EML 3034: Modeling Methods in MMAE Name:______________________

EXAM 2 - Spring 2011

Open books and open notes. Mark your answers on the bubble sheet.

Problem #1: (60 points) Answer all 15 short answer questions: ***bubble in all choices that are correct (there may be more than one correct choice).***

1. In solving the system of equations, [A]{x}={b}

$$\begin{bmatrix} 5 & 1 \\ 2 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

if the inverse of the coefficient matrix is

$$A^{-1} = \begin{bmatrix} 0.231 & -0.077 \\ -0.154 & 0.385 \end{bmatrix}$$

The conditioning number of the matrix [A] using the L_∞ norm is
(a) not computable because A^{-1} has negative entries.
(b) K(A)=3.231, the system is well conditioned, and small residuals mean an accurate solution has been obtained.
(c) K(A)=3.231, the system is ill-conditioned, and small residuals mean an accurate solution has been obtained.
(d) K(A)=591,267.25, the system is ill-conditioned and small residuals do not mean accurate solution has been obtained.
(e) none of the above

2. In solving a system of simultaneous equations [A]{x}={b} where [A] is a 100x100 matrix and it takes k=20 iterations to converge, then, the order of floating point operations that it took to obtain the solution is of the order of
(a) 200,000
(b) 1,000,000
(c) 2,000,000
(d) 1,500,000
(e) none of the above.

3. A sufficient but not necessary condition (also called the Scarsborough criteria) for the iterative solution of the system of linear simultaneous equations [A]{x}={b} to converge when solved by the Gauss-Siedel iteration method is that
(a) the coefficient matrix [A] is subordinate.
(b) the coefficient matrix [A] is diagonally dominant.
(c) the coefficient matrix [A] is symmetric.
(d) the coefficient matrix [A] is Hermetian.
(e) none of the above.

4. For the iterative solution of the linear simultaneous equations [A]{x}={b} by the methods we discussed in class which can be re-arranged in the form $\{x\}^{n+1} = [T]\{x\}^n + \{c\}$, it is necessary that the matrix [T]
(a) has eigenvalues that have all magnitudes less than 1.
(b) has all positive entries that are smaller than one.
(c) is convergent, that is $\lim_{n\to\infty} T^{(n)} = 0$, where $T^{(n)}$=$TT...T$ n-times.
(d) is Hermetian symmetric.
(e) none of the above.

5. If for the system of equations: $\begin{bmatrix} 5 & 1 \\ 2 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

that are solved iteratively, the current solution estimate is $\{x_1, x_2\} = \{1, 2\}$, then the residual norm is using the L_2 or Euclidean norm:

(a) 6.00
(b) 8.00
(c) 2.00
(d) 6.325
(e) none of the above

6. In solving an 2x2 system of simultaneous linear equations, you find at two subsequent iterations (n=11 and n=12) the following approximation for the solution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}^{11} = \begin{pmatrix} 0.25 \\ 0.29 \end{pmatrix} \quad and \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}^{12} = \begin{pmatrix} 0.26 \\ 0.30 \end{pmatrix}$$

what is the iterative convergence criteria using the L_2 or Euclidean norm?

(a) 0.014
(b) 0.020
(c) 0.114
(d) 0.590
(e) 0.510

7. Given the following two coupled non-linear equations

$$f_1(\underset{\sim}{x}) = x_1 + 2x_1x_2$$
$$f_2(\underset{\sim}{x}) = x_1 - x_1^3\, x_2$$

whose root we seek by the Newton-Raphson method, and the current estimate of the root as $\underset{\sim}{x}^T = \{1,2\}$, the element of the Jacobian $J_{2,2}(\underset{\sim}{x})$ is:

(a) -6
(b) 12
(c) -12
(d) 3
(e) none of the above.

8. When solving the non-linear system of equations, $\underset{\sim}{f}(\underset{\sim}{x}) = \underset{\sim}{0}$, the iteration is stopped when the residual norm at the current iteration update for the solution, $\underset{\sim}{x}^{n+1}$, reaches a given tolerance, and the residual norm, $||\underset{\sim}{R}(\underset{\sim}{x}^{n+1})||$, is evaluated as

(a) $||\underset{\sim}{R}(\underset{\sim}{x}^{n+1})|| = ||\underset{\sim}{f}(\underset{\sim}{x}^{n+1})||$.
(b) $||\underset{\sim}{R}(\underset{\sim}{x}^{n+1})|| = ||\underset{\approx}{A}\,\underset{\sim}{x}^{n+1} - \underset{\sim}{b}||$.
(c) $||\underset{\sim}{R}(\underset{\sim}{x}^{n+1})|| = ||\underset{\approx}{J}(\underset{\sim}{x}^{n+1})||$.
(d) both (a) and (b).
(e) none of the above.

9. When using interpolation we propose to fit real data with a function that

(a) has real and imaginary parts to best fit through the data.
(b) passes through all data points because the data are exact.
(c) best fits the data but does not pass necessarily through all data points because the data are noisy.
(d) is designed to have the minimum maximum deviation from the sum of the data.
(e) none of the above.

