FACE AND DIGIT CLASSIFICATION

Erica Cai, Boning Ding, and Shreya Jahagirdar December 7, 2019

1 File Structure

Our project folder contains six files and two folders with images and labels. The six files are:

- a. util.py
- b. prepareFeatures.py
- c. NaiveBayes.py
- d. Perceptron.py
- e. kNN.py
- f. featureFuncLib.py

We wrote prepareFeatures.py, featureFuncLib.py, NaiveBayes.py, Perceptron.py and kNN.py. We used util.py to store all of the code from the Berkeley website that helps us parse each image and find black and white pixels in each image.

In NaiveBayes.py, Perceptron.py, and kNN.py, we implement the Naïve Bayes, Perceptron, and kNN algorithms respectively.

In featureFuncLib.py, we define several feature functions.

In prepareFeatures.py, we wrote a main program to run each machine learning algorithm on the dataset and to analyze each run by calculating the accuracy of the predictions and the amount of time needed to train data.

For this project, there are six machine learning algorithms:

- a. Naïve Bayes for predicting if an image is a face
- b. Naïve Bayes for predicting which digit an image is
- c. Perceptron for predicting if an image is a face
- d. Perceptron for predicting which digit an image is

- e. kNN for predicting if an image is a face
- f. kNN for predicting which digit an image is

2 How to Test the Algorithms

We perform the same set of steps to test each algorithm.

- 1. Store the train images, test images, train labels, and test labels in respective arrays.
- 2. Apply the feature function on the images to get a train features and test features array.
- 3. Train the machine learning algorithm on the train features and train labels.
- 4. Predict the labels for the test features.
- 5. Calculate the percentage of labels that the machine learning algorithm guessed correctly.

Only the Naïve Bayes and Perceptron algorithms do step 3; the kNN algorithm does not have a train function and is discussed more in section 7.

3 How to Store Images

The images that we use in our train set and test set belong to the folders

- a. digitdata, in the files
 - digitdatatest
 - digitdatatrain
- b. facedata, in the files
 - facedatatest
 - facedatatrain

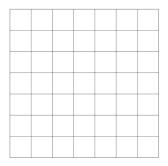
We extract each image and convert it into a "datum" object, which stores information about whether pixels in the image are black, white, or gray.

We store an array of "datum" objects and create features based on each "datum" object.

4 How to generate and store features

We convert each datum into a list of features.

Our feature function for digits splits the datum into 49 parts and counts the number of black pixels in each part. The regions of the data look like below.



Our feature function for faces splits the datum into 42 parts and counts the number of black pixels in each part. The regions of the data look like below.



To count the number of black pixels in each part, we traverse through each pixel of a part and update the number of black pixels in that part.

5 Naive Bayes Algorithm

5.1 Overview for Face Detection

We want to calculate $L(x) = \frac{p(y = true \mid x)}{p(y = false \mid x)} = \frac{p(x \mid y = true)p(y = true)}{p(x \mid y = false)p(y = false)}$. We return true if $L(x) \ge 1$ and false otherwise.

Our train function prepares all of the values necessary for calculating $p(x \mid$

y = true), p(y = true), $p(x \mid y = false)$, p(y = false) based on information in the train set.

Our predict function calculates L(x) for each item in the test set. Based on the L(x) value of an item, the predict function assigns a label of 0 or 1 to that item. A 0 represents that the item is not a face and 1 represents that the item is a face.

5.2 Data Structures for Face Detection

We have one probability array containing information necessary to calculate $p(x \mid y = true)$ and one probability array containing information necessary to calculate $p(x \mid y = false)$.

Each probability array is an array of dictionaries.

We use dictionaries because searching is O(1) time while searching in an array takes O(trainingsize) time.

The probability array has the following form:

```
{dictionary containing (value, probability) pairs for feature 1},
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{dictionary containing (value, probability) pairs for feature 2},

{dictionary containing (value, probability) pairs for feature 3},...

5.3 Method Definitions for Face Detection

Train

Input: train features, train labels

Output: P(face=true), P(face=not true), array1 as described below, array2 as described below

Where array1 is an array containing probabilities given that the image is a face. We organized the array to be an array of dictionaries; each dictionary represents a feature and contains (value, probability) pairs for the likelihood that the feature has that value, given that the image is a face.

Where array2 is an array containing probabilities given that the image is NOT a face. We organized the array to be an array of dictionaries; each dictionary represents a feature and contains (value, probability) pairs for the likelihood that the feature has that value, given that the image is not a face.

Predict

Input: test features, pFace, pNotFace, array1, array2

Output: predicted labels

5.4 Train Algorithm for Face Detection

