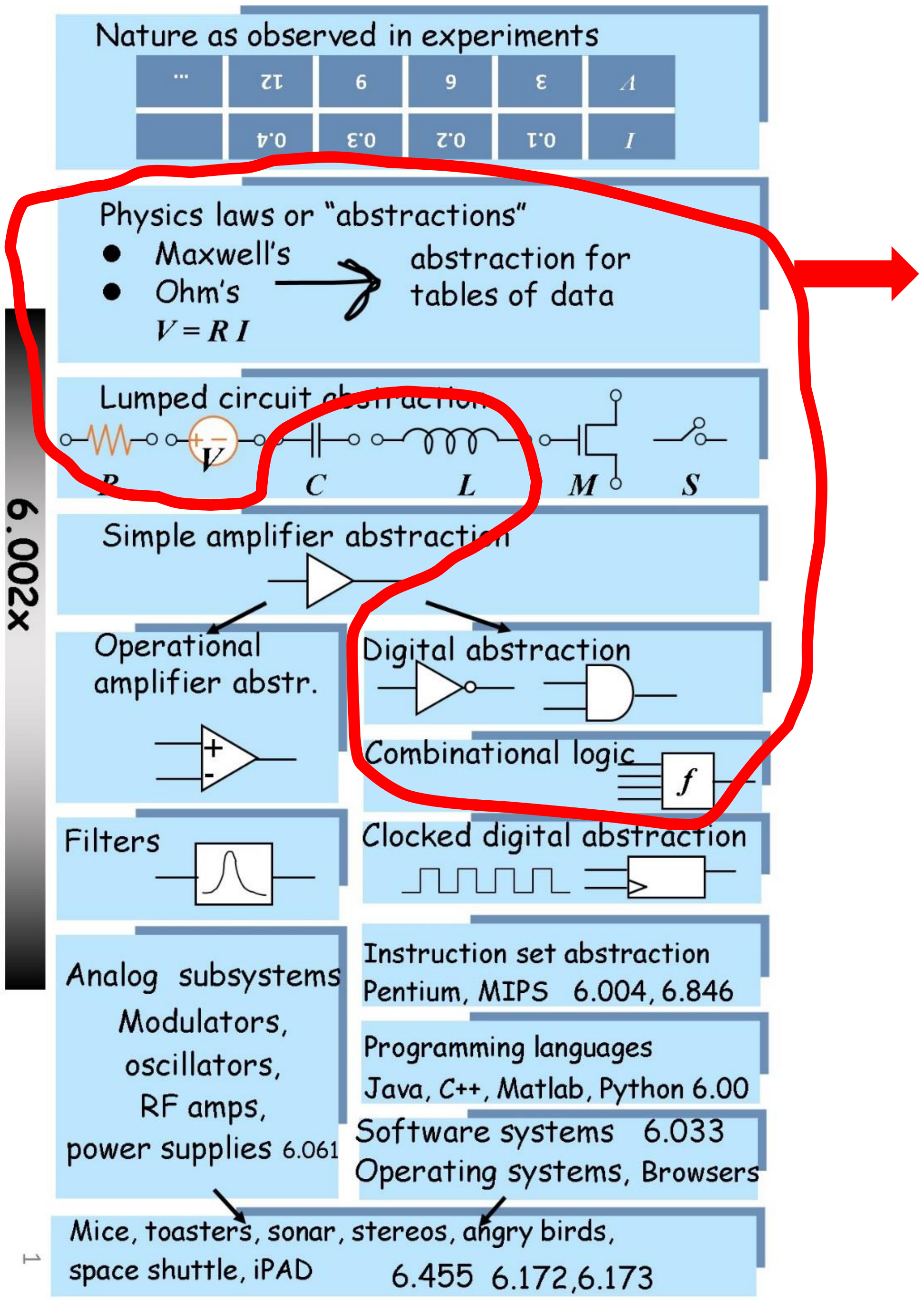
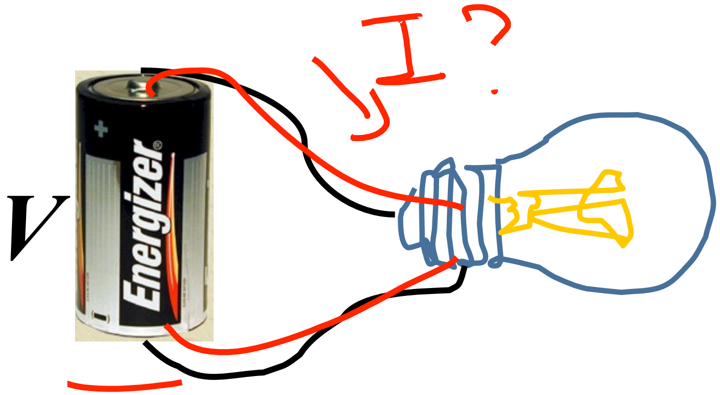


