FIR filter

→ Finite impulse response filter

→ will have linear phase

We know that $H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; |\omega| \leq \omega_c \\ 0 & ; \omega_c \leq |\omega| \leq \pi \end{cases}$

If the sym is LTI then for i/p $x(n)$ & impulse response $h(n)$, the o/p $y(n)$ is given by

$$y(n) = x(n) * h(n)$$

Taking DTFT on both side

$$\text{DTFT}\{y(n)\} = Y(\omega) = X(\omega) \cdot H(\omega) \quad \text{Sub } H(\omega) \text{ from above eqn}$$

$$Y(\omega) = \begin{cases} e^{-j\omega\alpha} \cdot X(\omega) & ; |\omega| \leq \omega_c \\ 0 & ; \omega_c \leq |\omega| \leq \pi \end{cases}$$

Taking inverse DTFT

$$y(n) = x(n-\alpha)$$ → from this eqn it's clear that

the o/p $y(n)$ is a delayed/shifted version of i/p $x(n)$, it's not distorted ∴ phase expected is a means that linear.

→ From Paley-Wiener theorem, we can say that $|H_d(\omega)|$ can be zero at some frequencies, but it cannot be zero over any finite band of frequencies

→ The necessary and sufficient condition for a FIR filter to have a linear phase is to have a finite duration and

① N must have a finite duration

② $h(n) = +h(N-1-n)$; $n = 0, 1, \ldots N-1$ — Symmetric

$= -h(N-1-n)$; $n = 0, 1, \ldots N-1$ — Asymmetric

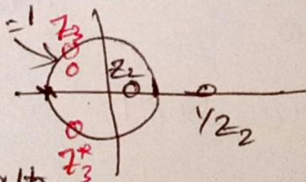
→ Location of zeros of linear phase FIR filters:

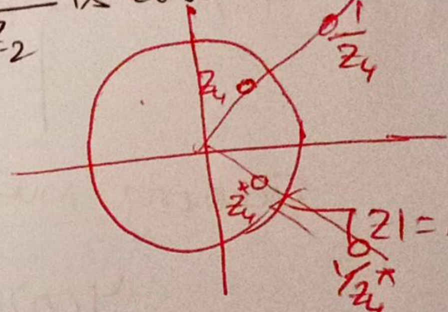
→ roots of the polynomial $H(z)$ are identical to the roots of the polynomial $H(z^{-1})$.

→ Zeros of $H(z)$ must occur in reciprocal pairs
ie if z_1 is a zero, then $\frac{1}{z_1}$ is also a zero

① If $z_1 = -1$, then $\frac{1}{z_1} = -1$

② If z_2 is a real zero with $|z_2| < 1$, then $\frac{1}{z_2}$ is also a zero
thus there are 2 zeros





③ If z_3 is a complex zero with $|z_3| = 1$ then $\frac{1}{z_3} = z_3^*$, 2 zeros

④ If z_4 is a complex zero with $|z_4| \neq 1$, then we have 4 zeros
$z_4, \frac{1}{z_4}, z_4^*$ and $\frac{1}{z_4^*}$

FIR filter

Design a linear phase LP FIR filter using rectangular window by taking 7 samples of window sequence and with a cut off freq $\omega_c = 0.2\pi$ r/s

* chosen symmetric impulse response ie $h(n) = h(N-1-n)$

WKT, the ideal freq. response of LP FIR filter is

$$\boxed{\begin{aligned} H_d(e^{j\omega}) &= e^{-j\alpha\omega} \quad ; -\omega_c \le \omega \le \omega_c \\ &= 0 \quad \text{otherwise} \end{aligned}} \Rightarrow \boxed{h_d(n) = \begin{cases} \dfrac{\sin \omega_c (n-\alpha)}{\pi(n-\alpha)} & ; n \ne \alpha \\ \dfrac{\omega_c}{\pi} & ; n = \alpha \end{cases}}$$