10. For the given data: $x_o = 1.25$ $x_1 = 2.35$ $y_0 = 2$ $y_1 = 3$, the Lagrange interpolator $L_{1,1}(x)$ is

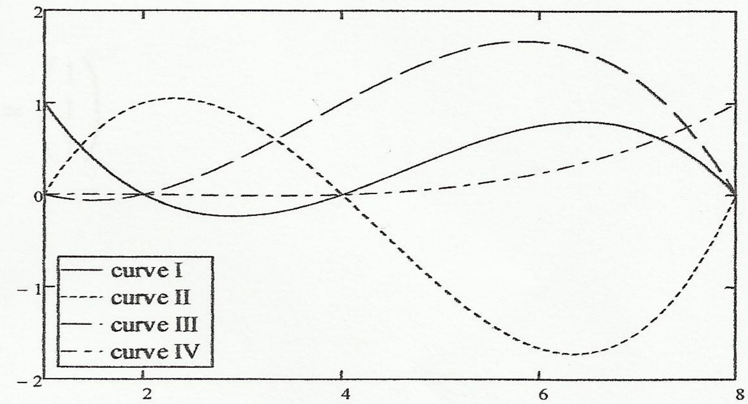
$$\frac{X - X_0}{X_1 - X_0}$$

(a) $(x - 1.25)/(1.10)$
(b) $x + (1.35/2.35)$
(c) $(2.5/1.5) - x$
(d) $(2/2.25) + x$
(e) none of the above

11. In generating a cubic Lagrange interpolant over the given set of x-values for which data are available,

$x_o = 1$ $x_1 = 2$ $x_2 = 4$ $x_3 = 8$

the $L_{3,2}(x)$ cubic interpolant is plot is



(a) curve I (solid)
(b) curve II (dotted)
(c) curve III (dashed)
(d) curve IV (dot-dashed)
(e) none of the above

12. When interpolating with the cubic spline, we approximate the underlying data with a function that is
(a) piecewise linear function that passes through the data.
(b) a global cubic polynomial that approximates the data but does not necessarily pass through the data points.
(c) a general approximation of the data but does not necessarily pass through the data points.
(d) piecewise cubic polynomial that passes through the data points.
(e) able to maintain continuity of the first and second derivative at the data points.

13. When interpolating with the cubic spline through a set of $i = 1, 2...N$ data pairs (x_i, y_i),
(a) specifying the data pairs is the only requirement.
(b) in addition to supplying the data pairs one must supply two additional boundary conditions.
(c) natural conditions refer to additionally setting the second derivatives at the left and right endpoints to zero.
(d) clamped conditioned refer to additionally specifying the slopes at the left and right endpoints.
(e) none of the above.

14. When data are noisy, the best way to determine the data fit is to use
(a) interpolation.
(b) min-max approximation.
(c) least-squares.
(d) collocation.
(e) none of the above.

15. As we solve for an increasing number of coefficients in linear least squares fit, the coefficient matrix of the system of equations for the unknown coefficients is known to
(a) have a conditioning number that is complex and cannot be computed with increasing number of sought-after coefficients.
(b) become well-conditioned very quickly with increasing of number of sought-after coefficients.
(c) become ill-conditioned very quickly with increasing number of sought-after coefficients.
(d) have a conditioning number that is relatively insensitive with increasing number of sought-after coefficients.
(e) none of the above.

Problem #2: (20 points) Carry out the 1st two iterations for the iterative solution of the following system of equations using Gauss Seidel iteration

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 2 \\ 1 & 2 & -8 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

using the initial guess

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^{(0)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

16. Using the L_∞ norm initial residual is evaluated as
(a) 3.4
(b) 8.0
(c) 4.5
(d) 1.0
(e) none of the above.

17. After one iteration, the L_∞ norm of the residual is evaluated as
(a) 1.0
(b) -2.5
(c) 0.34
(d) 2.5
(e) none of the above.

18. After one iteration, the solution is evaluated as
(a) (x_1, x_2, x_3)=$(1.0, 0.0, -0.25)$
(b) (x_1, x_2, x_3)=$(0.0, -0.34, -0.25)$
(c) (x_1, x_2, x_3)=$(-1.0, 0.56, 1.25)$
(d) (x_1, x_2, x_3)=$(1.2, 1.3, 1.25)$
(e) none of the above.

19. After two iterations, the L_∞ norm of the residual is evaluated as
(a) 0.26
(b) 0.58
(c) -0.37
(d) 1.2
(e) none of the above.

20. After two iterations, the solution is evaluated as
(a) (x_1, x_2, x_3)=$(1.0, 2.0, 3.0)$
(b) (x_1, x_2, x_3)=$(0.045, -0.45, -0.45)$
(c) (x_1, x_2, x_3)=$(-1.0, -2.0, -3.0)$
(d) (x_1, x_2, x_3)=$(0.375, 0.883, -0.12)$
(e) none of the above.

<u>Problem #3</u>: (20 points) Given the following two coupled non-linear equations

$$f_1(\underset{\sim}{x}) = 3x_1^2 + x_2 - 5$$
$$f_2(\underset{\sim}{x}) = x_1 + x_2^3$$

whose root we seek by the <u>Newton-Raphson</u> method, and the <u>initial guess</u> $\underset{\sim}{x}^T = \{1,1\}$

21. what is the value of the initial residual norm using the L_∞ norm
(a) -1
(b) 2.236
(c) 3
(d) 2
(e) none of the above.

22. what is the initial value of the Jacobian element $J_{1,1}(x)$
(a) 1
(b) 6
(c) 4.5
(d) 3
(e) none of the above.

23. after one step of the Newton-Rapshon method, what is the value of the residual norm using the L_∞ norm
(a) 1.307
(b) 2.358
(c) 0.132
(d) 1.569
(e) none of the above.

24. after two steps of the Newton-Rapshon method, what is the value of the solution
(a) $\underset{\sim}{x}^T = \{5.568, -33.911\}$
(b) $\underset{\sim}{x}^T = \{-15.735, -303.146\}$
(c) $\underset{\sim}{x}^T = \{2.668, 13.333\}$
(d) $\underset{\sim}{x}^T = \{12.825, 3.105\}$
(e) none of the above.

25. after two steps of the Newton-Rapshon method, what is the norm the iterative convergence criteria using the L_∞ norm
(a) 1.573
(b) -23.496
(c) 65.396
(d) 34.146
(e) none of the above.