### FACE AND DIGIT CLASSIFICATION

## Erica Cai, Boning Ding, and Shreya Jahagirdar December 9, 2019

### 1 File Structure

Our project folder contains seven Python files and two folders with images and labels. The seven Python files are:

- a. util.py
- b. prepareFeatures.py
- c. NaiveBayes.py
- d. Perceptron.py
- e. kNN.py
- f. featureFuncLib.py
- g. demo.py

We wrote prepareFeatures.py, featureFuncLib.py, NaiveBayes.py, Perceptron.py and kNN.py. We used util.py to store all of the code from the Berkeley website that helps us to convert each image into a data structure in which we can identify black and white pixels.

In NaiveBayes.py, Perceptron.py, and kNN.py, we implement the Naïve Bayes, Perceptron, and kNN algorithms respectively.

In featureFuncLib.py, we define several feature functions.

In prepareFeatures.py, we wrote a main program to run each machine learning algorithm on the dataset and to analyze each run by calculating the accuracy of the predictions and the amount of time needed for training the data.

In demo.py, we wrote a main program to run each machine learning algorithm on 100% of the training data, which we can show during the demo since it does not run too many trials. For this project, there are six machine learning algorithms:

- a. Naïve Bayes for predicting whether an image is a face
- b. Naïve Bayes for predicting which digit an image is
- c. Perceptron for predicting whether an image is a face
- d. Perceptron for predicting which digit an image is
- e. kNN for predicting whether an image is a face
- f. kNN for predicting which digit an image is

### 2 How We Tested the Algorithms

To run the algorithms, type "python prepareFeatures.py" in a Python shell. The main program in prepareFeatures.py will execute all six algorithms on different training set sizes, outputting statistics such as the accuracy for prediction 100 labels and the average training time per run.

We perform the same set of steps to test each algorithm.

- 1. Store the train images, test images, train labels, and test labels from data files in respective arrays.
- 2. Apply the feature function on the images to get a train features and test features array.
- 3. Select a certain percentage of the training features and labels to use for training the machine learning algorithm
- 4. Train the machine learning algorithm on the train features and train labels.
- 5. Predict the labels for the test features.
- 6. Calculate the percentage of labels that the machine learning algorithm guessed correctly.

Only the Naïve Bayes and Perceptron algorithms complete step 4; the kNN algorithm does not have a train function and is discussed more in section 7.

### 3 How We Stored the Images

We have a total of 450 images for face training and have 5000 images for digit training. The images that we use in our train set and test set belong to the folders

a. digitdata, in the files digitdatatest digitdatatrain

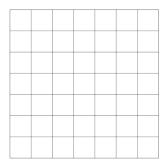
We extract each image and convert it into a data structure which stores information about whether pixels in the image are black, white, or gray.

We store an array of the data structure and create features based on the information stored in each data structure.

### 4 How We Generated and Stored Features

We implement a datum as a data structure that stores information about an image. We convert each datum into a list of features.

Our feature function for digits splits the datum into 49 parts and counts the number of black pixels in each part. The regions of the data look like below.



Our feature function for faces splits the datum into 42 parts and counts the number of black pixels in each part. The regions of the data look like below.



To count the number of black pixels in each part, we traverse through each pixel of a part and update the number of black pixels in that part.

We implemented several other feature functions in featureFuncLib.py but used the two above because they helped to give the best accuracy.

#### Naive Bayes Algorithm 5

### Overview for Face Detection

We want to calculate  $L(x) = \frac{p(y = true \mid x)}{p(y = false \mid x)} = \frac{p(x \mid y = true)p(y = true)}{p(x \mid y = false)p(y = false)}$ . We return true if  $L(x) \ge 1$  and false otherwise.

Our train function prepares all of the values necessary for calculating  $p(x \mid$ y = true), p(y = true),  $p(x \mid y = false)$ , p(y = false) based on information in the train set.

Our predict function calculates L(x) for each item in the test set. Based on the L(x) value of an item, the predict function assigns a label of 0 or 1 to that item. A 0 represents that the item is not a face and 1 represents that the item is a face.

#### 5.2 Data Structures for Face Detection

We have one probability array containing information necessary to calculate  $p(x \mid y = true)$  and one probability array containing information necessary to calculate  $p(x \mid y = false)$ .

Each probability array is an array of dictionaries.

We use dictionaries because searching is O(1) time while searching in an array takes O(trainingsize) time.

The probability array has the following form:

```
{dictionary containing (value, probability) pairs for feature 1},
{dictionary containing (value, probability) pairs for feature 2},
{dictionary containing (value, probability) pairs for feature 3},...
```

### 5.3 Method Definitions for Face Detection

Train

Input: train features, train labels

Output: P(face=true), P(face=not true), array1 as described below, array2 as described below

Where array1 is an array containing probabilities given that the image is a face. We organized the array to be an array of dictionaries; each dictionary represents a feature and contains (value, probability) pairs for the likelihood that the feature has that value, given that the image is a face.

Where array2 is an array containing probabilities given that the image is NOT a face. We organized the array to be an array of dictionaries; each dictionary represents a feature and contains (value, probability) pairs for the likelihood that the feature has that value, given that the image is not a face.