where $0 \le n \le N-1$

& $\alpha = \frac{N-1}{2} = 3$

The impulse response of the given filter specification $\omega_c = 0.2\pi$ r/s

$$h_d(0) = \frac{\sin \omega_c (n-3)}{\pi(n-3)} = 0.1009$$

$$h_d(n) = \{0.1009, 0.1514, 0.1871, 0.2, 0.1871, 0.1514, 0.1009\}$$

→ The impulse response of the FIR filter is det. by

$h(n) = h_d(n) . W_R(n)$

$h(n)$: $H(\omega)$ $H(z)$

Since $W_R(n) = 1$ for $0 \le n \le N-1$

$h(n) = h_d(n)$; $0 \le n \le N-1$

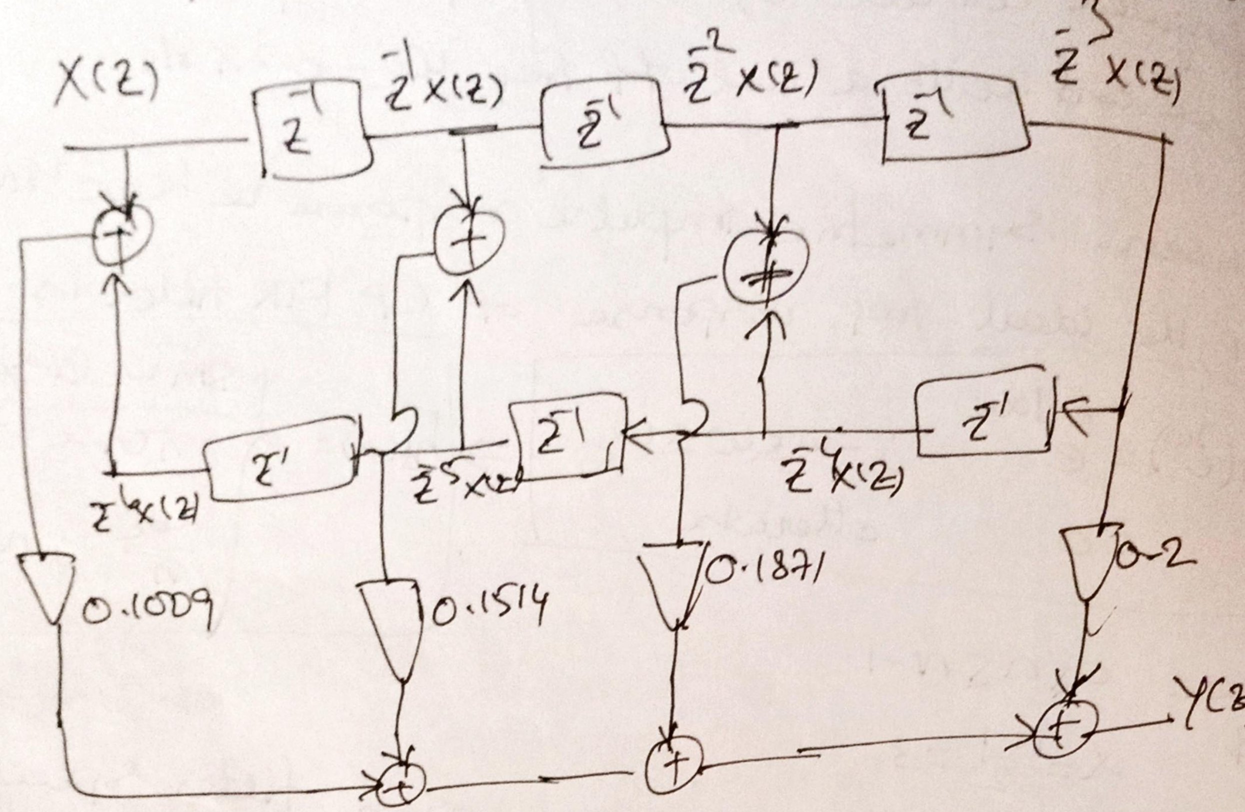
$$\Rightarrow \boxed{h(n) = \{0.1009, 0.1514, 0.1871, 0.2, 0.1871, 0.1514, 0.1009\}}$$