```
Part 0: This part initializes variables
numberOfFaces=0
numberOfNotFaces{=}0
Array1=[\ ,\ ,\ldots]
Array2{=}[\phantom{1},\phantom{1},\phantom{1},\ldots]
Part 1: This part finds counts
For each feature array in the training set
    If the label shows that the image IS A FACE
          numberOfFaces++
          for feature i in the array of features
                in array1, add feature i to the dictionary that corresponds to feature i and
                update the count that corresponds to feature i
    If the label shows the image IS NOT A FACE
          numberOfNotFaces++
          for feature i in the array of features
                in array2, add feature i to the dictionary that corresponds to feature i and
```

Part 2: This part turns counts into probabilities
For every count in array1, turn it into count/numberOfFaces
For every count in array2, turn it into count/numberOfNotFaces
pFace=numberOfFaces/total
pNotFace=numberOfNotFaces/total
return pFace, pNotFace, array1, array2

5.5 Predict Algorithm for Face Detection

Need to calculate $p(x \mid y = true)$, p(y = true), $p(x \mid y = false)$, p(y = false)

update the count that corresponds to feature i

For each item in test features

Use the formulas below to compute $p(x \mid y = true)$ from array1 and $p(x \mid y = false)$ from array2 Then compute L(x)

Then add label based on the L(x) value into the predict array

$$\begin{split} p(x|y=true) &= \prod_{j=1}^{l} p(\phi_{j}(x)|y=true) \\ p(x|y=false) &= \prod_{j=1}^{l} p(\phi_{j}(x)|y=false) \end{split}$$

5.6 Overview for Digit Identification

Instead of calculating L(x) as for face, calculate $p(x \mid y = true) * p(y = true)$ for each digit and predict the digit with the maximum result in this calculation.

The train and predict algorithm are very similar to that for Face.

5.7 Method Definitions for Digit Identification

Train

Input: train features, train labels

Output: P(digit 0=true), P(digit 1=true), P(digit 2=true), P(digit 3=true), P(digit 4=true), P(digit 5=true), P(digit 6=true), P(digit 7=true), P(digit 8=true), P(digit 9=true), array0, array1, array2, array3, array4, array5, array6, array7, array8, array9

Predict

Input: test features, test labels, P(digit 0=true), P(digit 1=true), P(digit 2=true), P(digit 3=true), P(digit 4=true), P(digit 5=true), P(digit 6=true), P(digit 7=true), P(digit 8=true), P(digit 9=true), array0, array1, array2, array3, array4, array5, array6, array7, array8, array9

Output: predict

6 Perceptron

6.1 Overview for Face Detection

We want to calculate $f(x_i, w) = w_0 + w_1 \Phi_1(x_i) + w_2 \Phi_2(x_i) + w_3 \Phi_3(x_i) + ... + w_l \Phi_l(x_i)$. If ≥ 0 , we predict that it is a face; otherwise, we predict that it is not a face.

Our train function prepares all of the weights necessary for calculating $f(x_i, w) = w_0 + w_1 \Phi_1(x_i) + w_2 \Phi_2(x_i) + w_3 \Phi_3(x_i) + ... + w_l \Phi_l(x_i)$ based on information in the train set.

Our predict function calculates $f(x_i, w)$ for each item in test set assigns a label of 0 or 1 to that item; a 0 represents that the item is not a face and 1 represents

that the item is a face.

6.2 Method Definition and Algorithm of Train for Face Detection

Input: train features, train labels

Output: array of weights

Initialize weights as [0.0001]

Iterate through the train features and labels until you don't have to adjust the

weights anymore, or until a specific time has passed

Return weights

6.3 Method Definition and Algorithm of Predict for Face Detection

Input: test features, array of weights

Output: predict labels

For each test feature array, calculate f(x)

Based on f(x), append 0 or 1 to the predict array, where 0 represents that the item is not a face and 1 represents that the item is a face.

6.4 Overview for Digit Identification

