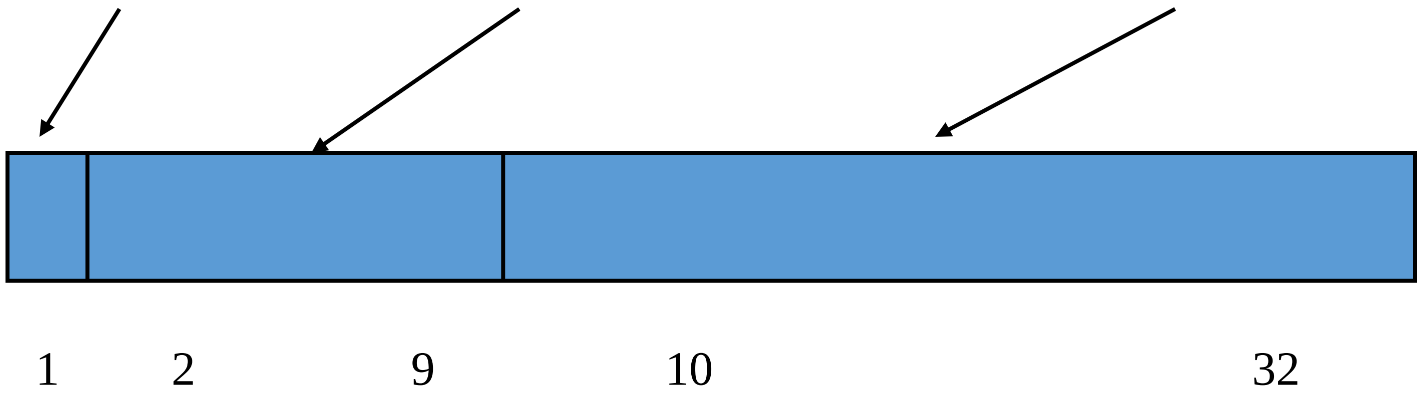
Problem storing binary form

- We have no way to store the radix point!
- Standards committee came up with a way to store floating point numbers (that have a decimal point)

IEEE Floating Point Representation

- Floating point numbers can be stored into 32-bits, by dividing the bits into three parts:

the **sign**, the **exponent**, and the **mantissa.**



IEEE Floating Point Representation

- The first (leftmost) field of our floating point representation will STILL be the sign bit:
  - 0 for a positive number,
  - 1 for a negative number.

IEEE Floating-Point Format

	e bits	*m* bits
s	exponent	mantissa

$$x = M \times 2^{E} = (-1 \times s) \times 1.mantissa \times 2^{exponent - 2^{e-1} + 1}$$

- s: sign bit (0 $\Rightarrow$ non-negative, 1 $\Rightarrow$ negative)
- Normalize: $1.0 \leq |M| < 2.0$
  - M always has a leading pre-binary-point 1 bit, so no need to represent it explicitly (*hidden bit*)
- Exponent: excess representation: $E + 2^{e-1} - 1$

Storing the Binary Form

How do we store a radix point?

- All we have are zeros and ones…

Make sure that the radix point is ALWAYS in the same position within the number.

Use the IEEE 32-bit standard

→ the **leftmost** digit must be a 1

Solution is Normalization

Every binary number, **except the one corresponding to the number zero**, can be normalized by choosing the exponent so that the radix point falls to the right of the leftmost 1 bit.

$37.25_{10} = 100101.01_2 = 1.0010101 \times \mathbf{2^5}$

$7.625_{10} = 111.101_2 = 1.11101 \times \mathbf{2^2}$

$0.3125_{10} = 0.0101_2 = 1.01 \times \mathbf{2^{-2}}$

IEEE Floating Point Representation

- The second field of the floating point number will be the **exponent**.
- The exponent is stored as an unsigned 8-bit number, RELATIVE to a **bias of 127.**
  - Exponent 5 is stored as (127 + 5) or 132
    - 132 = 10000100
  - Exponent -5 is stored as (127 + (-5)) or 122
    - 122 = 01111010

Try It Yourself

How would the following exponents be stored (8-bits, 127-biased):

2^{-10}

2^{8}

Answers

2^{-10}

exponent	-10		8-bit
bias	+127	value	
	117	→	01110101

2^{8}

exponent	8		8-bit
bias	+127	value	
	135	→	10000111

IEEE Floating Point Representation

- The **mantissa** is the set of 0's and 1's to the right of the radix point of the **normalized** (when the digit to the left of the radix point is 1) binary number.

  Ex: $1.\mathbf{00101} \times 2^3$

  (The mantissa is 00101)

- The mantissa is stored in a 23 bit field, so we add zeros to the right side and store:

  00101000000000000000000

Decimal Floating Point to IEEE standard Conversion

Ex 1: Find the IEEE FP representation of 40.15625

Step 1.

