Math 1231 Summer 2024
Mastery Quiz 8
Due Monday, July 29

This week's mastery quiz has three topics. If you already have a 4/4 on M3, a 2/2 on S5, or a 2/2 on S6 (check Blackboard—grades may have changed after the midterm), you don't need to submit these topics again.

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn in this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name is clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on this quiz:

- Major Topic 3: Extrema and optimization
- Secondary Topic 5: Curve sketching
- Secondary Topic 6: Applied optimization

Name:

Name:

Major Topic 3: Extrema and optimization

(a) The function $f(x) = \dfrac{x^3 - 5x^2}{x + 3}$ has absolute extrema either on the interval $[-4, -1]$ or on the interval $[-1, 4]$. Pick one of those intervals, explain why f has extrema on that interval, and find the absolute extrema.

(b) Find and classify all the critical points of $f(x) = x^4 + 8x^3 + 10x^2 + 1$, that is, for each critical point you find, say whether it is a maximum, minimum, or neither.

Name:

Secondary Topic 5: Curve sketching

Let $g(x) = \dfrac{x^2-7}{x^2-4}$. We can compute that $g'(x) = \dfrac{6x}{(x+2)^2(x-2)^2}$ and also that $g''(x) = \dfrac{-6(3x^2+4)}{(x+2)^3(x-2)^3}$. Sketch a graph of the function $g(x)$. Your answer should state

(a) the domain of the function
(b) any horizontal or vertical asymptotes
(c) the roots of the function
(d) the critical points of the function
(e) intervals on which the function is increasing or decreasing
(f) any relative minima or maxima
(g) intervals on which the function is concave up or concave down
(h) any inflection points.

Name:

Secondary Topic 6: Applied optimization

Suppose you are running a toy shop. It costs $C(x) = 200 + 10x$ dollars to produce x toys in a day, and you make a revenue of $R(x) = 26x - .2x^2$ dollars if you sell x toys in a day. How many toys should you produce per day to maximize your profit?