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New Interplanetary Proton Fluence Model

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A new predictive engineering model for the interplanetary fluence of protons with energies >10 MeV and >30 MeV is described. The data set used is a combination of observations made from the Earth's surface and from above the atmosphere between 1956 and 1963 and observations made from spacecraft in the vicinity of Earth between 1963 and 1985. The data cover a time period three times as long as the period used in earlier models. With the use of this data set the distinction between "ordinary proton events" and "anomalously large events" made in earlier work disappears. This permitted the use of statistical analysis methods developed for "ordinary events" on the entire data set. The >10 MeV fluences at 1 AU (astronomical unit) calculated with the new model are about twice those expected on the basis of models now in use. At energies >30 MeV, the old and new models agree. In contrast to earlier models, the results do not depend critically on the fluence from any one event and are independent of sunspot number. Mission probability curves derived from the fluence distribution are presented.

Nomenclature

E = energy per nucleon, MeV
F = log of integral proton fluence
f_p = integral proton fluence, particle /cm^{-2}
n = number of events; $>10^7$ particle cm^{-2} for >10 MeV, $>10^6$ particle cm^{-2} for >30 MeV
r = heliocentric distance, AU (astronomical unit)
w = average number of events per year
μ = mean of the log fluence
σ = standard deviation, unitless
τ = mission length, yr

Introduction

THIS study and development of the new model was carried out in response to a request for an estimate of the natural proton environment in interplanetary space. Such an estimate is often needed when spacecraft spend a significant amount of time in the interplanetary environment. In this case the radiation environment of electronic parts for such a mission may be dominated by the high energy protons found in interplanetary space. Typical spacecraft shielding is of the order of 100 mils of aluminum and so excludes protons with energies less than about 10 MeV. Therefore the energy range of interest for this study was in the tens of MeV per proton. The interplanetary proton fluence in the energy range 5–100 MeV is dominated by solar proton events. A review of the proton event model[1] in use when this study was undertaken showed it to be inadequate as will be discussed. We were able to increase the model accuracy considerably by developing a new model using a proton event data set about three times as large as the data set used to develop the then current model.[1] The new model for 10- and 30-MeV fluences is described here.

The interplanetary high energy solar proton fluence model currently used to evaluate hazards to spacecraft systems is that developed by King in 1974.[1] That model was designed specifically to predict fluence during the period from 1977–1983, i.e., the 21st solar cycle. Because of this specificity we undertook a review of the King model and, as a result of the review, we have developed an updated model. The purpose of this paper is to provide an overview of our approach to this problem.

The King[1] model for 1977–1983 was based on two assumptions. First, King noted that the fluence during the solar cycle that maximized in 1957 (cycle 19, maximum annual sunspot number 190) was much larger than the fluence during the 20th cycle that had just been completed. The major contribution to the 19th cycle fluence was from 4 or 5 major events. In contrast, the fluence during cycle 20 was dominated by a single event with fluence comparable to the major events of cycle 19, the great proton event of August 1972. This lower fluence during cycle 20 (maximum annual sunspot number 107) was in agreement with the notion that was widely held at the time, i.e., that the number of major solar particle events during a solar cycle was a function of the cycle's maximum sunspot number. Furthermore, the predictions King used for sunspot maximum for cycle 21 indicated that it would resemble or be smaller than cycle 20. With these assumptions about the relation between sunspot number and major proton events and about the intensity of cycle 21, it was very reasonable to ignore the cycle 19 data and to use the cycle 20 data base to make a conservative prediction of cycle 21 fluence. Thus, King developed his model using only cycle 20 data. However, neither of these assumptions have proved valid for cycle 21. There were no major (i.e., >30 MeV fluence more than 1×10^9 particle

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cm^{-2}) proton events at all during cycle 21 despite the fact that the maximum annual sunspot number in cycle 21 was 155, compared to cycle 20's maximum of 107. The failure of these assumptions indicates the importance of reviewing the data and producing a new model.

Note also, as mentioned earlier, our experience with the last three solar cycles shows that neither the cycle integrated proton fluence nor the number of very intense events in a cycle are a function of the amplitude of the cycle measured by the maximum sunspot number. Hence, the expected sunspot number does not influence expectations for the proton fluence in our model.

Data Base

Data on proton fluences used in this study come from two major sources, observations using riometers, rockets and balloons from 1956 to 1963, and observations taken in space since 1963.

The first data set is that used by Yucker[2] and consists of the events between 1956 and 1962. As is well known, several of these earlier events were said to have fluences comparable to and even larger than the great event of August 1972. Because these events occurred before the space era had really begun, and because they were not observed from interplanetary space, it is widely believed that the fluences reported for them were highly inaccurate and exaggerated. To check on the validity of this data set, a careful review of the original papers was undertaken. The care with which these early events were studied can be illustrated by noting that a conference was held on the November 1960 solar-terrestrial events[3] at the then Air Force Cambridge Research Laboratories (now known as the Air Force Geophysics Laboratory). A second thorough review of the known high fluence events between 1949 and 1961 was reported on in the *Solar Proton Manual*. In that publication Malitson and Webber[4] report that events since 1956 had been carefully studied, and 1956 was chosen here as the beginning of our data set. Malitson and Webber reviewed their data in the *Solar Proton Manual* as did Fichtel et al.[5] Fichtel et al. had as their goal to determine the fluences of individual solar particle events within a factor of two. Fichtel et al. claim that the accuracy obtained is frequently much better. Furthermore, although the Ref. 4 evaluations of the integral intensities were carried out independently of those in Ref. 5, the agreement was excellent, except for the February 23, 1956 event. As a nonscientific aside we would like to mention that the fact that these events were extremely large is not doubted by those observers who are still active in the field and who were concerned with proton events and aurora at the time they occurred. On the basis of our reviews of the 1956–1962 data set we have included that data in our data base.