Transfer function of FIR filter $H(z) = Z\{h(n)\} = 0.1009 + 0.1514 z^{-1} + 0.1871 z^{-2} + 0.2 z^{-3} + 0.1871 z^{-4} + 0.1514 z^{-5} + 0.1009 z^{-6}$

$$\frac{Y(z)}{X(z)} = 0.1009[1 + z^{-6}] + 0.1514[z^{-1} + z^{-5}] + 0.1871[z^{-2} + z^{-4}] + 0.2 z^{-3}$$

$$Y(z) = X(z)\, 0.1009\,(1 + z^{-6}) + X(z)\, 0.1514\,(z^{-1} + z^{-5}) + X(z)\, 0.1871\,(z^{-2} + z^{-4}) + X(z)\, 0.2 z^{-3}$$

Corresponding Structure is



Hamming window fn

N = 7

LPF

Hamming window

$\omega_c = 0.2\pi$ r/s

$H_d(e^{j\omega}) = e^{-j\alpha\omega}$; $\omega \le \omega \le \omega_c$

$= 0$ other

$$\boxed{h_d(n) = \frac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)}, \; n \neq \alpha \qquad = \frac{\omega_c}{\pi}; \; n = \alpha}$$

$w_{hamm}(n) = 0.54 - 0.46\cos\left(\frac{n\pi}{\alpha}\right)$

$w_{ham}(0) = 0.08$

$\boxed{w_{ham}(n) = \{0.08, 0.31, 0.76, 1, 0.76, 0.311, 0.08}$

$h(2) = 0.1871 \times 0.76 = 0.1421$

$h_d(3) = 0.2 \times 1 = 0.2$

$h(4) = 0.1871 \times 0.76 = 0.1421$

$h_d(n) = \{0.1009, 0.1574, 0.1871, 0.2, 0.1871, 0.1574, 0.1009\}$

$h(n) = h_d(n) \cdot w_{Hamming}(n)$

$h(0) = 0.1009 \times 0.08 = 0.00807$

$h(1) = 0.1574 \times 0.31 = 0.04879$

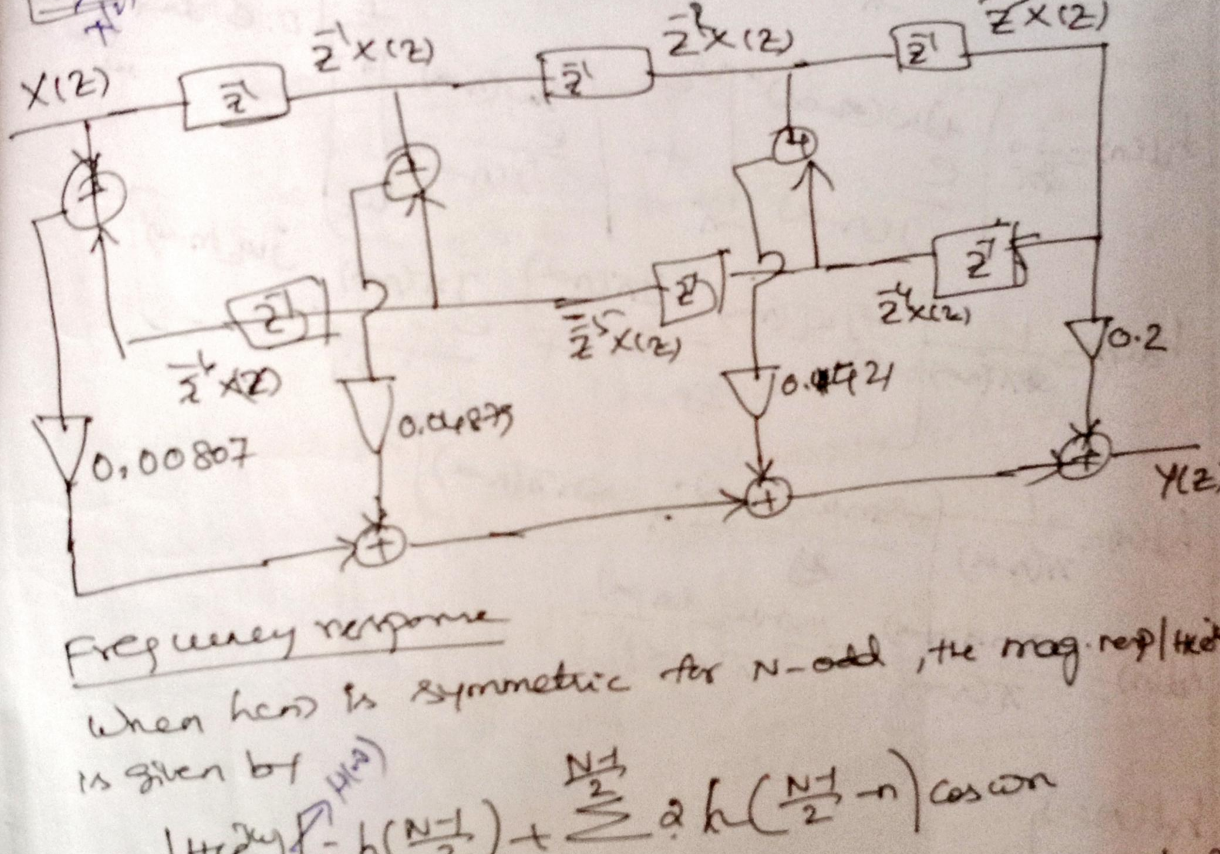
$\Rightarrow h(n) = \{0.00807, 0.04879, 0.1421, 0.2, 0.1421, 0.04879, 0.00807\}$