6.002.1x

Circuits and Electronics 1

Lumped Element Abstraction

Consider



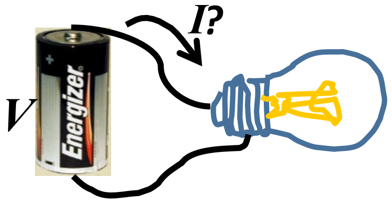
The Big Jump from physics to EECS

Suppose we wish to answer this question:

What is the current through the bulb?

Reading: Skim through Chapter 1 of A&L

We could do it the Hard Way…



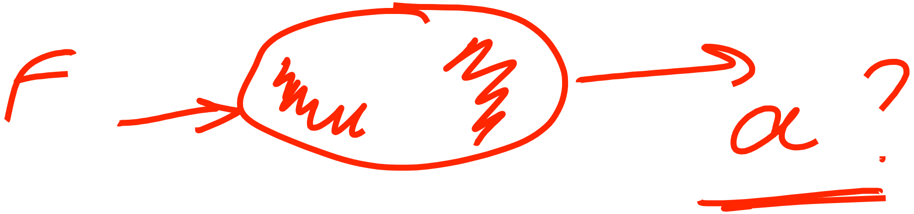
Apply Maxwell's

	Differential form	Integral form
Faraday's	$\nabla \times E = -\frac{\partial B}{\partial t}$	$\oint E \cdot dl = -\frac{\partial \phi_B}{\partial t}$
Continuity	$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$	$\oint J \cdot dS = -\frac{\partial q}{\partial t}$
Others	$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$	$\oint E \cdot dS = \frac{q}{\varepsilon_0}$
	$\vdots$	$\vdots$

Instead, there is an Easy Way…

First, let us build some insight:

Analogy



I ask you: What is the acceleration?

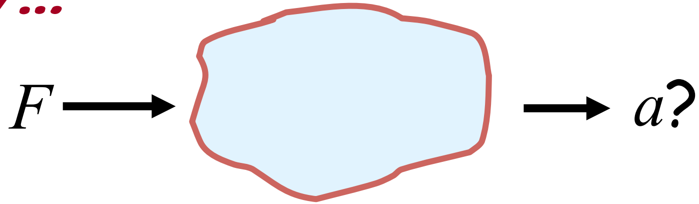
You quickly ask me: What is the mass? m

I tell you: m

You respond: $a = \frac{f}{m}$

Done !!!

Instead, there is an Easy Way...



m

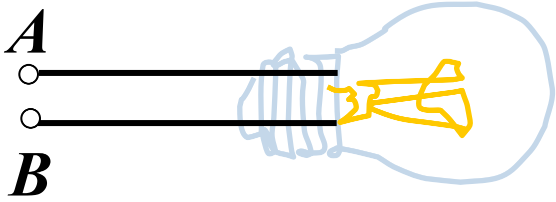
In doing so, you ignored

- the object's shape
- its temperature
- its color
- point of force application
- ...

→ Point-mass discretization

The Easy Way...

Consider the filament of the light bulb.



I ?

We do not care about

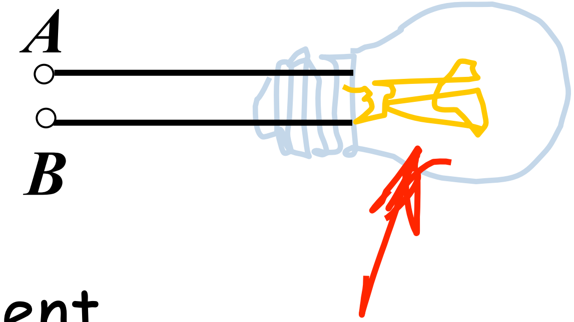
- how current flows inside the filament
- its temperature, shape, orientation, etc.