Since 1963, instruments have been observing proton fluxes in space. All of the feasible data from satellite observations have been collected and edited for valid solar particle responses.[6] A nearly time-continuous record of daily average fluxes of particles above the thresholds of 10, 30, and 60 MeV has been constructed. The details of the production of this data set are described in Armstrong et al.[6] These data form the second of the two sets used.

Table 1a List of solar particle events

	Solar cosmic ray data (integral fluence—/cm²)		
Date	10 MeV	30 MeV	100 MeV
02/23/56	1.8+09	1.0+09	3.5+08
03/10/56	—	1.1+08	—
08/31/56	—	2.5+07	6.0+06
11/13/56	—	1.0+08	—
01/20/57	—	3.0+08	1.0+07
04/03/57	—	5.0+07	—
04/06/57	—	3.8+07	—
06/21/57	—	1.5+08	—
07/03/57	—	2.0+07	—
07/24/57	—	7.5+06	7.7+05
08/09/57	—	1.5+06	—
08/29/57	—	1.2+08	3.0+06
08/31/57	—	8.0+07	8.0+06
09/02/57	—	5.0+07	4.5+06
09/12/57	—	6.0+06	5.0+05
09/21/57	—	1.5+06	—
09/26/57	—	—	2.0+04
10/20/57	—	5.0+07	1.0+07
11/04/57	—	9.0+06	—
02/09/58	—	5.0+06	4.0+05
03/23/58	2.0+09	2.5+08	1.0+07
03/25/58	—	6.0+08	—
04/10/58	—	5.0+06	—
07/07/58	1.8+09	2.5+08	9.0+06
07/29/58	—	8.5+06	7.0+05
08/16/58	4.0+08	4.0+07	1.6+06
08/21/58	—	—	1.9+04
08/22/58	8.0+08	7.0+07	1.8+06
08/26/58	1.5+09	1.1+08	2.0+06
09/22/58	9.0+07	6.0+06	1.0+05
02/13/59	—	2.8+07	—
05/10/59	5.5+09	9.6+08	8.5+07
06/13/59	—	8.5+07	—
07/10/59	4.5+09	1.0+09	1.0+08
07/14/59	7.5+09	1.3+09	1.0+08
07/16/59	3.3+09	9.1+08	1.3+08
08/18/59	—	1.8+06	—
09/02/59	—	1.2+07	—
01/11/60	—	4.0+05	—
03/29/60	—	6.0+06	—
03/30/60	—	6.0+06	4.0+05
04/01/60	1.5+07	5.0+06	8.5+05
04/05/60	1.4+07	1.1+06	—
04/28/60	1.3+07	5.0+06	7.0+05
04/29/60	—	7.0+06	—
05/04/60	1.2+07	6.0+06	1.2+06
05/06/60	—	4.0+06	—
05/13/60	1.5+07	4.0+06	4.5+05
06/01/60	—	4.0+05	—
08/12/60	—	6.0+05	—
09/03/60	9.0+07	3.5+07	7.0+06
09/26/60	2.0+07	2.0+06	1.2+05
11/12/60	3.2+10	9.0+09	2.4+08
11/15/60	2.5+09	7.2+08	1.2+08
11/20/60	1.4+08	4.5+07	8.0+06
07/11/61	1.7+07	3.0+06	3.0+05
07/12/61	5.0+08	4.0+07	1.0+06
07/15/61	—	1.3+07	1.0+06
07/18/61	1.0+09	3.0+08	4.0+07
07/20/61	1.5+07	5.0+06	9.0+05
07/28/61	—	4.4+06	—
09/08/61	—	3.0+06	—
09/10/61	5.0+07	—	—
09/28/61	5.0+07	6.0+06	1.1+06
11/10/61	3.0+07	—	—
10/23/62	6.0+05	1.2+05	1.0+04
09/21/63	5.0+07	—	—
09/26/63	5.0+07	—	—