Predict

Input: test features, pFace, pNotFace, array1, array2 Output: predicted labels

### 5.4 Train Algorithm for Face Detection

```
\begin{tabular}{ll} Part 0: This part initializes variables \\ numberOfFaces=0 \\ numberOfNotFaces=0 \\ Array1=[\phantom{+},\phantom{+},\ldots] \\ Array2=[\phantom{+},\phantom{+},\ldots] \\ \end{tabular}
```

```
Part 1: This part finds counts

For each feature array in the training set

If the label shows that the image IS A FACE

numberOfFaces++

for feature i in the array of features

in array1, add feature i to the dictionary that corresponds to feature i and update the count that corresponds to feature i
```

If the label shows the image IS NOT A FACE

numberOfNotFaces++

for feature i in the array of features

in array2, add feature i to the dictionary that corresponds to feature i and update the count that corresponds to feature i

Part 2: This part turns counts into probabilities

For every count in array1, turn it into count/numberOfFaces

For every count in array2, turn it into count/numberOfNotFaces

pFace=numberOfFaces/total

pNotFace = numberOfNotFaces/total

return pFace, pNotFace, array1, array2

### 5.5 Predict Algorithm for Face Detection

Need to calculate  $p(x \mid y = true)$ , p(y = true),  $p(x \mid y = false)$ , p(y = false)

For each item in test features

Use the formulas below to compute  $p(x \mid y = true)$  from array1 and  $p(x \mid y = false)$  from array2 Then compute L(x)

Then add label based on the L(x) value into the predict array

$$\begin{split} p(x|y=true) &= \prod_{j=1}^{l} p(\phi_{j}(x)|y=true) \\ p(x|y=false) &= \prod_{j=1}^{l} p(\phi_{j}(x)|y=false) \end{split}$$

### 5.6 Overview for Digit Identification

Instead of calculating L(x) as for face, calculate  $p(x \mid y = true) * p(y = true)$  for each digit and predict the digit with the maximum result in this calculation.

The train and predict algorithm are very similar to that for face detection.

### 5.7 Method Definitions for Digit Identification

Train

Input: train features, train labels

Output: P(digit 0=true), P(digit 1=true), P(digit 2=true), P(digit 3=true), P(digit 4=true), P(digit 5=true), P(digit 6=true), P(digit 7=true), P(digit 8=true), P(digit 9=true), array0, array1, array2, array3, array4, array5, array6, array7, array8, array9

### Predict

Input: test features, test labels, P(digit 0=true), P(digit 1=true), P(digit 2=true), P(digit 3=true), P(digit 4=true), P(digit 5=true), P(digit 6=true), P(digit 7=true), P(digit 8=true), P(digit 9=true), array0, array1, array2, array3, array4, array5, array6, array7, array8, array9

Output: predict

Where array1,2,...10 is an array containing probabilities given that the image is a face. We organized the array to be an array of dictionaries; each dictionary represents a feature and contains (value, probability) pairs for the likelihood that the feature has that value, given that the image is a face.

### 6 Perceptron

### 6.1 Overview for Face Detection

We calculate  $f(x_i, w) = w_0 + w_1 \Phi_1(x_i) + w_2 \Phi_2(x_i) + w_3 \Phi_3(x_i) + ... + w_l \Phi_l(x_i)$ . If  $f(x_i, w) \ge 0$ , we predict that it is a face; otherwise, we predict that it is not a face.

Our train function prepares all of the weights necessary for calculating  $f(x_i, w) = w_0 + w_1 \Phi_1(x_i) + w_2 \Phi_2(x_i) + w_3 \Phi_3(x_i) + ... + w_l \Phi_l(x_i)$  based on information in the train set.

Our predict function calculates  $f(x_i, w)$  for each item in test set assigns a label of 0 or 1 to that item; a 0 represents that the item is not a face and 1 represents that the item is a face.

## 6.2 Method Definition and Algorithm of Train for Face Detection

Input: train features, train labels

Output: array of weights

Initialize weights as [0.0001]

Iterate through the train features and labels until you don't have to adjust the weights anymore, or until a specific time has passed Return weights

We specified the time limit to be 136 iterations. After the train function has completed 136 iterations of adjusting weights, it will return the array of weights

as it exists after the 136 iterations.

## 6.3 Method Definition and Algorithm of Predict for Face Detection

Input: test features, array of weights

Output: predict labels

For each test feature array, calculate f(x)

Based on f(x), append 0 or 1 to the predict array, where 0 represents that the item is not a face and 1 represents that the item is a face.

### 6.4 Overview for Digit Identification

Instead of calculating one  $f(x_i, w)$  function as in face indentification, calculate 10 of the them and choose the digit that corresponds to the maximum f value as the digit that the algorithm predicts for an image

### 6.5 Method Definitions for Digit Identification

Train

Input: train features, train labels

Output: array of weights for digit 0, array of weights for digit 1, array of weights for digit 2, array of weights for digit 3, array of weights for digit 4, array of weights for digit 5, array of weights for digit 6, array of weights for digit 7, array of weights for digit 8, array of weights for digit 9

### Predict

Input: test features, array of weights for digit 0, array of weights for digit 1, array of weights for digit 2, array of weights for digit 3, array of weights for digit 4, array of weights for digit 5, array of weights for digit 6, array of weights for digit 7, array of weights for digit 8, array of weights for digit 9

Output: predict labels

We specified the time limit to be 175 iterations. After the train function has completed 175 iterations of adjusting weights, it will return the array of weights as it exists after the 175 iterations.

