Dayananda Sagar College of Engineering

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(An Autonomous Institute Affiliated to VTU, Approved by AICTE & ISO 9001:2008 Certified)
(Accredited by National Assessment & Accreditation Council (NAAC) with 'A' grade)

Dayananda Sagar College of Engineering

Department of Electronics & Communication Engg.

Continuous Internal Evaluation – I

Course Name : Digital Signal Processing	Date : 05.10.2020
Course Code : 18EC5DCDSP	Day : Monday
Semester : 5	Timings : 01.00 to 2.30pm
Max Marks : 50 M	Duration : 1½ Hrs.

			Marks	CO & Levels
Q1	(a)	If $X(k)$ and C_k are the coefficients of DFT and Fourier series respectively, then the relation between them is given as i) $X(k) = \frac{1}{N} C_k$ ii) $X(k) = N + C_k$ iii) $C_k = \frac{1}{N} X(k)$ iv) $C_k = X(k)$	1	
	(b)	Given W_N is a twiddle factor matrix with any element a_{ij}, choose the correct statement (a) $a_{ij} = a_{ji}$ (b) $a_{ij} = -a_{ji}$ (c) $a_{1j} = a_{i1}$ (d) $a_{ij} = a_{ji}^*$ i) (a) and (b) are correct ii) (a) and (c) are correct iii) (a) and (d) are correct iv) (c) and (d) are correct	1	
	(c)	Which of the following is/are incorrect statement/s? i) $W_N^a = W_N^{a+N}$ ii) $W_N^{-a} = (W_N^a)^*$ iii) $W_N^{-a} = -(W_N^a)^*$ iv) both ii) and iii)	1	
	(d)	The term $N\delta(k)$ is the DFT of ___________ i) $N\delta(n)$ ii) $u(n)$ iii) $Nu(n)$ iv) $\delta(n)$	1	
	(e)	For a real valued 4 point sequence $x(n)$, the 4 point DFT is $X(k)$. If $X(1) = 1 - 2j$ then $X(7) =$________ i) 1-2j ii) 1+2j iii) 2+j iv) 2-j	1	
	(f)	Which of the following is an example for odd sequence? i) $x(n) = [0, 5, -6, -6, 5]$ ii) $x(n) = [0, 5, 6, 6, 5]$ iii) $x(n) = [0, 5, 6, -6, -5]$ iv) $x(n) = [0, -5, 6, 6, -5]$	1	
	(g)	If $[W]_N$ is a twiddle factor matrix then its inverse is given by i) $N\,[W]_N$ ii) $\frac{1}{N}[W]_N^*$ iii) $N\,[W]_N^*$ iv) $\frac{1}{N}\,[W]_N$	1	
	(h)	For a real sequence $x(n)$, the N (even number) point DFT is denoted as $X(k)$. Which of the following is a wrong statement? i) $X(0)$ is real ii) $X\left(\frac{N}{2}\right)$ is real iii) $X(N-1)$ is always real iv) $X(N-k) = X(k)^*$	1	
	(i)	DFT of a real and even sequence is i) purely real ii) purely imaginary iii) combination of real and imaginary iv) None of these	1	
	(j)	Which of the following is a wrong statement as concerned to DFT/FFT? i) $W_N^{2kr} = W_{N/2}^{kr}$ ii) $-W_N^k = W_N^{k+\frac{N}{2}}$ iii) $X(k) = X(k+N)$ iv) $W_N^{2kr} = -W_{N/2}^{kr}$	1	
Q2		Let $x(n)$ be a finite length sequence with $X(k) = [0, 1+j,\ 1,\ 1-j]$ using the relation between DFT and DTFS and DFT properties find DTFS coefficients of the following sequences. i) $x_1(n) = e^{\frac{j\pi n}{2}} x(n)$ ii) $x_2(n) = x((n-1))_4$ iii) $x_3(n) = (0,0,1,0)\ (*)_N$ x(n), where $(*)_N$ indicates circular convolution.	10	CO 1,2/L4
Q3	(a)	Compute 4 point DFT of the sequence $x(n) = [1,3,5,7]$ using DIT-FFT approach.	5	CO1/L3
	(b)	Find the total number of multiplications and additions required for 2048 point DFT using conventional DFT and FFT approach. Also find the speed improvement factor.	5	CO1/L3
Q4	(a)	Find 5 point circular convolution of the following sequences using concentric circle graphical method: $x_1(n) = [1,2,-1,4,3]$ and $x_2(n) = [2,0,-1,4]$	6	CO1/L3
	(b)	A 4 point sequence $x(n) = [1,2,3,4]$ has DFT $X(k)$ for $0 \le k \le 3$. Find the sequence which has DFT $X((K-1))_4$ without performing DFT and IDFT.	4	CO1/L4