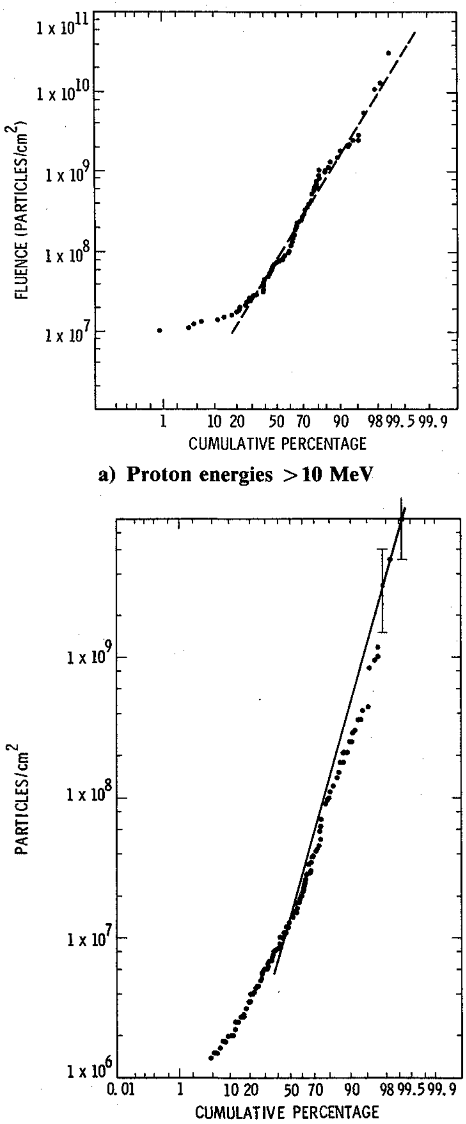
Table 1b List of solar particle events

Solar cosmic ray data (integral fluence—/cm^2)

Year	Begin-end date		Total days	Fluence, >10 MeV	Fluence, >30 MeV
65	36	38	3	0.16E+08	0.25E+07
65	276	279	4	0.26E+07	0.40E+06
66	82	85	4	0.11E+08	0.87E+06
66	123	128	6	0.15E+07	0.25E+06
66	188	192	5	0.64E+08	0.30E+07
66	240	266	27	0.10E+10	0.11E+08
67	11	12	2	0.36E+07	0.81E+05
67	28	47	20	0.11E+10	0.16E+08
67	58	66	9	0.71E+07	0.17E+07
67	70	73	4	0.16E+08	0.28E+07
67	144	154	11	0.78E+09	0.58E+08
67	157	169	13	0.24E+08	0.14E+08
67	303	324	22	0.30E+08	0.19E+08
67	337	341	5	0.25E+08	0.10E+08
67	350	356	7	0.15E+08	0.80E+07
68	161	165	5	0.29E+09	0.14E+08
68	189	198	10	0.47E+08	0.99E+07
68	270	281	12	0.74E+08	0.21E+08
68	303	312	10	0.21E+09	0.22E+08
68	323	330	8	0.10E+10	0.21E+09
68	337	347	11	0.23E+09	0.42E+08
69	56	61	6	0.76E+08	0.29E+08
69	80	82	3	0.71E+07	0.26E+07
69	89	101	13	0.78E+08	0.38E+08
69	102	117	16	0.22E+10	0.21E+09
69	268	273	6	0.18E+08	0.41E+07
69	306	314	9	0.64E+09	0.21E+09
69	328	337	10	0.71E+07	0.73E+07
69	352	356	5	0.52E+07	0.42E+07
70	28	34	7	0.28E+08	0.99E+07
70	65	70	6	0.68E+08	0.45E+07
70	82	99	18	0.94E+08	0.39E+08
70	150	154	5	0.14E+08	0.35E+07
70	166	170	5	0.28E+07	0.32E+07
70	188	190	3	0.41E+07	0.23E+07
70	202	207	6	0.36E+08	0.40E+07
70	223	237	15	0.19E+09	0.14E+08
70	309	317	9	0.66E+08	0.85E+07
70	346	349	4	0.42E+07	0.30E+07
70	358	365	8	0.16E+08	0.81E+07
71	24	39	16	0.15E+10	0.35E+09
71	91	94	4	0.30E+07	0.33E+07
71	96	100	5	0.32E+08	0.68E+07
71	110	114	5	0.43E+07	0.42E+07
71	132	142	11	0.14E+08	0.99E+07
71	244	260	17	0.39E+09	0.18E+09
71	277	281	5	0.70E+07	0.62E+07
72	108	113	6	0.30E+08	0.78E+07
72	149	157	9	0.76E+08	0.15E+08
72	160	174	15	0.40E+08	0.18E+08
72	201	215	15	0.54E+08	0.24E+08
72	216	238	23	0.11E+11	0.50E+10
72	303	307	5	0.60E+08	0.15E+08
73	102	108	7	0.82E+07	0.26E+07
73	119	129	11	0.16E+08	0.11E+08
73	210	215	6	0.72E+07	0.37E+07
73	250	254	5	0.19E+08	0.44E+07
73	307	309	3	0.47E+07	0.17E+07
74	159	162	4	0.44E+07	0.14E+07
74	184	191	8	0.24E+09	0.26E+08
74	254	278	25	0.33E+09	0.43E+08
74	309	312	4	0.13E+08	0.35E+07
75	232	235	4	0.66E+07	0.28E+07
76	83	91	9	0.54E+07	0.37E+07
76	121	125	5	0.10E+09	0.30E+08
76	235	237	3	0.10E+08	0.25E+07
77	203	211	9	0.61E+07	0.34E+07
77	251	274	24	0.43E+09	0.96E+08
77	285	287	3	0.39E+07	0.16E+07
77	325	331	7	0.28E+09	0.63E+08
78	2	12	11	0.12E+08	0.57E+07
78	44	53	10	0.15E+10	0.13E+09
78	98	105	8	0.70E+08	0.18E+08
78	107	134	28	0.24E+10	0.29E+09
78	151	155	5	0.18E+08	0.20E+07
78	174	180	7	0.53E+08	0.45E+07
78	193	199	7	0.32E+08	0.31E+07
78	205	209	5	0.27E+07	0.16E+07
78	250	252	3	0.28E+07	0.12E+07
78	266	280	15	0.29E+10	0.44E+09
78	282	288	7	0.86E+07	0.29E+07
78	314	318	5	0.18E+08	0.20E+07
78	346	350	5	0.62E+07	0.17E+07
79	48	53	6	0.16E+08	0.45E+07
79	61	76	16	0.21E+08	0.64E+07
79	93	96	4	0.21E+08	0.18E+07
79	157	165	9	0.21E+09	0.15E+08
79	187	192	6	0.21E+08	0.22E+07
79	213	227	15	0.12E+08	0.69E+07
79	231	242	12	0.60E+09	0.95E+08
79	251	275	25	0.36E+09	0.12E+09
79	320	322	3	0.32E+08	0.27E+07
80	11	13	3	0.28E+07	0.77E+06
80	37	40	4	0.30E+07	0.11E+07
80	91	98	8	0.87E+07	0.29E+07
80	198	209	12	0.12E+09	0.12E+08
80	245	250	6	0.37E+07	0.13E+07
80	289	297	9	0.30E+08	0.40E+07
80	320	325	6	0.63E+07	0.16E+07
80	328	338	11	0.14E+08	0.27E+07
81	62	67	6	0.34E+07	0.13E+07
81	89	96	8	0.28E+08	0.56E+07
81	100	111	12	0.85E+08	0.19E+08
81	114	143	30	0.10E+10	0.14E+09
81	201	207	7	0.81E+08	0.12E+08
81	220	224	5	0.14E+08	0.14E+07
81	250	253	4	0.73E+07	0.92E+06
81	262	270	9	0.15E+08	0.27E+07
81	281	298	18	0.21E+10	0.42E+09
81	314	320	6	0.56E+07	0.13E+07
81	326	329	4	0.38E+07	0.83E+06
81	339	347	9	0.77E+08	0.58E+07
81	361	364	4	0.75E+07	0.10E+07
82	30	42	13	0.11E+10	0.18E+09
82	65	68	4	0.11E+08	0.20E+07
82	155	171	17	0.70E+08	0.23E+08
82	190	201	12	0.84E+09	0.91E+08
82	203	207	5	0.12E+09	0.13E+08
82	247	251	5	0.14E+08	0.16E+07
82	297	299	3	0.33E+08	0.91E+06
82	325	337	13	0.25E+09	0.46E+08
82	338	347	10	0.57E+09	0.12E+09
82	348	356	9	0.13E+09	0.30E+08
82	359	365	7	0.21E+09	0.29E+08
83	34	38	5	0.10E+09	0.83E+07
83	166	178	13	0.21E+08	0.84E+07
84	31	32	2	0.24E+07	0.65E+06
84	46	58	13	0.16E+09	0.42E+08
84	71	79	9	0.29E+08	0.71E+07
84	115	128	13	0.13E+10	0.36E+09
85	21	24	4	0.87E+07	0.29E+07
85	114	120	7	0.28E+09	0.11E+08
85	185	192	8	0.23E+08	0.69E+07

