

Title

Explicit Pieri Inclusions and Minimal Graded Free Resolutions of Modules of Covariants of Several Vectors and Covectors for a General Linear Group

Abstract

By the Pieri rule, the tensor product of an exterior power and a finite-dimensional irreducible representation of a general linear group has a multiplicity-free decomposition. The embeddings of the constituents are called Pieri inclusions and were first studied by Weyman in his thesis and described explicitly by Olver. More recently, these maps have appeared in the work of Eisenbud, Fløstad, and Weyman and of Sam and Weyman to compute pure free resolutions for classical groups. In this talk we give a new closed form, non-recursive description of Pieri inclusions. For partitions with a bounded number of distinct parts, the resulting algorithm has polynomial time complexity whereas the previously known algorithm has exponential time complexity.

We will use Pieri inclusions to compute syzygies for modules of covariants. The fundamental problem of classical invariant theory is to find generators and relations (syzygies) for rings of invariants and, more generally, for modules of covariants. For the general linear group, this problem is partially answered by Weyl's first and second fundamental theorems for the rings of invariants of several vectors and covectors. The higher syzygies of these rings of invariants are given by Lascoux's resolution of determinantal ideals. We extend the results of Lascoux to give minimal free resolutions of modules of covariants in characteristic zero. These resolutions are obtained from Bernstein–Gelfand–Gelfand resolutions of unitary highest weight modules and the differentials are explicitly described in terms of Pieri inclusions.