OR

Q5	(a)	Show that multiplication of two DFTs in frequency domain corresponds to circular convolution in time domain.	4	CO1/L3
	(b)	Compute the 5 point DFT of the sequence $x(n) = [1, 3, 5, 7, 9]$. Also plot magnitude spectrum.	6	CO1/L3
Q6		Consider a sequence $x(n) = [1,1,0,3,-2,-4,6,5]$ with DFT $X(k)$. Evaluate the following functions without evaluating the DFT. i) $X(0)$ ii) $X(4)$ iii) $\sum_{k=0}^{7} e^{-j\frac{2\pi k}{4}} X(k)$ iv) $\sum_{k=0}^{7} \lvert X(k)\rvert^2$ v) $\sum_{k=0}^{7} X(k)$	10	CO1/L4
		OR		
Q7		Given the impulse response of a system as $h(n) = [1,-1,2]$, find the output of the system $y(n)$ for an input $x(n) = [1,3]$ using DFT-IDFT formula method.	10	CO2/L4

Staff : STM/KSG/CU/KP

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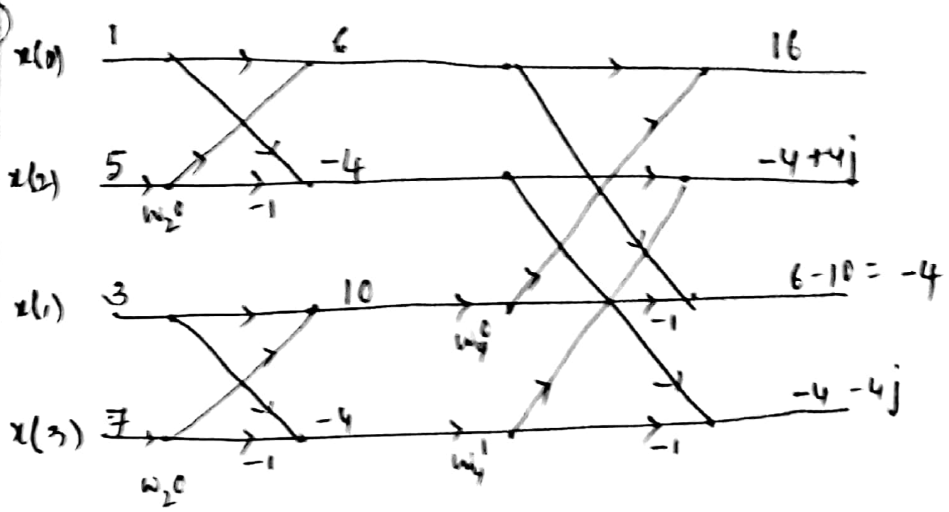
Date of test : 05.10.2018	Title of the subject	Max Marks : 50 M.
Day : day : Monday		Sub Mentor : Dr. STM
Branch : ECE	Sub initials DSP	Sub Mentor Sign :
Semester : 5	Sub Code : 18EC5DCDSP	Staff i/c of sec : KP
Section : A, B, C D.	Internal Test	Staffs i/c sign :
Timings : 09.30 AM–11.00 AM.	I / II / IMPT	HOD Name : Dr. TCM
Test Duration : 1½ Hrs.	Test Solutions	HOD's sign :