The data in this set can be compared with the data given by King[1] for 24 events during the same period. We have calculated the ratios of the fluences given by King to the fluences used in this study for both the $E > 10$ MeV and the $E > 30$ MeV cases. The agreement was generally good. For the $E > 10$ MeV case, 20 of 24 events had ratios between 1.6 and 0.8, and for the $E > 30$ MeV case, 18 of 24 events had ratios between 1.3 and 0.4. There was a slightly smaller average slope to the particle spectrum between > 10 MeV and > 30 MeV in the data used in this study than in the data used by King. We have not attempted to determine which of the two data sets is more correct because the systematic parts of the differences are so small they will not affect the findings of this study.

Because of the importance of the August 1972 event, we have compared the values on our data tape[6] with the values given by King in some detail. For both the $E > 10$ MeV and the $E > 30$ MeV cases, the King values are 1.6 times the values used here. However, the uncertainty in the observations is probably of the same order as the differences between the two data sets. For example, the King values are derived from the data published by Bostrom et al.,[7] who state that the 10-MeV channel counts electrons of energy > 700 keV with about 25% efficiency but that no corrections had been made to the published data on that account. Although no corrections have been attempted for the electrons in the data we used here, the uncertainty in both data sets due to electrons alone is comparable to the overall differences between the two data sets.[8] In this study the values from the data tape are used but, as will be seen, none of the results would be changed if King's values had been used.



a) Proton energies > 10 MeV

b) Proton energies > 30 MeV

Fig. 1 Distribution of fluences for complete data set, 1956–1986 (the dashed line is to guide the eye in comparing the data to a straight line).

A complete list of all the events used in our data base is given in Table 1(a) and 1(b). These events are the periods of high proton fluences observed in space[6,8] as discussed next.

Method of Analysis

To analyze the data, we followed the general approach used earlier by Yucker[9] and King.[1] That is, we first studied the distributions of event magnitudes. Malitson and Webber[4] had stressed that solar flares producing protons occur in groups with several flares occurring over a period of days in the same active center. Since these grouped events cannot be assumed to be occurring independently of one another, the distribution of fluences in a data set that considers each flare to be a separate event cannot be expected to be a random sample of any underlying parent population. We therefore decided to integrate over each group of flares in our definition of "event fluence." Initially we were concerned that there would be a certain amount of arbitrariness in choosing the beginning and ending times of events. To check this, beginning and ending times for events were chosen independently by Feynman and Dao-Gibner, but no significant differences were found between the two resulting lists.

