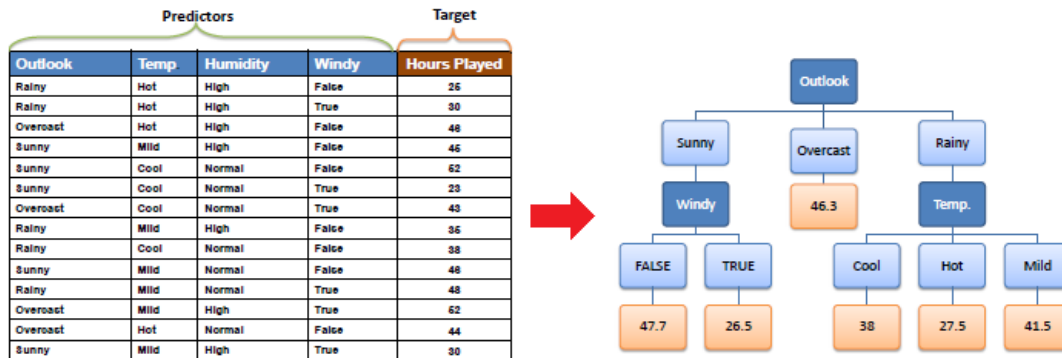


## Decision Tree - Regression

Decision tree builds regression or classification models in the form of a tree structure. It breaks down a dataset into smaller and smaller subsets while at the same time an associated decision tree is incrementally developed. The final result is a tree with **decision nodes** and **leaf nodes**. A decision node (e.g., Outlook) has two or more branches (e.g., Sunny, Overcast and Rainy), each representing values for the attribute tested. Leaf node (e.g., Hours Played) represents a decision on the numerical target. The topmost decision node in a tree which corresponds to the best predictor called **root node**. Decision trees can handle both categorical and numerical data.



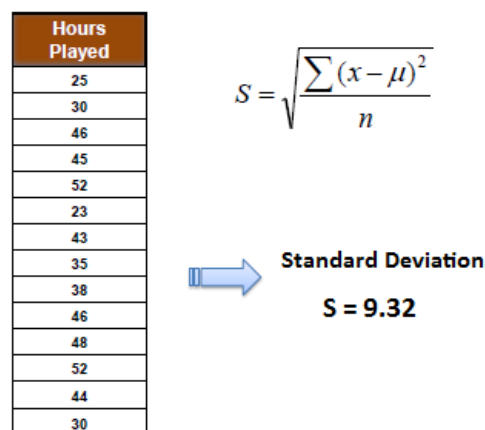
### Decision Tree Algorithm

The core algorithm for building decision trees called **ID3** by J. R. Quinlan which employs a top-down, greedy search through the space of possible branches with no backtracking. The ID3 algorithm can be used to construct a decision tree for regression by replacing Information Gain with *Standard Deviation Reduction*.

### Standard Deviation

A decision tree is built top-down from a root node and involves partitioning the data into subsets that contain instances with similar values (homogenous). We use standard deviation to calculate the homogeneity of a numerical sample. If the numerical sample is completely homogeneous its standard deviation is zero.

a) Standard deviation for **one** attribute:



b) Standard deviation for **two** attributes:

$$S(T, X) = \sum_{c \in X} P(c) S(c)$$

		Hours Played (StDev)	Count
Outlook	Overcast	3.49	4
	Rainy	7.78	5
	Sunny	10.87	5
			14



$$\begin{aligned}
 S(\text{Hours, Outlook}) &= P(\text{Sunny}) * S(\text{Sunny}) + P(\text{Overcast}) * S(\text{Overcast}) + P(\text{Rainy}) * S(\text{Rainy}) \\
 &= (4/14) * 3.49 + (5/14) * 7.78 + (5/14) * 10.87 \\
 &= 7.66
 \end{aligned}$$

### Standard Deviation Reduction

The standard deviation reduction is based on the decrease in standard deviation after a dataset is split on an attribute. Constructing a decision tree is all about finding attribute that returns the highest standard deviation reduction (i.e., the most homogeneous branches).