Q. No.	Test question paper solutions with steps	Marks Allocation
1 a)	iii) $C_k = \frac{1}{N} x(k)$	
b)	ii) a) & c) are correct	
c)	iii) $W_N^{-a} = (W_N^{a})^*$	
d)	ii) $u(n)$	
e)	ii) $1+2j$	
f)	iii) $x(n) = [0, 5, 6, -6, -5]$	
g)	ii) $\frac{1}{N}[W]_N^*$	
h)	iii) $x(N-1)$ is always real.	
i)	i) purely real.	
j)	iv) $W_N^{2k_0} = -W_{N/2}^{k_0}$	
2)	i) $x_1(n) = e^{\frac{j\pi n}{2}} x(n) = e^{\frac{j\pi n}{2} \times \frac{2}{2}}$ $x_1(n) = e^{\frac{j2\pi n}{4}} x(n) = W_4^{-n} x(n)$ $x_1(k) = x((k-1))_4$ $x_1(k) = [1-j, 0, 1+j, 1]$ — 2.	03 M
	$c_1(k) = \frac{1}{N} x_1(k) = \frac{1}{4} x_1(k)$ $= \frac{1}{4}[1-j, 0, 1+j \; 1]$ $c_1(k) = \frac{1}{4} - \frac{1}{4}j, \; 0 \; \frac{1}{4}+\frac{1}{4}j, \; \frac{1}{4}$ — 01	

Q. No.	Test question paper solutions with steps	Marks Allocation
	ii) $x_2(n) = x((n-1))_4$ $X_2(k) = W_4^{1k} X(k)$ $X_2(0) = W_4^0 X(0) = 1 \times 0 = 0$. $X_2(1) = W_4^1 X(1) = -j(1+j) = 1-j$ $X_2(2) = W_4^2 X(2) = -1(1) = -1$ $X_2(3) = W_4^3 X(3) = j(1-j) = 1+j$ 02 $c_2(0) = \frac{1}{4}[0] = 0$ $c_2(1) = \frac{1}{4}[1-j] = \frac{1}{4} - \frac{1}{4}j$ $c_2(2) = \frac{1}{4}[-1] = -1/4$ $c_2(3) = 1/4(1+j) = 1/4 + 1/4 j$ 01	03M
	iii) $x_3(n) = (0,0,1,0) \circledast x(n)$ $x_3(n) = y(n) \circledast x(n)$ $X_3(k) = Y(k)\, X(k)$ $X_3(k) = [0 + 0 + W_4^{2k} + 0]\, X(k) = W_4^{2k} X(k)$ 01 $X_3(k) = W_4^{2k} X(k)$ $X_3(0) = 1\, X(0) = 1 \times 0 = 0$ $X_3(1) = W_4^2 X(1) = -1(1+j) = -1-j$ $X_3(2) = W_4^4 X(2) = 1 \times 1 = 1$ $X_3(3) = W_4^6 X(3) = -1(1-j) = -1+j$ 02 $c_3(k) = \frac{1}{N} X_3(k)$ $= \frac{1}{4}[0,\ -1-j,\ 1,\ -1+j]$ $c_3(k) = 0,\ -\frac{1}{4} - \frac{1}{4}j\ \ \frac{1}{4}\ \ -1/4 + \frac{1}{4}j$ 01	04M
		10M

Q. No.	Test question paper solutions with steps	Marks Allocation

3) (a)



Structure 01
Labeling 01
calculation 02
05 M

$x(k) = [\,16, \; -4+4j \; -4, \; -4-4j\,]$

3 (b) $N = 2048$

No. of complex multiplications $= \frac{N}{2}\log_2 N = \frac{2048}{2}\log_2 2048$

$\boxed{\text{Conventional DFT} \quad \text{Mul} \; 4194304}$

$= 1024 \log_2 2^{11}$

$\boxed{\text{Speed Imp} \; 372.36.}$

No. of complex multiplication $= 1024 \times 11 = 11264$

No. of complex additions: $N \log_2 N = 2048 \log_2 2^{11}$

$= 2048 \times 11$

$\boxed{\text{Conventional DFT} \quad \text{Adds} \; 4192256}$

$= 22528$

No. of complex additions

$\boxed{\text{Speed}: 186.09}$

0 formula (02)
0 Calculation (02)
0 Speed Imp (01)
05

4 (a). $x_1(n) = [\,1, 2, -1, 4 \; 3\,]$ $x_2(n) = [\,2, 0, -1, 4, 0\,]$

$x_2((-n)) = [\,2 \; 0 \; 4 \; -1 \; 0\,]$

Use concentric circle method.

$$\begin{bmatrix} 2 & 0 & 4 & -1 & 0 \\ 0 & 2 & 0 & 4 & -1 \\ -1 & 0 & 2 & 0 & 4 \\ 4 & -1 & 0 & 2 & 0 \\ 0 & 4 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 4 \\ 3 \end{bmatrix}$$