Using the event fluences determined in this way, and following Yucker[9] and King,[1] we compared the distribution of fluences to a log normal distribution. The events were ordered according to the log of the magnitude, and this was plotted vs the percent of observed events that have a magnitude less than the given event. To be more exact, fluences were plotted against $(i \times 100)/(n+1)$, where i is the rank value of the events ordered from smallest to largest and n is the total number of events used in the data set. The graph paper used to plot the results is ruled so that a log normal distribution will appear as a straight line. The result for the $E > 10$ MeV data set is shown in Fig. 1a. Most of the data lie on a straight line. However, for the lowest fluences shown, the data turn up and the observed fluences become much larger than those expected from any straight line. There are at least two contributing effects. First, this upturn is expected whenever a data set is truncated.[10] In our case we have included only those events for which the fluence was greater than 1×10^7 particles cm^{-2}. Second, the log normal distribution is expected to underestimate the number of events at very low fluences because the number of actual events on the sun increases as fluence decreases[8] whereas, for a log normal distribution, the number of events at fluences less than the median fluence decreases as fluence decreases. However, for a data set with a large enough range of fluences, the contribution of the low fluence events to the total fluence occurring in time intervals long enough to be of interest to modern space missions is negligible. In this study care was taken to use data sets with a large enough fluence range so that this criterion was satisfied. Figure 1b shows the data for the $E > 30$ MeV case. The low fluence cutoff was at 1×10^6 particles cm^{-2}. The error bars have been added to two of the pre-1963 events to show the effect of a factor of 2 uncertainty in the event fluence.

In Figs. 1a and 1b, the distribution of fluences for events having fluences above the median fluences are well fit by a straight line. This is in contrast to King's[1] results where only the data from cycle 20 were considered (for the reasons discussed in the Introduction). In that case, the 1972 event was more than a factor of 10 larger than any other event in the set and could not be considered part of the same distribution. King had to treat the 1972 event separately from other events. He called the 1972 event an AL (anomolously large) event and all other events OR (ordinary). In the present study, the 1972 event is not outstanding and, in fact, is not the event with the highest fluence.[11] Instead it is the second highest fluence event.

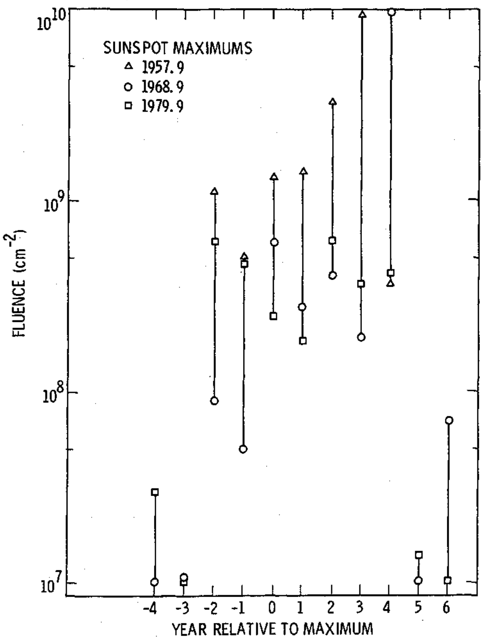


Fig. 2 Solar cycle dependence of annual fluences, 1956–1986 (see text for definition of "years").

These results encourage us to use a single method of analysis for all events in the data set.

Solar Cycle Variation

In King's treatment he distinguished between the "maximum" and "minimum" phases of the sunspot cycle. However, maximum and minimum phases were not clearly defined. This would have caused difficulty if the 1972 event were to have been predicted. The maximum of cycle 20 occurred in 1968. Thus the event occurred four years after solar maximum and three years before solar minimum. If a prediction was to have been made from say 1965, would the appropriate model have been considered to be the maximum or minimum model?

To examine the solar cycle dependence in more detail, we used a superposed epoch/analysis of the annual fluence for the 30 years covered by our data set. Our approach differed from that of other workers in that we defined the time-of-cycle maximum accurately to 0.1 years instead of the usual 1 year accuracy. The times of maximum of the 13-month running average sunspot number were supplied by Heckman.[12] The "zero years" of the cycles were then defined as 365-day periods centered on the sunspot maximum correct to 0.1 years. The other "years" of the cycle were defined in a corresponding manner, i.e., "years" are not calendar years.

The results of this analysis for the three cycles for which we have data are shown in Fig 2. Notice the clear difference between the seven years of high fluence and the four years of low fluence in each cycle. The annual fluence for the solar minimum that occurred early in the 1960s is not shown because the values were less than 10^7 particles cm^{-2}. With only two exceptions, the annual fluences exceeded 10^8 particles cm^{-2} during the three sets of seven hazardous years cycle^{-1} and was less than that during the other three sets of four years cycle^{-1}. This is true even if no major proton events occurred during a hazardous year of a particular cycle. Furthermore, note that the hazardous period is not centered on sunspot maximum but extends from two years before maximum to four years after maximum.