ransfer function of FIR filter is

$$H(z) = \{0.00807 + 0.04879 z^{-1} + 0.1421 z^{-2} + 0.2 z^{-3} + 0.1421 z^{-4} + 0.4879 z^{-5} + 0.00807 z^{-6}\}$$

$$\boxed{\begin{aligned} Y(z) = X(z)\, 0.00807 [1 + z^{-6}] &+ X(z).0.04879[z^{-1} + z^{-5}] \\ &+ X(z).0.1421[z^{-2} + z^{-4}] + X(z)[0.2 z^{-3}] \end{aligned}}$$

$\frac{Y(z)}{X(z)} = H(z)$



Frequency response

When h(n) is symmetric for N-odd, the mag. resp $|H(e^{j\omega})|$ is given by

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right)\cos \omega n$$

$$|H(e^{j\omega})| = h(3) + 2h(2)\cos\omega + 2h(1)\cos 2\omega + 2h(0)\cos 3\omega$$

$\cancel{|H(e^{j\omega})| = 0.2 + 2(0.1871)\cos\omega + 2(0.1574.}$

$$|H(e^{j\omega})| = 0.2 + 2(0.1421)\cos\omega + 2(0.04879)\cos 2\omega + 2(0.00807)\cos 3\omega$$

$$|H(e^{j\omega})| = 0.2 + 0.02842\cos\omega + 0.0975\cos 2\omega + 0.01614\cos 3\omega$$

$$z = e^{j\omega}$$

Design a linear phase FIR HPF using Rect. window with a cut off freq $\omega_c = 0.8\pi$ r/sample $N = 7$

WKT

$$H_d(e^{j\omega}) = e^{-j\alpha\omega} \; ; \quad \omega_c \le |\omega| \le \pi$$

$$= 0 \quad \text{otherwise}$$

$$\Rightarrow h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{-j\alpha\omega} \cdot e^{j\omega n}\, d\omega + \int_{-\omega_c}^{\omega_c} 0 \cdot e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} e^{-j\alpha\omega} \cdot e^{j\omega n}\, d\omega\right]$$