Instead of calculating one $f(x_i, w)$ function as in face indentification, calculate 10 of the them and choose the digit that corresponds to the maximum f value as the digit that the algorithm predicts for an image

6.5 Method Definitions for Digit Identification

TRAIN

Input: train features, train labels

Output: array of weights for digit 0, array of weights for digit 1, array of weights for digit 2, array of weights for digit 3, array of weights for digit 4, array of weights for digit 5, array of weights for digit 6, array of weights for digit 7, array of weights for digit 8, array of weights for digit 9

PREDICT

Input: test features, array of weights for digit 0, array of weights for digit 1, array of weights for digit 2, array of weights for digit 3, array of weights for digit 4, array of weights for digit 5, array of weights for digit 6, array of weights for

digit 7, array of weights for digit 8, array of weights for digit 9 Output: predict labels

7 kNN

Unlike the Naïve Bayes and Perceptron algorithms, kNN only has a predict function. The kNN algorithm takes in training features, training labels, test features, test labels, and a k value and returns an array of predicted labels for the test features.

For each array of features in the test set, the kNN algorithm looks through all of the arrays of features in the training set and chooses the k images that are most similar to the image in the test set. Then it uses the most frequent label of the k images to predict whether the current image in the test set is a face or not.

8 How We Compared the Algorithms

In total, we wanted to compare the performace for 6 algorithms: Naive Bayes for face detection and digit identification, Perceptron for face detection and digit identification, and kNN for face detection and digit identification. We also wanted to compare the performance of these algorithms when using 10%, 20%... 100% of the training set.

We ran each algorithm five times on each training set size. When the training set size was 40% of the entire training data, we implemented an algorithm to randomly select 40% of the entire training data during each of the five trials. As a result of the randomness, our algorithm has slightly different results in accuracy and training time.

Therefore, for each algorithm and training set size, we performed five trials. For each trial, we recorded the accuracy and the time necessary to train the algorithm. kNN does not have a training function; we could not calculate the time necessary to train the algorithm so we recorded the amount of time that the algorithm spent to predict one label for one image.

For each algorithm and training set size, we recorded the average accuracy, the standard deviation of the accuracy, and the average time needed to train the algorithm over all five trials.

9 Analysis of the Naive Bayes Face Detection Algorithm on Each Training Set Size

Our results for the Naive Bayes Face Detection Algorithm are below.

Trial	Accuracy	Error	Training time (seconds)
0	0.76	0.24	0.001
1	0.73	0.27	0.002
2	0.7	0.3	0.001
3	0.64	0.36	0.001
4	0.79	0.21	0.001

Table 1: TRAINING SIZE: 10%

Summary for Table 1 on a Training Size of 10%: Mean Accuracy=0.72, Standard Deviation=0.05, Mean Error=0.28, Average Training Time (Seconds)=0.001

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.8	0.2	0.002
1	0.7	0.3	0.002
2	0.7	0.3	0.004
3	0.74	0.26	0.002
4	0.72	0.28	0.002

Table 2: TRAINING SIZE: 20%

Summary for Table 2 on a Training Size of 20%: Mean Accuracy=0.73, Standard Deviation=0.04, Mean Error=0.27, Average Training Time (Seconds)=0.002

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.76	0.24	0.003
1	0.8	0.2	0.003
2	0.77	0.23	0.003
3	0.82	0.18	0.003
4	0.76	0.24	0.003

Table 3: TRAINING SIZE: 30%

Summary for Table 3 on a Training Size of 30%: Mean Accuracy=0.78, Standard Deviation=0.02, Mean Error=0.22, Average Training Time (Seconds)=0.003