This clear result has important implications to space missions. In comparing the fluences to be expected during different missions, it is very important to take into account the actual launch date, since we can now be quite secure in predicting negligible fluences during the four minimum years of each cycle. Also notice that the dates of the last three cycle maxima occurred 11 years apart to the 0.1 year, so that we can be reasonably confident in predicting the time of the next maximum (about 1991). There is much more variance in the times between minima. The first spots of the new cycle (22) appeared during September 1986.[13]

Solar Cycle Corrected Proton Fluences

With the establishment of such a clear solar cycle variation, the approach to the determination of the best fit to the fluence distribution must be changed somewhat. The distribution should be constructed using data from only the seven hazardous years in each cycle. The few small events that occurred during the four-year quiet periods should be dropped from the data set.

The hazardous years fluence distribution for protons with $E > 10$ MeV is shown in Fig. 3 and that for $E > 30$ MeV in Fig. 4. Again there is an upturn of the points at low fluence due to truncation of the data set and the underestimation of the number of small fluence events. However, even after this is taken into account, the rest of the data deviate somewhat from a single straight line. This problem was also evident in the $E > 30$ MeV data set before the solar cycle minimum events were excluded. We have also looked at other types of distribution functions such as type II and III extreme value distributions, but the fits to the data were not improved.

Our approach to the problem of the deviation of the data from a straight line is to note that it is only those events with large fluences that influence the total fluence during a year. It is therefore more important to fit the large fluence part of the distribution than the low fluence part. We have carried out our analysis using the straight-line fits shown in Figs. 3 and 4. Both of these fits were made by eye. Note that in fact these fits do not depend crucially on the accuracy of the determination of the fluence from any one event. For example, the fits would not be appreciably different if the King value had been used for the August 1972 event. This is an advantage when compared with the situation faced by King who had to use fluences from only one solar cycle during which there was only a single event with fluences greater than 2×10^{10} particles cm^{-2} for $E > 10$ MeV.

Statistical Analyses

Since the high fluence portion of the data can be fit quite well with a straight line, the analysis was carried out along the

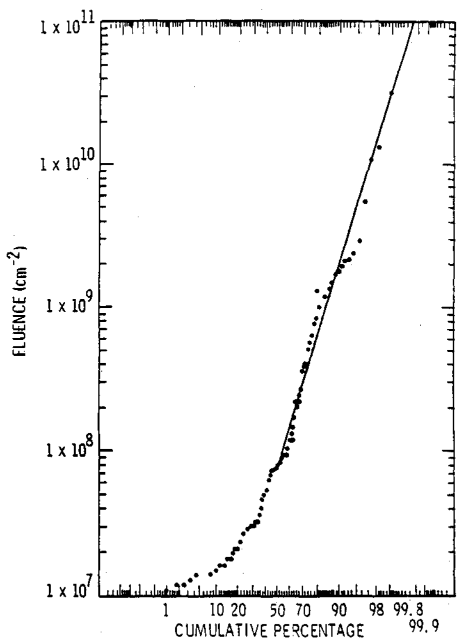


Fig. 3 Distribution of fluences for solar cycle active years for proton energies >10 MeV.

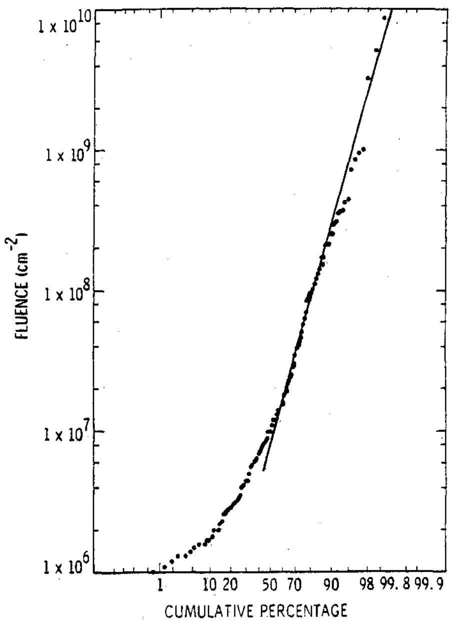


Fig. 4 Distribution of fluences for solar cycle active years for proton energies >30 MeV.

lines used by King for the so called "ordinary flares." Let f_p be the proton fluence of an event; f_p can be written as $f_p = 10^F$. If f_p is distributed log normally, then F is distributed normally and its density function is commonly expressed as

$$f(F) = (1/\sqrt{2\pi}\sigma)\exp - \tfrac{1}{2}[(F-\mu)/\sigma]^2 \qquad (1)$$

where σ is standard deviation, and μ is the mean log fluence. These are obtained from the straight-line fit to the data. The probability that during a mission length τ the fluence level will exceed f_p is

$$P(>F,\tau) = \sum_{n=1}^{\infty} p(n,w\tau)Q(F,n) \qquad (2)$$

where $p(n,w\tau)$ is the probability of n events occurring during mission length τ if an average of w events occurred per year during the observation period. The probability is assumed to follow a Poisson distribution and is calculated as

$$p(n,w\tau) = e^{-w\tau}(w\tau)^n/n! \qquad (3)$$

This choice of occurrence distribution is somewhat different from that of King who used an extension of the Poisson method introduced by Burrell[14] to account for the small size of the sample of events available to King. Since our sample consists of over 50 events, we have not used the Burrell extension.

