Bonus: 數學實作 Math Application Project 3

Fourier Series and Transforms

Logistics:

- This project can be worked out and submitted individually or in a team of ≤3. Submit one report per team. No two reports may be identical/verbatim word-replacement.
- Project requires very basic Matlab. The university provides free Matlab license.
- This project is due *before June 13*. Submit report *and* code on e3.
- All parts of the report must be typed and presented using a font/size/format identical to this document. Graphs must be computer-generated and contain labels and gridlines where appropriate. Hand-written parts or pasting photos of a computer screen/paper will score zero, while graphs with unprofessional formatting will also be points-deducted.
- All writing must be in English. Provide answer to questions **in blue.**

Project Goal:

To give you an intuitive feel for Fourier Series and Transforms through examples.

What Do You Need To Do:

Part 1: Synthetic Signals:

1. **Simple cosine signal:**
    a. Given a cosine signal of amplitude = 1 and frequency = 10Hz. Write a function $g(t)$ for this signal. What assumptions did you make to ensure the units are correct?
    b. Solve the Fourier Transform of $g(t)$ by hand, getting $\hat{g}(f)$, where f denotes frequency. Show step-by-step derivation.
    c. Plot magnitude spectrum of $\hat{g}(f)$ as a function of f in Matlab. Does the plot correctly show a peak at 10Hz?
    d. From the attached example Matlab code, generate a discrete cosine signal of the same amplitude and frequency as (a). Take the DFT of the signal. Plot both the signal and its magnitude spectrum. Does it agree with your hand-derived solution?
2. **Phasing signal:**
    a. Add a sine signal of identical amplitude and frequency to the signal in (1a). Plot its spectrum in Matlab.
    b. Did the spectrum's peak change in frequency? Why?
    c. How about amplitude? Why?
    d. Plot the signal-vs-time for $g(t) = \cos(\dots t)$ and the new $g(t) = \cos(\dots t) + \sin(\dots t)$. How are they different? How does this difference show up in the phase plots?
    *Note: the phase plot shows phases across the entire frequency range. But only the phase where your spectrum has a peak is important.

Part 2: Audio Manipulation:

3. **Application: This is the fun part, illustrating the application of FFT in signal-processing. Particularly: How do you separate two superimposed "voices" based on their frequencies.**
    a. Two audio files from https://www.fisheries.noaa.gov/national/science-data/sounds-ocean have been included: (i) Dolphin, (ii) Whale + ocean noise.
    b. Write Matlab code that reads both audio and plot their signals.

c. Plot the spectrum of these audio signals. Observe any differences (Dolphin should be higher freq).

d. To simulate a situation where both Dolphin and Whale are recorded simultaneously, add the two signals together (note: since the audios are different length, you may need to repeat the shorter signal). Plot this new signal and its spectrum. Compare to (3b).

e. Can you identify which region of the frequency-domain belong to the Dolphin, the Whale and the ocean background noise, respectively?

f. Try to separate the signal in (5d-e) into ***Dolphin vs. Whale vs. Background*** in the frequency domain, then use inverse-FFT to reconstruct the time-domain audio files. Save these files and play. Was the audio manipulation successful? (I.e. did you clearly isolate Dolphin, Whale and Background sounds using FFT?)