We can replace the bulb with a

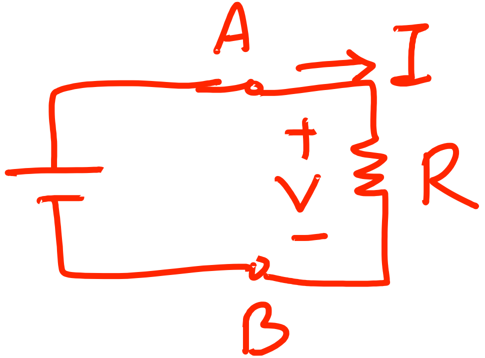
discrete resistor

for the purpose of calculating the current.

The Easy Way…

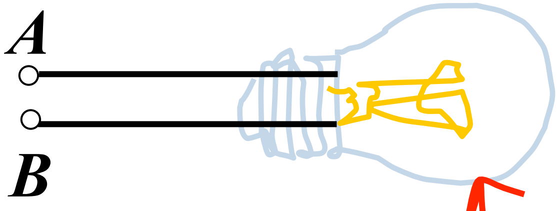


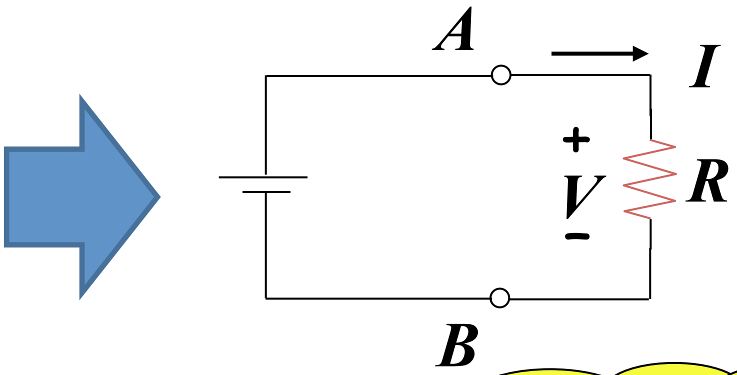
Replace the bulb with a
discrete resistor
for the purpose of calculating the current.



$$I = \frac{V}{R}$$

The Easy Way…





$$I = \frac{V}{R}$$

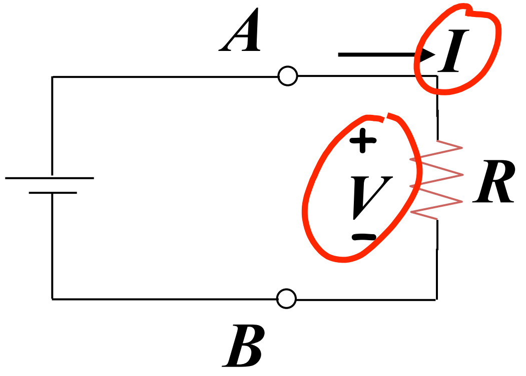
In EECS, we do things the easy way…

R represents the only property of interest!

Like with point-mass:

replace objects with their mass m to find $a = \frac{F}{m}$

V-I Relationship



and $I = \dfrac{V}{R}$

$\boldsymbol{R}$ represents the only property of interest!

$\boldsymbol{R}$ relates element V and I

$I = \dfrac{V}{R}$

$$I = \frac{V}{R}$$ called element v-i relationship

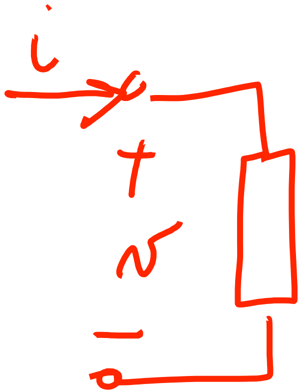
int f(- -)

f()

int

R is a lumped element abstraction for the bulb.