### 7 kNN

Unlike the Naïve Bayes and Perceptron algorithms, kNN only has a predict function. The kNN algorithm takes in training features, training labels, test features, test labels, and a k value and returns an array of predicted labels for the test features.

For each array of features in the test set, the kNN algorithm looks through all of the arrays of features in the training set and chooses the k images that are most similar to the image in the test set. Then it uses the most frequent label of the k images to predict whether the current image in the test set is a face or not.

In our trials, we noticed that the prediction accuracy is highest when k=1. As a result, we recorded data from trials only when we assigned k to be 1.

However, our code works for any k-value.

### 8 How We Compared the Algorithms

In total, we wanted to compare the performace for 6 algorithms: Naive Bayes for face detection and digit identification, Perceptron for face detection and digit identification, and kNN for face detection and digit identification. We also wanted to compare the performance of these algorithms when using 10%, 20%... 100% of the training set.

We ran each algorithm five times on each training set size. When the training set size was 40% of the entire training data, we implemented an algorithm to randomly select 40% of the entire training data during each of the five trials. As a result of the randomness, our algorithm has slightly different results in accuracy and training time.

Therefore, for each algorithm and training set size, we performed five trials. For each trial, we recorded the accuracy and the time necessary to train the algorithm. kNN does not have a training function; we could not calculate the time necessary to train the algorithm so we recorded the amount of time that the algorithm spent to predict one label for one image.

For each algorithm and training set size, we recorded the average accuracy, the standard deviation of the accuracy, and the average time needed to train the algorithm over all five trials.

## 9 Analysis of the Naive Bayes Face Detection Algorithm on Each Training Set Size

Our results for the Naive Bayes Face Detection Algorithm are below.

Trial	Accuracy	Error	Training time (seconds)
0	0.76	0.24	0.001
1	0.73	0.27	0.002
2	0.7	0.3	0.001
3	0.64	0.36	0.001
4	0.79	0.21	0.001

Table 1: TRAINING SIZE: 10%

Summary for Table 1 on a Training Size of 10%: Mean Accuracy=0.72, Standard Deviation=0.05, Mean Error=0.28, Average Training Time (Seconds)=0.001

Trial	Accuracy	Error	Training time (seconds)
0	0.8	0.2	0.002
1	0.7	0.3	0.002
2	0.7	0.3	0.004
3	0.74	0.26	0.002
4	0.72	0.28	0.002

Table 2: TRAINING SIZE: 20%

Summary for Table 2 on a Training Size of 20%: Mean Accuracy=0.73, Standard Deviation=0.04, Mean Error=0.27, Average Training Time (Seconds)=0.002

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.76	0.24	0.003
1	0.8	0.2	0.003
2	0.77	0.23	0.003
3	0.82	0.18	0.003
4	0.76	0.24	0.003

Table 3: TRAINING SIZE: 30%

Summary for Table 3 on a Training Size of 30%: Mean Accuracy=0.78, Standard Deviation=0.02, Mean Error=0.22, Average Training Time (Seconds)=0.003

Trial	Accuracy	Error	Training time (seconds)
0	0.81	0.19	0.004
1	0.84	0.16	0.004
2	0.71	0.29	0.005
3	0.69	0.31	0.004
4	0.76	0.24	0.004

Table 4: TRAINING SIZE: 40%

Summary for Table 4 on a Training Size of 40%: Mean Accuracy=0.76, Standard Deviation=0.06, Mean Error=0.24, Average Training Time (Seconds)=0.004

Trial	Accuracy	Error	Training time (seconds)
0	0.78	0.22	0.005
1	0.86	0.14	0.005
2	0.8	0.2	0.005
3	0.82	0.18	0.005
4	0.78	0.22	0.005

Table 5: TRAINING SIZE: 50%

Summary for Table 5 on a Training Size of 50%: Mean Accuracy=0.81, Standard Deviation=0.03, Mean Error=0.19, Average Training Time (Seconds)=0.005

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.76	0.24	0.007
1	0.85	0.15	0.006
2	0.8	0.2	0.007
3	0.79	0.21	0.006
4	0.78	0.22	0.006

Table 6: TRAINING SIZE: 60%

Summary for Table 6 on a Training Size of 60%: Mean Accuracy=0.8, Standard Deviation=0.03, Mean Error=0.2, Average Training Time (Seconds)=0.006

Trial	Accuracy	Error	Training time (seconds)
0	0.84	0.16	0.007
1	0.8	0.2	0.007
2	0.79	0.21	0.008
3	0.83	0.17	0.007
4	0.78	0.22	0.007

Table 7: TRAINING SIZE: 70%

Summary for Table 7 on a Training Size of 70%: Mean Accuracy=0.81, Standard Deviation=0.02, Mean Error=0.19, Average Training Time (Seconds)=0.007