Trial	Accuracy	Error	Training time (seconds)
0	0.81	0.19	0.004
1	0.84	0.16	0.004
2	0.71	0.29	0.005
3	0.69	0.31	0.004
4	0.76	0.24	0.004

Table 4: TRAINING SIZE: 40%

Summary for Table 4 on a Training Size of 40%: Mean Accuracy=0.76, Standard Deviation=0.06, Mean Error=0.24, Average Training Time (Seconds)=0.004

Trial	Accuracy	Error	Training time (seconds)
0	0.78	0.22	0.005
1	0.86	0.14	0.005
2	0.8	0.2	0.005
3	0.82	0.18	0.005
4	0.78	0.22	0.005

Table 5: TRAINING SIZE: 50%

Summary for Table 5 on a Training Size of 50%: Mean Accuracy=0.81, Standard Deviation=0.03, Mean Error=0.19, Average Training Time (Seconds)=0.005

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.76	0.24	0.007
1	0.85	0.15	0.006
2	0.8	0.2	0.007
3	0.79	0.21	0.006
4	0.78	0.22	0.006

Table 6: TRAINING SIZE: 60%

Summary for Table 6 on a Training Size of 60%: Mean Accuracy=0.8, Standard Deviation=0.03, Mean Error=0.2, Average Training Time (Seconds)=0.006

Trial	Accuracy	Error	Training time (seconds)
0	0.84	0.16	0.007
1	0.8	0.2	0.007
2	0.79	0.21	0.008
3	0.83	0.17	0.007
4	0.78	0.22	0.007

Table 7: TRAINING SIZE: 70%

Summary for Table 7 on a Training Size of 70%: Mean Accuracy=0.81, Standard Deviation=0.02, Mean Error=0.19, Average Training Time (Seconds)=0.007

Trial	Accuracy	Error	Training time (seconds)
0	0.84	0.16	0.008
1	0.81	0.19	0.008
2	0.86	0.14	0.009
3	0.84	0.16	0.009
4	0.84	0.16	0.008

Table 8: TRAINING SIZE: 80%

Summary for Table 8 on a Training Size of 80%: Mean Accuracy=0.84, Standard Deviation=0.02, Mean Error=0.16, Average Training Time (Seconds)=0.008

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.88	0.12	0.009
1	0.84	0.16	0.009
2	0.85	0.15	0.009
3	0.86	0.14	0.009
4	0.88	0.12	0.01

Table 9: TRAINING SIZE: 90%

Summary for Table 9 on a Training Size of 90%: Mean Accuracy=0.86, Standard Deviation=0.02, Mean Error=0.14, Average Training Time (Seconds)=0.009

Trial	Accuracy	Error	Training time (seconds)
0	0.88	0.12	0.011
1	0.88	0.12	0.01
2	0.88	0.12	0.011
3	0.88	0.12	0.01
4	0.88	0.12	0.011

Table 10: TRAINING SIZE: 100%

Summary for Table 10 on a Training Size of 100%: Mean Accuracy=0.88, Standard Deviation=0.0, Mean Error=0.12, Average Training Time (Seconds)=0.011

10 Analysis of the Naive Bayes Digit Identification Algorithm on Each Training Set Size

Our results for the Naive Bayes Digit Identification Algorithm are below.

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.64	0.36	0.014
1	0.69	0.31	0.014
2	0.68	0.32	0.013
3	0.58	0.42	0.013
4	0.59	0.41	0.014

Table 11: TRAINING SIZE: 10%

Summary for Table 11 on a Training Size of 10%: Mean Accuracy=0.64, Standard Deviation=0.04, Mean Error=0.36, Average Training Time (Seconds)=0.014

Trial	Accuracy	Error	Training time (seconds)
	0.69	0.31	0.028
1	0.69	0.31	0.028
2	0.69	0.31	0.028
3	0.68	0.32	0.028
4	0.72	0.28	0.028

Table 12: TRAINING SIZE: 20%

Summary for Table 12 on a Training Size of 20%: Mean Accuracy=0.69, Standard Deviation=0.01, Mean Error=0.31, Average Training Time (Seconds)=0.028

Trial	Accuracy	Error	Training time (seconds)
0	0.7	0.3	0.041
1	0.83	0.17	0.042
2	0.76	0.24	0.041
3	0.74	0.26	0.042
4	0.75	0.25	0.042

Table 13: TRAINING SIZE: 30%

Summary for Table 13 on a Training Size of 30%: Mean Accuracy=0.76, Standard Deviation=0.04, Mean Error=0.24, Average Training Time (Seconds)=0.042

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.75	0.25	0.055
1	0.76	0.24	0.055
2	0.76	0.24	0.055
3	0.76	0.24	0.055
4	0.74	0.26	0.057

Table 14: TRAINING SIZE: 40%

Summary for Table 14 on a Training Size of 40%: Mean Accuracy=0.75, Standard Deviation=0.01, Mean Error=0.25, Average Training Time (Seconds)=0.055

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.76	0.24	0.069
1	0.78	0.22	0.069
2	0.8	0.2	0.069
3	0.77	0.23	0.069
4	0.78	0.22	0.07

Table 15: TRAINING SIZE: 50%