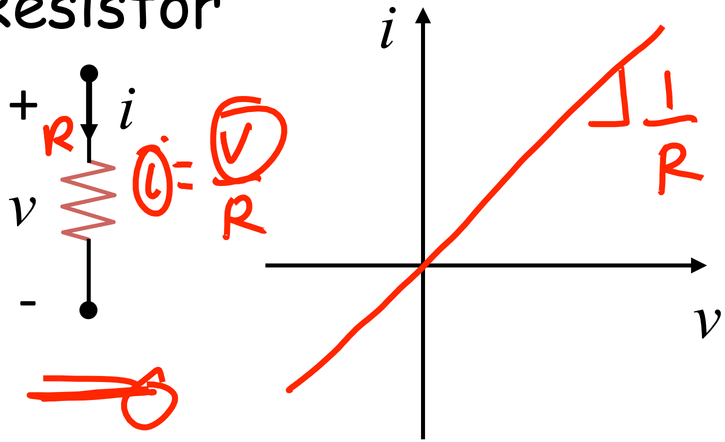
Lumped Elements



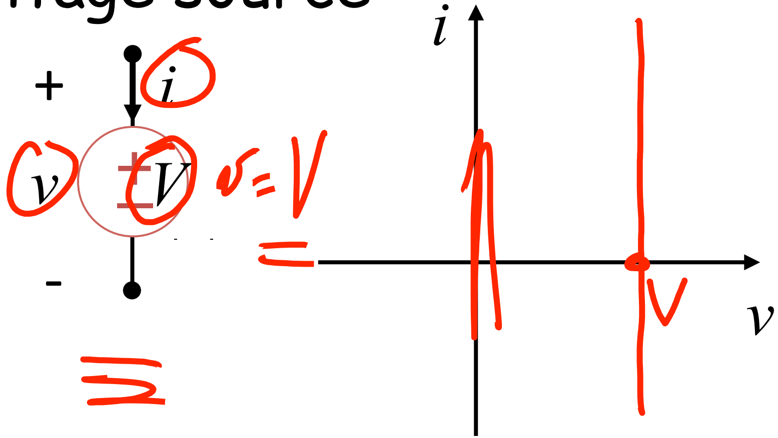
Lumped circuit element
described by its vi relation

Power consumed by element = vi

Resistor



Voltage source



Demo → Lumped element examples whose behavior is completely captured by their V–I relationship.

only for the sorts of questions we as EEs would like to ask!

Demo →

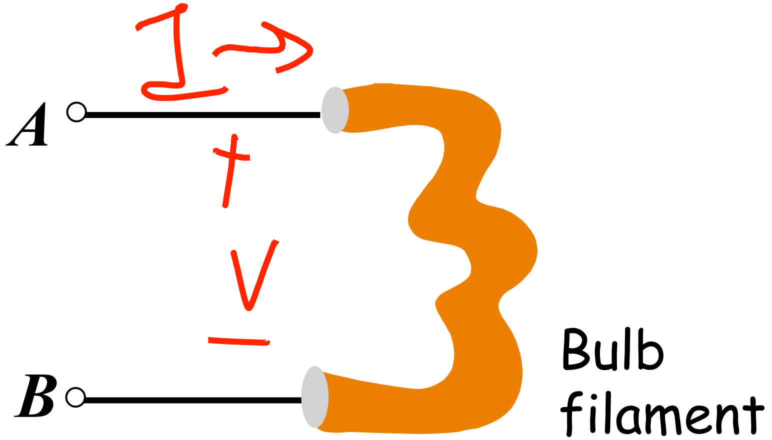
Exploding resistor demo

→ can't predict that!

Pickle demo

→ can't predict light, smell

Not so fast, though …

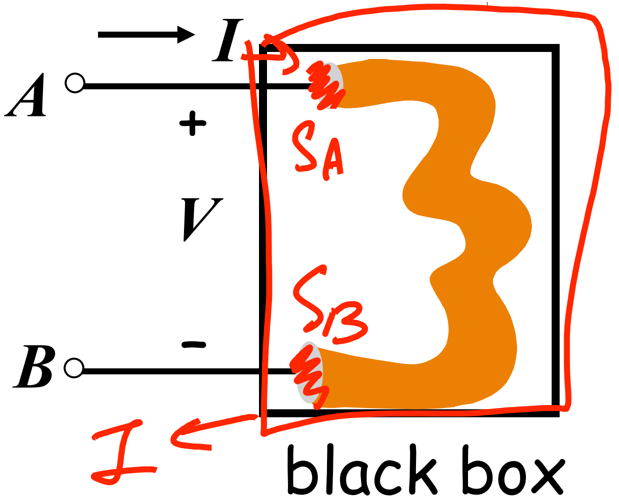


Although we will take the easy way using lumped abstractions for the rest of this course, we must make sure (at least for the first time) that our abstraction is reasonable.