Trial	Accuracy	Error	Training time (seconds)
0	0.84	0.16	0.008
1	0.81	0.19	0.008
2	0.86	0.14	0.009
3	0.84	0.16	0.009
4	0.84	0.16	0.008

Table 8: TRAINING SIZE: 80%

Summary for Table 8 on a Training Size of 80%: Mean Accuracy=0.84, Standard Deviation=0.02, Mean Error=0.16, Average Training Time (Seconds)=0.008

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.88	0.12	0.009
1	0.84	0.16	0.009
2	0.85	0.15	0.009
3	0.86	0.14	0.009
4	0.88	0.12	0.01

Table 9: TRAINING SIZE: 90%

Summary for Table 9 on a Training Size of 90%: Mean Accuracy=0.86, Standard Deviation=0.02, Mean Error=0.14, Average Training Time (Seconds)=0.009

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.88	0.12	0.011
1	0.88	0.12	0.01
2	0.88	0.12	0.011
3	0.88	0.12	0.01
4	0.88	0.12	0.011

Table 10: TRAINING SIZE: 100%

Summary for Table 10 on a Training Size of 100%: Mean Accuracy=0.88, Standard Deviation=0.0, Mean Error=0.12, Average Training Time (Seconds)=0.011

## 10 Analysis of the Naive Bayes Digit Identification Algorithm on Each Training Set Size

Our results for the Naive Bayes Digit Identification Algorithm are below.

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.64	0.36	0.014
1	0.69	0.31	0.014
2	0.68	0.32	0.013
3	0.58	0.42	0.013
4	0.59	0.41	0.014

Table 11: TRAINING SIZE: 10%

Summary for Table 11 on a Training Size of 10%: Mean Accuracy=0.64, Standard Deviation=0.04, Mean Error=0.36, Average Training Time (Seconds)=0.014

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.69	0.31	0.028
1	0.69	0.31	0.028
2	0.69	0.31	0.028
3	0.68	0.32	0.028
4	0.72	0.28	0.028

Table 12: TRAINING SIZE: 20%

Summary for Table 12 on a Training Size of 20%: Mean Accuracy=0.69, Standard Deviation=0.01, Mean Error=0.31, Average Training Time (Seconds)=0.028

Trial	Accuracy	Error	Training time (seconds)
0	0.7	0.3	0.041
1	0.83	0.17	0.042
2	0.76	0.24	0.041
3	0.74	0.26	0.042
4	0.75	0.25	0.042

Table 13: TRAINING SIZE: 30%

Summary for Table 13 on a Training Size of 30%: Mean Accuracy=0.76, Standard Deviation=0.04, Mean Error=0.24, Average Training Time (Seconds)=0.042

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.75	0.25	0.055
1	0.76	0.24	0.055
2	0.76	0.24	0.055
3	0.76	0.24	0.055
4	0.74	0.26	0.057

Table 14: TRAINING SIZE: 40%

Summary for Table 14 on a Training Size of 40%: Mean Accuracy=0.75, Standard Deviation=0.01, Mean Error=0.25, Average Training Time (Seconds)=0.055

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.76	0.24	0.069
1	0.78	0.22	0.069
2	0.8	0.2	0.069
3	0.77	0.23	0.069
4	0.78	0.22	0.07

Table 15: TRAINING SIZE: 50%

Summary for Table 15 on a Training Size of 50%: Mean Accuracy=0.78, Standard Deviation=0.01, Mean Error=0.22, Average Training Time (Seconds)=0.069

Trial	Accuracy	Error	Training time (seconds)
0	0.8	0.2	0.083
1	0.79	0.21	0.089
2	0.81	0.19	0.084
3	0.79	0.21	0.085
4	0.8	0.2	0.085

Table 16: TRAINING SIZE: 60%

Summary for Table 16 on a Training Size of 60%: Mean Accuracy=0.8, Standard Deviation=0.01, Mean Error=0.2, Average Training Time (Seconds)=0.085

Trial	Accuracy	Error	Training time (seconds)
0	0.77	0.23	0.097
1	0.82	0.18	0.097
2	0.81	0.19	0.096
3	0.82	0.18	0.098
4	0.82	0.18	0.098

Table 17: TRAINING SIZE: 70%

Summary for Table 17 on a Training Size of 70%: Mean Accuracy=0.81, Standard Deviation=0.02, Mean Error=0.19, Average Training Time (Seconds)=0.097

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.78	0.22	0.115
1	0.77	0.23	0.112
2	0.8	0.2	0.116
3	0.81	0.19	0.113
4	0.79	0.21	0.112

Table 18: TRAINING SIZE: 80%

Summary for Table 18 on a Training Size of 80%: Mean Accuracy=0.79, Standard Deviation=0.01, Mean Error=0.21, Average Training Time (Seconds)=0.114

Trial	Accuracy	Error	Training time (seconds)
			,
0	0.82	0.18	0.126
1	0.82	0.18	0.127
2	0.8	0.2	0.126
3	0.79	0.21	0.127
4	0.8	0.2	0.126

Table 19: TRAINING SIZE: 90%

Summary for Table 19 on a Training Size of 90%: Mean Accuracy=0.81, Standard Deviation=0.01, Mean Error=0.19, Average Training Time (Seconds)=0.126

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.81	0.19	0.145
1	0.81	0.19	0.141
2	0.81	0.19	0.141
3	0.81	0.19	0.142
4	0.81	0.19	0.139

Table 20: TRAINING SIZE: 100%

Summary for Table 20 on a Training Size of 100%: Mean Accuracy=0.81, Standard Deviation=0.0, Mean Error=0.19, Average Training Time (Seconds)=0.142

## 11 Analysis of the Perceptron Face Detection Algorithm on Each Training Set Size

Our results for the Perceptron Face Detection Algorithm are below.