Compute the binary equivalent of the whole part and the fractional part. (i.e. convert 40 and .15625 to their binary equivalents)

Decimal Floating Point to IEEE standard Conversion

$$\begin{array}{r} 40 \\ -\ 32 \\ \hline 8 \\ -\ \ 8 \\ \hline 0 \end{array}$$	**Result:** 101000	$$\begin{array}{r} .15625 \\ -.12500 \\ \hline .03125 \\ -.03125 \\ \hline .0 \end{array}$$	**Result:** .00101

So: $40.15625_{10} = 101000.00101_2$

Decimal Floating Point to IEEE standard Conversion

Step 2. Normalize the number by moving the decimal point to the right of the leftmost one.

$$101000.00101 = 1.0100000101 \times \mathbf{2}^{5}$$

Decimal Floating Point to IEEE standard Conversion

Step 3. Convert the exponent to a biased exponent

$$127 + 5 = 132$$

And convert biased exponent to 8-bit unsigned binary:

$$132_{10} = 10000100_2$$

Decimal Floating Point to IEEE standard Conversion

Step 4. Store the results from steps 1-3:

Sign	Exponent (from step 3)	Mantissa (from step 2)
0	**10000100**	**01000001010000000000000**

Decimal Floating Point to IEEE standard Conversion

Ex 2: Find the IEEE FP representation of **–24.75**

Step 1. Compute the binary equivalent of the whole part and the fractional part.

24			**.75**	
- 16	**Result:**	**- .50**	**Result:**	
8	**11000**	**.25**	**.11**	
- 8		**- .25**		
0		**.0**		

So: $-24.75_{10} = -11000.11_2$

Decimal Floating Point to IEEE standard Conversion

Step 2.

Normalize the number by moving the decimal point to the right of the leftmost one.

$$-11000.11 = -1.100011 \times 2^4$$

Decimal Floating Point to IEEE standard Conversion.

Step 3. Convert the exponent to a biased exponent

$$127 + 4 = 131$$

$$\Longrightarrow \quad 131_{10} = 10000011_2$$

Step 4. Store the results from steps 1-3

Sign	Exponent	mantissa
1	**10000011**	**1000110..0**

IEEE standard to Decimal Floating Point Conversion.

- Do the steps in reverse order

- In reversing the normalization step move the radix point the number of digits equal to the exponent:
  - If exponent is positive, move to the right
  - If exponent is negative, move to the left

IEEE standard to Decimal Floating Point Conversion.

Ex 2: Convert the following 32 bit binary number to its decimal floating point equivalent:

Sign	Exponent	Mantissa
0	10000011	10011000..0

IEEE standard to Decimal Floating Point Conversion..

Step 1: Extract the biased exponent and unbias it

Biased exponent = 1000011_2 = 131_{10}

Unbiased Exponent: 131 – 127 = 4

IEEE standard to Decimal Floating Point Conversion..

Step 2: Write Normalized number in the form:

Exponent

1 . ___Mantissa___ x 2 ----

For our number:

1.10011×2^{4}

IEEE standard to Decimal Floating Point Conversion.

Step 3: Denormalize the binary number from step 2 (i.e. move the decimal and get rid of (x 2^n) part:

11001.1_2 (positive exponent – move right)

Step 4: Convert binary number to the FP equivalent (i.e. Add all column values with 1s in them)

11001.1 = 16 + 8 + 1 +.5

= 25.5_{10}

IEEE standard to Decimal Floating Point Conversion.

Ex 1: Convert the following 32-bit binary number to its decimal floating point equivalent:

Sign	Exponent	Mantissa
1	01111101	010..0

IEEE standard to Decimal Floating Point Conversion..

Step 1: Extract the biased exponent and unbias it

Biased exponent = 01111101_2 = 125_{10}

Unbiased Exponent: 125 – 127 = -2

IEEE standard to Decimal Floating Point Conversion..

Step 2: Write Normalized number in the form:

$$1.\underline{\qquad\qquad\qquad} \times 2^{----}$$

Exponent

Mantissa

For our number:

-1.01×2^{-2}

IEEE standard to Decimal Floating Point Conversion.

Step 3: Denormalize the binary number from step 2 (i.e. move the decimal and get rid of ($x\ 2^n$) part):

-0.0101_2 (negative exponent – move left)

Step 4: Convert binary number to the FP equivalent (i.e. Add all column values with 1s in them)

$$-0.0101_2 = -(0.25 + 0.0625)$$

$$= -0.3125_{10}$$

Floating-Point Range

- Exponents 000...0 and 111...1 reserved
- Smallest value
  - exponent: 000...01 $\Rightarrow E = -2^{e-1} + 2$
  - mantissa: 0000...00 $\Rightarrow M = 1.0$
- Largest value
  - exponent: 111...10 $\Rightarrow E = 2^{e-1} - 1$
  - mantissa: 111...11 $\Rightarrow M \approx 2.0$
- Range:

$$2^{-2^{e-1}+2} \le |x| < 2^{2^{e-1}}$$

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Floating-Point Precision

- Relative precision approximately 2^{-m}
  - all mantissa bits are significant
- m bits of precision
  - $m \times \log_{10} 2 \approx m \times 0.3$ decimal digits

Example Formats

- IEEE single precision, 32 bits
  - e = 8, m = 23
  - range ≈ $\pm 1.2 \times 10^{-38}$ to $\pm 1.7 \times 10^{38}$
  - precision ≈ 7 decimal digits
- Application-specific, 22 bits
  - e = 5, m = 16
  - range ≈ $\pm 6.1 \times 10^{-5}$ to $\pm 6.6 \times 10^{4}$
  - precision ≈ 5 decimal digits

Floating-Point Operations

- Considerably more complicated than integer operations
  - E.g., addition
    - unpack, align binary points, adjust exponents
    - add mantissas, check for exceptions
    - round and normalize result, adjust exponent
- Combinational circuits not feasible
  - Pipelined sequential circuits