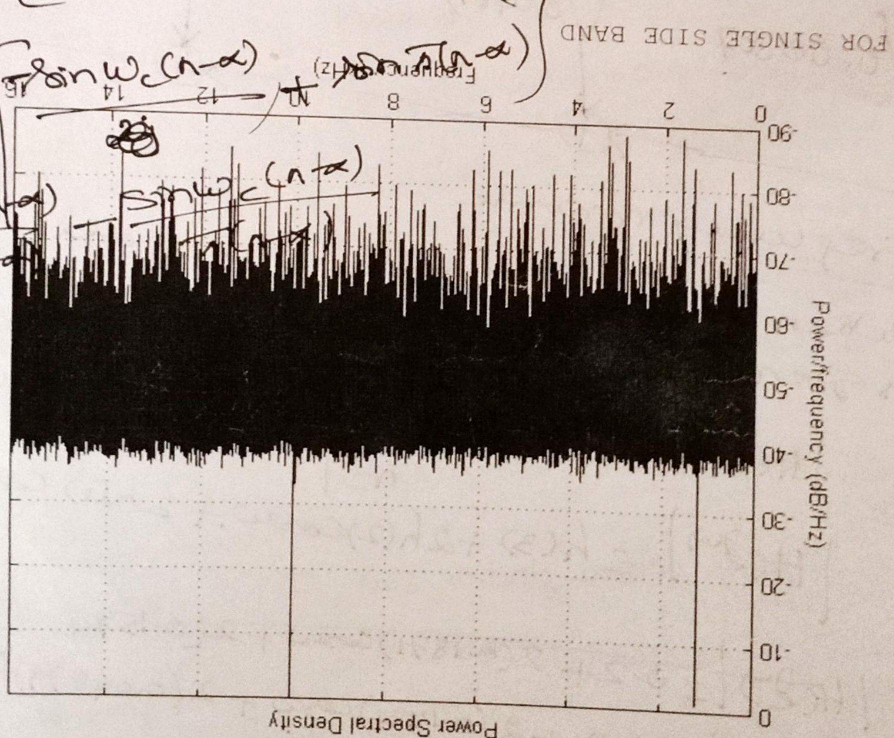
$$h_d(n) = \frac{1}{2\pi}\left[\left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\pi}^{-\omega_c} + \left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{\omega_c}^{\pi}\right]$$

$$h_d(n) = \frac{1}{\pi(n-\alpha)}\left[\frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)}}{2j} + \frac{e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{2j}\right]$$

$$h_d(n) = \frac{1}{\pi(n-\alpha)}\left[-\sin\omega_c(n-\alpha) + \sin\pi(n-\alpha)\right]$$

$$h_d(n) = \frac{\sin\pi(n-\alpha)}{\pi(n-\alpha)} - \frac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)}$$

$h_d(n) = \{$
-0.1
0.15
-0.187
0.2
-0.187
0.15
-0.1 $\}$



$$\boxed{\begin{aligned} h_d(n) &= \frac{\sin\pi(n-\alpha)}{\pi(n-\alpha)} - \frac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)} \quad n\neq\alpha \\ &= 1-\left(\frac{\omega_c}{\pi}\right) \quad ; n=\alpha \end{aligned}}$$

$\alpha = \frac{N-1}{2}$

$\boxed{\alpha = 3}$

$-\cancel{4.77}\times10^{-3}(0.94981) - 0.1008$

$$h_d(0) = \frac{\cancel{4.77} - 0.878}{-9.42} = \cancel{+0.093}$$

$\omega_c = 2.512$ r/s

$\sin\pi(-3) = 0$

$\sin\pi(-2) = 0$

$\sin\pi = 0$

$$h_d(1) = \frac{-\cancel{0.003185} - 0.9518}{-\cancel{6.28}} = \cancel{0.1010}\ 0.1525$$

$$h_d(2) = \frac{\cancel{0.001592} + 0.5888}{\cancel{9.42}\ 3.14} = \cancel{0.0626}\ 0.1875$$