Summary for Table 15 on a Training Size of 50%: Mean Accuracy=0.78, Standard Deviation=0.01, Mean Error=0.22, Average Training Time (Seconds)=0.069

Trial	Accuracy	Error	Training time (seconds)
0	0.8	0.2	0.083
1	0.79	0.21	0.089
2	0.81	0.19	0.084
3	0.79	0.21	0.085
4	0.8	0.2	0.085

Table 16: TRAINING SIZE: 60%

Summary for Table 16 on a Training Size of 60%: Mean Accuracy=0.8, Standard Deviation=0.01, Mean Error=0.2, Average Training Time (Seconds)=0.085

Trial	Accuracy	Error	Training time (seconds)
	0.77	0.23	0.097
1	0.82	0.18	0.097
2	0.81	0.19	0.096
3	0.82	0.18	0.098
4	0.82	0.18	0.098

Table 17: TRAINING SIZE: 70%

Summary for Table 17 on a Training Size of 70%: Mean Accuracy=0.81, Standard Deviation=0.02, Mean Error=0.19, Average Training Time (Seconds)=0.097

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.78	0.22	0.115
1	0.77	0.23	0.112
2	0.8	0.2	0.116
3	0.81	0.19	0.113
4	0.79	0.21	0.112

Table 18: TRAINING SIZE: 80%

Summary for Table 18 on a Training Size of 80%: Mean Accuracy=0.79, Standard Deviation=0.01, Mean Error=0.21, Average Training Time (Seconds)=0.114

Trial	Accuracy	Error	Training time (seconds)
0	0.82	0.18	0.126
1	0.82	0.18	0.127
2	0.8	0.2	0.126
3	0.79	0.21	0.127
4	0.8	0.2	0.126

Table 19: TRAINING SIZE: 90%

Summary for Table 19 on a Training Size of 90%: Mean Accuracy=0.81, Standard Deviation=0.01, Mean Error=0.19, Average Training Time (Seconds)=0.126

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.81	0.19	0.145
1	0.81	0.19	0.141
2	0.81	0.19	0.141
3	0.81	0.19	0.142
4	0.81	0.19	0.139

Table 20: TRAINING SIZE: 100%

Summary for Table 20 on a Training Size of 100%: Mean Accuracy=0.81, Standard Deviation=0.0, Mean Error=0.19, Average Training Time (Seconds)=0.142

11 Analysis of the Perceptron Face Detection Algorithm on Each Training Set Size

Our results for the Perceptron Face Detection Algorithm are below.

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.66	0.34	0.011
1	0.71	0.29	0.002
2	0.61	0.39	0.047
3	0.65	0.35	0.018
4	0.7	0.3	0.009

Table 21: TRAINING SIZE: 10%

Summary for Table 21 on a Training Size of 10%: Mean Accuracy=0.67, Standard Deviation=0.04, Mean Error=0.33, Average Training Time (Seconds)=0.017

Trial	Accuracy	Error	Training time (seconds)
0	0.75	0.25	0.043
1	0.69	0.31	0.109
2	0.73	0.27	0.114
3	0.73	0.27	0.062
4	0.74	0.26	0.103

Table 22: TRAINING SIZE: 20%

Summary for Table 22 on a Training Size of 20%: Mean Accuracy=0.73, Standard Deviation=0.02, Mean Error=0.27, Average Training Time (Seconds)=0.086

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.78	0.22	0.169
1	0.76	0.24	0.188
2	0.77	0.23	0.172
3	0.69	0.31	0.182
4	0.62	0.38	0.195

Table 23: TRAINING SIZE: 30%

Summary for Table 23 on a Training Size of 30%: Mean Accuracy=0.72, Standard Deviation=0.06, Mean Error=0.28, Average Training Time (Seconds)=0.181

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.71	0.29	0.241
1	0.74	0.26	0.244
2	0.74	0.26	0.229
3	0.69	0.31	0.247
4	0.77	0.23	0.231

Table 24: TRAINING SIZE: 40%

Summary for Table 24 on a Training Size of 40%: Mean Accuracy=0.73, Standard Deviation=0.03, Mean Error=0.27, Average Training Time (Seconds)=0.238

Trial	Accuracy	Error	Training time (seconds)
0	0.71	0.29	0.292
1	0.75	0.25	0.305
2	0.73	0.27	0.297
3	0.71	0.29	0.312
4	0.78	0.22	0.296

Table 25: TRAINING SIZE: 50%

Summary for Table 25 on a Training Size of 50%: Mean Accuracy=0.74, Standard Deviation=0.03, Mean Error=0.26, Average Training Time (Seconds)=0.3

Trial	Accuracy	Error	Training time (seconds)
0	0.59	0.41	0.388
1	0.76	0.24	0.503
2	0.76	0.24	0.385
3	0.78	0.22	0.384
4	0.79	0.21	0.359

Table 26: TRAINING SIZE: 60%

Summary for Table 26 on a Training Size of 60%: Mean Accuracy=0.74, Standard Deviation=0.07, Mean Error=0.26, Average Training Time (Seconds)=0.404

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.77	0.23	0.449