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.68	0.32	0.004
1	0.8	0.2	0.011
2	0.7	0.3	0.013
3	0.72	0.28	0.007
4	0.67	0.33	0.02

Table 21: TRAINING SIZE: 10%

Summary for Table 21 on a Training Size of 10%: Mean Accuracy=0.71, Standard Deviation=0.05, Mean Error=0.29, Average Training Time (Seconds)=0.011

Trial	Accuracy	Error	Training time (seconds)
0	0.69	0.31	0.113
1	0.68	0.32	0.084
2	0.63	0.37	0.113
3	0.8	0.2	0.109
4	0.73	0.27	0.052

Table 22: TRAINING SIZE: 20%

Summary for Table 22 on a Training Size of 20%: Mean Accuracy=0.71, Standard Deviation=0.06, Mean Error=0.29, Average Training Time (Seconds)=0.094

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.67	0.33	0.181
1	0.76	0.24	0.178
2	0.71	0.29	0.169
3	0.75	0.25	0.178
4	0.68	0.32	0.181

Table 23: TRAINING SIZE: 30%

Summary for Table 23 on a Training Size of 30%: Mean Accuracy=0.71, Standard Deviation=0.04, Mean Error=0.29, Average Training Time (Seconds)=0.177

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.79	0.21	0.218
1	0.73	0.27	0.226
2	0.72	0.28	0.238
3	0.8	0.2	0.242
4	0.77	0.23	0.24

Table 24: TRAINING SIZE: 40%

Summary for Table 24 on a Training Size of 40%: Mean Accuracy=0.76, Standard Deviation=0.03, Mean Error=0.24, Average Training Time (Seconds)=0.233

Trial	Accuracy	Error	Training time (seconds)
0	0.65	0.35	0.322
1	0.75	0.25	0.298
2	0.75	0.25	0.301
3	0.76	0.24	0.296
4	0.72	0.28	0.31

Table 25: TRAINING SIZE: 50%

Summary for Table 25 on a Training Size of 50%: Mean Accuracy=0.73, Standard Deviation=0.04, Mean Error=0.27, Average Training Time (Seconds)=0.305

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.74	0.26	0.363
1	0.77	0.23	0.366
2	0.71	0.29	0.358
3	0.73	0.27	0.364
4	0.75	0.25	0.361

Table 26: TRAINING SIZE: 60%

Summary for Table 26 on a Training Size of 60%: Mean Accuracy=0.74, Standard Deviation=0.02, Mean Error=0.26, Average Training Time (Seconds)=0.362

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.75	0.25	0.437
1	0.77	0.23	0.448
2	0.77	0.23	0.427
3	0.8	0.2	0.429
4	0.74	0.26	0.433

Table 27: TRAINING SIZE: 70%

Summary for Table 27 on a Training Size of 70%: Mean Accuracy=0.77, Standard Deviation=0.02, Mean Error=0.23, Average Training Time (Seconds)=0.435

Trial	Accuracy	Error	Training time (seconds)
0	0.77	0.23	0.496
1	0.74	0.26	0.493
2	0.7	0.3	0.499
3	0.72	0.28	0.496
4	0.71	0.29	0.49

Table 28: TRAINING SIZE: 80%

Summary for Table 28 on a Training Size of 80%: Mean Accuracy=0.73, Standard Deviation=0.02, Mean Error=0.27, Average Training Time (Seconds)=0.495

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.76	0.24	0.562
1	0.71	0.29	0.56
2	0.75	0.25	0.552
3	0.77	0.23	0.57
4	0.78	0.22	0.558

Table 29: TRAINING SIZE: 90%

Summary for Table 29 on a Training Size of 90%: Mean Accuracy=0.75, Standard Deviation=0.02, Mean Error=0.25, Average Training Time (Seconds)=0.56

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.81	0.19	0.63
1	0.78	0.22	0.628
2	0.78	0.22	0.627
3	0.76	0.24	0.629
4	0.79	0.21	0.629

Table 30: TRAINING SIZE: 100%

Summary for Table 30 on a Training Size of 100%: Mean Accuracy=0.78, Standard Deviation=0.02, Mean Error=0.22, Average Training Time (Seconds)=0.629

## 12 Analysis of the Perceptron Digit Identification Algorithm on Each Training Set Size

Our results for the Perceptron Digit Identification Algorithm are below.