Q. No.	Test question paper solutions with steps	Marks Allocation
	$y(n) = \sum_{m=0}^{N-1} x(m)\, h((n-m))_N$	formula = 01
	$y(0) = 2+0-4-4+0 = 2-8 = -6$	Each value with diagram 01×5 = 5
	$y(1) = 0+4+0+16-3 = 17$	
	$y(2) = -1+0+(-2)+0+12 = 09$	
	$y(3) = 4-2+0+8+0 = 10$	
	$y(4) = 0+8+1+0+6 = 15$	06 M
	$y(n) = [-6, 17, 9, 10, 15]$	
4 (b)	Let $y(k) = x((k-1))_4$	
	$y(n) = W_4^{-1n}\, x(n)$	1×4 = 4
	$y(0) = W_4^{0}\, x(0) = 1 \times 1 = 1$	
	$y(1) = W_4^{-1}\, x(1) = j \times 2 = 2j$	4M
	$y(2) = W_4^{-2}\, x(2) = -1 \times 3 = -3$	
	$y(3) = W_4^{-3}\, x(3) = -j \times 4 = -4j$	
	$y(n) = (1, 2j, -3, -4j)$	
5 (a)	Let the two DFTs be $X(k)$ & $H(k)$	
	$X(k) = \sum_{i=0}^{N-1} x(i)\, W_N^{ik} \longrightarrow (1) \quad k = 0, 1 \ldots N-1$	01
	$H(k) = \sum_{m=0}^{N-1} h(m)\, W_N^{mk} \longrightarrow (2) \quad k = 0, 1 \ldots N-1$	
	The product $Y(k) = X(k)\, H(k)$	
	But $y(n) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\, W_N^{-kn}$	
	$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, H(k)\, W_N^{-kn}$	01
	$= \frac{1}{N}\sum_{k=0}^{N-1}\sum_{i=0}^{N-1} x(i)\, W_N^{ik} \sum_{m=0}^{N-1} h(m)\, W_N^{mk}\, W_N^{-kn}$	

Q. No.	Test question paper solutions with steps	Marks Allocation

$$y(n) = \frac{1}{N}\sum_{i=0}^{N-1} x(i) \sum_{m=0}^{N-1} h(m) \sum_{k=0}^{N-1} W_N^{(i-(n-m))k}$$

Since $\sum_{k=0}^{N-1} W_N^{(i-(n-m))k} = N\,\delta(i-(n-m))$ we get

01

$$y(n) = \frac{1}{\cancel{N}}\sum_{i=0}^{N-1} x(i) \sum_{m=0}^{N-1} h(m)\, \cancel{N}\,\delta(i-(n-m))$$

$$= \sum_{m=0}^{N-1} h(m) \sum_{i=0}^{N-1} x(i)\,\delta(i-(n-m))$$

$$y(n) = \sum_{m=0}^{N-1} x(i)\Big|_{i=n-m} \cdot h(m)$$

01

$$y(n) = \sum_{m=0}^{N-1} x(n-m)\, h(m) = \sum_{m=0}^{N-1} h(m)\, x((n-m))_N.$$

04 M

5 (b) $x(n) = [1,\ 3,\ 5,\ 7,\ 9]$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad k = 0, 1, \ldots, N-1$$

$$W_5 = e^{-j\frac{2\pi}{5}}; \quad W_5^0 = 1 \quad W_5^1 = e^{-j\frac{2\pi}{5}}$$

$$W_5^2 = e^{-j\frac{4\pi}{5}} \qquad W_5^3 = e^{-j\frac{6\pi}{5}}$$

$$W_5^4 = e^{-j\frac{8\pi}{5}}$$

$$X(k) = 1 + 3W_5^{k} + 5W_5^{2k} + 7W_5^{3k} + 9W_5^{4k}$$

$$X(0) = 1 + 3 + 5 + 7 + 9 = \underline{25}$$

$$X(1) = 1 + 3W_5^1 + 5W_5^2 + 7W_5^3 + 9W_5^4$$
$$= -5 + 6.88j$$

$$X(2) = 1 + 3W_5^2 + 5W_5^4 + 7W_5^1 + 9W_5^3$$
$$= -5 + 1.62j$$

Q. No.	Test question paper solutions with steps	Marks Allocation

$X(3) = 1 + 3W_5^3 + 5W_5^1 + 7W_5^0 + 9W_5^2$ formula 01M

$= -5 - 1.62j$

$X(4) = 1 + 3W_5^4 + 5W_5^3 + 7W_5^2 + 9W_5^1$ $5 \times 1 = 5$

$= -5 - 6.88j$ 06M

Q6. i) $X(0) = \sum_{n=0}^{N-1} x(n) = 10 \longrightarrow$ 01

ii) $X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$ put $k = \frac{N}{2} = \frac{8}{2}$

