WEIGHTED GRADIENT ESTIMATES OF SOLUTIONS TO DEGENERATE AND SINGULAR ELLIPTIC EQUATIONS

DAT CAO

Abstract. We study the weight-$W^{1,p}$ estimates of weak solutions to the equation

$$\begin{cases} \operatorname{div}[\mathbb{A}(x)\nabla u] & = & \operatorname{div}[\mathbf{F}] & \text{in} & \Omega, \\ u & = & 0 & \text{on} & \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ is an open bounded domain, $\mathbf{F} : \Omega \to \mathbb{R}^n$ is a given vector field, and the coefficient matrix $\mathbb{A} : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is symmetric, measurable, and satisfying the degenerate/singular elliptic condition:

$$\Lambda\mu(x)|\xi|^2 \leq \langle \mathbb{A}(x)\xi, \xi\rangle \leq \Lambda^{-1}\mu(x)|\xi|^2, \quad \forall\, \xi \in \mathbb{R}^n, \quad \text{a.e.} \quad x \in \mathbb{R}^n,$$

with fixed $\Lambda > 0$, and a non-negative weight μ in some Muckenhoupt class.

We obtain weighted estimates of solutions under a smallness condition on the mean oscillation of the coefficients with the weight μ and a flatness condition on the boundary of the domain Ω.

This is a joint work with Tadele Mengesha (University of Tennessee) and Tuoc Phan (University of Tennessee).