Math 1231 Summer 2024
Mastery Quiz 2
Due Monday, July 8

This week's mastery quiz has one topic. Please do your best on that topic. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn in this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name is clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on this quiz:

- Major Topic 1: Limits

Name: Solutions

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Major Topic 1: Limits

(a) $\lim_{x\to 3} \frac{\sqrt{7-x}-2}{x-3}$

Solution. If we try plugging in 3, we get a 0/0 indeterminate form, so we need some algebriac trick to produce an almost identical function. Since there is a square root here, we try multiplying by the conjugate:

$$\boxed{\begin{aligned}\lim_{x\to 3} \frac{\sqrt{7-x}-2}{x-3} &= \lim_{x\to 3} \frac{\sqrt{7-x}-2}{x-3} \cdot \frac{\sqrt{7-x}+2}{\sqrt{7-x}+2} = \lim_{x\to 3} \frac{7-x-2^2}{(x-3)(\sqrt{7-x}+2)} \\ &= \lim_{x\to 3} \frac{-(x-3)}{(x-3)(\sqrt{7-x}+2)} \overset{\text{AIF}}{=} \lim_{x\to 3} \frac{-1}{\sqrt{7-x}+2} = -\frac{1}{\sqrt{7-3}+2} = -\frac{1}{4}.\end{aligned}}$$

(b) $\lim_{x\to 0} \frac{\sin(3x)\sin(4x)}{x\sin(2x)}$

Solution. Given the presence of the trig functions, we want to find a way to use the Small Angle Approximation (SAA) that $\lim_{\theta\to 0}\sin(\theta)/\theta = 1$. We also know by the quotient law that

$$\lim_{\theta\to 0} \frac{\theta}{\sin(\theta)} = \lim_{\theta\to 0} \frac{1}{\frac{\sin(\theta)}{\theta}} = \frac{1}{\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta}} = \frac{1}{1} = 1.$$

Looking at the arguments of the sine functions in this problem, we'll want to use the SAA three times, taking $\theta = 3x$, $\theta = 4x$, and $\theta = 2x$.

$$\boxed{\begin{aligned}\lim_{x\to 0} \frac{\sin(3x)\sin(4x)}{x\sin(2x)} &= \lim_{x\to 0} \frac{\sin(3x)}{x} \cdot \sin(4x) \cdot \frac{1}{\sin(2x)} \cdot \frac{3}{3} \cdot \frac{2x}{4x} \cdot \frac{4}{2} \\ &= \lim_{x\to 0} \frac{3\cdot 4}{2} \cdot \frac{\sin(3x)}{3x} \cdot \frac{\sin(4x)}{4x} \cdot \frac{2x}{\sin(2x)} = 6\cdot 1\cdot 1\cdot 1 = 6.\end{aligned}}$$

We can check our answer using the informal version of the SAA, that $\sin(\theta) \approx \theta$ when θ is near 0. Then we might say that

$$\frac{\sin(3x)\sin(4x)}{x\sin(2x)} \approx \frac{3x\cdot 4x}{x\cdot 2x},$$

which is almost identical to the constant function 6. This informal version of the SAA is a good way to check our work, but not in itself a complete answer.

Name: Solutions

(c) $\lim_{x \to -2} \frac{x-2}{(x+2)^2}$

Solution. When we try plugging in -2 here, we get -4/0, which is *not* an indeterminate form. Instead, we're dividing some nonzero number by something increasingly small, so we get something back that is increasingly large. That is, this is going to be some kind of infinite limit.

$$\boxed{\lim_{x \to -2} \frac{x-2^{\nearrow -4}}{(x+2)^2_{\searrow 0+}} = -\infty.}$$

Since the number we divide by is always positive (due to the even exponent 2), we are always dividing a negative number by a positive number, so in addition to growing without bound, the output of this function near -2 is always negative. Hence the limit is $-\infty$.