1	0.74	0.26	0.435
2	0.79	0.21	0.453
3	0.74	0.26	0.48
4	0.75	0.25	0.45

Table 27: TRAINING SIZE: 70%

Summary for Table 27 on a Training Size of 70%: Mean Accuracy=0.76, Standard Deviation=0.02, Mean Error=0.24, Average Training Time (Seconds)=0.453

Trial	Accuracy	Error	Training time (seconds)
0	0.75	0.25	0.511
1	0.79	0.21	0.513
2	0.8	0.2	0.552
3	0.75	0.25	0.521
4	0.74	0.26	0.531

Table 28: TRAINING SIZE: 80%

Summary for Table 28 on a Training Size of 80%: Mean Accuracy=0.77, Standard Deviation=0.02, Mean Error=0.23, Average Training Time (Seconds)=0.526

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.67	0.33	0.568
1	0.77	0.23	0.563
2	0.77	0.23	0.558
3	0.79	0.21	0.567
4	0.74	0.26	0.567

Table 29: TRAINING SIZE: 90%

Summary for Table 29 on a Training Size of 90%: Mean Accuracy=0.75, Standard Deviation=0.04, Mean Error=0.25, Average Training Time (Seconds)=0.565

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.67	0.33	0.64
1	0.77	0.23	0.633
2	0.77	0.23	0.636
3	0.7	0.3	0.635
4	0.65	0.35	0.644

Table 30: TRAINING SIZE: 100%

Summary for Table 30 on a Training Size of 100%:Mean Accuracy=0.71, Standard Deviation=0.05, Mean Error=0.29, Average Training Time (Seconds)=0.638

12 Analysis of the Perceptron Digit Identification Algorithm on Each Training Set Size

Our results for the Perceptron Digit Identification Algorithm are below.

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.71	0.29	8.239
1	0.73	0.27	7.127
2	0.77	0.23	8.173
3	0.62	0.38	8.107
4	0.64	0.36	7.247

Table 31: TRAINING SIZE: 10%

Summary for Table 31 on a Training Size of 10%: Mean Accuracy=0.69, Standard Deviation=0.06, Mean Error=0.31, Average Training Time (Seconds)=7.779

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.72	0.28	18.35
1	0.74	0.26	17.803
2	0.76	0.24	17.615
3	0.76	0.24	17.5
4	0.78	0.22	15.661

Table 32: TRAINING SIZE: 20%

Summary for Table 32 on a Training Size of 20%: Mean Accuracy=0.75, Standard Deviation=0.02, Mean Error=0.25, Average Training Time (Seconds)=17.386

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.79	0.21	26.515
1	0.74	0.26	26.519
2	0.71	0.29	26.719
3	0.71	0.29	26.493
4	0.78	0.22	26.458

Table 33: TRAINING SIZE: 30%

Summary for Table 33 on a Training Size of 30%: Mean Accuracy=0.75, Standard Deviation=0.03, Mean Error=0.25, Average Training Time (Seconds)=26.54

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.7	0.3	35.74
1	0.65	0.35	35.296
2	0.69	0.31	35.645
3	0.75	0.25	35.338
4	0.77	0.23	35.317

Table 34: TRAINING SIZE: 40%

Summary for Table 34 on a Training Size of 40%: Mean Accuracy=0.71, Standard Deviation=0.04, Mean Error=0.29, Average Training Time (Seconds)=35.467

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.73	0.27	44.312
1	0.78	0.22	44.512
2	0.77	0.23	44.602
3	0.8	0.2	44.578
4	0.82	0.18	44.261

Table 35: TRAINING SIZE: 50%

Summary for Table 35 on a Training Size of 50%: Mean Accuracy=0.78, Standard Deviation=0.03, Mean Error=0.22, Average Training Time (Seconds)=44.453

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.77	0.23	53.416
1	0.76	0.24	53.193
2	0.69	0.31	54.296
3	0.86	0.14	53.523
4	0.78	0.22	53.393

Table 36: TRAINING SIZE: 60%

Summary for Table 36 on a Training Size of 60%: Mean Accuracy=0.77, Standard Deviation=0.05, Mean Error=0.23, Average Training Time (Seconds)=53.564

Trial	Accuracy	Error	Training time (seconds)
0	0.8	0.2	62.602
1	0.77	0.23	62.664
2	0.81	0.19	62.719
3	0.77	0.23	63.035
4	0.81	0.19	63.752

Table 37: TRAINING SIZE: 70%

Summary for Table 37 on a Training Size of 70%: Mean Accuracy=0.79, Standard Deviation=0.02, Mean Error=0.21, Average Training Time (Seconds)=62.954

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.78	0.22	72.469
1	0.78	0.22	72.138
2	0.79	0.21	73.232
3	0.73	0.27	72.964
$\parallel 4$	0.79	0.21	74.094

Table 38: TRAINING SIZE: 80%