$$h_d(3) = 1-\left(\frac{2.512}{3.14}\right) = 0.2$$

$w_{hamm}(0) = 0.08$

$w_{hamm}(1) = 0.309$

$w_{ham}(2) = 0.31$

$w_{hamm}(3) = 1$

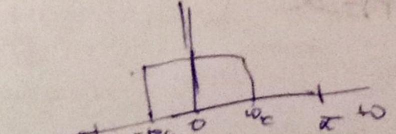
$w_{hamm}(4) = 0.77$

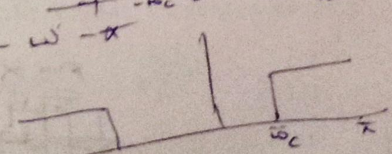
$h(0) = 0.008104$

$h(1) = \cancel{0.03107}\ 0.04712$

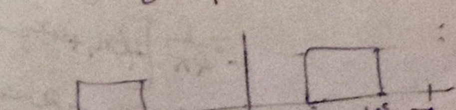
$h(2) = 0.019$

$h(3) = 0.2$





$0 < |\omega| < \omega_c$



$$\int^{-\omega_{c_1}} + \int_{-\omega_{c_2}}^{-\pi}$$

$$F(\omega) = \int_{-\infty}^{\omega} S_{xx}(\omega') \, d\omega'$$

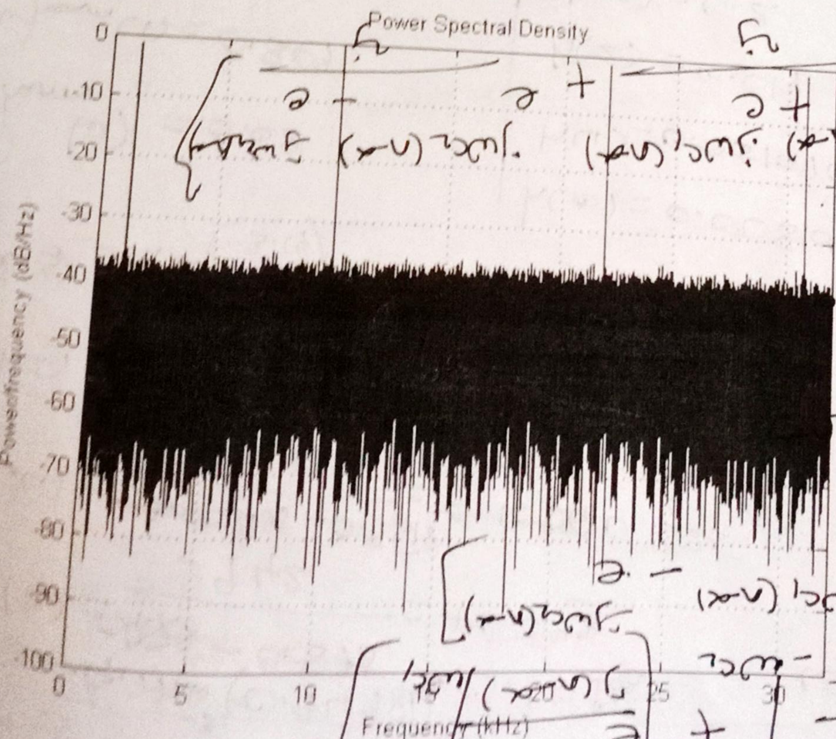
MATLAB CODE

```
Fs = 32e3;
t = 0:1/Fs:2.96;
x = sin(2*pi*t*1.24e3)+ sin(2*pi*t*10e3)+ randn(size(t));
nfft = 2^nextpow2(length(x));
Pxx = abs(fft(x,nfft)).^2/length(x)/Fs;

% Create a single-sided spectrum
Hpsd = dspdata.psd(Pxx(1:length(Pxx)/2),'Fs',Fs);
plot(Hpsd)
```

FOR DOUBLESIDE BAND

```
% Create a double-sided spectrum
Hpsd = dspdata.psd(Pxx,'Fs',Fs,'SpectrumType','twosided');
plot(Hpsd)
```



r double side band

$$H_d(e^{j\omega}) = e^{-j\alpha\omega} \quad \omega_{c1} \le |\omega| \le \omega_{c2}$$
$$= 0 \quad \text{ow}$$

$$h_d(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n} d\omega\right] = \frac{1}{2\pi}\left[\int_{-\omega_{c2}}^{-\omega_{c1}} e^{-j\alpha\omega} \cdot e^{j\omega n} d\omega + \int_{\omega_{c1}}^{\omega_{c2}} e^{-j\alpha\omega} \cdot e^{j\omega n} d\omega\right]$$

