Math 1231 Summer 2024
Mastery Quiz 7
Due Wednesday, July 24

This week's mastery quiz has three topics, and everyone should submit all of them.

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn in this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name is clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on this quiz:

- Major Topic 3: Extrema and optimization
- Secondary Topic 5: Curve sketching
- Secondary Topic 6: Applied optimization

Name:

Name:

Major Topic 3: Extrema and optimization

(a) Find and classify all the critical points of $f(x) = x^4 + 3x^3 + x^2 - 3$, that is, for each critical point you find, say whether it is a maximum, minimum, or neither.

(b) The function $g(x) = \dfrac{x^2 - 4x + 8}{2x - 1}$ has absolute extrema either on the interval $[-3, 0]$ or on the interval $[0, 3]$. Pick one of those intervals, explain why g has extrema on that interval, and find the absolute extrema.

Name:

Secondary Topic 5: Curve sketching

Sketch the graph of $g(x) = 3x^4 - 4x^3 - 36x^2 + 64 = (x+2)^2(3x-4)(x-4)$. We have $g'(x) = 12x^3 - 12x^2 - 72x = 12x(x-3)(x+2)$ and $g''(x) = 36x^2 - 24x - 72 = 12(3x^2 - 2x - 6)$. Your answer should state

(a) the domain of the function
(b) any horizontal or vertical asymptotes
(c) the roots of the function
(d) the critical points of the function
(e) intervals on which the function is increasing or decreasing
(f) any relative minima or maxima
(g) intervals on which the function is concave up or concave down
(h) any inflection points.

Name:

Secondary Topic 6: Applied optimization

We wish to build a rectangular pen with two parallel internal partitions, using 1000 feet of fencing. We want to maximize the total area of the pen.

(a) What is your objective function, and why?
(b) What constraint equation(s) can you use?
(c) What dimensions maximize the total area of the pen? (Prove this is a max.)