The $Q(F,n)$ is the probability that the sum of all fluences due to n events will exceed 10^F. The $Q(F,1)$ is the probability that the fluence given by that 1 event that occurred is $\geq 10^F$. The $Q(F,2)$ is the probability that the 2 events occurring had the sum of their fluences $\geq 10^F$, and $Q(F,3)$, etc.

The values of $Q(F,n)$ were simulated using a Monte Carlo method. The Monte Carlo program used two subroutines.[15] One is a random number subroutine that generates random numbers with a uniform distribution in the interval of [0,1]. The other is a subroutine that applies the Box-Muller method of inverse transformation to obtain a Gaussian distribution. The inverse transformed method is discussed in detail in Yost.[16]

The random numbers are assumed to be the inverse function of $p(F)$, which is defined as

$$p(F) = \int_{-\infty}^{F} \frac{1}{\sqrt{2\pi}\sigma} \exp -\frac{1}{2}\left(\frac{F^*-\mu}{\sigma}\right)^2 \mathrm{d}F^* \qquad (4)$$

Table 2 Fitting parameters

Parameter	>10 MeV	>30 MeV
w	5.8	7.5
μ	7.8×10^7	8.6×10^6
σ	1.125	1.193

which can be written as

$$p(F) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp -\frac{1}{2} t^2 \,\mathrm{d}t \qquad (5)$$

where $z = (F-\mu)/\sigma$.

The values of μ and σ used in Eq. (4) are those obtained from the straight-line fit to the log fluence F distribution. They are shown in Table 2. As explained previously, the events below the median fluence portion of the distribution were ignored in making the fit and, since the largest fluence events were very important in calculating the total expected fluences, the largest events were given greater weight than the remaining small fluence events in determining the fitted straight line. Generating these random numbers and performing the inverse transformed calculations on them will result in a set of numbers that are random samples of the fit to the log fluence F distribution.

The actual simulation of $Q(F,n)$ consists basically of two steps. In step 1, N sets of random samples from a Gaussian distribution are generated. The N is a large number to insure the randomness (100,000). Each set j is a collection of n ran-

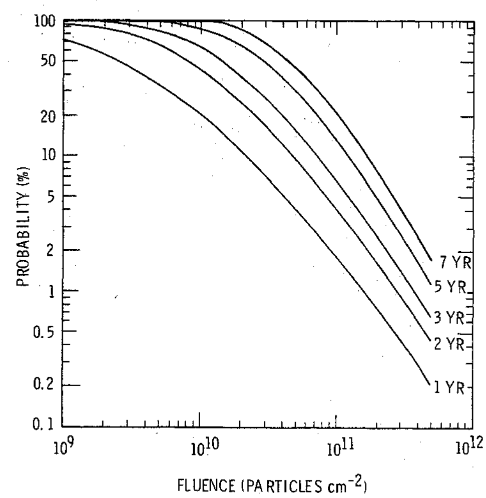


Fig. 5 Probability of exceeding selected fluences for different mission lengths for proton energies >10 MeV (all curves approach 100% asymptotically).

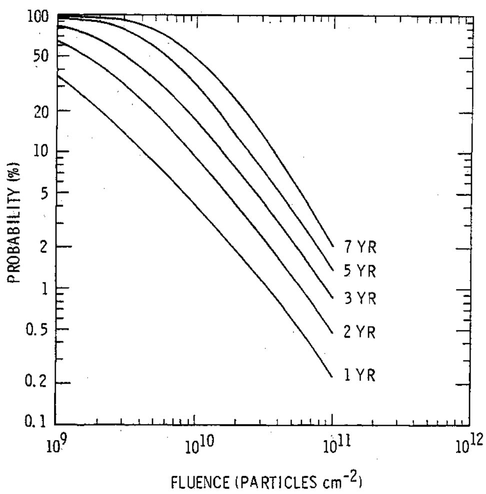


Fig. 6 Probability of exceeding selected fluences for different mission lengths for proton energies >30 MeV.

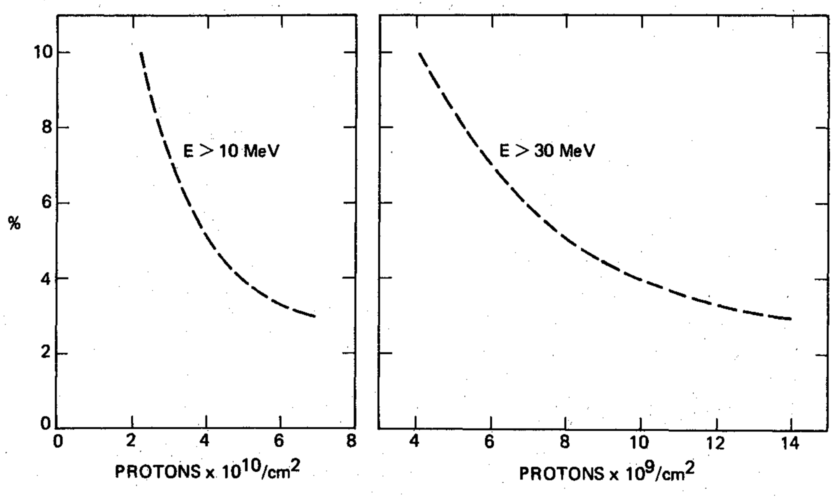


Fig. 7 Fluence as a function of confidence level for a one-year mission at 1 AU.

dom numbers x_i. In step 2, each set j is assigned a value of 1 if

$$\sum_{i=1}^{n}(10^{x_i^j\sigma+\mu}) \geq 10^F \tag{6}$$

The ratio of the cumulative numbers of set j with value of 1 over the total numbers of generated sets N is the probability of exceeding fluence f_p due to n events. This procedure is repeated to determine the value of each $Q(F,n)$ of interest.