In this case, ensuring that $\boxed{V}$ $\boxed{I}$ are defined for the element

$\boxed{I}$ must be defined.

I into S_A = I out of S_B



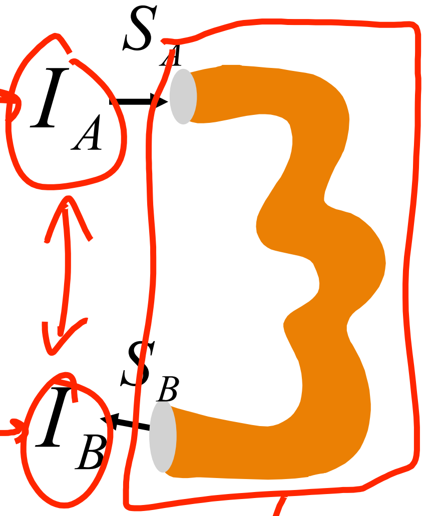
$\boxed{I}$ must be defined. True when

I into S_A = I out of S_B

True only when $\frac{\partial q}{\partial t} = 0$ in the filament!

$\int_{S_A} J \cdot ds \rightarrow I_A$

$\int_{S_B} J \cdot ds \rightarrow I_B$



from Maxwell

$\underbrace{\int_{S_A} J \cdot ds}_{I_A} - \underbrace{\int_{S_B} J \cdot ds}_{I_B} = -\frac{\partial q}{\partial t}$

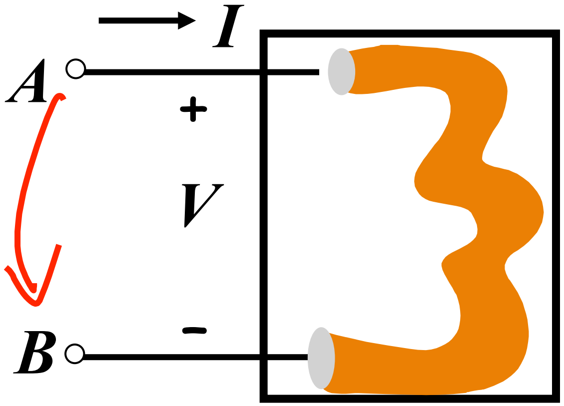
$I_A = I_B$ only if $\frac{\partial q}{\partial t} = 0$ → 0

EECS $\frac{\partial q}{\partial t} = 0$

So, are we stuck?

We're engineers! So, let's make it true!

$\boxed{V}$ Must also be defined.



V_{AB} defined $\frac{\partial q_B}{\partial t}$

V_{AB} defined when $\frac{\partial \phi_B}{\partial t} = 0$

So $V_{AB} = \int_{AB} E \cdot dl$ outside elements

So $V_{AB} = \int E \cdot dl$ outside elements

① $\frac{\partial q}{\partial t} = 0$

② $\frac{\partial \phi_B}{\partial t} = 0$

see A & L Appendix A.3

So let's assume this too!

③ Also, signal speeds of interest should be way lower than speed of light

Welcome to the EECS Playground

The world

The EECS playground

Our self imposed constraints in this playground

$$\frac{\partial \phi_B}{\partial t} = 0$$ Outside

$$\frac{\partial q}{\partial t} = 0$$ Inside elements
Bulb, wire, battery

Where good things happen

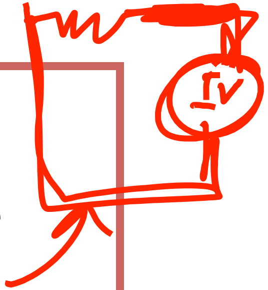
Lumped Matter Discipline (LMD)

Or self imposed constraints:

- $\frac{\partial \phi_B}{\partial t} = 0$ outside
- $\frac{\partial q}{\partial t} = 0$ inside elements bulb, wire, battery

More in Chapter 1 of A & L

Connecting using ideal wires lumped elements that obey LMD to form an assembly results in the lumped circuit abstraction

