Title: Optimal Obstacle Problems: The case of Harmonic Measure.

Abstract

Suppose we are interested in solutions u of a **Boundary Value Problem** on a domain D. We suppose further that a domain D is changed by inserting into D an *"obstacle"* E from some class $\mathcal{E} = \{E\}$ where each obstacle E carries its own boundary data. Then the **Optimal Obstacle Problem** is to find an obstacle $E_0 \in \mathcal{E}$ which distorted the original solution u as minimally as possible.

The main goal of my talk is to discuss the Optimal Obstacle Problem for Harmonic Measures. More precisely, we will consider the following problem.

Let D be a domain on $\mathbb{C}$ and let E be a Borel set on ∂D. Then $\omega(z, E, D)$ will denote the harmonic measure of E with respect to D; i.e. $\omega(z, E, D)$ is the Perron solution to the Dirichlet problem in D with boundary values 1 on E and 0 on $E' = \partial D \setminus E$. If E is a closed set on $\overline{\mathbb{C}}$ and $z \in D \setminus E$, then $\omega(z, E, D)$ will denote the harmonic measure of the set $E \cap \partial(D_z)$ with respect to the connected component D_z of $D \setminus E$, which contains z. The harmonic measure can be thought as the steady-state distribution of heat on D created by the heating element E.

Let $A = \{a_1, \ldots, a_n\}$ and $B = \{b_1, \ldots, b_m\}$ be two disjoint sets of distinct points in D. Let $\mathcal{L}_{A,B}(D) = \{L\}$ be the family of all sets L in $\overline{D}$ of the form $L = \cup_{k=1}^{n} l_k$, where l_k is a continuum in $\overline{D} \setminus B$ connecting a_k and ∂D. Let $\omega_L(z) = \omega(z, L, D)$. Let $F(x_1, \ldots, x_n)$ be a continuous real-valued function increasing in each variable.

Obstacle problem. The obstacle problem on the harmonic measure on D for the function F and sets A and B is to find the infimum

$$\inf F(\omega_L(b_1), \ldots, \omega_L(b_m)),$$

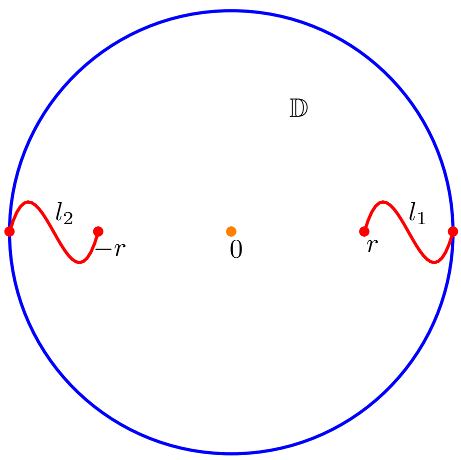
taken over all $L \in \mathcal{L}_{A,B}(D)$ and identify all *obstacles* L in $\mathcal{L}_{A,B}$ extremal for this problem.

One particular goal of this talk is to discuss solution of the obstacle problem in one of the simplest non-trivial cases, when D is the unit disk $\mathbb{D} = \{z : |z| < 1\}$, $B = \{0\}$, and $A = \{-r, r\}$ with some $0 < r < 1$. In this case, the problem can be stated as follows.

Obstacle problem for two heating elements. For fixed $r \in (0, 1)$, find

$$m(r) = \inf \omega_L(0),$$

taken over all sets $L = l_1 \cup l_2$, where l_1 connects r with $\mathbb{T} = \partial\mathbb{D}$ and l_2 connects $-r$ with $\mathbb{T}$ and identify all extremal obstacles.



Round stove with two heating elements.