Math 1231 Summer 2024
Mastery Quiz 1
Due Wednesday, July 3

This week's mastery quiz has one topic. Please do your best on that topic. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name is clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on this quiz:

- Major Topic 1: Limits

Name: Solutions

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Major Topic 1: Limits

(a) $\lim_{x\to 3} \dfrac{x^2-5x+6}{x^2-x-6}$

Solution. Trying to plug in 3 doesn't work since the denominator is zero there. The numerator is also zero, and whenever we have a 0/0 situation we want to look for an Almost Identical Function. To do this, we factor:

$$\boxed{\lim_{x\to 3} \frac{x^2-5x+6}{x^2-x-6} = \lim_{x\to 3} \frac{(x-3)(x-2)}{(x-3)(x+2)} \overset{\text{AIF}}{=} \lim_{x\to 3} \frac{x-2}{x+2} = \frac{3-2}{3+2} = \frac{1}{5}.}$$

(b) $\lim_{x\to 1} \dfrac{\sqrt{x+3}-2}{x-1}$

Solution. Again plugging in 1 gives a 0/0 situation, and because we have a square root in the expression, we should try the trick of multiplying by the conjugate:

$$\boxed{\begin{aligned} \lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} &= \lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} = \lim_{x\to 1} \frac{x+3-2^2}{(x-1)(\sqrt{x+3}+2)} \\ &= \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x+3}+2)} \overset{\text{AIF}}{=} \lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}. \end{aligned}}$$

(c) Let $f(x) = \begin{cases} x^2-2 & x<1 \\ 3-4x & x>1 \end{cases}$. Compute $\lim_{x\to 1} f(x)$.

Solution. Because this is a piecewise function whose pieces are nice continuous functions, we can compute the one-sided limits just by plugging in 1:

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2-2 = (1)^2-2 = -1$$
$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 3-4x = 3-4(1) = -1.$$

Since the one-sided limits exist and are equal, the two-sided limit exists and we have

$$\lim_{x\to 1} f(x) = -1.$$