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.71	0.29	8.239
1	0.73	0.27	7.127
2	0.77	0.23	8.173
3	0.62	0.38	8.107
4	0.64	0.36	7.247

Table 31: TRAINING SIZE: 10%

Summary for Table 31 on a Training Size of 10%: Mean Accuracy=0.69, Standard Deviation=0.06, Mean Error=0.31, Average Training Time (Seconds)=7.779

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.72	0.28	18.35
1	0.74	0.26	17.803
2	0.76	0.24	17.615
3	0.76	0.24	17.5
4	0.78	0.22	15.661

Table 32: TRAINING SIZE: 20%

Summary for Table 32 on a Training Size of 20%: Mean Accuracy=0.75, Standard Deviation=0.02, Mean Error=0.25, Average Training Time (Seconds)=17.386

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.79	0.21	26.515
1	0.74	0.26	26.519
2	0.71	0.29	26.719
3	0.71	0.29	26.493
4	0.78	0.22	26.458

Table 33: TRAINING SIZE: 30%

Summary for Table 33 on a Training Size of 30%: Mean Accuracy=0.75, Standard Deviation=0.03, Mean Error=0.25, Average Training Time (Seconds)=26.54

Trial	Accuracy	Error	Training time (seconds)
0	0.7	0.3	35.74
1	0.65	0.35	35.296
2	0.69	0.31	35.645
3	0.75	0.25	35.338
4	0.77	0.23	35.317

Table 34: TRAINING SIZE: 40%

Summary for Table 34 on a Training Size of 40%: Mean Accuracy=0.71, Standard Deviation=0.04, Mean Error=0.29, Average Training Time (Seconds)=35.467

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.73	0.27	44.312
1	0.78	0.22	44.512
2	0.77	0.23	44.602
3	0.8	0.2	44.578
4	0.82	0.18	44.261

Table 35: TRAINING SIZE: 50%

Summary for Table 35 on a Training Size of 50%: Mean Accuracy=0.78, Standard Deviation=0.03, Mean Error=0.22, Average Training Time (Seconds)=44.453

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.77	0.23	53.416
1	0.76	0.24	53.193
2	0.69	0.31	54.296
3	0.86	0.14	53.523
4	0.78	0.22	53.393

Table 36: TRAINING SIZE: 60%

Summary for Table 36 on a Training Size of 60%: Mean Accuracy=0.77, Standard Deviation=0.05, Mean Error=0.23, Average Training Time (Seconds)=53.564

Trial	Accuracy	Error	Training time (seconds)
0	0.8	0.2	62.602
1	0.77	0.23	62.664
2	0.81	0.19	62.719
3	0.77	0.23	63.035
4	0.81	0.19	63.752

Table 37: TRAINING SIZE: 70%

Summary for Table 37 on a Training Size of 70%: Mean Accuracy=0.79, Standard Deviation=0.02, Mean Error=0.21, Average Training Time (Seconds)=62.954

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.78	0.22	72.469
1	0.78	0.22	72.138
2	0.79	0.21	73.232
3	0.73	0.27	72.964
4	0.79	0.21	74.094

Table 38: TRAINING SIZE: 80%

Summary for Table 38 on a Training Size of 80%: Mean Accuracy=0.77, Standard Deviation=0.02, Mean Error=0.23, Average Training Time (Seconds)=72.98

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.77	0.23	84.443
1	0.76	0.24	83.723
2	0.78	0.22	84.17
3	0.75	0.25	83.871
4	0.78	0.22	84.22

Table 39: TRAINING SIZE: 90%

Summary for Table 39 on a Training Size of 90%: Mean Accuracy=0.77, Standard Deviation=0.01, Mean Error=0.23, Average Training Time (Seconds)=84.085

Trial	Accuracy	Error	Training time (sec-
			onds)
0	0.73	0.27	93.306
1	0.78	0.22	91.718
2	0.7	0.3	89.916
3	0.72	0.28	89.939
4	0.76	0.24	90.01

Table 40: TRAINING SIZE: 100%

Summary for Table 40 on a Training Size of 100%: Mean Accuracy=0.74, Standard Deviation=0.03, Mean Error=0.26, Average Training Time (Seconds)=90.978

# 13 Analysis of the kNN Face Detection Algorithm on Each Training Set Size

Our results for the kNN Face Detection Algorithm are below.

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.7	0.3	0.001
1	0.65	0.35	0.001
2	0.65	0.35	0.001
3	0.69	0.31	0.001
4	0.62	0.38	0.001

Table 41: TRAINING SIZE: 10%

Summary for Table 41 on a Training Size of 10%: Mean Accuracy=0.66, Standard Deviation=0.03, Mean Error=0.34, Average Prediction Time Per Image (Seconds)=0.001

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.69	0.31	0.001
1	0.71	0.29	0.001
2	0.59	0.41	0.001
3	0.61	0.39	0.002
4	0.63	0.37	0.002

Table 42: TRAINING SIZE: 20%

Summary for Table 42 on a Training Size of 20%: Mean Accuracy=0.65, Standard Deviation=0.05, Mean Error=0.35, Average Prediction Time Per Image (Seconds)=0.001

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.74	0.26	0.002
1	0.68	0.32	0.002
2	0.74	0.26	0.002
3	0.74	0.26	0.002
4	0.65	0.35	0.002