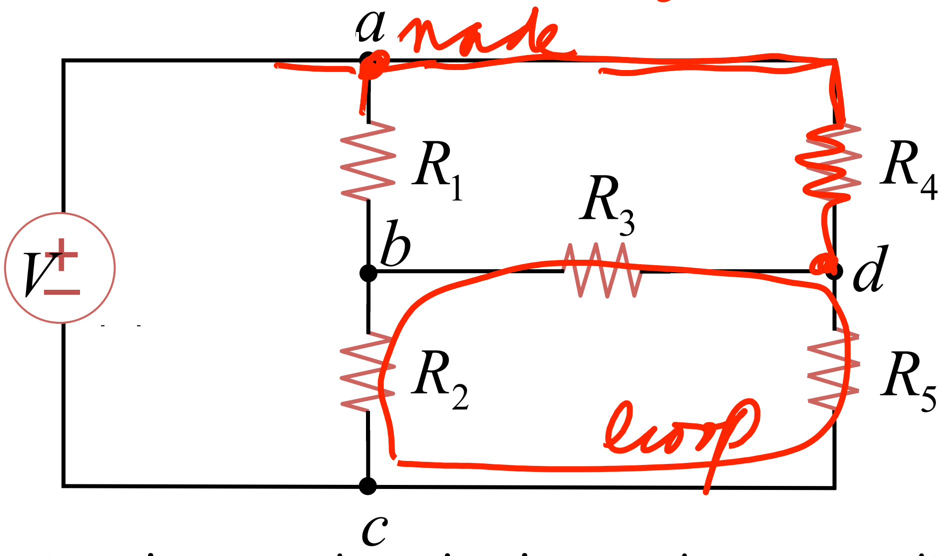


So, what does LMD buy us?

Replace the differential equations with simple algebra using lumped circuit abstraction (LCA).

$2a + 3b = 0 \cdots$

For example:



What can we say about voltages in a loop under the lumped matter discipline?

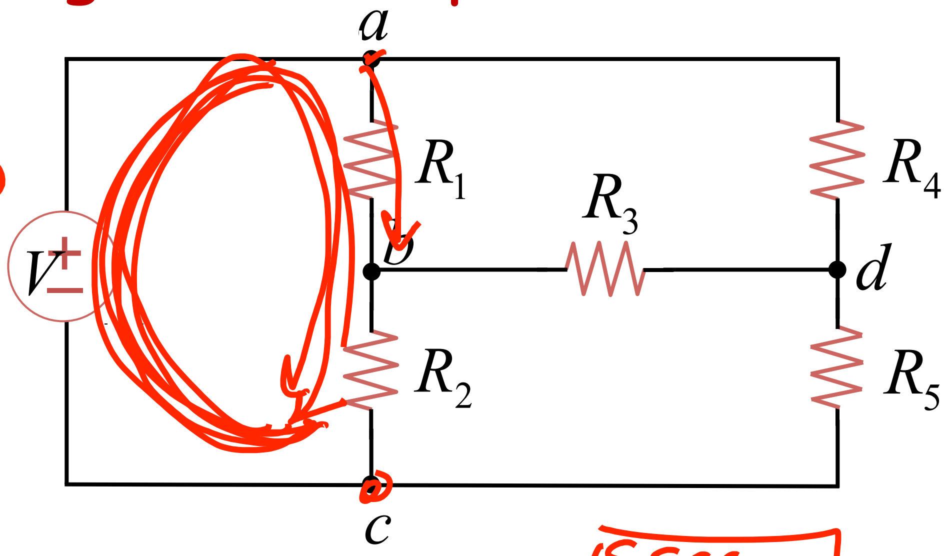
Reading: Chapter 2.1 – 2.2.2 of A&L

What can we say about voltages in a loop under LMD?