$$= \frac{1}{2\pi}\left[\left|\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\omega_{c2}}^{-\omega_{c1}} + \left|\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{\omega_{c1}}^{\omega_{c2}}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{j(n-\alpha)}\left[e^{-j\omega_{c1}(n-\alpha)} - e^{-j\omega_{c2}(n-\alpha)}\right] + \frac{1}{j(n-\alpha)}\left[e^{j\omega_{c2}(n-\alpha)} - e^{j\omega_{c1}(n-\alpha)}\right]\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{-e^{j\omega_{c1}(n-\alpha)} + e^{-j\omega_{c1}(n-\alpha)} + e^{j\omega_{c2}(n-\alpha)} - e^{-j\omega_{c2}(n-\alpha)}}{2j}\right]$$

$$= \frac{\sin\omega_{c2}(n-\alpha) - \sin\omega_{c1}(n-\alpha)}{\pi(n-\alpha)} \quad n \ne \alpha$$

$$= \frac{\omega_{c2}-\omega_{c1}}{\pi} \; ; \; n=\alpha$$

$$n=\alpha: \quad \frac{1}{2\pi}\left[\int_{-\omega_{c2}}^{-\omega_{c1}} d\omega + \int_{\omega_{c1}}^{\omega_{c2}} d\omega\right] = \frac{1}{2\pi}\left[-\omega_{c1}+\omega_{c2}+\omega_{c2}-\omega_{c1}\right] = \frac{2\omega_{c2}-2\omega_{c1}}{2\pi}$$

$h_d(n) =$

$h_d(n) = \{$

$$H_d(e^{j\omega}) = e^{-j\alpha\omega} \quad \omega c_1 \le |\omega| \le \omega c_2$$
$$0 \quad \text{ow}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\left[\int_{-\omega c_2}^{-\omega c_1} e^{-j\alpha\omega} e^{j\omega n}\, d\omega + \int_{\omega c_1}^{\omega c_2} e^{-j\alpha\omega} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\left|\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\omega c_2}^{-\omega c_1} + \left|\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{\omega c_1}^{\omega c_2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{j(n-\alpha)}\left[e^{-j\omega c_1(n-\alpha)} - e^{-j\omega c_2(n-\alpha)}\right] + \frac{1}{j(n-\alpha)}\left[e^{j\omega c_2(n-\alpha)} - e^{j\omega c_1(n-\alpha)}\right]\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{-e^{-j\omega c_1(n-\alpha)} + e^{j\omega c_1(n-\alpha)}}{2j} + \frac{e^{j\omega c_2(n-\alpha)} - e^{-j\omega c_2(n-\alpha)}}{2j}\right]$$

$$= \frac{\sin\omega c_2(n-\alpha) - \sin\omega c_1(n-\alpha)}{\pi(n-\alpha)}$$

$$= \frac{\omega c_2 - \omega c_1}{\pi}; \quad n = \alpha$$

$h_d(n) =$

$$h_d(n) = \frac{1}{2\pi}\left[\int_{-\omega c_2}^{-\omega c_1} d\omega + \int_{\omega c_1}^{\omega c_2} d\omega\right]_{n=\alpha}$$

