Mixed Boundary Value Problem for Non-divergence Elliptic Equation

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Abstract

We consider solutions of non-divergence elliptic equations

$$\mathcal{L}u := -\sum_{i,j=1}^{n} a_{ij}(x)D_iD_ju + \sum_{i=1}^{n} b_i(x)D_iu + c(x)u = 0 \qquad \text{in} \quad \Omega. \qquad (0.1)$$ equ

Such equations arise in theory of stochastic processes and various applications.

In (0.1) Ω is a domain in $\mathbb{R}^n$, $n \geq 3$, and D_i stands for the differentiation with respect to x_i. Also we suppose that the boundary $\partial\Omega$ is split to two parts: a closed set Γ_1 is support of the Dirichlet condition, and $\Gamma_2 = \partial\Omega \setminus \Gamma_1$ is support of the oblique derivative condition:

$$u(x) = \Phi(x) \ \text{ on } \ \Gamma_1; \quad \partial_\ell u(x) := \lim_{\delta \to +0} \frac{u(x) - u(x - \delta\ell)}{\delta} = \Psi(x) \ \text{ on } \ \Gamma_2, \ (0.2)$$ BC

where $\ell = \ell(x)$ is a measurable, strictly and uniformly non-tangential outward vector field on Γ_2. Without loss of generality we can suppose $|\ell| \equiv 1$.

We discuss the regularity of a junction boundary point $X \in \Gamma_1 \cap \overline{\Gamma_2}$ with respect to the operator $\mathcal{L}$. In first part of the talk we will consider "Neumann" boundary Γ_2 to be Lipschitz, satisfying some structural constraint. We will present proof of Wiener type test for regularity of point of junction of Dirichlet and Neumann type boundaries. To formulate structural conditions on Γ_2 we define so called accessibility condition, which allows in finite number of steps cover domain in the spherical layer around junction point X without intersecting Neumann boundary Γ_2. The accessibility of boundary serves as analog of "isoperimetric" properties of the domain in case of non-divergent equations. Note that in contrast to divergent equation the case of Lipschitz boundary, mixed boundary value problem for non-divergent equation can not be reduced to Dirichlet problem only .