*Step 1:* The standard deviation of the target is calculated.

**Standard deviation (Hours Played) = 9.32**

*Step 2:* The dataset is then split on the different attributes. The standard deviation for each branch is calculated. The resulting standard deviation is subtracted from the standard deviation before the split. The result is the standard deviation reduction.

		Hours Played (StDev)
Outlook	Overcast	3.49
	Rainy	7.78
	Sunny	10.87
SDR=1.66		

		Hours Played (StDev)
Temp.	Cool	10.51
	Hot	8.95
	Mild	7.65
SDR=0.17		

		Hours Played (StDev)
Humidity	High	9.36
	Normal	8.37
SDR=0.28		

		Hours Played (StDev)
Windy	False	7.87
	True	10.59
SDR=0.29		

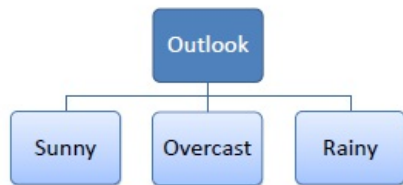
$$SDR(T, X) = S(T) - S(T, X)$$

$$\begin{aligned}
 SDR(\text{Hours, Outlook}) &= S(\text{Hours}) - S(\text{Hours, Outlook}) \\
 &= 9.32 - 7.66 = 1.66
 \end{aligned}$$

*Step 3:* The attribute with the largest standard deviation reduction is chosen for the decision node.

★		Hours Played (StDev)
Outlook	Overcast	3.49
	Rainy	7.78
	Sunny	10.87
SDR=1.66		

*Step 4a:* Dataset is divided based on the values of the selected attribute.



Outlook	Temp	Humidity	Windy	Hours Played
Sunny	Mild	High	FALSE	45
Sunny	Cool	Normal	FALSE	52
Sunny	Cool	Normal	TRUE	23
Sunny	Mild	Normal	FALSE	46
Sunny	Mild	High	TRUE	30
Rainy	Hot	High	FALSE	25
Rainy	Hot	High	TRUE	30
Rainy	Mild	High	FALSE	35
Rainy	Cool	Normal	FALSE	38
Rainy	Mild	Normal	TRUE	48
Overcast	Hot	High	FALSE	46
Overcast	Cool	Normal	TRUE	43
Overcast	Mild	High	TRUE	52
Overcast	Hot	Normal	FALSE	44

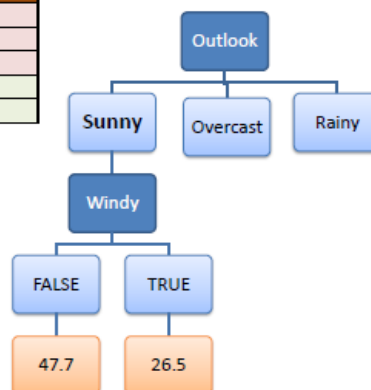
Step 4b: A branch set with standard deviation more than 0 needs further splitting.

In practice, we need some termination criteria. For example, when standard deviation for the branch becomes smaller than a certain fraction (e.g., 5%) of standard deviation for the full dataset OR when too few instances remain in the branch (e.g., 3).

Temp	Humidity	Windy	Hours Played
Mild	High	FALSE	45
Cool	Normal	FALSE	52
Mild	Normal	FALSE	46
Cool	Normal	TRUE	23
Mild	High	TRUE	30

★		Hours Played (StDev)
Windy	False	3.09
	True	3.50
SDR= 7.62		

$$SDR = 10.87 - ((3/5) * 3.09 + (2/5) * 3.5)$$



Step 5: The process is run recursively on the non-leaf branches, until all data is processed.

When the number of instances is more than one at a leaf node we calculate the *average* as the final value for the target.

### Exercise



Try to invent a new algorithm to construct a decision tree from data using [MLR](#) instead of average at the leaf node.