$$= \frac{2\omega c_2 - 2\omega c_1}{2\pi}$$

$$F(\omega) = \int_{-\infty}^{\omega} S_{xx}(\omega)\, d\omega$$

MATLAB CODE

```
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t = 0:1/Fs:2.96;
x = sin(2*pi*t*1.24e3)+ sin(2*pi*t*10e3)+ randn(size(t));
nfft = 2^nextpow2(length(x));
Pxx = abs(fft(x,nfft)).^2/length(x)/Fs;

% Create a single-sided spectrum
Hpsd = dspdata.psd(Pxx(1:length(Pxx)/2),'Fs',Fs);
plot(Hpsd)
```
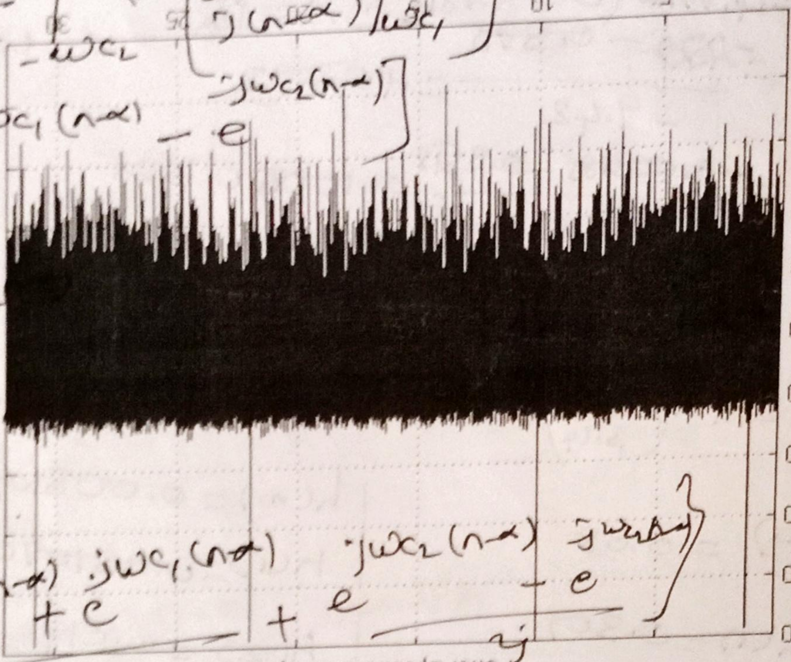

FOR DOUBLESIDE f AND

```
% Create a double-sided spectrum
Hpsd = dspdata.psd(Pxx,'Fs',Fs,'SpectrumType','twosided');
plot(Hpsd)
```



For double side band

Design a linear phase FIR BPF to pass freq. in the range 0.4π to 0.65π rds by taking 7 samples of hanning

$$H_d(e^{j\omega}) = e^{-j\alpha\omega} \quad \omega_{c_1} \le |\omega| \le \omega_{c_2}$$
$$= 0 \quad \text{otherwise}$$

$$h_d(n) = \frac{\sin\omega_{c_2}(n-\alpha) - \sin\omega_{c_1}(n-\alpha)}{\pi(n-\alpha)} \quad ; n \ne \alpha$$

$$= \frac{\omega_{c_2} - \omega_{c_1}}{\pi} \quad ; n = \alpha$$

$\omega_{c_1} = 1.2566$ r/s
$\omega_{c_2} = 2.0420$ r/s

$$w_{hann}(n) = 0.5 - 0.5\cos\left(\frac{n\pi}{\alpha}\right)$$

$w_{hann}(0) = 0$
$w_{hann}(1) = 0.25$
$w_{hann}(2) = 0.75$
$w_{hann}(3) = 1$
$w_{hann}(4) = 0.75$
$w_{hann}(5) = 0.25$
$w_{hann}(6) = 0$

$h_d(0) = \frac{0.156 - 0.5876}{-9.42} = +0.0456$

$h_d(1) = \frac{0.8086 + 0.5878}{-6.28} = -0.222$

$h_d(2) = \frac{-0.8910 + 0.9510}{-3.14} = -0.019$

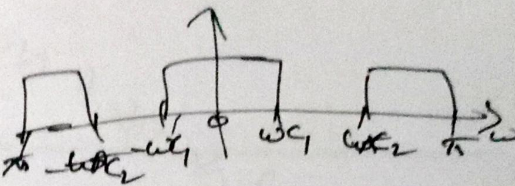
$h_d(3) = 0.25$

$h(0) = 0$, $h(1) = -0.0556$, $h(2) = -0.01425$, $h(3) = 0.25$

$h(4) = -0.0143$, $h(5) = -0.0556$, $h(6) = 0$

FIR BSF

$$H_d(e^{j\omega}) = e^{-j\alpha\omega} \; ; \; \pi \le |\omega| \le \omega_{c2}$$
$$= e^{-j\alpha\omega} \; ; \; -\omega_{c1} \le \omega \le \omega_{c1}$$



$$h_d(n) = \frac{\sin\omega_{c1}(n-\alpha) + \sin\pi(n-\alpha) - \sin\omega_{c2}(n-\alpha)}{\pi(n-\alpha)} \quad n \ne \alpha$$
$$= 1 - \left(\frac{\omega_{c2} - \omega_{c1}}{\pi}\right) \; ; \; n = \alpha$$