Boundary Estimates for Solutions to the Two-Phase Parabolic Obstacle Problem

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Consider the two-phase parabolic obstacle problem with non-trivial Dirichlet condition

$$\begin{aligned} \Delta u - \partial_t u &= \lambda^+ \chi_{\{u>0\}} - \lambda^- \chi_{\{u<0\}} \quad \text{in } Q = \Omega \times (0;T), \\ u &= \varphi \quad \text{on} \quad \partial_p Q. \end{aligned}$$

Here $T < +\infty$, $\Omega \subset \mathbb{R}^n$ is a given domain, $\partial_P Q$ denotes the parabolic boundary of Q, and $\lambda^\pm$ are non-negative constants satisfying $\lambda^+ + \lambda^- > 0$. The problem arises as limiting case in the model of temperature control through the interior.

In this talk we discuss the L^∞-estimates for the second-order space derivatives $D^2 u$ near the parabolic boundary $\partial_p Q$. Observe that the case of general Dirichlet data cannot be reduced to zero ones due to non-linearity and discontinuity at $u = 0$ of the right-hand side of the first equation.

The lecture is based on works in collaboration with Nina Uraltseva.