Equation (2) has been evaluated for various mission lengths τ, and the result is shown in Fig. 5 for an energy threshold > 10 MeV. In this figure all curves approach 100% asymptotically, but the values are so close to 100% for fluences less than 10^{10} particles and mission lengths of seven years that the asymptotic nature of the curve may not be obvious. The result for > 30 MeV is shown in Fig. 6.

Results and Use of the Model

The procedure described previously has been carried out for the active years of the solar cycle and for various mission lengths. Figures 5 and 6 show the results for energies > 10 MeV and > 30 MeV. These figures give the probability of exceeding a given fluence level over the life of the mission assuming a constant heliocentric distance of 1 AU (astronomical unit). Figures 5 and 6 show four mission lengths. In calculating mission length, only the time that the spacecraft spends in interplanetary space during solar cycle active years should be included.

To use Figs. 5 and 6 to estimate mission fluences, find the line in the figure that corresponds to the number of years the mission will be in space during active solar cycle years. Then locate the "confidence level" required, recalling that a confidence level of 95% means that only 5% of missions identical to the one being considered will have fluences larger than that determined for a confidence level of 95%. That is, the ordinate on the figures gives the probability of exceeding a given level of fluence and that probability plus the confidence level is 100%. If a mission will be in space during more than one solar cycle, the best method for finding the total expected fluence from the figures is to estimate the additional fluence per year from the seven-year fluence curve and to add the appropriate number of yearly fluences to the seven-year line. We recommend against using the one-year curve to estimate the additional fluence expected on a long mission because it will overestimate the fluence.

In Table 3 we compare our new expected fluences with the King value for a mission length of two years at 1 AU. The fluences are unchanged for energies > 30 MeV, but the new fluences are about twice the King fluences at energies > 10 MeV. The confidence levels should be interpreted as meaning

Table 3 Two-year mission fluences, p cm^{-2}

Energy range	Confidence level, %	King	New
> 10 MeV	80	1.3×10^{10}	2.5×10^{10}
> 10 MeV	95	4.0×10^{10}	7.7×10^{10}
> 30 MeV	80	4.9×10^{9}	5×10^{9}
> 30 MeV	95	1.7×10^{10}	1.5×10^{10}

that if 1000 two-year missions were flown consecutively during solar cycle active years, then 800 of them (or 950 depending on the chosen confidence level) would have fluences no larger then the fluences shown in the table. The confidence level does not include changes that would come about from using slightly different straight-line fits to the data on event fluences.

To find the fluence for a mission in which the vehicle does not remain at 1 AU, the 1-AU fluence should be multiplied by a factor that depends on the actual trajectory, i.e., the integration of the radial dependence of the fluence over the mission trajectory. We suggest using the radial dependence recommended by the working group on solar particle events of a workshop on Interplanetary Charged Particle Environment[17] held at the Jet Propulsion Laboratory in Pasadena, California in March of 1987. The group recommended using an r^{-3} fluence dependence for $r < 1$ AU and r^{-2} fluence dependence for $r > 1$ AU. These radial dependences are worst-case choices and are the best estimates that can be made based on the inadequate data set now in hand. Future work may indicate that they are unnecessarily conservative.

In the models discussed here the estimated fluences are a strong function of the selected confidence level. In some applications a small lowering of the confidence level requirement may be acceptable and result in a large enough decrease in estimated fluence to eliminate an otherwise important problem. For this reason we have included Fig. 7 showing the percent of missions exceeding selected fluences for a one-year mission at 1 AU. The confidence limit of course is 100 – %. Analogous graphs can be constructed for other mission lengths from Figs. 5 and 6.

Summary

Using a data set of proton events from the last three solar cycles, we have developed a statistical model designed to be used for the prediction of proton fluences for space mission analysis. The energy ranges considered are $E > 10$ MeV and $E > 30$ MeV. The use of this data set, which is three times longer than the data set used in earlier models, permits a much more reliable (in the statistical sense) model to be constructed. Our model predictions do not depend on the expected maximum sunspot number in a cycle because no relation has been observed between cycle amplitude and integrated proton fluence during the last three solar cycles. Furthermore, the distinction often made between "ordinary" events and "anomalously large" events was not required by the data set. We found that the hazardous period for enhanced proton fluences is seven years long and extends from two years before sunspot maximum to four years after maximum when the sunspot maximum epoch is defined to 0.1 years. Figures have been given that permit the determination of the probability of exceeding given levels of fluence accumulated over mission lifetimes of one year or more. In these figures the spacecraft is assumed to be at 1 AU throughout the mission. Recipes to extend the predictions to larger and smaller heliocentric distances are also given. Finally we discuss the sensitivity of the percent of missions exceeding selected fluences on the fluences selected. The figure is expected to be very useful for engineers faced with a serious design problem at a previously chosen confidence level since it permits an estimate of the impact of a changed confidence level on the prospects for mission success.

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