Summary for Table 38 on a Training Size of 80%: Mean Accuracy=0.77, Standard Deviation=0.02, Mean Error=0.23, Average Training Time (Seconds)=72.98

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.77	0.23	84.443
1	0.76	0.24	83.723
2	0.78	0.22	84.17
3	0.75	0.25	83.871
4	0.78	0.22	84.22

Table 39: TRAINING SIZE: 90%

Summary for Table 39 on a Training Size of 90%: Mean Accuracy=0.77, Standard Deviation=0.01, Mean Error=0.23, Average Training Time (Seconds)=84.085

Trial	Accuracy	Error	Training time (seconds)
0	0.73	0.27	93.306
1	0.78	0.22	91.718
2	0.7	0.3	89.916
3	0.72	0.28	89.939
4	0.76	0.24	90.01

Table 40: TRAINING SIZE: 100%

Summary for Table 40 on a Training Size of 100%: Mean Accuracy=0.74, Standard Deviation=0.03, Mean Error=0.26, Average Training Time (Seconds)=90.978

13 Analysis of the kNN Face Detection Algorithm on Each Training Set Size

Our results for the kNN Face Detection Algorithm are below.

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.7	0.3	0.001
1	0.65	0.35	0.001
2	0.65	0.35	0.001
3	0.69	0.31	0.001
4	0.62	0.38	0.001

Table 41: TRAINING SIZE: 10%

Summary for Table 41 on a Training Size of 10%: Mean Accuracy=0.66, Standard Deviation=0.03, Mean Error=0.34, Average Prediction Time Per Image (Seconds)=0.001

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.69	0.31	0.001
1	0.71	0.29	0.001
2	0.59	0.41	0.001
3	0.61	0.39	0.002
4	0.63	0.37	0.002

Table 42: TRAINING SIZE: 20%

Summary for Table 42 on a Training Size of 20%: Mean Accuracy=0.65, Standard Deviation=0.05, Mean Error=0.35, Average Prediction Time Per Image (Seconds)=0.001

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.74	0.26	0.002
1	0.68	0.32	0.002
2	0.74	0.26	0.002
3	0.74	0.26	0.002
4	0.65	0.35	0.002

Table 43: TRAINING SIZE: 30%

Summary for Table 43 on a Training Size of 30%: Mean Accuracy=0.71, Standard Deviation=0.04, Mean Error=0.29, Average Prediction Time Per Image (Seconds)=0.002

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.67	0.33	0.003
1	0.66	0.34	0.003
2	0.72	0.28	0.003
3	0.68	0.32	0.003
4	0.68	0.32	0.003

Table 44: TRAINING SIZE: 40%

Summary for Table 44 on a Training Size of 40%: Mean Accuracy=0.68, Standard Deviation=0.02, Mean Error=0.32, Average Prediction Time Per Image (Seconds)=0.003

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.72	0.28	0.004
1	0.69	0.31	0.004
2	0.69	0.31	0.004
3	0.69	0.31	0.004
4	0.69	0.31	0.004

Table 45: TRAINING SIZE: 50%

Summary for Table 45 on a Training Size of 50%: Mean Accuracy=0.7, Standard Deviation=0.01, Mean Error=0.3, Average Prediction Time Per Image (Seconds)=0.004

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.72	0.28	0.004
1	0.72	0.28	0.004
2	0.75	0.25	0.004
3	0.72	0.28	0.004
4	0.69	0.31	0.004

Table 46: TRAINING SIZE: 60%

Summary for Table 46 on a Training Size of 60%: Mean Accuracy=0.72, Standard Deviation=0.02, Mean Error=0.28, Average Prediction Time Per Image (Seconds)=0.004

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.69	0.31	0.005
1	0.67	0.33	0.005
2	0.71	0.29	0.005
3	0.65	0.35	0.005
4	0.76	0.24	0.005

Table 47: TRAINING SIZE: 70%

Summary for Table 47 on a Training Size of 70%: Mean Accuracy=0.7, Standard Deviation=0.04, Mean Error=0.3, Average Prediction Time Per Image (Seconds)=0.005

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.73	0.27	0.006
1	0.7	0.3	0.006
2	0.7	0.3	0.006
3	0.74	0.26	0.006
4	0.68	0.32	0.006

Table 48: TRAINING SIZE: 80%

Summary for Table 48 on a Training Size of 80%: Mean Accuracy=0.71, Standard Deviation=0.02, Mean Error=0.29, Average Prediction Time Per Image (Seconds)=0.006

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.72	0.28	0.007
1	0.72	0.28	0.007
2	0.74	0.26	0.007
3	0.75	0.25	0.007
4	0.7	0.3	0.007

Table 49: TRAINING SIZE: 90%

Summary for Table 49 on a Training Size of 90%: Mean Accuracy=0.73, Standard Deviation=0.02, Mean Error=0.27, Average Prediction Time Per Image (Seconds)=0.007

