Math 1231 Summer 2024
Practice Final Exam

- You will have 90 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, two-sided, handwritten cheat sheet you have made ahead of time.
- You may not use a calculator. You may leave answers unsimplified, except you should compute trigonometric functions as far as possible.
- The exam has 6 required problems, one on each major topic we've covered, and two on each secondary topic covered in the last two weeks. The exam has 10 pages total.
- Each part of each topic is worth ten points. The whole test is scored out of 100 points, with the opportunity to gain 4 bonus points by answering up to two of the earlier secondary topics (S1–S6).
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all. When in doubt, show more work and write complete sentences.
- If you need more paper to show work, I have extra at the front of the room.
- Good luck!

M1:			M2:		
M3:			M4:		
S7:		S8:			/100

Name:

Name:

Problem 1 (M1). Compute the following limits if they exist. Show enough work to justify your computation.

(a) $\displaystyle\lim_{x\to+\infty} \frac{\sqrt{3x^5+2x}}{x^{5/2}-x^{3/2}+1}$

(b) $\displaystyle\lim_{x\to 3} \frac{1}{x-3} - \frac{3}{x^2-3x}$

Name:

Problem 2 (M2). Compute the following derivatives.

(a) $\dfrac{d}{dx} \csc^4\left(\dfrac{x^3+4}{\sqrt[3]{x^2-3}}\right)$

(b) $\dfrac{d}{dx} x^2 \sin\left(\sec(\cos(x)) + x^7\right)$

Name:

Problem 3 (M3).

(a) The function $f(x) = \dfrac{x^2+5}{x+2}$ has absolute extrema either on the interval $[-3, 0]$ or on the interval $[0, 3]$. Pick one of those intervals, explain why f has extrema on that interval, and find the absolute extrema.

(b) Find and classify the critical points of $g(x) = x^4 + 4x^3 + 2$

Name:

Problem 4 (M4). Compute the following integrals.

(a) **By changing the bounds of the integral** compute $\int_0^{\sqrt{\pi}} x \sin(x^2)\, dx$

(b) Compute $\int \frac{x^2}{(x^3-3)^3}\, dx$

Name:

Problem 5 (S7). Using **only the definition of Riemann sum** and your knowledge of limits, compute the exact (signed) area under the curve $x^3 + 2x$ on the interval $[-1, 2]$.

Name:

Problem 6 (S8). Sketch and clearly label the region bounded by the curves $x = y^2 - 1$, $y = 0$, $y = 1$, and $x = \sqrt{y}$, and find the area of that region.

Name: ____________________

Problems 7 & 8 (S1–S6). Choose **two** of the following problems, and solve them on the blank page attached to this exam (front and back). The two problems you pick are both worth up to two bonus points on this exam, and will also count as mastery points on the chosen topics. For example, if you currently have a 0/2 or a 1/2 on S4, it might be a good idea to choose the S4 problem below. If you get a 2/2 on it on this exam, your exam grade will increase by 2 points, and your final mastery grade for S4 will be a 2/2.

(S1) If $f(x) = x^2 + \sqrt{x}$, compute $f'(4)$ directly from the definition of the derivative.

(S2) If $f(x) = \frac{x^2+3}{x-2}$, use a linear approximation to estimate $f(2.9)$.

(S3) Find a tangent line to the curve given by $x^4 - 2x^2y^2 + y^4 = 16$ at the point $(\sqrt{5}, 1)$.

(S4) The surface area of a cube is given by the formula $A = 6s^2$ where s is the length of a side. If the side lengths are increasing by 2 inches per second, how fast is the surface area increasing when the area is 54 square inches?

(S5) Let $f(x) = \sqrt[3]{x^2 - 2x} = \sqrt[3]{x(x-2)}$. We compute that

$$\begin{aligned} f'(x) &= \frac{2(x-1)}{3x^{2/3} \cdot (x-2)^{2/3}} \\ f''(x) &= \frac{-2(x^2 - 2x + 4)}{9(x-2)^{5/3} \cdot x^{5/3}}. \end{aligned}$$

Sketch a graph of f and find all of the following:

(a) the domain of the function
(b) any horizontal or vertical asymptotes
(c) the roots of the function
(d) the critical points of the function
(e) intervals on which the function is increasing or decreasing
(f) any relative minima or maxima
(g) intervals on which the function is concave up or concave down
(h) any inflection points.

(S6) Find the point on the line $y = 2x + 5$ that is closest to the origin.

Name: