Math 1231 Summer 2024
Midterm 2

- You will have 90 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, one-sided, handwritten cheat sheet you have made for yourself ahead of time.
- You may not use a calculator. You may leave answers unsimplified, except you should compute trigonometric functions as far as possible.
- The exam has 5 problems: one review problem, and one problem on each mastery topic we've covered since the first midterm. The exam has 6 pages total.
- The whole test is scored out of 100 points, with the points for individual questions indicated on the exam.
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all. When in doubt, show more work and write complete sentences.
- If you need more paper to show work, I have extra at the front of the room.
- Good luck!

M1:		M2:	
M3a:		M3b:	
S4:		S5:	
S6:		Tot:	/100

Name:

Name:

Problem 1 (Midterm 1 Review). These are very similar to the trickier limit and derivative problems on the first midterm. Part (a) is worth 10 points, and part (b) 15 points.

(a) Evaluate $\lim_{x \to 0} \dfrac{\sin(x^2)\sin(4x)}{x^3}$ **without** using any shortcuts or L'Hôpital's rule.

(b) Compute $\dfrac{d}{dx} \csc\left(\dfrac{\tan(x^2)}{4x\sin(x)}\right)$.

Name:

Problem 2 (M3). Each part is worth 20 points.

(a) The function $f(x) = \dfrac{(x-1)^{\frac{2}{3}}}{x+1}$ has absolute extrema either on $[-2, 0]$ or on $[0, 2]$. Pick one of those intervals, explain why f has extrema on that interval, and find the absolute extrema.

(b) Find and classify all the critical points of $g(x) = \dfrac{x^2 - 3x - 4}{x+5}$. That is, for each critical point you find, say whether it is a relative maximum, a relative minimum, or neither.

Name:

Problem 3 (S4). A street light is mounted at the top of a 12-foot-tall pole. A six-foot-tall man walks straight away from the pole at 4 feet per second. How fast is the length of his shadow changing when he is twenty feet from the pole?

Name:

Problem 4 (S5). Sketch the graph of $f(x) = \dfrac{x^3}{(x-1)^2}$. We have $f'(x) = \dfrac{x^2(x-3)}{(x-1)^3}$ and $f''(x) = \dfrac{6x}{(x-1)^4}$. Your answer should state

(a) the domain of the function
(b) any horizontal or vertical asymptotes
(c) the roots of the function
(d) the critical points of the function
(e) intervals on which the function is increasing or decreasing
(f) any relative minima or maxima
(g) intervals on which the function is concave up or concave down
(h) any inflection points.

Name:

Problem 5 (S6). To check a bag on a certain airplane, the length plus width plus height must be less than or equal to 63in. Assuming the suitcase should be twice as long as it is wide, what height maximizes the volume of the suitcase? **Your answer should include some justification for how you know this is really a maximum.**