Math 1231 Summer 2024
Final Exam Syllabus

The final exam is cumulative, and so will cover all the major and secondary topics we've seen this summer. M1-M4, along with S7 and S8 will be required topics, with S1-S6 being optional. You can attempt up to two of the earlier secondary topics, and doing well on one of these topics will potentially increase your mastery score on that topic, as well as give you bonus points on the final exam. **Since each mastery point translates into 1% of your final grade, I'd highly encourage you to think in advance about which bonus secondary topics you'd like to attempt, focusing on ones that you do not yet have a 2/2 on.** Here's a brief review of things you should be able to do on the exam, categorized by topic:

M1: Computing limits

- Compute limits by direct substitution
- Compute algebraic limits with a $0/0$ indeterminate form, e.g. by factoring or multiplying by the conjugate
- Use the small angle approximation to compute trig limits
- Compute limits with a $\infty \pm \infty$ indeterminate form by finding a common denominator
- Compute infinite limits
- Compute limits at infinity

M2: Computing derivatives

- Use all derivative rules fluently

M3: Optimization

- Determine whether or not the Extreme Value Theorem applies to a given function on a particular domain
- Find absolute extrema in cases when the Extreme Value Theorem applies
- Locate the crtical points of a function
- Classify critical points as relative minima, relative maxima, or neither
  - Use the first derivative test
  - Use the second derivative test

M4: Computing integrals

- Find antiderivatives of a given function
- Perform u-substitutions in both the definite and indefinite case
- Use the Fundamental Theorems of Calculus

S1: Definition of derivative

- Use the $h \to 0$ limit definition to compute derivatives
- Use the $t \to a$ limit definition to compute derivatives

S2: Linear approximation

- Employ the linear approximation formula
- Given a number to approximate, determine the function being approximated and find a useful point at which to perform the approximation

S3: Implicit differentiation

- Take first and second derivatives of implicitly defined functions
- Find tangent lines to curves given by implicitly defined functions

S4: Related rates

- Translate word problems into information about functions and their derivatives
- Use various area and volume formulas, the Pythagorean Theorem, and facts about similar triangles to relate various functions
- Interpret derivatives as rates of change

S5: Curve sketching

- Determine various pieces of information about a function, including
  - its domain
  - its asymptotes, both vertical and horizontal
  - its roots
  - its relative extrema
  - its inflection points
  - intervals on which it's increasing or decreasing
  - intervals on which it's concave up or down
- Use the above information to accurately sketch the graph of a function

S6: Applied optimization

- Translate word problems into an objective function
- Use constraints given in the problem to write the objective function as a function of a single variable
- Use tools from M3 to maximize or minimize the objective function
- Give an argument to classify any critical points as true maxima or minima

S7: Riemann sums

- Approximate the area under a function's graph using some given number of rectangles
- More generally, approximate the area under a function's graph using n rectangles (i.e. given a function, find a Riemann sum approximating its area)
- Compute the exact area by taking the limit of a Riemann sum

S8: Applications of integration

- Find the area of a region in the plane by integrating with respect to either x or y
- Compute the average value of a function over a given interval
- Apply the Net Change Theorem to various physical and economic phenomena, computing, for example
  - the displacement of a moving object
  - the total work done on a moved object, such as a spring
  - the (total/producer/consumer) surplus in a market