Orthogonal polynomials in evolutionary theory

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Abstract: I will discuss the application of orthogonal polynomials to problems in evolutionary biology. We often need to represent some value, such as fitness or offspring phenotype, as a function of the phenotype of individuals in a population. Standard polynomial regression will not do in this case, because the regression coefficients are functions of the degree of the polynomial that we choose to fit. Since this degree is arbitrary, the coefficients of a standard nonlinear regression do not have any clear biological meaning. The solution to this problem is to construct a space of orthogonal polynomials that are defined by the distribution of variation in the population that we are studying. I will show how we construct such polynomials for both the cases of univariate and multivariate evolution. Multivariate orthogonal polynomials pose a problem not encountered in the univariate case - the need to choose an ordering on the variables - that has limited their applicability. I will show how we get around this problem in the case of evolutionary theory, and how orthogonal polynomials allow us to solve problems, such as evolution with nonlinear inheritance, that could not be addressed using traditional quantitative genetics.