Texas Tech University - Department of Mathematics and Statistics
Seminar in Applied Mathematics

Zagier polynomials and modified Nörlund polynomials

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ABSTRACT. In 1998, Don Zagier studied the numbers B_n^* which he called the 'modified Bernoulli numbers'. They satisfy amusing variants of the properties of the ordinary Bernoulli numbers. Recently, Victor H. Moll, Christophe Vignat and I studied an obvious generalization of the modified Bernoulli numbers, which we call 'Zagier polynomials'. These polynomials are also rich in structure, and we have shown that a theory parallel to that of ordinary Bernoulli polynomials exists. Zagier showed that his asymptotic formula for B_{2n}^* can be replaced by an exact formula. In an ongoing joint work with M. L. Glasser and K. Mahlburg, we have shown that a similar thing is true for the Zagier polynomials. This exact formula involves Chebyshev polynomials and infinite series of Bessel function $Y_n(z)$. Through a motivation coming from diffraction theory, C. M. Linton has already proved this, but in a disguised form, and our proof is new and gives new results along the way. We also derive Zagier's formula as a limiting case of this general formula, which is interesting in itself. In the second part of my talk, I will discuss another generalization of the modified Bernoulli numbers that we recently studied along with A. Kabza, namely 'modified Nörlund polynomials' $B_n^{(\alpha)*}$, $\alpha \in \mathbb{N}$, and obtain their generating function along with applications. The talk will include an interesting mix of special functions, number theory, probability and umbral calculus.