$$\oint \vec{E}\cdot d\vec{l} = \cancel{-\frac{\partial \Phi_B}{\partial t}} \xrightarrow{LMD} 0$$

$$\Rightarrow \int_{ca} \vec{E}\cdot d\vec{l} + \int_{ab} \vec{E}\cdot d\vec{l} + \int_{bc} \vec{E}\cdot d\vec{l} = 0$$

$$\rightarrow V_{ca} + V_{ab} + V_{bc} = 0$$



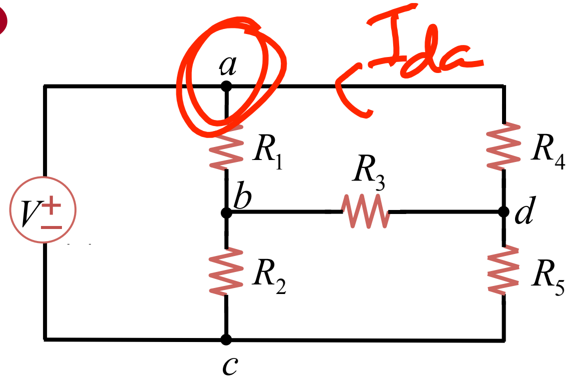
EECS

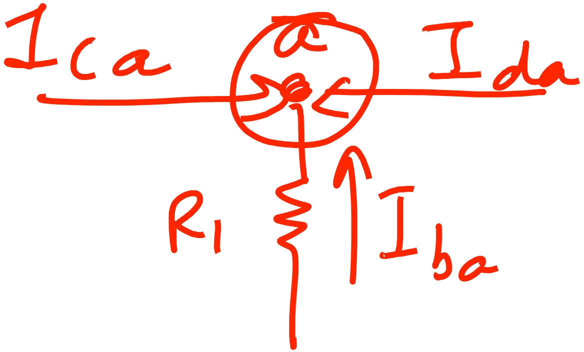
Kirchhoff's Voltage Law (KVL):

The sum of the voltages in a loop is 0.

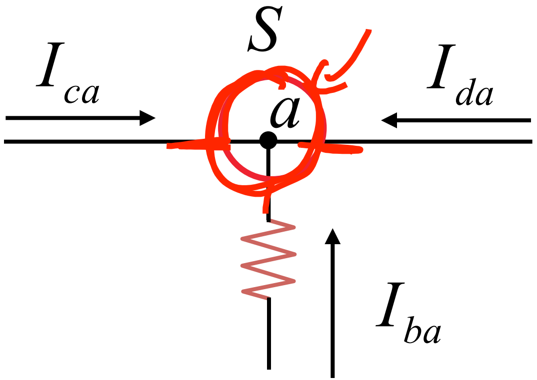
Remember, this is not true everywhere, only in our EECS playground

What can we say about currents?





What can we say about currents?



$$\oint_S J \cdot ds = -\frac{\partial q}{\partial t} \rightarrow 0 \quad \text{LMD!}$$

$$\Rightarrow I_{ca} + I_{da} + I_{ba} = 0$$

Kirchhoff's Current Law (KCL):
The sum of the currents into a node is 0.

simply conservation of charge

KVL and KCL Summary

KVL:

$$\sum_{j\ \text{loop}} v_j = 0$$

KCL:

$$\sum_{i\ \text{node}} i_i = 0$$

Summary

Lumped Matter Discipline LMD:

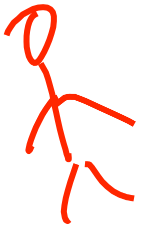
Constraints we impose on ourselves to simplify our analysis

$$\frac{\partial \phi_B}{\partial t} = 0$$ Outside elements

$$\frac{\partial q}{\partial t} = 0$$ Inside elements

wires resistors sources

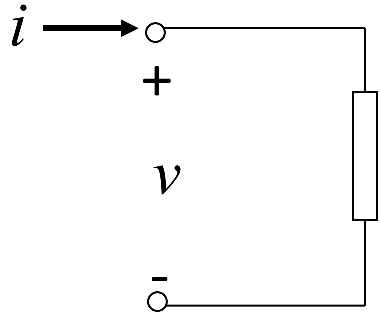
Also, signals speeds of interest should be way lower than speed of light



Allows us to create the lumped circuit abstraction

Remember, our EECS playground

Summary



Lumped circuit element

$i = f(v)$

$i = \frac{V}{R}$

$i = 6.7v + \frac{v^2}{2} + 1$

power consumed by element = vi

Summary

Maxwell's equations simplify to algebraic KVL and KCL under LMD.

KVL:

$$\sum_j v_j = 0$$

loop

KCL:

$$\sum_j i_j = 0$$

node

This is amazing!