Potential theory for quasilinear elliptic equations

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ABSTRACT

In this talk, we study quasilinear elliptic equations of the type

$$-\Delta_p u = \sigma u^q \quad \text{in } \mathbb{R}^n, \tag{1}$$

where $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian, $1 < p < \infty$, and $\sigma \geq 0$ is an arbitrary locally integrable function, or measure, in the case $0 < q < p-1$.

Necessary and sufficient conditions on σ for the existence of finite energy and weak solutions to (1) are given. Sharp global pointwise estimates of solutions are obtained as well. We also discuss the uniqueness and regularity properties of solutions.

Our main tools are Wolff potential estimates, dyadic models, and related integral inequalities. Special nonlinear potentials of Wolff type associated with "sublinear" problems are constructed to obtain sharp bounds of solutions. We also treat equations with the fractional Laplacians $(-\Delta)^\alpha$. This is a joint work with Igor Verbitsky.