$X(4) = \sum_{n=0}^{7} x(n)\, W_8^{4n} = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi}{8}(4n)}$

$= \sum_{n=0}^{7} x(n)\, e^{-j\pi n} = \sum_{n=0}^{7} (-1)^n x(n)$ 02

$X(4) = \cancel{1} - \cancel{1} + 0 - 3 - 2 + 4 + 6 - 5 = 0$

iii). $\sum_{k=0}^{7} e^{-j\frac{2\pi k}{4}} X(k) = ?$

$IDFT\{e^{-j\frac{2\pi k}{4}} X(k)\} = IDFT\{e^{-j\frac{2\pi}{4}k\frac{2}{2}} X(k)\}$

$IDFT\{e^{-j\frac{2\pi k}{4}} X(k)\} = IDFT\{W_8^{2k} X(k)\}$

$IDFT\{e^{-j\frac{2\pi k}{4}} X(k)\} = x((n-2))_8 \longrightarrow (1)$

New $IDFT\{e^{-j\frac{2\pi k}{4}} X(k)\} = \frac{1}{N}\sum_{k=0}^{7} e^{-j\frac{2\pi k}{4}} X(k)\, W_8^{-kn} \longrightarrow (2)$ 03

compare (1) & (2)

$x((n-2))_8 = \frac{1}{8}\sum_{k=0}^{7} e^{-j\frac{2\pi k}{4}} X(k)\, W_8^{-kn}$

put $n = 0$

$x((-2))_8 = \frac{1}{8}\sum_{k=0}^{7} e^{-j\frac{2\pi k}{4}} X(k)$

Q. No.	Test question paper solutions with steps	Marks Allocation
	$\sum_{k=0}^{7} e^{-j\frac{2\pi k}{4}} x(k) = 8\, x((-2))_8 = 8 \times x(6)$	
	$\sum_{k=0}^{7} e^{-j\frac{2\pi k}{4}} x(k) = 8 \times 6 = \underline{48}$	
	(iv) $\sum_{k=0}^{N-1} \lvert X(k) \rvert^2 = N \sum_{n=0}^{N-1} \lvert x(n) \rvert^2$	
	$= 8 \sum_{n=0}^{7} \lvert x(n) \rvert^2$	02
	$\sum_{k=0}^{N-1} \lvert X(k) \rvert^2 = 8[1+1+9+4+16+36+25]$	
	$= 8[2+29+61]$	
	$\sum_{k=0}^{7} \lvert X(k) \rvert^2 = 8 \times 92 = \underline{736}$	
	v) $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-kn}$	
	put $n=0$ $x(0) = \frac{1}{8} \sum_{k=0}^{7} X(k)\, W_8^{0}$	
	$\sum_{k=0}^{7} X(k) = 8 \times x(0) = \underline{8}$ $\longrightarrow$	02
7	$x(n) = [1, 3]$ $h(n) = [1, -1, 2]$	
	$x(n) = [1, 3, 0, 0]$ $h(n) = [1, -1, 2, 0]$	
	$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$ $X(k)$	3M
	$X(k) = 1 + 3W_4^{k}$	
	$H(k) = \sum_{n=0}^{N-1} h(n)\, W_N^{kn}$ $H(k)$	3M
	$H(k) = 1 - W_4^{k} + 2W_4^{2k}$	

Q. No.	Test question paper solutions with steps	Marks Allocation
	$y(k) = x(k)\,H(k)$	
	$= [1 + 3W_4^k][1 - W_4^k + 2W_4^{2k}]$	
	$= 1 - W_4^k + 2W_4^{2k}$	
	$3W_4^k - 3W_4^{2k} + 6W_4^{3k}$	
	$Y(k) = 1 + 2W_4^k - W_4^{2k} + 6W_4^{3k}$	y(n) - 04M
	On taking IDFT	
	$y(n) = [1, 2, -1, 6]$	10M