Table 43: TRAINING SIZE: 30%

Summary for Table 43 on a Training Size of 30%: Mean Accuracy=0.71, Standard Deviation=0.04, Mean Error=0.29, Average Prediction Time Per Image (Seconds)=0.002

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.67	0.33	0.003
1	0.66	0.34	0.003
2	0.72	0.28	0.003
3	0.68	0.32	0.003
4	0.68	0.32	0.003

Table 44: TRAINING SIZE: 40%

Summary for Table 44 on a Training Size of 40%: Mean Accuracy=0.68, Standard Deviation=0.02, Mean Error=0.32, Average Prediction Time Per Image (Seconds)=0.003

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.72	0.28	0.004
1	0.69	0.31	0.004
2	0.69	0.31	0.004
3	0.69	0.31	0.004
4	0.69	0.31	0.004

Table 45: TRAINING SIZE: 50%

Summary for Table 45 on a Training Size of 50%: Mean Accuracy=0.7, Standard Deviation=0.01, Mean Error=0.3, Average Prediction Time Per Image (Seconds)=0.004

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.72	0.28	0.004
1	0.72	0.28	0.004
2	0.75	0.25	0.004
3	0.72	0.28	0.004
4	0.69	0.31	0.004

Table 46: TRAINING SIZE: 60%

Summary for Table 46 on a Training Size of 60%: Mean Accuracy=0.72, Standard Deviation=0.02, Mean Error=0.28, Average Prediction Time Per Image (Seconds)=0.004

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.69	0.31	0.005
1	0.67	0.33	0.005
2	0.71	0.29	0.005
3	0.65	0.35	0.005
4	0.76	0.24	0.005

Table 47: TRAINING SIZE: 70%

Summary for Table 47 on a Training Size of 70%: Mean Accuracy=0.7, Standard Deviation=0.04, Mean Error=0.3, Average Prediction Time Per Image (Seconds)=0.005

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.73	0.27	0.006
1	0.7	0.3	0.006
2	0.7	0.3	0.006
3	0.74	0.26	0.006
4	0.68	0.32	0.006

Table 48: TRAINING SIZE: 80%

Summary for Table 48 on a Training Size of 80%: Mean Accuracy=0.71, Standard Deviation=0.02, Mean Error=0.29, Average Prediction Time Per Image (Seconds)=0.006

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.72	0.28	0.007
1	0.72	0.28	0.007
2	0.74	0.26	0.007
3	0.75	0.25	0.007
4	0.7	0.3	0.007

Table 49: TRAINING SIZE: 90%

Summary for Table 49 on a Training Size of 90%: Mean Accuracy=0.73, Standard Deviation=0.02, Mean Error=0.27, Average Prediction Time Per Image (Seconds)=0.007

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.72	0.28	0.007
1	0.72	0.28	0.007
2	0.72	0.28	0.007
3	0.72	0.28	0.007
4	0.72	0.28	0.007

Table 50: TRAINING SIZE: 100%

Summary for Table 50 on a Training Size of 100%: Mean Accuracy=0.72, Standard Deviation=0.0, Mean Error=0.28, Average Prediction Time Per Image (Seconds)=0.007

# 14 Analysis of the kNN Digit Identification Algorithm on Each Training Set Size

Our results for the kNN Digit Identification Algorithm are below.

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.77	0.23	0.009
1	0.77	0.23	0.009
2	0.77	0.23	0.009
3	0.78	0.22	0.009
4	0.82	0.18	0.009

Table 51: TRAINING SIZE: 10%

Summary for Table 51 on a Training Size of 10%: Mean Accuracy=0.78, Standard Deviation=0.02, Mean Error=0.22, Average Prediction Time Per Image (Seconds)=0.009

Trial	Accuracy	Error	Prediction Time Per Image (sec-
0	0.83	0.17	onds) 0.018
1	0.85	0.15	0.018
2	0.82	0.18	0.018
3	0.79	0.21	0.018
4	0.84	0.16	0.018

Table 52: TRAINING SIZE: 20%

Summary for Table 52 on a Training Size of 20%: Mean Accuracy=0.83, Standard Deviation=0.02, Mean Error=0.17, Average Prediction Time Per Image (Seconds)=0.018

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.85	0.15	0.027
1	0.87	0.13	0.027
2	0.86	0.14	0.027
3	0.88	0.12	0.027
4	0.85	0.15	0.027

Table 53: TRAINING SIZE: 30%

Summary for Table 53 on a Training Size of 30%: Mean Accuracy=0.86, Standard Deviation=0.01, Mean Error=0.14, Average Prediction Time Per Image (Seconds)=0.027

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.83	0.17	0.036
1	0.89	0.11	0.036
2	0.91	0.09	0.036
3	0.89	0.11	0.036
4	0.84	0.16	0.036

Table 54: TRAINING SIZE: 40%

Summary for Table 54 on a Training Size of 40%: Mean Accuracy=0.87, Standard Deviation=0.03, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.036

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.85	0.15	0.045
1	0.88	0.12	0.045
2	0.87	0.13	0.046
3	0.86	0.14	0.045
4	0.85	0.15	0.045

