

Matlab Exercise 3

Question [a]:

The DH parameters:

i	\mathbf{a}_{i-1}	α_{i-1}	\mathbf{d}_i	θ_i
1	0	0°	0	θ_1
2	L_1	0°	0	θ_2
3	L_2	0°	0	θ_3
4	L_3	0°	0	0°

Question [b]:

Link Transformations:

$${}^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} C_2 & -S_2 & 0 & L_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2_3T = \begin{bmatrix} C_3 & -S_3 & 0 & L_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_HT = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question [c]:

The forward pose kinematics solutions:

$$\therefore {}^0_3T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T = \begin{bmatrix} C_{123} & -S_{123} & 0 & L_1C_1 + L_2C_{12} \\ S_{123} & C_{123} & 0 & L_1S_1 + L_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore {}^0_HT = {}^0_3T \cdot {}^3_HT = \begin{bmatrix} C_{123} & -S_{123} & 0 & L_1C_1 + L_2C_{12} + L_3C_{123} \\ S_{123} & C_{123} & 0 & L_1S_1 + L_2S_{12} + L_3S_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i)

Matlab program is as follows:

```
clear all
clc
```

```

%Constants
L1=4;
L2=3;
L3=2;
theta1=30;
theta2=0;
theta3=0;

%DH parameters
alpha(1)=0; a(1)=0; d(1)=0; theta(1)=theta1;
alpha(2)=0; a(2)=L1; d(2)=0; theta(2)=theta2;
alpha(3)=0; a(3)=L2; d(3)=0; theta(3)=theta3;
alpha(4)=0; a(4)=L3; d(4)=0; theta(4)=0;

%Transformation matrices

T0_1=trotx(alpha(1), 'deg') * transl(a(1),0,0) * trotz(theta(1), 'deg')
* transl(0,0,d(1));
T1_2=trotx(alpha(2), 'deg') * transl(a(2),0,0) * trotz(theta(2), 'deg')
* transl(0,0,d(2));
T2_3=trotx(alpha(3), 'deg') * transl(a(3),0,0) * trotz(theta(3), 'deg')
* transl(0,0,d(3));
T3_H=trotx(alpha(4), 'deg') * transl(a(4),0,0) * trotz(theta(4), 'deg')
* transl(0,0,d(4));

T0_3 = T0_1 * T1_2 * T2_3

T0_H = T0_3 * T3_H

```

Results:

T0_3 =

```

1  0  0  7
0  1  0  0
0  0  1  0
0  0  0  1

```

T0_H =

```

1  0  0  9

```

```

0  1  0  0
0  0  1  0
0  0  0  1

```

ii)

Matlab program is as follows:

```

clear all
clc

%Constants
L1=4;
L2=3;
L3=2;
theta1=10;
theta2=20;
theta3=30;

%DH parameters
alpha(1)=0; a(1)=0; d(1)=0; theta(1)=theta1;
alpha(2)=0; a(2)=L1; d(2)=0; theta(2)=theta2;
alpha(3)=0; a(3)=L2; d(3)=0; theta(3)=theta3;
alpha(4)=0; a(4)=L3; d(4)=0; theta(4)=0;

%Transformation matrices

T0_1=trotx(alpha(1),'deg') * transl(a(1),0,0) * trotx(theta(1),'deg')
* transl(0,0,d(1));
T1_2=trotx(alpha(2),'deg') * transl(a(2),0,0) * trotx(theta(2),'deg')
* transl(0,0,d(2));
T2_3=trotx(alpha(3),'deg') * transl(a(3),0,0) * trotx(theta(3),'deg')
* transl(0,0,d(3));
T3_H=trotx(alpha(4),'deg') * transl(a(4),0,0) * trotx(theta(4),'deg')
* transl(0,0,d(4));

T0_3 = T0_1 * T1_2 * T2_3

T0_H = T0_3 * T3_H

```

Results:

T0_3 =

0.5000	-0.8660	0	6.5373
0.8660	0.5000	0	2.1946
0	0	1.0000	0
0	0	0	1.0000

T0_H =

0.5000	-0.8660	0	7.5373
0.8660	0.5000	0	3.9266
0	0	1.0000	0
0	0	0	1.0000

ii)

Matlab program is as follows:

```
clear all
clc

%Constants
L1=4;
L2=3;
L3=2;
theta1=90;
theta2=90;
theta3=90;

%DH parameters
alpha(1)=0; a(1)=0; d(1)=0; theta(1)=theta1;
alpha(2)=0; a(2)=L1; d(2)=0; theta(2)=theta2;
alpha(3)=0; a(3)=L2; d(3)=0; theta(3)=theta3;
alpha(4)=0; a(4)=L3; d(4)=0; theta(4)=0;

%Transformation matrices

T0_1=trotx(alpha(1),'deg') * transl(a(1),0,0) * trotx(theta(1),'deg')
* transl(0,0,d(1));
T1_2=trotx(alpha(2),'deg') * transl(a(2),0,0) * trotx(theta(2),'deg')
* transl(0,0,d(2));
T2_3=trotx(alpha(3),'deg') * transl(a(3),0,0) * trotx(theta(3),'deg')
* transl(0,0,d(3));
T3_H=trotx(alpha(4),'deg') * transl(a(4),0,0) * trotx(theta(4),'deg')
* transl(0,0,d(4));

T0_3 = T0_1 * T1_2 * T2_3
```

$$T0_H = T0_3 * T3_H$$

Results:

$$T0_3 =$$

$$\begin{pmatrix} 0 & 1.0000 & 0 & -3.0000 \\ -1.0000 & 0 & 0 & 4.0000 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{pmatrix}$$

$$T0_H =$$

$$\begin{pmatrix} 0 & 1.0000 & 0 & -3.0000 \\ -1.0000 & 0 & 0 & 2.0000 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{pmatrix}$$

where :

$$T_{0_3} \equiv {}^0_3T$$

$$T_{0_H} \equiv {}^0_HT$$

Question [d]:

Using Robotics Toolbox :

```
clear all
clc

%Constants
L1=4;
L2=3;
L3=2;
theta1=0;
theta2=0;
theta3=0;

%DH parameters
alpha(1)=0; a(1)=0; d(1)=0; theta(1)=theta1;
alpha(2)=0; a(2)=L1; d(2)=0; theta(2)=theta2;
alpha(3)=0; a(3)=L2; d(3)=0; theta(3)=theta3;
alpha(4)=0; a(4)=L3; d(4)=0; theta(4)=0;

%Link() function
L(1) = Link([alpha(1) a(1) theta(1) d(1)], 'standard');
L(2) = Link([alpha(2) a(2) theta(2) d(2)], 'standard');
L(1) = Link([alpha(3) a(3) theta(3) d(3)], 'standard');

%robot() function "same as SerialLink"
threelink = SerialLink(L, 'name', 'threelink')
```


Results:

```
threelink =
```

```
threelink (2 axis, RR, stdDH, slowRNE)
```

+-----+				
j	theta	d	a	alpha
+-----+				
1	q1	3	0	0
2	q2	4	0	0
+-----+				

```
grav =    0  base = 1  0  0  0  tool = 1  0  0  0
          0          0  1  0  0          0  1  0  0
        9.81        0  0  1  0          0  0  1  0
                  0  0  0  1          0  0  0  1
```