Notions of genericity:

1. Bayer - Stillman: a homogeneous ideal $I \subset S = k[x_1, \dots, x_n]$ is in generic coordinates iff $(x_n, \dots, x_{n-r+1}) \in U_r$ where $r = \dim(S/I)$

2. Caminata - Gorla: a homogeneous ideal $I \subset S = k[x_1, \dots, x_n, h]$ w/ $|Z_+(I)| < \infty$ is in generic coordinates iff $h \nmid 0$ on S/I^{sat}.

3. Fröberg / Trung: a homogeneous ideal $I \subset R = K[x_1, \dots, x_n]$ w/ $I = (f_1, \dots, f_r)$ and K an extension of F is generic iff
   - each f_i is generic, meaning its coeffs are algebraically independent over F
   - the set of all coeffs is algebraically independent over F.

4. Pardue: a property P is generic if it holds on a nonempty open $U \subset \prod_{i=1}^{r} S_{d_i}$ in the Zariski topology. A sequence $(f_1, \dots, f_r) \in U$ is called a generic sequence.

→ Note: The precise meaning of a "generic sequence" depends on the generic property it satisfies — $(f_1, \dots, f_r)$ could be generic wrt property P but not property Q.

5. Old versions of Caminata-Gorla: A homogeneous ideal $I \subset R = k[x_1, \ldots, x_n]$ is in generic coordinates iff $\mathrm{gin}_\tau I = \mathrm{in}_\tau I$.

→ <u>This</u> is the meaning of "generic coordinates" referenced in the 2020 paper w/ both Gorla and Minko as authors — specifically version 4.

Questions: 1. Caminata-Gorla used both definitions at different points — are they equivalent?

2. Minko/Gorla/etc. paper says that "since I is generated by generic polynomials, then it is in generic coordinates" (pg 11). They are using Pardue's meaning of "generic polynomials," so the suggestion seems to be that $(4) \Longrightarrow (5)$.

3. Main, overarching question is which of these are equivalent. What other hypotheses might be needed?

→ Known for now is that $(2) \Longrightarrow (1)$ and $(1) \not\Longrightarrow (2)$.