Trial	Accuracy	Error	Prediction Time Per Image (sec-
			onds)
0	0.72	0.28	0.007
1	0.72	0.28	0.007
2	0.72	0.28	0.007
3	0.72	0.28	0.007
4	0.72	0.28	0.007

Table 50: TRAINING SIZE: 100%

Summary for Table 50 on a Training Size of 100%: Mean Accuracy=0.72, Standard Deviation=0.0, Mean Error=0.28, Average Prediction Time Per Image (Seconds)=0.007

14 Analysis of the kNN Digit Identification Algorithm on Each Training Set Size

Our results for the kNN Digit Identification Algorithm are below.

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.77	0.23	0.009
1	0.77	0.23	0.009
2	0.77	0.23	0.009
3	0.78	0.22	0.009
4	0.82	0.18	0.009

Table 51: TRAINING SIZE: 10%

Summary for Table 51 on a Training Size of 10%: Mean Accuracy=0.78, Standard Deviation=0.02, Mean Error=0.22, Average Prediction Time Per Image (Seconds)=0.009

Trial	Accuracy	Error	Prediction Time Per Image (sec-
0	0.83	0.17	onds) 0.018
1	0.85	0.15	0.018
2	0.82	0.18	0.018
3	0.79	0.21	0.018
4	0.84	0.16	0.018

Table 52: TRAINING SIZE: 20%

Summary for Table 52 on a Training Size of 20%: Mean Accuracy=0.83, Standard Deviation=0.02, Mean Error=0.17, Average Prediction Time Per Image (Seconds)=0.018

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.85	0.15	0.027
1	0.87	0.13	0.027
2	0.86	0.14	0.027
3	0.88	0.12	0.027
4	0.85	0.15	0.027

Table 53: TRAINING SIZE: 30%

Summary for Table 53 on a Training Size of 30%: Mean Accuracy=0.86, Standard Deviation=0.01, Mean Error=0.14, Average Prediction Time Per Image (Seconds)=0.027

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.83	0.17	0.036
1	0.89	0.11	0.036
2	0.91	0.09	0.036
3	0.89	0.11	0.036
4	0.84	0.16	0.036

Table 54: TRAINING SIZE: 40%

Summary for Table 54 on a Training Size of 40%: Mean Accuracy=0.87, Standard Deviation=0.03, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.036

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.85	0.15	0.045
1	0.88	0.12	0.045
2	0.87	0.13	0.046
3	0.86	0.14	0.045
4	0.85	0.15	0.045

Table 55: TRAINING SIZE: 50%

Summary for Table 55 on a Training Size of 50%: Mean Accuracy=0.86, Standard Deviation=0.01, Mean Error=0.14, Average Prediction Time Per Image (Seconds)=0.045

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.87	0.13	0.054
1	0.89	0.11	0.054
2	0.86	0.14	0.055
3	0.86	0.14	0.054
4	0.86	0.14	0.054

Table 56: TRAINING SIZE: 60%

Summary for Table 56 on a Training Size of 60%: Mean Accuracy=0.87, Standard Deviation=0.01, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.054

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.9	0.1	0.066
1	0.87	0.13	0.063
2	0.84	0.16	0.063
3	0.89	0.11	0.063
4	0.86	0.14	0.063

Table 57: TRAINING SIZE: 70%

Summary for Table 57 on a Training Size of 70%: Mean Accuracy=0.87, Standard Deviation=0.02, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.064

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.88	0.12	0.072
1	0.88	0.12	0.073
2	0.86	0.14	0.072
3	0.87	0.13	0.072
4	0.88	0.12	0.074

Table 58: TRAINING SIZE: 80%

Summary for Table 58 on a Training Size of 80%: Mean Accuracy=0.87, Standard Deviation=0.01, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.073

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.88	0.12	0.082
1	0.87	0.13	0.082
2	0.87	0.13	0.083
3	0.88	0.12	0.082
4	0.87	0.13	0.082

Table 59: TRAINING SIZE: 90%

Summary for Table 59 on a Training Size of 90%: Mean Accuracy=0.87, Standard Deviation=0.0, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.082

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.88	0.12	0.091
1	0.88	0.12	0.094
2	0.88	0.12	0.092
3	0.88	0.12	0.092
4	0.88	0.12	0.092

Table 60: TRAINING SIZE: 100%

Summary for Table 60 on a Training Size of 100%: Mean Accuracy=0.88, Standard Deviation=0.0, Mean Error=0.12, Average Prediction Time Per Image (Seconds)=0.092

15 Analysis