Table 55: TRAINING SIZE: 50%

Summary for Table 55 on a Training Size of 50%: Mean Accuracy=0.86, Standard Deviation=0.01, Mean Error=0.14, Average Prediction Time Per Image (Seconds)=0.045

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.87	0.13	0.054
1	0.89	0.11	0.054
2	0.86	0.14	0.055
3	0.86	0.14	0.054
4	0.86	0.14	0.054

Table 56: TRAINING SIZE: 60%

Summary for Table 56 on a Training Size of 60%: Mean Accuracy=0.87, Standard Deviation=0.01, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.054

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.9	0.1	0.066
1	0.87	0.13	0.063
2	0.84	0.16	0.063
3	0.89	0.11	0.063
4	0.86	0.14	0.063

Table 57: TRAINING SIZE: 70%

Summary for Table 57 on a Training Size of 70%: Mean Accuracy=0.87, Standard Deviation=0.02, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.064

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.88	0.12	0.072
1	0.88	0.12	0.073
2	0.86	0.14	0.072
3	0.87	0.13	0.072
4	0.88	0.12	0.074

Table 58: TRAINING SIZE: 80%

Summary for Table 58 on a Training Size of 80%: Mean Accuracy=0.87, Standard Deviation=0.01, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.073

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.88	0.12	0.082
1	0.87	0.13	0.082
2	0.87	0.13	0.083
3	0.88	0.12	0.082
4	0.87	0.13	0.082

Table 59: TRAINING SIZE: 90%

Summary for Table 59 on a Training Size of 90%: Mean Accuracy=0.87, Standard Deviation=0.0, Mean Error=0.13, Average Prediction Time Per Image (Seconds)=0.082

Trial	Accuracy	Error	Prediction Time
			Per Image (sec-
			onds)
0	0.88	0.12	0.091
1	0.88	0.12	0.094
2	0.88	0.12	0.092
3	0.88	0.12	0.092
4	0.88	0.12	0.092

Table 60: TRAINING SIZE: 100%

Summary for Table 60 on a Training Size of 100%: Mean Accuracy=0.88, Standard Deviation=0.0, Mean Error=0.12, Average Prediction Time Per Image (Seconds)=0.092

### 15 Reflection on the Data

### 15.1 Our Observations

For each of the six algorithms, the general trend is that the larger the training set size is, the more accurate the algorithms are in predicting labels for images in the test set. This trend is the most obvious for the Naive Bayes algorithm, as the accuracy increases almost every time the training size increases by 10%. For the perceptron algorithm, the accuracy does not always increase every time that the training size increases and actually fluctuates around the low 70 percent range. However, for the perceptron algorithm, when the training size is 10%, the accuracy is around the high 60th percentile and low 70th percentile on average, but when the training size is close to 100%, the accuracy changes to consistently be in the 70% range. For the kNN algorithm, the accuracy is similar over different training sizes, but almost consistently increases in accuracy

by a very small amount as the training size increases.

The standard deviation over all of the trials in the Naive Bayes algorithm for digit identification and kNN algorithms is very small; close to 0. This shows that the algorithms consistently have a similar accuracy for the same training size. On the other hand, the Naive Bayes algorithm for face detection and Perceptron algorithms have a slightly larger standard deviation in accuracy per training size that is around 0.05.

For each of the algorithms, as the training size gets larger, the time to train the machine learning algorithms becomes much longer. Also, the time to train the machine learning algorithms that identify a digit takes much longer than the time to train the machine learning algorithms that detect a face. Furthermore, the Naive Bayes algorithms are much faster than the Perceptron algorithms. In the worst case, the Perceptron algorithm to identify a digit could take 93.3 seconds to train. On the other hand, the worst case for training the Naive Bayes algorithm to identify a digit is 0.145 seconds.

The time that we calculated for the kNN algorithm is not comparable to the time that we calculated for the Naive Bayes and Perceptron algorithms since the kNN algorithm does not have a train function. However, we were able to observe that it is faster than the Perceptron algorithms.

## 15.2 Our Explanation For These Observations and the Learning Curve

For the Naive Bayes algorithm, as the training size increases, the probabilities that the algorithm stores in the probability tables become more accurate. Since the probabilities are more accurate and the algorithm uses these probabilities to make predict a label, the algorithm is able to predict a label with higher accuracy.

For the kNN algorithm, as the training size increases, the algorithm has a higher likelihood of finding an image that is more similar to an image in the test set. As a result, the kNN algorithm accuracy increases.

For the Perceptron algorithm, as the training size increases, the accuracy generally increases but sometimes fluctuates. One explanation is that the accuracy of the Perceptron algorithm also depends on other factors such as the order in which the training images are examined in the feature weight adjustments process, and the composition of the training images selected. For example, let's say that we have a feature that is less effective, and both the face and non-face images have that feature. If we have a training set that consists of mostly face images, after training, the weight of that less effective feature will tend to be higher, hinting that an image with that feature is a face, which clearly is not the

case. This example shows that the Perceptron algorithm accuracy is impacted not only by the size of the training set, but also by other factors such as the composition of the training set (the balance between true and false images).